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Anomalous Energy Transport in Many-particle Systems: Levy-walk Approach

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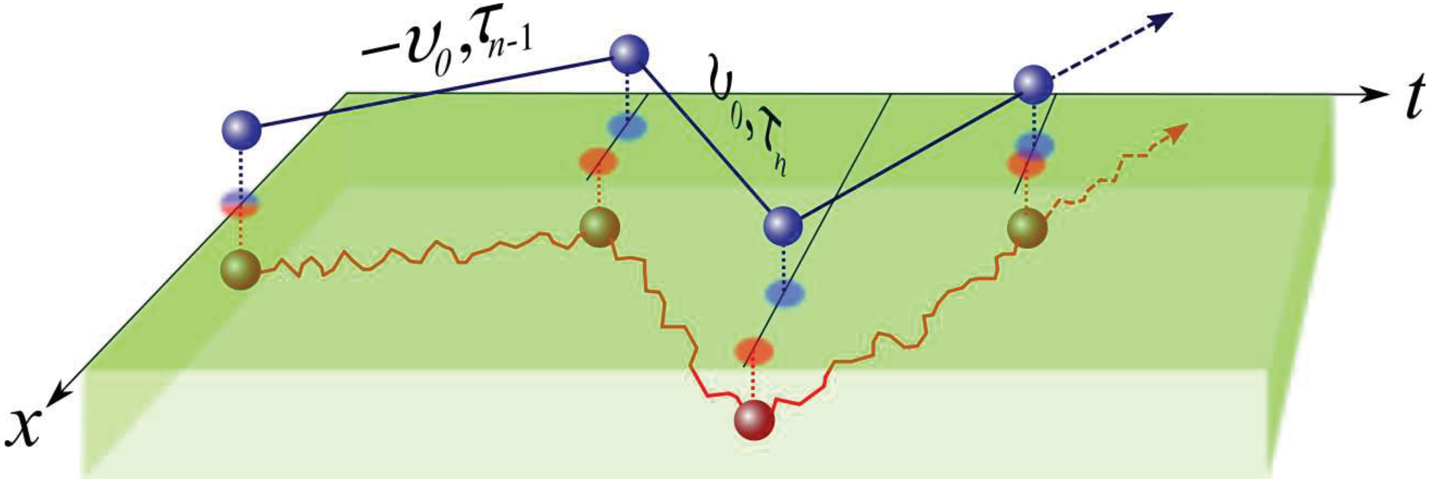
Levy walk

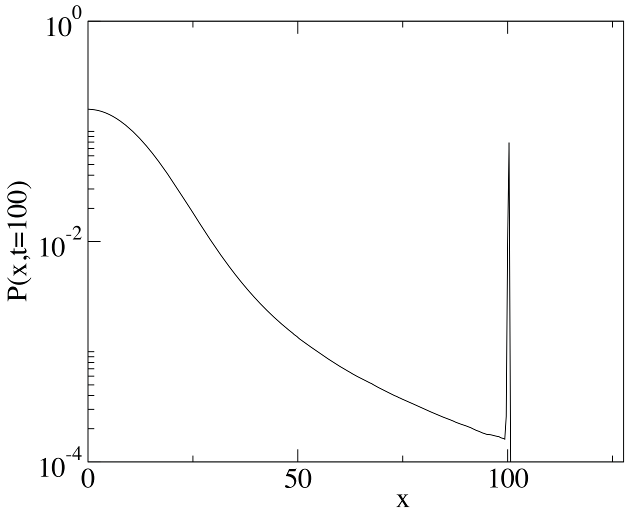


probability density function (pdf) of flight time

$$\psi(\tau) \propto (\tau/\tau_0)^{-\mu-1}, \; 1 < \mu \leq 2$$

M. F. Shlesinger, J. Klafter, and B. West, Physica A, 140, 212 (1986)

Levy walk



anomalous diffusion,
$\sigma^2(t) \propto t^\alpha$,

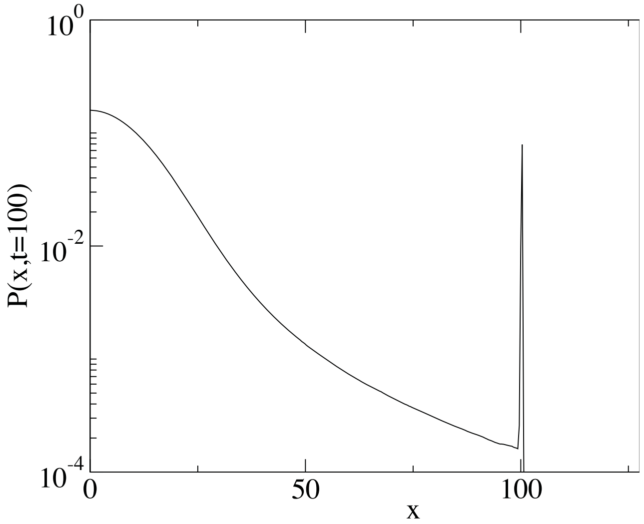
$$\alpha = \begin{cases} 2, & 0 < \mu < 1 \\ 3 - \mu, & 1 < \mu < 2, \\ 1, & 2 < \mu \end{cases}$$

Propagator

- central part is given by the Levy distribution,
  $P_c(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(t) e^{-ixt} dt$,
  $\phi(t;\gamma) = \exp(-|t|^\mu)$;

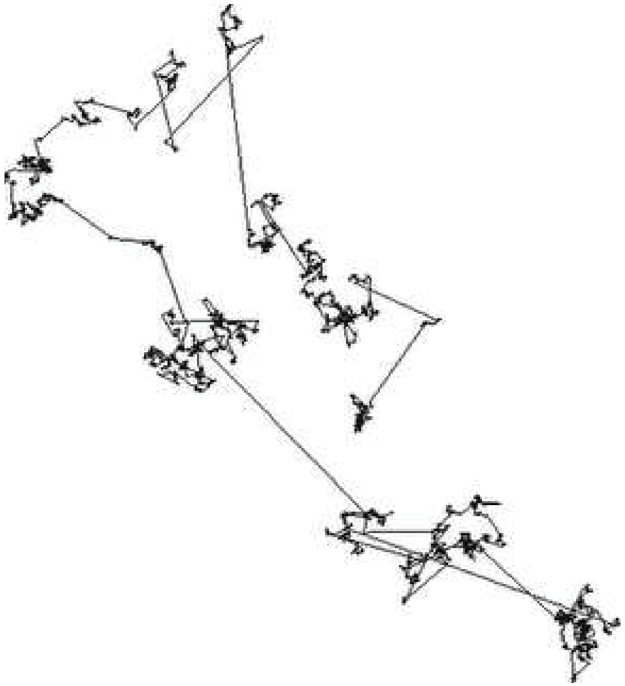
- ballistic fronts: δ-like peaks.

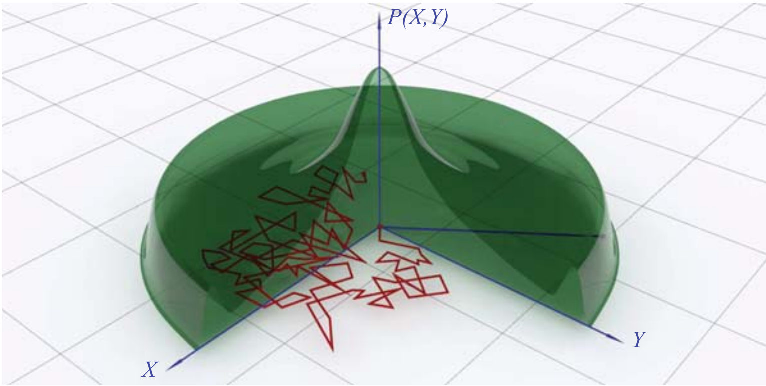
Levy walk



$$P(x,t) \propto \begin{cases} t^{-1/\mu} \exp\left(\frac{-ax^2}{t^{2/\mu}}\right) & |x| \lesssim t^{1/\mu} \\ tx^{-\mu-1} & t^{1/\mu} \lesssim |x| < t \\ t^{1-\mu} & |x| = t \\ 0 & |x| > t \end{cases}$$

Levy walk in higher dimensions



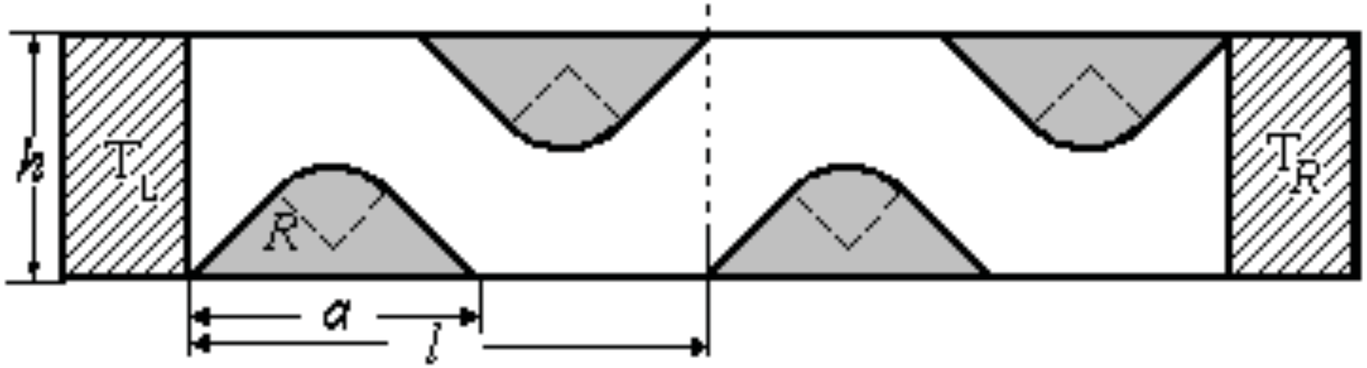


2d generalization of Levy walk

Levy walk: applications

- anomalous kinetics of low-dimensional Hamiltonian systems with mixed phase space (1993-...);
- diffusion of a tracer in a turbulent flow (Solomon & Swinney, 1993);
- motion of cold atoms in optical lattices (H. Katori et al., 1997; Y. Sagi et al., 2011)
- walking patterns of human beings (2007-...);
- patterns of animal foraging (monkeys, bees, albatrosses, deers, mussels, etc) (2005-...);
- ...

Dynamical heat channels



A typical billiard heat channel (from Jun-Wen Mao et al., 2005)

B. Li, L. Wang, and B. Hu, PRL 88, 223901 (2002)
D. Alonso, A. Ruiz, and I. de Vega, PRE 66, 066131 (2002)
B. Li, G. Casati, and J. Wang, PRE 67, 021204 (2003)
Jun-Wen Mao, You-Quan Li, and Yong-Yun Ji, PRE 71, 061202 (2005)

Anomalous energy transport in many-particle systems: Levy-walk approach

S. Denisov

Dynamical heat channels

Features

- it is a single-particle process;
- the trajectory of a particle is independent of the particle velocity, v_{Th}. The last only scales time, $t \to t/v_{Th}$.

A model

A particle moves along direction X, following equations of motion,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \qquad X \in \mathbf{x}.$$

The evolution leads to a particle diffusion along the relevant direction, X, with the asymptotic time-dependence of the mean square displacement (msd) given by

$$\langle X^2(t) \rangle \sim t^{\alpha}.$$

SD, J. Klafter, and M. Urbakh, PRL 91, 194301 (2003).

Dynamical heat channels

Heat transfered by a particle through the channel up to time t

$$Q(t) = \sum_{j=1}^{M(t)} \Delta E_j = \sum_{j=1}^{M(t)} q_j \cdot (E_j^{in} - E_j^{out}),$$

Heat flux

$$J = lim_{t\to\infty} \frac{Q(t)}{t}.$$

is given by

$$J(L) = \tau^{-1} \frac{\int_0^\infty v_{Th}^2 [P_{T_+}(v_{Th}) - P_{T_-}(v_{Th})] dv_{Th}}{\int_0^\infty \frac{1}{v_{Th}} [P_{T_+}(v_{Th}) + P_{T_-}(v_{Th})] dv_{Th}},$$

where $\tau(L)$ is the mean first passage time of the diffusion process $X(t)$.

Dynamical heat channels

For a Levy-walk diffusion process single-particle heat flux scales

$$J(L) \propto \tau^{-1} \propto L^{-\beta}, \quad \beta = \begin{cases} 1, & \alpha = 2 \\ \mu = 3 - \alpha, & 1 < \alpha < 2, \\ 2, & \alpha = 1 \end{cases}$$

Thermal conductivity, $\kappa(L) = -J_{ens}(N)/\nabla T \propto J(L)L^2$, scales like

$$k \propto L^{2-\mu} \propto L^{\alpha-1}.$$

Dynamical heat channels

Comparison with the results obtained for billiard heat channels

B. Li, G. Casati, and J. Wang, PRE 67, 021204 (2003):

$$\alpha = 1.8, \quad \beta = 1.178$$

D. Alonso, A. Ruiz, and I. de Vega, PRE 66, 066131 (2002):

$$\alpha = 1.3, \quad \beta = 1.72$$

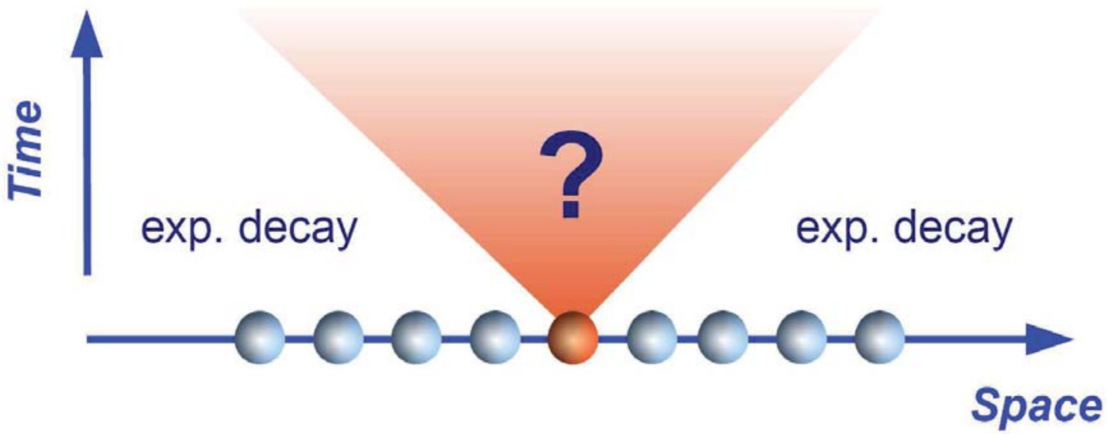
But real dynamics in billiard channels could be much more complicated than that...

Perturbation spreading in many-body systems

Following the linear response theory, transport coefficients can be determined by looking at the way the system of interest responds to small perturbations. To measure heat conductivity, one can follow the evolution of an initially localized energy perturbation, $\triangle e(x,t)$, in time. If the perturbation is weak enough, for the system to be locally in thermodynamic equilibrium, a reasonable measure of the local temperature field $\triangle T(x,t)$ is $\triangle e(x,t) = C_P \triangle T(x,t)$, where C_P is the heat capacity per unit length at constant pressure.

E. Helfand, Phys. Rev. 119, 1 (1960).

Perturbation spreading: Causal cone



- casual cone is a general feature of many-body systems (Lieb & Robinson (1972), Marciolo *et al.* (1978))
- what is going on within the cone?
- ballistic fronts: how do they look like?

Setup

Hamiltonian $H_{\text{total}}(\{x_i, p_i\}) = \sum_{i=1}^{N} H_i$

$$H_i = H(x_i, x_{i-1}, x_{i+1}, p_i)$$

perturbation,

$$\triangle E(i,t) = H_i^{\text{replica}}(t) - H_i(t), \qquad \Sigma_{i=1}^{N} \triangle E(i,t) = E_p$$

Perturbation spreading on average

distribution function $\varrho(i,t) = \overline{\triangle E(i,t)}/E_p$

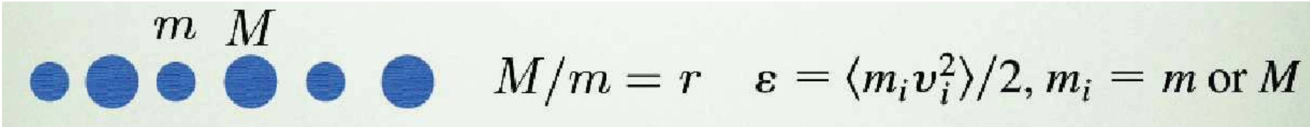
spreading as a certain (not yet known) one-dimensional diffusion process

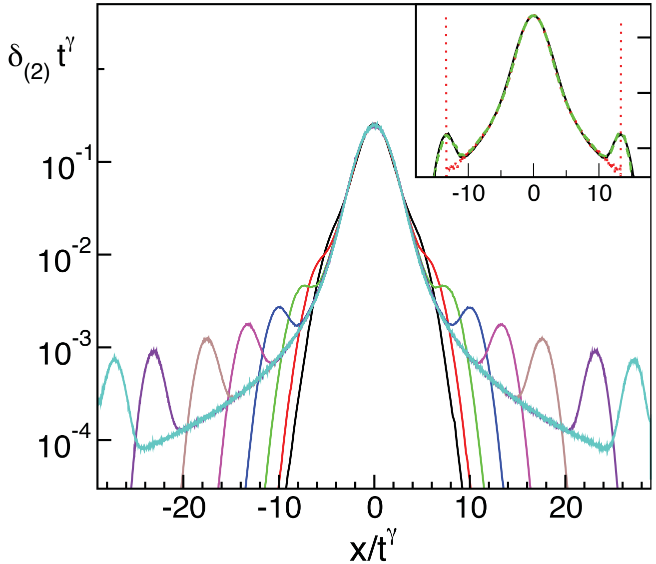
$\sigma^2(t) = \sum_{-\infty}^{\infty} i^2 \varrho(i,t)$

issues

- non-negativity, continuity, smoothness...
- anomalous diffusion, $\sigma^2(t) \propto t^{\alpha}$, $1 > \alpha > 2$
- casual cone

Test: Hard-point gas



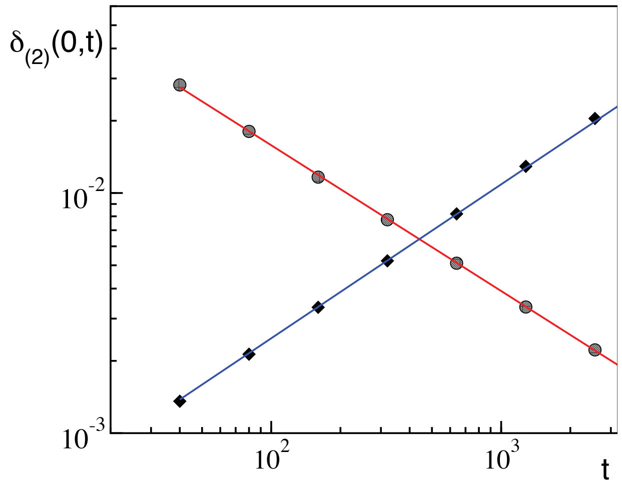


Looks almost like a Levy walk with the exponent

$$\mu = 1/\gamma = 5/3$$

P. Cipriani, SD, and A. Politi, PRL 94, 244301 (2005).

Test: Hard-point gas

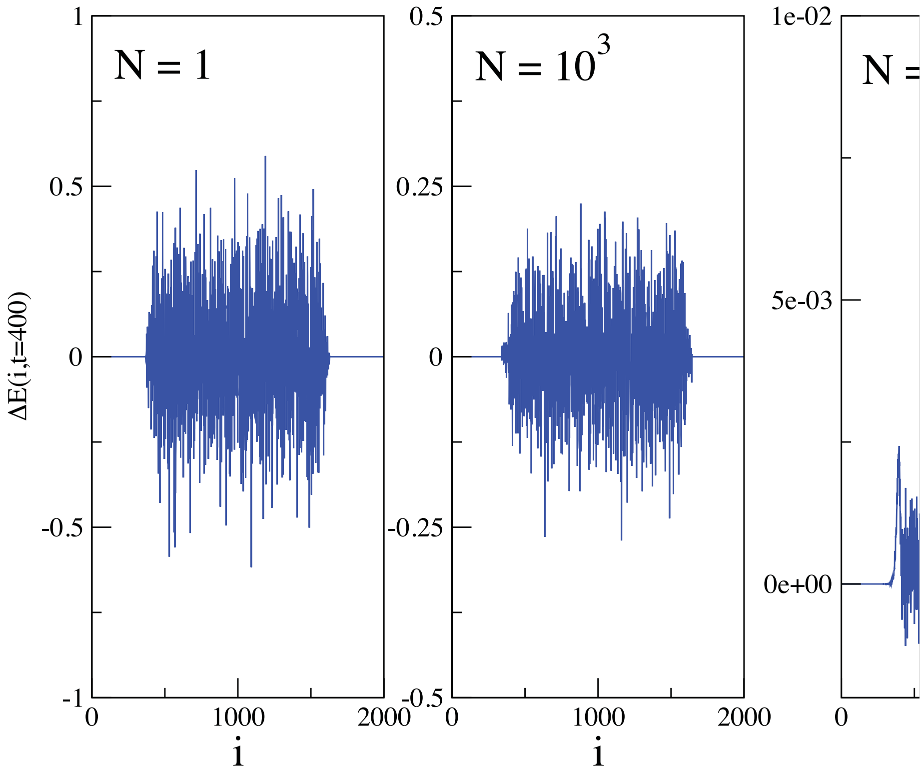


Maximum height of the infinitesimal perturbations profile (circles) and mean square displacement $\sigma^2(t)$ (diamonds) versus time. Lines correspond to a Levy walk process with the exponent $\mu = 5/3$.

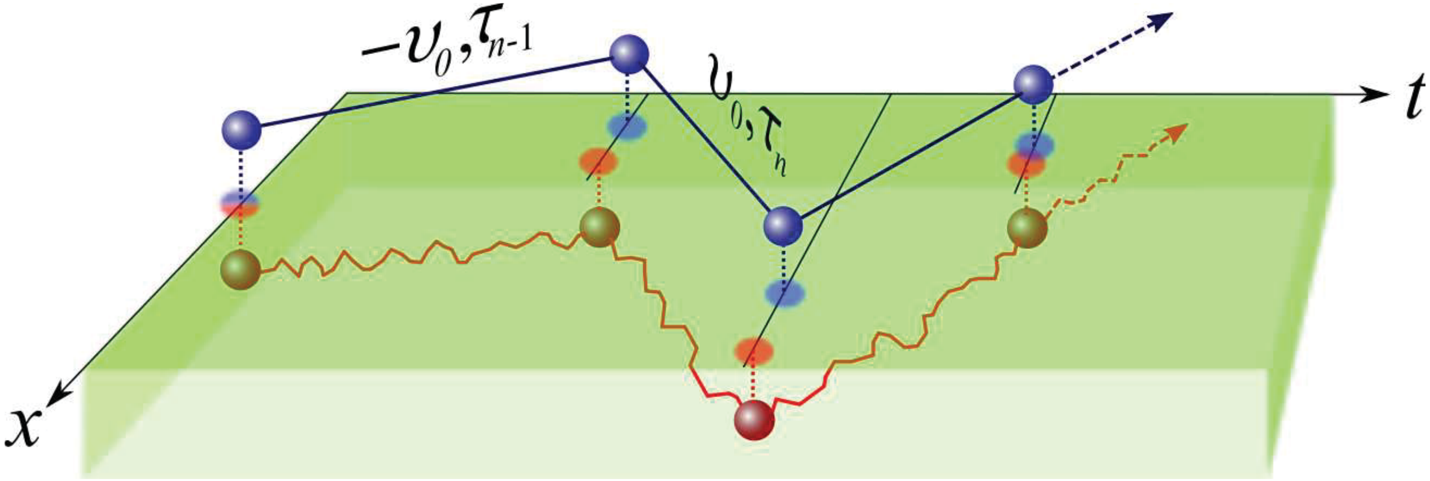
More results in L. Delfini, SD, S. Lepri, R. Livi, P. K. Mohanty, and A. Politi, Euro. Phys. J. 146, 21 (2007).

Test: βFPU chain

finite perturbation



Generalized Levy walk: a tracer in an active medium



single flight as a Wiener process with the drift

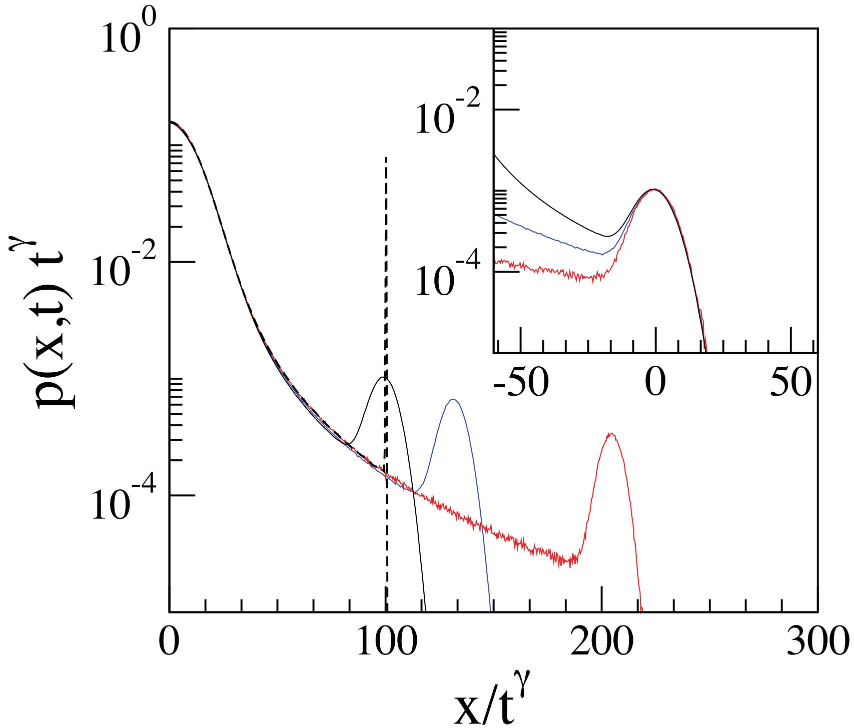
$$\dot{x} = v_0 + \xi(t),$$

$$\langle \xi(t) \rangle = 0, \; \langle \xi(t)\xi(t') \rangle = D_v \delta(t - t')$$

V. Zaburdaev, SD, and P. Hanggi, PRL 106, 180601 (2011).

Generalized Levy walk: a tracer in an active medium

pdf of finding a tracer in x at time t

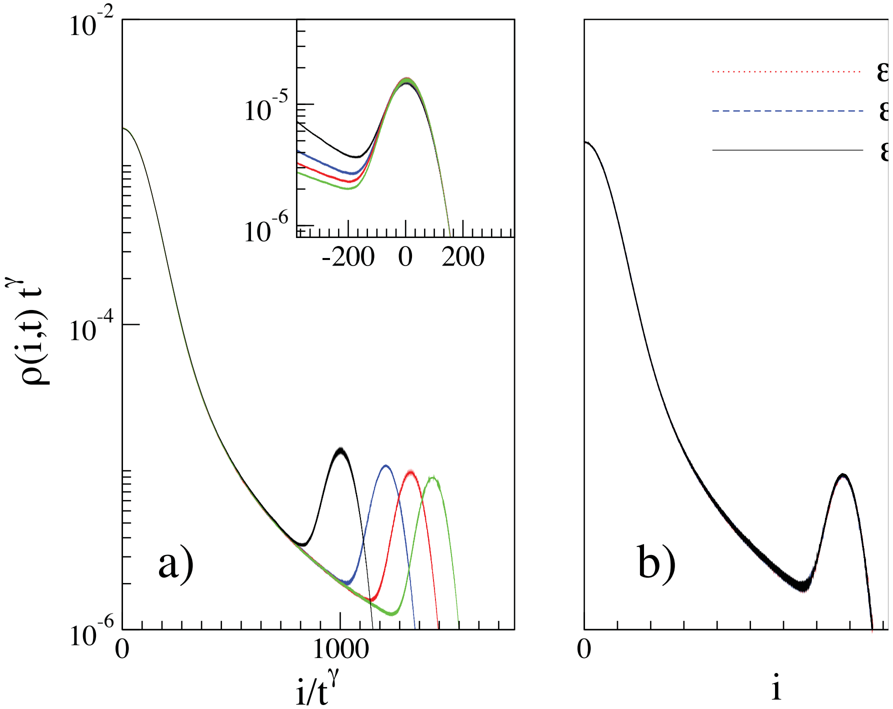


scaling of ballistic humps

$$P_{\text{hump}}(\bar{x}, t') \simeq s^{1/2-\mu} P_{\text{hump}}(\bar{x}/s^{1/2}, t)$$

$$s = t'/t, \; \bar{x} = x - v_0 t$$

Test: Hard-point gas

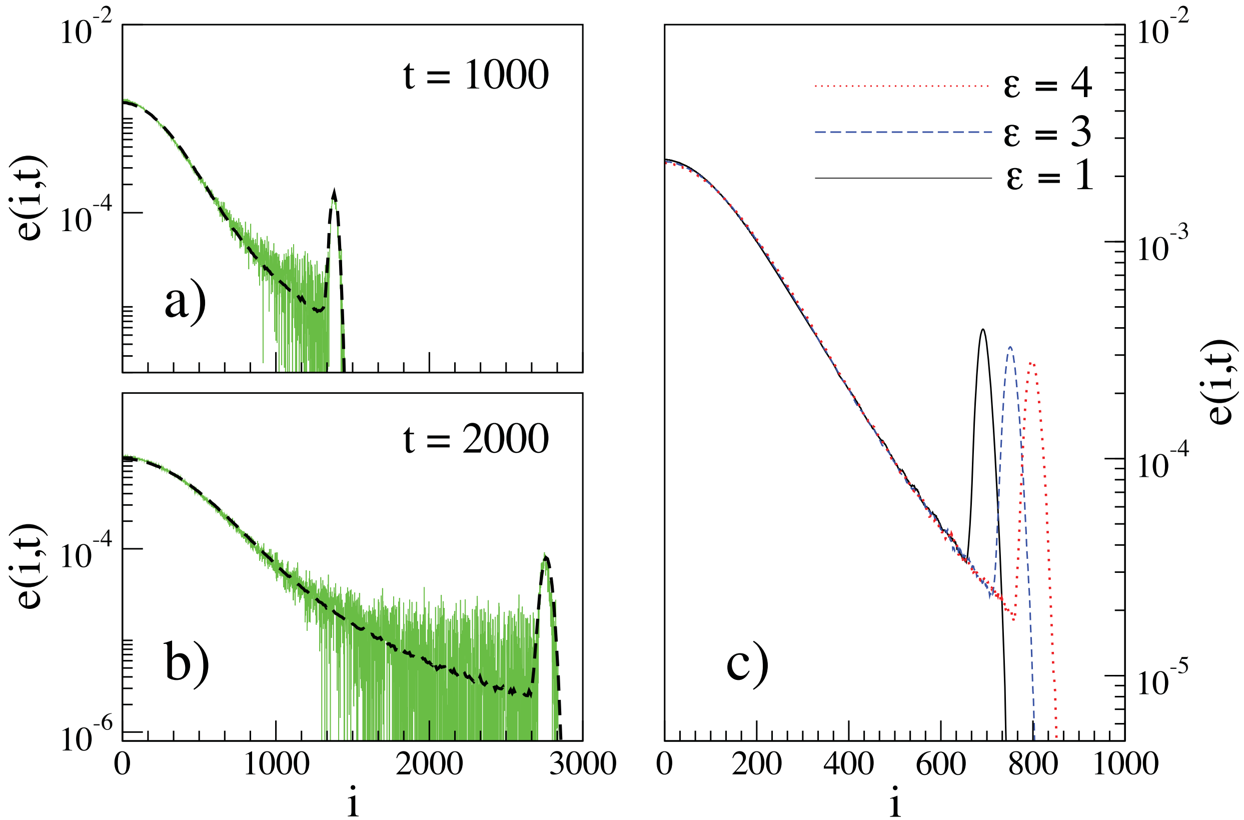


$$v_0, D_v \propto \sqrt{\varepsilon}, \qquad \varepsilon = E/N$$

Test: βFPU chain

energy-energy correlation function

Hong Zhao, PRL 96, 140602 (2006)



$$\psi(\tau) \propto (\tau/\tau_0)^{-\mu-1}, \qquad \tau_0 \propto v_0^{\mu/(1-\mu)}$$

Conclusions

- FPU: $v_0(\varepsilon)$?
  effective thermal phonons (N. Li, B. Li, and S. Flach, PRL **105**, 054102 (2010))

- necessary condition: ergodic dynamics

- sufficient conditions: directions for further studies