

2384-4

**ICTP Latin-American Advanced Course on FPGA Design for Scientific
Instrumentation**

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**Digital Design
(Part 4)**

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Outline

□ Digital CMOS design

- Boolean algebra
- Basic digital CMOS gates
- Combinational and sequential circuits
- Coding - Representation of numbers

Representing Numbers

How values can be coded ?

In a digital circuit each signal can take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to 2^n values



Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character 'F'

The character 'Φ'

The number 46

The number 70

The number 123

Any symbol in a set where the *Card* = 256



Representing Numbers

How values can be coded ?

by itself a code has no signification



Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers
Relative numbers
Rational numbers
Real numbers



Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to $2^n - 1$

Standards



Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$$\begin{array}{c} 0100\ 0110 = 2^1 + 2^2 + 2^6 = 70 \\ \swarrow \quad \quad \quad \nwarrow \\ 2^7 \quad \quad \quad 2^0 \end{array}$$



Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2

The quartets represent the successive powers of 10

$$\begin{array}{c} \text{0100 0110} \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 2^3 \quad 10^1 \quad 10^0 \quad 2^0 \end{array} = (2^1 + 2^2) \times 10^0 + 2^2 \times 10^1 = 46$$

packed



Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2

The bytes represent the successive powers of 10

In each byte the 4 Msb are 0

0100 0110 = Illegal 0000 0110 = 6

10^0 2^0



Representing Numbers

How can I represent a Relative number ?

Sign + Value

The bits represent the successive powers of 2

The Msb represents the sign (1 means negative)



Representing Numbers

How can I represent a Relative number ?

Sign+Value :

$$+ \begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \end{array} \begin{array}{c} 0100 \\ 0110 \end{array} = 2^1 + 2^2 + 2^6 = 70$$

Diagram showing the binary representation 0100 0110. A '+' sign is to the left. Three arrows point from the '+' to the bits: the first arrow points to the 6th bit (0), the second to the 7th bit (1), and the third to the 0th bit (0). Below the 7th bit is 2^6 and below the 0th bit is 2^0 .

$$- \begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \end{array} \begin{array}{c} 1100 \\ 0110 \end{array} = -1 \times (2^1 + 2^2 + 2^6) = -70$$

Diagram showing the binary representation 1100 0110. A '-' sign is to the left. Three arrows point from the '-' to the bits: the first arrow points to the 6th bit (1), the second to the 7th bit (1), and the third to the 0th bit (0). Below the 7th bit is 2^6 and below the 0th bit is 2^0 .

Representing Numbers

How can I represent a Relative number ?

2's complement

The bits represent the successive powers of 2

The Msb represents -2^{n-1}



Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$\begin{array}{cccc} & & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ & \nearrow & & \nearrow & & & & \nwarrow & & & \\ -2^7 & & & & & & & & & & 2^0 \end{array} \quad = 2^1 + 2^2 + 2^6 = 70$$

$$1100 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$$



Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

$$\begin{array}{c}
 70 \\
 \swarrow \quad | \quad \searrow \\
 2^7 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 1 \\
 2^7 = 2^0 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad + 1 + 70
 \end{array}$$

$$\begin{array}{l}
 1011\ 1010 \quad -70 = 2^0 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad + 1 - 2^7 \\
 -70 = \quad 2^1 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad - 2^7
 \end{array}$$



Representing Numbers

How can I represent a Real number ?

Range

Precision



Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$0100.0110 = 2^{-3} + 2^{-2} + 2^2 = 4.325$$

The diagram illustrates the bit weights for the binary number 0100.0110. Arrows point from the bits to their respective powers of 2: the first '0' to -2^3 , the '1' to 2^0 , the second '0' to 2^{-1} , and the first '0' after the decimal to 2^{-4} .

Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point :
Logarithmic representation



Representing Numbers

How can I represent a Real number ?

$$R = (-1)^S \times M \times 2^E$$

Normalized scientific representation

S : Sign (1 if negative)

M : Mantissa ($\in [1, 2[$)

E : Exponent (Relative number)



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

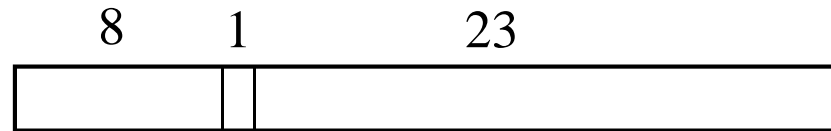
	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ($\in [1, 2[$)	23 bits	52 bits
E : Exponent	8 bits	11 bits



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :



Exponent

Number between
-127 and 127

The code 0000 0000
means -127

Sign

Mantissa

Fixed point positive real number
1. . . .

The 1 is not represented !!

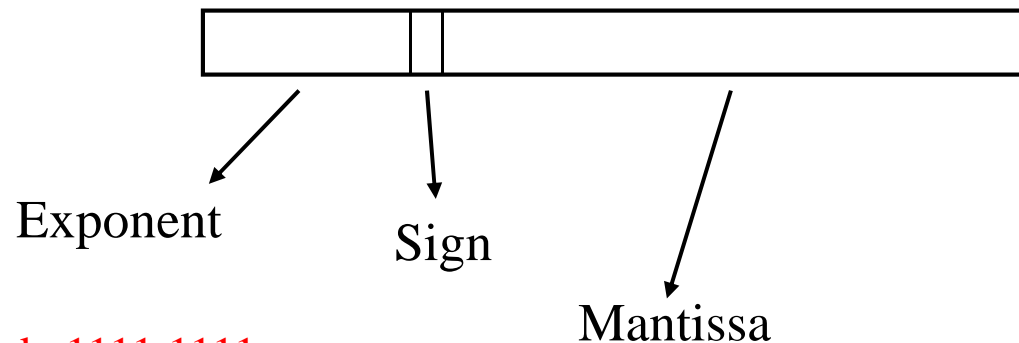
Natural Binary Code
by Excess of 127



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



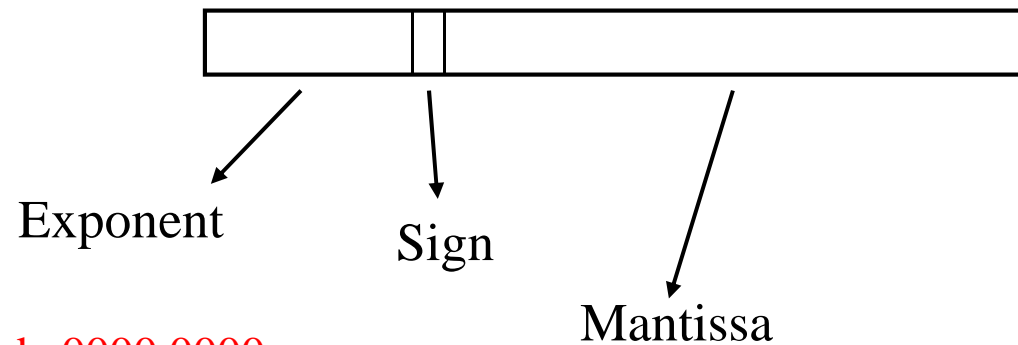
The code 1111 1111
(128) means $\pm\infty$ or
an error

.111 ... 111 means $\pm\infty$
other values mean error (NaN)

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 0000 0000
(-127) indicates
denormalized
Mantissa

0.00 ... 000 means 0

