

2384-6

**ICTP Latin-American Advanced Course on FPGA Design for Scientific
Instrumentation**

19 November - 7 December, 2012

**Digital Design
(Part 6)**

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Outline

- Digital CMOS design

- Arithmetic operators

 - Adders

 - Comparators

 - Shifters

 - Multipliers



Comparators

Comparing a natural number to a constant : =

Let consider a natural number a coded on 8 bits using Natural Binary Code

a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0

= ?

0 0 0 0 0 0 0 0



0 / 1

Comparators

Comparing a natural number to zero : =

Boolean function

Null = 1 if

$$\bar{a}_7 \cdot \bar{a}_6 \cdot \bar{a}_5 \cdot \bar{a}_4 \cdot \bar{a}_3 \cdot \bar{a}_2 \cdot \bar{a}_1 \cdot \bar{a}_0 = 1$$

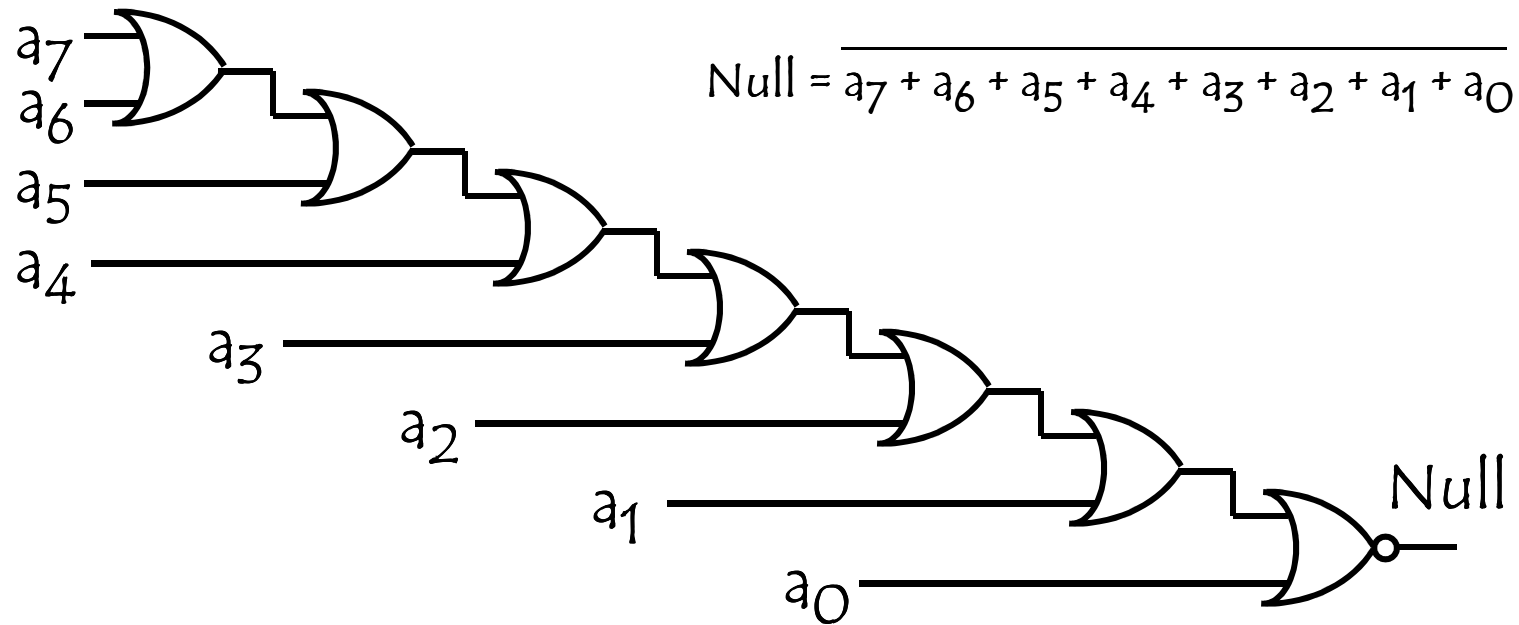
$$\text{Null} = a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0$$



Comparators

Comparing a natural number to zero :=

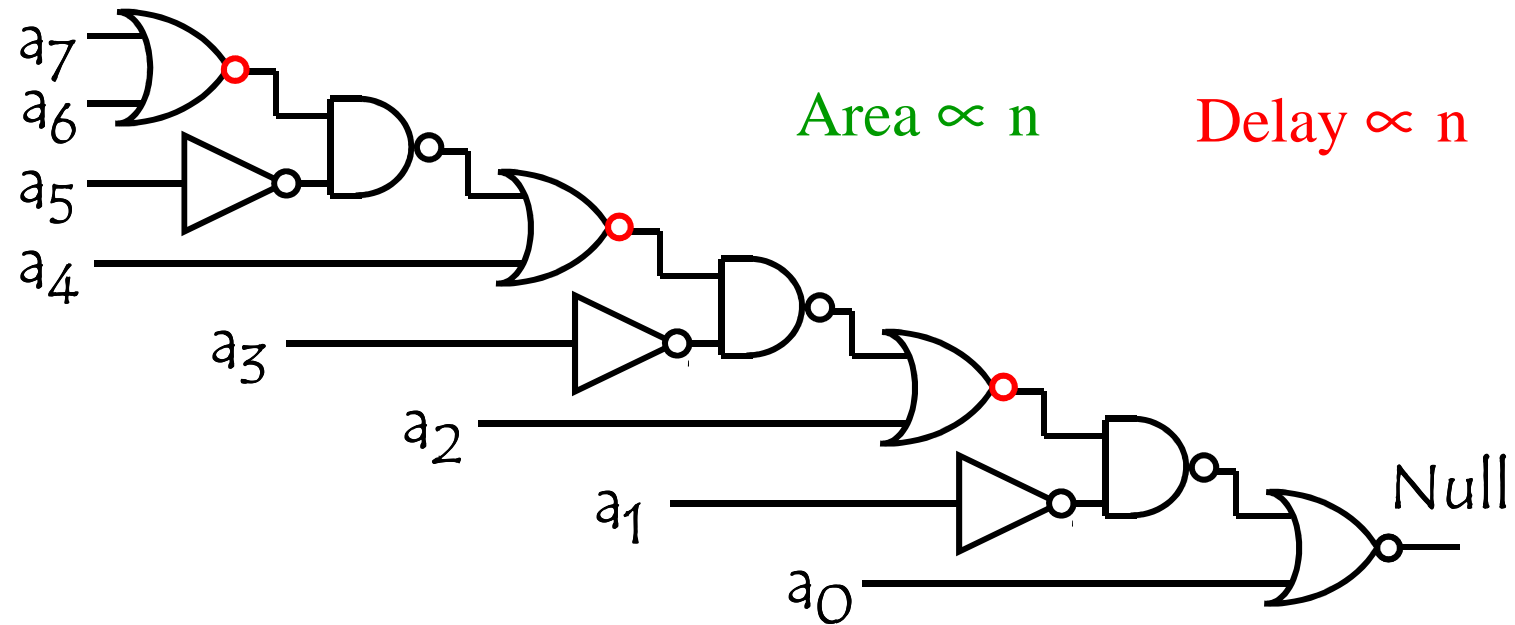
Implementation



Comparators

Comparing a natural number to zero : =

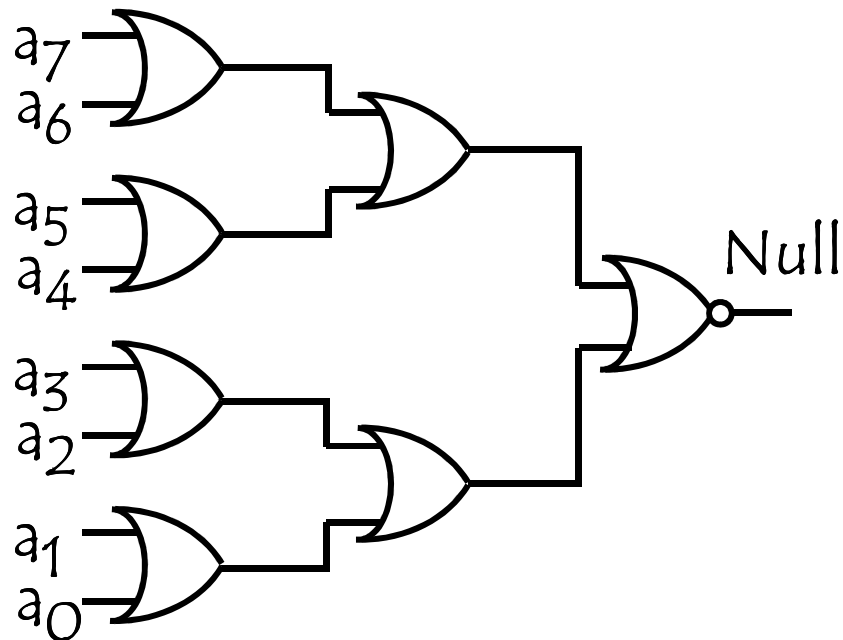
Implementation



Comparators

Comparing a natural number to zero : =

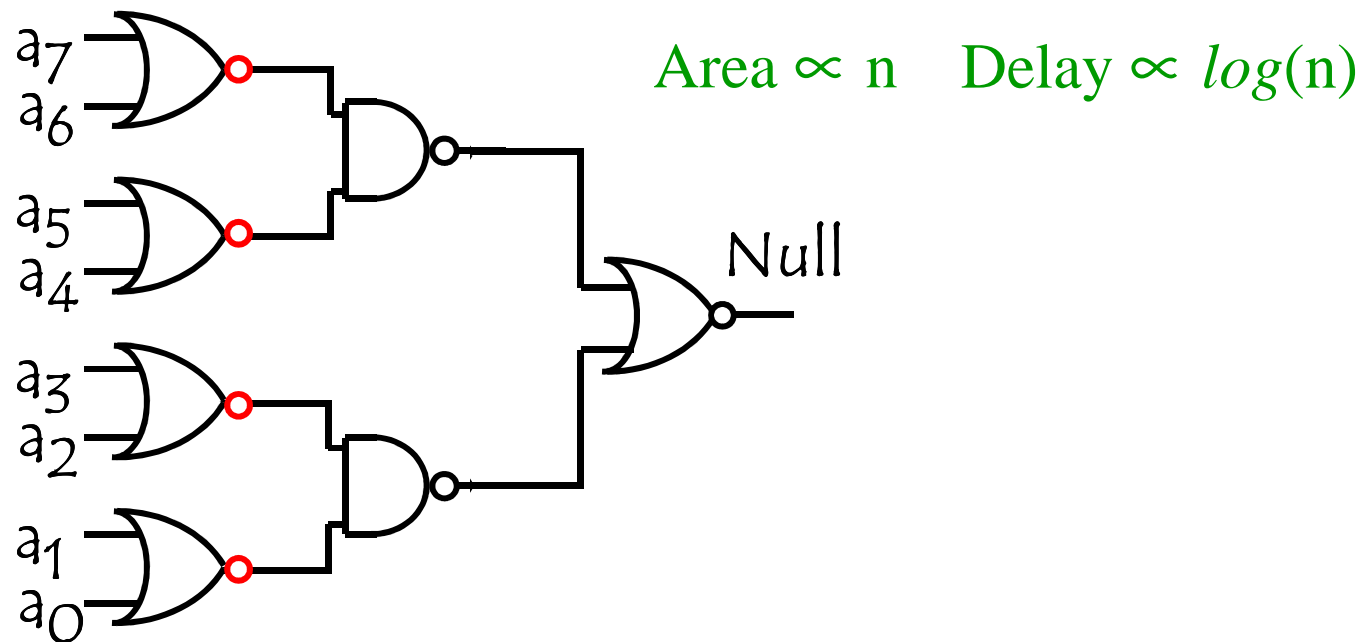
Implementation improvement



Comparators

Comparing a natural number to zero :=

Implementation improvement



Comparators

Comparing two natural numbers : =

Let consider two natural numbers a and b
coded on 8 bits using Natural Binary Code

$$\begin{array}{cccccccc} a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ & & & & = ? & & & \\ b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \end{array}$$

↓

0 / 1

Comparators

Comparing two natural numbers :=

Boolean function

a Equal b if : $a_7=b_7$ and $a_6=b_6$ and ... and $a_0=b_0$

a Equal b if : $\overline{(a_7 \oplus b_7)} \cdot \dots \cdot \overline{(a_0 \oplus b_0)} = 1$

$$\text{Equal} = \overline{(a_7 \oplus b_7)} + \dots + (a_0 \oplus b_0)$$

$$\text{Equal} = (e_7) + \dots + (e_0)$$

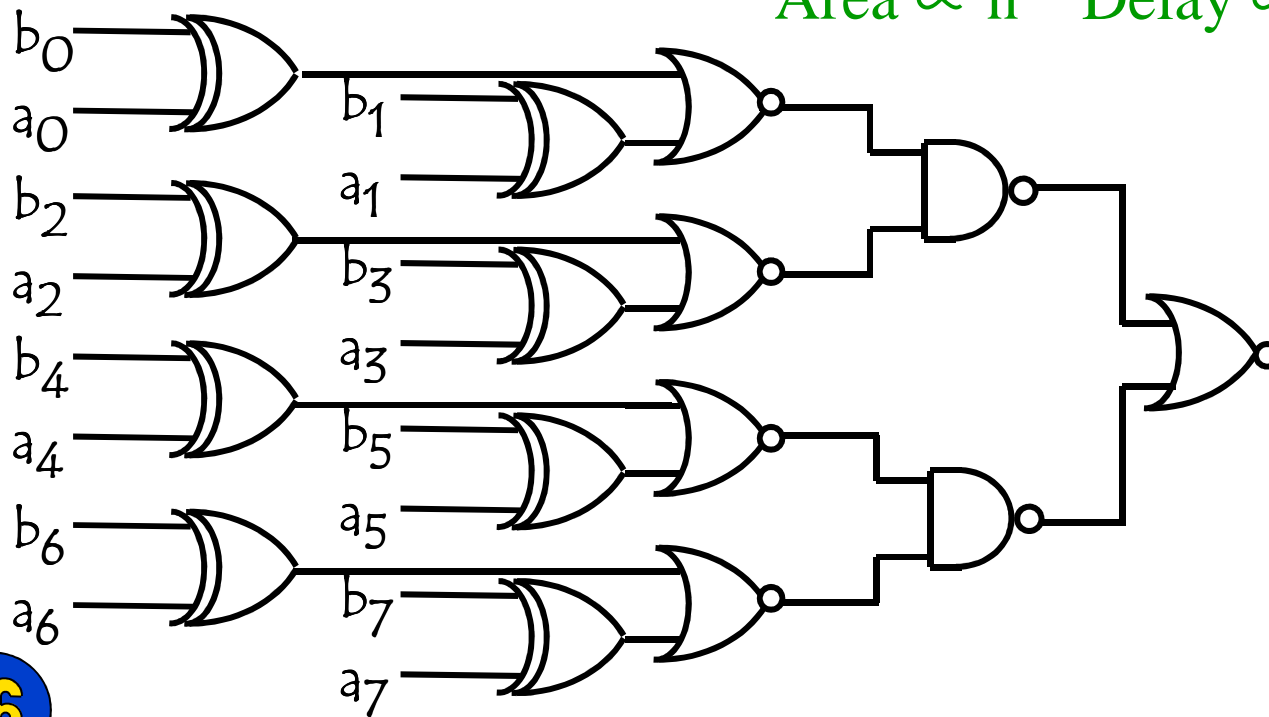


Comparators

Comparing two natural numbers :=

Implementation

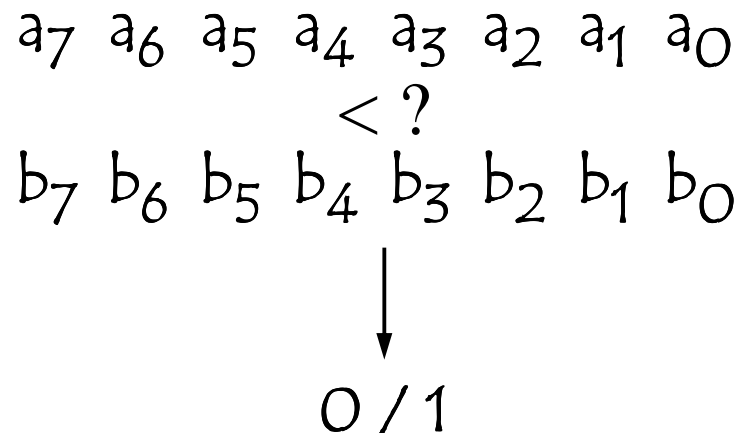
Area $\propto n$ Delay $\propto \log(n)$



Comparators

Comparing two natural numbers : <

Let consider two natural numbers a and b
coded on 8 bits using Natural Binary Code



Comparators

Comparing two natural numbers : <

Boolean function

$a < b$ if : $a_7 < b_7$ or $(a_7 = b_7$ and $(a_6 < b_6$ or $(a_6 = b_6$ and ...)))

$a_7 \ a_6 \ a_5 \ a_4 \ a_3 \ a_2 \ a_1 \ a_0$

< ?

$b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0$



0 / 1



Comparators

Comparing two natural numbers : <

Boolean function

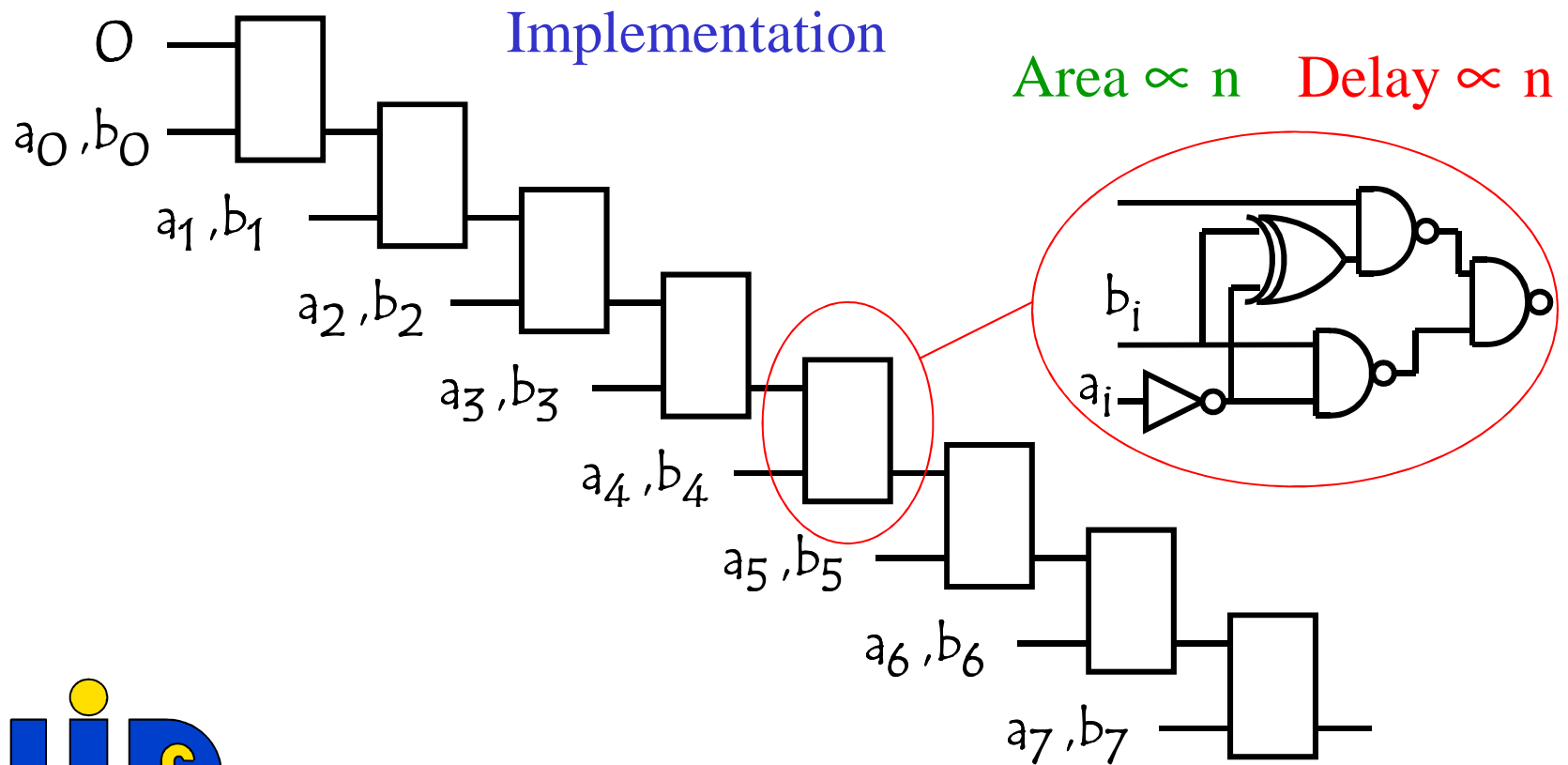
$a < b$ if : $a_7 < b_7$ or $(a_7 = b_7$ and $(a_6 < b_6$ or $(a_6 = b_6$ and ...)))

$a < b$ if : $\overline{a_7}b_7 + ((\overline{a_7} \oplus \overline{b_7}) \cdot (\overline{a_6}b_6 + ((\overline{a_6} \oplus \overline{b_6}) \cdot \dots)))$



Comparators

Comparing two natural numbers : <



Comparators

Comparing two natural numbers : <

Implementation Improvement

$a < b$ if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and ...)))

$a < b$ if : $\overline{a_7}b_7 + ((\overline{a_7} \oplus b_7) \cdot (\overline{a_6}b_6 + ((\overline{a_6} \oplus b_6) \cdot \dots)))$

$$\overline{a_7}b_7 + (\overline{a_7} \oplus b_7) \cdot c_6$$

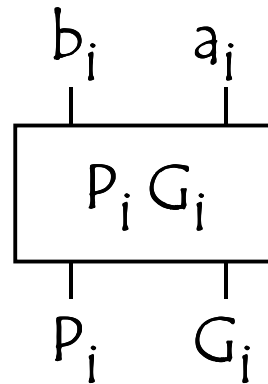
Propagation
Generation



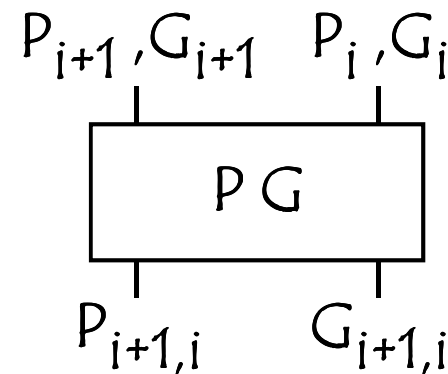
Comparators

Comparing two natural numbers : <

Implementation Improvement

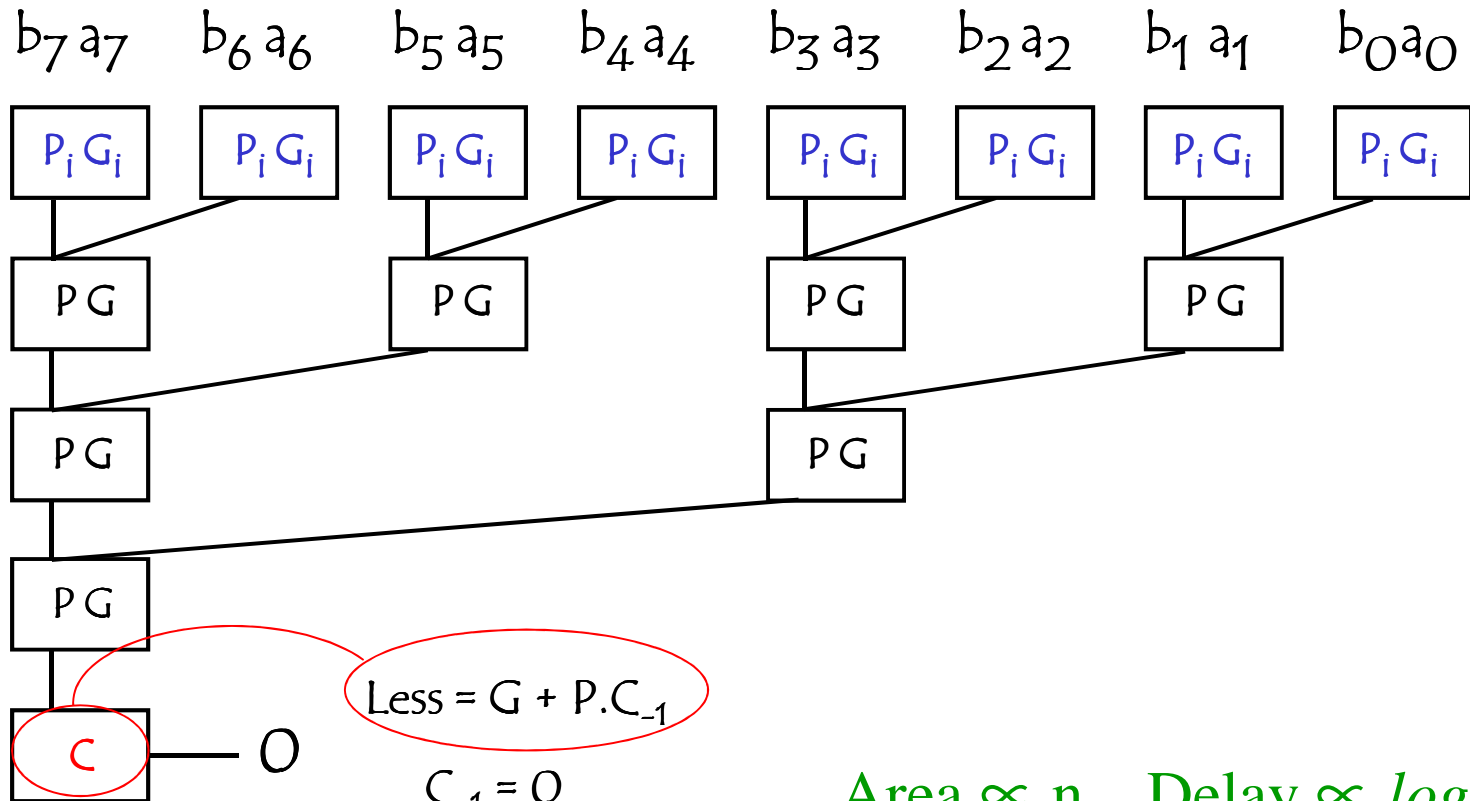


$$G_i = \overline{a_i} b_i$$
$$P_i = \overline{a_i} \oplus b_i$$



$$G_{i+1,i} = G_{i+1} + G_i \cdot P_{i+1}$$
$$P_{i+1,i} = P_i \cdot P_{i+1}$$

Comparators



Area $\propto n$ Delay $\propto \log(n)$

Comparators

