The Abdus Salam International Centre for Theoretical Physics

2384-19

ICTP Latin-American Advanced Course on FPGADesign for Scientific Instrumentation

19 November - 7 December, 2012

Convolución

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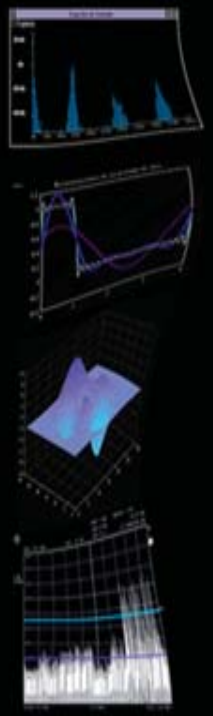
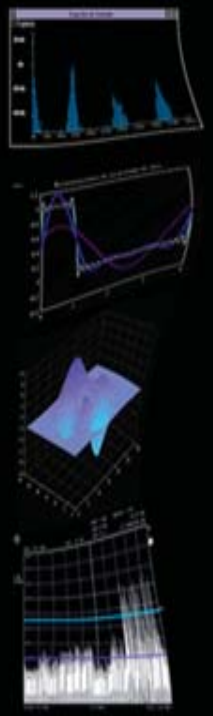
ARGENTINA

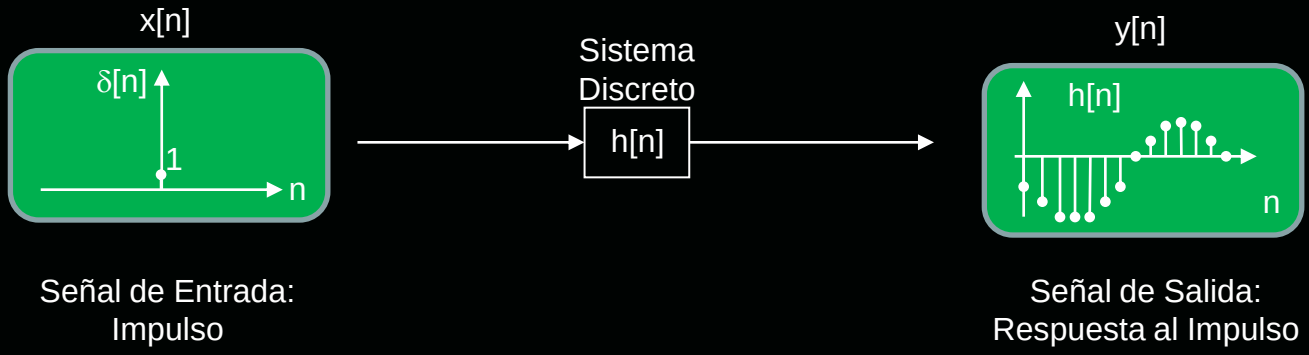
Convolución



Convolución

Respuesta al impulso





Señal de Entrada:
Impulso

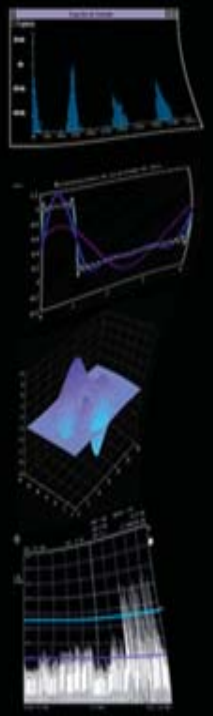
Señal de Salida:
Respuesta al Impulso

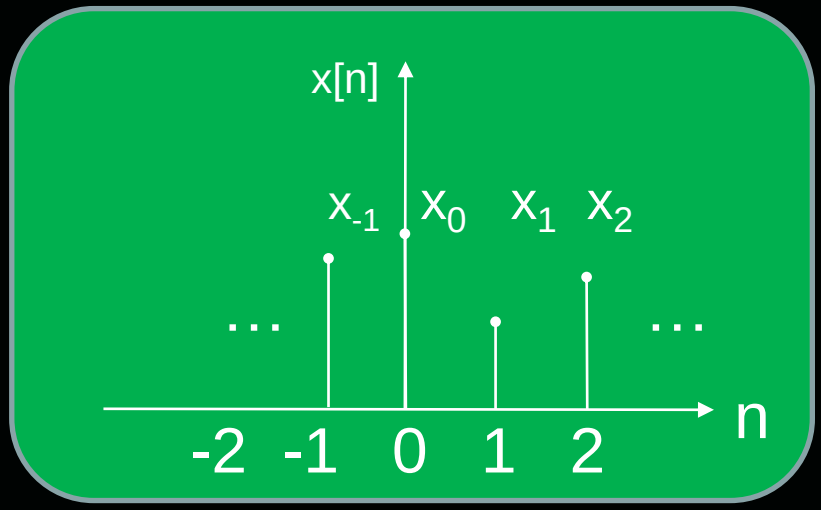
$$y[n] = \aleph\{\delta[n]\} = h[n]$$

Convolución

Ejemplos de señales

$$x[n] = ... + x_0\delta[n] + x_1\delta[n-1] + x_2\delta[n-2] + ... = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$





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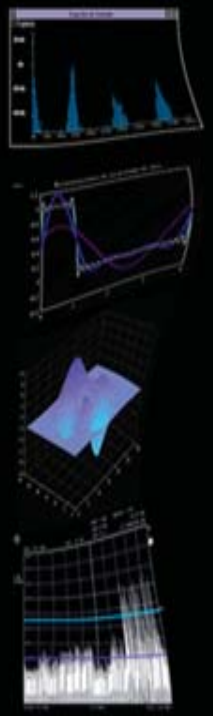
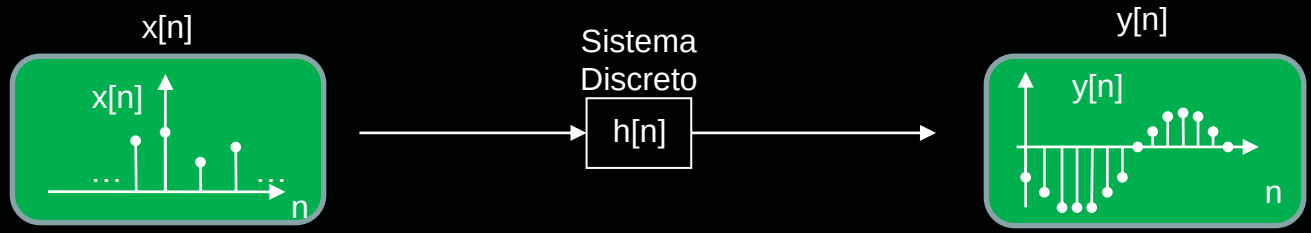




Señal de Entrada

Señal de Salida

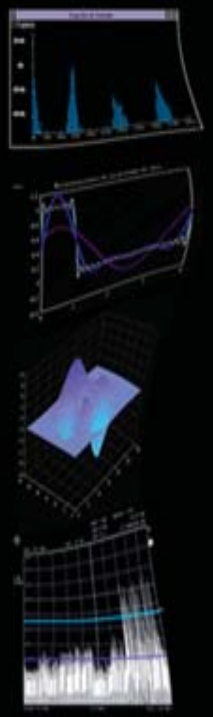
$$y[n] = \aleph\{x[n]\} = \aleph\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\} =$$

$$= \sum_{k=-\infty}^{\infty} \aleph\{x[k]\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k]\aleph\{\delta[n-k]\} =$$

$$= \sum_{k=-\infty}^{\infty} x[k]h[n,k] = x[n] * h[n,k]$$

Convolución

Sistemas lineales



Convolución para SLVT

$$h[n,k] = \aleph\{\delta[n-k]\} \Rightarrow y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n,k]$$

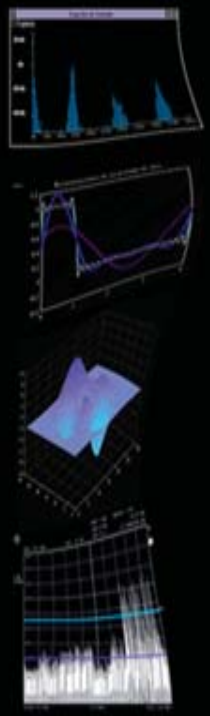
Convolución para SLIT

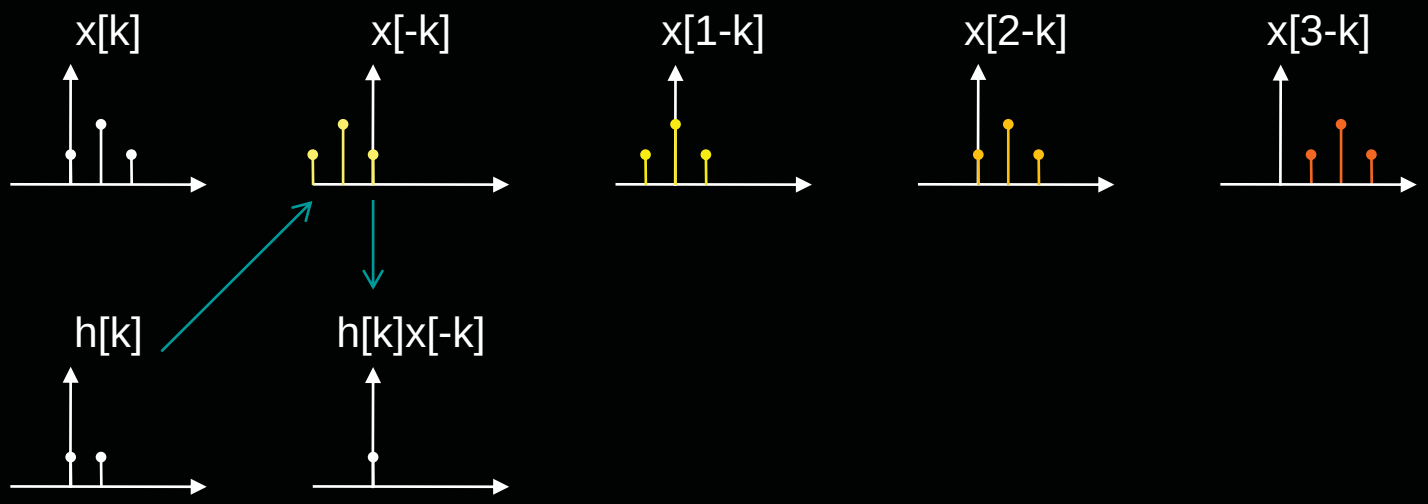
$$h[n-k] = \aleph\{\delta[n-k]\} \Rightarrow y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolución

Cálculo

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$





Convolución

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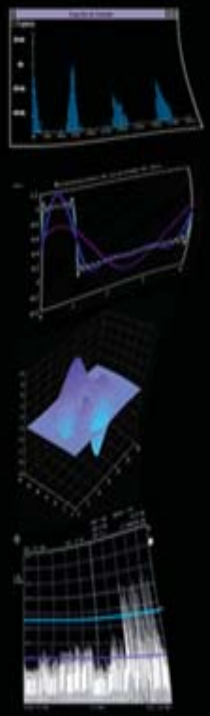

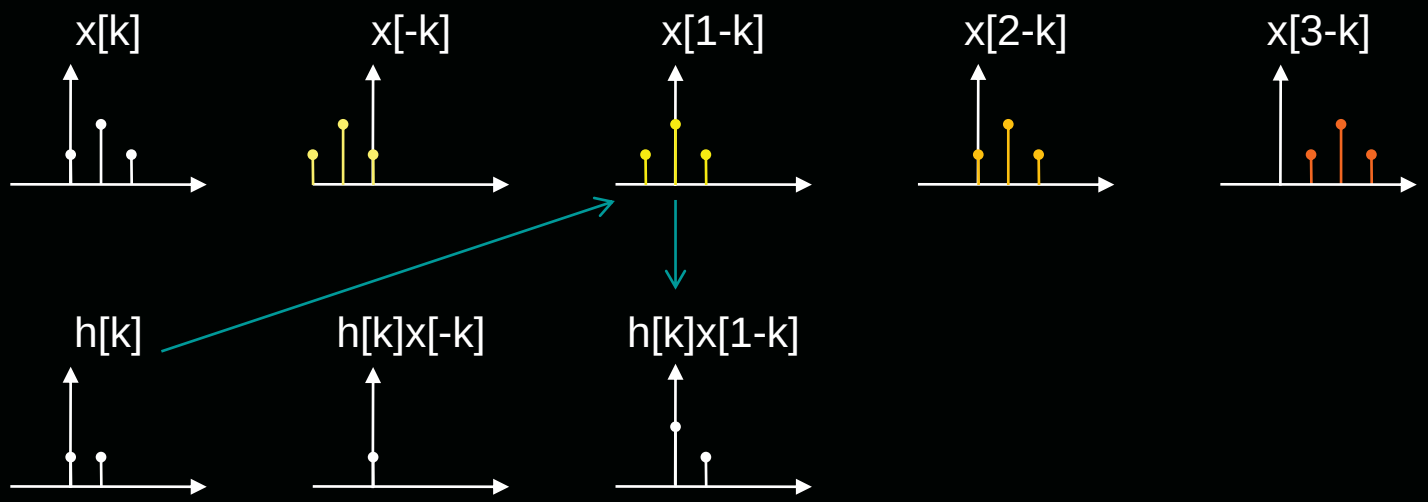

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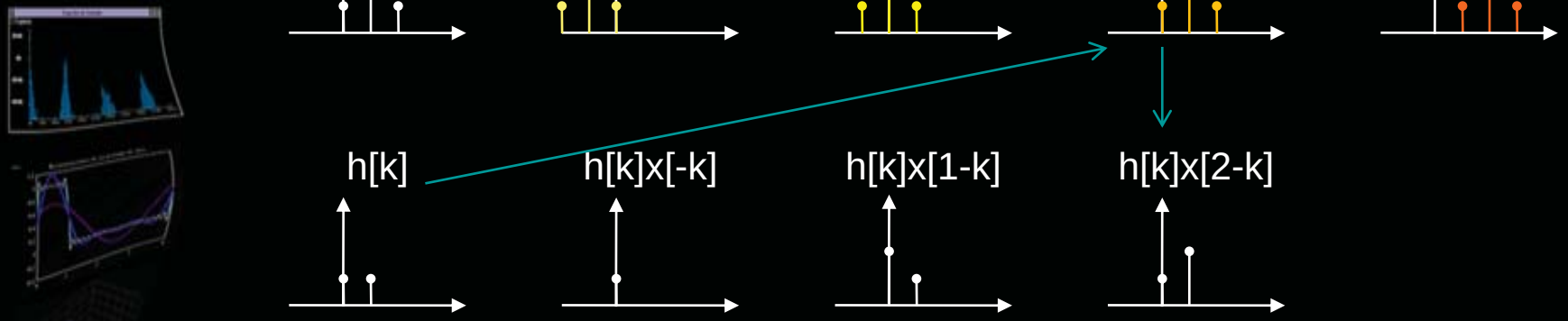

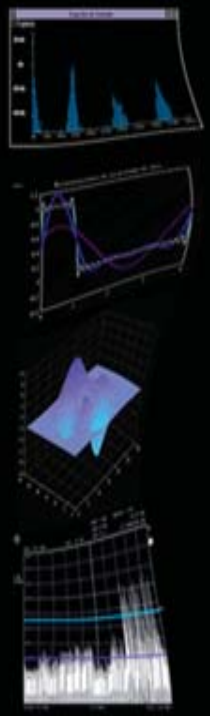

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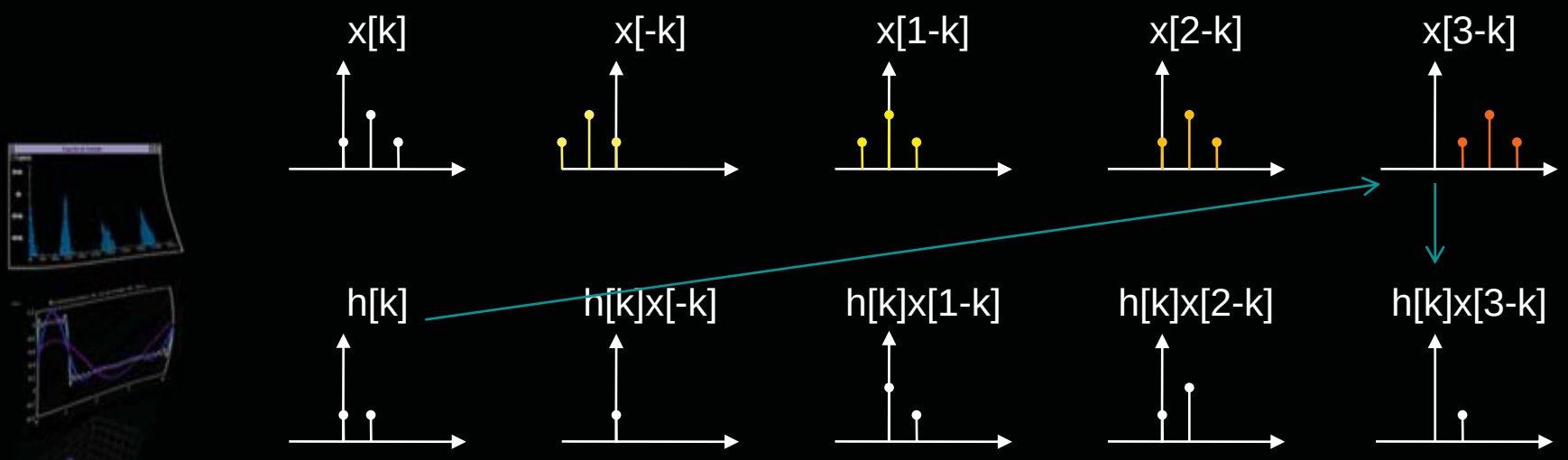

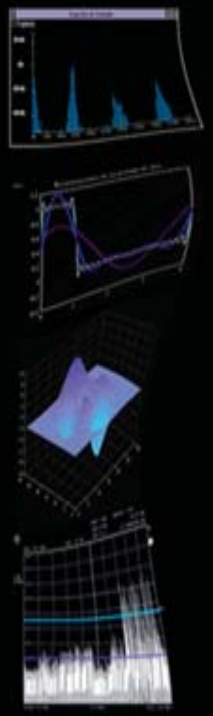

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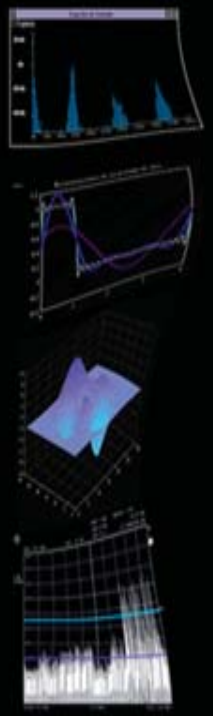

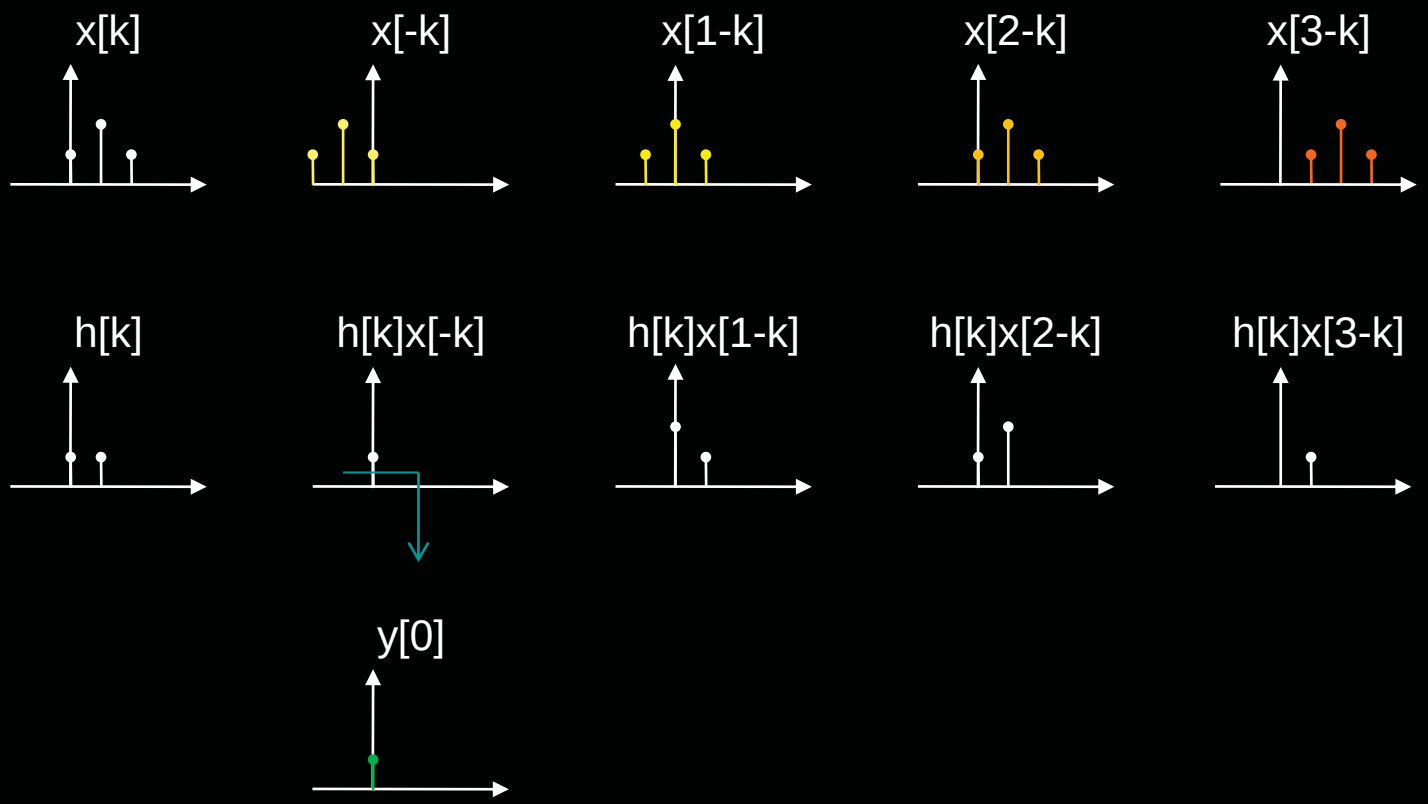

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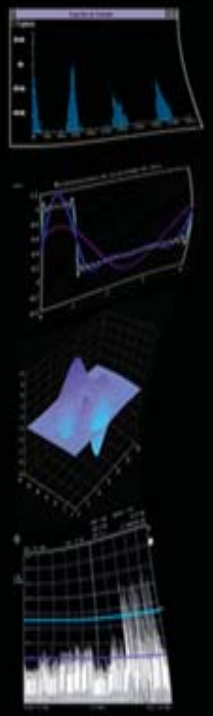

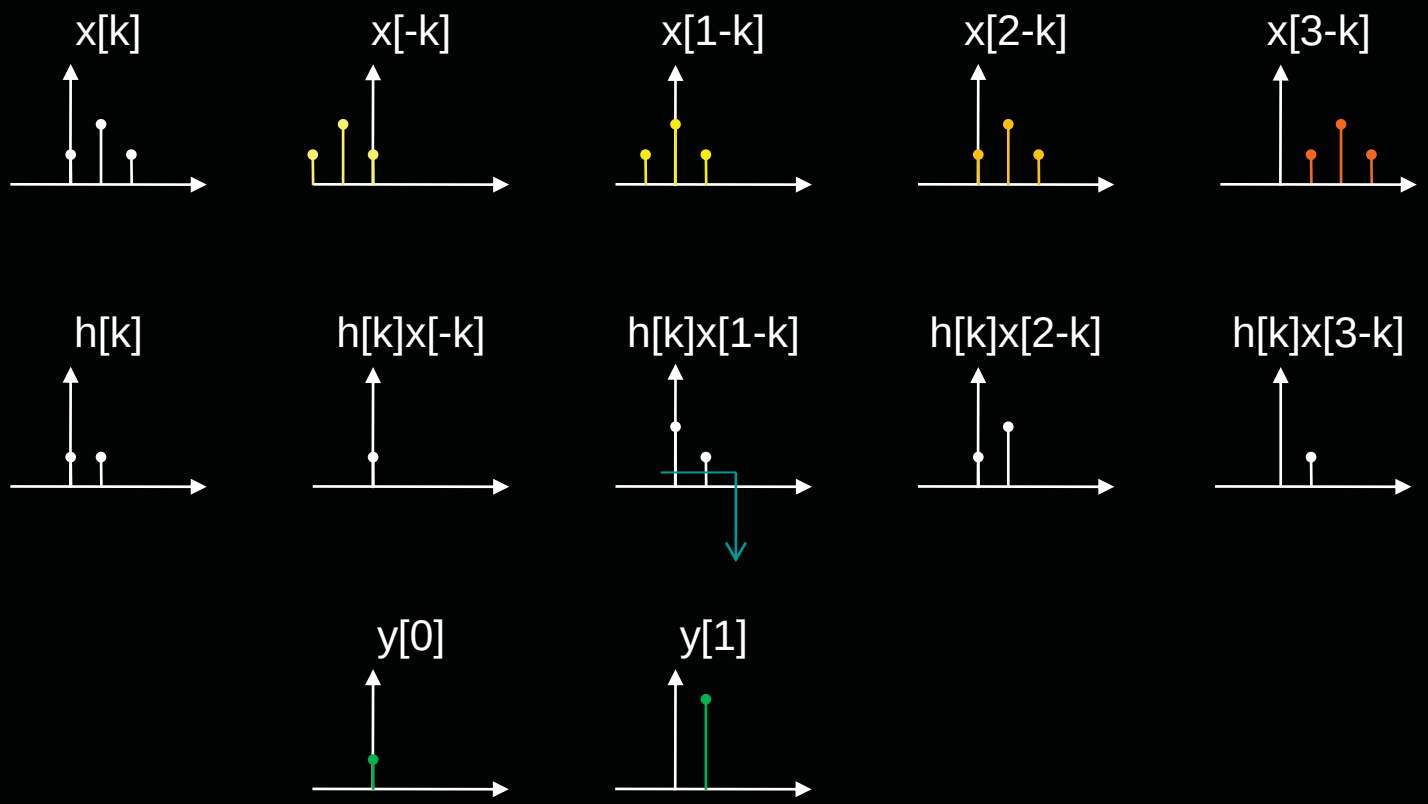

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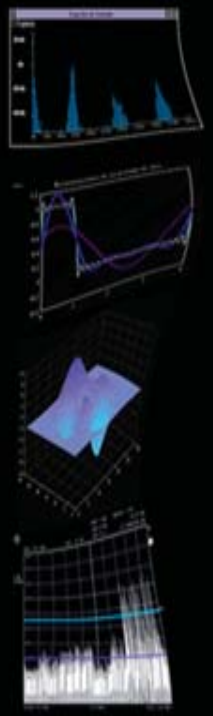

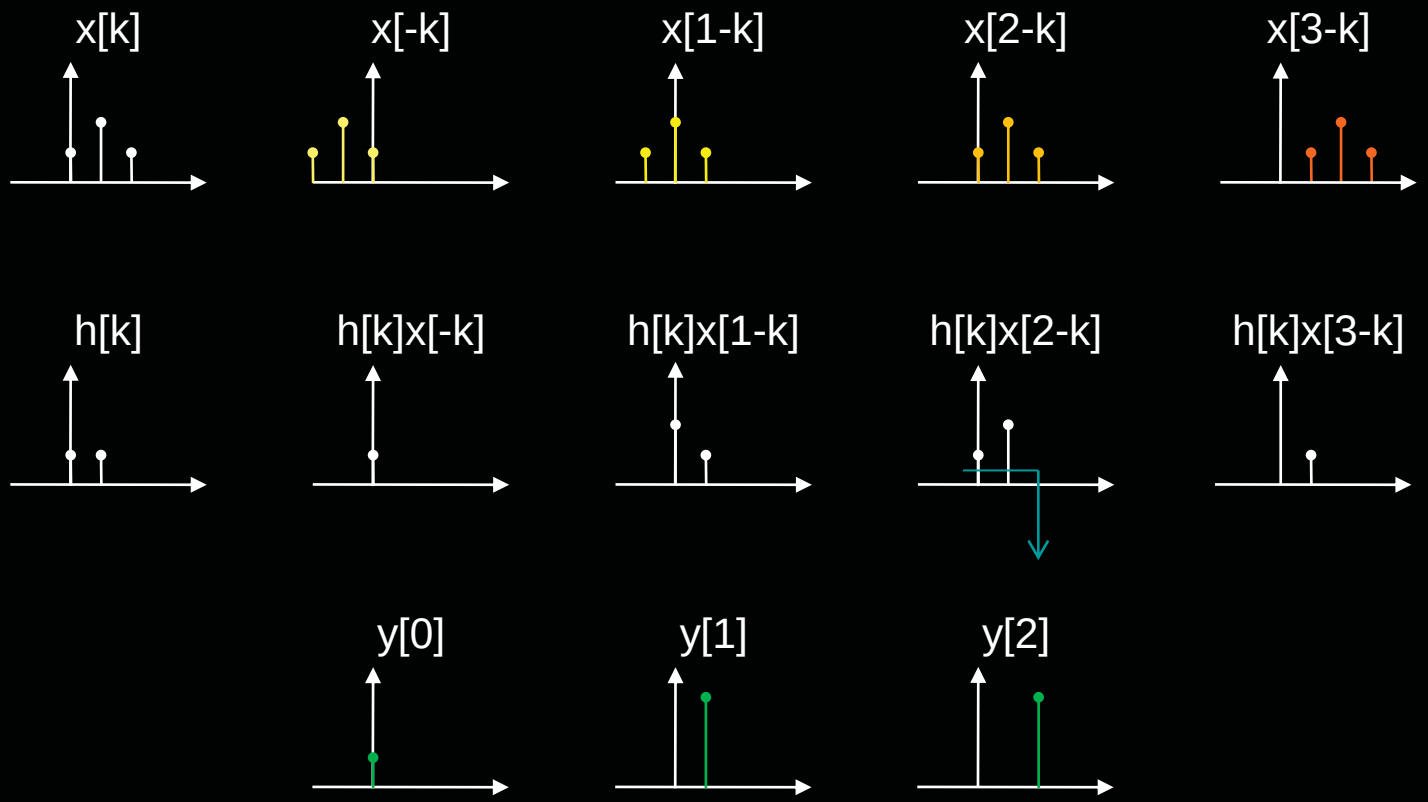

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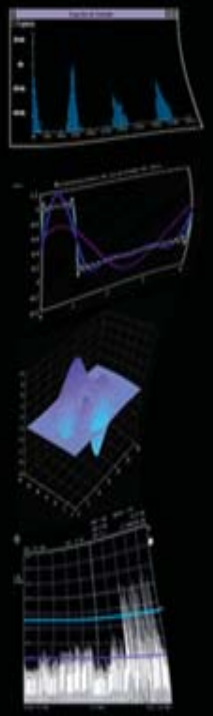

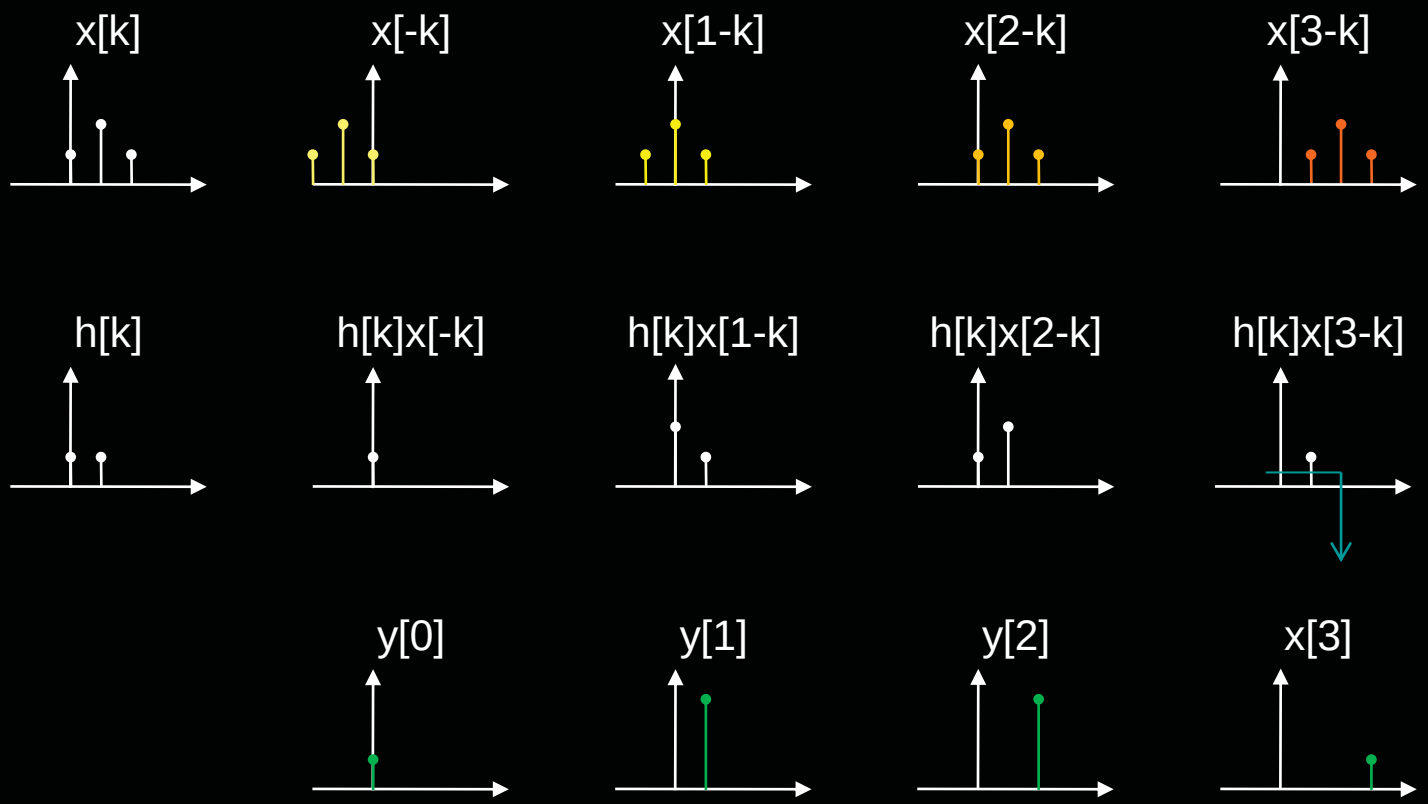

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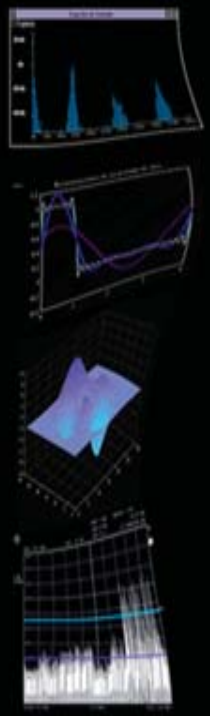

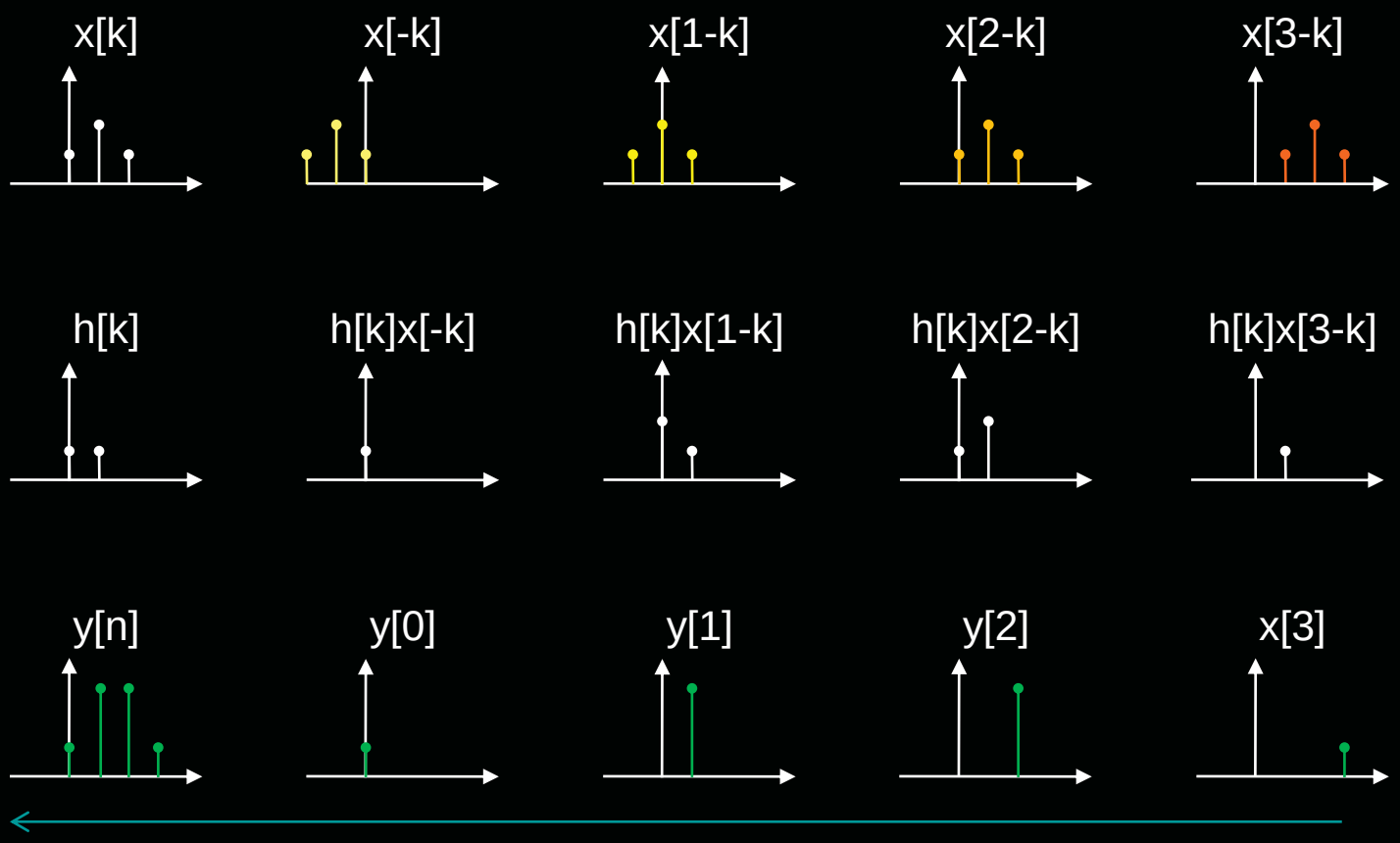

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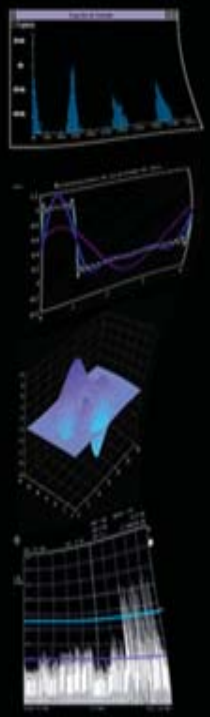

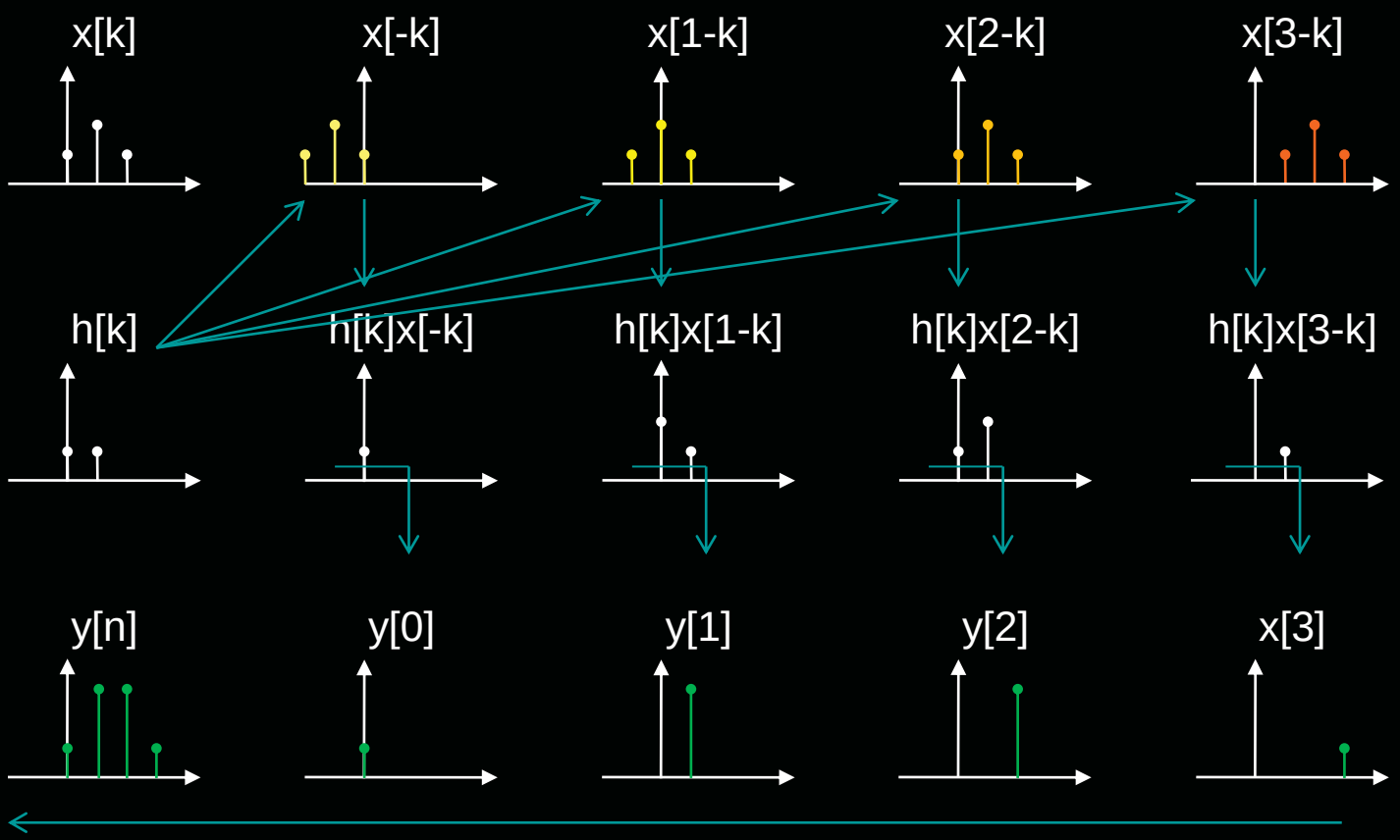

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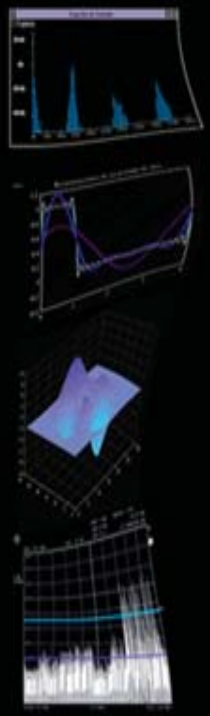
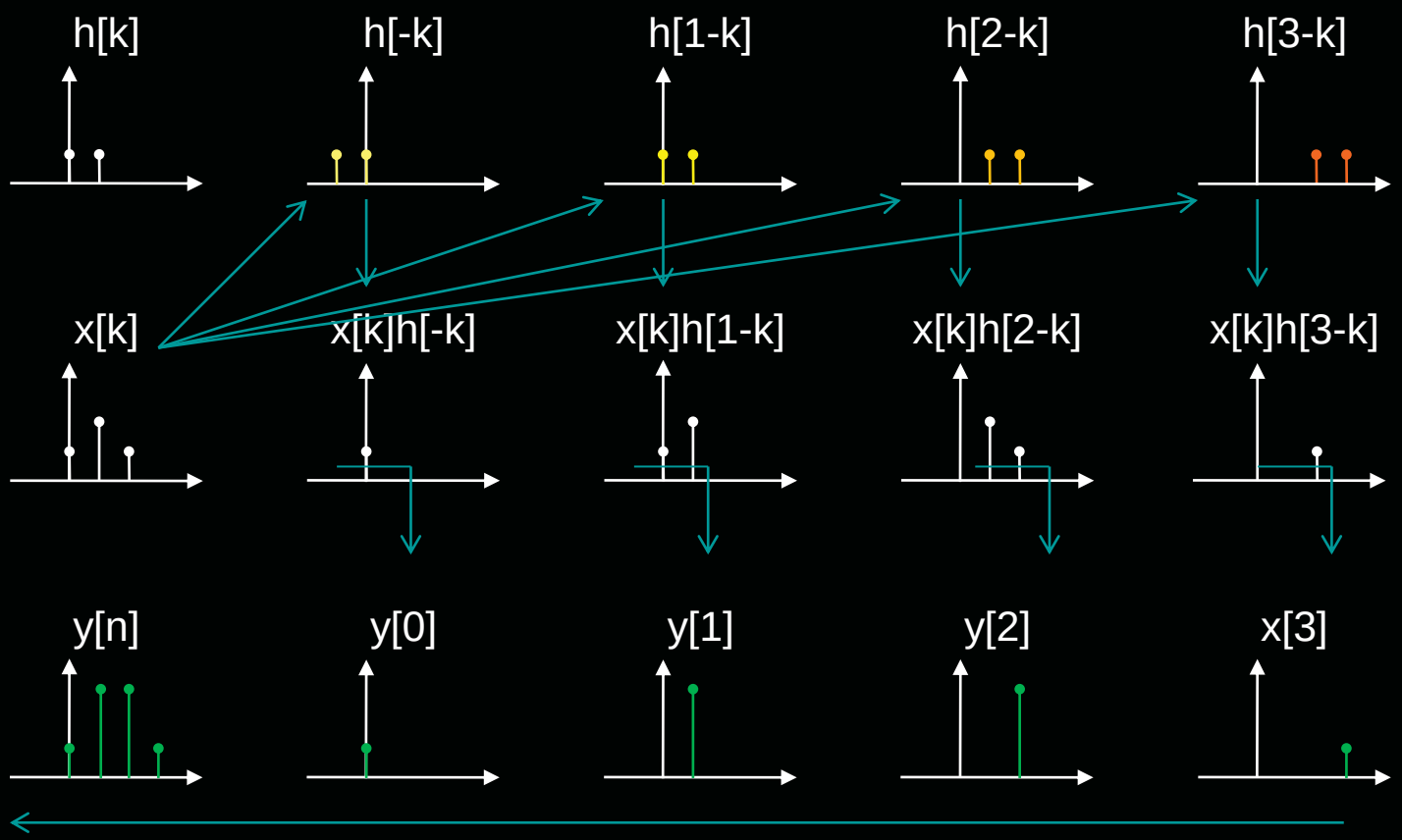
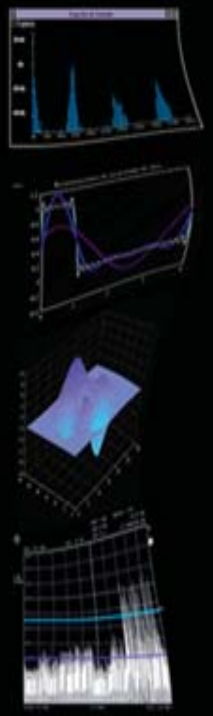
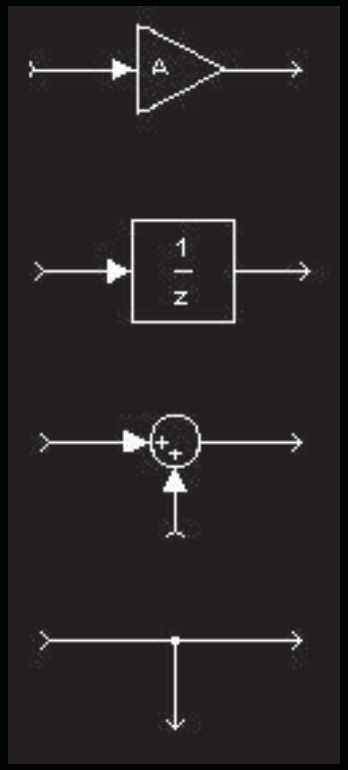




Convolución

Diagrama de bloques

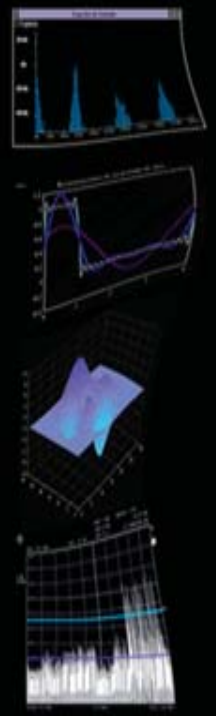


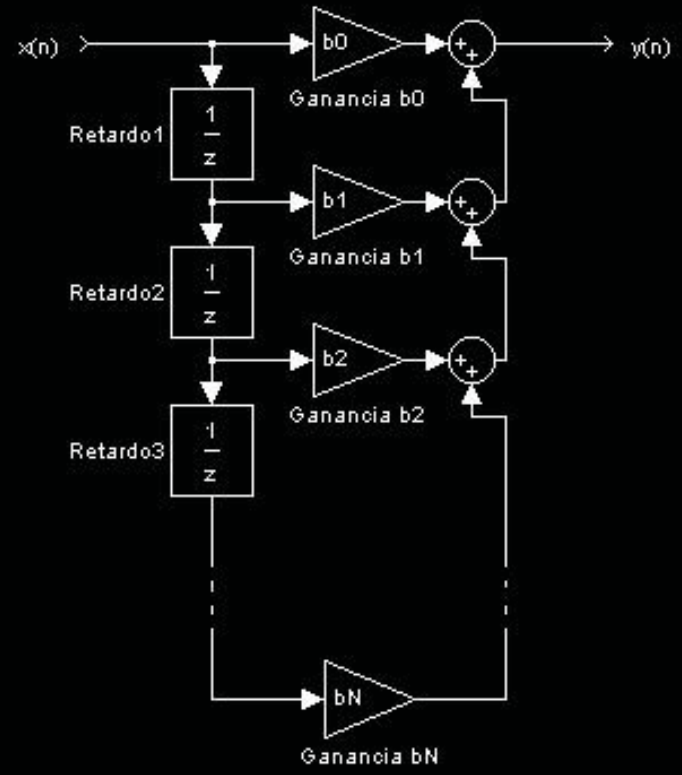


Convolución

FIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

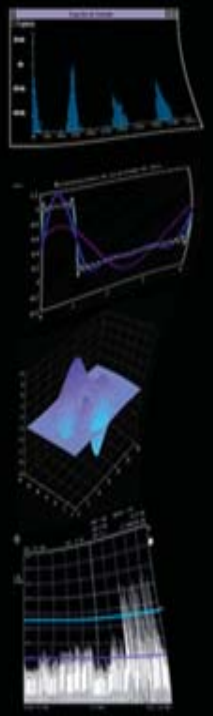


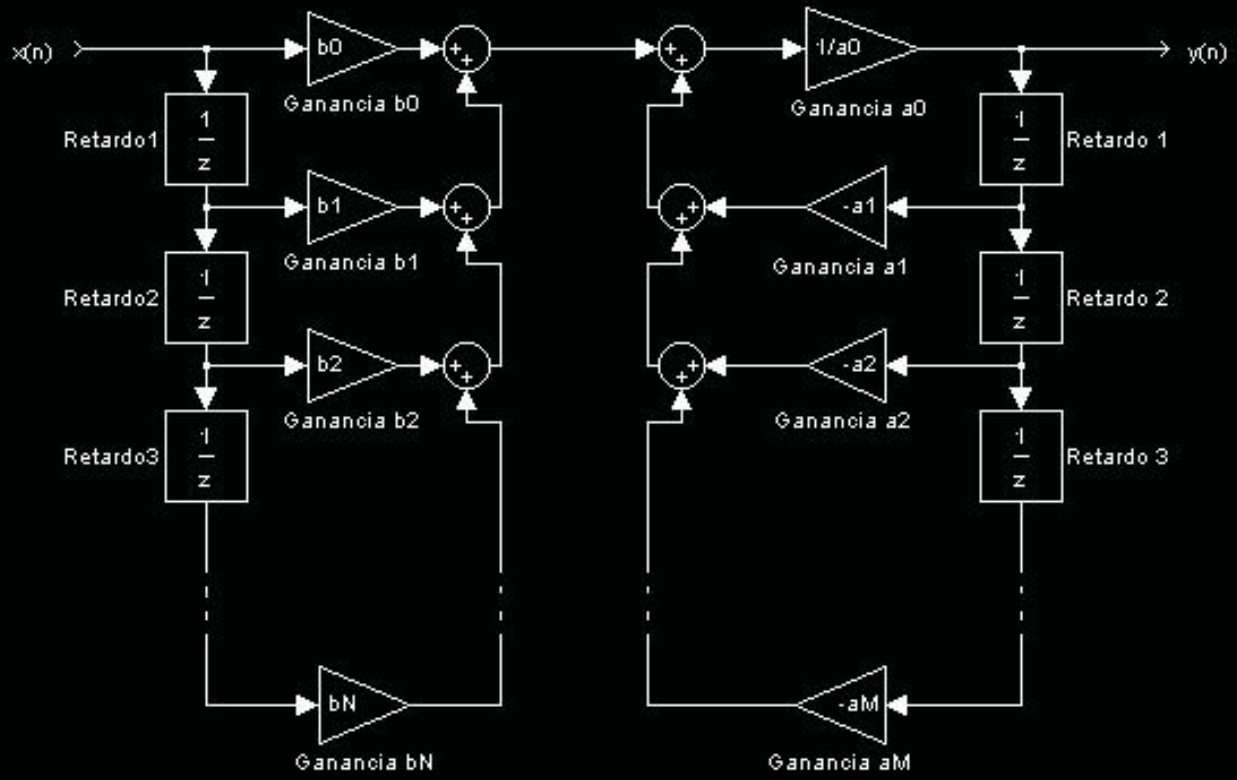


Convolución

IIR

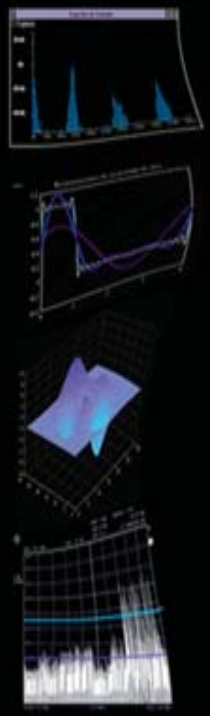
$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$





Convolución

Ecuación en diferencias



FIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

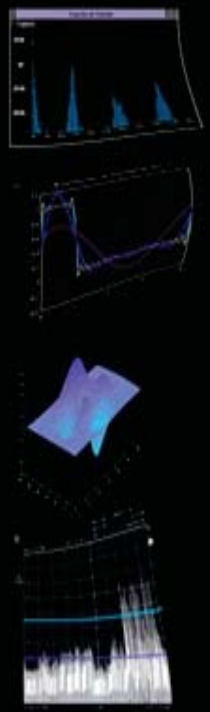
IIR

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Convolución

Ecuación en diferencias



$$y[n] = \sum_{k=0}^{M} b_k x[n-k] =$$

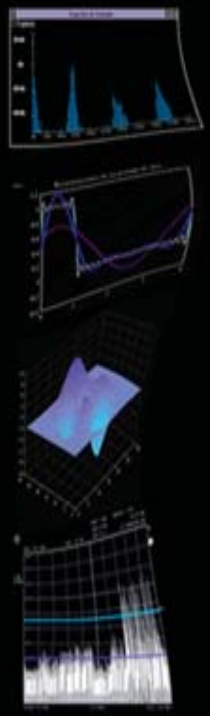
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + \ldots =$$

$$= h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + \ldots =$$

$$y[n] = \begin{cases} h[0]x[0] & si \quad n = 0 \\ h[0]x[1] + h[1]x[0] & si \quad n = 1 \\ h[0]x[2] + h[1]x[1] + h[2]x[0] & si \quad n = 2 \\ \vdots & \vdots \quad \vdots \\ h[0]x[M] + h[1]x[M-1] + \ldots + h[M]x[0] & si \quad n = M \end{cases}$$

Convolución

Propiedades de los SLIT



Duración

$$L\{x[n]*h[n]\} = L\{x[n]\} + L\{h[n]\} - 1$$

Inicio

$$x[n] = 0 \forall n < n_x \wedge h[n] = 0 \forall n < n_h$$

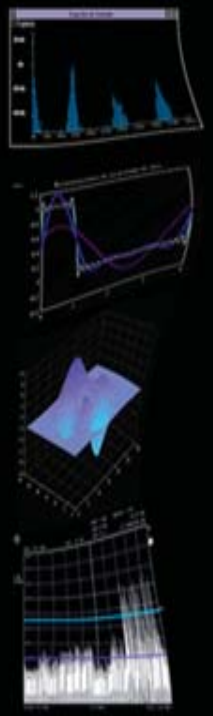
$$\Rightarrow x[n]*h[n] = 0 \forall n < n_x + n_h$$

Respuesta al impulso

$$\delta[n]*h[n] = h[n]$$

Convolución

Propiedades de los SLIT



Distributividad

$$y[n] = x[n] * \left(h_1[n] + h_2[n]\right) = x[n] * h_1[n] + x[n] * h_2[n]$$

Conmutatividad

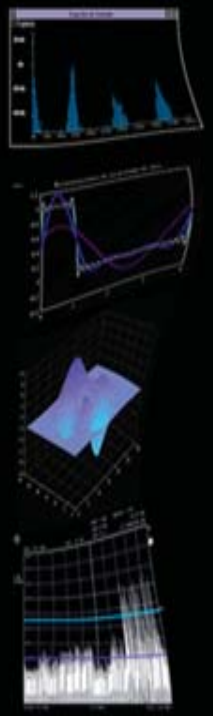
$$y[n] = x[n] * h[n] = h[n] * x[n]$$

Desplazamiento

$$x[n-\alpha] * h[n-\beta] = y[n-\alpha-\beta]$$

Convolución

Propiedades de los SLIT



Asociatividad

$$x_i[n], x_j[n], x_k[n] \in C :$$

$$x_{i,j,k}[n] = 0 \forall n < 0 \vee x_{i,j,k}[n] = 0 \forall n > 0$$

$$\wedge \quad \sum_{n=-\infty}^{\infty} x_{i,j,k}[n] \leq M_x < \infty$$

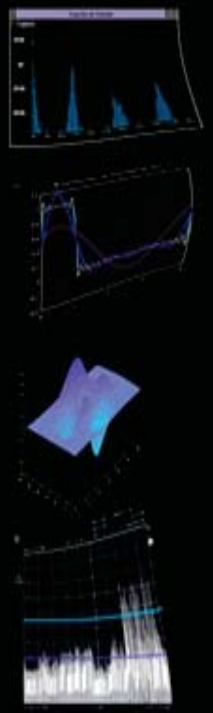
$$\Rightarrow x_i[n] * \left[x_j[n] * x_k[n] \right] =$$

$$= \left[x_i[n] * x_j[n] \right] * x_k[n] =$$

$$= x_j[n] * \left[x_i[n] * x_k[n] \right]$$

Convolución

Propiedades de los SLIT



Correlación

$$r_{xy}[n] = x[n] \bigstar y[n] = \sum_{k=-\infty}^{\infty} x^*[k]\, y[n+k] =$$

$$= \sum_{k=-\infty}^{\infty} x^*[k-n]\, y[k] = x[n] * y[-n]$$

$$r_{yx}[n] = y[n] \bigstar x[n] = \sum_{k=-\infty}^{\infty} y^*[k]\, x[n+k] =$$

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