The Abdus Salam
International Centre
for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2384-17

ICTP Latin-American Advanced Course on FPGADesign for Scientific Instrumentation

19 November - 7 December, 2012

Advanced topics in signal processing
(Part 2)

MAGNASCO Marcelo Osvaldo
The Rockefeller University, Mathematical Physics Lab
Box 212, 1230 YOrk Avenue
New York 10021
NY
U.S.A.

the sopranos

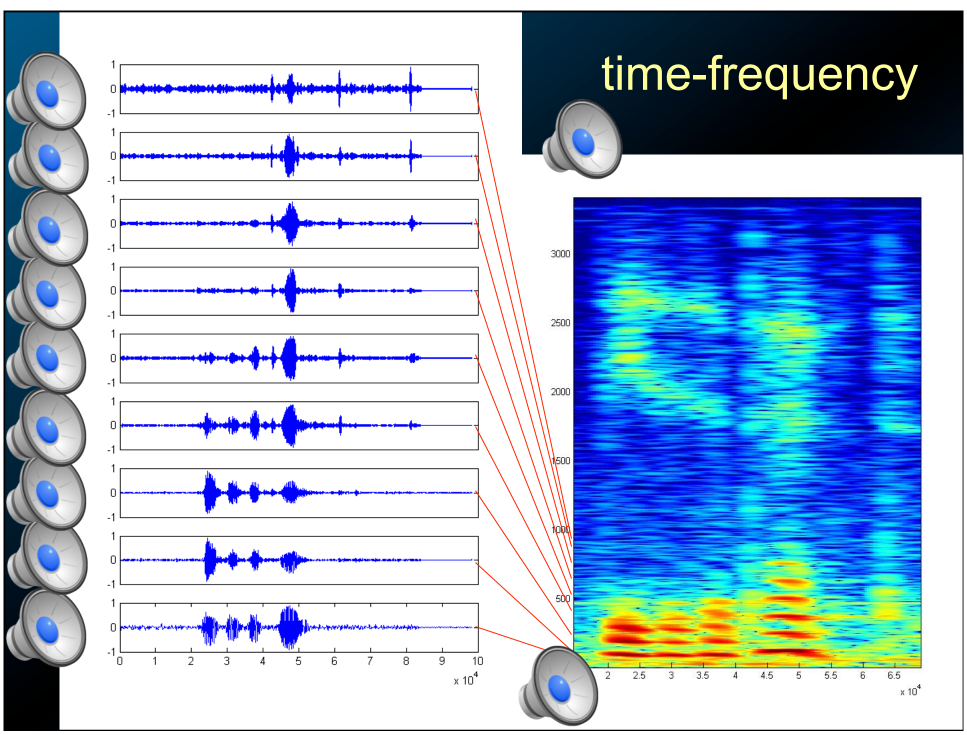
- cheryl studer
- edita gruberova
- florence foster-jenkins

what is time-frequency analysis?

time-frequency analysis refers to decomposing a signal *simultaneously* along a time axis and a frequency axis. such representations are extremely important in engineering, quantum mechanics, animal vocalizations, signal processing, ethnomusicography, radar, sound analysis and recognition, geophysics, radioastronomy and the physiology of hearing

well known to all is the musical score, which represents frequency vertically and time horizontally.


early XVth


time-frequency
1
0
-1
0 1 2 3 4 5 6 7 8 9 10
x 10^4
3000
2500
2000
1500
1000
500
2 2.5 3 3.5 4 4.5 5 5.5 6 6.5
x 10^4

the short time Fourier transform

(a.k.a. Gabor transform, a.k.a. single wavelet)

Fourier signal

$$\chi(\omega,t) = \int e^{i\omega(t-t')} e^{-\frac{(t-t')^2}{2\sigma^2}} x(t')\,dt'$$

time localiz

at a given frequency

and a given time

phase

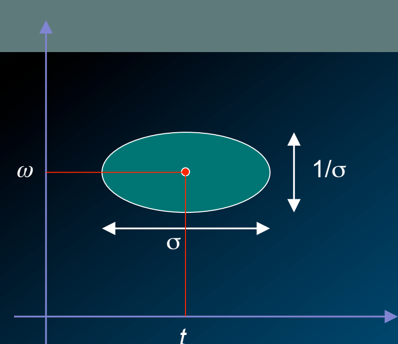
$$\chi(\omega,t) = |\chi|\, e^{i\phi}$$

amplitude

the short time Fourier transform

$$\chi(\omega,t) = \int e^{i\omega(t-t')} e^{-\frac{(t-t')^2}{2\sigma^2}} x(t')\,dt'$$

the stft depends *only* on nearby data: a neighbourhood of size σ in time, and of size 1/σ in frequency. therefore all nearby points are strongly correlated, and so the amplitude of the stft appears to be "out of focus". making σ smaller only succeeds, of course, in making 1/σ bigger, so the area of the "grain" is constant.



Time-frequency distributions:

- Linear: Gabor/STFT (sonogram), various windows (prolate spheroidal functions). Extension -> timescale (wavelets)
- Quadratic (Cohen's class): Wigner-Ville (Housimi), Choi-Williams.
- Huang-Hilbert
- Matching pursuit
- Multitapered spectral estimates / Spectral derivatives method
- Reassigned spectrogram

phase and intensity

the auditory system preserves phase information much more carefully than intensity information:

12% v 26% E

26% v 58% E

- discrimination in intensity: 1dB (26% change in energy)

445.7

441.9

- discrimination in frequency: 1/12th semitone (0.5 % change in f).
- discrimination in heading: 15 degrees = 20 μs. (needs headphones – echoes interfere)

auditory nerve fibers

the fibers of the eighth cranial nerve fire in response to sounds. each fiber has a frequency to which it is most sensitive. near the threshold of the fiber, the number of action potentials evoked by the stimulus increases with stimulus intensity, but it rapidly saturates.

for all stimuli intensity that evoke potentials, the fiber fires always at specific phases of the stimulus. they may not fire every cycle but in subharmonic lock, particularly at high frequencies. this phase lock has been shown in mammals up to 4 kHz, and up to 9 kHz in owls.

therefore any given fiber carefully encodes the driving frequency, and encodes very little information about intensity.

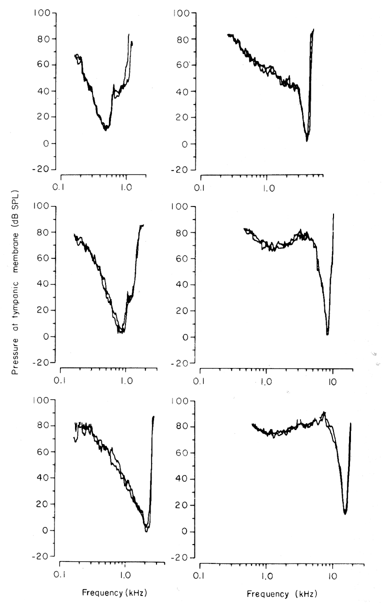


Fig. 4.3 Representative tuning curves (frequency threshold curves) of cat auditory nerve fibres are shown for six different frequency regions. In each panel, two fibres from the same animal, of similar characteristic frequency and threshold are shown, indicating the constancy of tuning under such circumstances. From Liberman and Kiang (1978, Fig. 1).

encoding in the auditory nerve

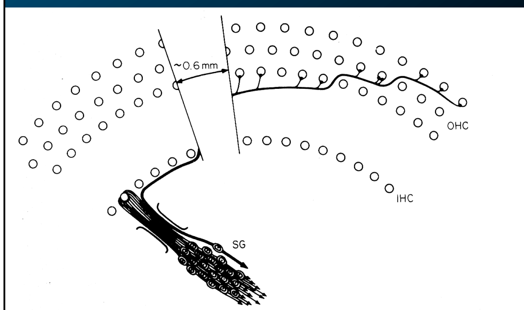


Fig. 3.6 The great majority of auditory nerve fibres (type I fibres) connect with inner hair cells. A few fibres (type II) pass to outer hair cells, after running basally for about 0.6 mm. IHC, Inner hair cells; OHC, Outer hair cells; SG, spiral ganglion. From Spoendlin (1978, Fig 8).

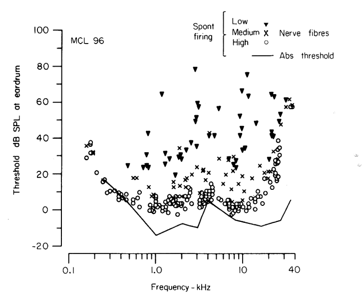


Fig 4.4 Distribution of best thresholds of auditory nerve fibres in one cat. Fibres with high spontaneous firing rates (○, ⩾18/s) have low thresholds, and those with low spontaneous firing rates (▼, <0.5/s) have high thresholds. Fibres with intermediate spontaneous firing rates (×) have thresholds in between. The behavioural absolute threshold of the cat, expressed in terms of the intensity at the eardrum, lies just below the lowest thresholds of the auditory nerve fibres. Neural data from Liberman and Kiang (1978, Fig. 2). Behavioural data from Elliott *et al.* (1960).

use the phase!

- can compute intervals within a fiber and across fibers
- should not make explicit use of fiber labels or time
- nervous system keeps accurate timing info all the way to primary auditory cortex

instantaneous time-frequency

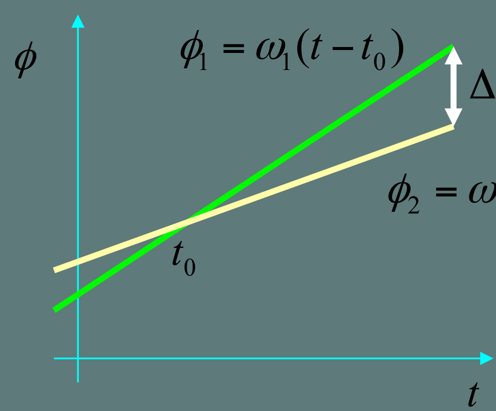
$$\varphi(\omega,t) = \operatorname{Im}\log\chi$$

simplest thing one can do with the phase is take derivatives

$$\omega_{ins}(\omega,t) = \frac{\partial\phi}{\partial t}$$

$$t_{ins}(\omega,t) = t - \frac{\partial\phi}{\partial\omega}$$

instantaneous time picture



$$\phi_1 = \omega_1(t - t_0)$$

$$\Delta\phi = (\omega_1 - \omega_2)(t - t_0)$$

$$\phi_2 = \omega_2(t - t_0)$$

$$\frac{\Delta\phi}{\Delta\omega} = t - t_0$$

$$t_0 = t - \frac{\Delta\phi}{\Delta\omega}$$

instantaneous time-frequency

$$\phi(\omega, t)$$

simplest thing one can do with the phase is take derivatives

$$\omega_{ins}(\omega, t) = \frac{\partial\phi}{\partial t}$$

$$t_{ins}(\omega, t) = t - \frac{\partial\phi}{\partial\omega}$$

now that we have these two estimates, the easiest thing to do is to plot one against the other and dispense with (w,t) entirely!

simple elements

	$x = e^{i\omega_0 t}$	$x = \delta(t - t_0)$	$x = e^{i\alpha t^2/2}$
$\omega_{ins}(\omega,t) \quad = \dfrac{\partial\phi}{\partial t}$	ω_0	ω	αt_{ins}
$t_{ins}(\omega,t) \quad = t - \dfrac{\partial\phi}{\partial\omega}$	t	t_0	...

any linear element in the time-frequency plane, like a tone, a click, or a linear frequency sweep (calculation mercifully spared and available upon request) is mapped by the instantaneous reassignment to a one-dimensional, perfectly thin line. meanwhile, the original stft is still out of focus, with this line blurred by (σ,1/σ). no linear transform has this property. only the (bilinear) wigner-ville distribution localizes these signals exactly.

the implications for the fourier uncertainty theorem are clear. uncertainty in the time-frequency plane refers to *resolution*, i.e., the ability to distinguish two objects as distinct. any *single* object can be tracked in both frequency and time to arbitrary accuracy.

K. Kodera, R. Gendrin, and C. de Villedary, ``Analysis of time-varying signals with small BT values", *IEEE Trans. ASSP*, **26.**1 64-76 (1978).
F. Auger and P. Flandrin, ``Improving the readability of time-frequency and time-scale representations by the reassignment method," IEEE Trans. on Signal Proc. {\bf 43}, 1068-1089 (1995).
E. Chassande-Mottin, I. Daubechies, F. Auger, P. Flandrin, ``Differential reassignment," *IEEE Signal Proc. Lett.*, **4**10, 293-294 (1997).

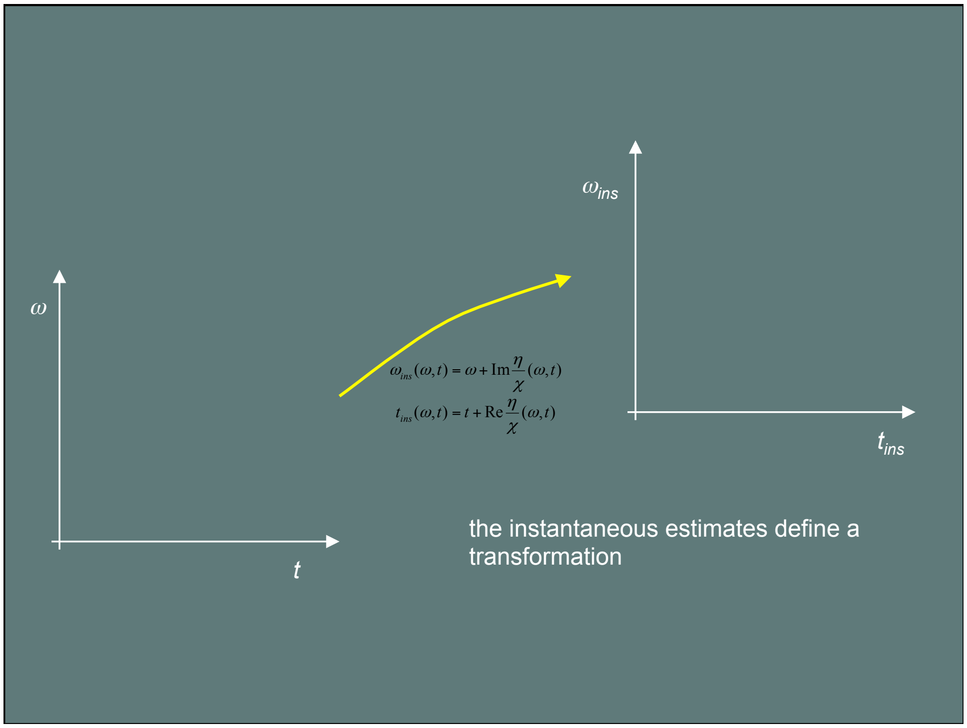
the map

$$\chi(t,\omega) = \int e^{-(t-t')^2/2} e^{iw(t-t')} x(t')\, dt'$$

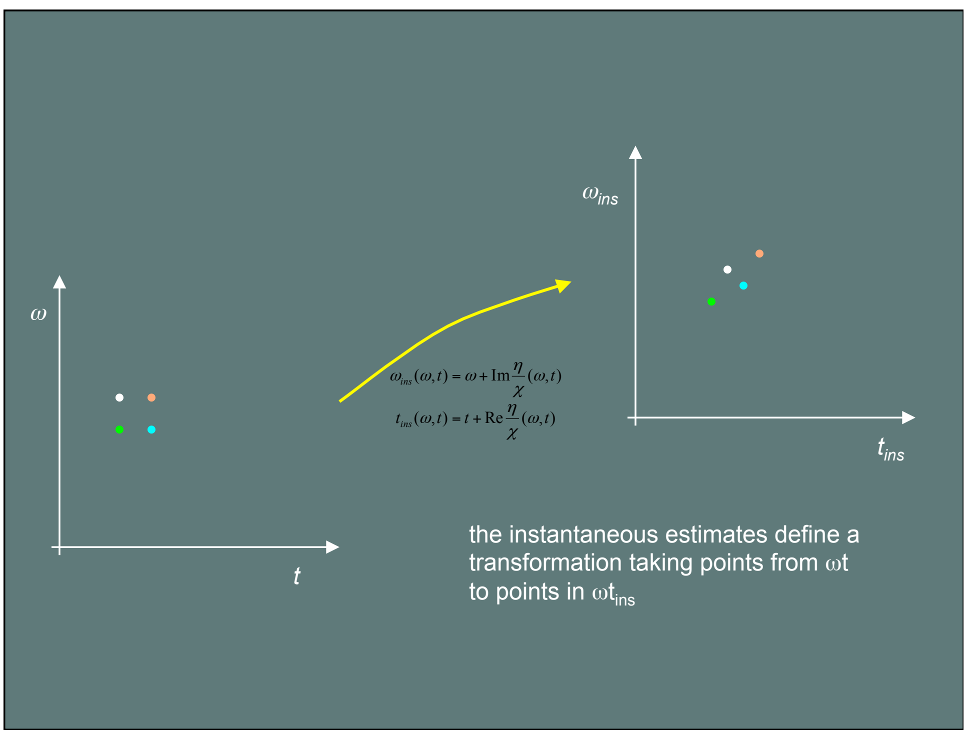
$$\eta(t,\omega) = \int (t'-t) e^{-(t-t')^2/2} e^{iw(t-t')} x(t')\, dt'$$

$$\omega_{ins}(\omega,t) \quad = \quad \partial_t \operatorname{Im}\ln\phi \quad = \quad \omega + \operatorname{Im}\frac{\eta}{\chi}(\omega,t)$$

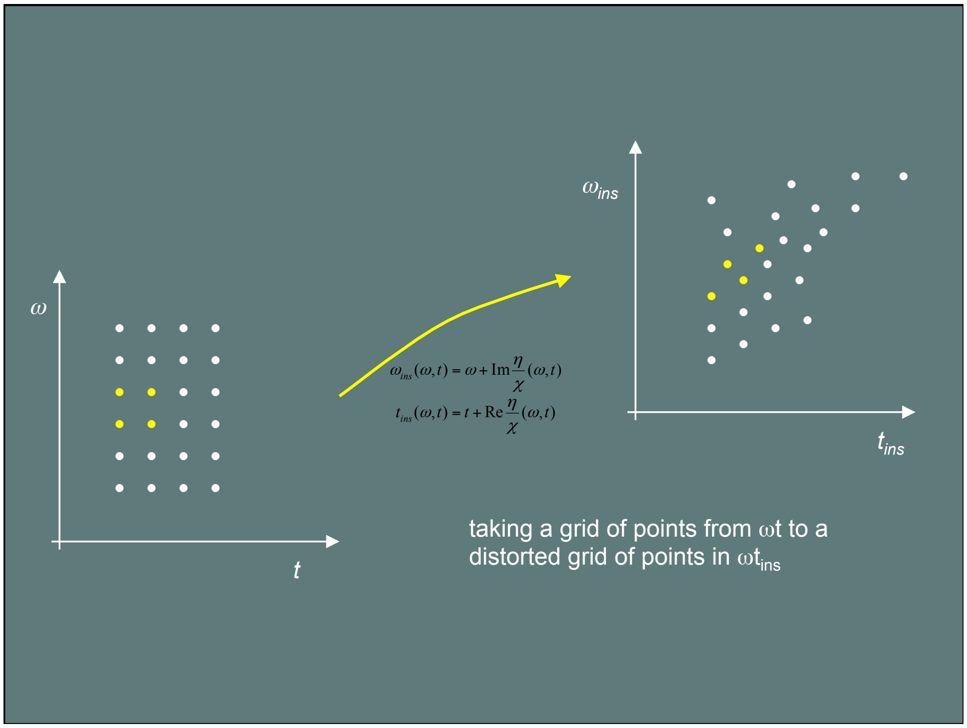
$$t_{ins}(\omega,t) \quad = \quad t - \partial_\omega \operatorname{Im}\ln\phi \quad = \quad t + \operatorname{Re}\frac{\eta}{\chi}(\omega,t)$$

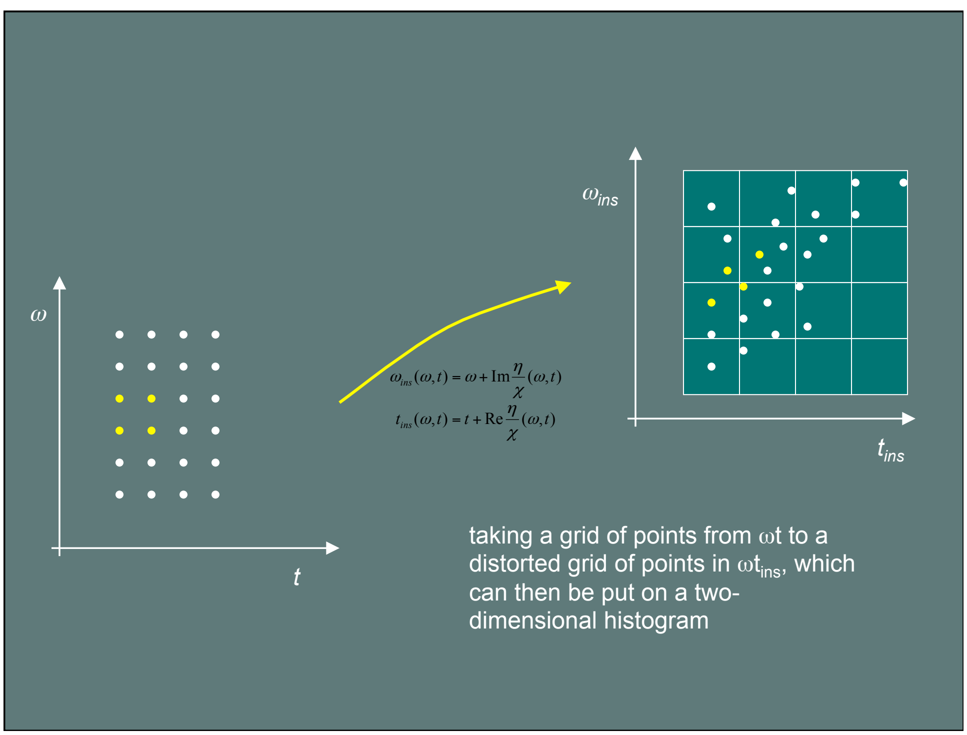


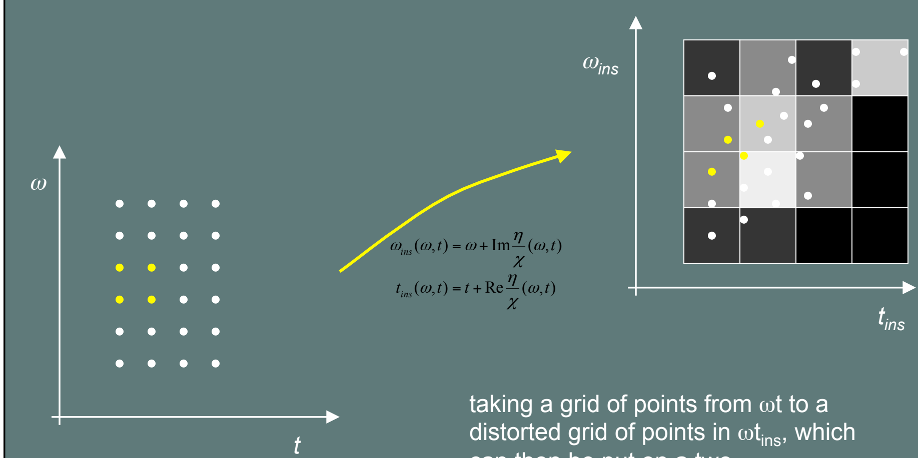
ω_{ins}
ω
$\omega_{ins}(\omega,t) = \omega + \mathrm{Im}\frac{\eta}{\chi}(\omega,t)$
$t_{ins}(\omega,t) = t + \mathrm{Re}\frac{\eta}{\chi}(\omega,t)$
t_{ins}
the instantaneous estimates define a transformation
t



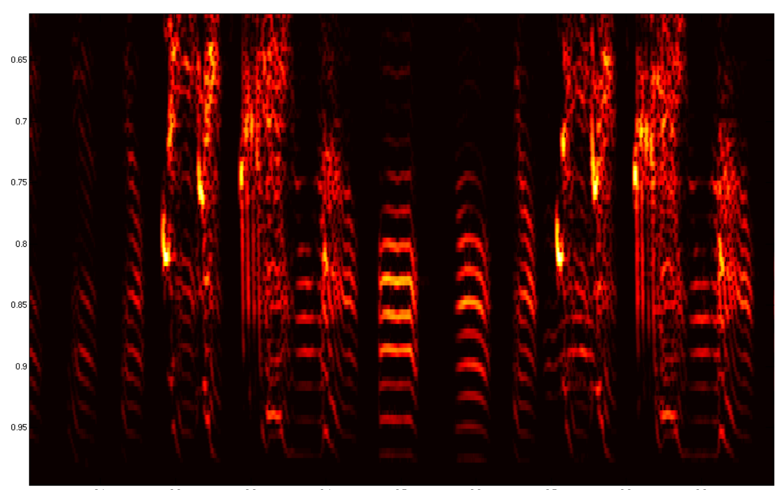
ω_{ins}
ω
$\omega_{ins}(\omega,t) = \omega + \mathrm{Im}\frac{\eta}{\chi}(\omega,t)$
$t_{ins}(\omega,t) = t + \mathrm{Re}\frac{\eta}{\chi}(\omega,t)$
t_{ins}
the instantaneous estimates define a transformation taking points from ωt to points in ωt_{ins}
t


ω
t
ω_{ins}
t_{ins}
$\omega_{ins}(\omega,t)=\omega+\mathrm{Im}\frac{\eta}{\chi}(\omega,t)$
$t_{ins}(\omega,t)=t+\mathrm{Re}\frac{\eta}{\chi}(\omega,t)$
taking a grid of points from ωt to a distorted grid of points in ωt_{ins}

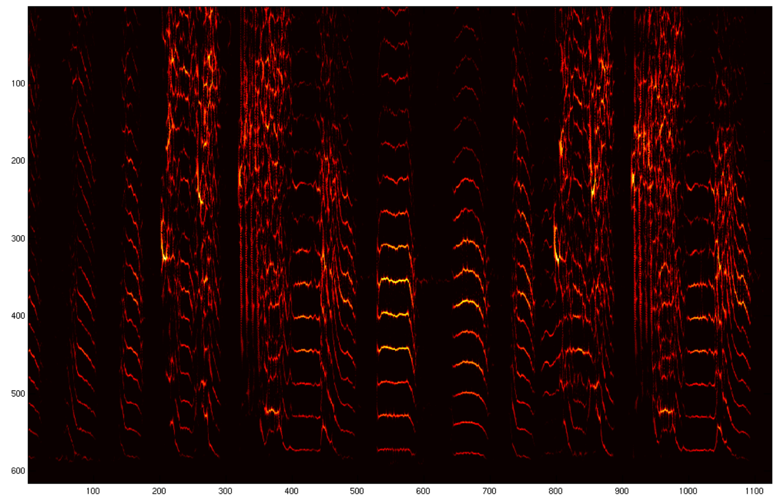

ω
t
ω_{ins}
t_{ins}
$\omega_{ins}(\omega,t)=\omega+\mathrm{Im}\frac{\eta}{\chi}(\omega,t)$
$t_{ins}(\omega,t)=t+\mathrm{Re}\frac{\eta}{\chi}(\omega,t)$
taking a grid of points from ωt to a distorted grid of points in ωt_{ins}, which can then be put on a two-dimensional histogram



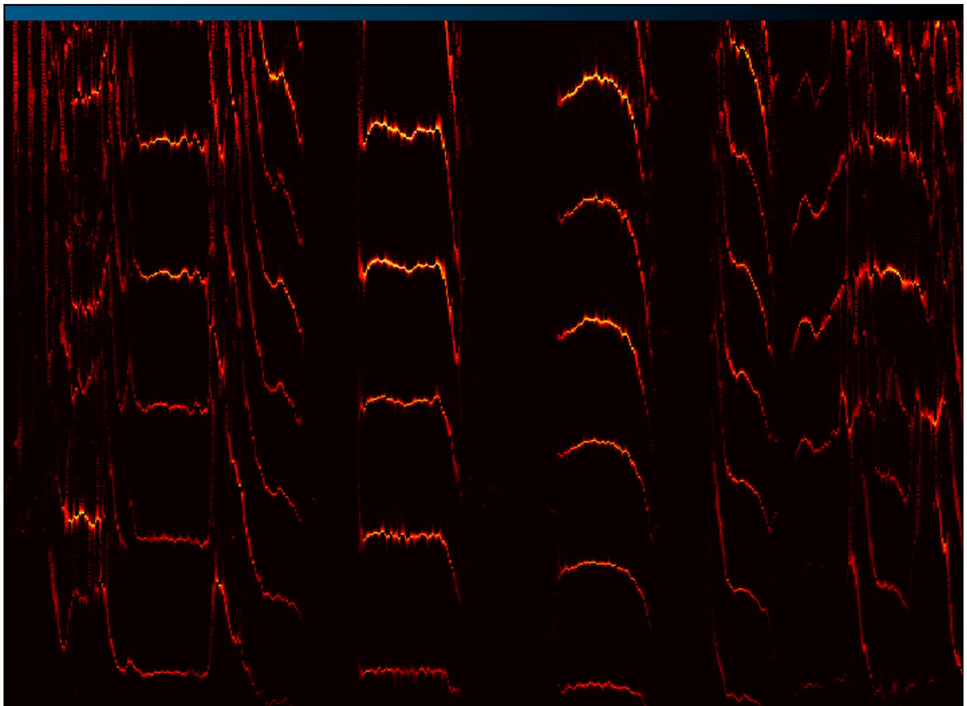
taking a grid of points from ωt to a distorted grid of points in ωt_{ins}, which can then be put on a two-dimensional histogram and counted

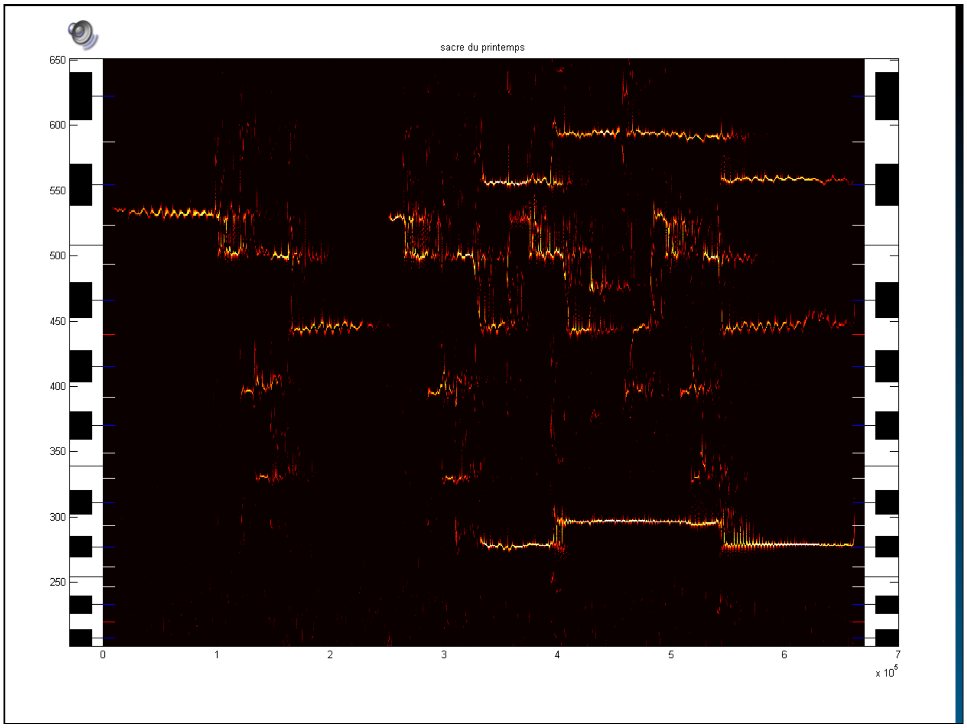


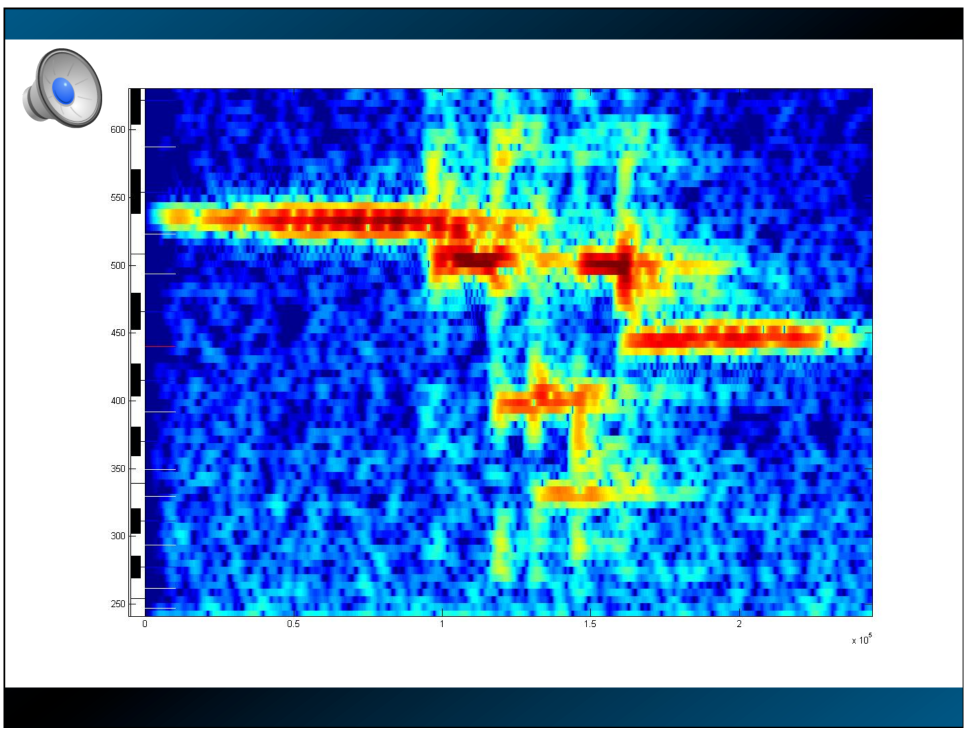
zebra finch, regular sonogram

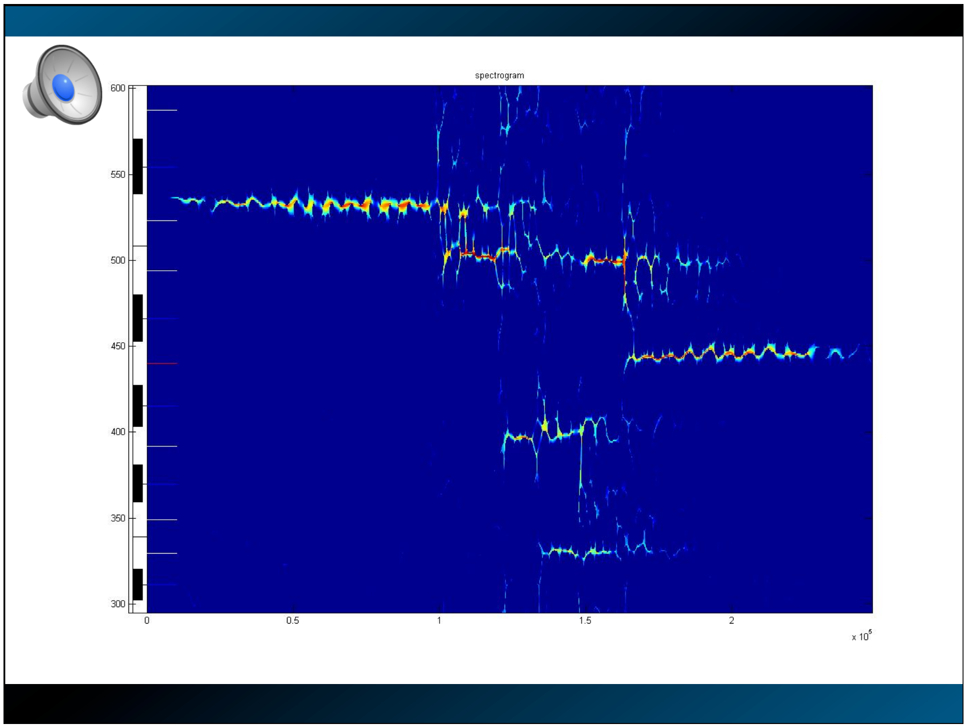


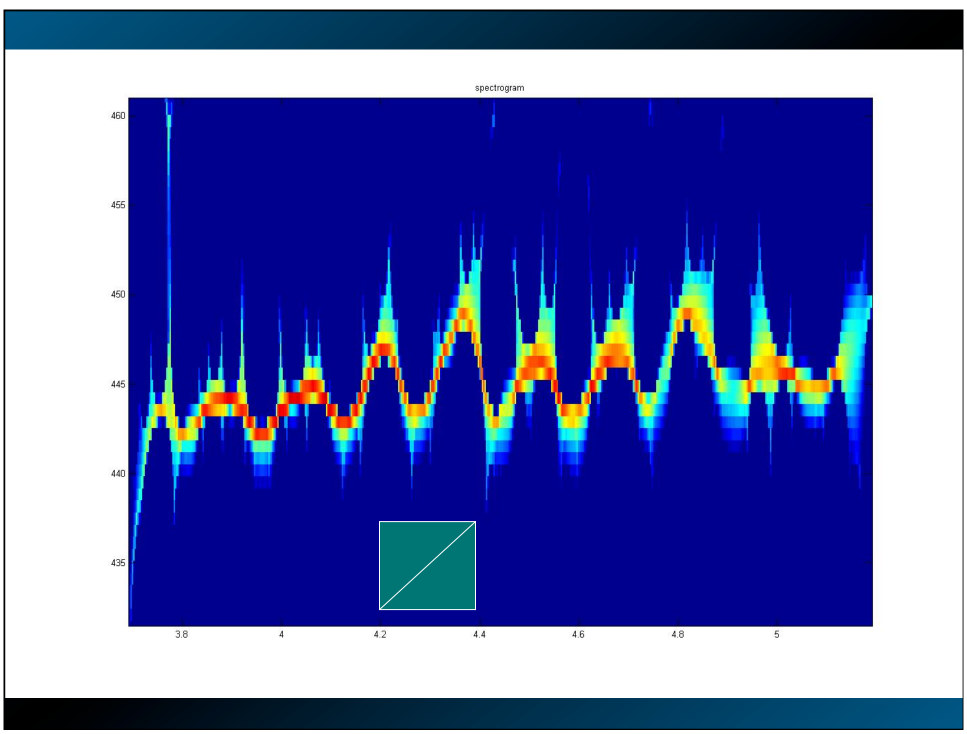
zebra finch, reassigned multiband

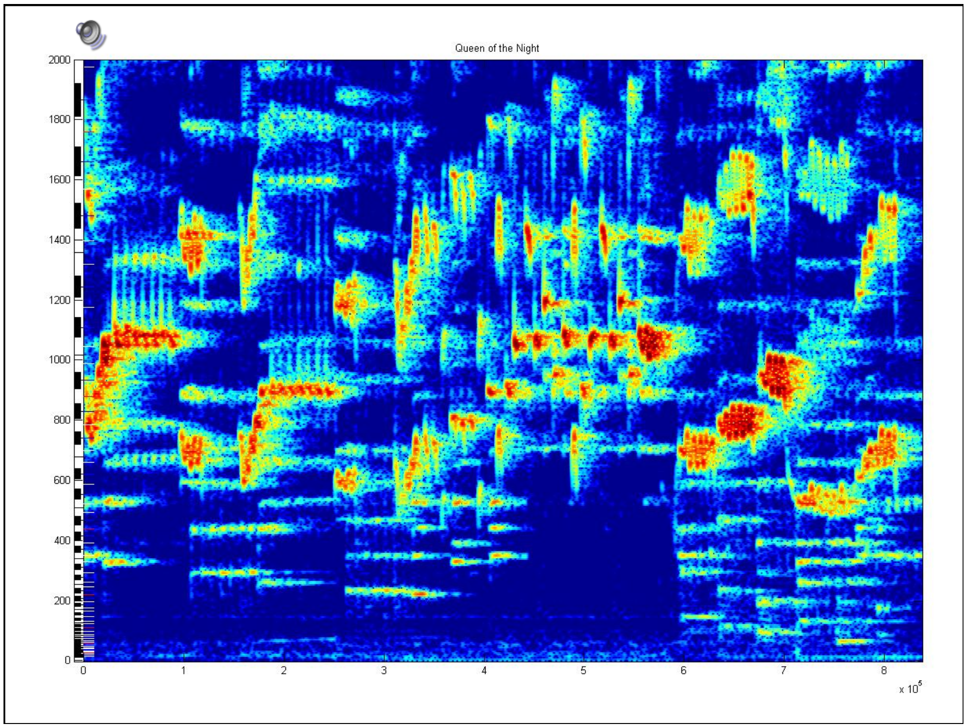


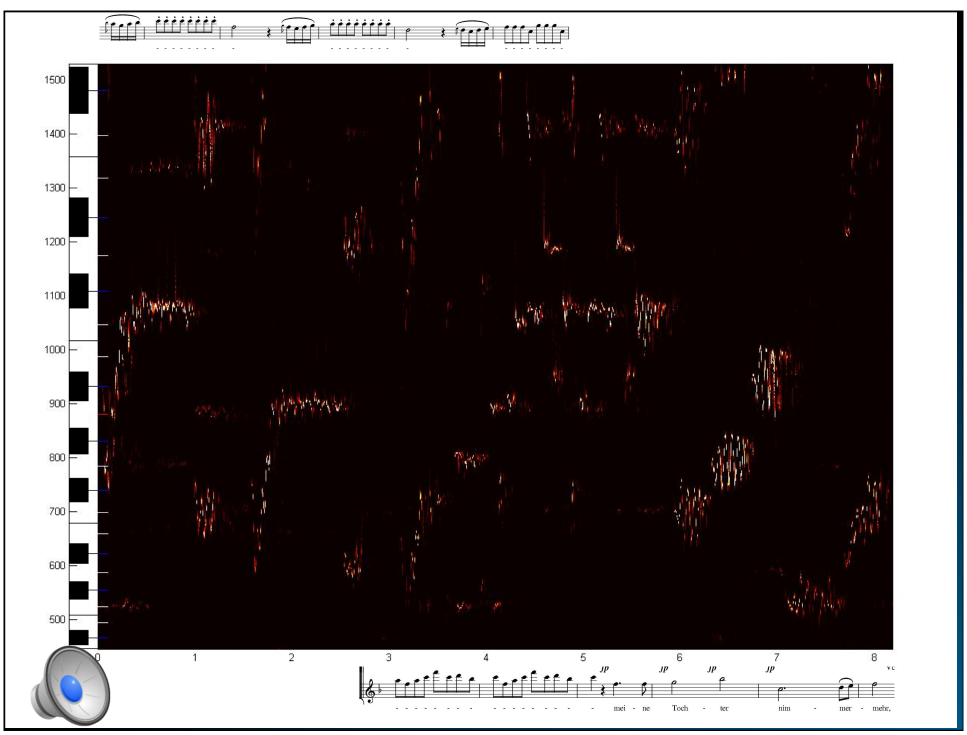

sacre du printemps
650
600
550
500
450
400
350
300
250
0
1
2
3
4
5
6
7
x 10^5

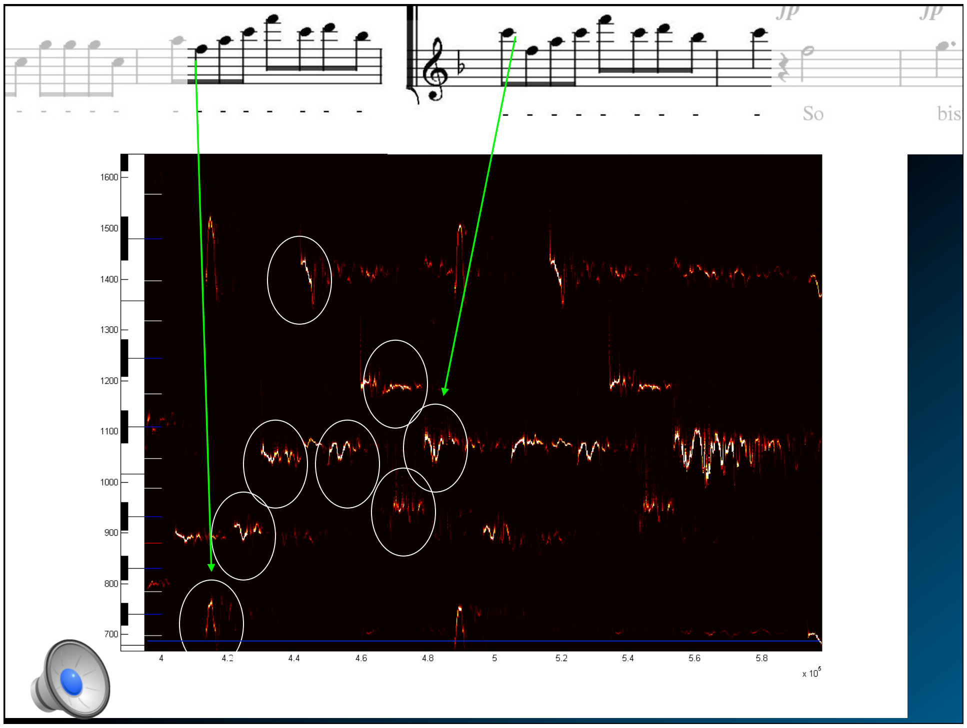

600
550
500
450
400
350
300
250
0
0.5
1
1.5
2
x 10^5

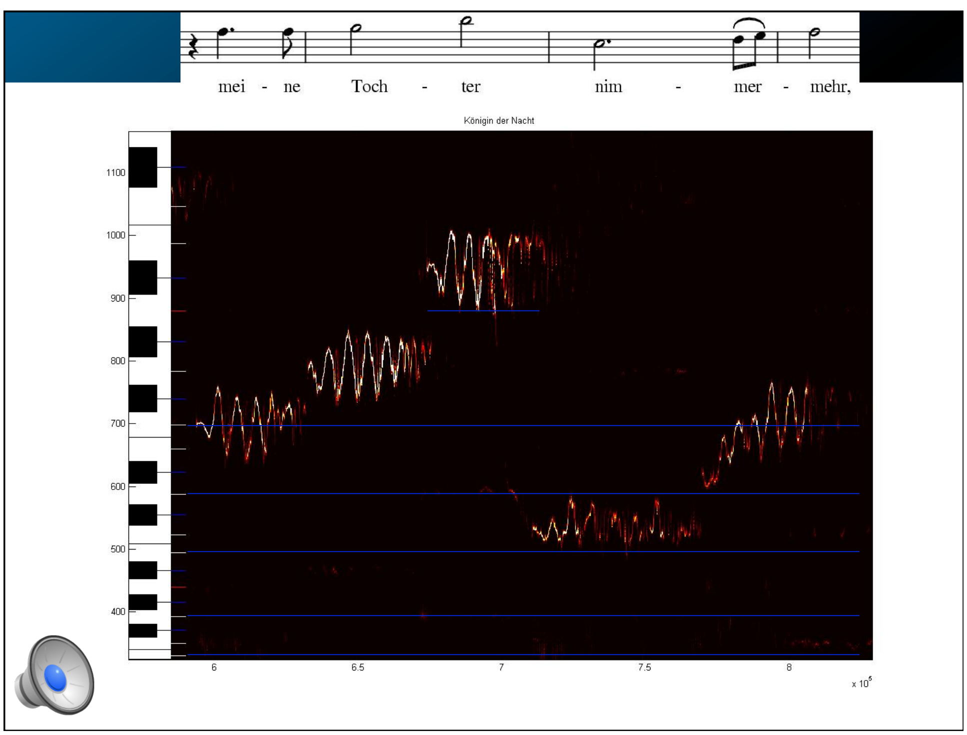

spectrogram
600
550
500
450
400
350
300
0
0.5
1
1.5
2
x 10^5

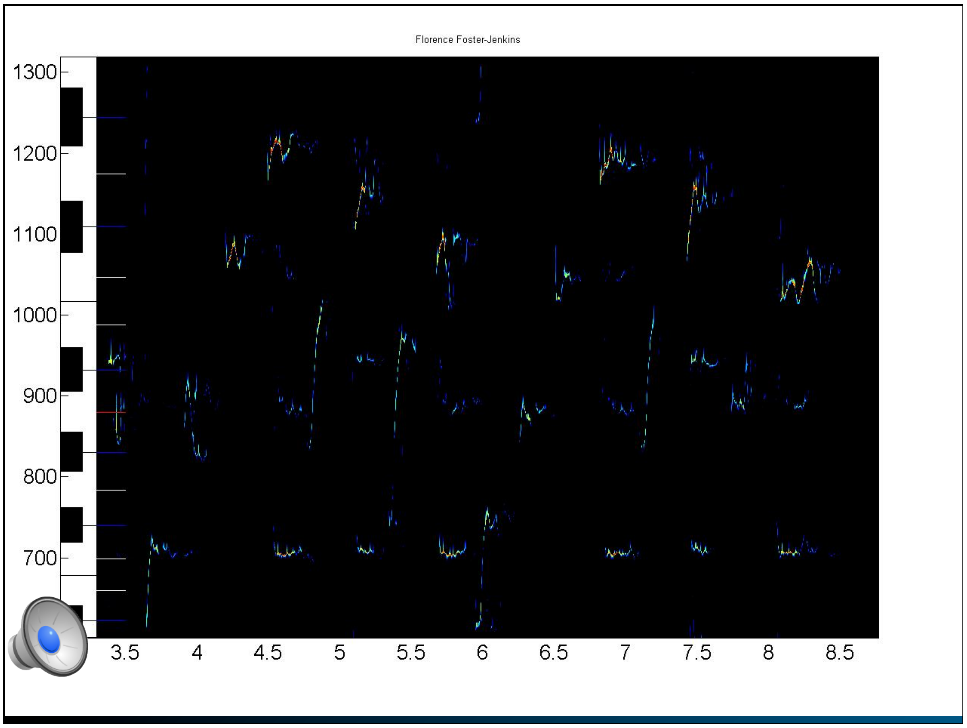

spectrogram
460
455
450
445
440
435
3.8
4
4.2
4.4
4.6
4.8
5

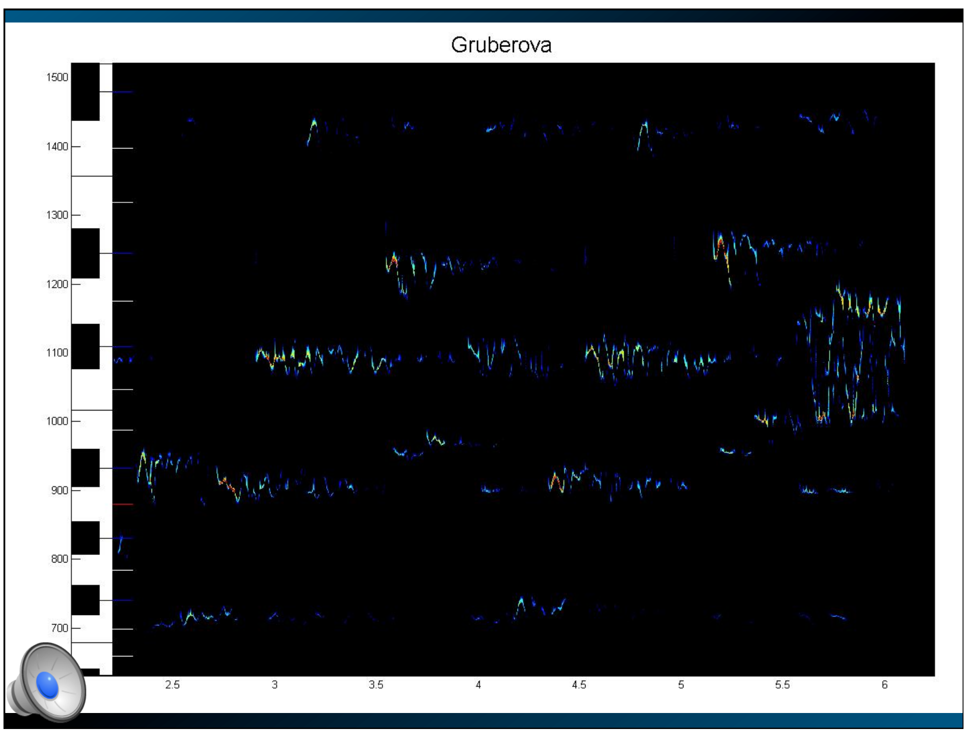

Queen of the Night
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
7
8
x 10^5


1500
1400
1300
1200
1100
1000
900
800
700
600
500
0
1
2
3
4
5
6
7
8
mei - ne Toch - ter nim - mer - mehr,


So
bis


mei - ne Toch - ter nim - mer - mehr,
Königin der Nacht


Florence Foster-Jenkins
1300
1200
1100
1000
900
800
700
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5


Gruberova
1500
1400
1300
1200
1100
1000
900
800
700
2.5
3
3.5
4
4.5
5
5.5
6

- there's a lot of auditory acuity that cannot be explained through most extant methods
- we do not understand the geometry of the space of sounds

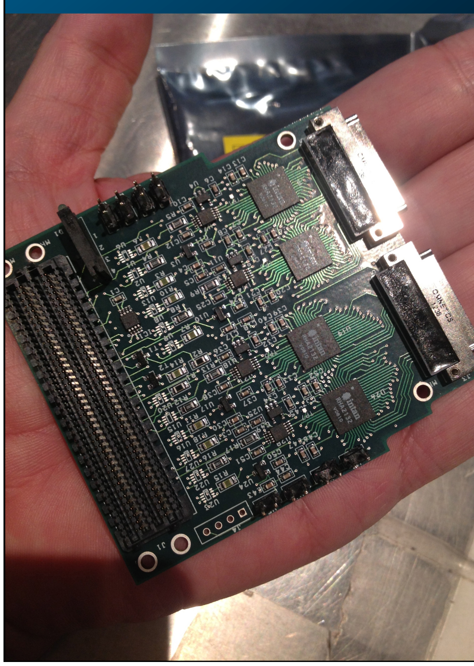
fast fourier transform algorithm and algorithmic complexity

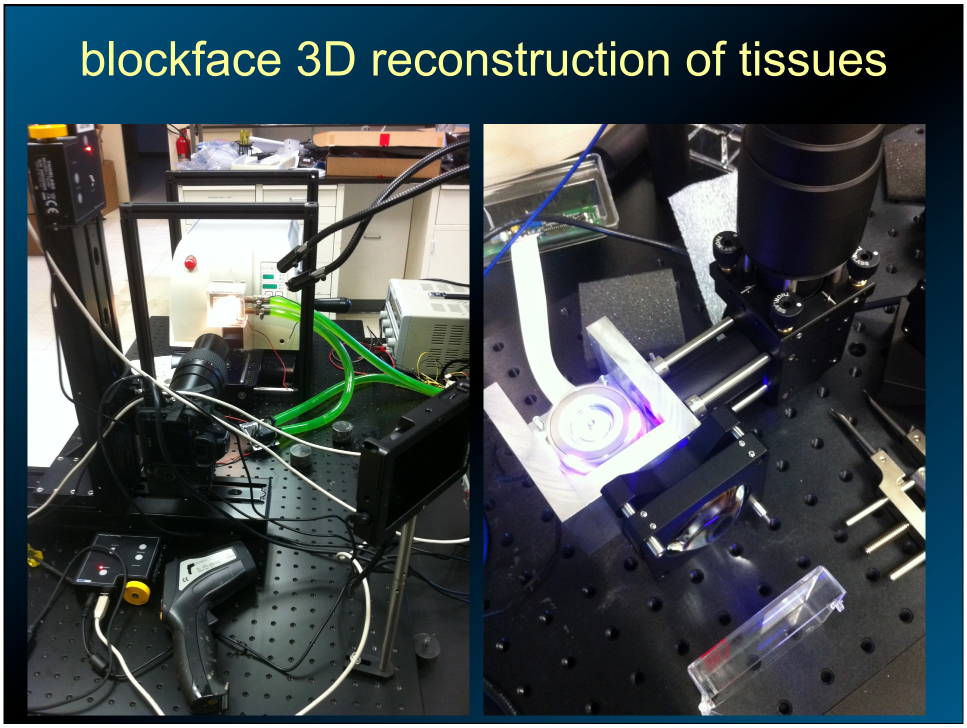
- divide and conquer: heapsort, quicksort; Cooley and Tukey
- time to do convolution, time to do an arbitrary precision multiplication.
- FFTW

HiCCE

(pronounced “Hickey”)
High Channel Count Electrophysiology.

http://www.ohwr.org/projects/hicce-fmc-128




blockface 3D reconstruction of tissues