

2384-2

**ICTP Latin-American Advanced Course on FPGA Design for Scientific
Instrumentation**

19 November - 7 December, 2012

**Digital Design
(Part 2)**

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FRANCE*

Outline

□ Digital CMOS design

- Boolean algebra

- Basic digital CMOS gates

- **Combinational and sequential circuits**

- Coding - Representation of numbers



CMOS Circuits

How to implement Boolean functions
in CMOS technology ?

- A complex function cannot be implemented using a single gate
- Use a network of gates

Boolean network



CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$$f = \bar{x}.y.z + x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z$$

$$f = (x+y+z) . (x+y+\bar{z}) .$$

$$(x+\bar{y}+z) . (\bar{x}+\bar{y}+z)$$

$$f = x.(yz+\bar{y}) + \bar{x}.(y.z)$$

$$f = \bar{x}.y.z + x.\bar{y}.\bar{z} + x.z$$

$$f = x.\bar{y} + y.z$$

There is not a unique expression



CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Which gates should I use ?

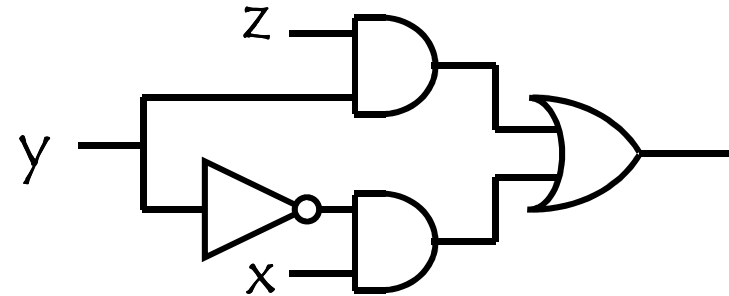
Cell library

$$f = x.\bar{y} + y.z$$

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



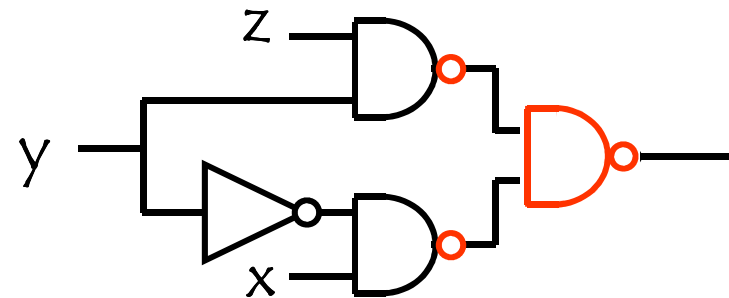
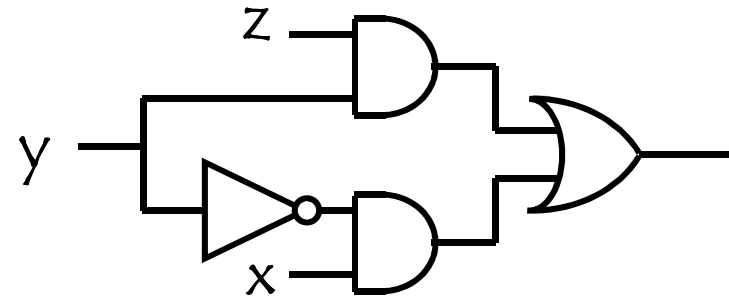
Non-inverting gates do NOT exist

$$f = x \cdot \bar{y} + y \cdot z$$

CMOS Circuits

Example :

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



CMOS Circuits

Example :

$$f = (\bar{x} + y) \oplus x$$

$$f = \overline{(\bar{x} + y)} \oplus \bar{x}$$

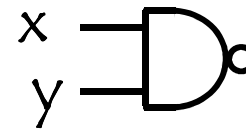
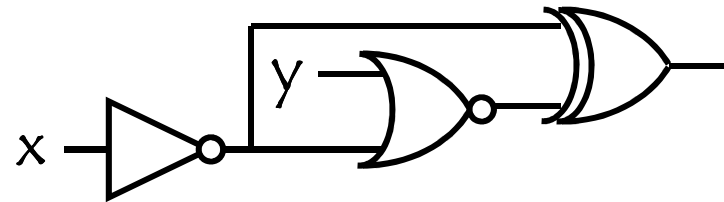
$$f = (\bar{x} + y) \cdot \bar{x} + \overline{(\bar{x} + y)} \cdot x$$

$$f = \bar{x} + y \cdot \bar{x} + x \cdot \bar{y}$$

$$f = \bar{x} + x \cdot \bar{y}$$

$$f = \bar{x} + \bar{y}$$

$$f = \overline{x \cdot y}$$



CMOS Circuits

How to implement Boolean functions
with a gate network ?

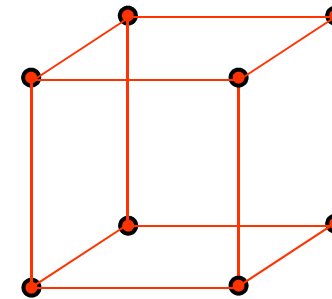
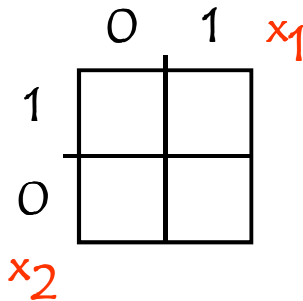
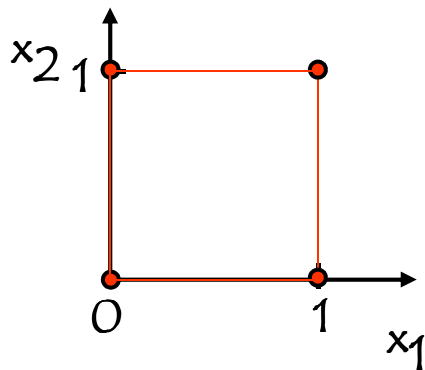
- Which expression ?

CMOS Circuits

A function can be defined by its Truth table

- Karnaugh representation gives a minimal expression

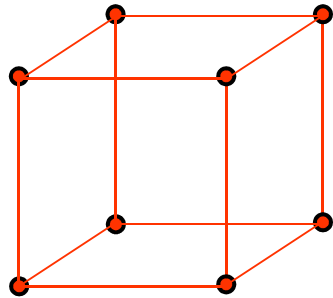
Representation of the function in a space of dimension n



Representation of vectors' adjacency

CMOS Circuits

A Karnaugh table is represented in a flat 2-dimension table



	00	01	11	10
0		○		○
1				

The table shows a 2x4 grid of cells. The top row is labeled with binary vectors 00, 01, 11, and 10. The left column is labeled with 0 and 1. The cell at (0, 01) contains a small circle with a red line extending downwards to the cell at (1, 01). The cell at (0, 10) contains a small circle with a blue line extending downwards to the cell at (1, 10). A blue oval highlights the top row, and a blue line connects the circles in the 0 row to the circles in the 1 row.

vectors' adjacency

CMOS Circuits

Example :

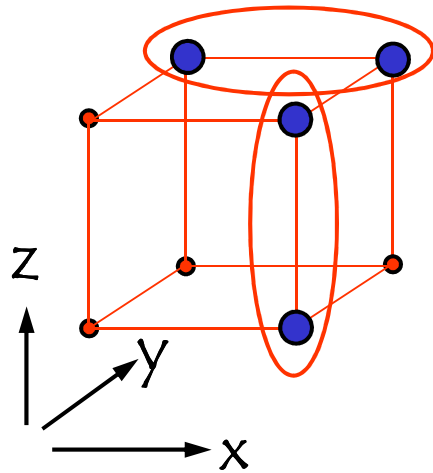
x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

	00	01	11	10	xy
0	0	0	0	1	
1	0	1	1	1	

z

CMOS Circuits

Example :



	00	01	11	10	xy
0	0	0	0	1	
1	0	1	1	1	
z					

$$f = x.\bar{y} + yz$$

CMOS Circuits

Example :

	00	01	11	10	xy
00	0	1	1	1	
01	0	1	1	0	
11	0	1	0	0	
10	0	1	1	1	

zV

$$\bar{x}.y + x.\bar{v} + y.\bar{z}$$

CMOS Circuits

Example :

A Karnaugh map for a function g(x,y,z) with variables x, y, and z. The map is a 2x8 grid. The columns are labeled with binary values 000, 001, 011, 010, 110, 111, 101, and 100. The rows are labeled 0 and 1. The cells contain 0 or 1. Red circles highlight the 1s in the following cells: (0,001), (0,101), (1,001), (1,011), (1,010), (1,110), and (1,101). Blue arrows show groupings: one from (0,001) to (0,011), one from (0,001) to (0,101), and one from (0,001) to (0,010). Red arrows point from the circled 1s to the Boolean expression below.

	000	001	011	010	110	111	101	100	xyv
0	0	1	0	0	0	0	1	0	
1	0	1	1	1	1	0	1	0	

z

$$g = \bar{x}.y.z + y.\bar{v}.z + \bar{y}.v$$

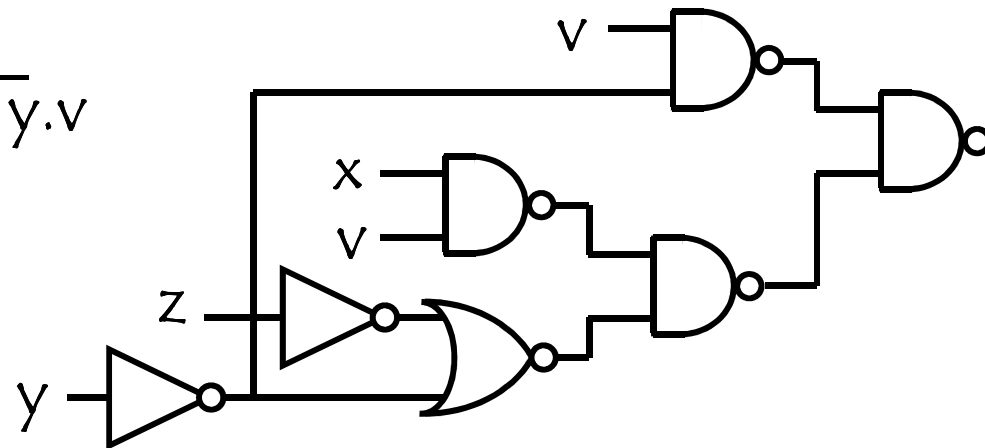
CMOS Circuits

Example :

$$g = \bar{x}.y.z + y.\bar{v}.z + \bar{y}.v$$

$$g = y.z.(\bar{x}+\bar{v}) + \bar{y}.v$$

$$g = \overline{(\bar{y}+\bar{z})}.(\bar{x}.v) + \bar{y}.v$$



CMOS Circuits

How to implement Boolean functions
with a gate network ?

- Local optimization using Karnaugh tables

A design includes several Boolean functions

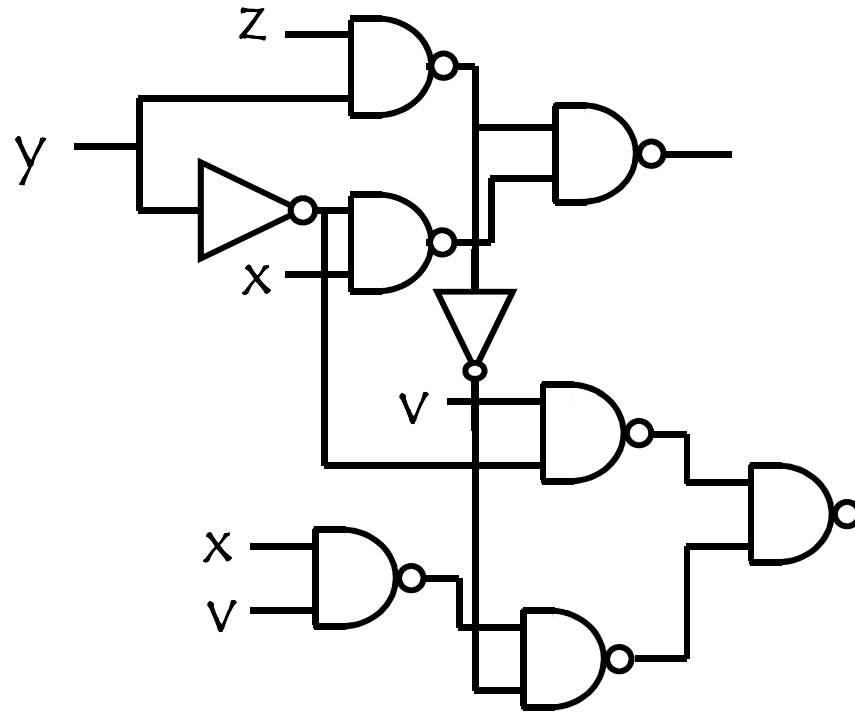


CMOS Circuits

Example :

$$g = \overline{\overline{\overline{y+z}} \cdot \overline{x \cdot v}} + \overline{y} \cdot v$$

$$f = x \cdot \overline{y} + y \cdot z$$



CMOS Circuits

How to implement Boolean functions
with a gate network ?

- Local optimization using Karnaugh tables

A design includes several Boolean functions

- Global optimization by sharing sub functions



Synthesis Tool

CMOS Circuits

- **Combinational logic**

The value of the output can be determined knowing the value of the inputs

- **Sequential logic**

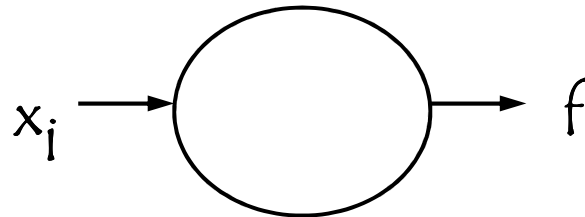
The value of the output depends on the value of the inputs **and the history**

Notion of memory



CMOS Circuits

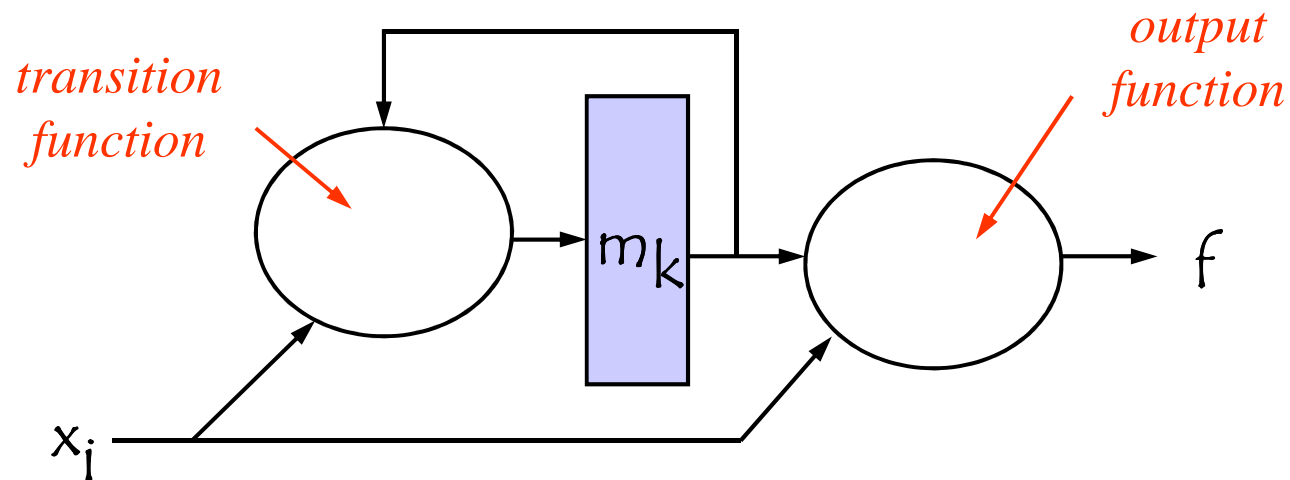
Sequential logic



$$f(x_1, \dots, x_i, \dots, x_n, m_1, \dots, m_k, \dots, m_p)$$
$$m_k(x_1, \dots, x_i, \dots, x_n, m_1, \dots, m_k, \dots, m_p)$$

CMOS Circuits

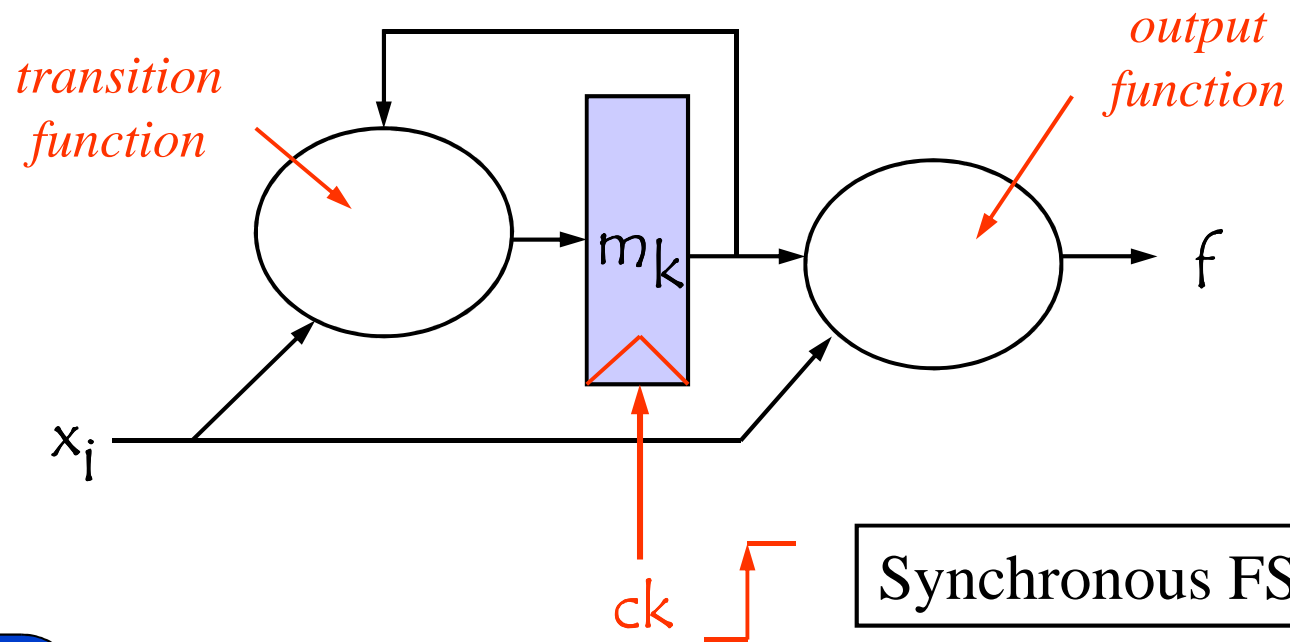
Sequential logic



Finite State Machine (FSM)

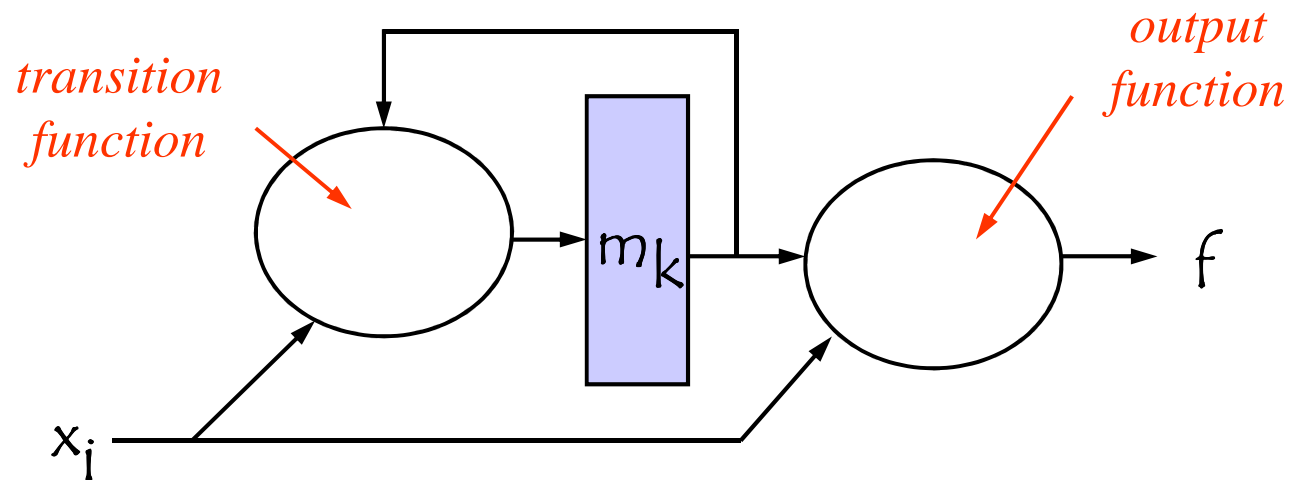
CMOS Circuits

Finite State Machine



CMOS Circuits

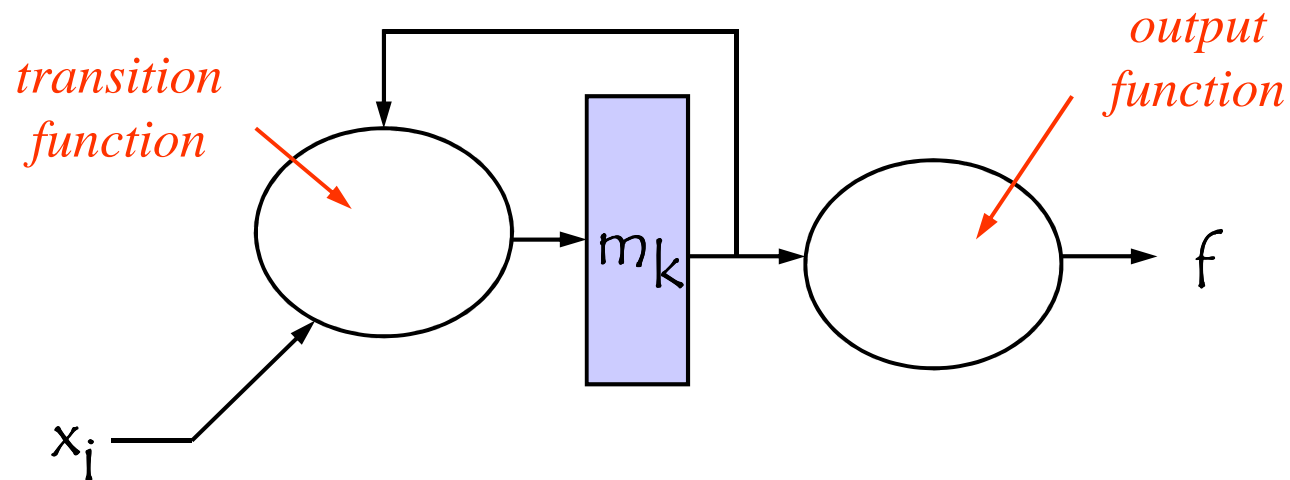
Finite State Machine



Mealy FSM

CMOS Circuits

Finite State Machine



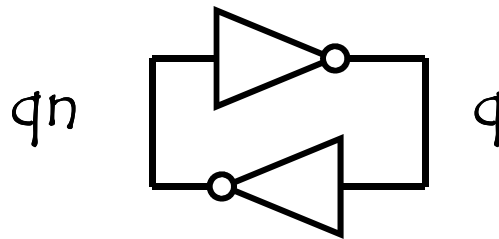
Moore FSM

CMOS Circuits

Memory :

Hold a data (0 or 1)

Write a data (0 or 1)

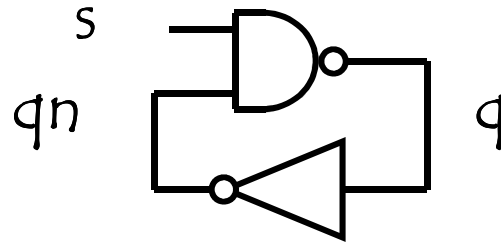


CMOS Circuits

Memory :

Hold a data (0 or 1)

Write a data (0 or 1)



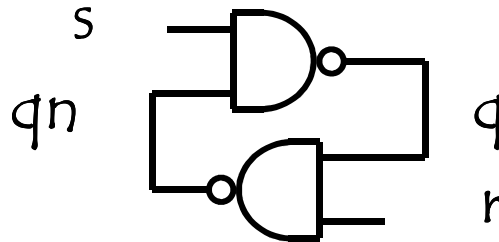
s	q	qn
0	1	0
1	q	qn

CMOS Circuits

Memory :

Hold a data (0 or 1)

Write a data (0 or 1)



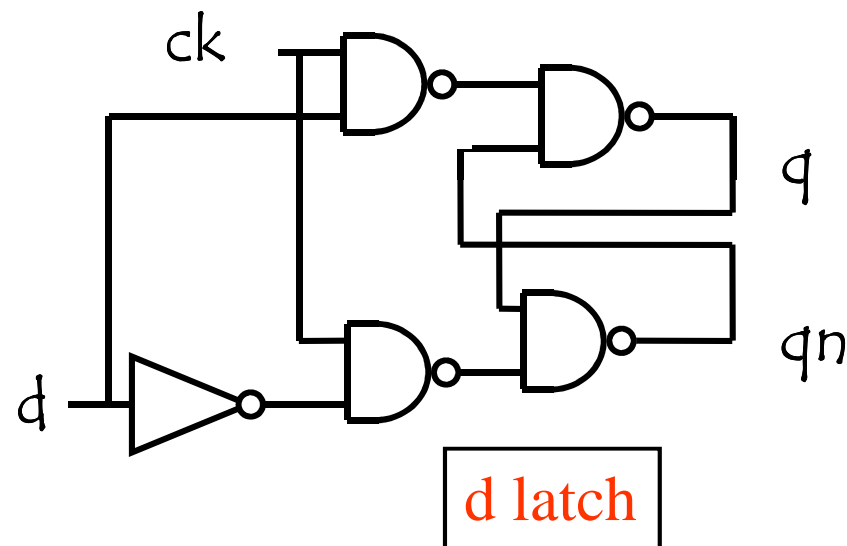
s	r	q	qn
0	1	1	0
1	0	0	1
1	1	q	qn
0	0	1	1

RS flip flop

CMOS Circuits

Synchronous Memory :

Write a data d when the clock $ck = 1$



s	r	q	qn
0	1	1	0
1	0	0	1
1	1	q	qn
0	0	1	1

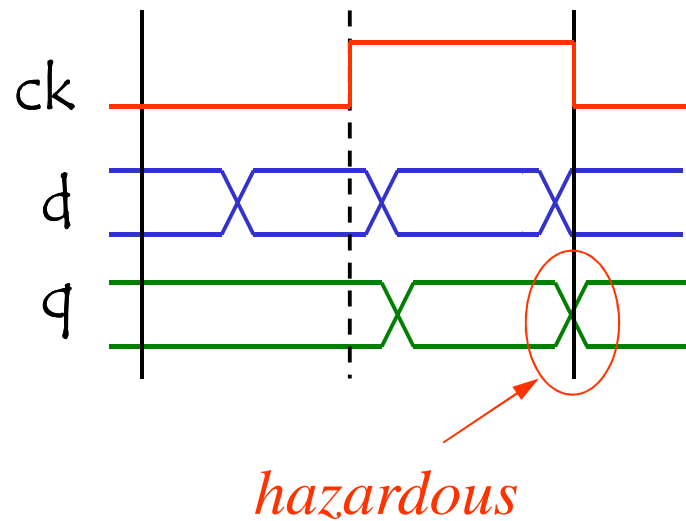
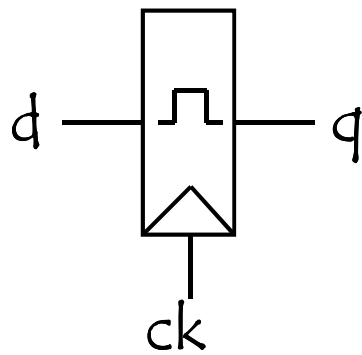
if $ck \cdot d = 1$ $s = 0$

if $ck \cdot \bar{d} = 1$ $r = 0$

CMOS Circuits

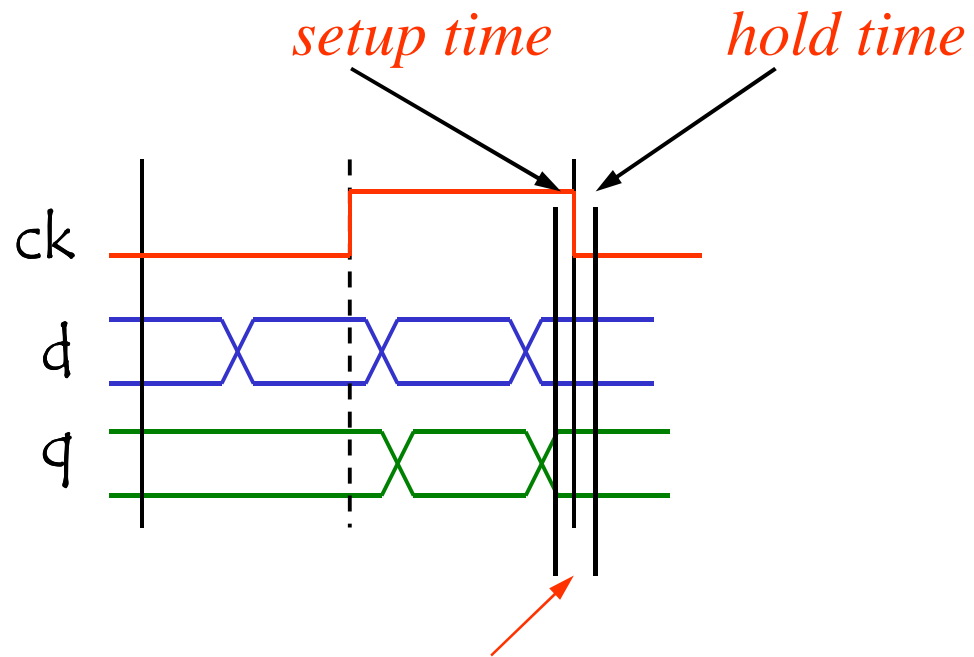
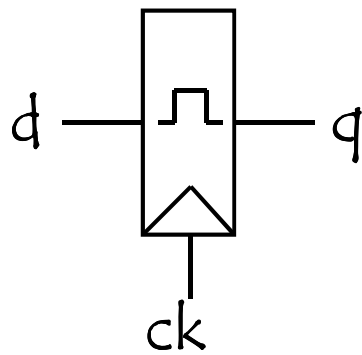
Synchronous Memory :

Write a data d when the clock $ck = 1$



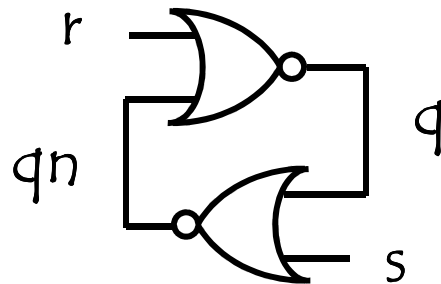
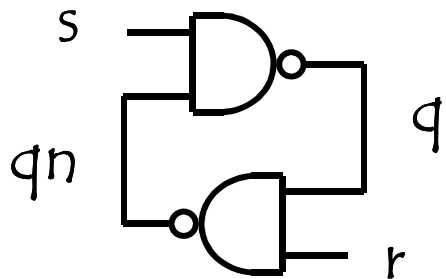
CMOS Circuits

Synchronous Memory :



CMOS Circuits

Memory :



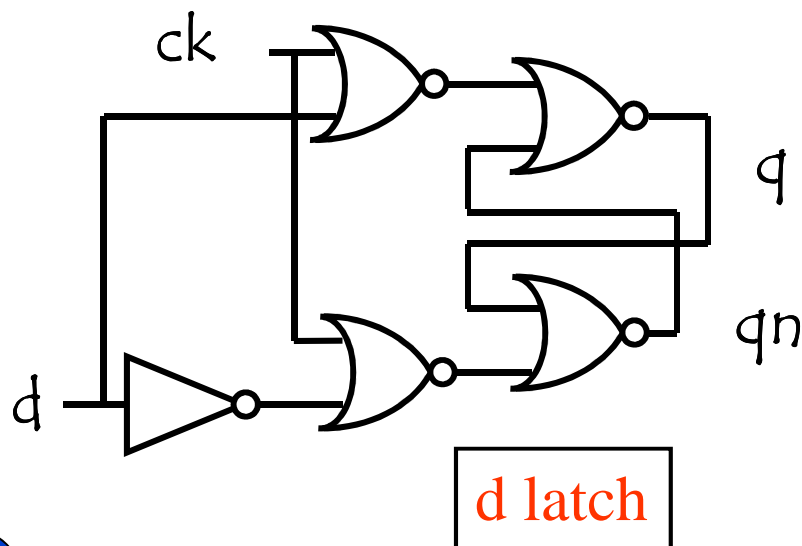
s	r	q	qn
0	1	0	1
1	0	1	0
1	1	0	0
0	0	q	qn

RS flip flop

CMOS Circuits

Synchronous Memory :

Write a data q when the clock $ck = 0$



s	r	q	qn
0	1	0	1
1	0	1	0
1	1	0	0
0	0	q	qn

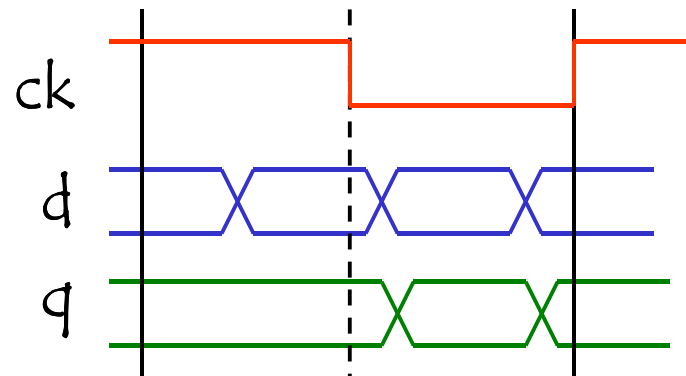
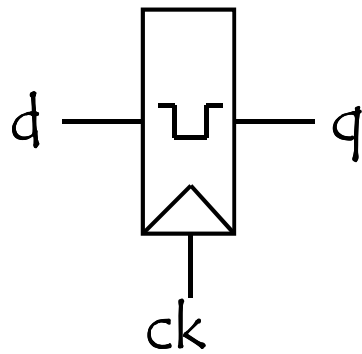
if $\overline{ck}.d = 1$ $s = 1$

if $\overline{ck}.\overline{d} = 1$ $r = 1$

CMOS Circuits

Synchronous Memory :

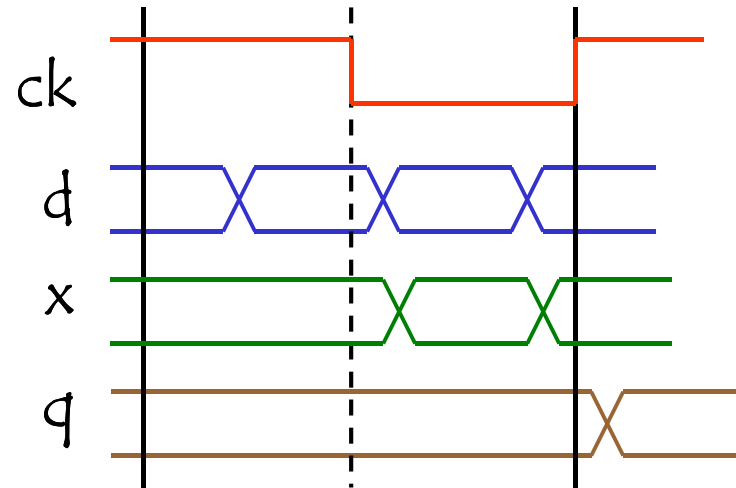
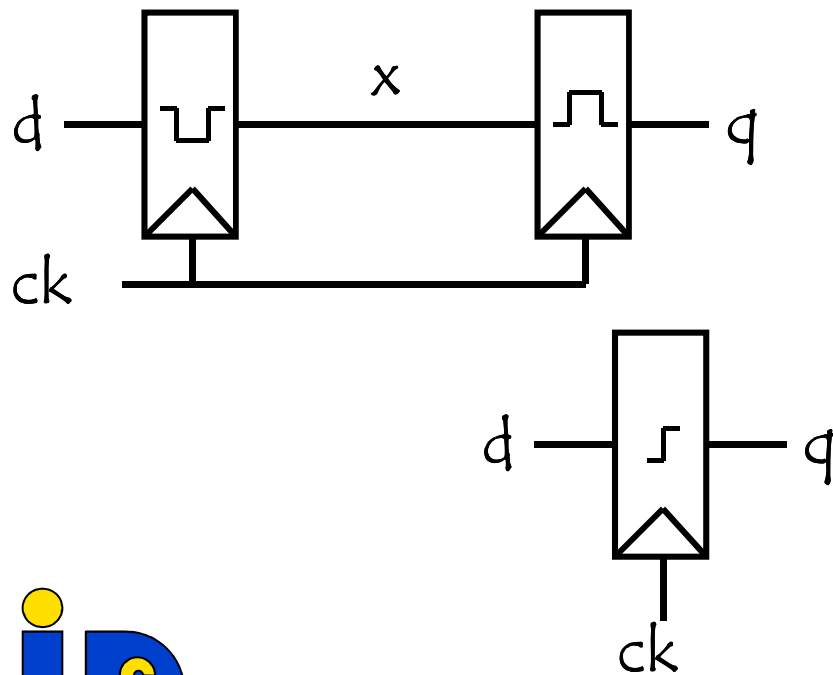
Write a data d when the clock $ck = 0$



CMOS Circuits

Synchronous Memory :

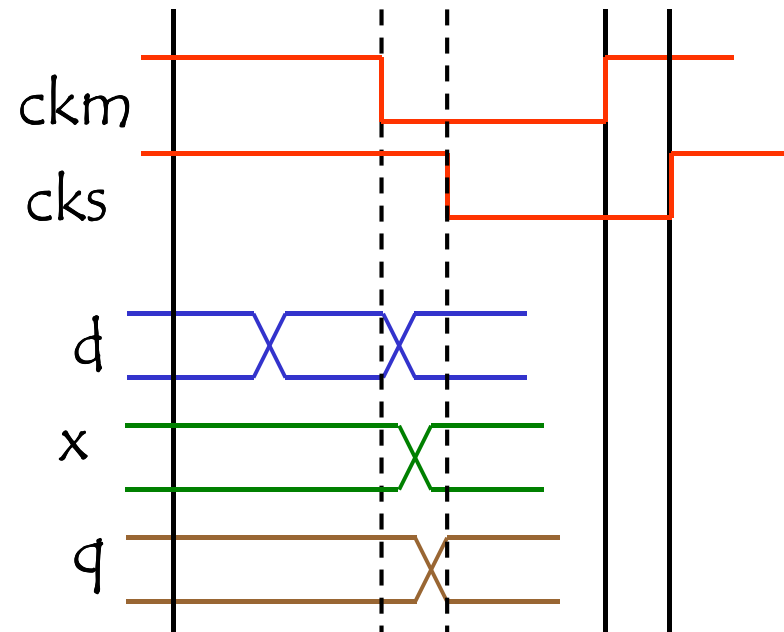
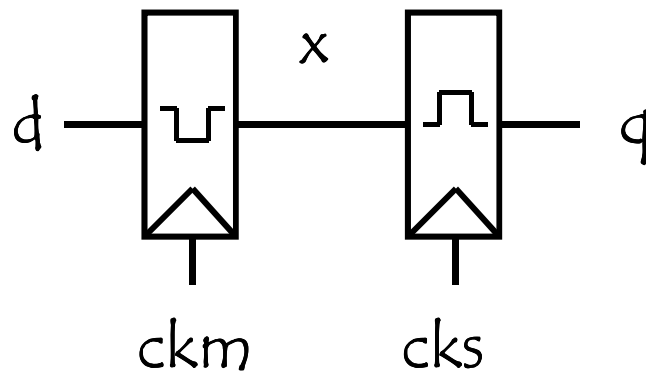
Write a data d on the rising edge of the clock ck



CMOS Circuits

Synchronous Memory :

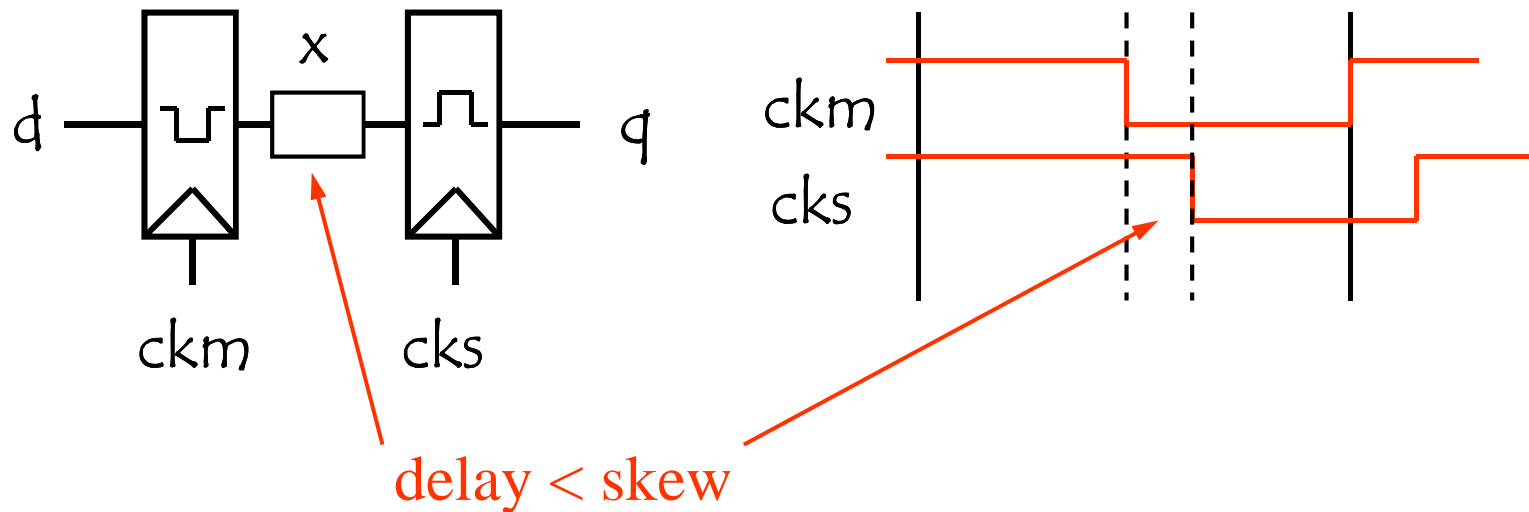
Write a data d on the rising edge of the clock ck



CMOS Circuits

Synchronous Memory :

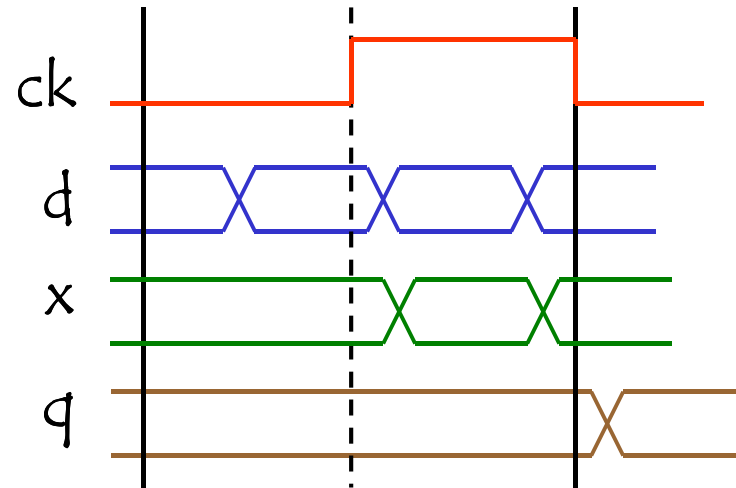
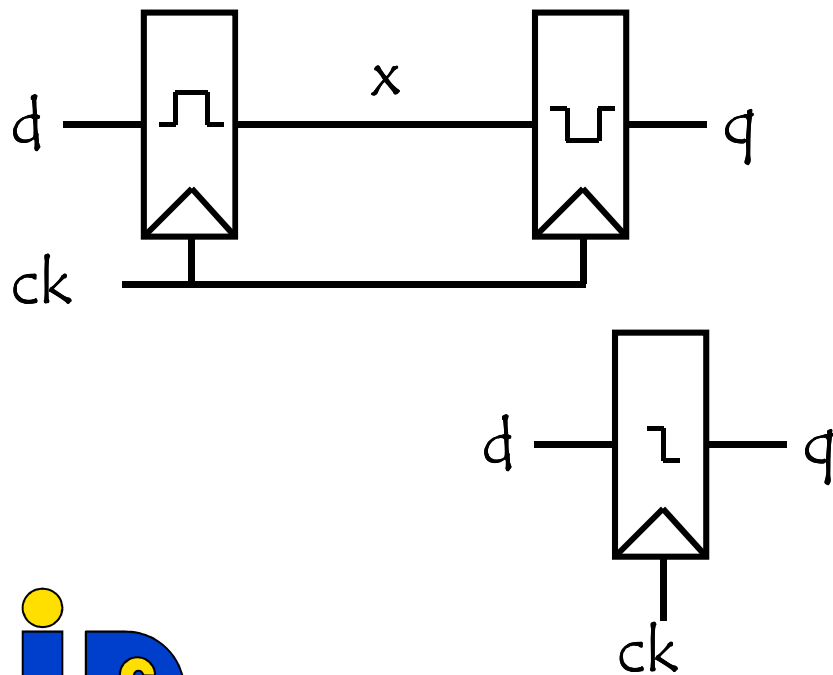
Write a data d on the rising edge of the clock ck



CMOS Circuits

Synchronous Memory :

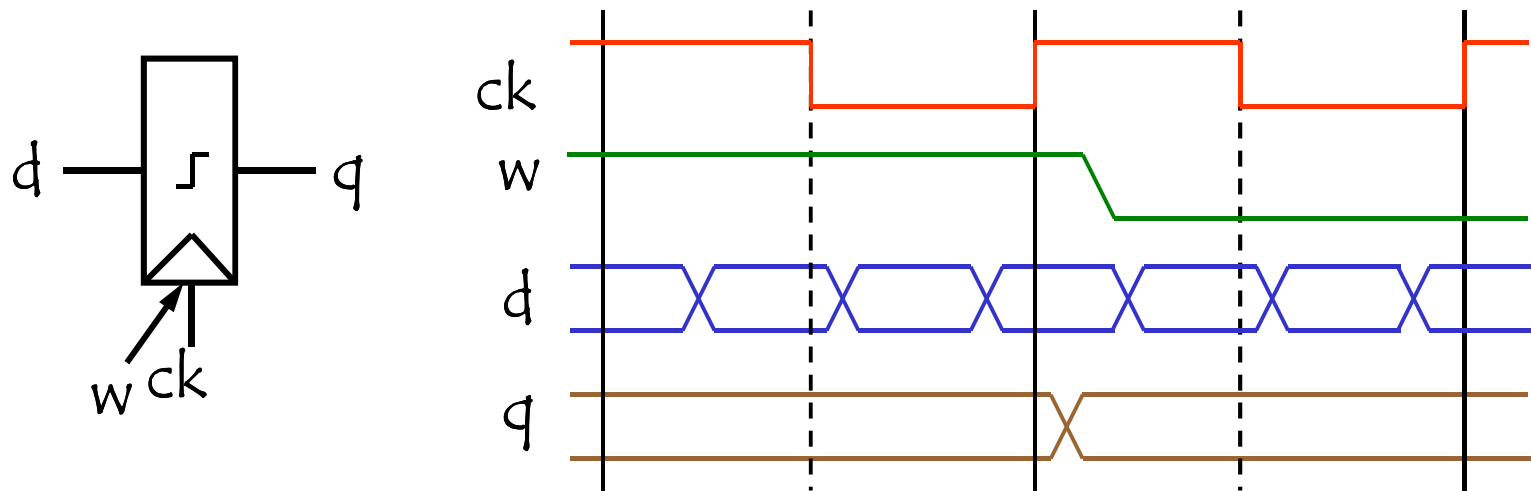
Write a data d on the falling edge of the clock ck



CMOS Circuits

Synchronous Memory :

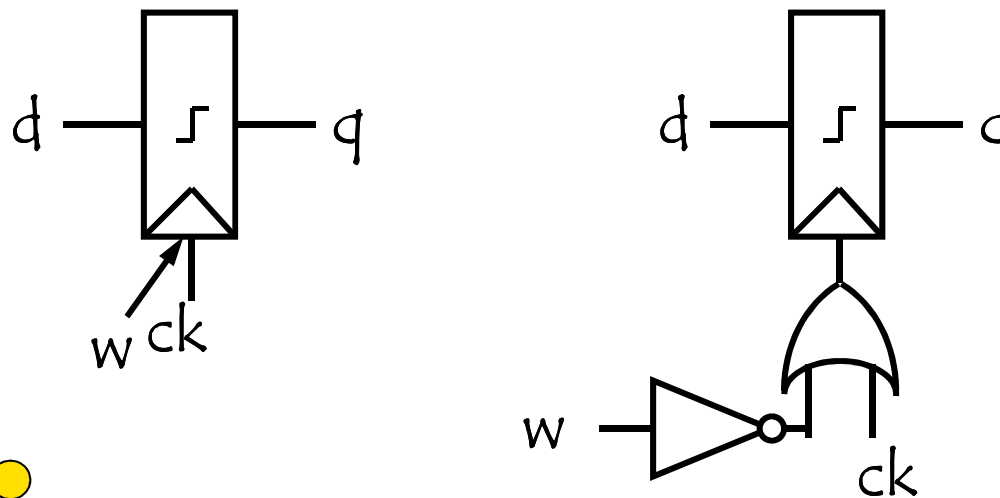
Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

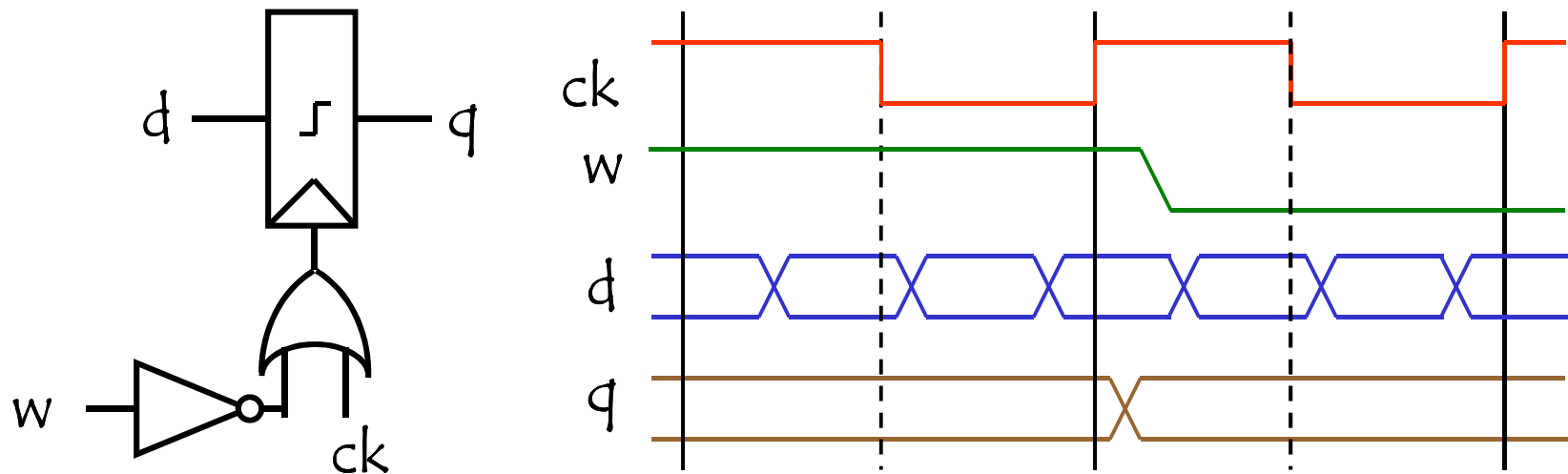
Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

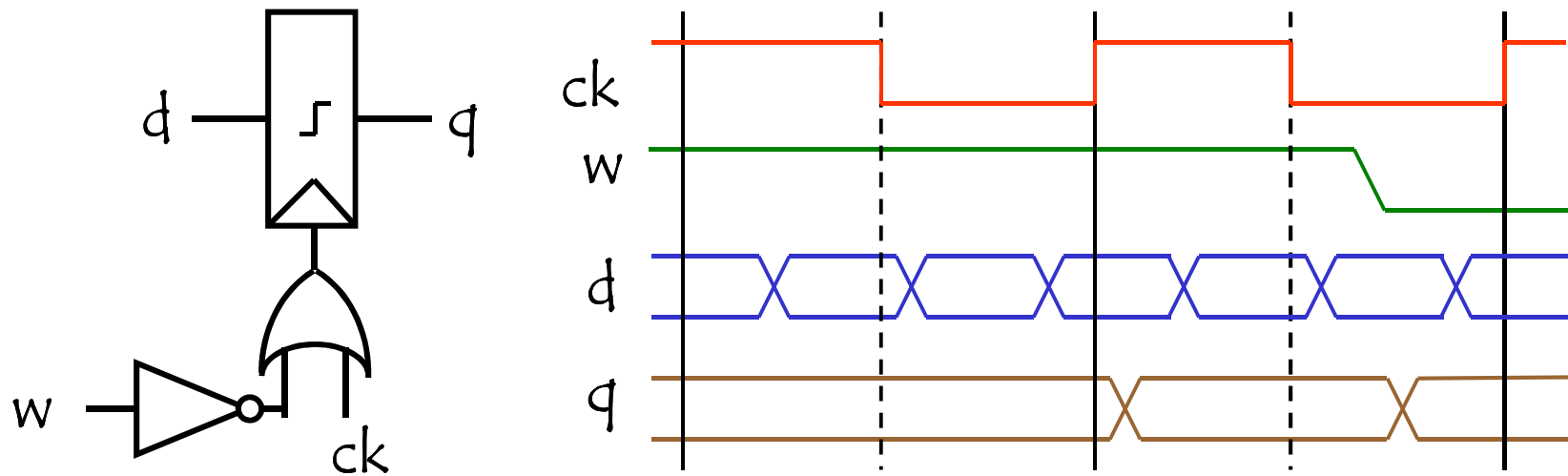
Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

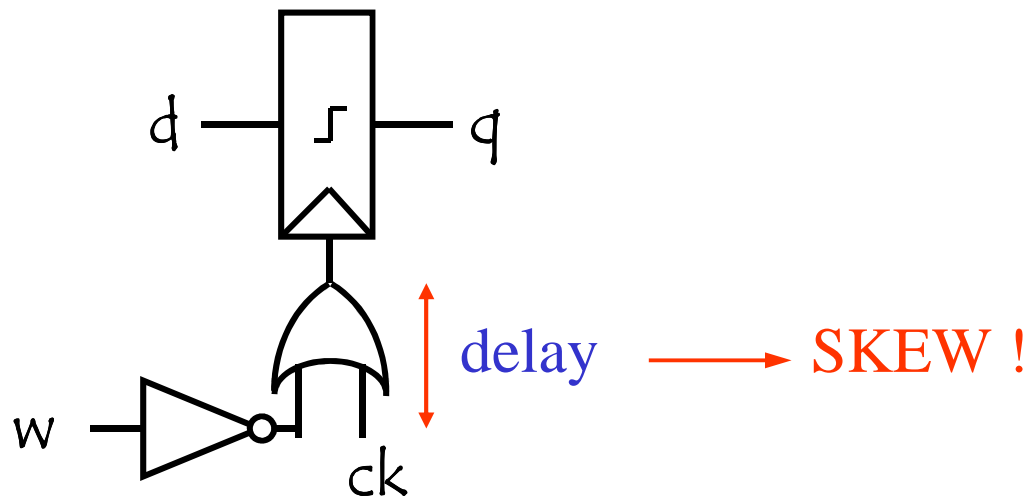
Write a data d on the rising edge of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

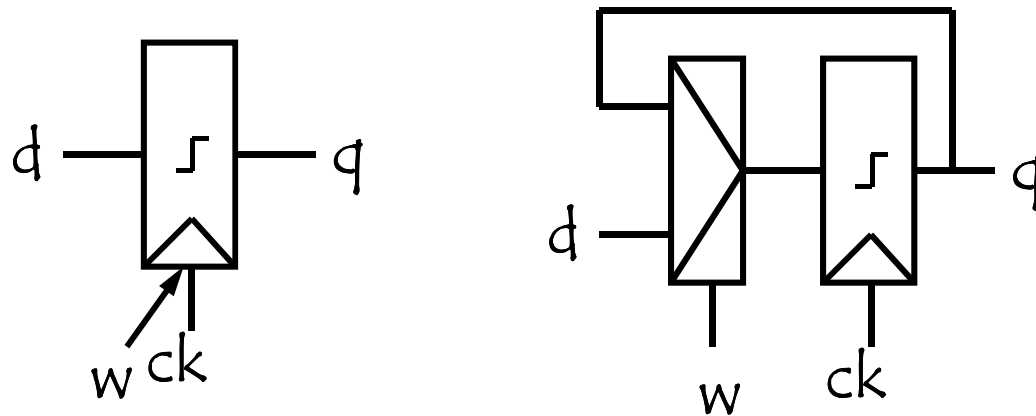
Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

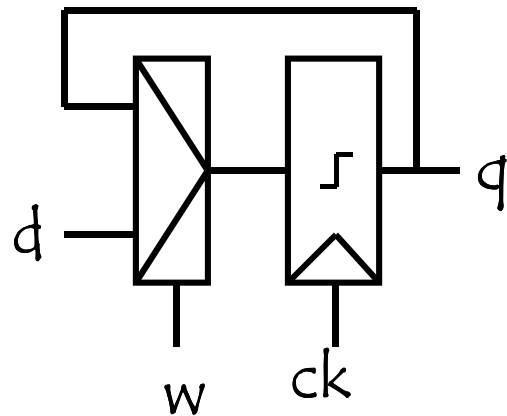
Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

Write a data d on the rising **edge** of the clock ck when a condition is true (write enable)



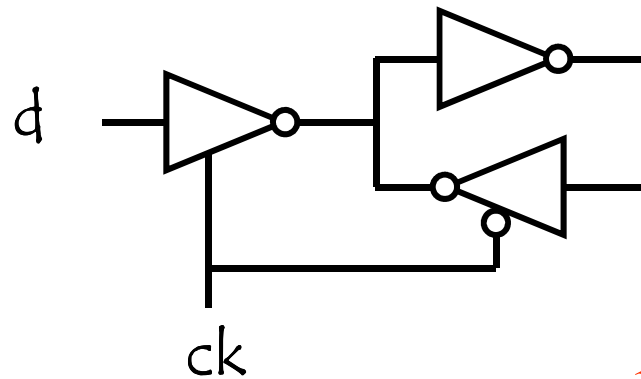
44 tr per register !!

CMOS Circuits

Memory : Latch

Hold a data (0 or 1)

Write a data (0 or 1)



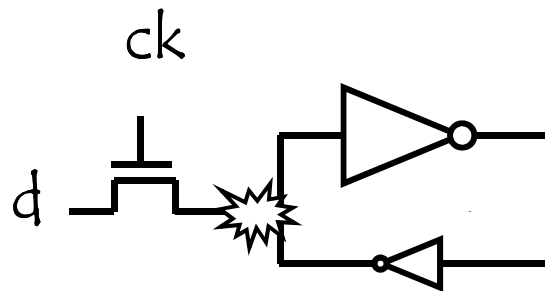
10 tr per latch

CMOS Circuits

Memory : Latch

Hold a data (0 or 1)

Write a data (0 or 1)



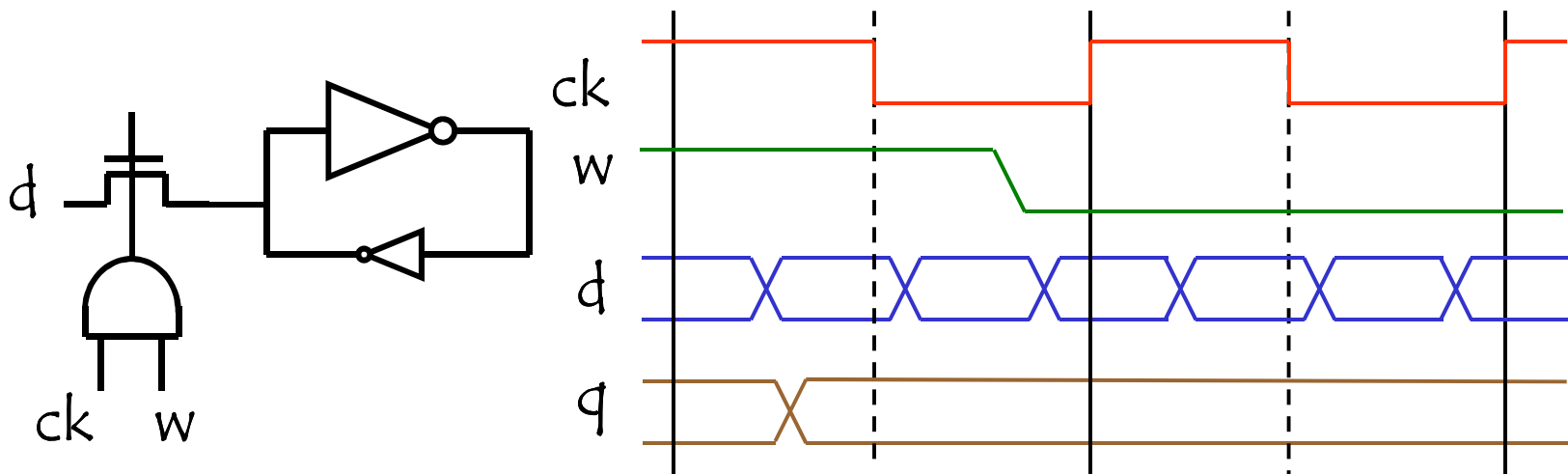
short circuit !

5 tr per latch

CMOS Circuits

Synchronous Memory :

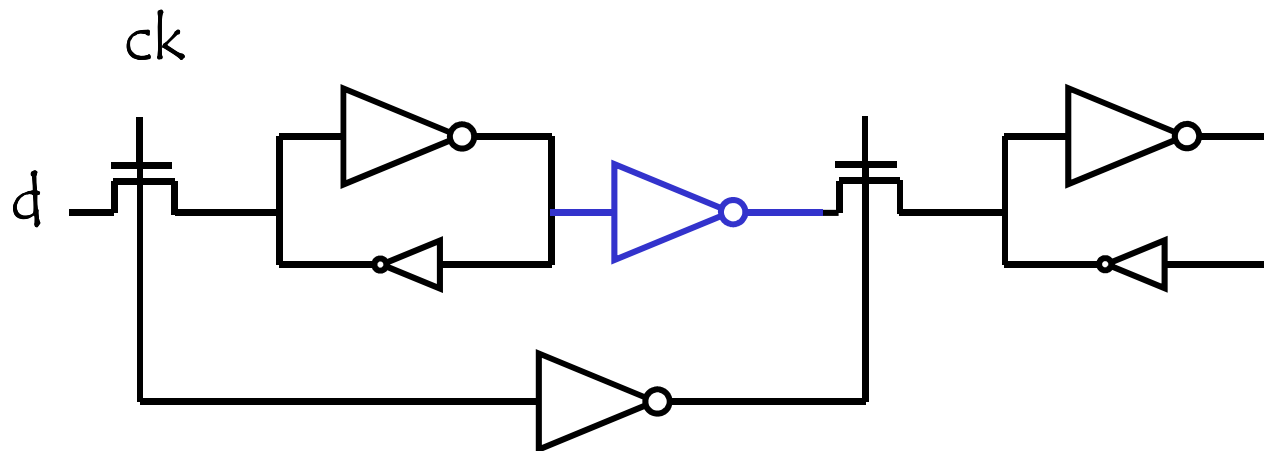
Write a data d when the clock $ck = 1$
when a condition is true (write enable)



CMOS Circuits

Synchronous Memory :

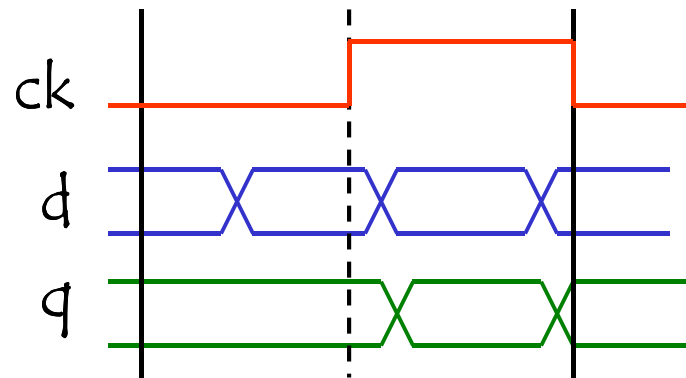
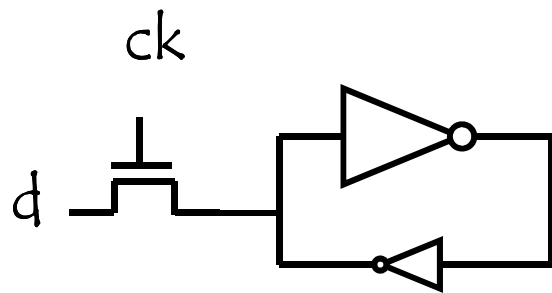
Write a data d on the falling edge of the clock ck



CMOS Circuits

Synchronous Memory :

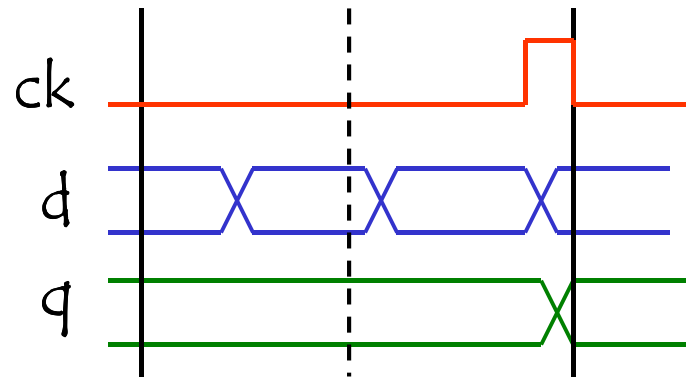
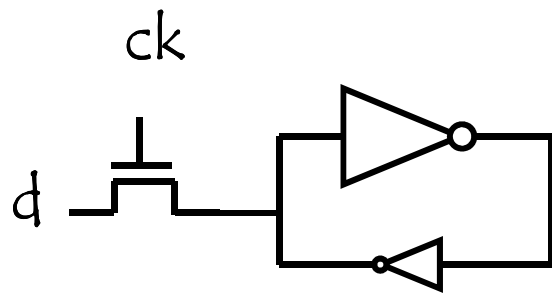
Write a data d when the clock $ck = 1$



CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck



CMOS Circuits

Synchronous Memory :

Write a data d on the rising edge of the clock ck

