The Abdus Salam International Centre for Theoretical Physics

2384-22

ICTP Latin-American Advanced Course on FPGADesign for Scientific Instrumentation

19 November - 7 December, 2012

Muestreo y Reconstrucción

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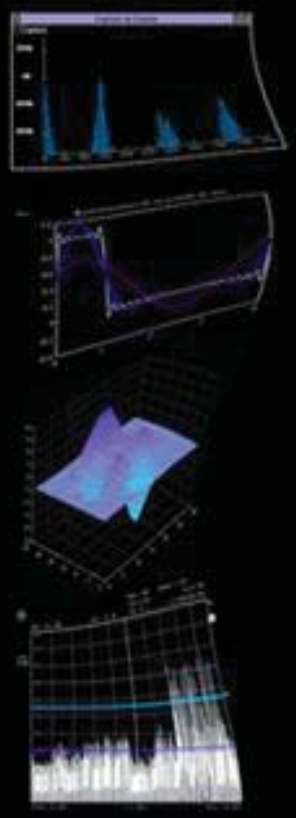
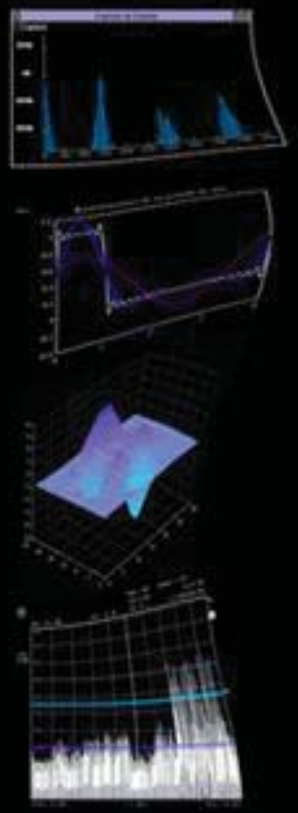
ARGENTINA

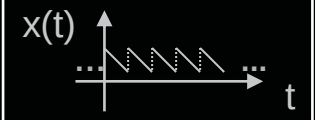
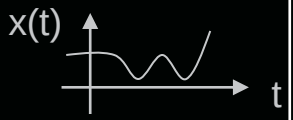
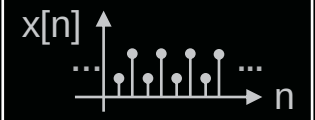
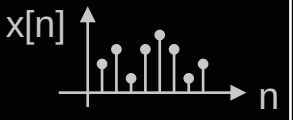
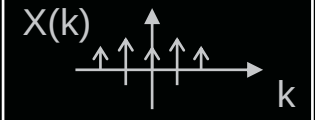
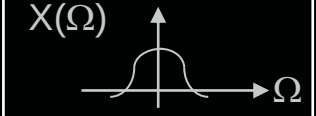
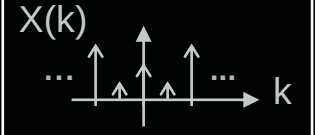
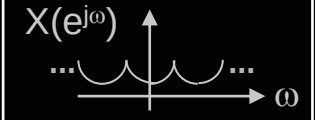


Muestreo y Reconstrucción

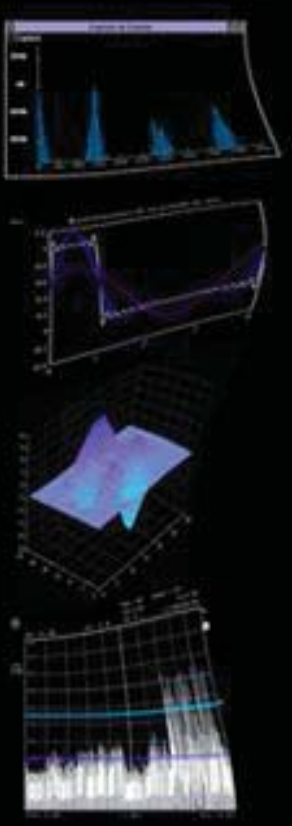
Muestreo y reconstrucción

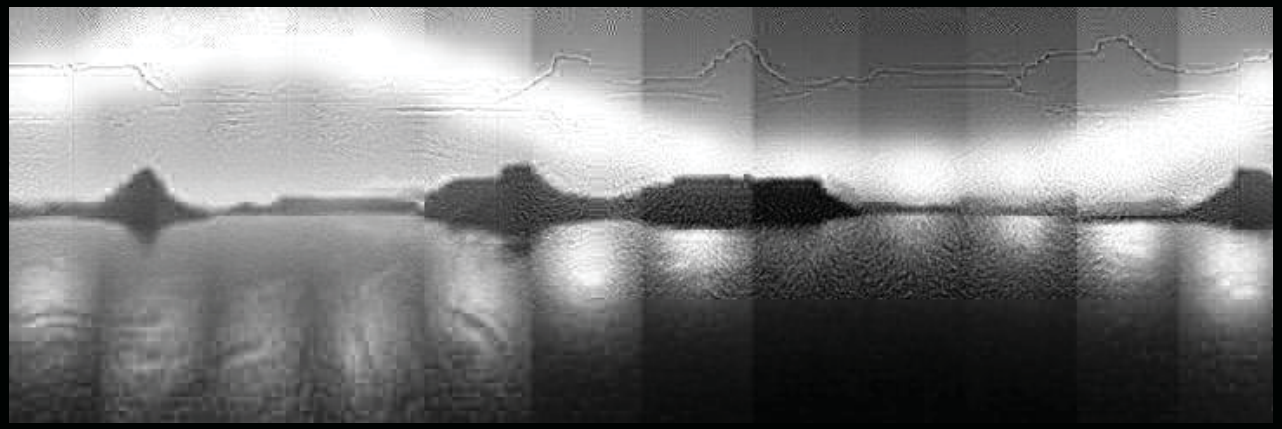
Señales y espectros



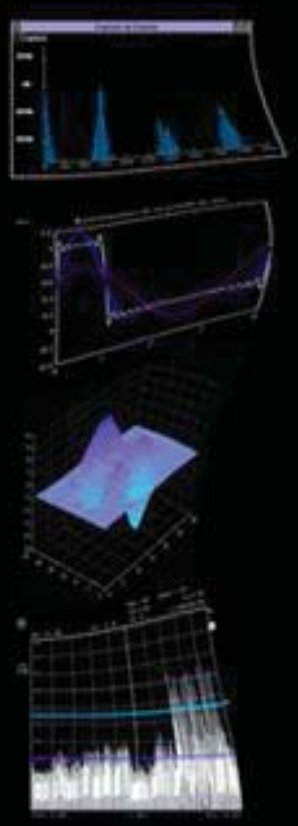
Señal	 Continua periódica	 Continua aperiódica	 Discreta periódica	 Discreta aperiódica
Espectro	 Discreto aperiódico	 Continuo aperiódico	 Discreto periódico	 Continua periódica

Muestreo y Reconstrucción



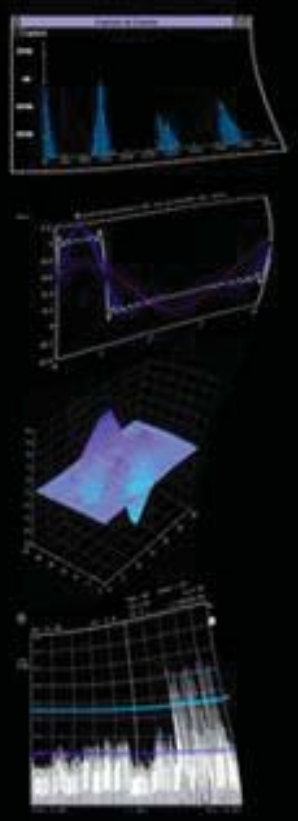


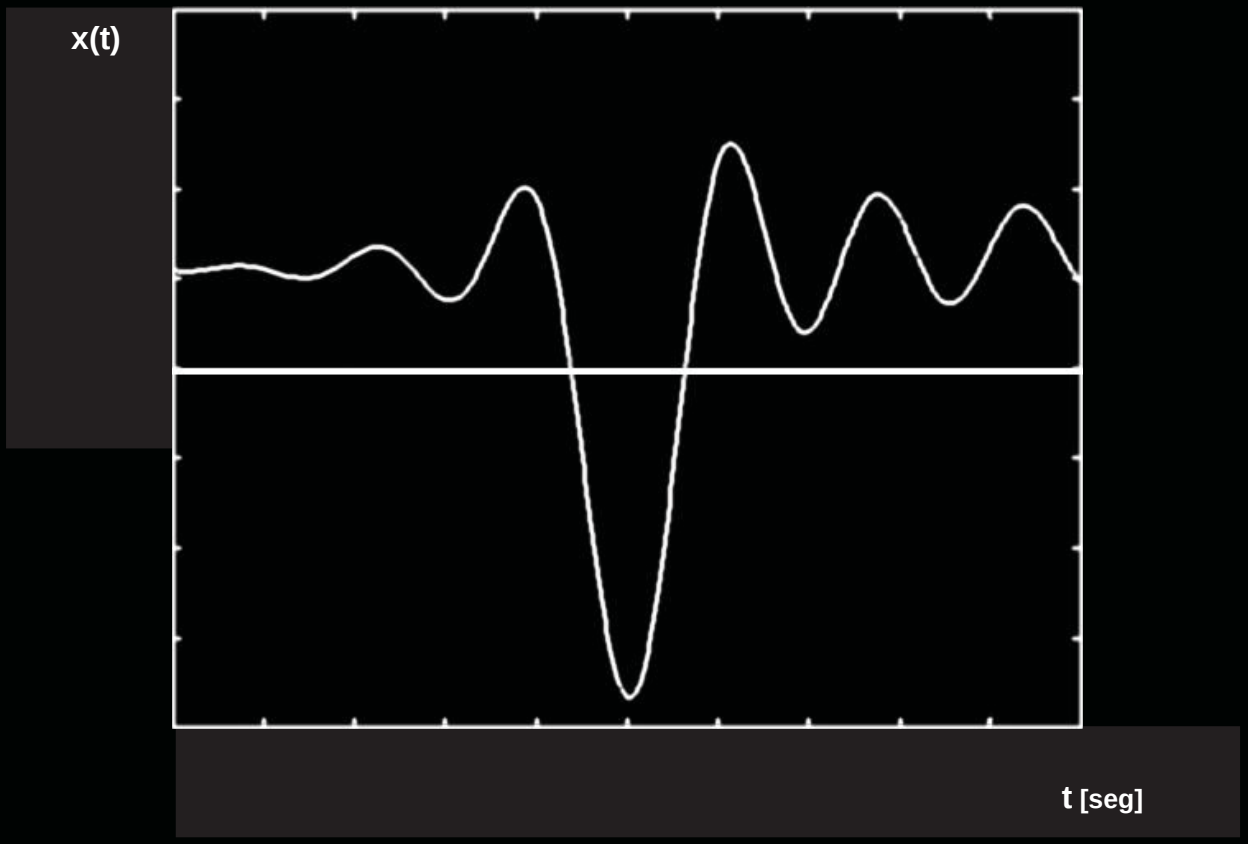
Muestreo y Reconstrucción



Muestreo y Reconstrucción

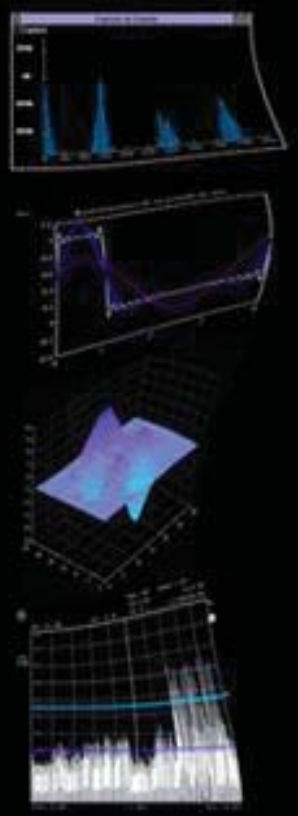
Muestreo

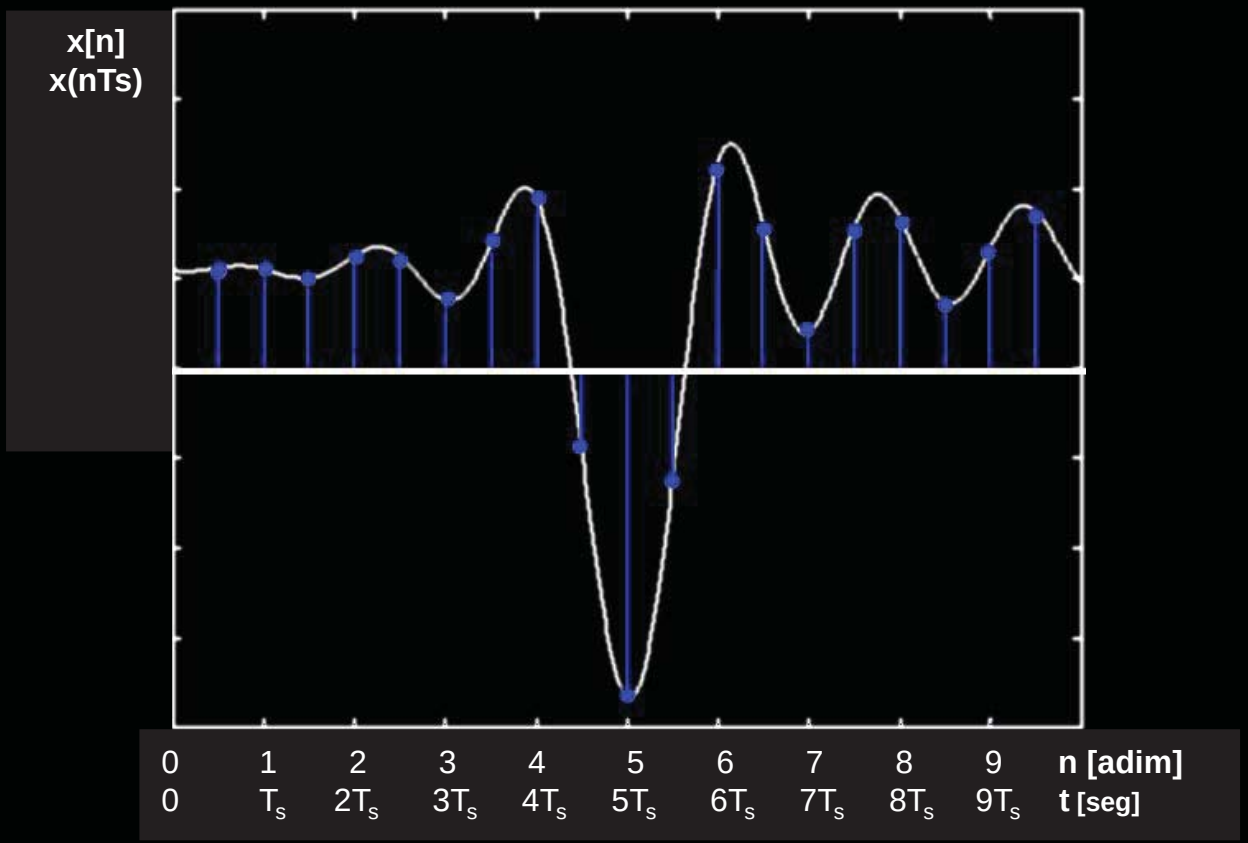




Muestreo y Reconstrucción

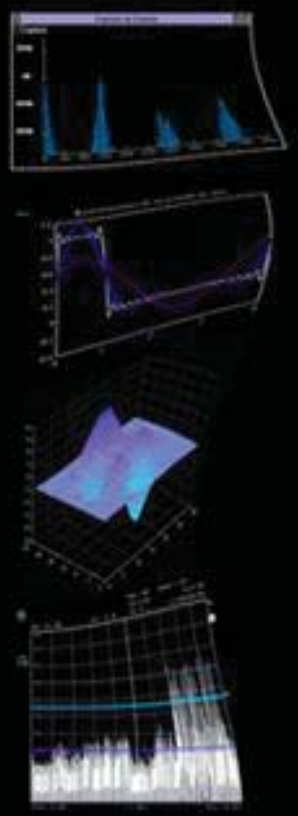
Muestreo

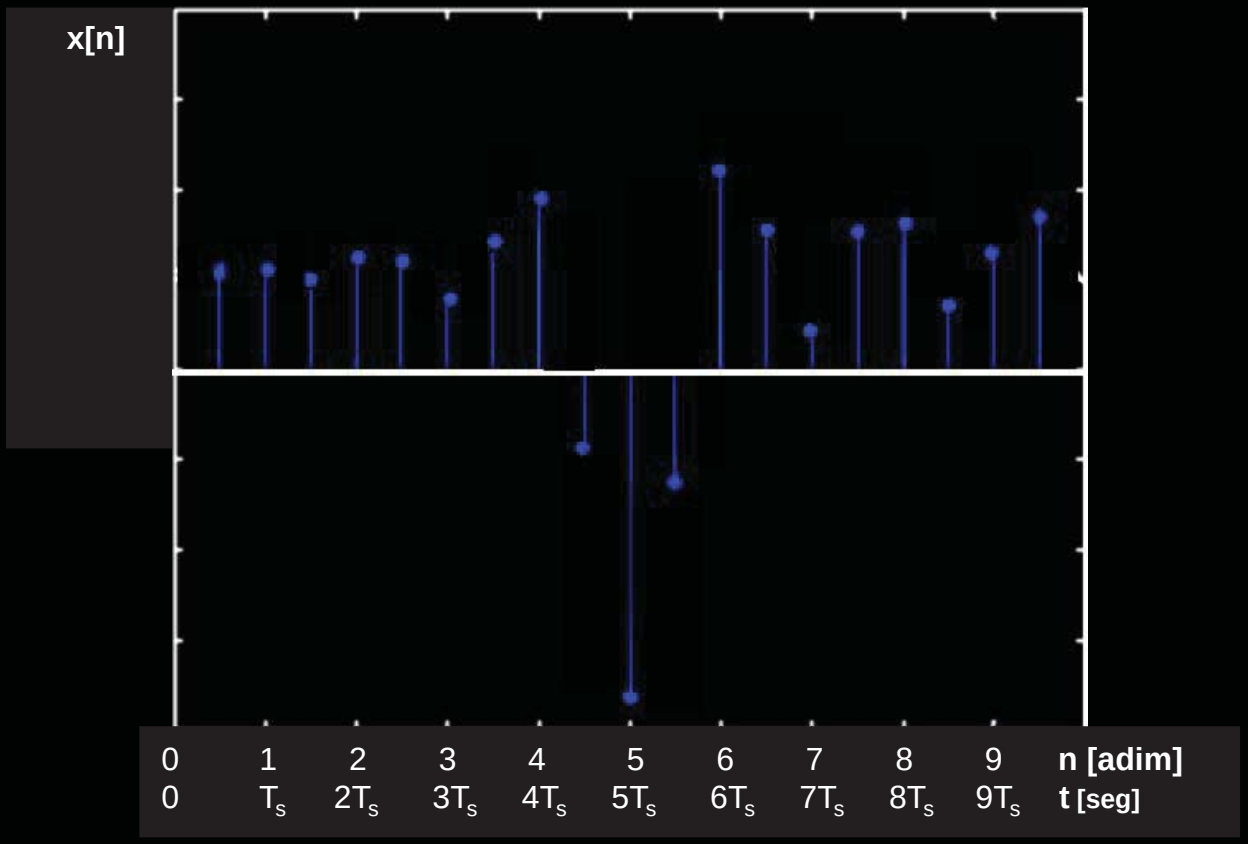




Muestreo y Reconstrucción

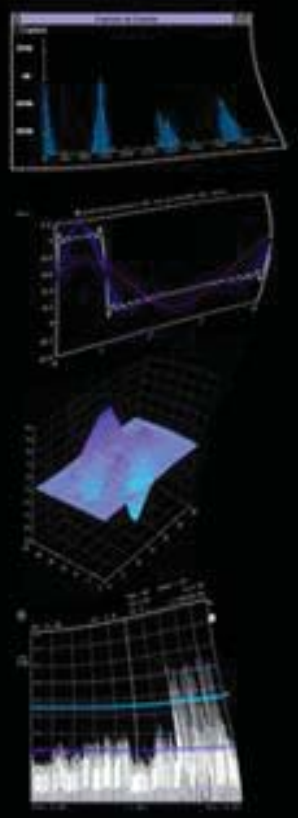
Muestreo

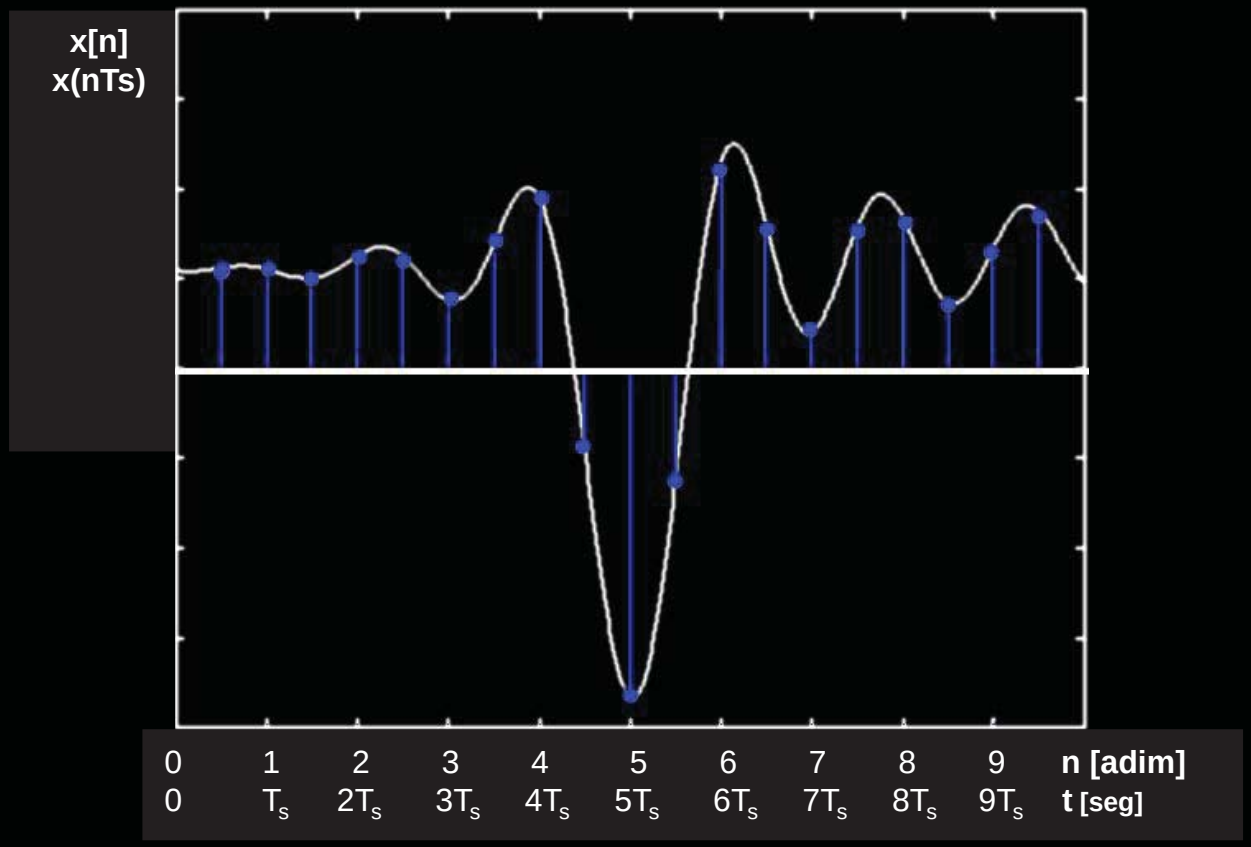




Muestreo y Reconstrucción

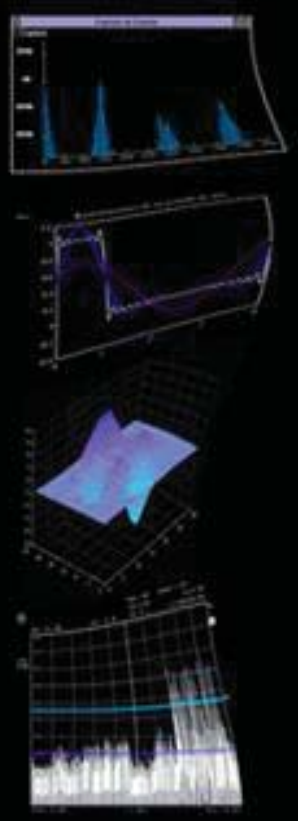
Reconstrucción

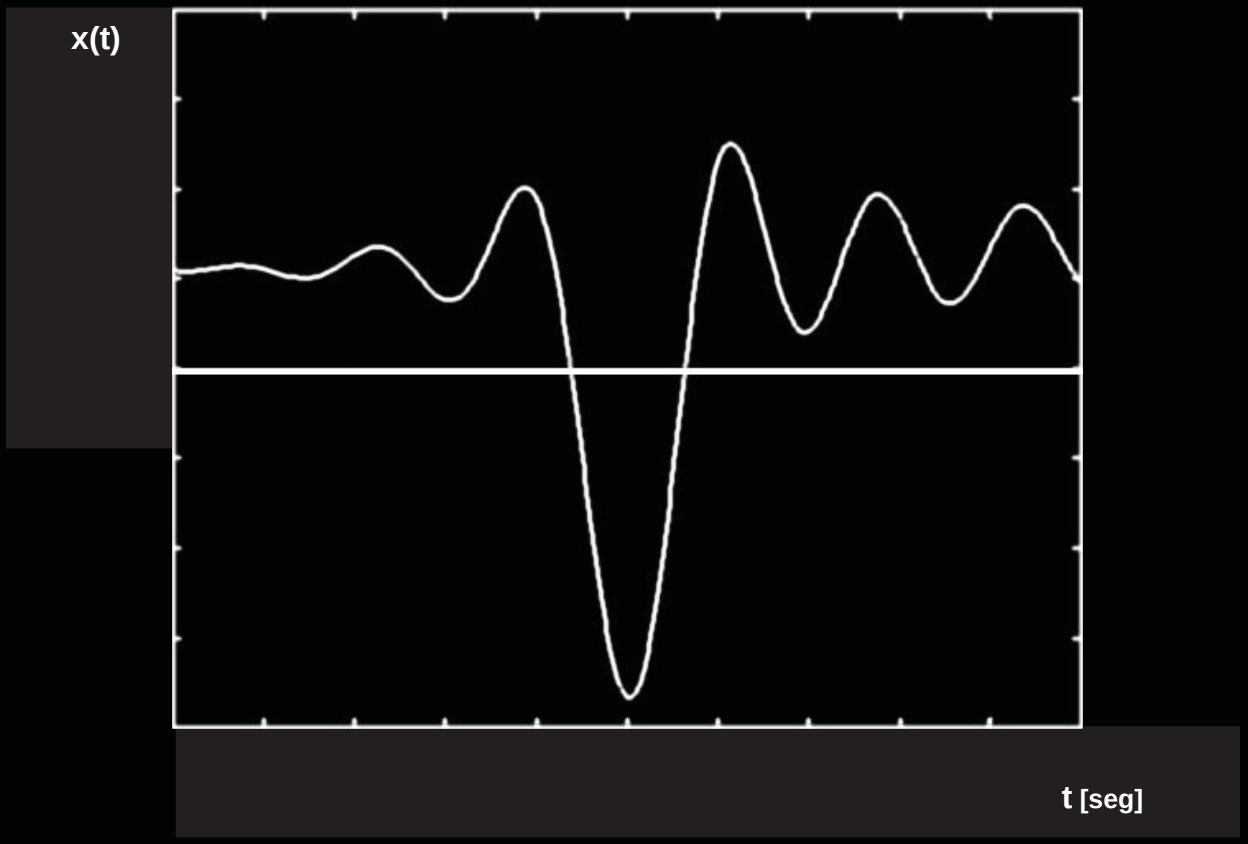




Muestreo y Reconstrucción

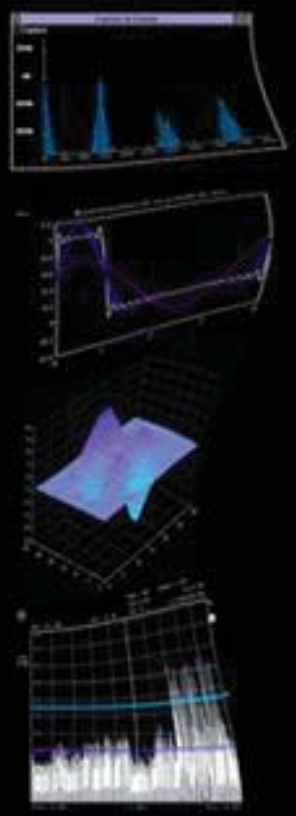
Reconstrucción

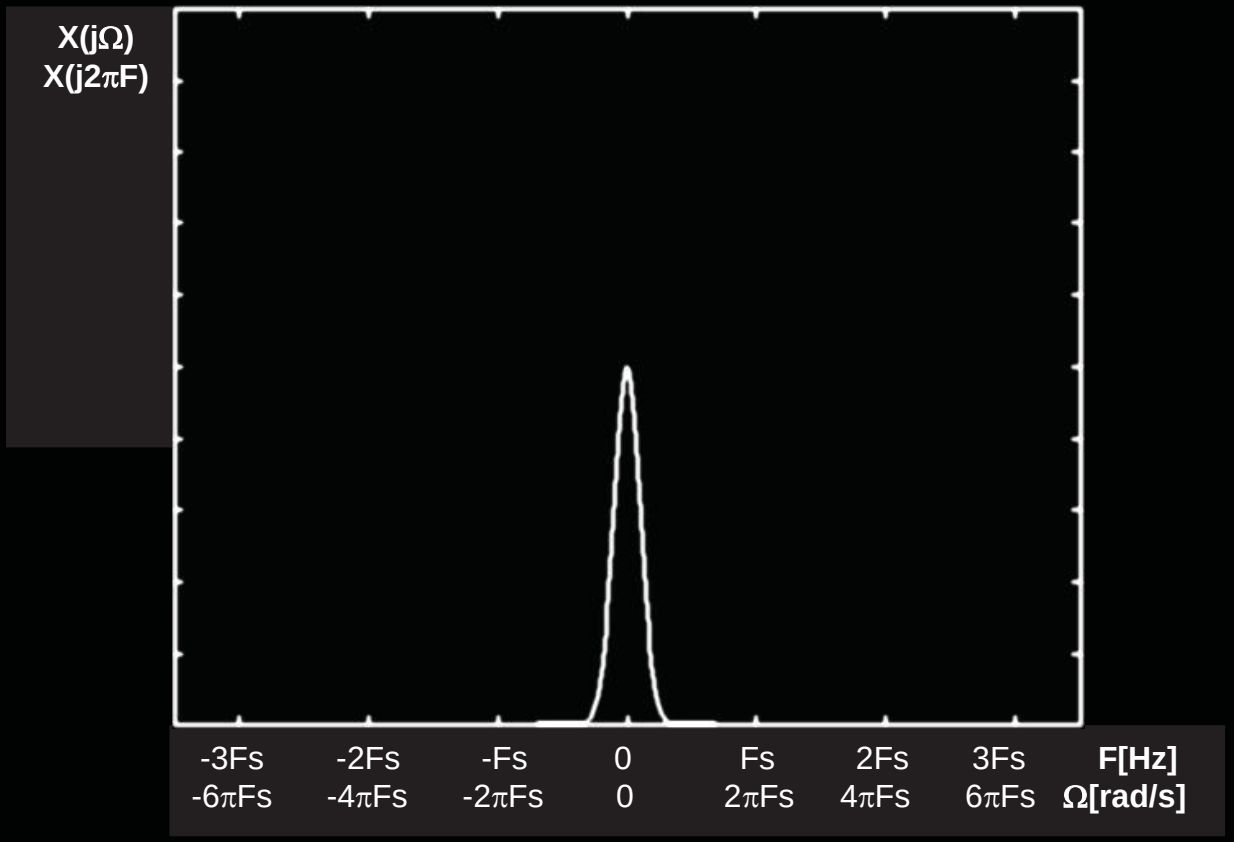




Muestreo y Reconstrucción

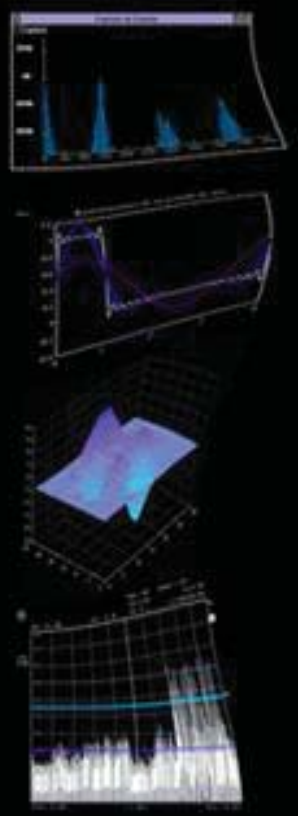
Muestreo



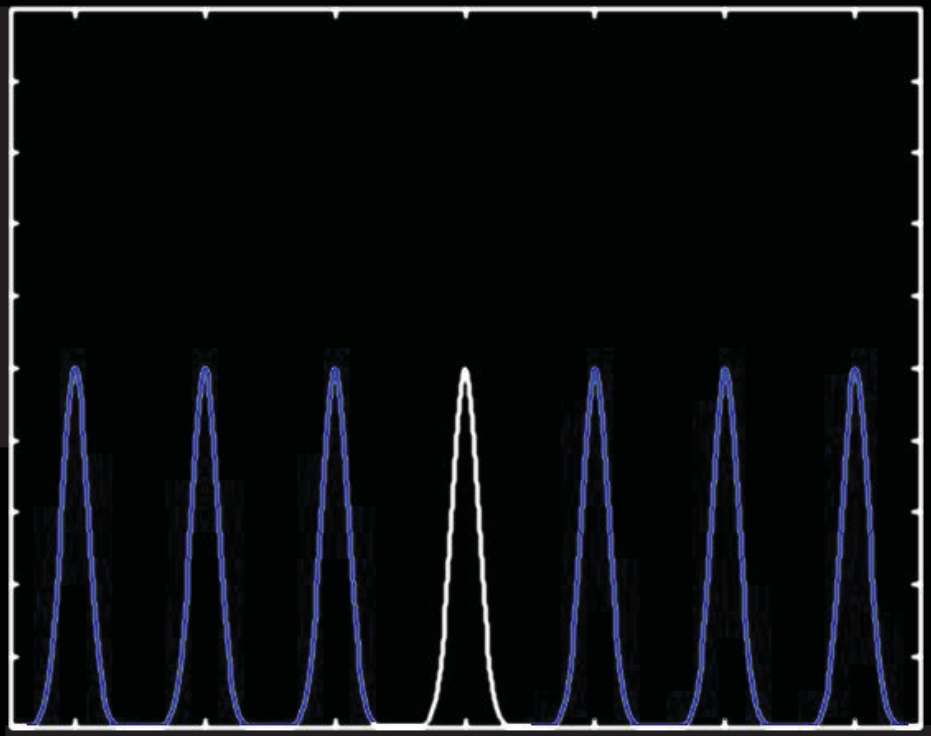


Muestreo y Reconstrucción

Muestreo



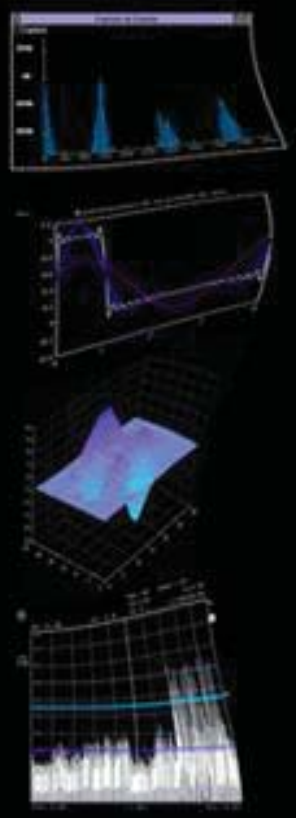
$X(j\Omega)$
$X(j2\pi F)$
$X(e^{j\omega})$
$X(e^{j2\pi f})$

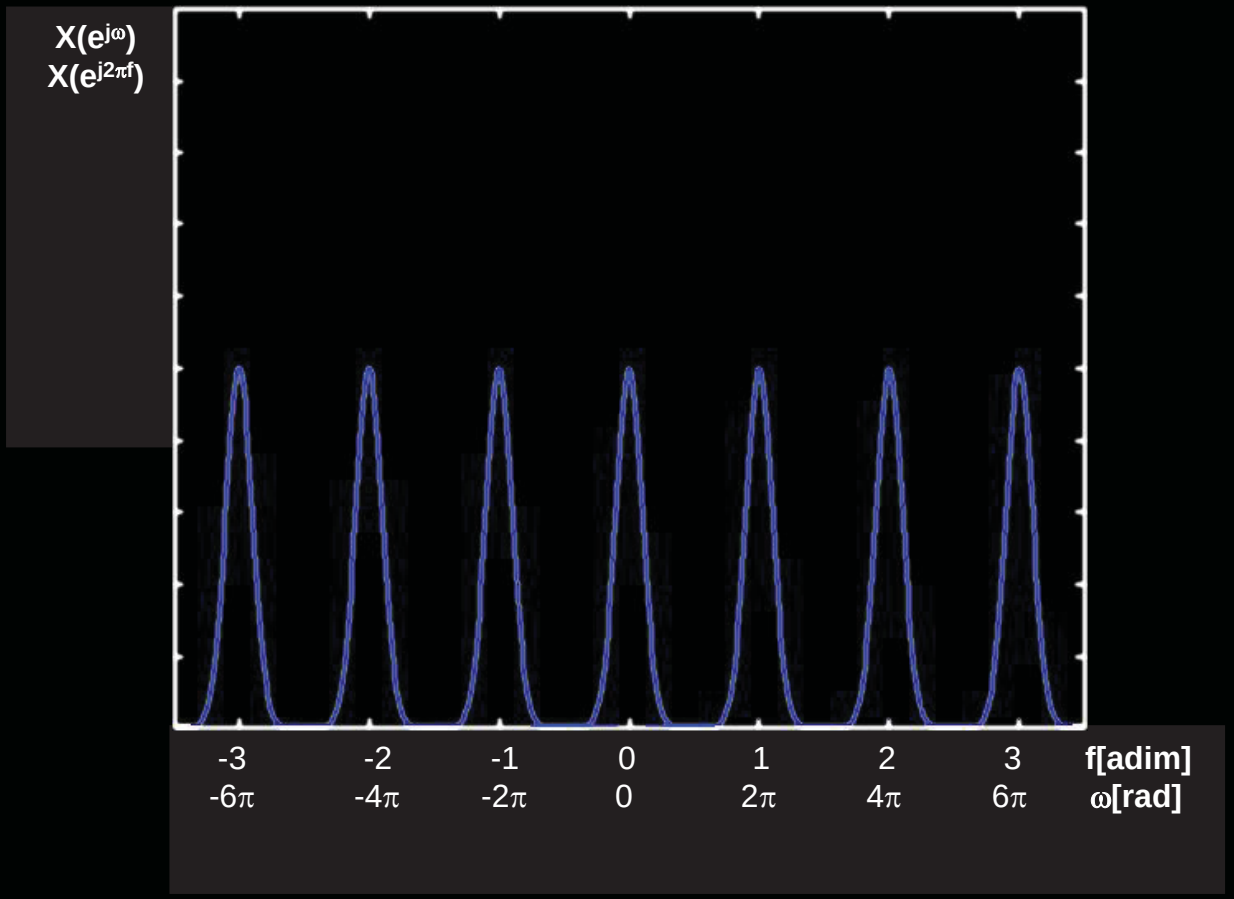


-3Fs	-2Fs	-Fs	0	Fs	2Fs	3Fs	**F[Hz]**
-6πFs	-4πFs	-2πFs	0	2πFs	4πFs	6πFs	**Ω[rad/s]**
-3	-2	-1	0	1	2	3	**f[adim]**
-6π	-4π	-2π	0	2π	4π	6π	**ω[rad]**

Muestreo y Reconstrucción

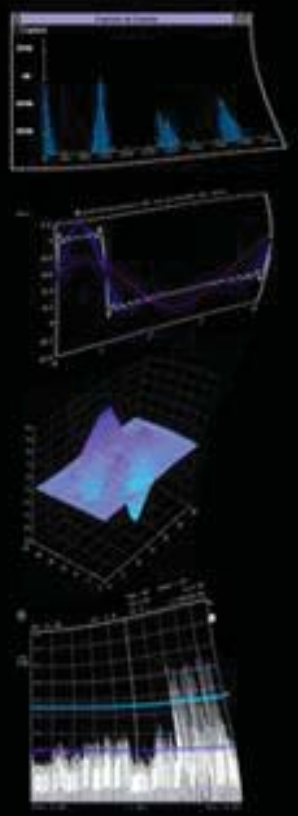
Muestreo



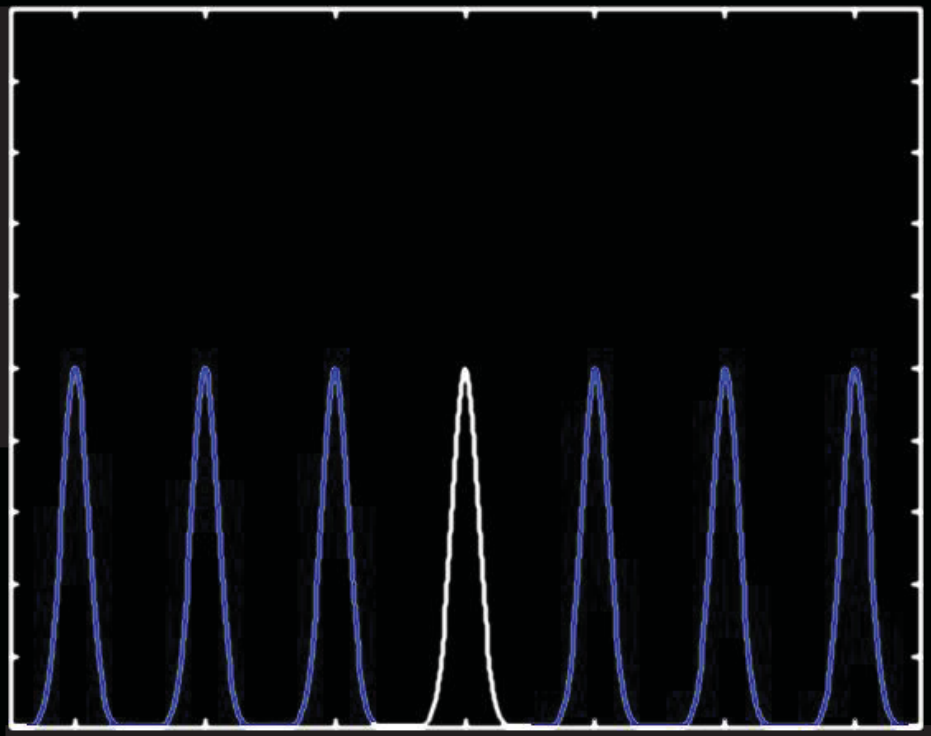


Muestreo y Reconstrucción

Reconstrucción



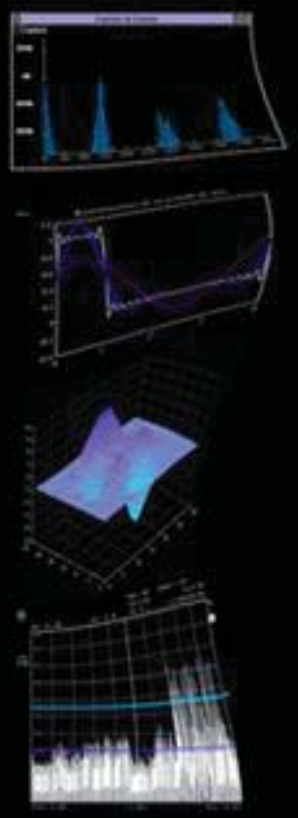
$X(j\Omega)$
$X(j2\pi F)$
$X(e^{j\omega})$
$X(e^{j2\pi f})$

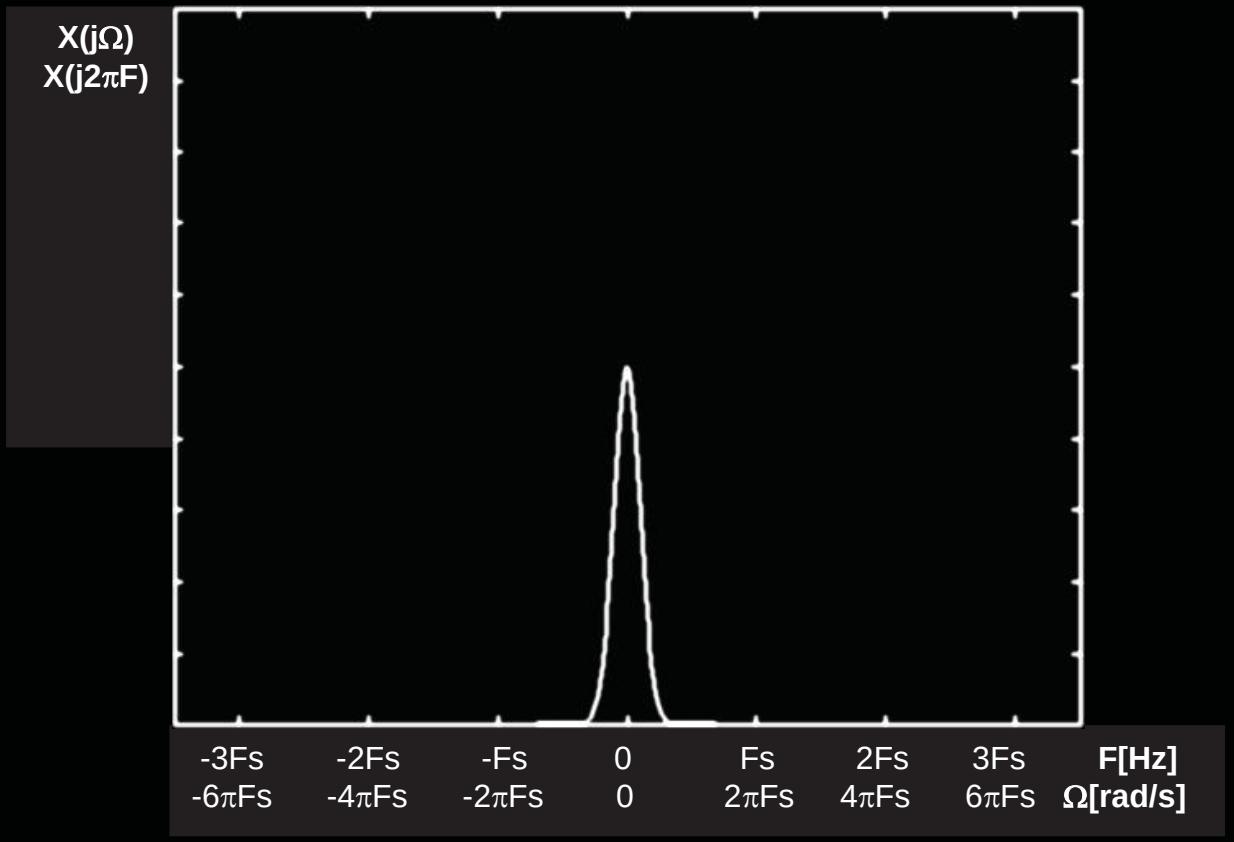


-3Fs	-2Fs	-Fs	0	Fs	2Fs	3Fs	**F[Hz]**
-6πFs	-4πFs	-2πFs	0	2πFs	4πFs	6πFs	**Ω[rad/s]**
-3	-2	-1	0	1	2	3	**f[adim]**
-6π	-4π	-2π	0	2π	4π	6π	**ω[rad]**

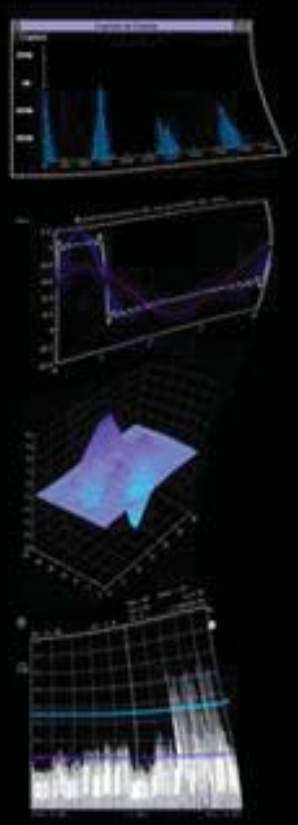
Muestreo y Reconstrucción

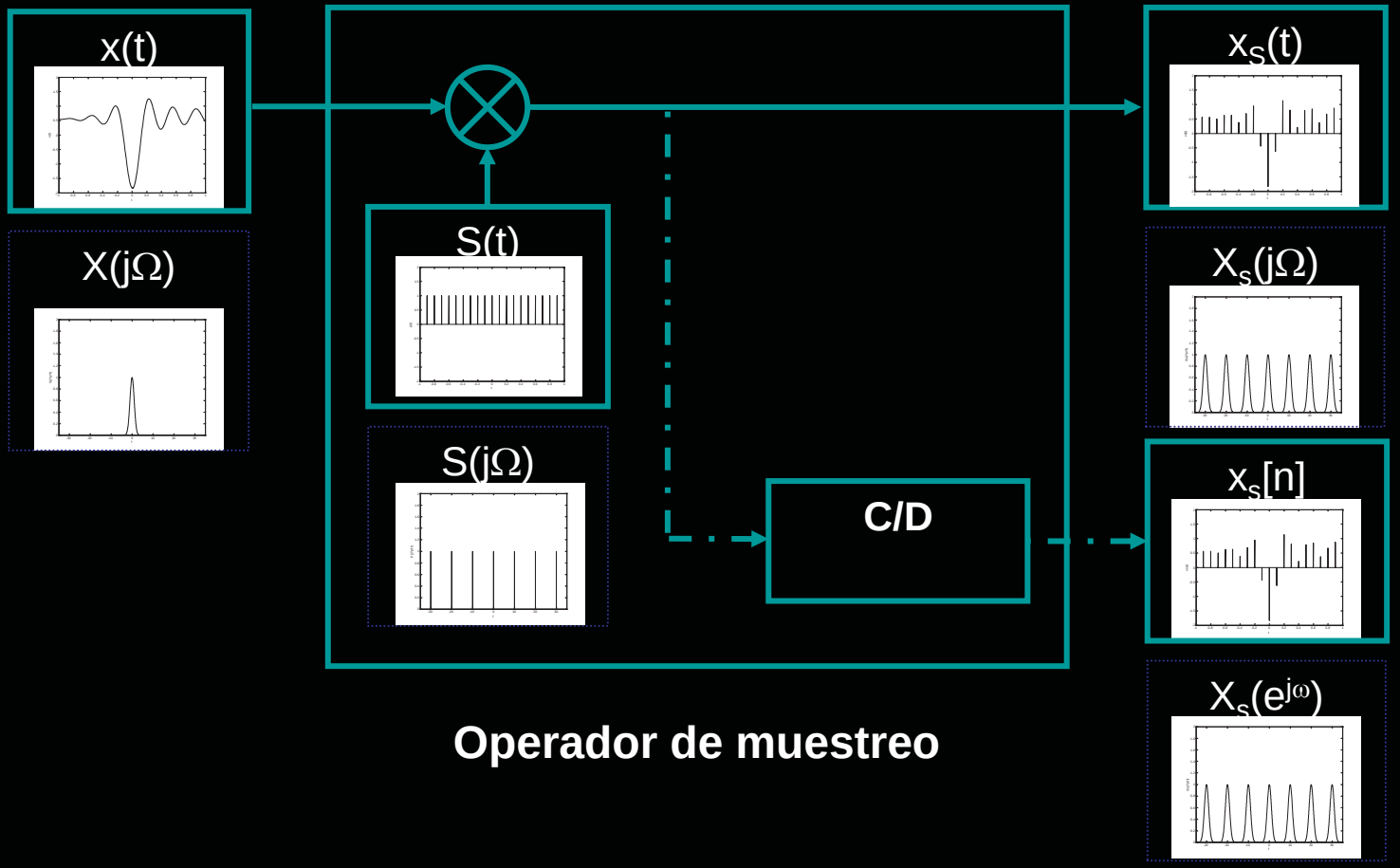
Reconstrucción



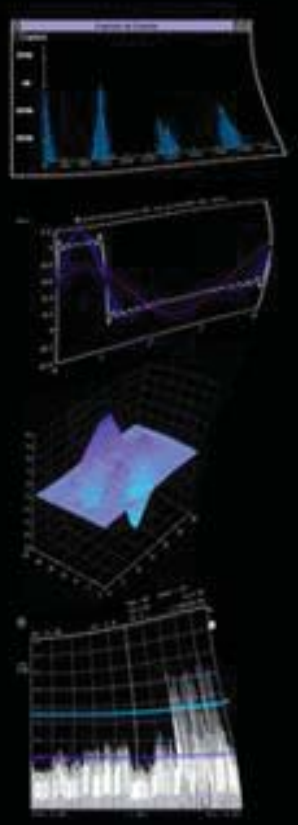


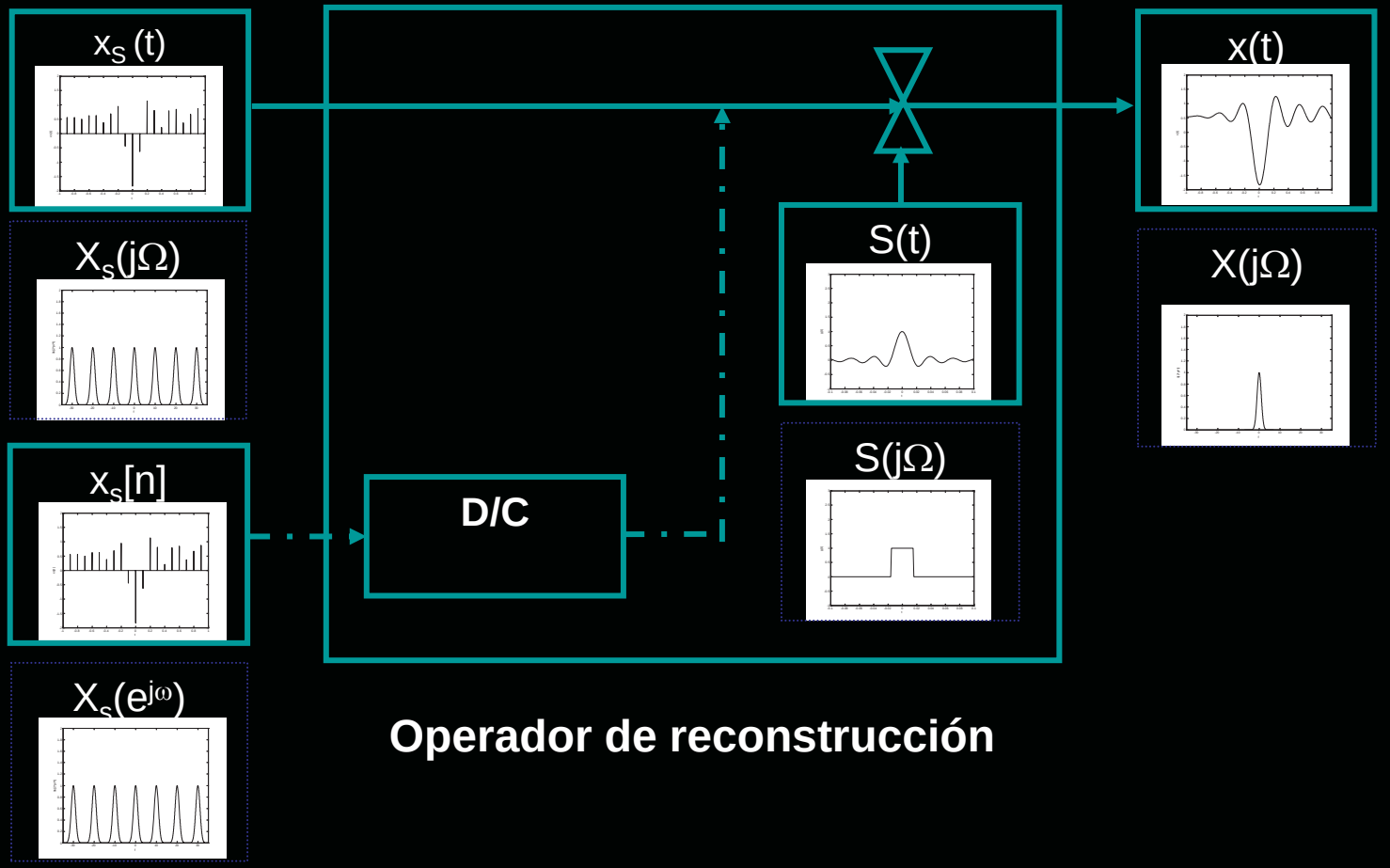
Muestreo y Reconstrucción





Muestreo y Reconstrucción

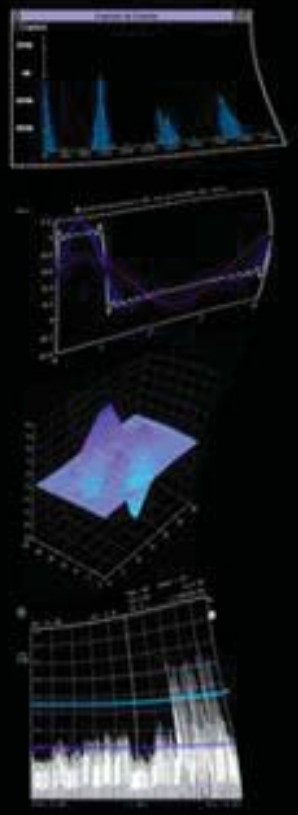


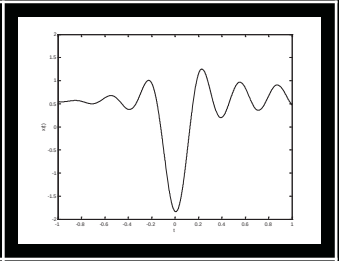
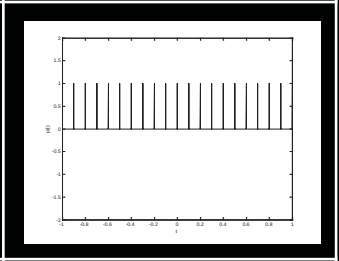
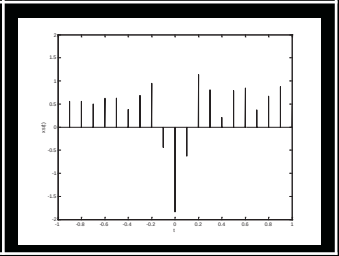


Operador de reconstrucción

Muestreo y Reconstrucción

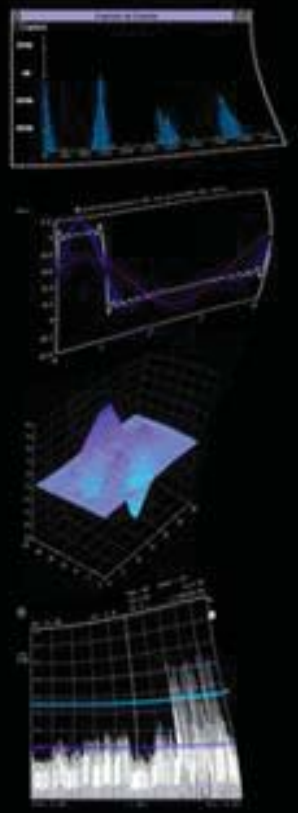
Secuencia muestreada

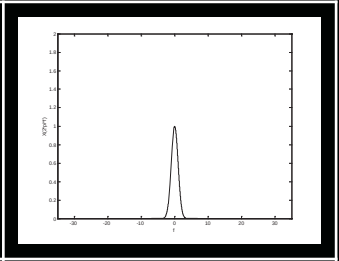
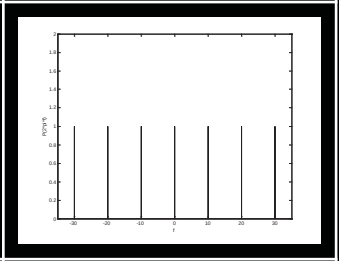
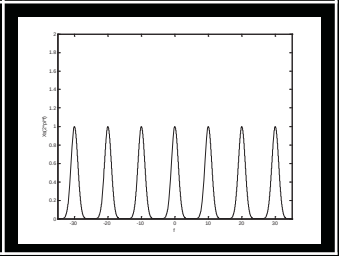


$x_a(t)$	
$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$	
$x_s(t) = x_a(t)s(t) = \sum_{n=-\infty}^{+\infty} x_a(t)\delta(t - nT_s)$	

Muestreo y Reconstrucción

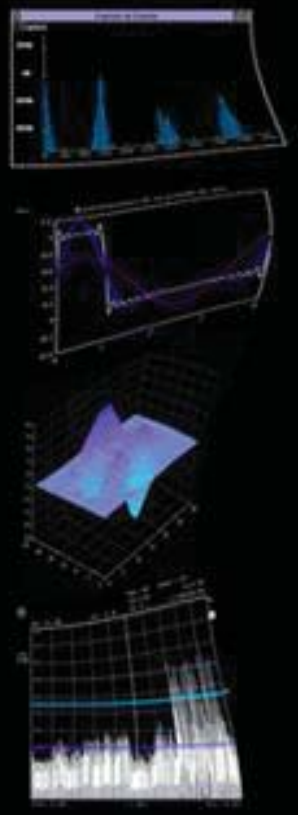
Espectro muestreado

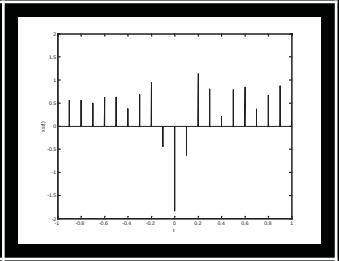
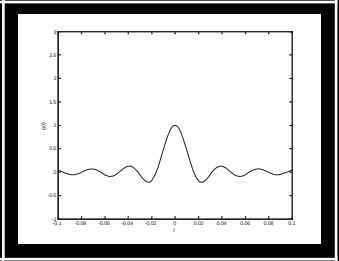
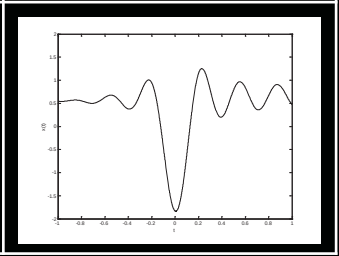


$X_a(j\Omega)$	
$S(j\Omega) = \Omega_s \sum_{k=-\infty}^{+\infty} \delta(\Omega - k\Omega_s)$	
$X_s(j\Omega) =$ $= \frac{1}{2\pi} X_a(j\Omega) * S(j\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(j\left(\Omega - k\Omega_s\right)\right)$	

Muestreo y Reconstrucción

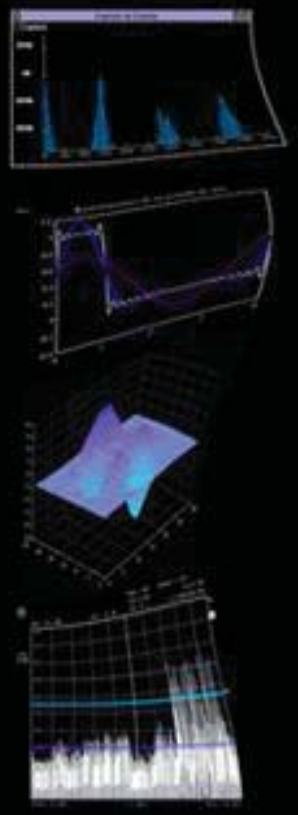
Secuencia reconstruida



$x_s(t) = x_a(t)s(t)$	
$h_r(t) = \text{sen}c\left(\frac{t}{T_s}\right) = \frac{sen\left(\pi t / T_s\right)}{\left(\pi t / T_s\right)}$	
$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\text{sen}c\left(\frac{t - nT_s}{T_s}\right)$	

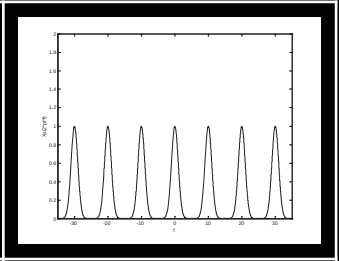
Muestreo y Reconstrucción

Espectro reconstruido

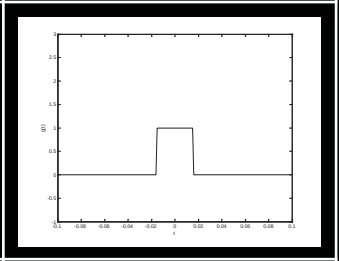


$$X_s(j\Omega) =$$

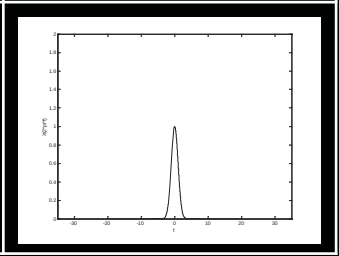
$$= \frac{1}{2\pi} X_a(j\Omega) * S(j\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_a\left(j\left(\Omega - k\Omega_s\right)\right)$$



$$H_r(j\Omega) = T_s \Pi\left(\frac{\Omega}{\Omega_s}\right) = \begin{cases} T_s & si \quad |\Omega| \le \frac{\Omega_s}{2} \\ 0 & cc \end{cases}$$

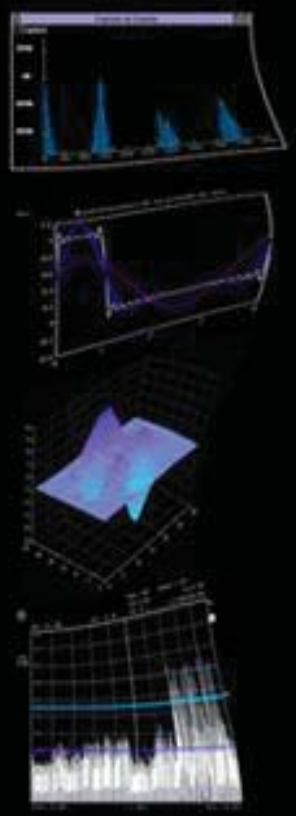


$$X(j\Omega) = \begin{cases} \sum_{k=-\infty}^{\infty} X_a\left(j\left(\Omega - k\Omega_s\right)\right) & si \quad |\Omega| \le \frac{\Omega_s}{2} \\ 0 & cc \end{cases}$$



Muestreo y Reconstrucción

Relación C/D en el tiempo

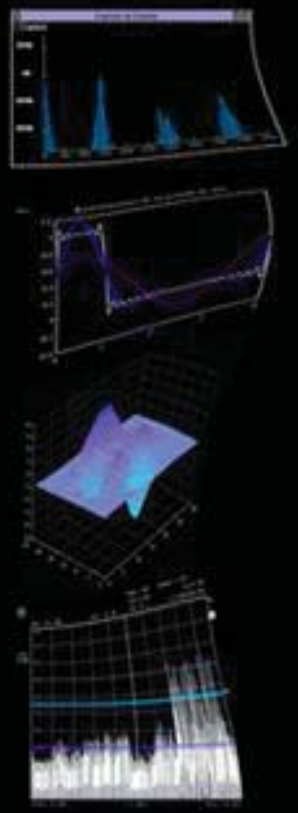


$$t = nT_s$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x_a(kT_s)\delta[n-k]$$

Muestreo y Reconstrucción

Relación C/D en el espectro

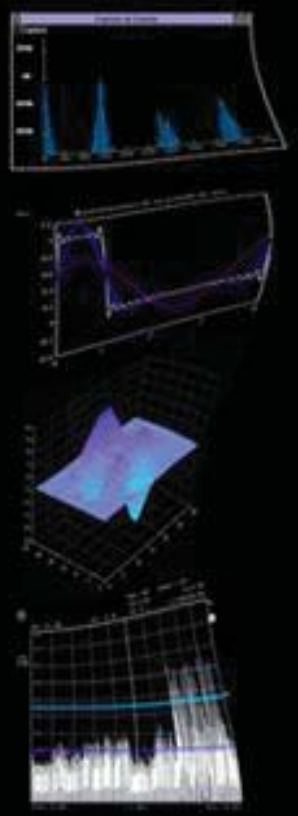


$$f = \frac{F}{F_s} = FT_s \qquad \omega = 2\pi f = \Omega T_s$$

$$X(e^{j\omega}) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_a\left(j\left(\frac{\omega}{T_s} - \frac{2\pi k}{T_s}\right)\right)$$

Muestreo y Reconstrucción

Teorema del muestreo



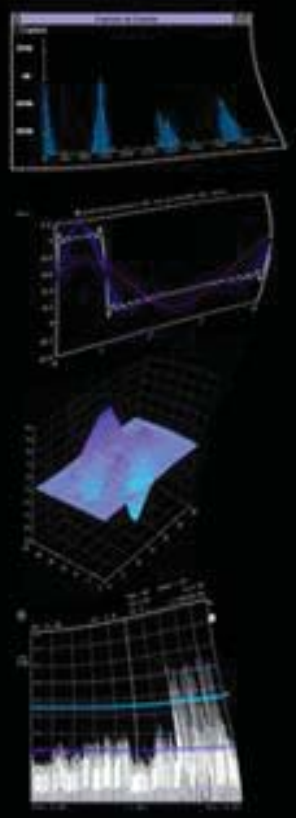
$$x(t) \overset{TF}{\leftrightarrow} X(j\Omega)$$

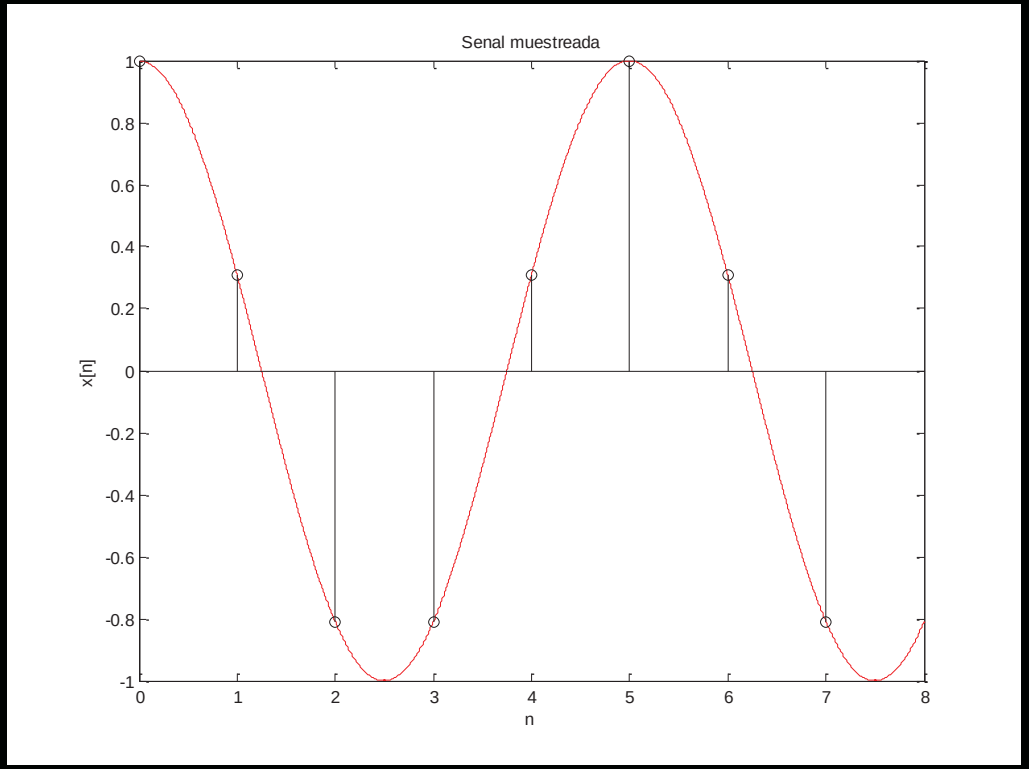
$$si \quad X(j\Omega) = 0 \quad \forall |\Omega| \geq \Omega_N \quad \wedge \quad \Omega_N \leq 2\Omega_s = 2(2\pi F_s)$$

$$\Rightarrow x(t) \overset{muestreo}{\leftrightarrow} x[n] \overset{reconstrucción}{\leftrightarrow} x(t)$$

Muestreo y Reconstrucción

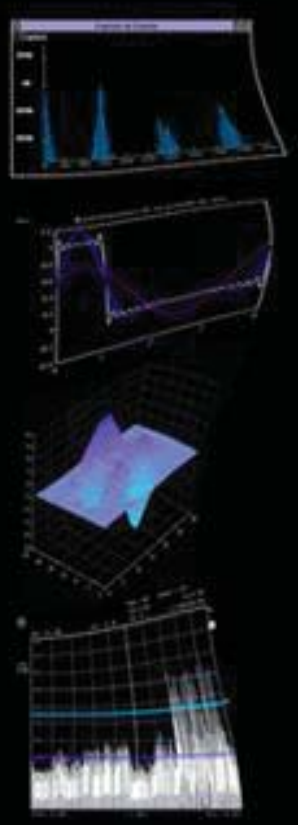
Teorema del muestreo

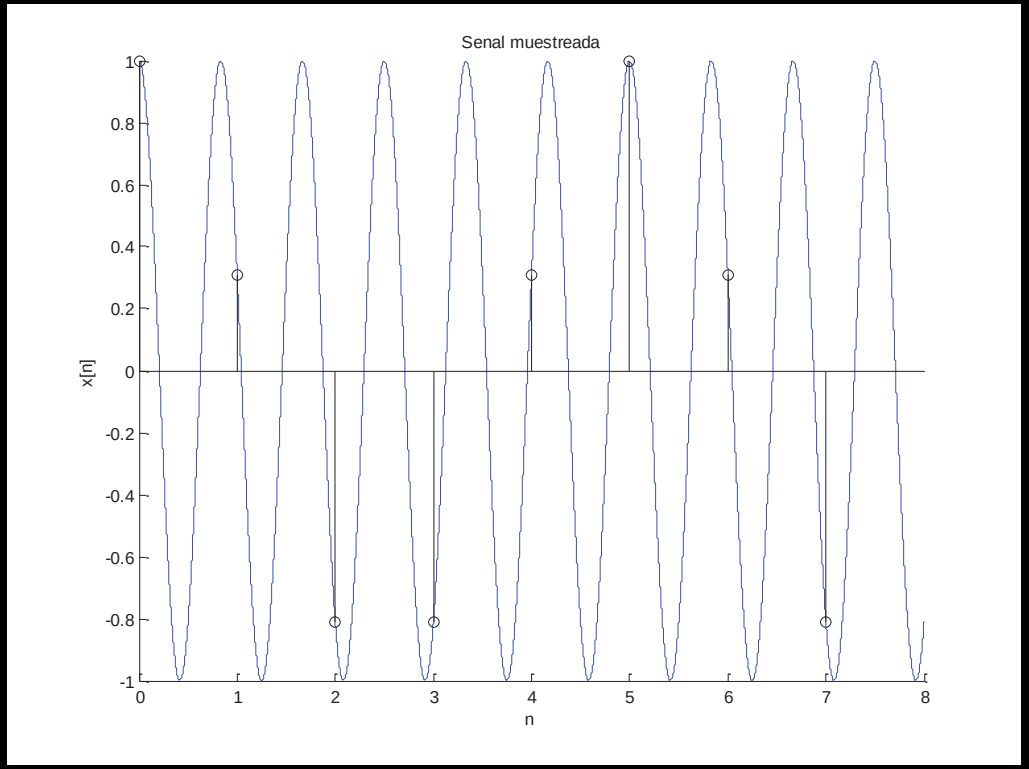




Muestreo y Reconstrucción

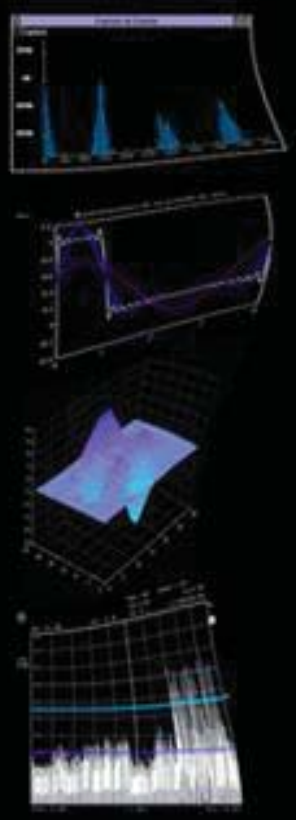
Teorema del muestreo

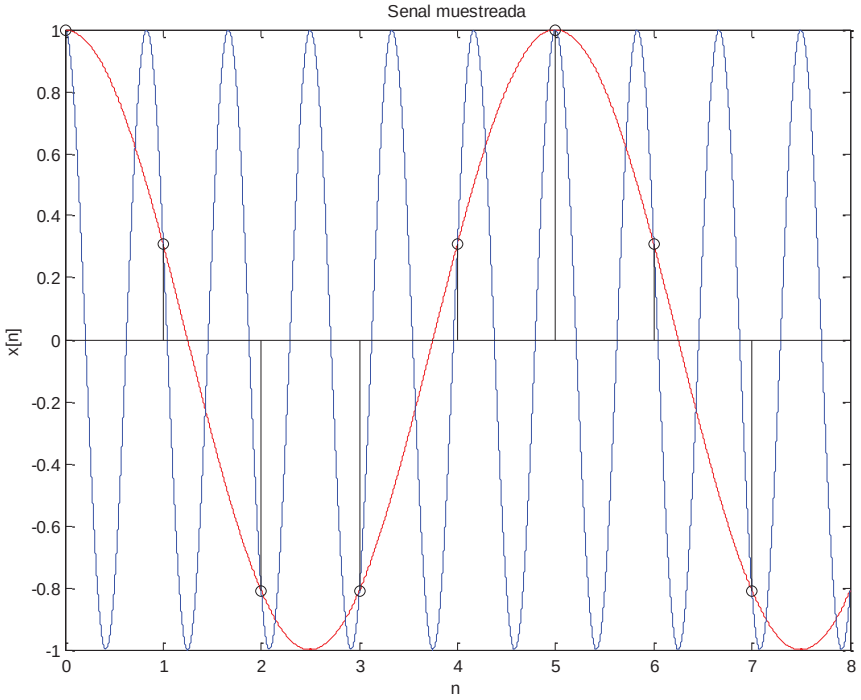




Muestreo y Reconstrucción

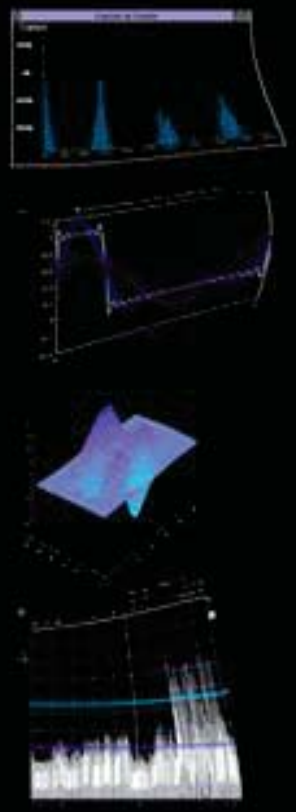
Teorema del muestreo





Muestreo y Reconstrucción

Teorema del muestreo



Ubicación de las réplicas

$$\pm\omega_k = \pm\omega_0 \pm 2k\pi \qquad \pm\Omega_k = \pm\Omega_0 \pm k\Omega_s$$

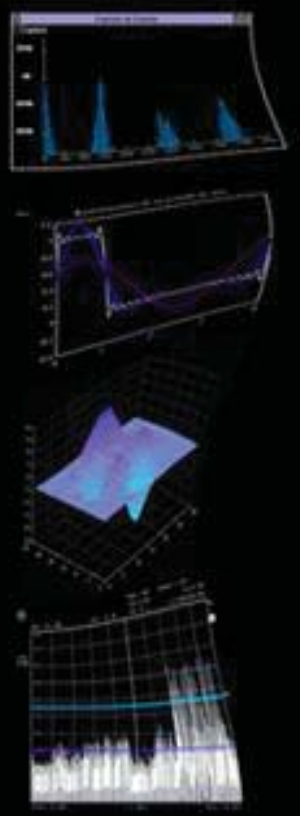
$\times Ts$ (↔)

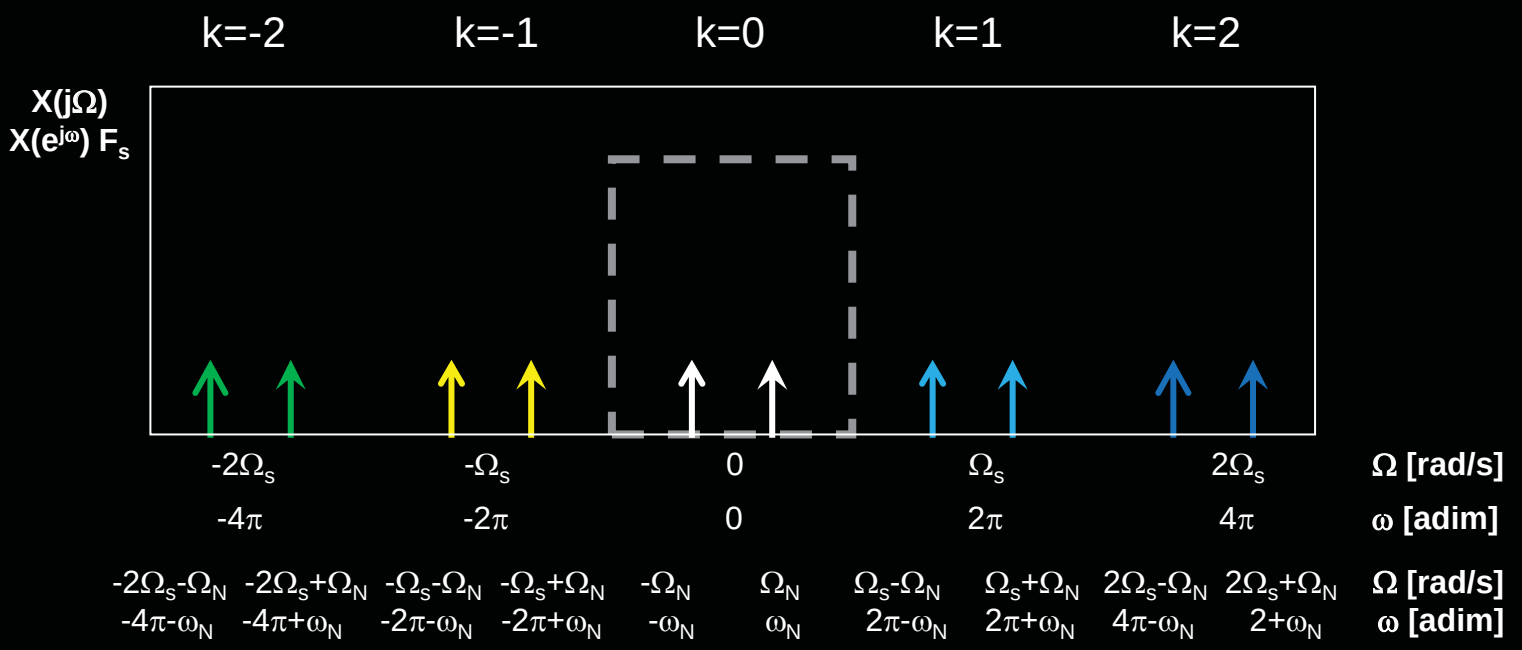
$\div 2\pi$ (↕) $\qquad\qquad$ $\div 2\pi$ (↕)

$$\pm f_k = \pm f_0 \pm k \qquad \pm F_k = \pm F_0 \pm kF_s$$

Muestreo y Reconstrucción

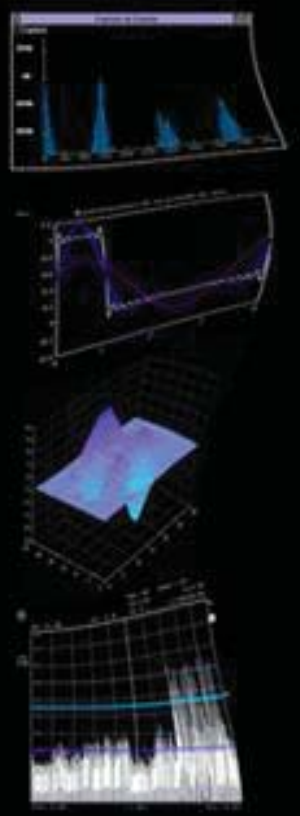
Teorema del muestreo

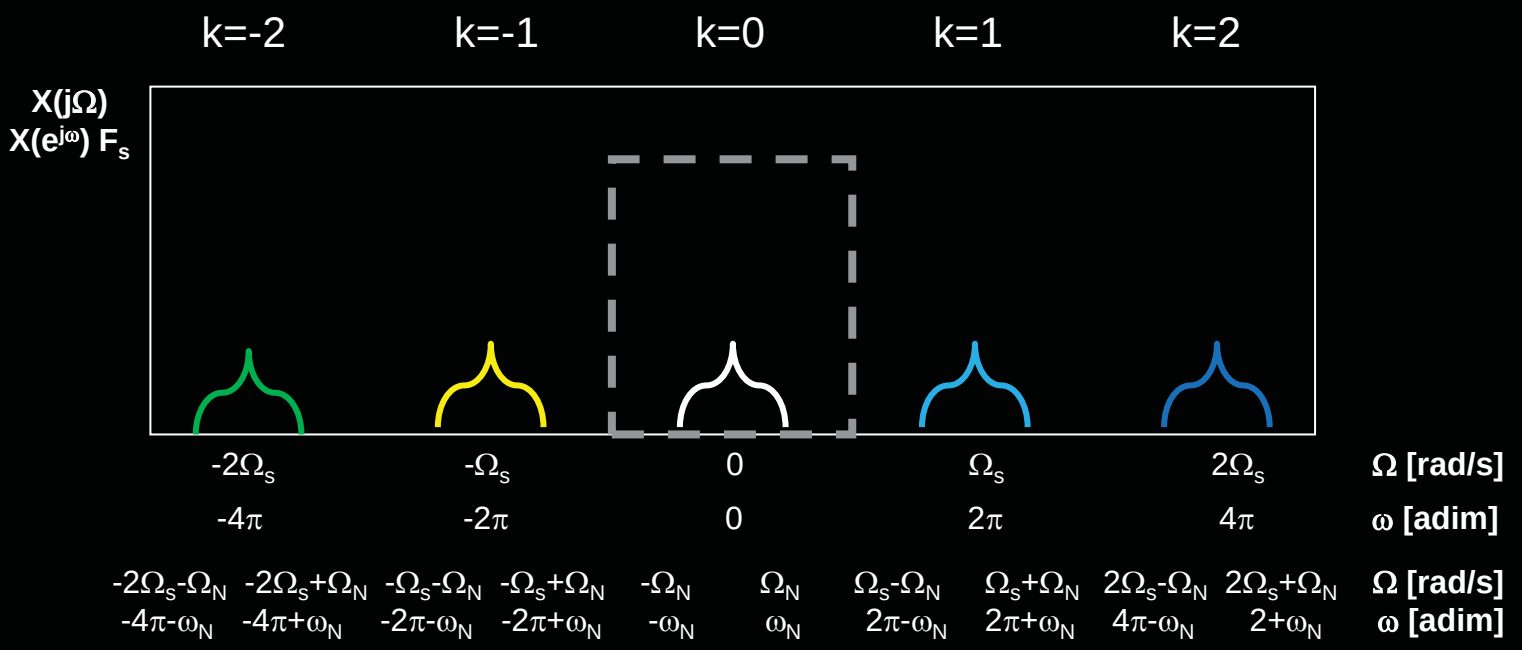




Muestreo y Reconstrucción

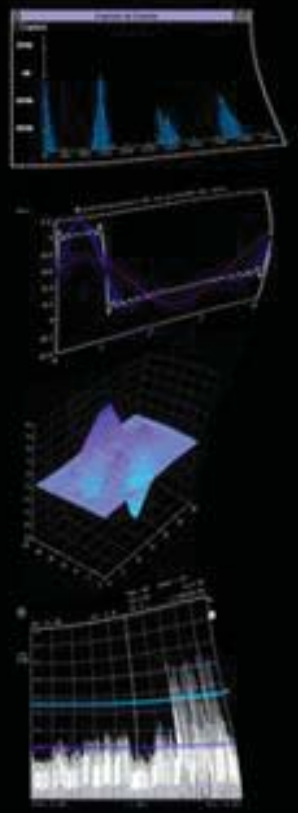
Teorema del muestreo

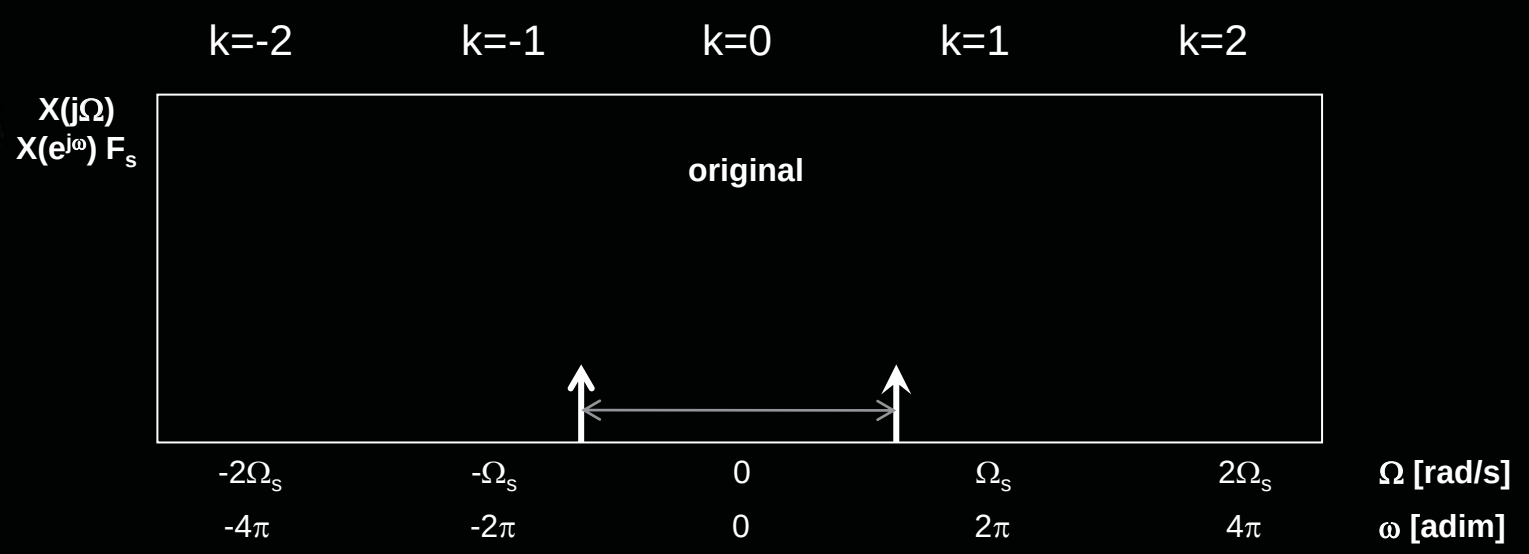




Muestreo y Reconstrucción

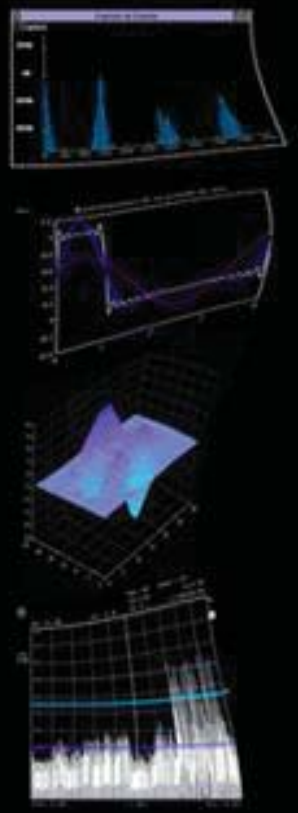
Teorema del muestreo

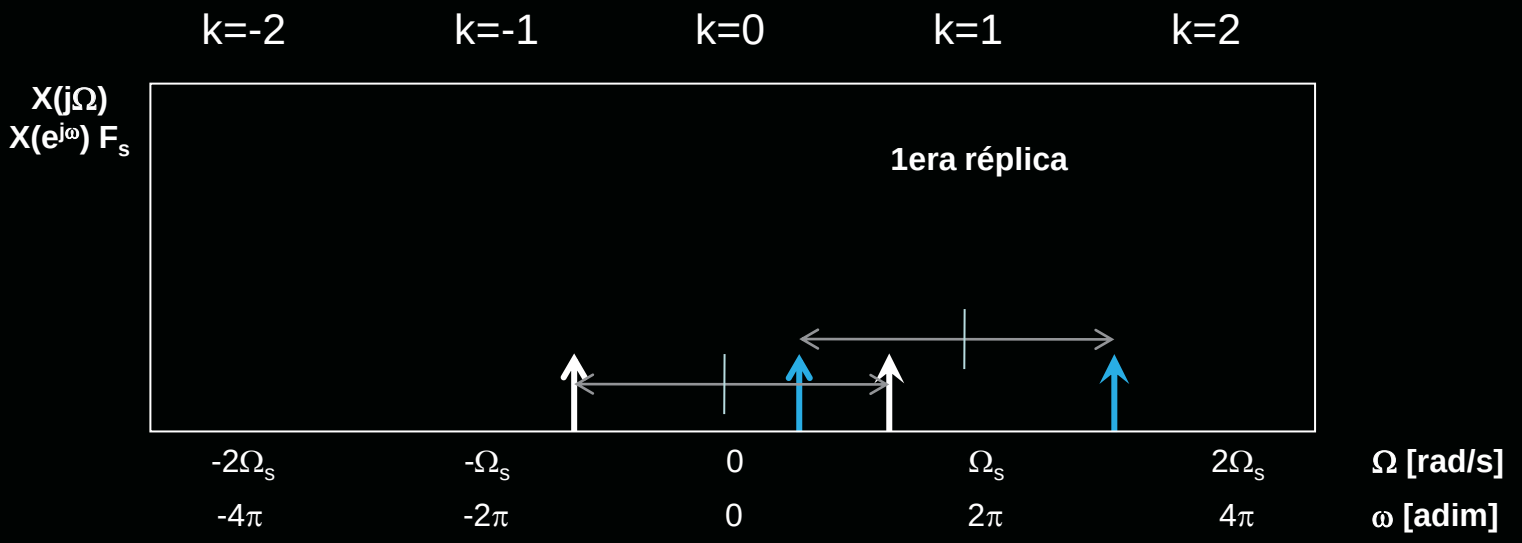




Muestreo y Reconstrucción

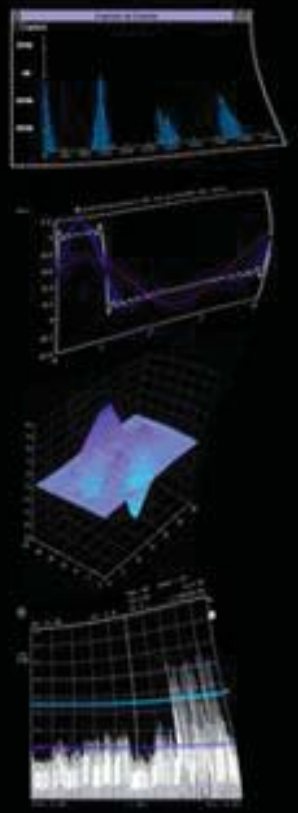
Teorema del muestreo

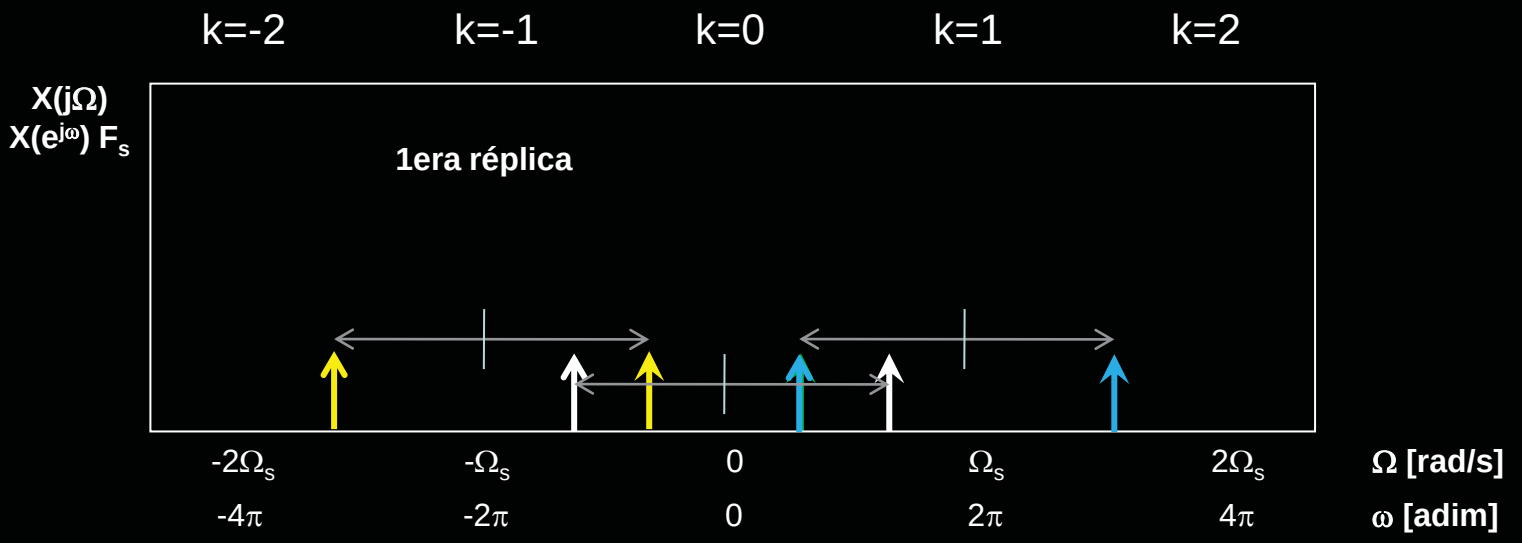




Muestreo y Reconstrucción

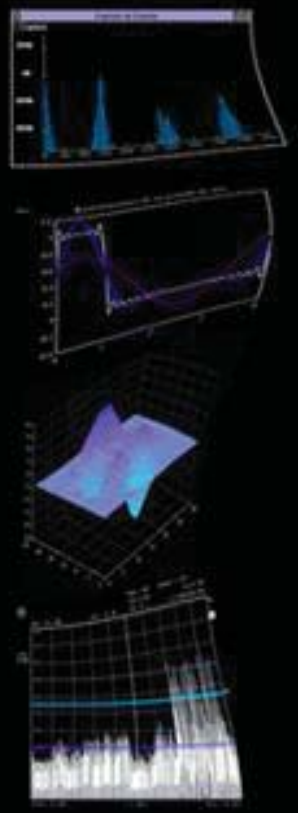
Teorema del muestreo

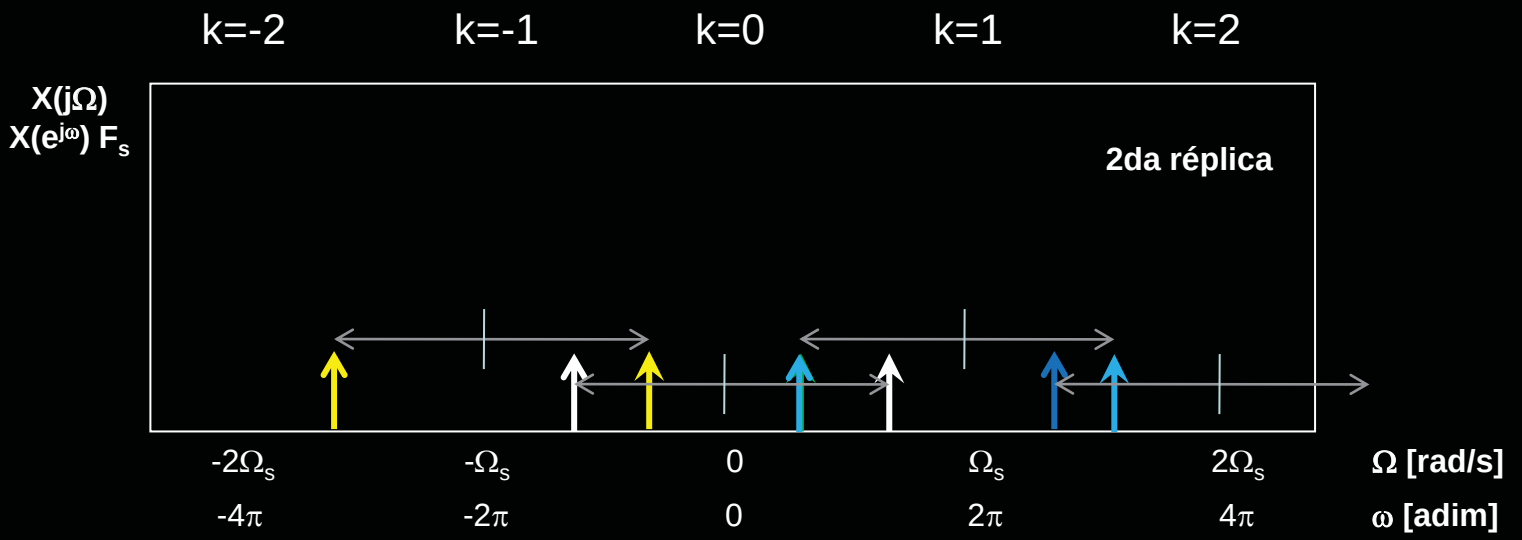




Muestreo y Reconstrucción

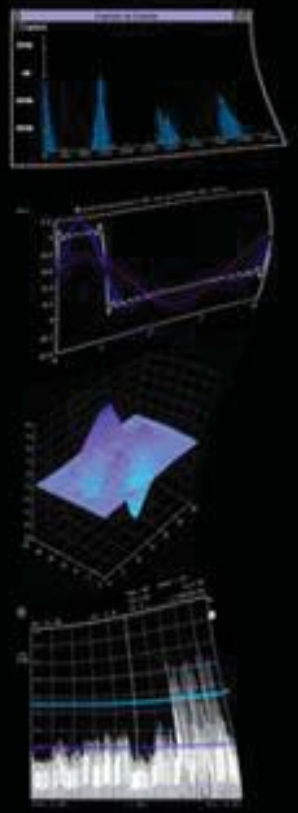
Teorema del muestreo

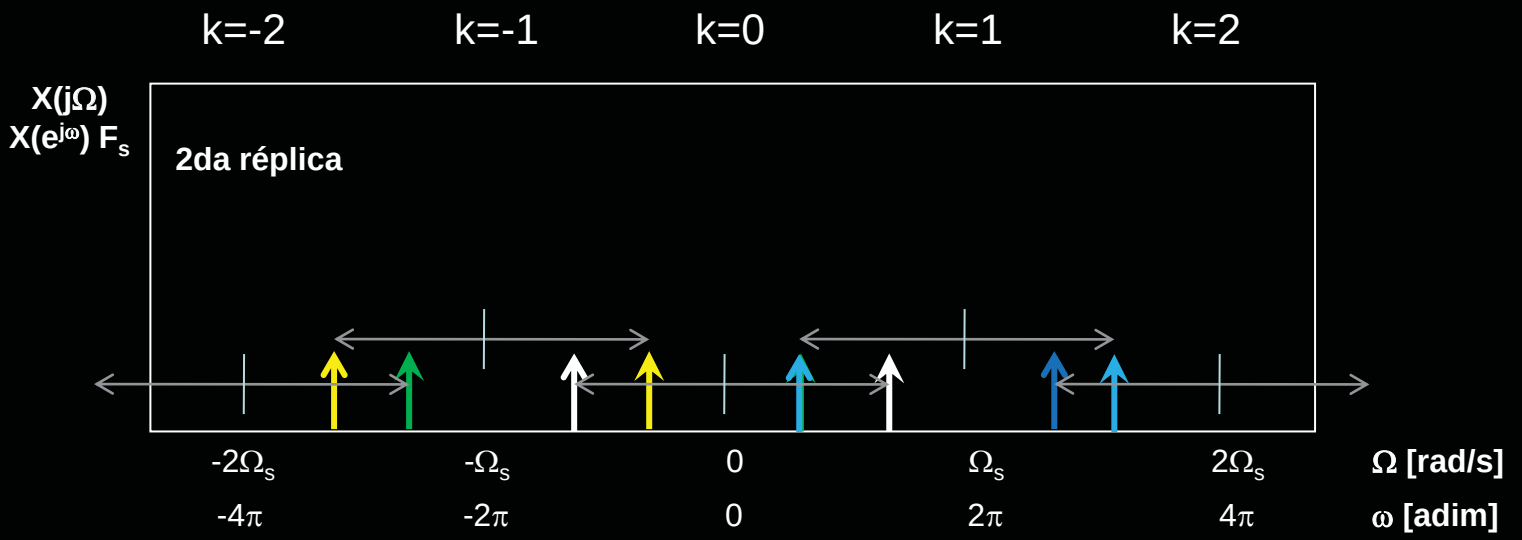




Muestreo y Reconstrucción

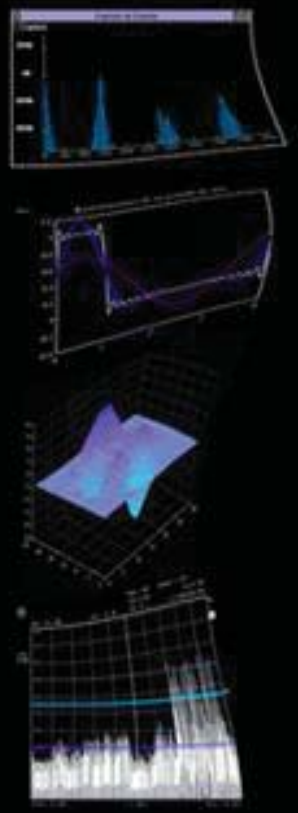
Teorema del muestreo

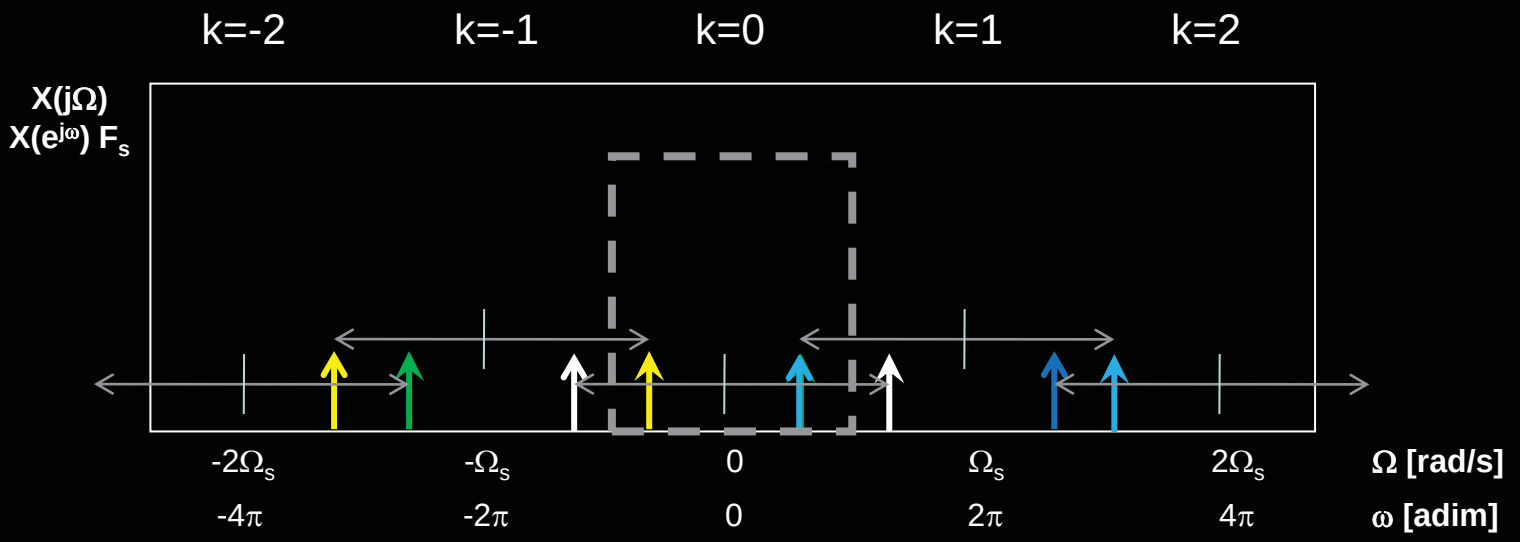




Muestreo y Reconstrucción

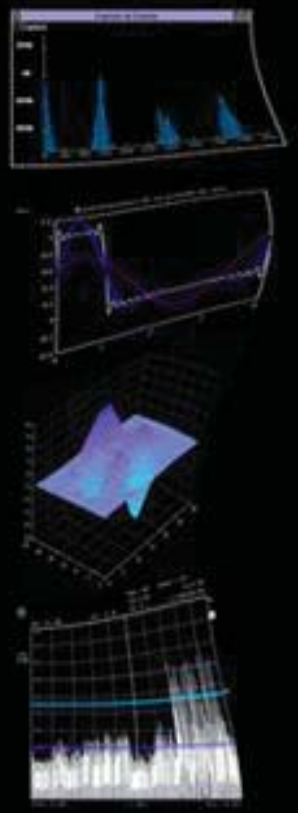
Teorema del muestreo

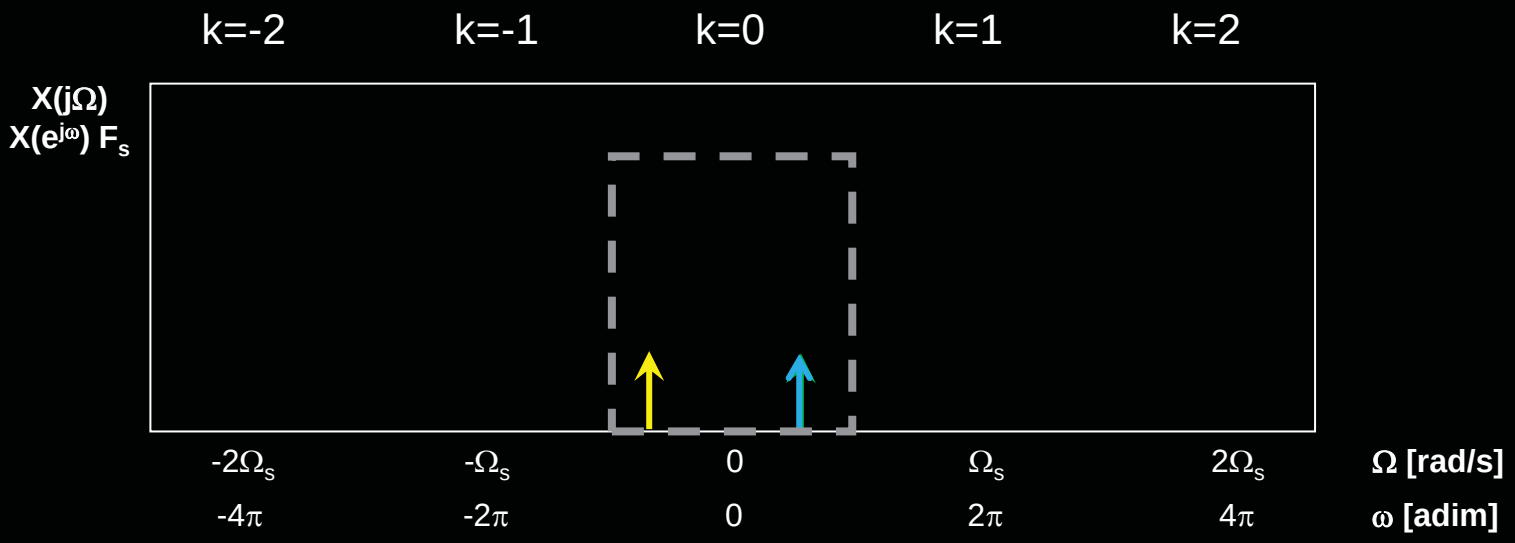




Muestreo y Reconstrucción

Teorema del muestreo





Muestreo y Reconstrucción

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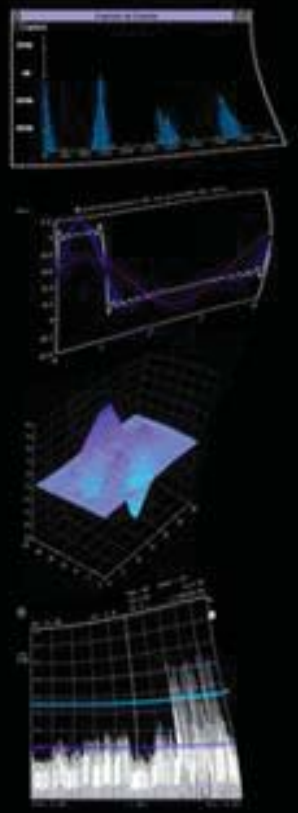
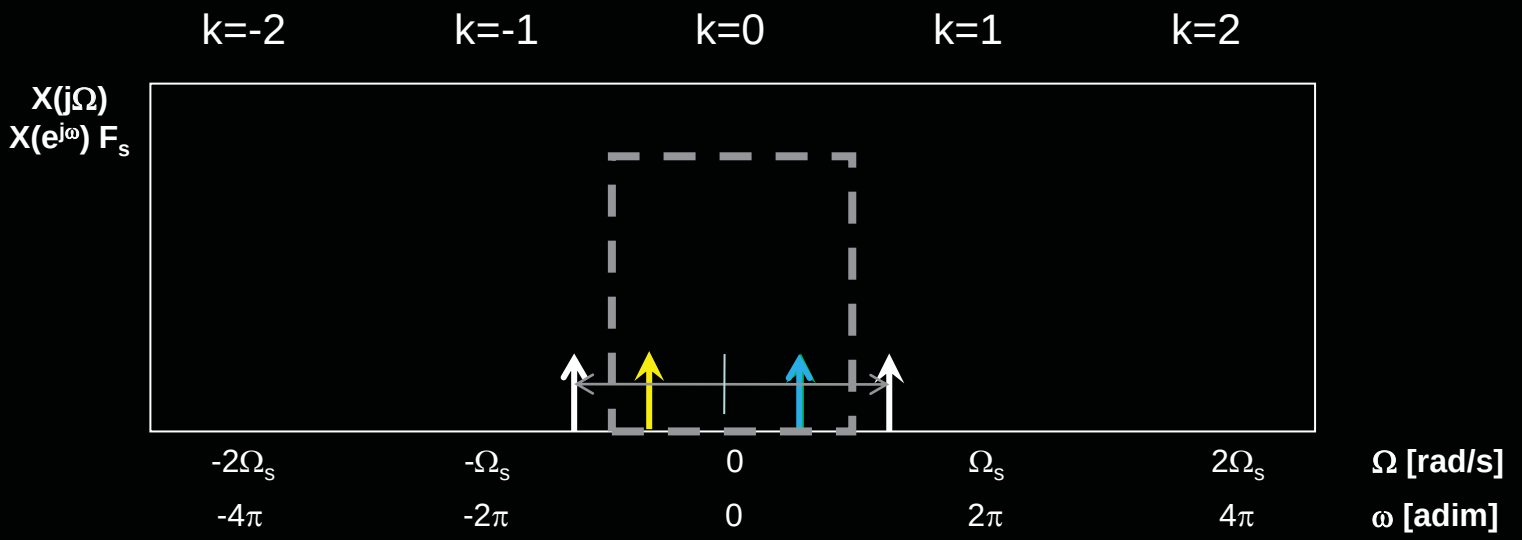
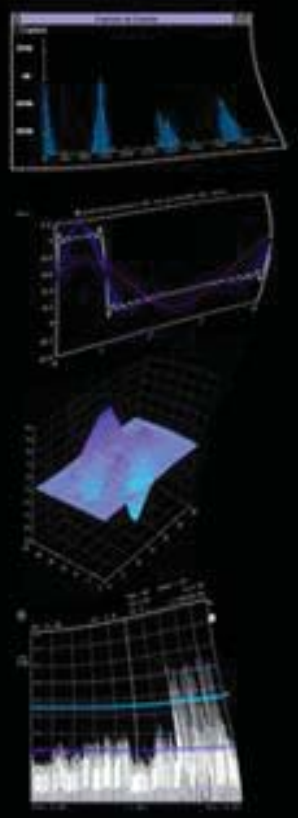
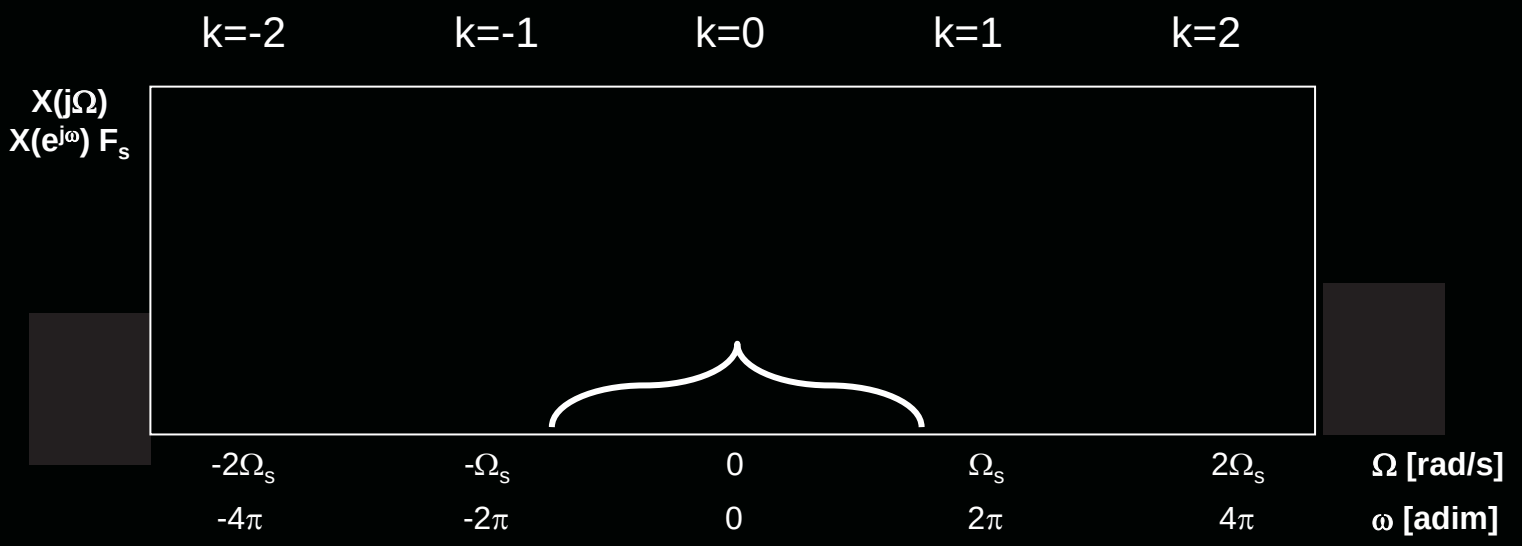




Muestreo y Reconstrucción

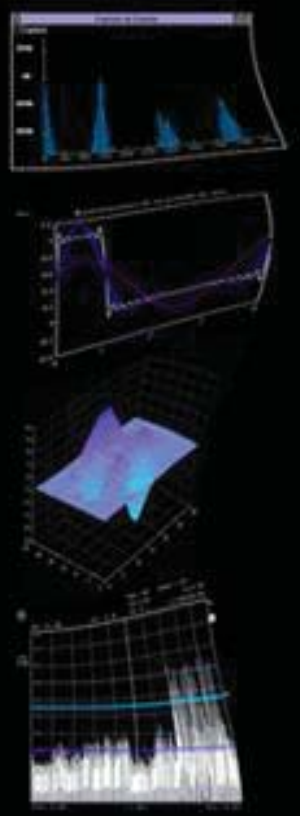
Teorema del muestreo

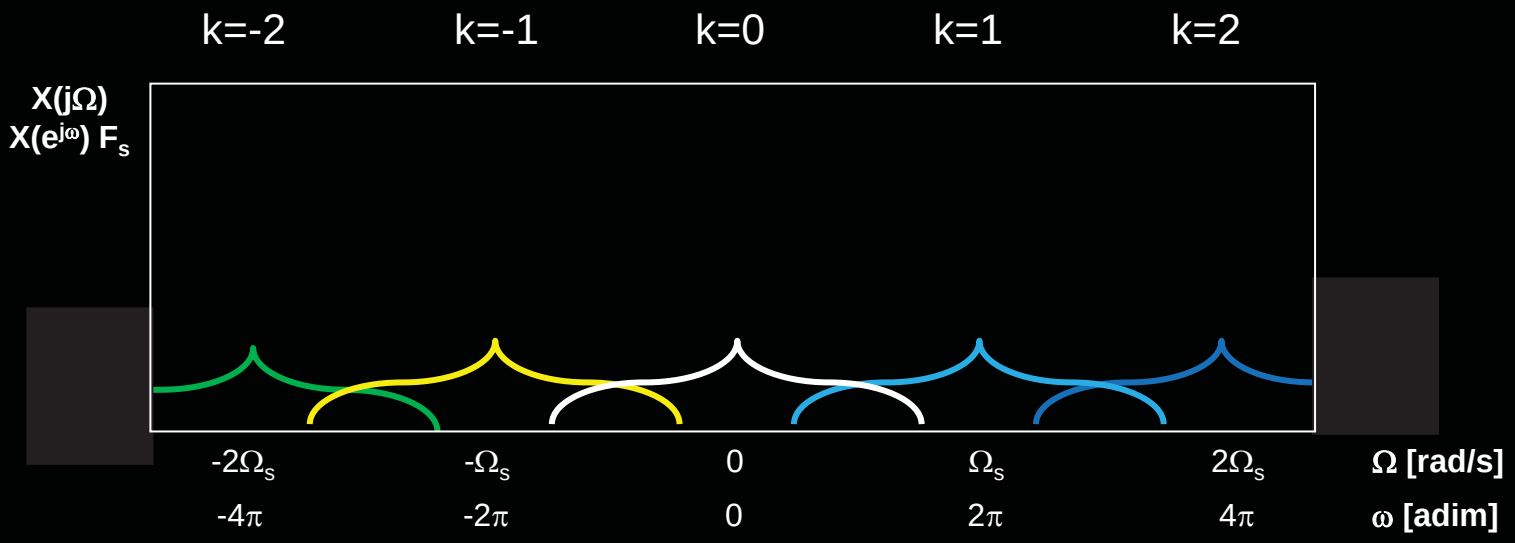




Muestreo y Reconstrucción

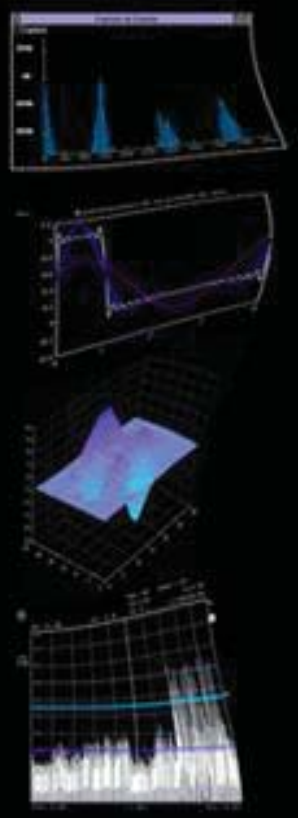
Teorema del muestreo

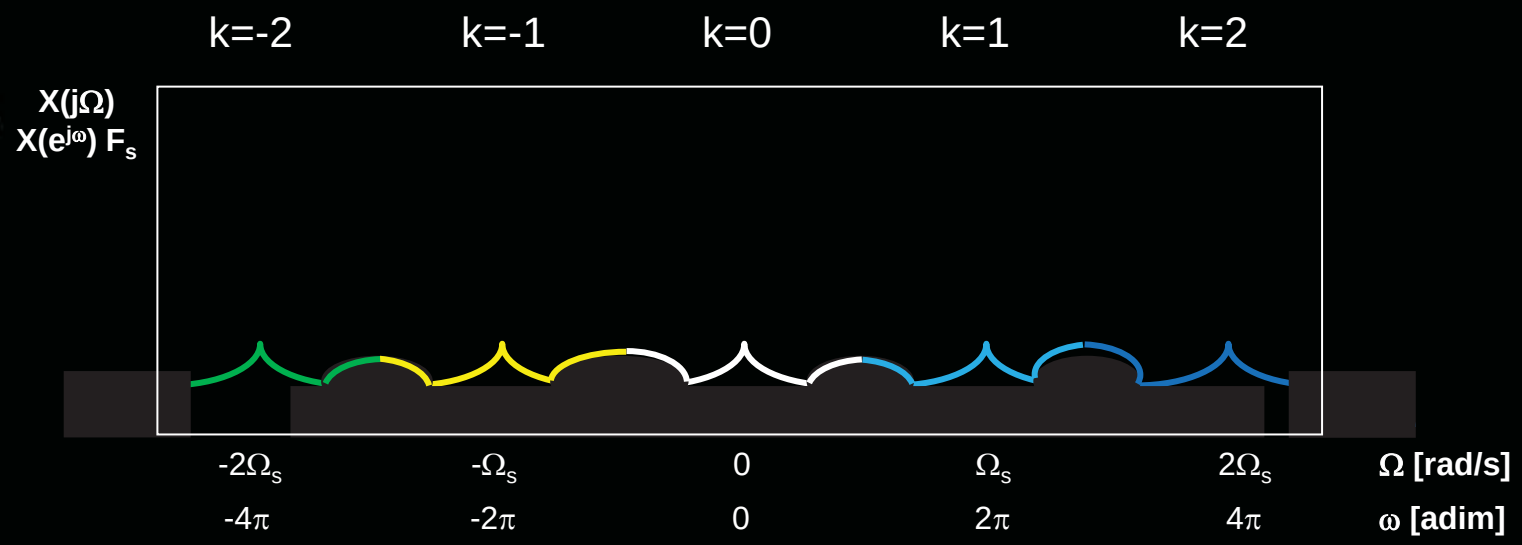




Muestreo y Reconstrucción

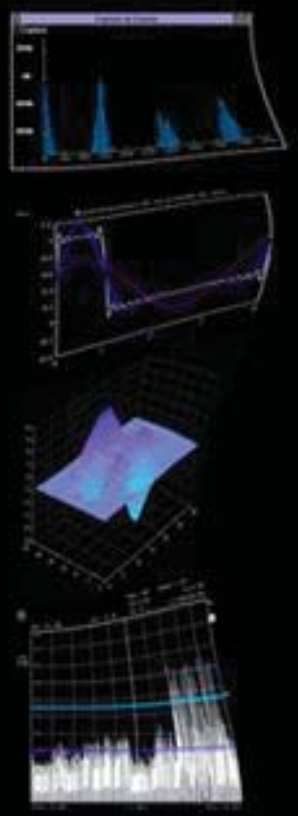
Teorema del muestreo

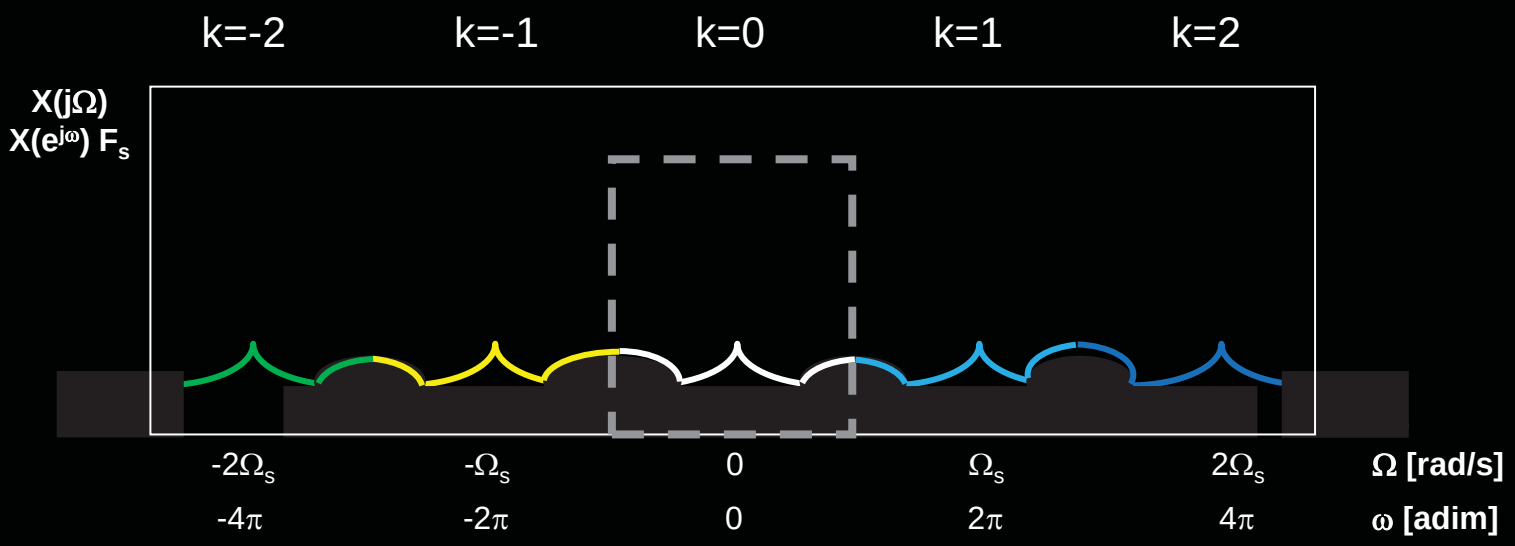




Muestreo y Reconstrucción

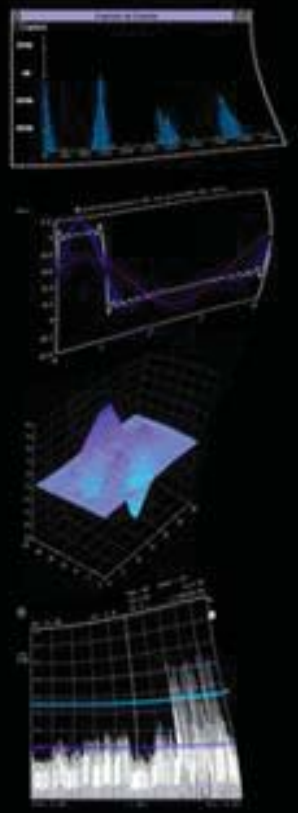
Teorema del muestreo

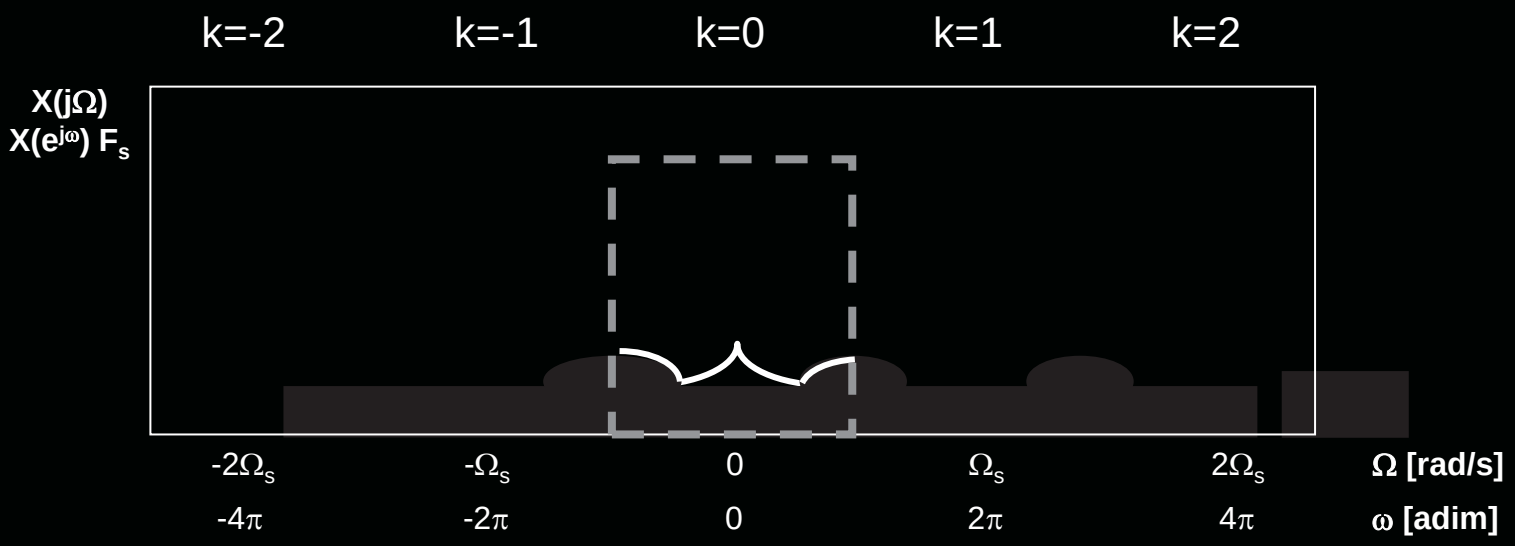




Muestreo y Reconstrucción

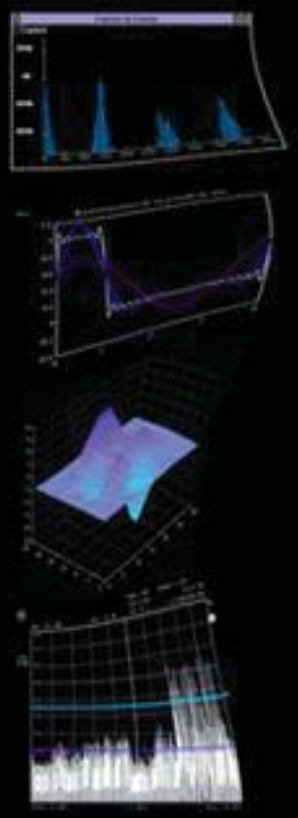
Teorema del muestreo

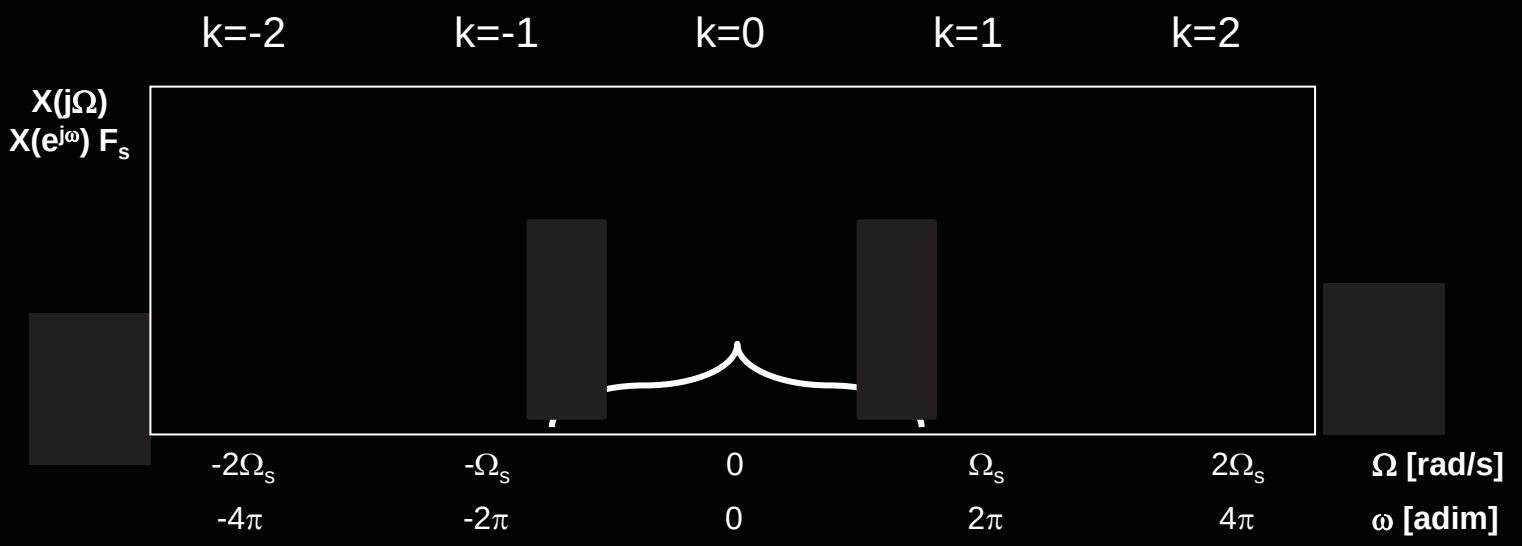




Muestreo y Reconstrucción

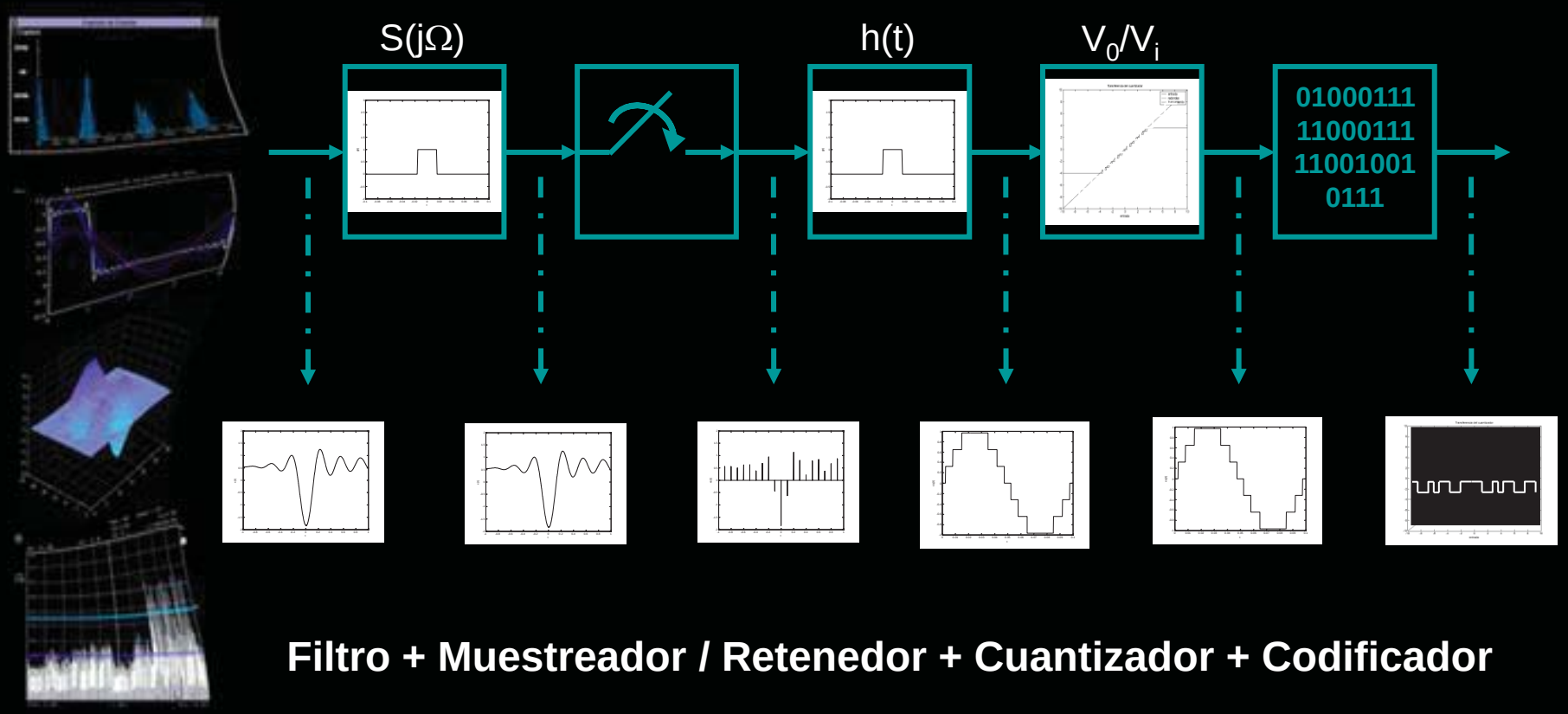
Teorema del muestreo





Muestreo y Reconstrucción

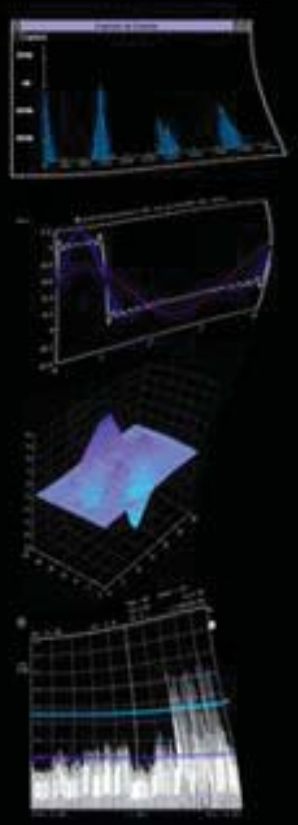
Conversor A/D

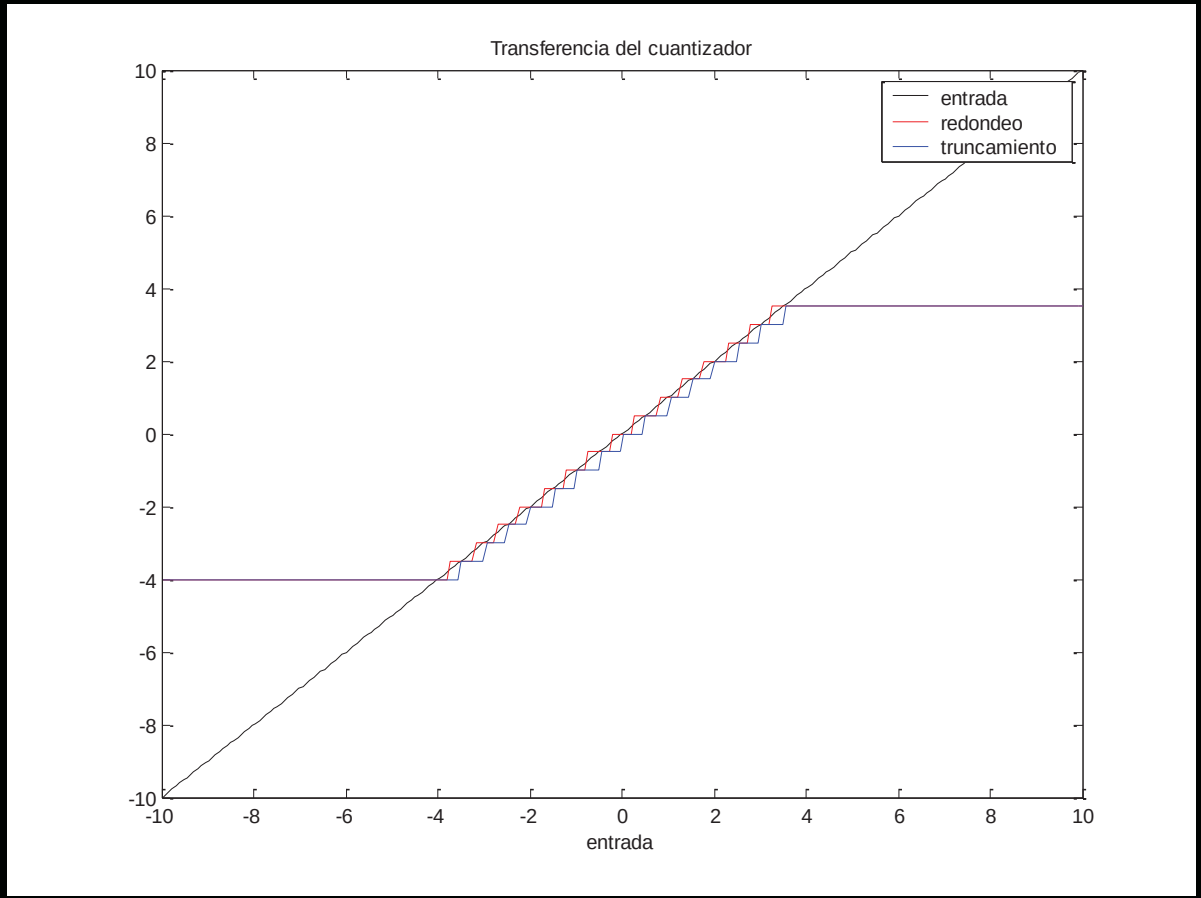


Filtro + Muestreador / Retenedor + Cuantizador + Codificador

Muestreo y Reconstrucción

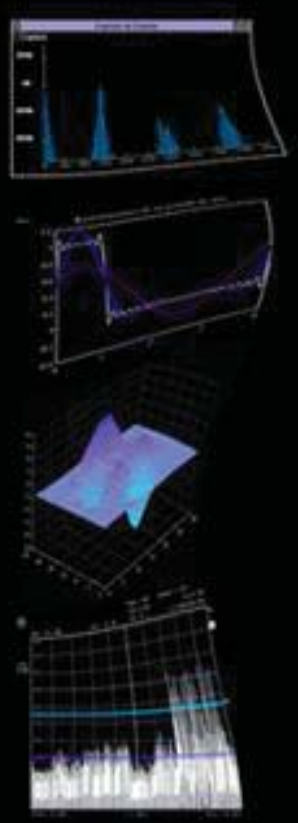
Transferencia de Cuantización

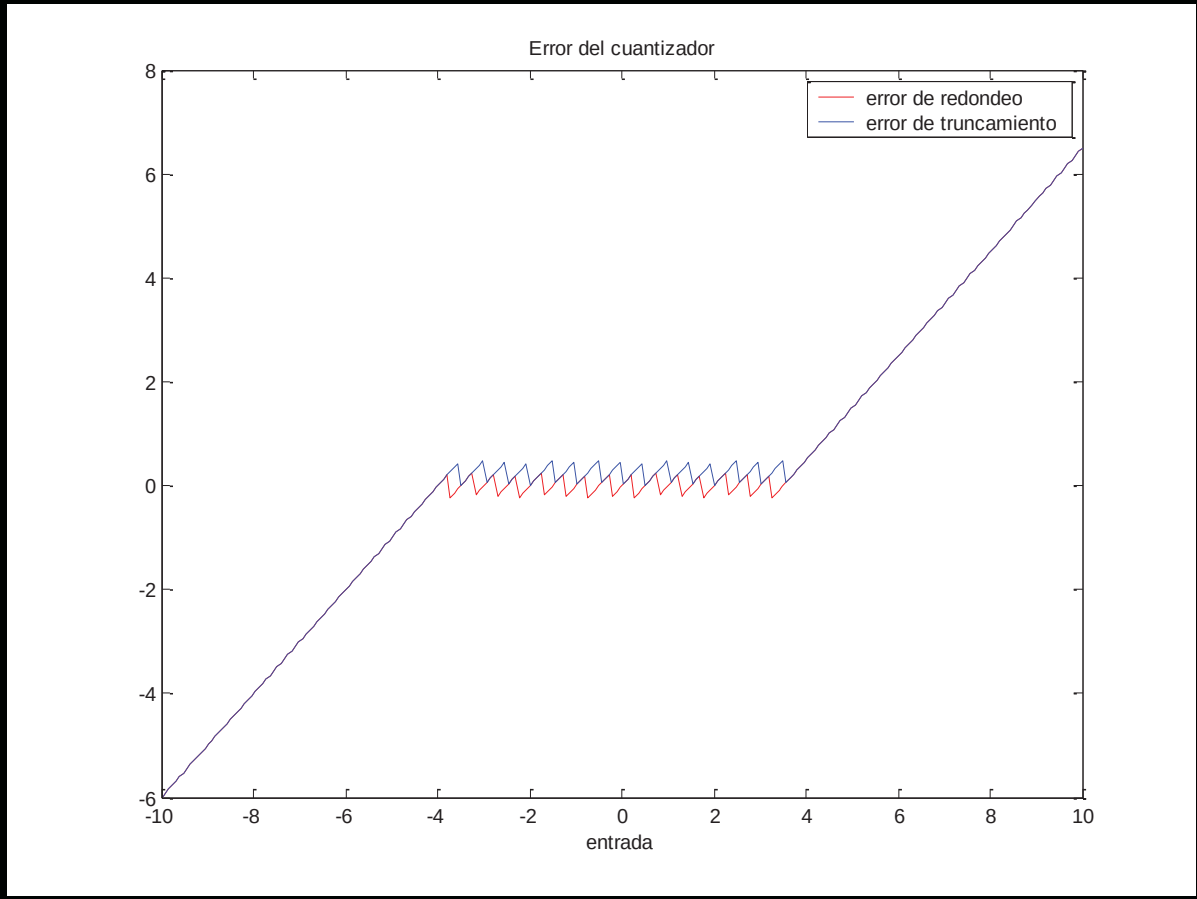




Muestreo y Reconstrucción

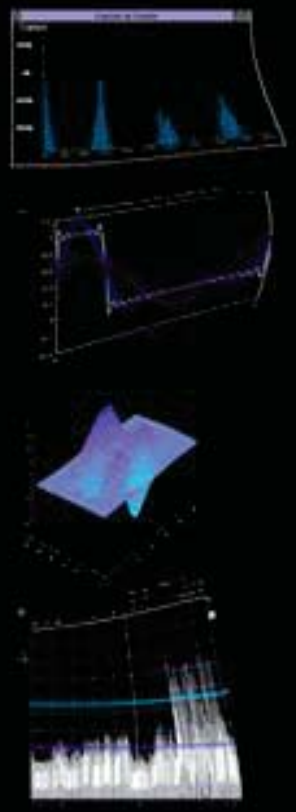
Error de Cuantización





Muestreo y Reconstrucción

Cuantización



Señal cuantizada

$$x_q[n] = \square\{x[n]\}$$

Error de cuantización

$$e_q[n] = x_q[n] - x[n]$$

$$-\frac{\Delta}{2} \le e_q[n] \le \frac{\Delta}{2}$$

Paso de cuantización

$$\Delta = \frac{x_{máx}[n] - x_{mín}[n]}{L-1} = \frac{2A}{L-1} = \frac{2A}{2^b}$$