

2384-24

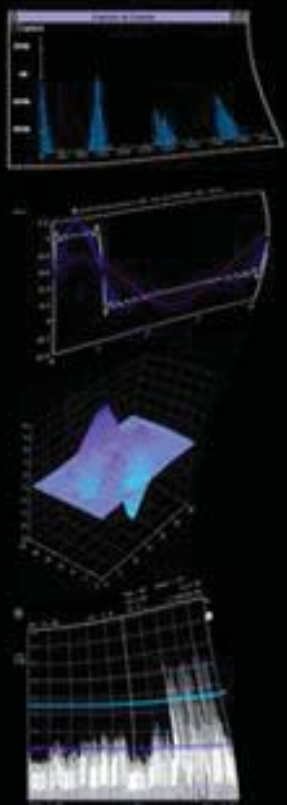
**ICTP Latin-American Advanced Course on FPGA Design for Scientific  
Instrumentation**

*19 November - 7 December, 2012*

**Estructura de filtros**

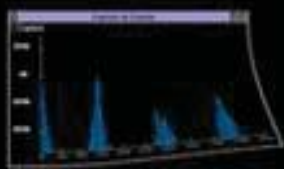
COSTA Diego Esteban  
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Av. Ejercito de los Andes, D5700HHW San Luis  
ARGENTINA*

# Estructura de filtros

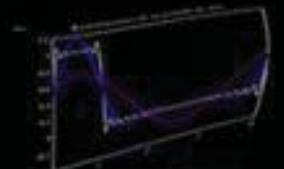


# Estructura de Filtros

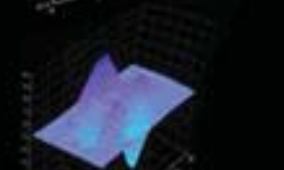
## Definición



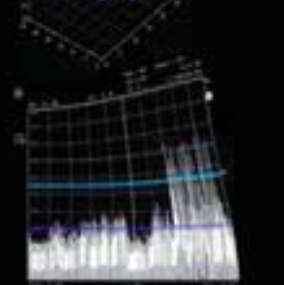
**Es la forma en la que se organiza la implementación**



**Elementos: Retardos, ganancias y sumadores**



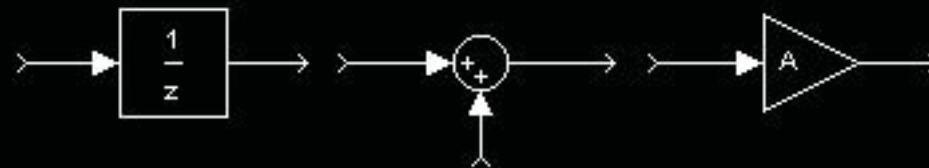
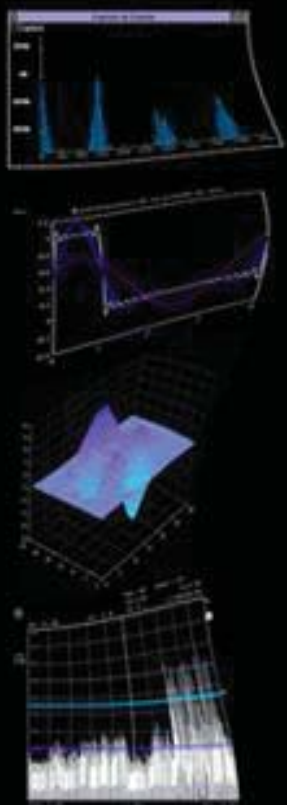
**Es definida por los elementos y la forma en que se vinculan**



**Puede ser hardware o software**

# Estructura de Filtros

## Diagrama de bloques



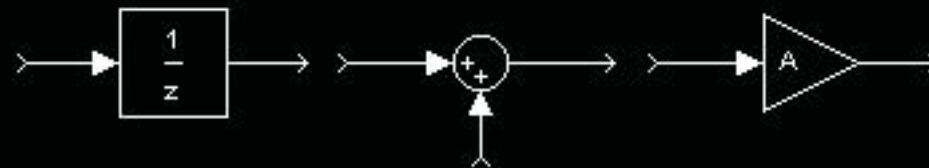
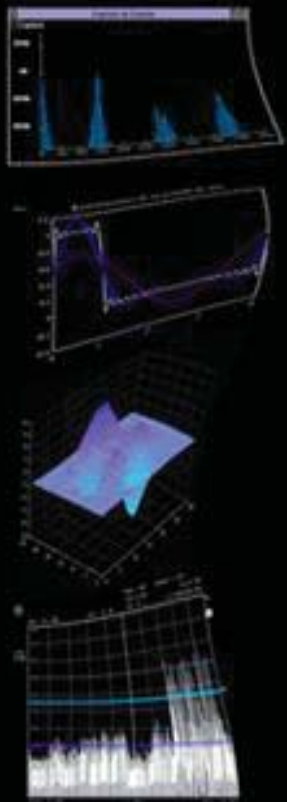
retardo

suma

producto

# Estructura de Filtros

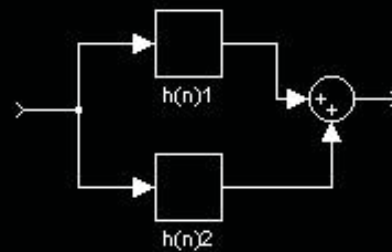
## Diagrama de bloques



retardo

producto

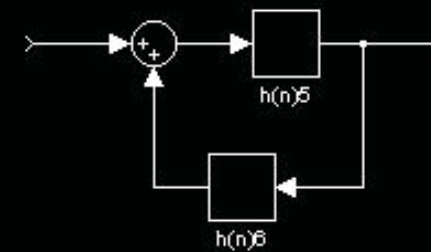
suma



paralelo



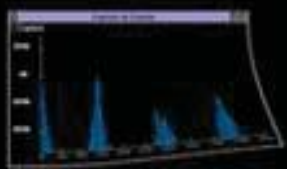
cascada



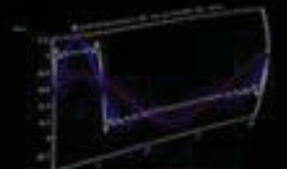
realimentación

# Estructura de Filtros

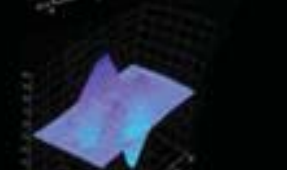
## Características



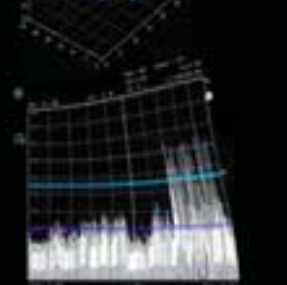
*et* Complejidad computacional



*et* Requisitos de memoria



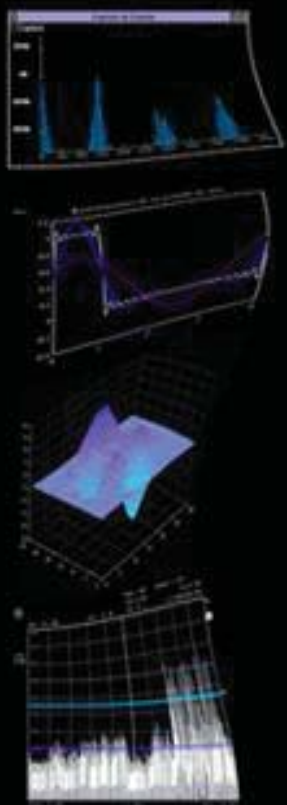
*et* Efectos de las palabras de longitud finita



*et* Factores adicionales (paralelización, pipeline, etc)

# Estructura de Filtros

## Análisis de sistemas LIT



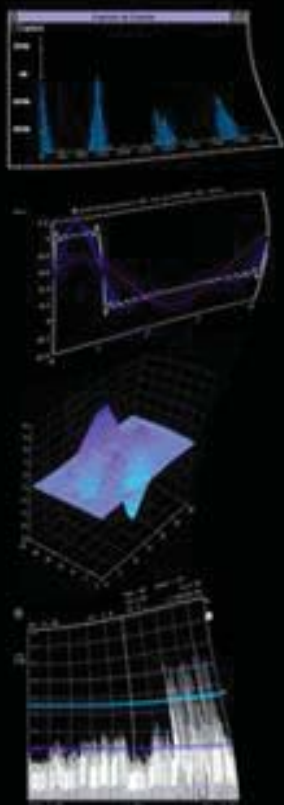
$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = \sum_{k=0}^M b_k x[n-k] - \sum_{k=1}^N a_k y[n-k]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{\left(1 + \sum_{k=1}^N a_k z^{-k}\right)}$$

# Estructura de Filtros

## Análisis de sistemas LIT



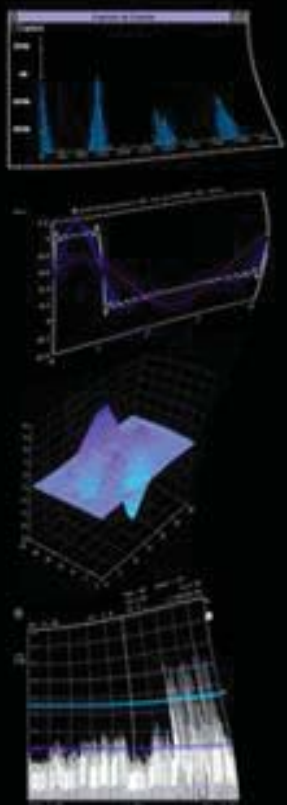
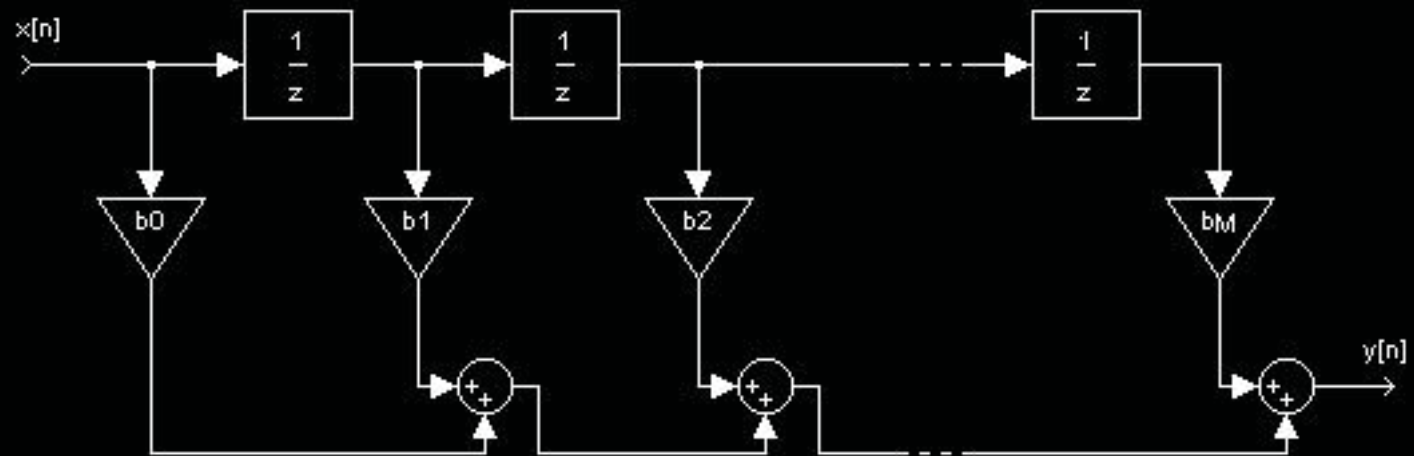
FIR	IIR
<p><b>MA (Promediador Móvil)</b></p> $H(z) = \sum_{k=0}^M b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^M b_k z^{M-k}$ <p>M ceros y 1 polo de orden M</p>	<p><b>AR (Autoregresivo)</b></p> $H(z) = \frac{b_0}{\sum_{k=0}^N a_k z^{-k}} = \frac{b_0 z^N}{\sum_{k=0}^N a_k z^{N-k}}$ <p>N polos y 1 cero de orden N</p>
	<p><b>ARMA (Promediador Móvil Autoregresivo)</b></p> $H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{\sum_{k=0}^N a_k z^{-k}}$ <p>M ceros y N polos no triviales</p>



# Estructura de Filtros

## FIR – Forma Directa

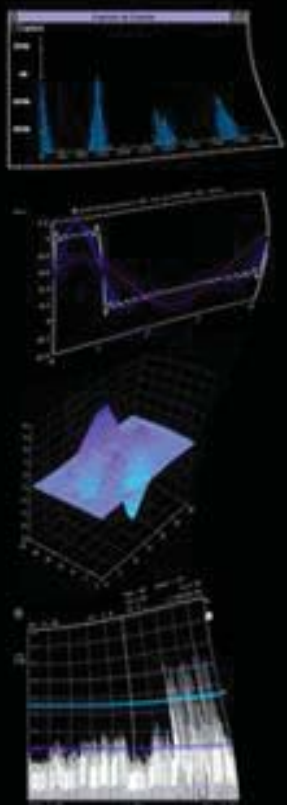
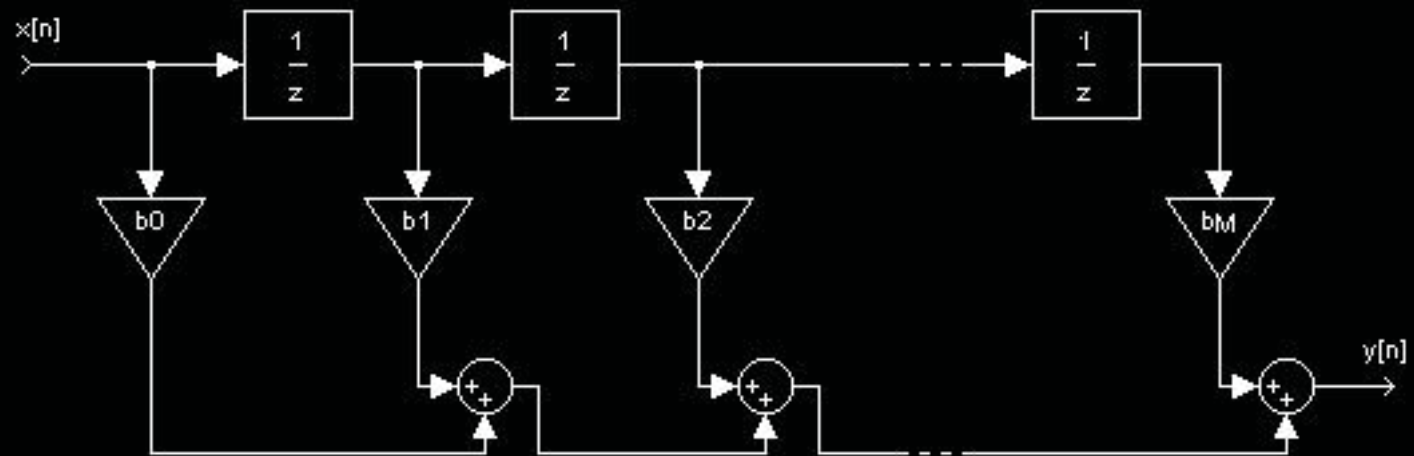
$$y[n] = \sum_{k=0}^M b_k x[n-k]$$



# Estructura de Filtros

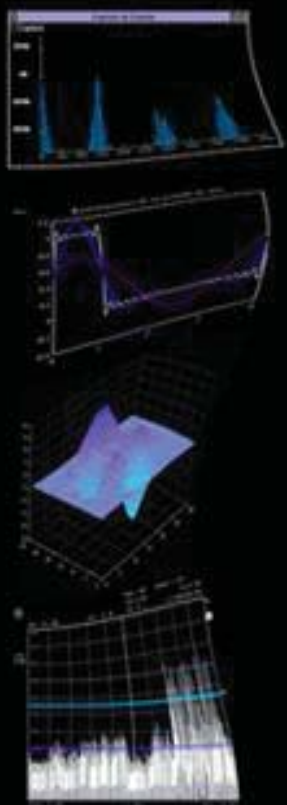
## FIR – Forma Directa

M+1 productos, M sumas y M retardos

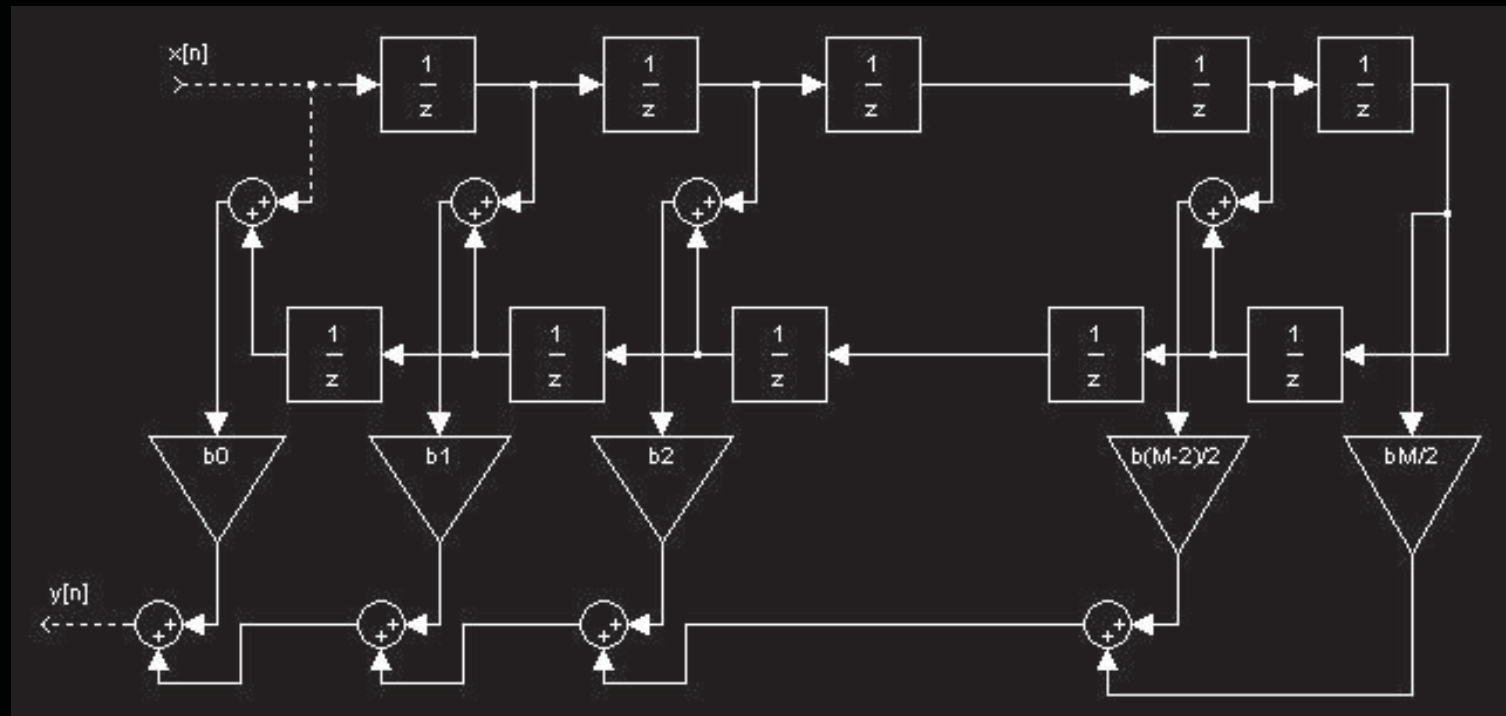


# Estructura de Filtros

## FIR – Forma Directa con simetría



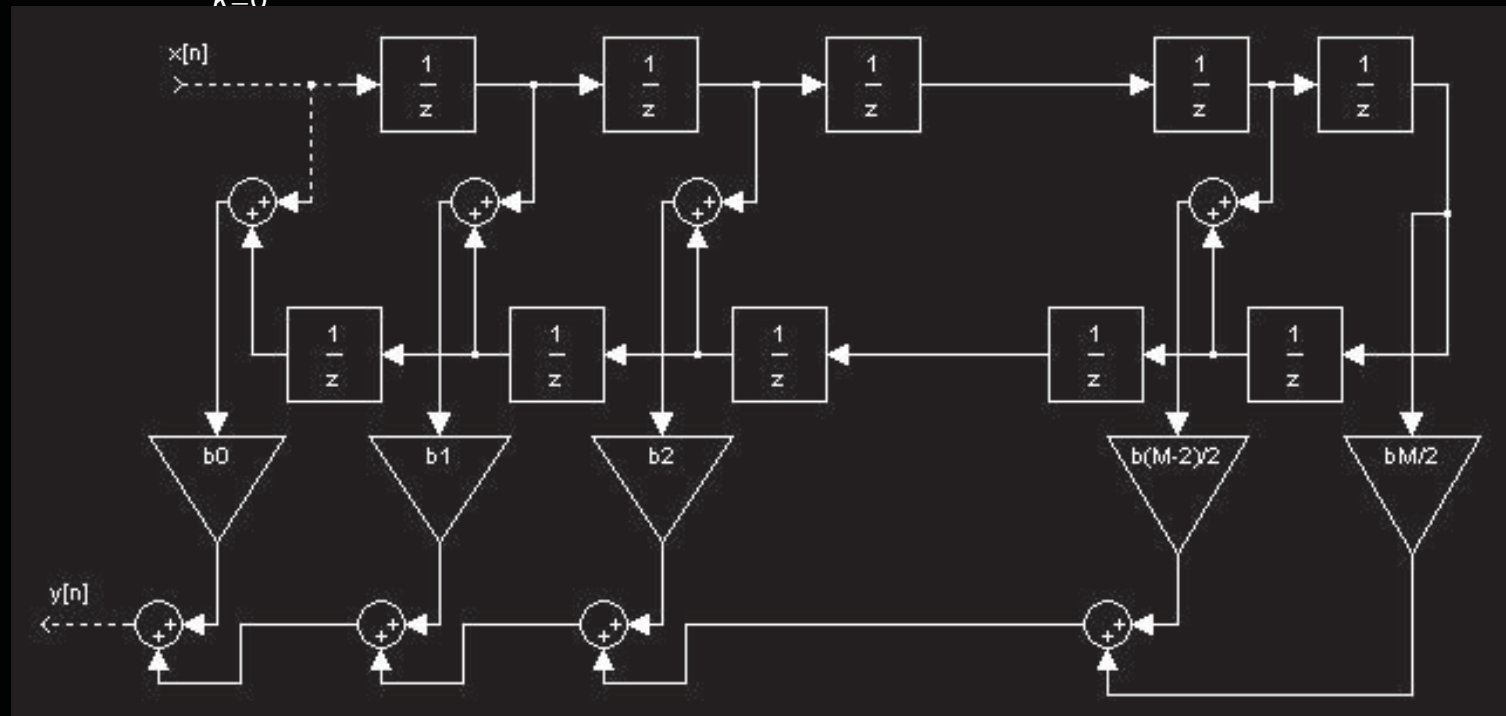
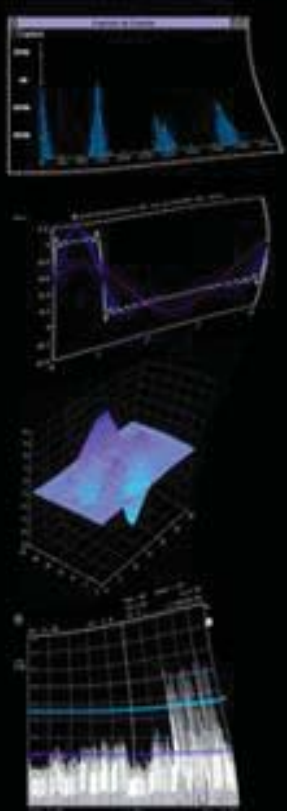
$$h[n] = \pm h[M-n]$$



# Estructura de Filtros

## FIR – Forma Directa con simetría

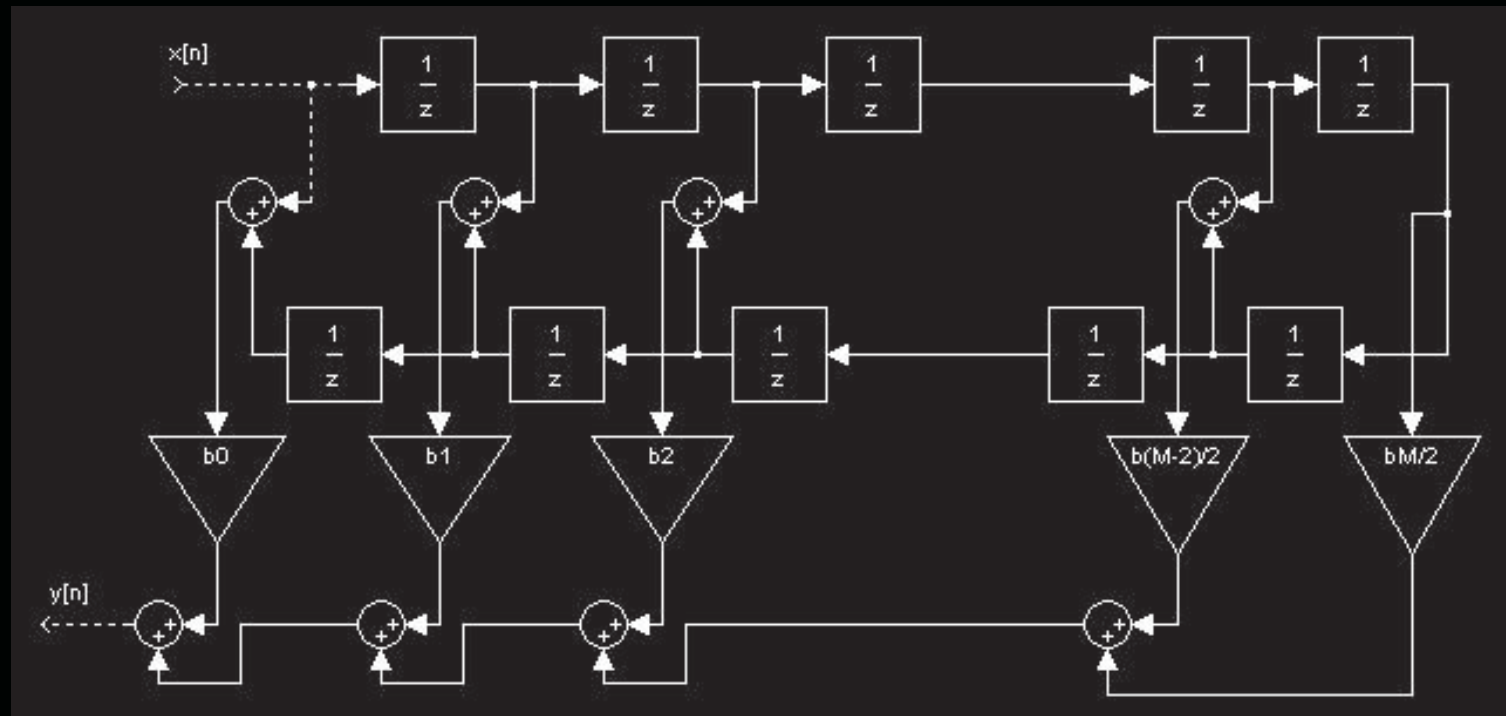
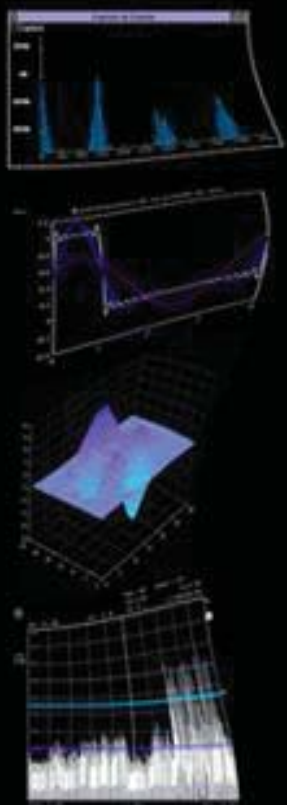
$$y[n] = \sum_{k=0}^K b_k (x[n-k] + x[n-(N-k)])$$



# Estructura de Filtros

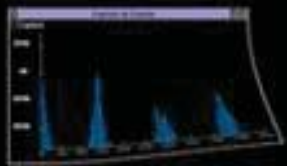
## FIR – Forma Directa con simetría

$(M+1)/2$  ó  $M/2$  productos,  $M$  sumas y  $M$  retardos

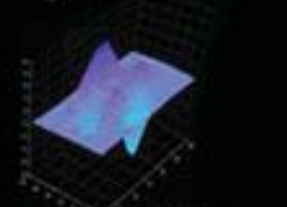
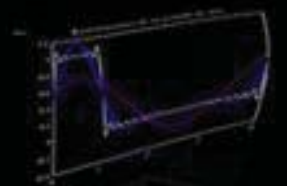


# Estructura de Filtros

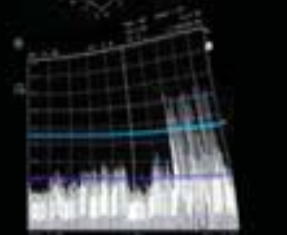
## IIR – Forma Directa I



$$\begin{cases} v[n] = \sum_{k=0}^M b_k x[n-k] \\ y[n] = \frac{1}{a_0} \left[ v[n] - \sum_{k=1}^N a_k y[n-k] \right] \end{cases}$$

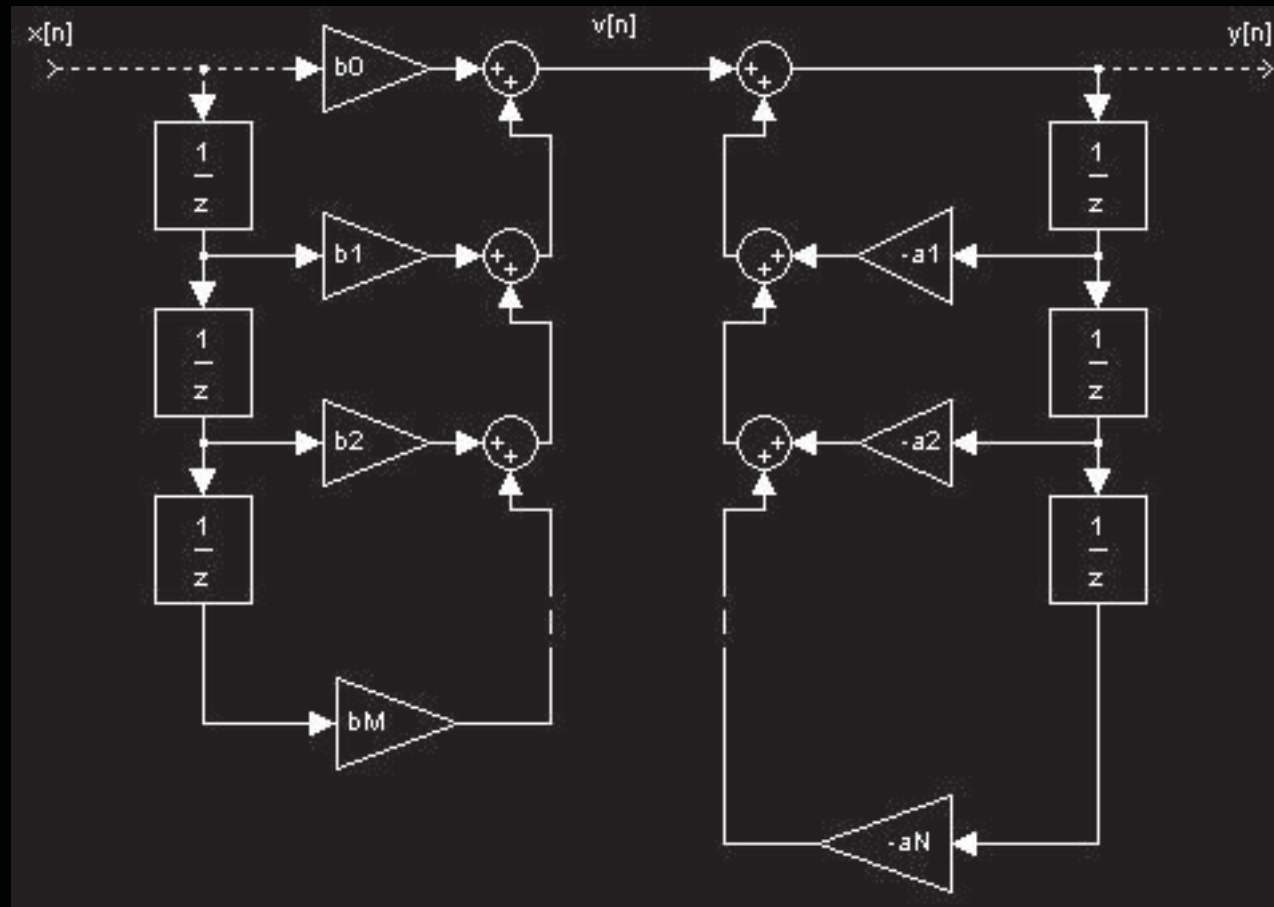
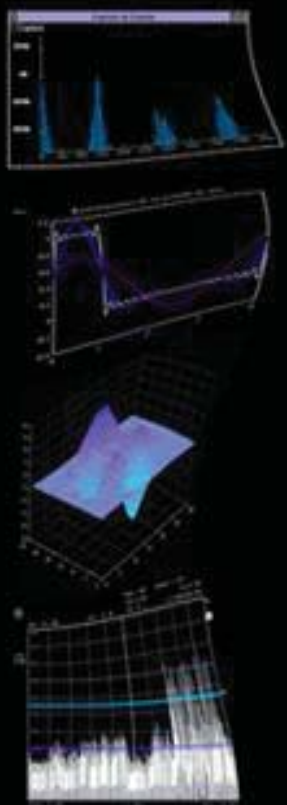


$$\begin{cases} V(z) = X(z)N(z) = X(z) \sum_{k=0}^M b_k z^{-k} \\ Y(z) = V(z) \frac{1}{D(z)} = \frac{W(z)}{\sum_{k=0}^N a_k z^{-k}} \end{cases}$$



# Estructura de Filtros

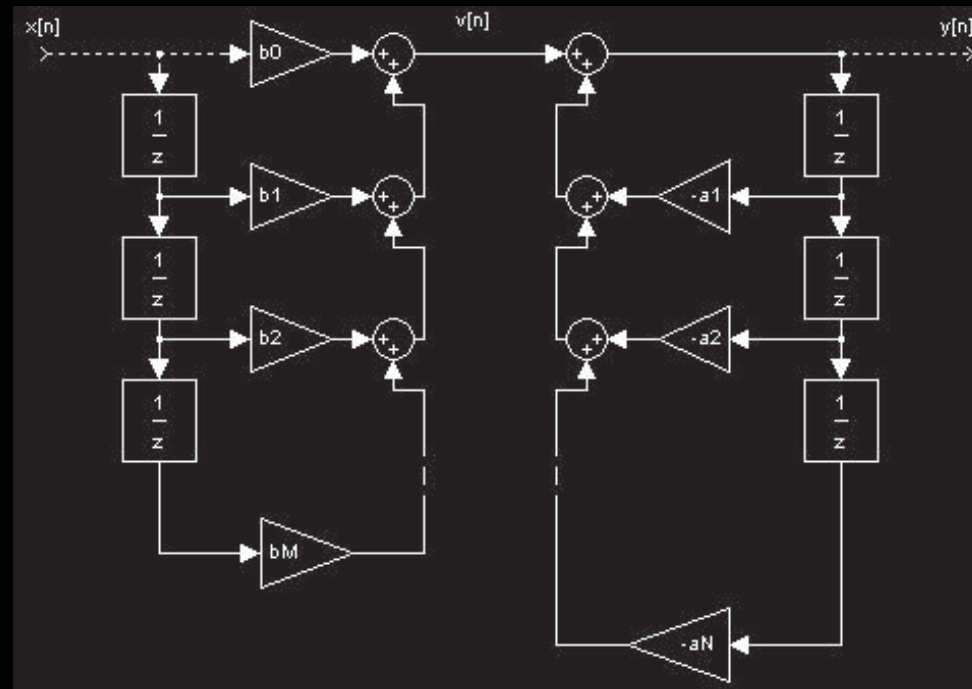
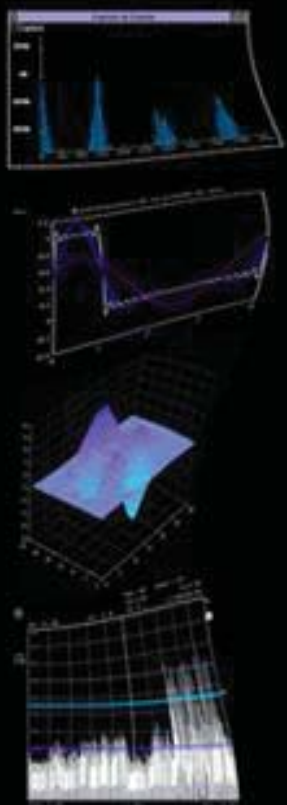
## IIR – Forma Directa I



# Estructura de Filtros

## IIR – Forma Directa I

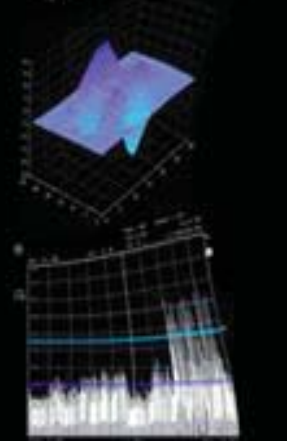
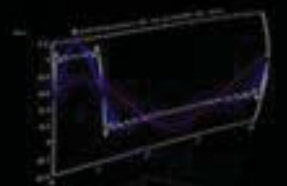
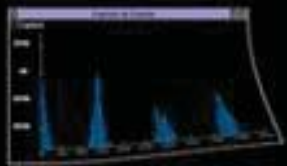
**M+N+1 productos, M+N sumas y M+N retardos**





# Estructura de Filtros

## IIR – Forma Directa II

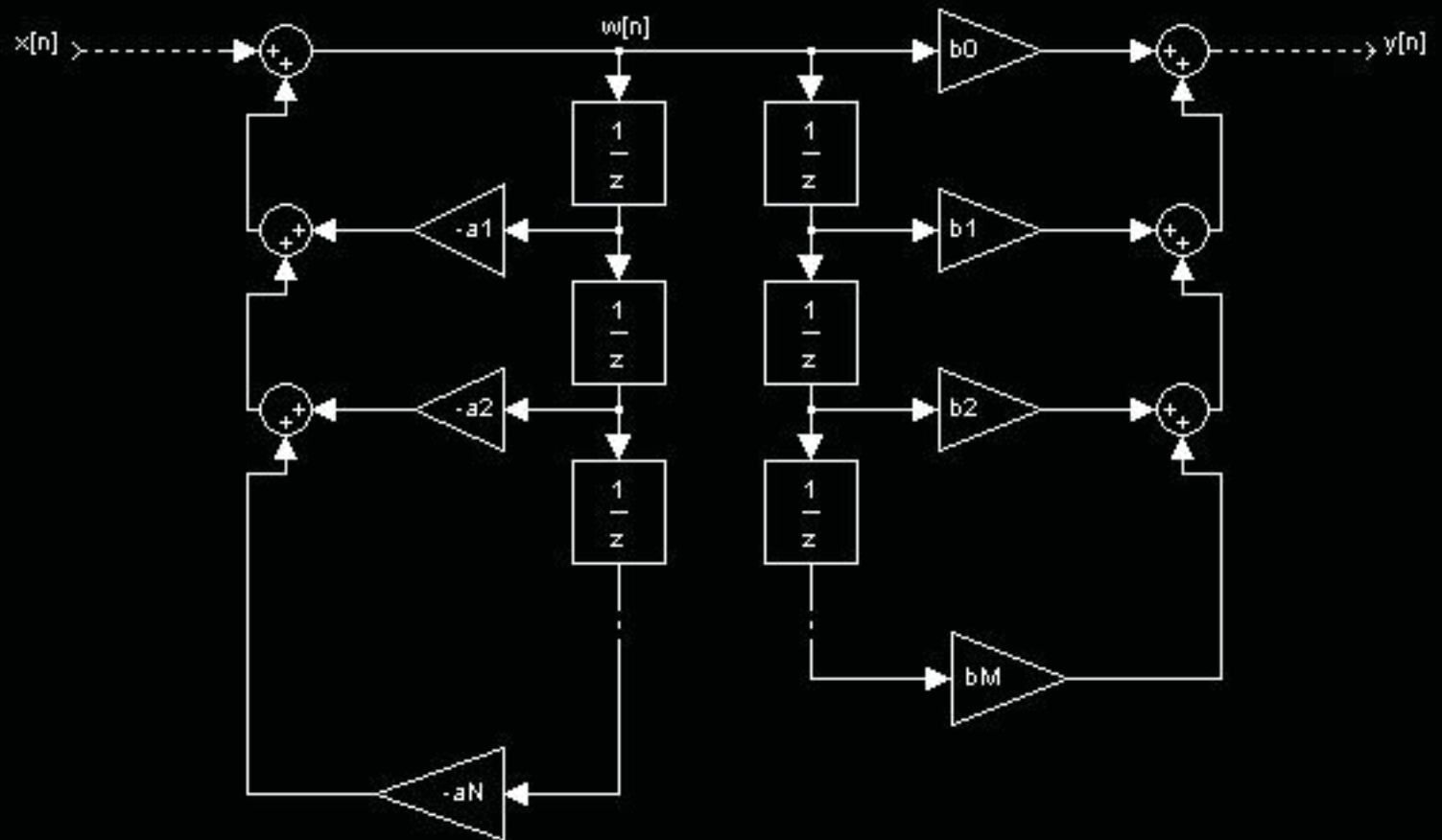
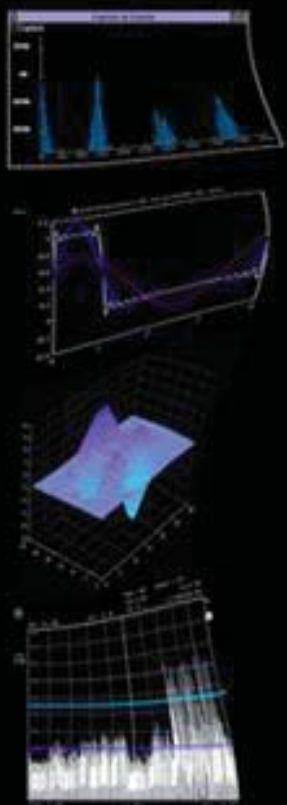


$$\begin{cases} w[n] = \frac{1}{a_0} \left[ x[n] - \sum_{k=1}^N a_k w[n-k] \right] \\ y[n] = \sum_{k=0}^M b_k w[n-k] \end{cases}$$

$$\begin{cases} W(z) = X(z) \frac{1}{D(z)} = \frac{X(z)}{\sum_{k=0}^N a_k z^{-k}} \\ Y(z) = W(z)N(z) = W(z) \sum_{k=0}^M b_k z^{-k} \end{cases}$$

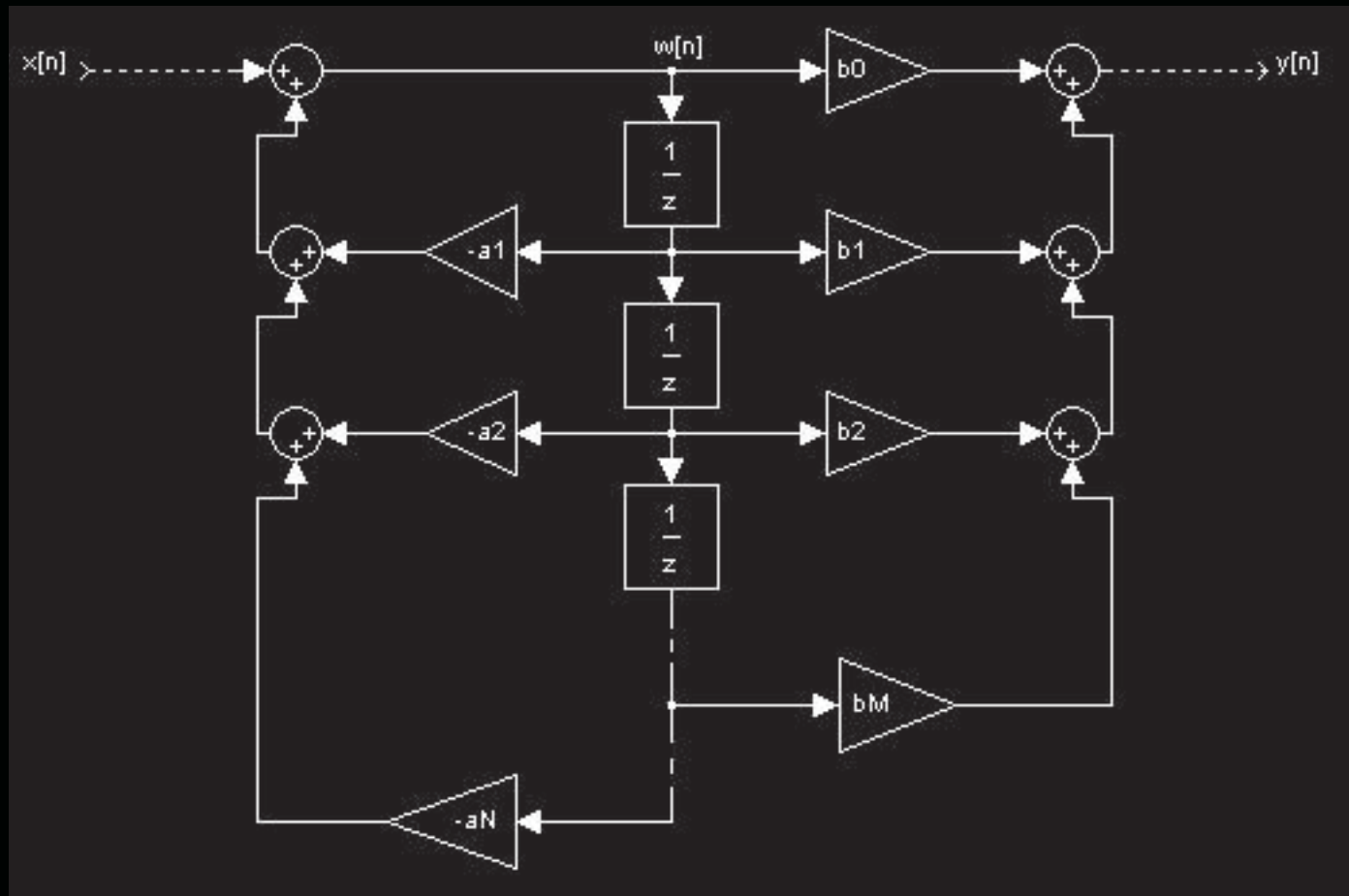
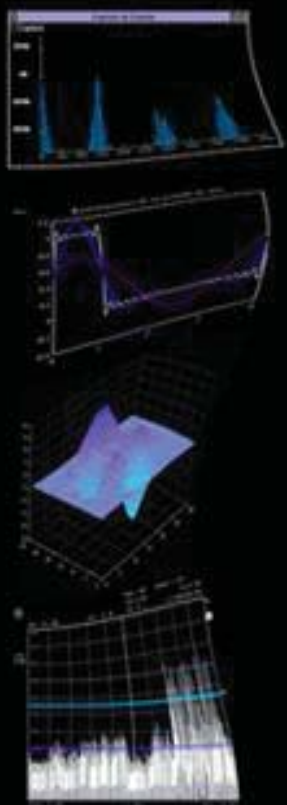
# Estructura de Filtros

## IIR – Forma Directa II



# Estructura de Filtros

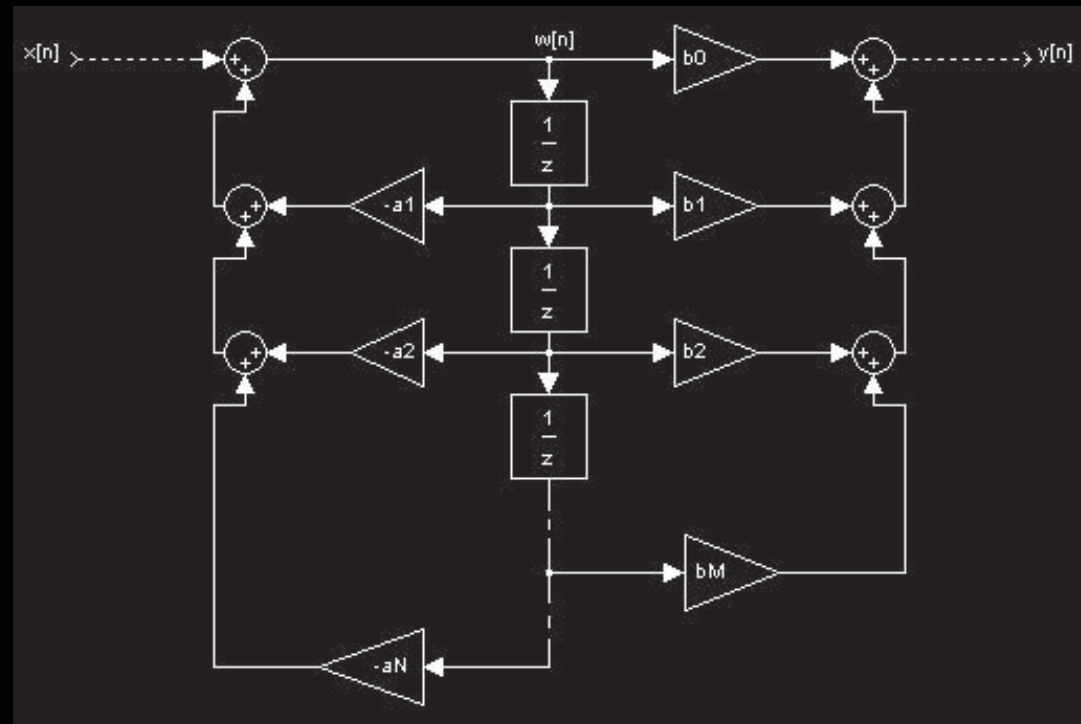
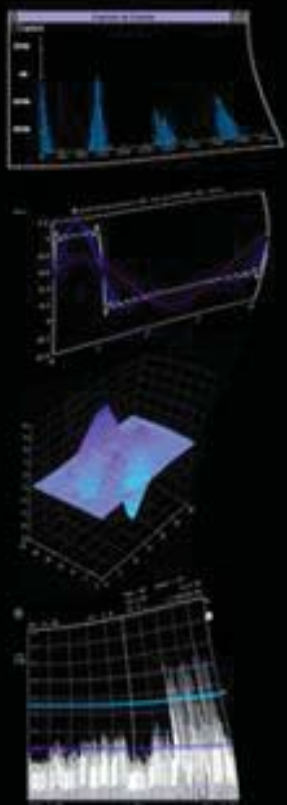
## IIR – Forma Directa II



# Estructura de Filtros

## IIR – Forma Directa II

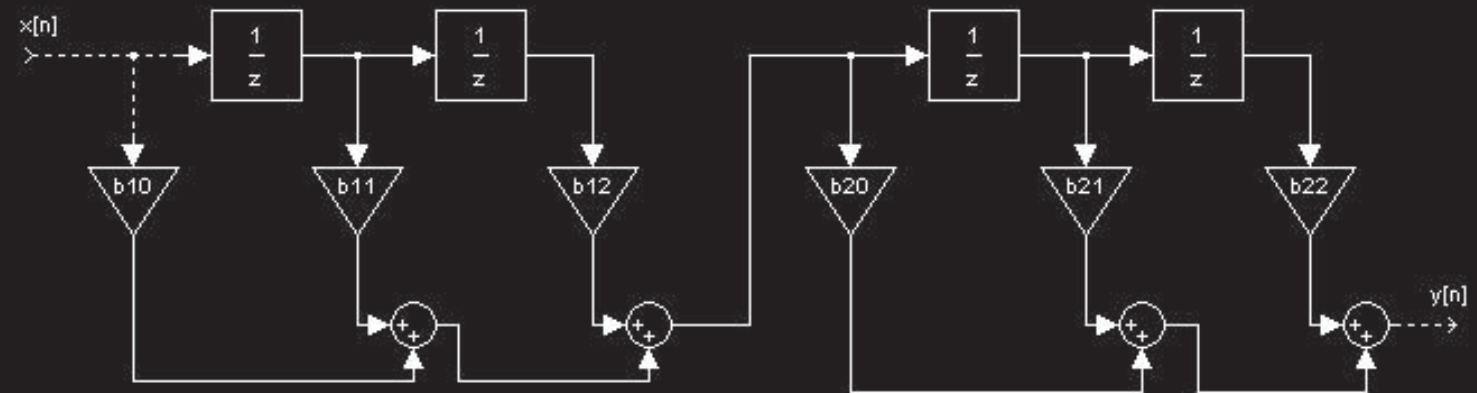
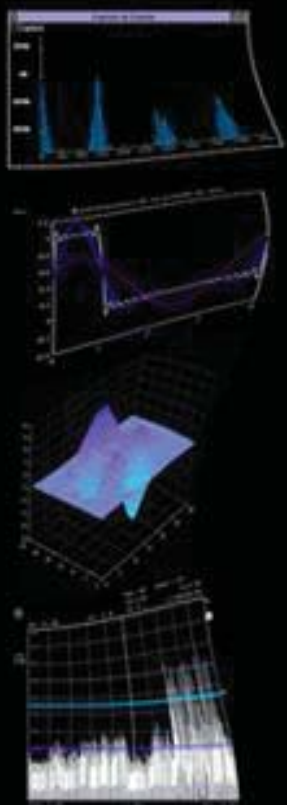
**$M+N+1$  productos,  $M+N$  sumas y  $\max\{M;N\}$  retardos**



# Estructura de Filtros

## FIR Cascada

$$H(z) = \prod_{k=1}^K H_k(z) = \prod_{k=1}^K (b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}) \quad ; \quad K = \text{int} \left\{ \frac{M+1}{2} \right\}$$



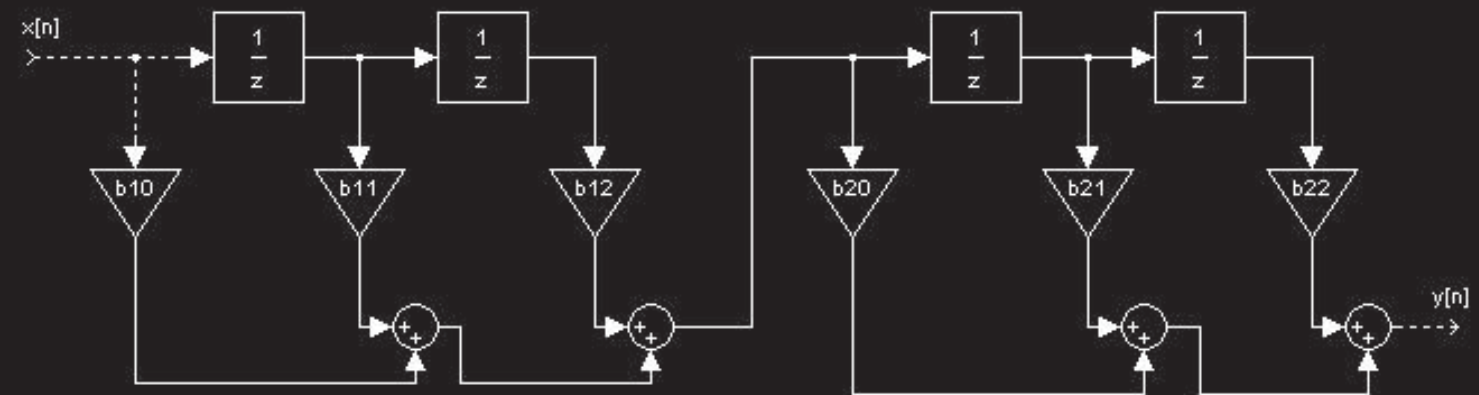
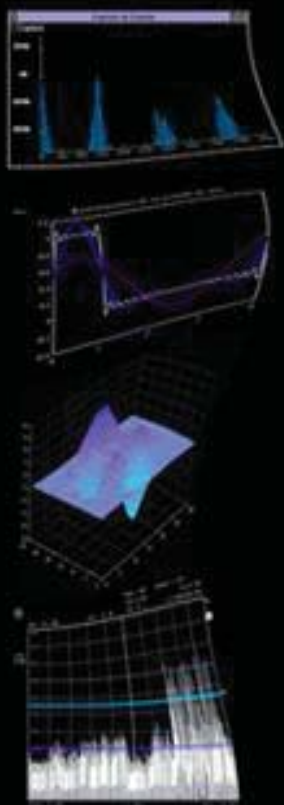
# Estructura de Filtros

## FIR Cascada

$$y_k[n] = b_{k0}x_k[n] + b_{k1}x_k[n-1] + b_{k2}x_k[n-2]$$

$$x[n] = x_1[n]$$

$$y[n] = y_k[n]$$

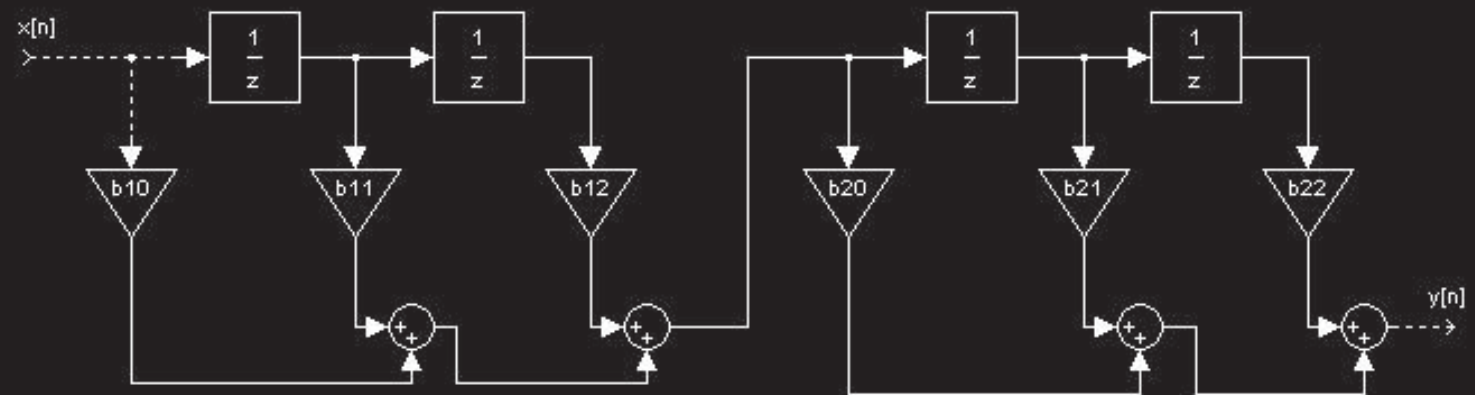


# Estructura de Filtros

## FIR Cascada

3K productos, 2K sumas y 2K retardos

$$K = \text{int} \left\{ \frac{M+1}{2} \right\}$$

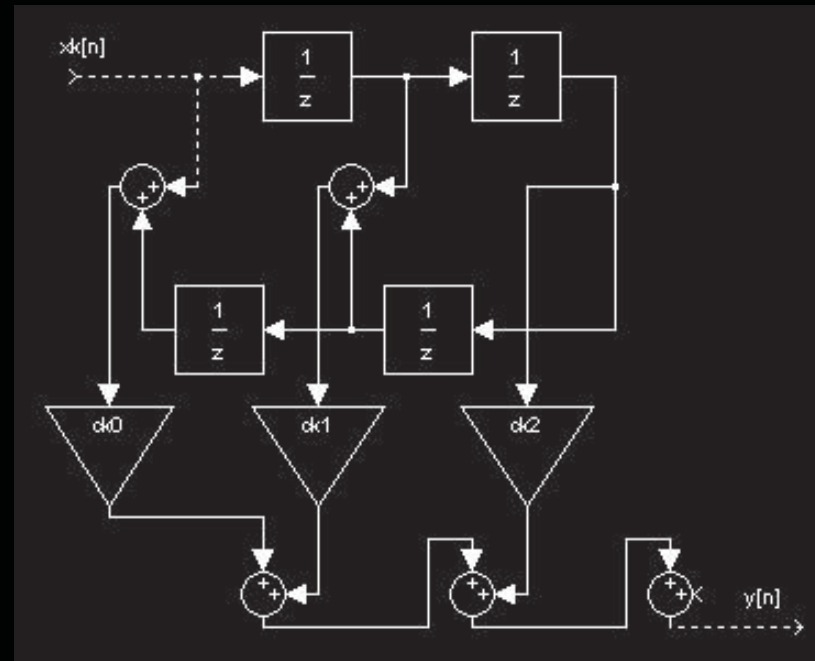
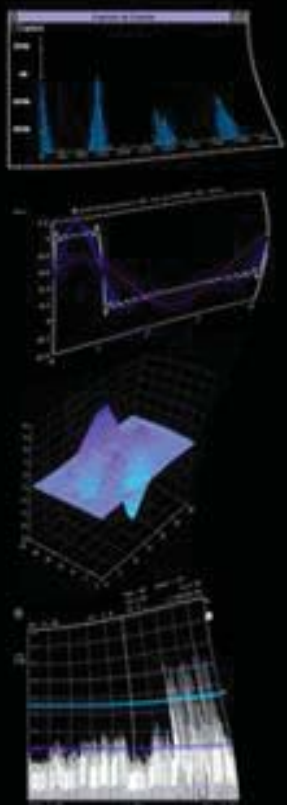


# Estructura de Filtros

## FIR Cascada con Simetría

$$H(z) = \prod_{k=1}^K H_k(z) = \prod_{k=1}^K \left[ c_{k0}(1 + z^{-4}) + c_{k1}(z^{-1} + z^{-3}) + c_{k2}z^{-2} \right]$$

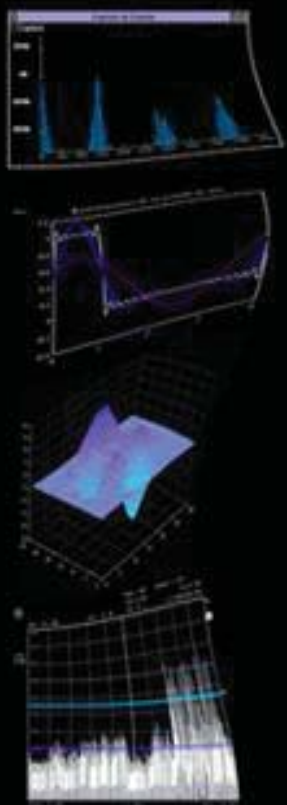
$$K = \text{int} \left\{ \frac{M+1}{2} \right\}$$





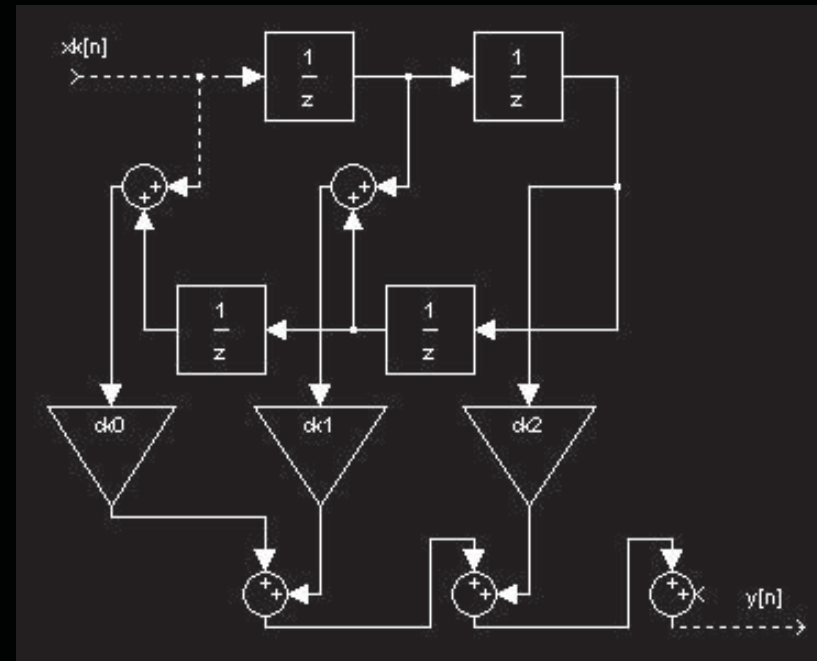
# Estructura de Filtros

## FIR Cascada con simetría



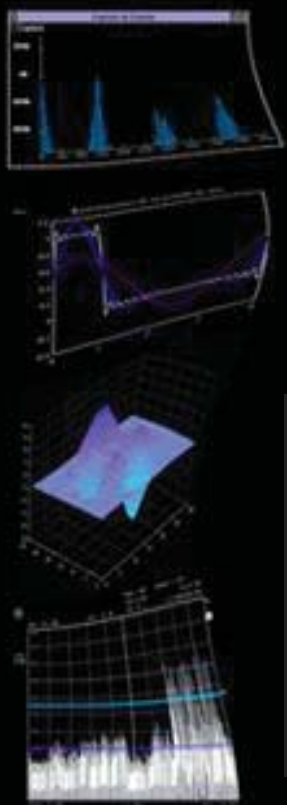
$\text{int}\{3/2K\}$  productos,  $2K$  sumas y  $2K$  retardos

$$K = \text{int}\left\{\frac{M+1}{2}\right\}$$

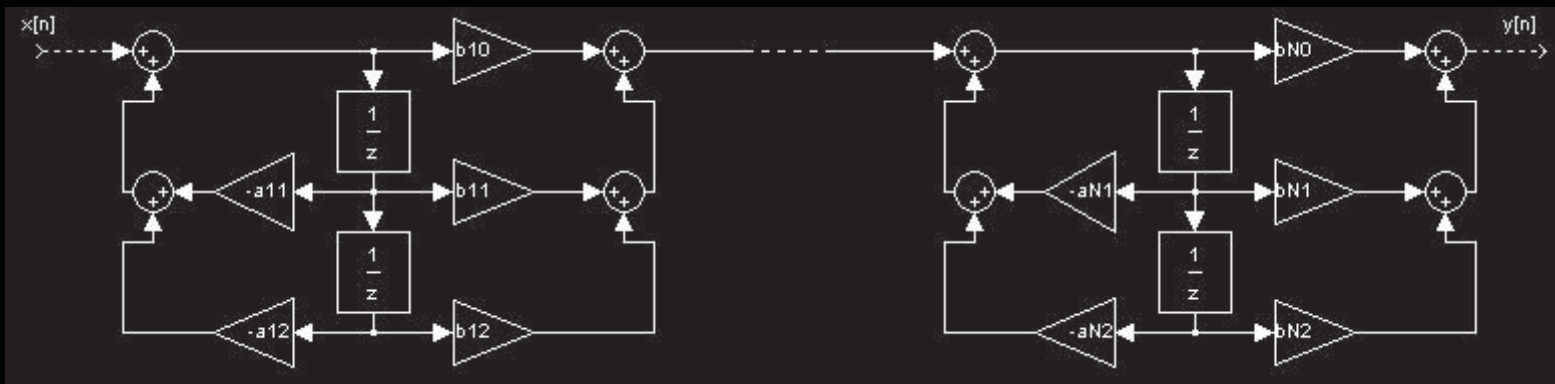


# Estructura de Filtros

## IIR Cascada

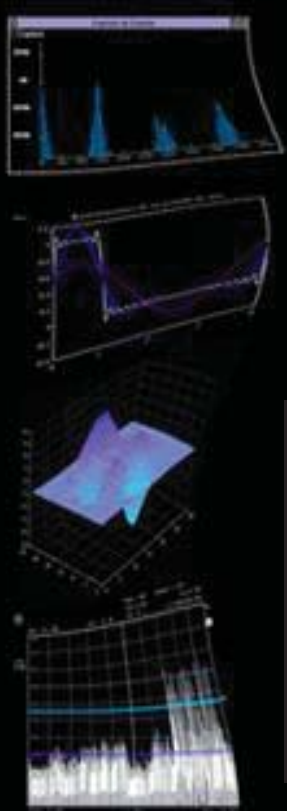


$$H(z) = \prod_{k=1}^K H_k(z) = \prod_{k=1}^K \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}} \quad ; \quad K = \text{int} \left\{ \frac{N+1}{2} \right\}$$



# Estructura de Filtros

## IIR Cascada

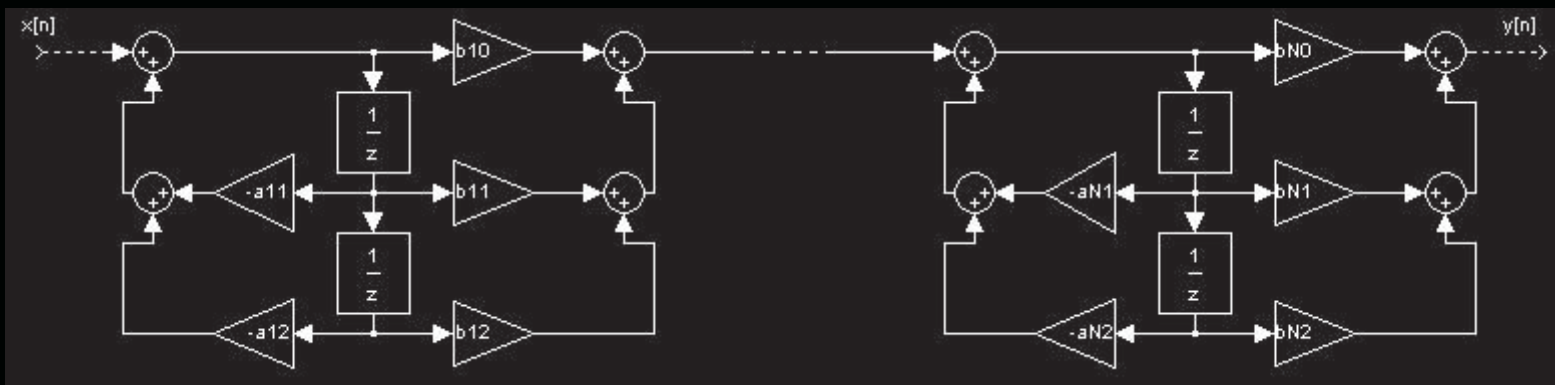


$$w_k[n] = [x_k[n] - a_{k1}w[n-1] - a_{k2}w[n-2]]$$

$$y_k[n] = b_{k0}w_k[n] + b_{k1}w_k[n-1] + b_{k2}w_k[n-2]$$

$$x[n] = x_1[n]$$

$$y[n] = y_k[n]$$

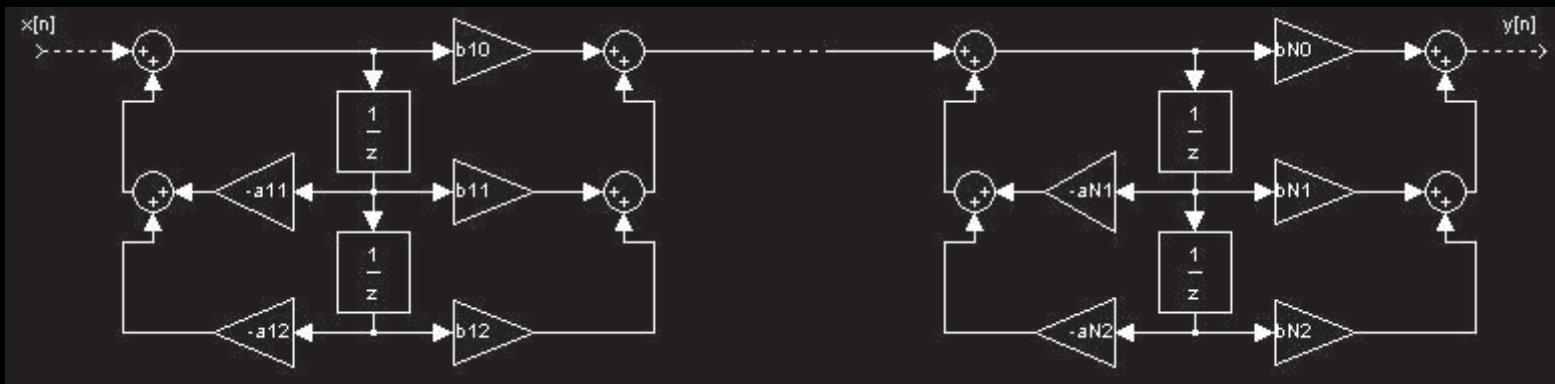


# Estructura de Filtros

## IIR Cascada

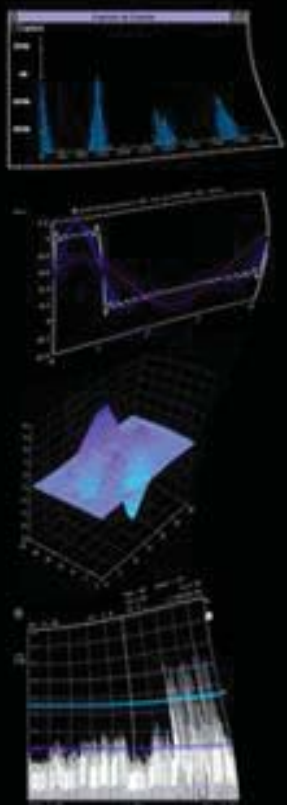
5K productos, 4K sumas y 2K retardos

$$K = \text{int} \left\{ \frac{N+1}{2} \right\}$$



# Estructura de Filtros

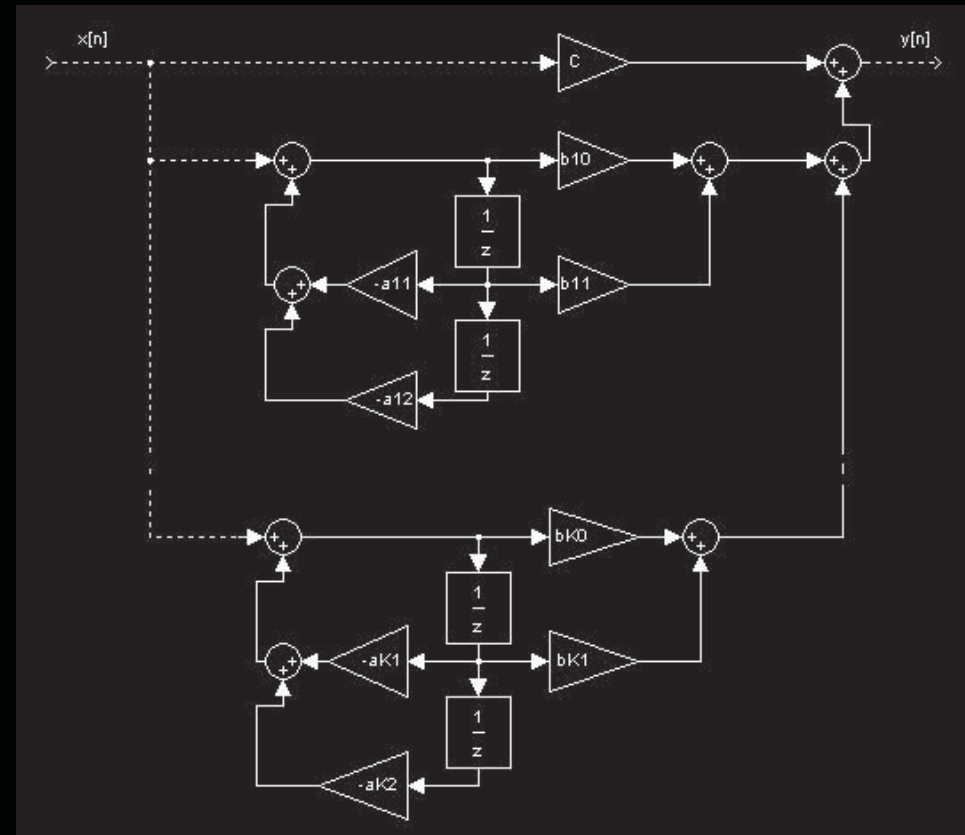
## IIR Paralelo



$$d = \frac{b_M}{a_N}$$

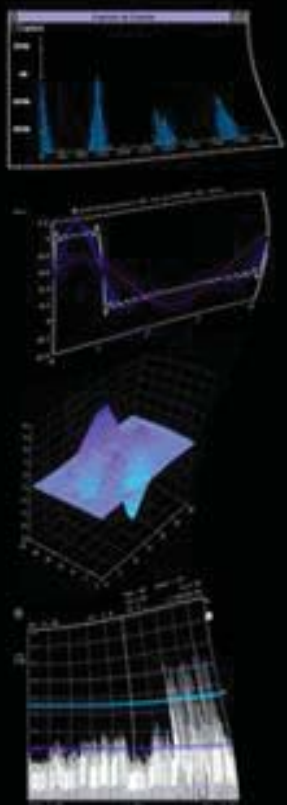
$$K = \text{int} \left\{ \frac{N+1}{2} \right\}$$

$$H(z) = d + \sum_{k=1}^K H_k(z) = d + \sum_{k=1}^K \frac{b_{k0} + b_{k1}z^{-1}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$$



# Estructura de Filtros

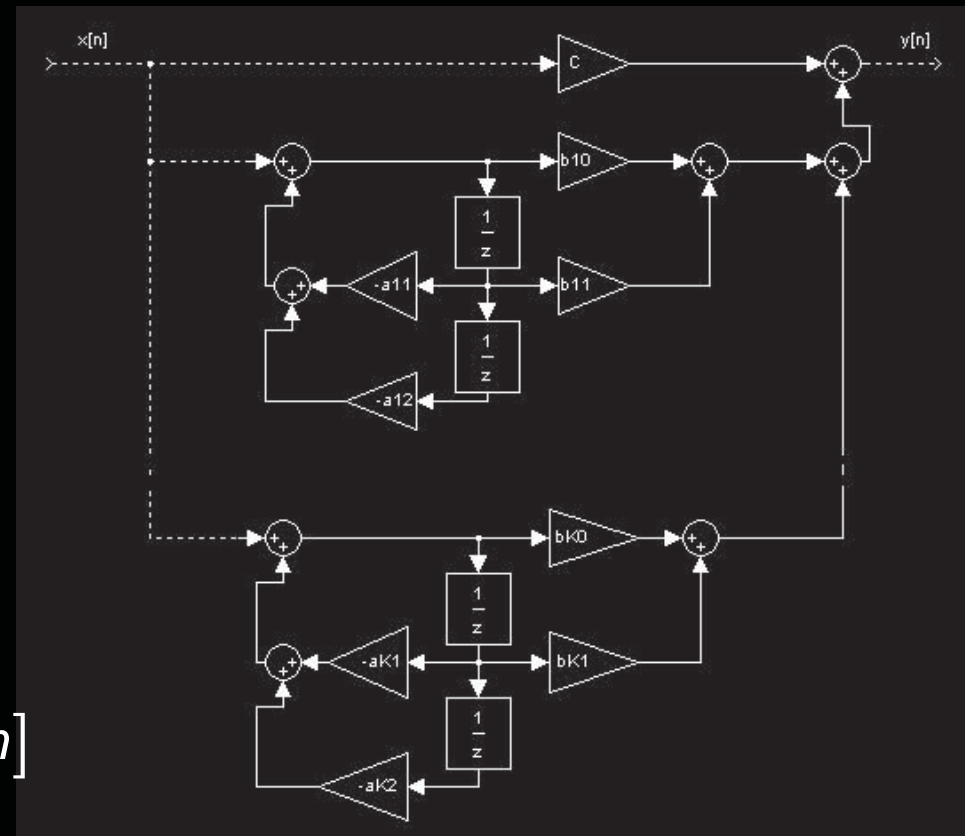
## IIR Paralelo



$$y[n] = dx[n] + \sum_{k=1}^K y_k[n]$$

$$y_k[n] = b_{k0}w_k[n] + b_{k1}w_k[n-1]$$

$$w_k[n] = x[n] - a_{k1}w_k[n-1] - a_{k2}w_k[n-2]$$



# Estructura de Filtros

## Transposición

