The Abdus Salam International Centre for Theoretical Physics

2384-25

ICTP Latin-American Advanced Course on FPGADesign for Scientific Instrumentation

19 November - 7 December, 2012

Filtros digitales

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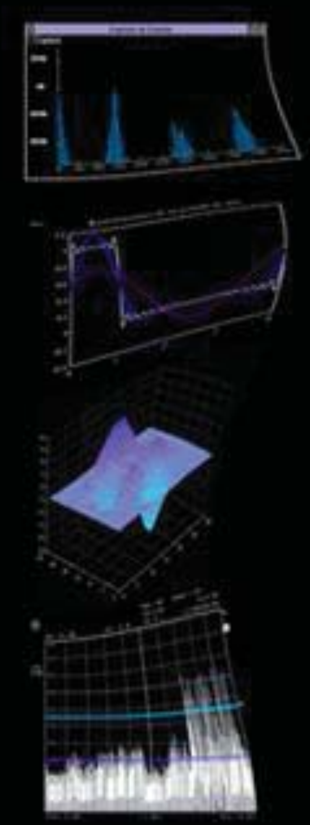
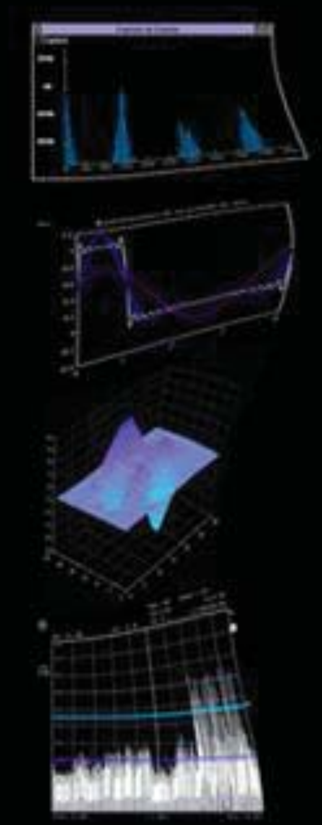
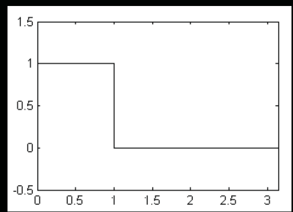
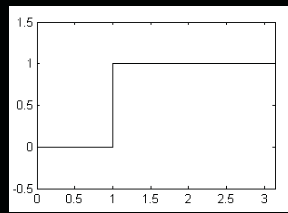
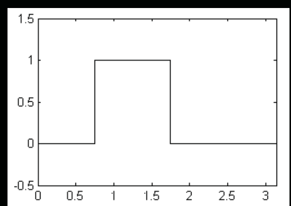
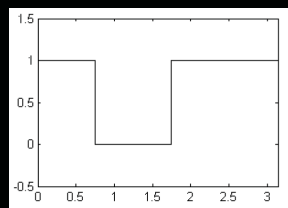


Filtros digitales

Filtros Digitales

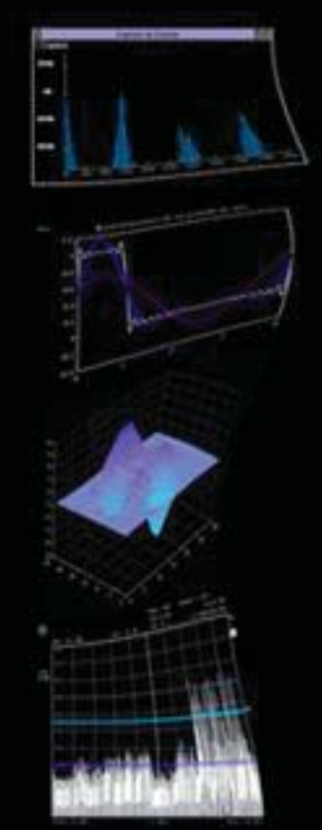
Filtros ideales

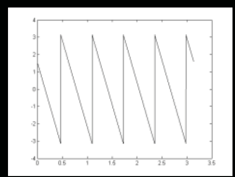
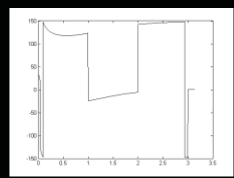


pasa-bajos (LP)	pasa-altos (HP)
	
pasa-banda (BP)	**rechaza-banda (SB)**
	

Filtros Digitales

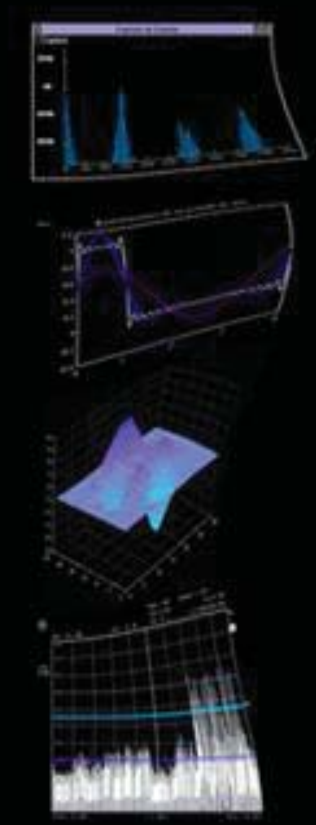
Filtros ideales



lineal	no lineal
 $\angle G(e^{j\omega}) = \beta - \alpha\omega$ $\beta = \begin{cases} 0 \\ \pm\frac{\pi}{2} \end{cases} \quad \alpha = \frac{M}{2}$	 $\angle G(e^{j\omega}) \neq \beta - \alpha\omega$

Filtros Digitales

Filtros realizables

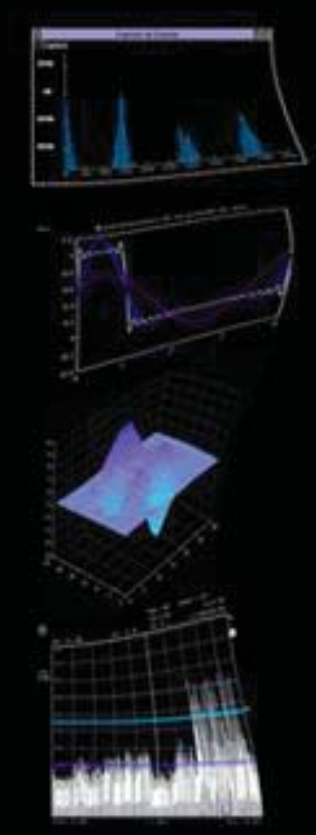


Teorema de causalidad de Pauli-Wienner:

- $|H(e^{j\omega})| = A$ en $\omega_1, \omega_2, ..., \omega_k$ con $k \in N$
- pero $|H(e^{j\omega})| \neq A = cte$ con $A \in R$ $\forall \omega_{k-1} < \omega < \omega_k$
- $\partial |H(e^{j\omega})| / \partial\omega < \infty$ $(\Leftrightarrow \Delta\omega \neq 0)$
- $|H(e^{j\omega})|$ y $\angle H(e^{j\omega})$ están relacionados por la TH

Filtros Digitales

Etapas de diseño



Especificaciones

Cálculo de coeficientes:

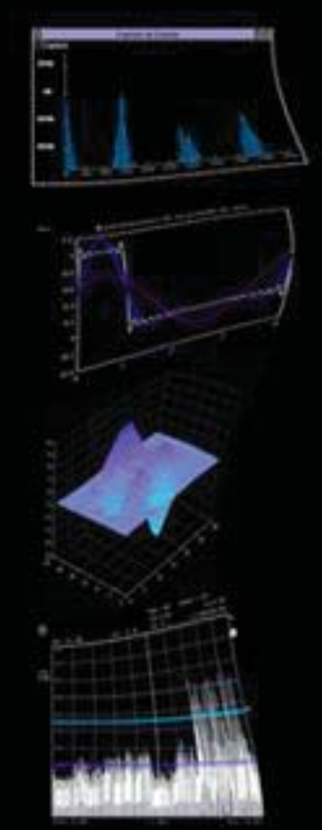
- FIR:
  - Optimo
  - Ventanas
  - Muestreo en frecuencia
- IIR:
  - Transformación de filtros analógicos
  - Optimo

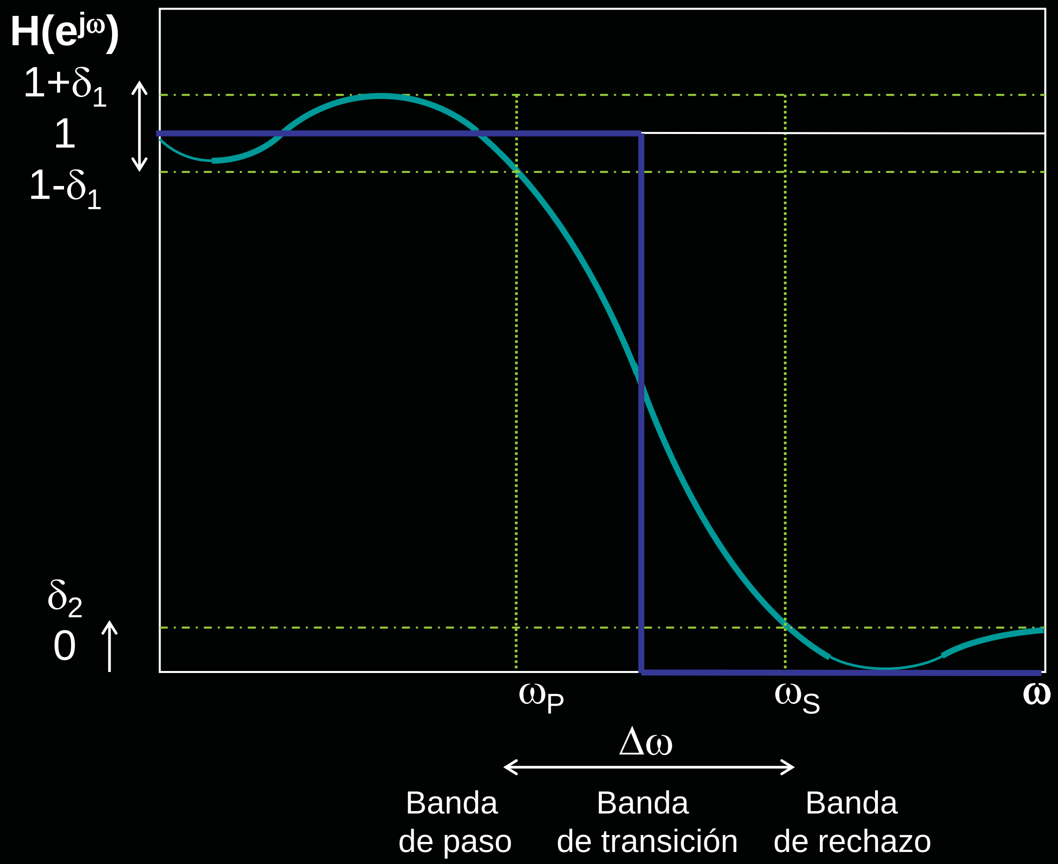
Cálculo de la estructura

Implementación

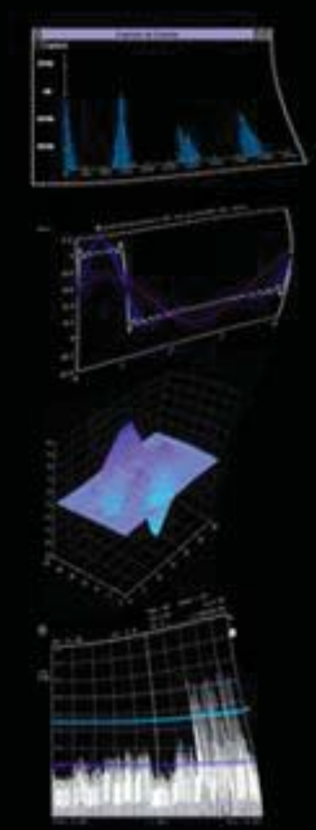
Filtros Digitales

Especificaciones

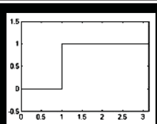
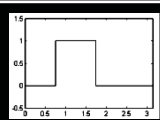
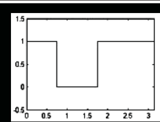


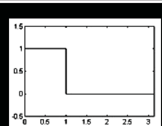


Filtros Digitales



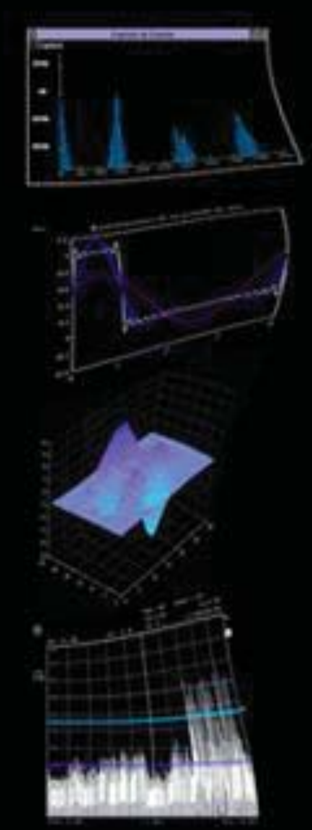
Conversión de la banda pasante

pasa-altos (HP)	pasa-banda (BP)	rechaza-banda (SB)
		
$\omega_{pLP} = \omega_{sHP}$ $\omega_{sLP} = \omega_{pHP}$	$\omega_{pLP} = \dfrac{\omega_{suSB} - \omega_{slSB}}{2}$ $\omega_{sLP} = \dfrac{\omega_{puSB} - \omega_{plSB}}{2}$	$\omega_{pLP} = \dfrac{\omega_{puBP} - \omega_{plBP}}{2}$ $\omega_{sLP} = \dfrac{\omega_{suBP} - \omega_{slBP}}{2}$

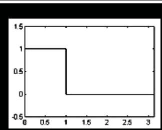
pasa-bajos (LP)


Filtros Digitales

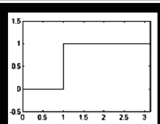
Conversión de la banda pasante



pasa-bajos (LP)



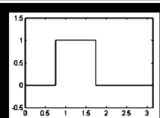
pasa-altos (HP)



$$\left|H_{HP}\left(e^{j\omega}\right)\right| = 1 - \left|H_{LP}\left(e^{j\omega}\right)\right|$$

$$h_{HP}[n] = \delta[n] - h_{LP}[n]$$

pasa-banda (BP)

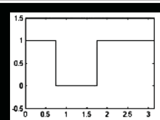


$$\left|H_{BP}\left(e^{j\omega}\right)\right| =$$

$$\left|H_{LP}\left(e^{j(\omega-\omega_0)}\right) + H_{LP}\left(e^{j(\omega+\omega_0)}\right)\right|$$

$$h_{BP}[n] = 2\cos[n\omega_0]\,h_{LP}[n]$$

rechaza-banda (SB)



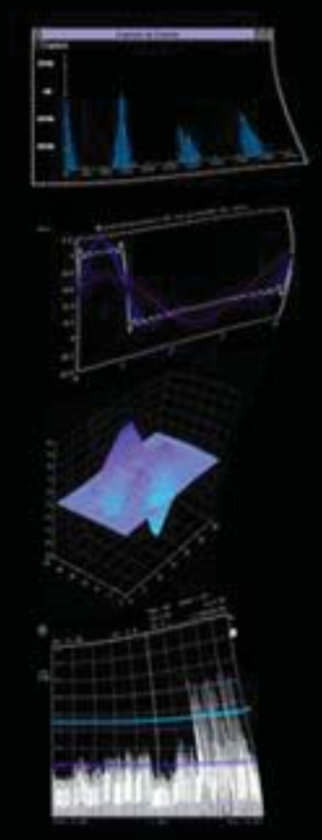
$$\left|H_{SB}\left(e^{j\omega}\right)\right| = 1 - \left|H_{BP}\left(e^{j\omega}\right)\right|$$

$$h_{SB}[n] =$$

$$\delta[n] - 2\cos[n\omega_0]\,h_{LP}[n]$$

Filtros Digitales

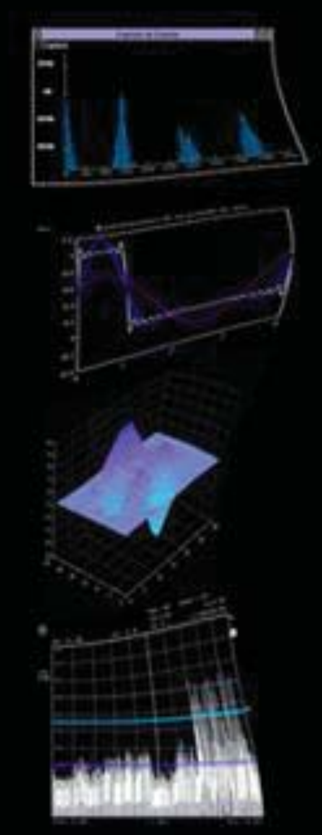
Especificaciones



Nombre	Expresión
Ancho de banda de transición	$\Delta\omega = \omega_s - \omega_p$
Rizado en la banda pasante	$A_p = 20 * \log\left(\frac{1+\delta_1}{1-\delta_1}\right)$
Atenuación en la banda suprimida	$A_s = 20 * \log(\delta_2)$

Filtros Digitales

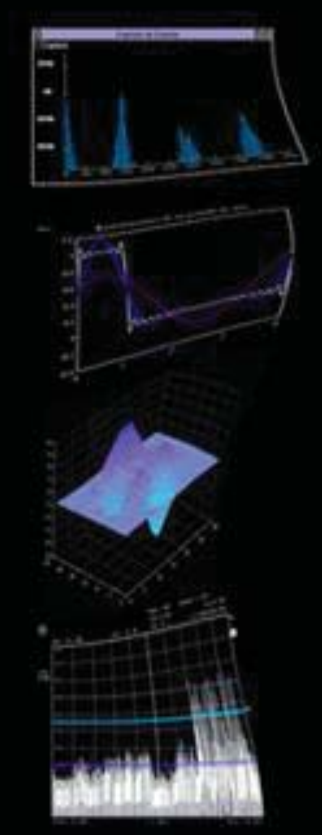
Cálculo de coeficientes



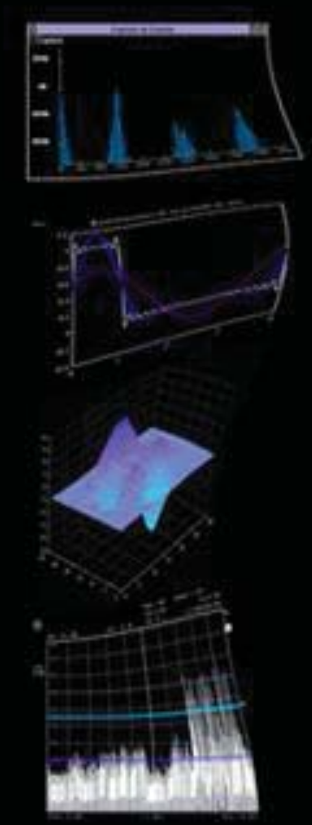
FIR	IIR
MA (Promediador Móvil) $H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^{M} b_k z^{M-k}$ M ceros y 1 polo de orden M	**AR (Autoregresivo)** $H(z) = \frac{b_0}{\sum_{k=0}^{N} a_k z^{-k}} = \frac{b_0 z^N}{\sum_{k=0}^{N} a_k z^{N-k}}$ N polos y 1 cero de orden N
	ARMA (Promediador Móvil Autoregresivo) $H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$ M ceros y N polos no triviales

Filtros Digitales

Cálculo de coeficientes



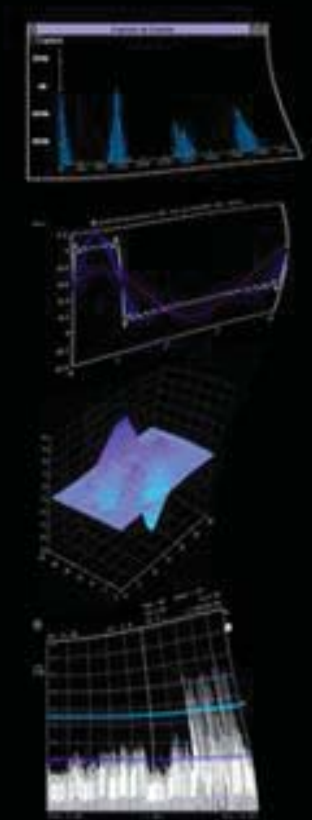
FIR	IIR
SLIT causal no recursivo	SLIT causal recursivo
$H(z)<\infty \ \forall z$	$H(z)<\infty \forall z \neq z_p$
$\angle H(e^{j\omega})=\beta-\alpha\omega$	$\downarrow\delta$ y $\downarrow\Delta\omega$ p/=M
Estructura sencilla	Requiere menos recursos

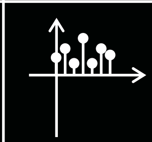
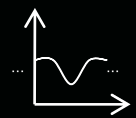
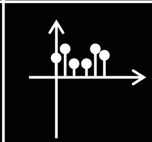
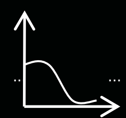
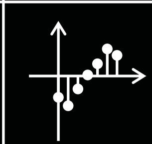
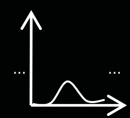
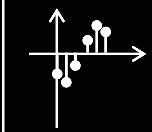
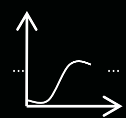


Filtros FIR

Filtros FIR

Simetría y Paridad



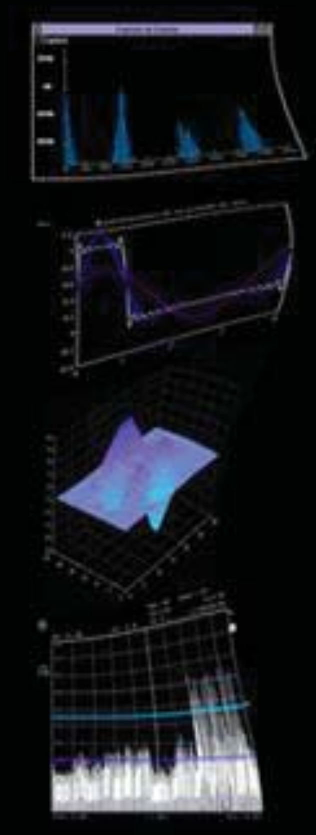
Tipo	h[n]	$H(e^{j\omega})$	Uso	
I			LP, HP, BP, SB	Filtros FIR de fase lineal multibandas
II			LP, BP	
III			BP	Diferenciadores y transformadores de Hilbert
IV			HP, BP	

remezord (firpmord)

Estima los parámetros de diseño óptimo para calcular los coeficientes de filtros FIR

Filtros FIR

Método óptimo



Especificaciones

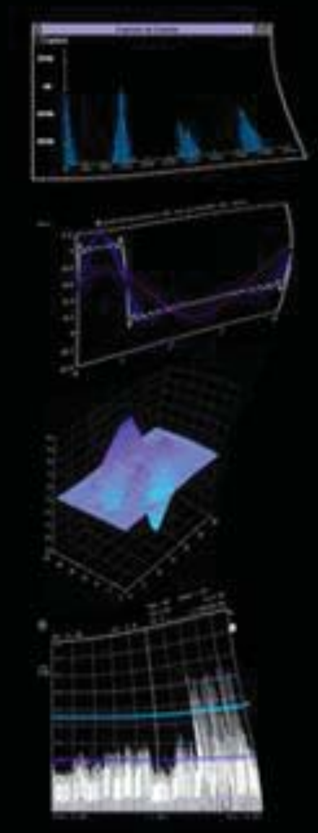
Estimación de parámetros

Algoritmo de Parks-McClelland

Filtros FIR

firpmord (remezord)

Estima los parámetros del filtro óptimo



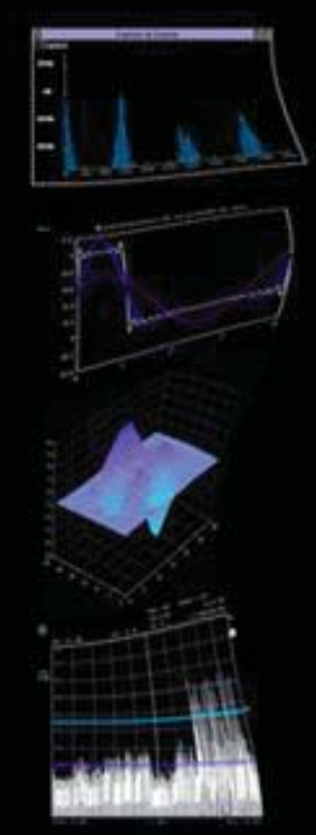
Sintaxis

```
[M,vc,Ao,W]=remezord(Fc,A,delta,Fm)
```

remezord (firpmord)

Estima los parámetros de diseño óptimo para calcular los coeficientes de filtros FIR

Filtros FIR



firpm (remez)

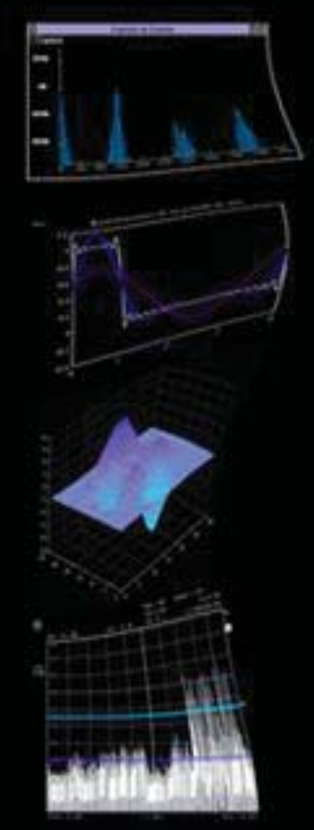
Calcula los coeficientes de un filtro óptimo

Sintaxis

```
b=remez(M,vc,A,'tipodebanda',W)
```

Filtros FIR

Método de las ventanas



Especificaciones

Conversión a prototipo pasabajos

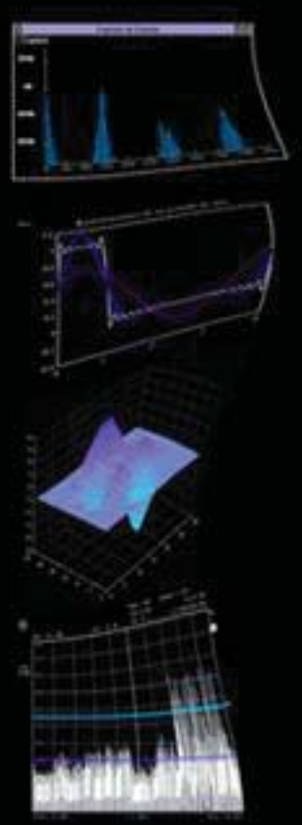
Elección de la ventana

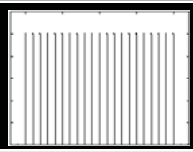
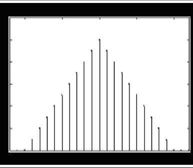
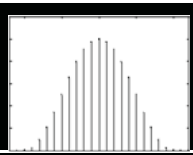
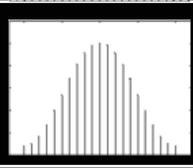
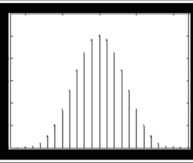
Enventanado

Conversión a la banda pasante deseada

Filtros FIR

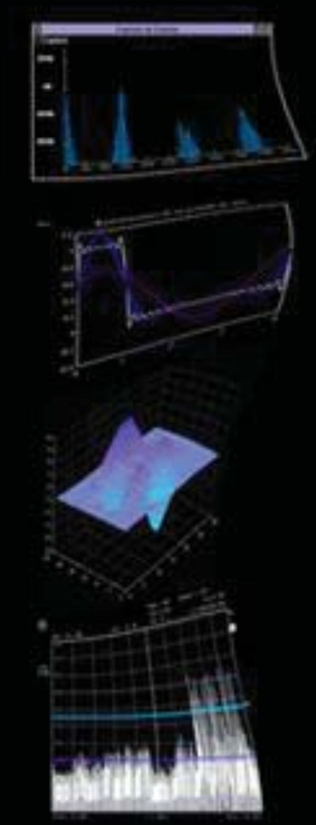
Ventanas

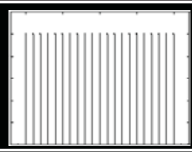
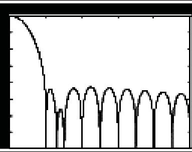
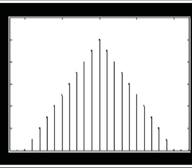
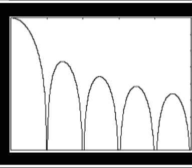
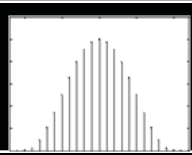
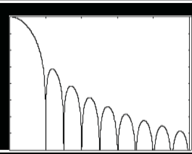
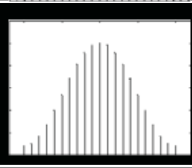
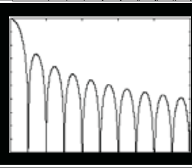
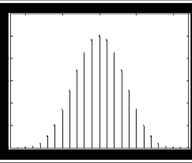
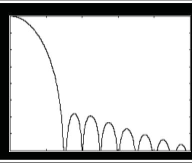


Tipo	w[n]
Rectangular	 $w[n]=\Pi\left[\frac{n-\frac{M}{2}}{M}\right]=\begin{cases}1 & si \quad 0\le n\le M\\ 0 & cc\end{cases}$
Bartlett	 $w[n]=\Lambda\left[\frac{n-\frac{M}{2}}{M}\right]=\begin{cases}\frac{2n}{M} & si \quad 0\le n\le \frac{M}{2}\\ 2-\frac{2n}{M} & si \quad \frac{M}{2}\le n\le M\end{cases}$
Hanning	 $w[n]=\begin{cases}\alpha-(1-\alpha)\cos\left[\frac{2\pi n}{M}\right] & si \quad 0\le n\le M\\ 0 & cc\end{cases} \qquad \alpha=0.5$
Hamming	 $w[n]=\begin{cases}\alpha-(1-\alpha)\cos\left[\frac{2\pi n}{M}\right] & si \quad 0\le n\le M\\ 0 & cc\end{cases} \qquad \alpha=0.54$
Blackman	 $w[n]=\begin{cases}0.42-0.5\cos\left[\frac{2\pi n}{M}\right]+0.08\cos\left(\frac{4\pi n}{M}\right) & si \quad 0\le n\le M\\ 0 & cc\end{cases}$

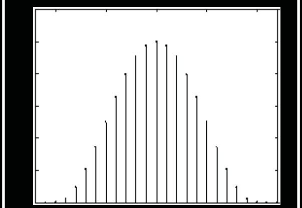
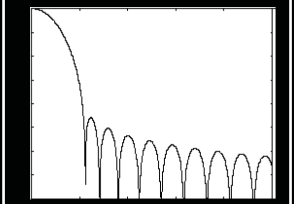
Filtros FIR

Ventanas

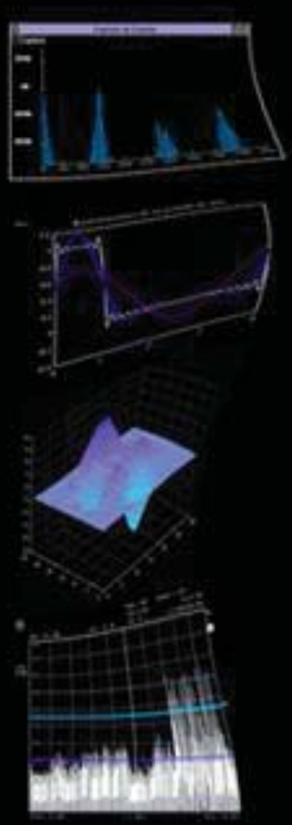


Tipo	w[n]	$W(e^{jw})$	A_s [dB]	$\Delta\omega$
Rectangular			21	$\frac{4\pi}{M+1}$
Bartlett			25	$\frac{8\pi}{M+1}$
Hanning			44	
Hamming			53	
Blackman			74	$\frac{12\pi}{M+1}$

Filtros FIR

w[n]	W(e^{jw})
	

Kaiser



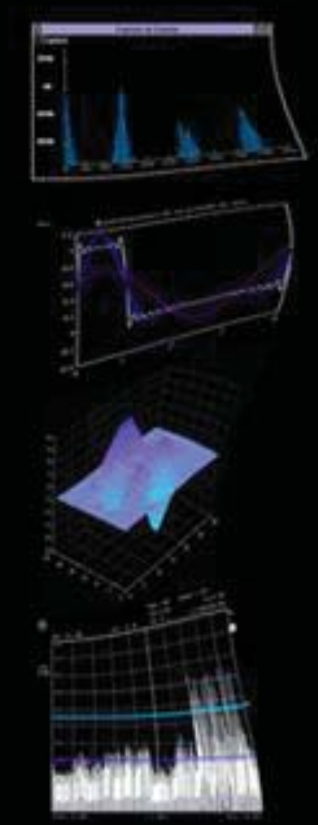
$$w[n] = \begin{cases} \dfrac{I_0\left(\beta\sqrt{1-\left[1-\dfrac{2n}{M}\right]^2}\right)}{I_0[\beta]} & si \quad 0 \le n \le M \\ 0 & cc \end{cases}$$

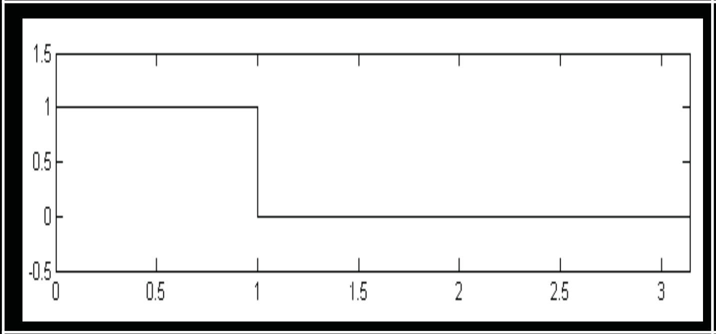
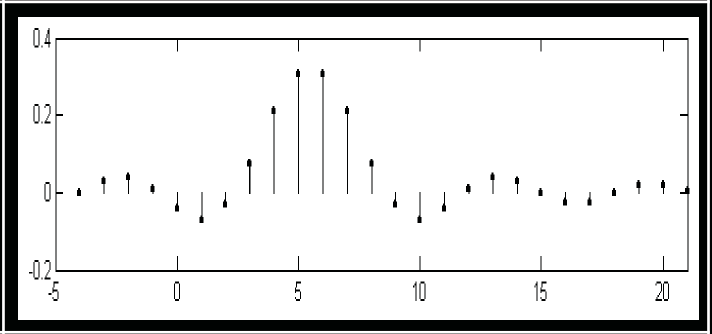
$$\beta = \begin{cases} 0 & si \quad A_s < 21 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21) & si \quad 21 \le A_s < 50 \\ 0.1102(A_s - 8.7) & si \quad 50 \le A_s \end{cases}$$

$$\Delta\omega = \frac{A_s - 8}{2.285M}$$

Filtros FIR

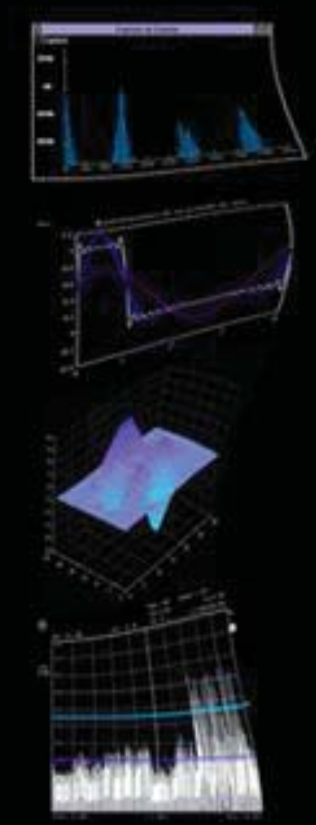
Enventanado

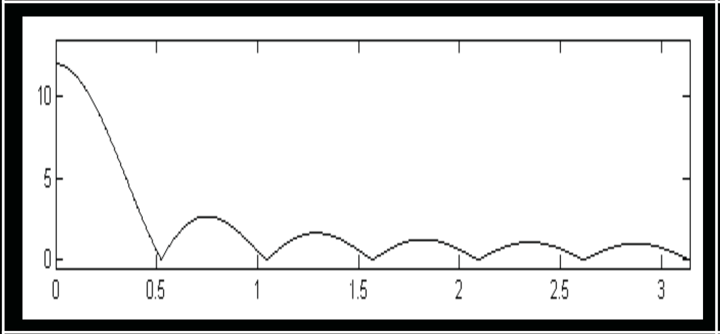
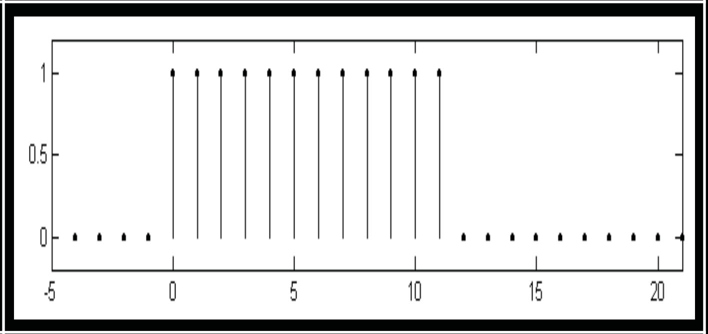


Respuesta espectral	Respuesta impulsiva
	
$H_d\left(e^{j\omega}\right) = \begin{cases} e^{-j\omega\frac{M}{2}} & si \quad \lvert\omega\rvert \leq \omega_c \\ 0 & si \quad 0 < \lvert\omega\rvert \end{cases}$	$h_d[n] = \frac{\omega_c}{\pi} senc\left[\left(n - \frac{M}{2}\right)\omega_c\right]$

Filtros FIR

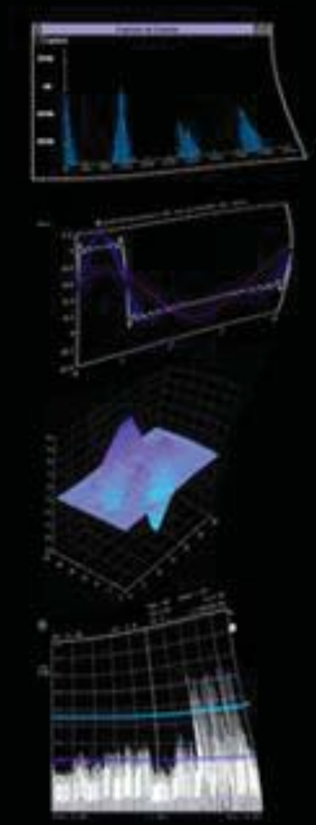
Enventanado

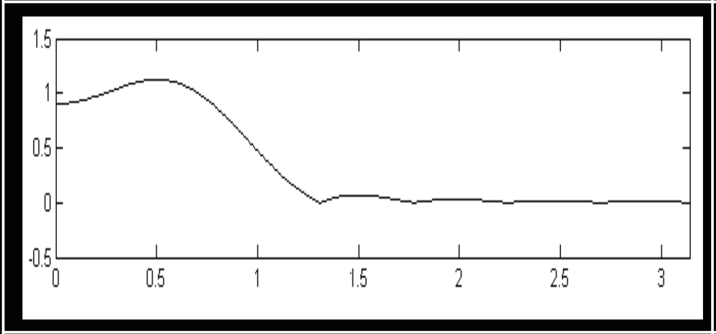
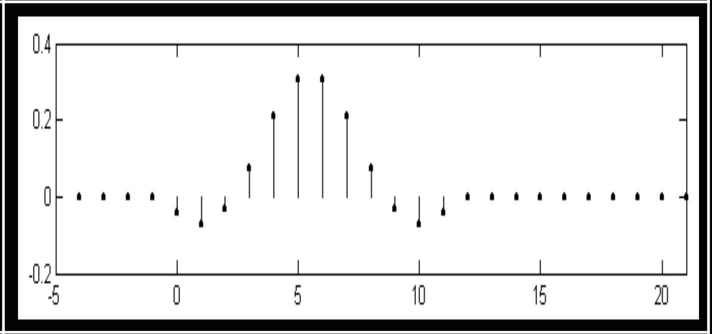


<table>
<tr><th>Respuesta espectral</th><th>Respuesta impulsiva</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$$W\left(e^{j\omega}\right)=e^{-j\omega\frac{M}{2}}\frac{sen\left[(M+1)\frac{\omega}{2}\right]}{sen\left(\frac{\omega}{2}\right)}$$</td><td>$$w[n]=\begin{cases}1 & si \quad 0\le n\le M\\ 0 & cc\end{cases}$$</td></tr>
</table>

Filtros FIR

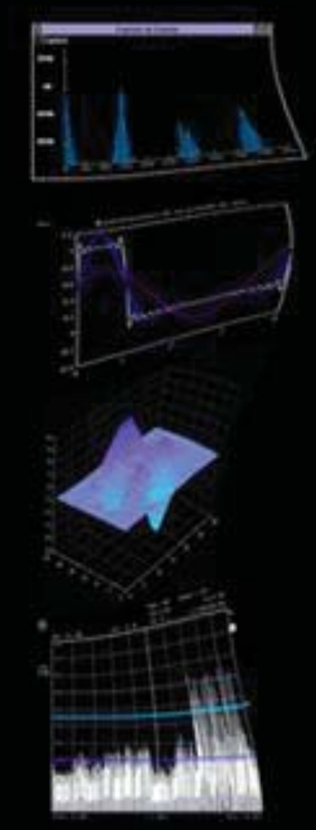
Enventanado



Respuesta espectral	Respuesta impulsiva
	
$H\left(e^{j\omega}\right) = H_d\left(e^{j\omega}\right) * W\left(e^{j\omega}\right)$	$h[n] = h_d[n]w[n]$

Filtros FIR

Método de Muestreo en Frecuencia



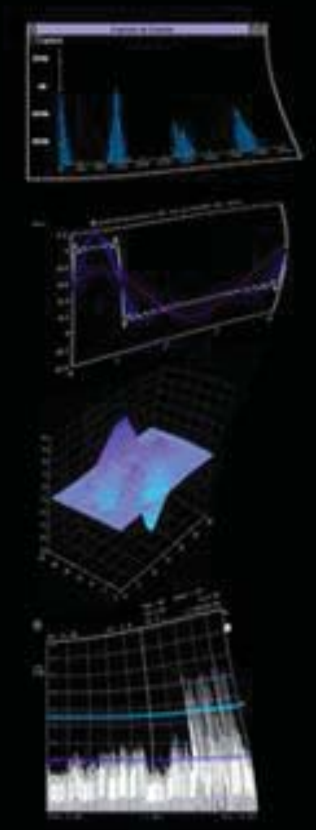
Especificaciones
- Muestreo de $H(e^{j\omega})$
- Optimización en $\Delta\omega$

Respuesta al impulso
- Determinación del tipo
- Elección de G[k]
- Cálculo de h[n]

Garantizar la estabilidad
- Traslación de raíces dentro de |z |=1

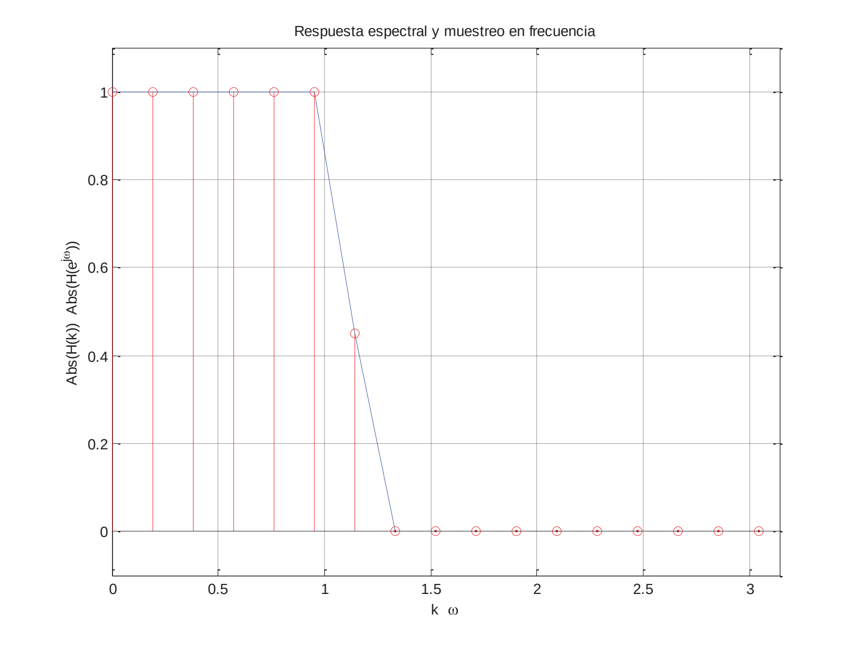
Filtros FIR

Método de Muestreo en Frecuencia



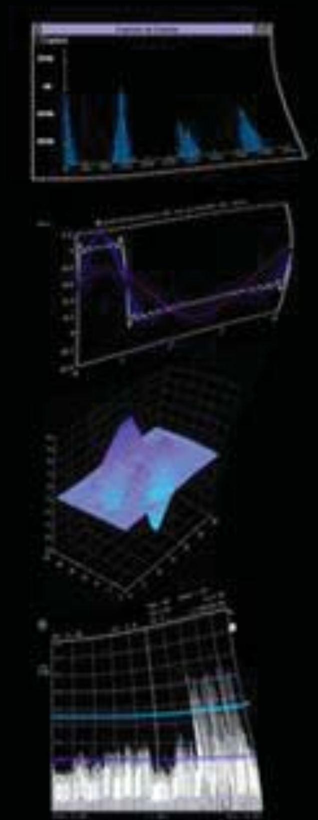
Especificaciones

Muestreo de H($e^{j\omega}$)



Filtros FIR

Método de Muestreo en Frecuencia



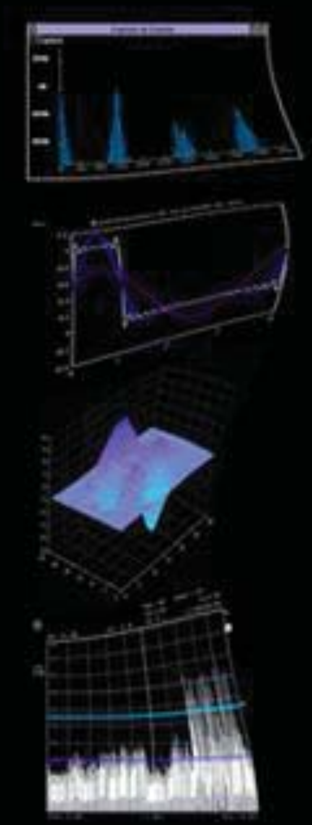
Especificaciones

Muestreo de $H(e^{j\omega})$

$$
\begin{cases}
k = 0;1;2;\ldots;\dfrac{M}{2} & L = M+1 \quad \textit{impar} \\
k = 0;1;2;\ldots;\dfrac{M}{2}-\dfrac{1}{2} & L = M+1 \quad \textit{par}
\end{cases}
$$

Filtros FIR

Método de Muestreo en Frecuencia



Especificaciones

Muestreo de H($e^{j\omega}$)

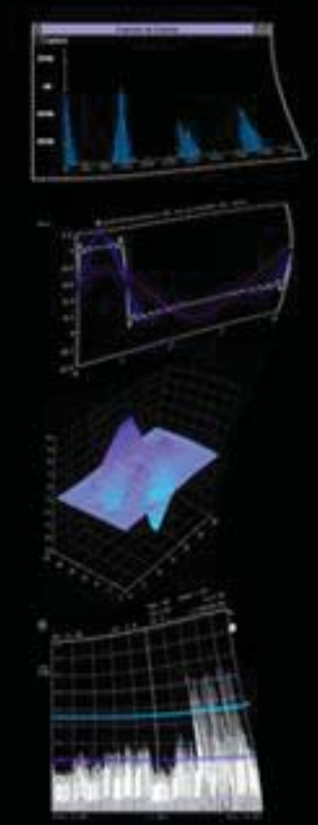
$$2\pi \,\rule{4cm}{0.4pt}\, M+1$$

$$\omega \,\rule{4cm}{0.4pt}\, k$$

$$k = \frac{\omega\left(M+1\right)}{2\pi} \quad ; \quad \omega = \frac{2\pi}{\left(M+1\right)}k$$

Filtros FIR

Método de Muestreo en Frecuencia



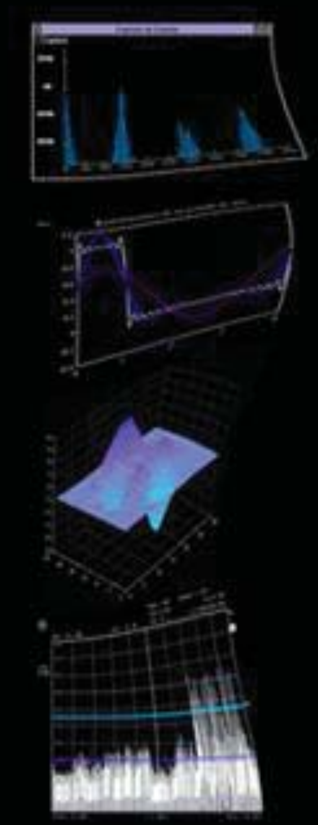
Especificaciones

Muestreo de H($e^{j\omega}$)

$$H[k+\alpha] \equiv H\left(e^{j\frac{2\pi}{M+1}(k+\alpha)}\right) = \sum_{n=0}^{M} h[n]e^{-j\frac{2\pi(k+\alpha)}{M+1}n} \quad ; \quad k = 0;1;2;\dots;M$$

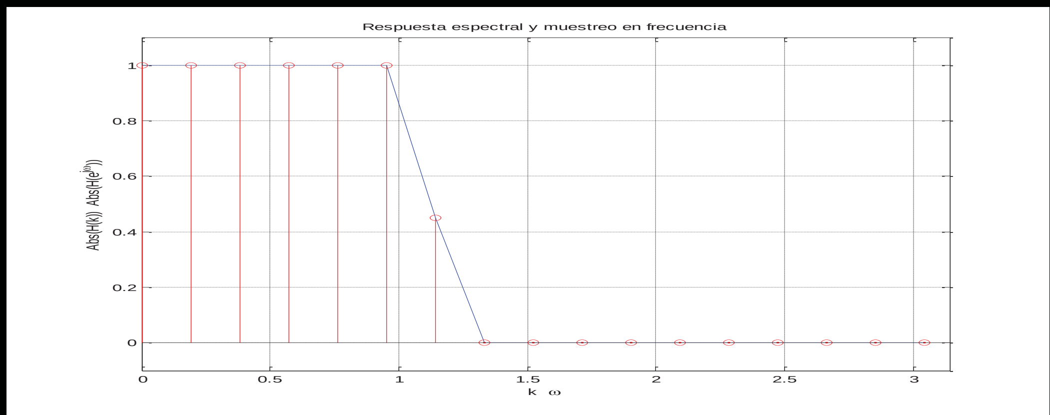
Filtros FIR

Método de Muestreo en Frecuencia



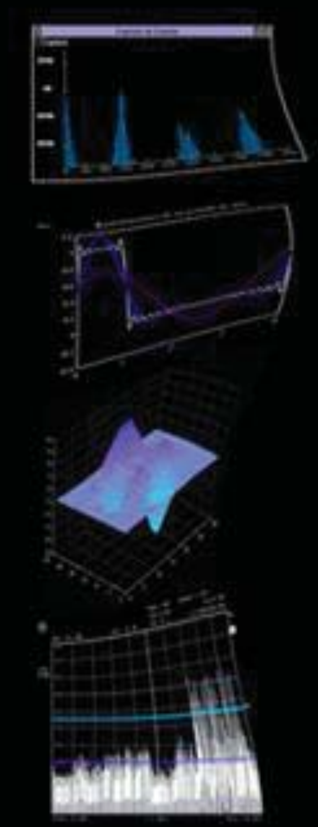
Especificaciones

- **Muestreo de $H(e^{j\omega})$**
  - **M debe cumplir las especificaciones de $\Delta\omega$**



Filtros FIR

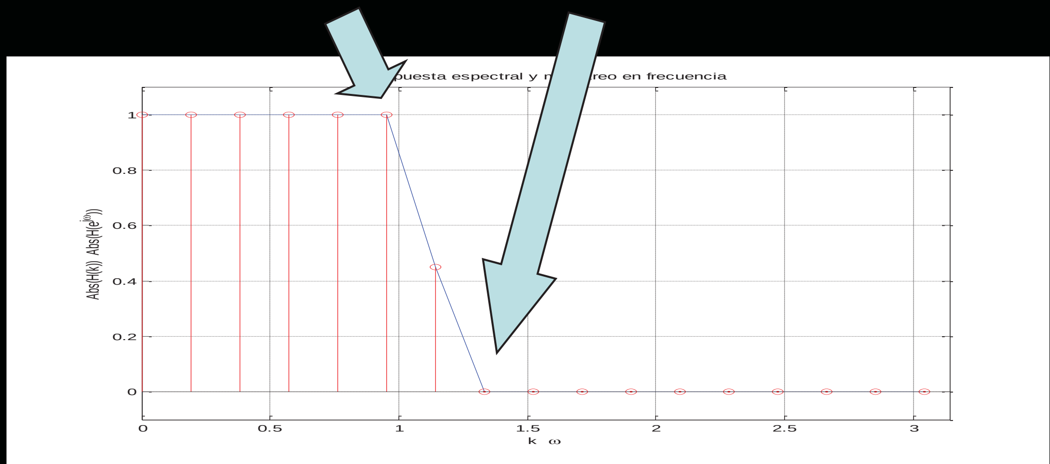
Método de Muestreo en Frecuencia



Especificaciones

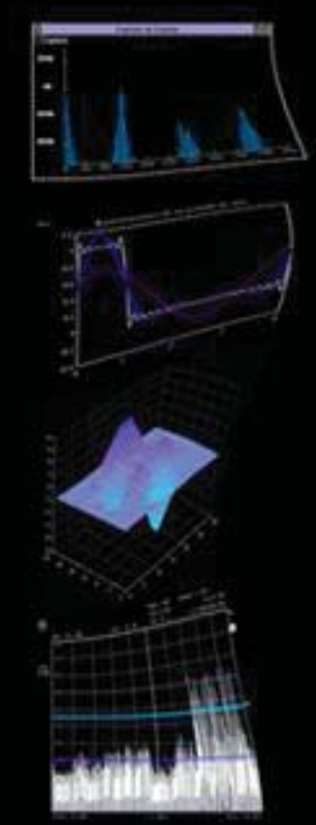
Muestreo de H($e^{j\omega}$)

$$k_p = \frac{\omega_p (M+1)}{2\pi} \quad ; \quad k_s = \frac{\omega_s (M+1)}{2\pi}$$



Filtros FIR

Método de Muestreo en Frecuencia



Respuesta impulsiva

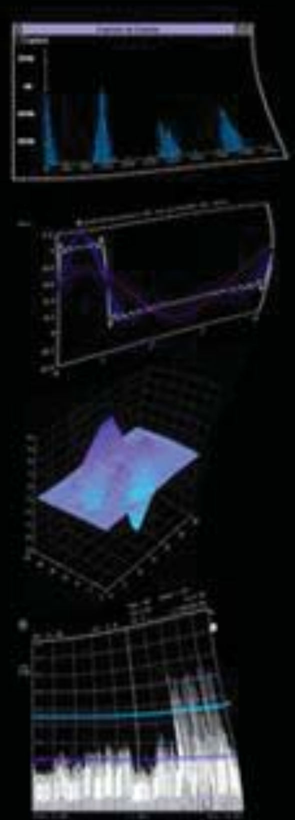
Muestreo de H($e^{j\omega}$)

Reducción de los lóbulos laterales

$$Hr\left[\frac{2\pi}{M}k\right]=\begin{cases}1 & si \quad k=0;1;\ldots;k_s\\ T_1 & si \quad k=k_s+1\\ 0 & si \quad k=k_s\ldots;\dfrac{M}{2}\end{cases} \quad con \quad H\left[k\right]=H\left[M-k\right]$$

Filtros FIR

Método de Muestreo en Frecuencia



Respuesta impulsiva

α=0 simetría par

$$H[k] = G[k]e^{j\frac{\pi k}{M+1}} \quad k = 0;1;\ldots;M$$

$$G[k] = (-1)^k H_r\left[\frac{2\pi k}{M+1}\right] \quad ; \quad G[k] = -G[M+1-k]$$

$$h[n] = \frac{1}{M+1}\left\{G[0] + 2\sum_{k=1}^{U} G[k]\cos\left[\frac{2\pi k}{M+1}\left(n+\frac{1}{2}\right)\right]\right\}$$

$$U = \begin{cases} \frac{M}{2} \quad M \quad impar \quad ó \quad \frac{M-1}{2} \quad M \quad par \end{cases}$$