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C-method for multilayered periodic structures

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The innovative method (hereafter, the C-method) proposed by Chandezon *et al.* in the early eighties is a mode-matching differential method particularly suited for the rigorous analysis of corrugated periodic surfaces with smooth profiles. Over the last two decades, further development and extensions of the method have been proposed by many researcher. Among them, Popov and Mashev [1] considered the conical diffraction mounting, Inchaussandague and Depine [2] presented a generalization to anisotropic media and Preist*et al.* [3] an adaptation to overhanging profiles. Moreover, the method has been improved for the analysis of multilayered structures with constant and arbitrarily varying profiles [4, 5, 6, 7] . The extended C-method described in this appendix refers mainly to the work of Li in Ref. [8].

1 Notations and statement of the problem

The purpose of the proposed method is to analyze a typical 2D diffraction problem. As depicted in Fig. 1, the periodic diffracting structure (period Λ) is composed by a stack of N media whose boundaries profiles are described by a set of $N-1$ functions $a_i(x)$, $i = 1, 2, \ldots N-1$. The structure is invariant along the z-direction. Each medium is optically characterized by a permittivity $\epsilon_i(x, y)$, $i = 1, 2, \ldots N$ that is a function of both cartesian coordinates x and y. The illuminating field is represented by a single plane wave ($k_0 = 2\pi\lambda^{-1}$ in vacuum) propagating in medium N and incident on interface $a_{N-1}(x)$ at an angle θ with respect to the y axis. In such a two-dimensional geometry, two polarization states are usually considered. In the first case, the electric field vector $\mathbf{E}$ of the electromagnetic wave has only one cartesian component, E_z, being always perpendicular to the xy plane (Transverse Electric -TE- configuration). In the second case, the magnetic field $\mathbf{H}$ is always perpendicular to the xy- plane (Transverse Magnetic -TH- configuration) and the electric field vector is described by the E_x and E_y components. In the following, the TH polarization will be considered, and the unknown field H_z to calculate will be generally indicated by $F(x, y; t)$.

The Maxwell equations referred to a scalar, harmonic field $F(x, y; t) = F(x, y)exp(-i\omega t)$ propagating in a inhomogeneous, non magnetic medium with no charges neither currents can be reduced to the Helmoltz equation:

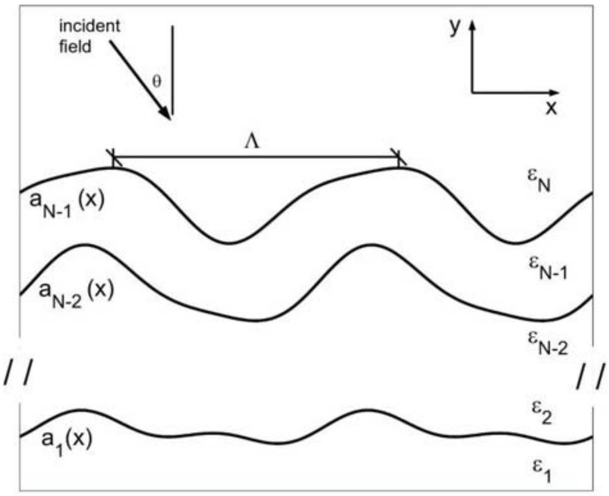

Figure 1: Schematic of the multilayered structure considered with the C-method.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2\epsilon(x,y)\right) F(x,y) = 0 \tag{1}$$

or, more simply

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}; x, y\right) F(x,y) = 0 \tag{2}$$

Solutions of equation (2) can not be written using the well-known Rayleigh expansion formula [9] since the permittivity $\epsilon = \epsilon(x,y)$ is not space-invariant. Nevertheless, a simple form for these solutions can be obtained if the differential operator $L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}; x, y\right)$ is reduced to $L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}; x\right)$. There are at least two possibilities of reformulating the problem in such a way.

A first possibility is to split the equation (2) into $n > N-1$ elementary problems where the permittivity can be locally considered as a function of the x coordinate only. This corresponds to cut the structure into n rectangular slices of thickness t_i $(i = 1, 2, \ldots n)$, in which the permittivity has constant value along the y direction . The accuracy of the method depends strongly on the quality of such a rectangular approximation to the real profile: better results are obtained as n increases and t_i decreases. Methods like the Fourier Modal Method (FMM) or the Rigorous Coupled Wave Analysis (RCWA) are based on this approach.

An alternative formulation requires the use of $N-1$ curvilinear coordinate transformations of the type:

$$\begin{cases} v = x \\ u_j = y - a_j(x) \qquad j = 1, 2, \ldots N-1 \end{cases} \tag{3}$$

In the new coordinate systems (v, u_j) , the j^{th} interface is described by a flat profile $u_j = 0$ and the two layers surrounding the interface have permittivities $\epsilon(v, u_j) = \epsilon_i \quad (i = j, j{+}1)$.

We call $F^{(i,j)}$ the magnetic field H_z in medium i expressed in the coordinate system (v, u_j). The general problem (2) is then simplified into the $2(N-1)$ elementary problems:

$$L_{i,j}\left(\frac{\partial}{\partial v}, \frac{\partial}{\partial u_j}; v\right) F^{(i,j)}(v, u_j) = 0 \qquad i = j, j+1; \quad j = 1, 2, \dots N-1 \tag{4}$$

As a final step, a proper mathematical connections between fields $F^{(j+1,j)}$ and $F^{(j+1,j+1)}$ $(j = 1, 2, \dots N-2)$ must be carried out. The C-method is based on this formulation of the problem.

2 The eigenvalue problem

Let us consider the interface $a_j(x)$ of the structure. If we apply the change of variables (3), the $L_{i,j}$ operators become:

$$L_{i,j} = \frac{\partial^2}{\partial v^2} - 2\dot{a}_j \frac{\partial^2}{\partial v \partial u_j} - \ddot{a}_j \frac{\partial}{\partial u_j} + (1 + \dot{a}_j^2)\frac{\partial^2}{\partial u_j^2} + k_0^2 \epsilon_i \quad i = j, j+1 \tag{5}$$

and the Helmoltz equation (4) in media j , $j+1$ can be rewritten as a double couple of differential equations of the first order:

$$\begin{bmatrix} k_0^2\epsilon_i + \frac{\partial^2}{\partial v^2} & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} F^{(i,j)} \\ f^{(i,j)} \end{pmatrix} = \begin{bmatrix} i\left(\frac{\partial}{\partial v}\dot{a}_j + \dot{a}_j \frac{\partial}{\partial v}\right) & 1 + \dot{a}_j^2 \\ 1 & 0 \end{bmatrix} \cdot \frac{1}{i}\frac{\partial}{\partial u_j} \begin{pmatrix} F^{(i,j)} \\ f^{(i,j)} \end{pmatrix} \quad i = j, j+1 \tag{6}$$

Let the fields $F^{(i,j)}(v, u_j)$ and $f^{(i,j)}(v, u_j)$ be expanded in Fourier series:

$$F^{(i,j)}(v, u_j) = \sum_{m=-\infty}^{\infty} F_m^{(i,j)}(u_j) e^{i\alpha_m v} \qquad f^{(i,j)}(v, u_j) = \sum_{m=-\infty}^{\infty} f_m^{(i,j)}(u_j) e^{i\alpha_m v} \tag{7}$$

where $\alpha_m = n_N k_0 \sin\theta + mK$, $n_N = \sqrt{\epsilon_N}$ and $K = 2\pi\Lambda^{-1}$. In a practical implementation, the summation over an infinite number of elements must be truncated. We will retain $2N_o + 1$ orders, in such a way that $m \in [-N_o, N_o]$. Since the differential operators in equation (6) do not depend explicitly on u_j, we can assume a dependence in the exponential form $\exp(i\rho u_j)$ for $F^{(i,j)}$ and $f^{(i,j)}$. As a consequence, the following transformations can be introduced:

$$\frac{\partial}{\partial v} \to i\boldsymbol{\alpha}, \qquad \frac{\partial}{\partial u_j} \to i\rho \tag{8}$$

and equation (6) can be re-written in the following matrix form

$$\begin{bmatrix} \boldsymbol{\beta}^{(i)^2} & 0 \\ 0 & \mathbb{I} \end{bmatrix} \begin{pmatrix} \boldsymbol{F}^{(i,j)} \\ \boldsymbol{f}^{(i,j)} \end{pmatrix} = \rho \begin{bmatrix} -(\boldsymbol{\alpha}\dot{\boldsymbol{a}}_j + \dot{\boldsymbol{a}}_j\boldsymbol{\alpha}) & \mathbb{I} + \dot{\boldsymbol{a}}_j\dot{\boldsymbol{a}}_j \\ \mathbb{I} & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{F}^{(i,j)} \\ \boldsymbol{f}^{(i,j)} \end{pmatrix} \quad i = j, j+1 \tag{9}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}^{(i)}$ are diagonal matrices whose elements are α_m and $\beta_m^{(i)} = \sqrt{\epsilon_i k_0^2 - \alpha_m^2}$, $\text{Re}[\beta_m^{(i)}] + \text{Im}[\beta_m^{(i)}] > 0$ $(m \in [-N_o, N_o])$ respectively; $\boldsymbol{F}^{(i,j)}$ and $\boldsymbol{f}^{(i,j)}$ are column vectors containing the $2N_o + 1$ Fourier components $F_m^{(i,j)}$ and $f_m^{(i,j)}$ and $\dot{\boldsymbol{a}}_j$ is the Toeplitz antisymmetric squared matrix of the Fourier coefficients of the the function $\dot{a}_j = \frac{da_j}{dx}$:

$$(\dot{a}_j)_{mn} = \frac{1}{\Lambda}\int_0^{\Lambda} \dot{a}_j\, e^{-i(m-n)Kx} dx \qquad n \in [-N_o, N_o] \tag{10}$$

After a little algebra, it is possible to reduce the differential equations (9) to two eigenvalues problems of the kind $B\Psi = \rho^{-1}\Psi$.

The set of $2(2N_o + 1)$ eigenvalues $\rho^{(i,j)}$ can be split into the following two groups:

$$\begin{array}{ll} \rho_p^{(i,j)+} = \rho_p^{(i,j)} & \forall p : \text{Re}[\rho_p^{(i,j)}] + \text{Im}[\rho_p^{(i,j)}] > 0 \\ \rho_p^{(i,j)-} = \rho_p^{(i,j)} & \forall p : \text{Re}[\rho_p^{(i,j)}] + \text{Im}[\rho_p^{(i,j)}] < 0 \end{array} \tag{11}$$

and the general solution to (4) can be written as:

$$F_+^{(i,j)}(v, u_j) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}_{mp}^{(i,j)+}\, e^{i\rho_p^{(i,j)+} u_j} C_p^{(i,j)+} \tag{12}$$

$$F_-^{(i,j)}(v, u_j) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}_{mp}^{(i,j)-}\, e^{i\rho_p^{(i,j)-} u_j} C_p^{(i,j)-} \tag{13}$$

$$F^{(i,j)}(v, u_j) = F_+^{(i,j)}(v, u_j) + F_-^{(i,j)}(v, u_j) \qquad i = j\,, j+1 \tag{14}$$

where $C_p^{(i,j)\pm}$ is the unknown amplitude and $\mathcal{F}_{mp}^{(i,j)\pm}$ is the generic m^{th} element of the p^{th} eigenvector (associated to $\rho_p^{(i,j)\pm}$).

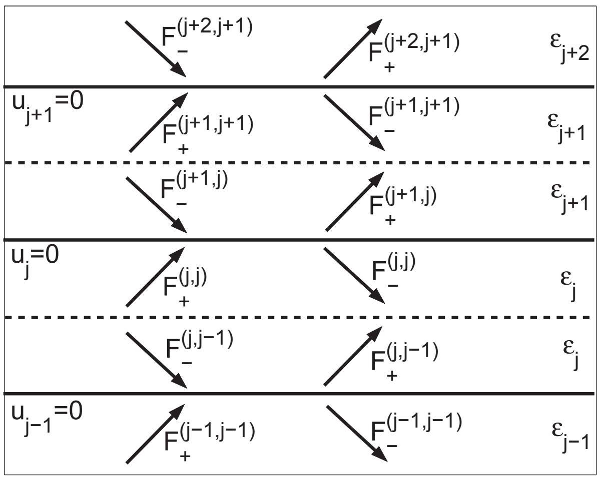

Figure 2: Schematic representation of up-going and down-going waves in the multilayered structure. Solid lines represent real medium boundaries, dashed lines represents fictitious interfaces where field connection must be imposed

Two terms contributes to the field $F^{(i,j)}(v, u_j)$: those terms denoted with the superscript "+" are associated to propagating or decaying waves traveling in the positive y-direction, while those denoted with the superscript "$-$" represent waves propagating or decaying in the negative y-direction. In the drawing of Fig. 2 an intuitive representation of the fields $F_{\pm}^{(i,j)}(v,u_j)$ is shown. Once the complete set of eigenvectors has been calculated for all interfaces, boundary conditions must be imposed.

3 Boundary conditions and field connection

In order to determine the unknown amplitudes $C_p^{(i,j)\pm}$, the continuity of the tangential electric and magnetic fields across each interface must be imposed. $H_z(x,y)$ is always tangential to the structure profiles, then:

$$F_-^{(j,j)} + F_+^{(j,j)} = F_-^{(j+1,j)} + F_+^{(j+1,j)} \qquad j = 1,2,\dots N-1 \tag{15}$$

At each interface, the tangential component $G^{(i,j)}(v,u_j)$ of the electric field is proportional to the scalar product of the tangential vector and the electric field vector:

$$G^{(i,j)}(v,u_j) = \boldsymbol{E}^{(i,j)}(v,u_j)\cdot\boldsymbol{t}^{(j)} = \boldsymbol{E}^{(i,j)}(v,u_j)\,\cdot\,\hat{\boldsymbol{x}} + \boldsymbol{E}^{(i,j)}(v,u_j)\,\cdot\,\dot{a}_j\hat{\boldsymbol{y}} \tag{16}$$

where

$$\boldsymbol{E}^{(i,j)}(v,u_j) = \left(\frac{i\eta_0}{k_0\epsilon_i}\frac{\partial F^{(i,j)}}{\partial y}\,,-\frac{i\eta_0}{k_0\epsilon_i}\frac{\partial F^{(i,j)}}{\partial x}\,,0\right)\;; \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \tag{17}$$

which gives

$$G^{(i,j)}(v,u_j) = -\frac{i\eta_0}{k_0\epsilon_i}\left[\dot{a}_j\frac{\partial F^{(i,j)}}{\partial v} - (1+\dot{a}_j^2)\frac{\partial F^{(i,j)}}{\partial u_j}\right] \tag{18}$$

For symmetry reasons, it is preferable to write $G^{(i,j)}$ in a form similar to (14):

$$G^{(i,j)}(v,u_j) = G_+^{(i,j)}(v,u_j) + G_-^{(i,j)}(v,u_j) \qquad i = j\,,j+1 \tag{19}$$

with

$$G_{\pm}^{(i,j)}(v,u_j) = \sum_m e^{i\alpha_m v}\sum_p \mathcal{G}_{mp}^{(i,j)\pm}\,e^{i\rho_p^{(i,j)\pm}u_j}C_p^{(i,j)\pm} \tag{20}$$

and

$$\mathcal{G}_{mp}^{(i,j)\pm} = \frac{\eta_0}{k_0\epsilon_i}\left(\sum_{n=-N_o}^{N_o}(\dot{a}_j)_{mn}\cdot\alpha_n\mathcal{F}_{np}^{(i,j)\pm} - (\delta_{mn} + (\dot{a}_j\cdot\dot{a}_j)_{mn})\,\rho_p^{(i,j)}\mathcal{F}_{np}^{(i,j)\pm}\right) \tag{21}$$

The continuity condition for the tangential electric field is, then:

$$G_-^{(j,j)} + G_+^{(j,j)} = G_-^{(j+1,j)} + G_+^{(j+1,j)} \qquad j = 1,2,\dots N-1 \tag{22}$$

The fields continuity conditions can be expressed using a scattering-matrix formalism. After having defined

$$s^{(j)} = \begin{pmatrix} \boldsymbol{\mathcal{F}}^{(j+1,j)+} & -\boldsymbol{\mathcal{F}}^{(j,j)-} \\ \boldsymbol{\mathcal{G}}^{(j+1,j)+} & -\boldsymbol{\mathcal{G}}^{(j,j)-} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{\mathcal{F}}^{(j,j)+} & -\boldsymbol{\mathcal{F}}^{(j+1,j)-} \\ \boldsymbol{\mathcal{G}}^{(j,j)+} & -\boldsymbol{\mathcal{G}}^{(j+1,j)-} \end{pmatrix} \tag{23}$$

equations (15) and (22) are summarized in the following relation

$$\begin{pmatrix} \boldsymbol{C}^{(j+1,j)+} \\ \boldsymbol{C}^{(j,j)-} \end{pmatrix} = s^{(j)} \begin{pmatrix} \boldsymbol{C}^{(j,j)+} \\ \boldsymbol{C}^{(j+1,j)-} \end{pmatrix} \qquad j = 1, 2, \ldots N-1 \tag{24}$$

Fields $F^{(j,j)}(v, u_j)$ and $F^{(j,j-1)}(v, u_{j-1})$ are defined over the same spatial region having permittiviy ϵ_j $(j = 2, 3, \ldots N-1)$. Nonetheless, they are expressed by means of two different coordinate systems. A connection between these two fields is, then, required. In order to deal with this problem, Granet et al. [7] proposed the following assumption: for any (v, u_j) and (v, u_{j-1}) describing the same point in the j^{th} layer:

$$F_{\pm}^{(j,j)}(v, u_j) = F_{\pm}^{(j,j-1)}(v, u_{j-1}) \tag{25}$$

If the two couples of coordinates (v, u_j) and (v, u_{j-1}) describe the same point, we have:

$$u_j + a_j(x) = u_{j-1} + a_{j-1}(x) \tag{26}$$

In particular, in the (v, u_{j-1}) coordinate system, the points along interface $u_j = 0$ are identified by $u_{j-1} = a_j(x) - a_{j-1}(x)$, while the points along interface $u_{j-1} = 0$ are described by $u_j = a_{j-1}(x) - a_j(x)$ in the (v, u_j) coordinate system. The fields connection relation in medium j can be expressed as

$$F_{+}^{(j,j)}(v, 0) = F_{+}^{(j,j-1)}(v, a_j - a_{j-1}) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}_{mp}^{(j,j-1)+} e^{i\rho_p^{(j,j-1)+}(a_j - a_{j-1})} C_p^{(j,j-1)+} \tag{27}$$

$$F_{-}^{(j,j-1)}(v, 0) = F_{-}^{(j,j)}(v, a_{j-1} - a_j) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}_{mp}^{(j,j)-} e^{i\rho_p^{(j,j)-}(a_{j-1} - a_j)} C_p^{(j,j)-} \tag{28}$$

In order to reduce the previous expressions to a simpler form, we build the following rank-2 tensors, formed by the Fourier coefficients

$$\left[L^{(j,j-1)+}\right]_{mlp} = \frac{1}{\Lambda} \int_0^{\Lambda} e^{i\rho_p^{(j,j-1)+}(a_j - a_{j-1})} \, e^{-i(m-l)Kx} dx\,; \quad l \in [-N_o, N_o] \tag{29}$$

$$\left[L^{(j,j)-}\right]_{mlp} = \frac{1}{\Lambda} \int_0^{\Lambda} e^{i\rho_p^{(j,j)-}(a_{j-1} - a_j)} \, e^{-i(m-l)Kx} dx\,; \qquad l \in [-N_o, N_o] \tag{30}$$

The previous expressions can be interpreted as two stacks of $2N_o + 1$ Toeplitz matrices. With the help of tensors (29) and (30), we can write a linear transformation of eigenvectors $\mathcal{F}_{mp}^{(j,j-1)+}$ and $\mathcal{F}_{mp}^{(j,j)-}$:

$$\tilde{\mathcal{F}}^{(j,j-1)+}_{mp} = \sum_l \left[L^{(j,j-1)+}\right]_{mlp} \mathcal{F}^{(j,j-1)+}_{mp} \tag{31}$$

$$\tilde{\mathcal{F}}^{(j,j)-}_{mp} = \sum_l \left[L^{(j,j)-}\right]_{mlp} \mathcal{F}^{(j,j)-}_{mp} \tag{32}$$

which allow us to re-write equations (27) and (28) in the following compact form:

$$F^{(j,j)}_{+}(v,0) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}^{(j,j-1)+}_{mp}\, e^{i\rho_p^{(j,j-1)+}(0)} C^{(j,j-1)+}_p = \tilde{F}^{(j,j-1)}_{+}(v,0) \tag{33}$$

$$F^{(j,j-1)}_{-}(v,0) = \sum_m e^{i\alpha_m v} \sum_p \mathcal{F}^{(j,j)-}_{mp}\, e^{i\rho_p^{(j,j)-}(0)} C^{(j,j)-}_p = \tilde{F}^{(j,j)}_{-}(v,0) \tag{34}$$

In a scattering-matrix formalism, expressions (33) and (34) can be written as

$$\begin{pmatrix} \boldsymbol{C}^{(j,j)+} \\ \boldsymbol{C}^{(j,j-1)-} \end{pmatrix} = t^{(j)} \begin{pmatrix} \boldsymbol{C}^{(j,j-1)+} \\ \boldsymbol{C}^{(j,j)-} \end{pmatrix} \qquad j = 2, 3, \ldots N-1 \tag{35}$$

with

$$t^{(j)} = \begin{pmatrix} \boldsymbol{\mathcal{F}}^{(j,j)+} & 0 \\ 0 & -\boldsymbol{\mathcal{F}}^{(j,j-1)-} \end{pmatrix}^{-1} \begin{pmatrix} \tilde{\boldsymbol{\mathcal{F}}}^{(j,j-1)+} & 0 \\ 0 & -\tilde{\boldsymbol{\mathcal{F}}}^{(j,j)-} \end{pmatrix} \tag{36}$$

The final step is to cascade all the scattering matrices $s^{(j)}$ and $t^{(j)}$:

$$j = N-1 \,\text{interface} \qquad \begin{pmatrix} \boldsymbol{C}^{(N,N-1)+} \\ \boldsymbol{C}^{(N-1,N-1)-} \end{pmatrix} = s^{(N-1)} \begin{pmatrix} \boldsymbol{C}^{(N-1,N-1)+} \\ \boldsymbol{C}^{(N,N-1)-} \end{pmatrix}$$

$$j = N-1 \,\text{medium} \qquad \begin{pmatrix} \boldsymbol{C}^{(N-1,N-1)+} \\ \boldsymbol{C}^{(N-1,N-2)-} \end{pmatrix} = t^{(N-1)} \begin{pmatrix} \boldsymbol{C}^{(N-1,N-2)+} \\ \boldsymbol{C}^{(N-1,N-1)-} \end{pmatrix}$$

$$j = N-2 \,\text{interface} \qquad \begin{pmatrix} \boldsymbol{C}^{(N-1,N-2)+} \\ \boldsymbol{C}^{(N-2,N-2)-} \end{pmatrix} = s^{(N-2)} \begin{pmatrix} \boldsymbol{C}^{(N-2,N-2)+} \\ \boldsymbol{C}^{(N-1,N-2)-} \end{pmatrix}$$

....

$$j = 2 \,\text{medium} \qquad \begin{pmatrix} \boldsymbol{C}^{(2,2)+} \\ \boldsymbol{C}^{(2,1)-} \end{pmatrix} = t^{(2)} \begin{pmatrix} \boldsymbol{C}^{(2,1)+} \\ \boldsymbol{C}^{(2,2)-} \end{pmatrix}$$

$$j = 1 \,\text{interface} \qquad \begin{pmatrix} \boldsymbol{C}^{(2,1)+} \\ \boldsymbol{C}^{(1,1)-} \end{pmatrix} = s^{(1)} \begin{pmatrix} \boldsymbol{C}^{(1,1)+} \\ \boldsymbol{C}^{(2,1)-} \end{pmatrix}$$

With some matrix algebra, it is possible to collapse all these matrices into a $2(2N_o+1) \times 2(2N_o+1)$ one, called *scattering matrix of the system* $\boldsymbol{S}$.

$$\begin{pmatrix} \boldsymbol{C}^{(N,N-1)+} \\ \boldsymbol{C}^{(1,1)-} \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \boldsymbol{C}^{(1,1)+} \\ \boldsymbol{C}^{(N,N-1)-} \end{pmatrix} \tag{37}$$

The amplitudes $C^{(1,1)+}$ and $C^{(N,N-1)-}$ represent the input of the scattering problem. In particular, $C^{(1,1)+} = 0$ since the structure is not illuminated from the bottom. Moreover, the incident field at interface $N-1$ expressed in the coordinate system (v, u_{N-1}) is $\psi = exp\,(in_N k_0 \sin\theta v) \cdot exp\,(in_N k_0 \cos\theta a_{N-1}(x))$ and must be expressed in the base of eigenvectors $\boldsymbol{\mathcal{F}}^{(N,N-1)-}$. This is readily done if we consider the Fourier decomposition of ψ:

$$\psi = \sum_m P_m e^{i\alpha_m v} \qquad \text{with} \quad P_m = \frac{1}{\Lambda}\int_0^\Lambda e^{in_N k_0 \cos\theta a_{N-1}}\, e^{-imKx} dv \tag{38}$$

and defining the matrix $\boldsymbol{\mathcal{F}}^{(N,N-1)-}$ with only one non-zero column:

$$\left[\boldsymbol{\mathcal{F}}^{(N,N-1)-}\right]_{m1} = P_m \tag{39}$$

By using the last representation of $\boldsymbol{\mathcal{F}}^{(N,N-1)-}$ in the definition of the scattering matrix $s^{(N-1)}$, the presence of the incident field can be taken into account by setting $C^{(N,N-1)-} = 1$.

The linear system of equations (37) is now completely defined. Once the solutions $C^{(N,N-1)+}$ and $C^{(1,1)-}$ are found, a classical iteration method can be used for the calculation of the whole set of amplitude values $C^{(j,i)\pm}$ $(i = j, j+1)$.

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