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Workshop on Strongly Coupled Physics Beyond the Standard Model

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Carving out the Space of 4D CFTs

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Carving Out the Space of 4D CFTs

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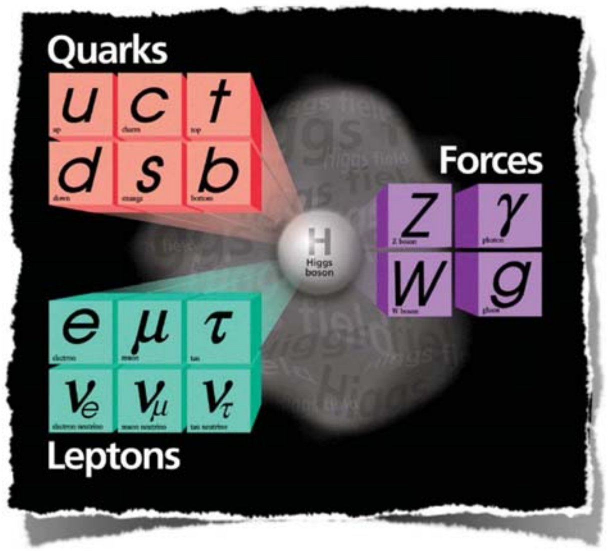
Workshop on Strongly Coupled Physics Beyond the Standard Model
ICTP – Trieste, Italy

Why Study Conformal Field Theories?

Many reasons to study Conformal Field Theories:

- QFTs often flow to conformal fixed points
- They describe quantum gravity via AdS/CFT
- They describe condensed matter systems
- ...

Why Study Conformal Field Theories?



- 4D CFTs could play a role in Beyond the Standard Model physics!
  - Walking/Conformal Technicolor [Holdom '81; ...]
  - Warped Extra Dimensions [Randall, Sundrum '99; ...]
  - Flavor Hierarchies [Georgi, Nelson, Manohar '83; Nelson, Strassler '00; DP, Simmons-Duffin '09; ...]
  - Conformal Sequestering [Luty, Sundrum '01]
  - Solution to $\mu/B\mu$ problem [Roy, Schmaltz '07; Murayama, Nomura, DP '07]
  - ...

Why Study Conformal Field Theories?

However, ideas often depend *crucially* on spectrum of operator dim's...

- Conformal Technicolor [Luty, Okui '04]:
  (previously "Strong ETC")
  - Higgs is CFT operator H, with couplings $\sim \left(\frac{1}{\Lambda}\right)^{d-1} H\overline{q}_i u_j$
  - Want $d = \dim(H) \sim 1$ to give top mass without low flavor scale Λ
  - Want $\dim(H^\dagger H) \gtrsim 4$ to solve hierarchy problem

Is this even possible???

Theories that don't work...

- Perturbative CFTs: $\dim(H) = 1 + \mathcal{O}(\epsilon)$, $\dim(H^\dagger H) = 2 + \mathcal{O}(\epsilon)$
- Large-N CFTs: $\dim(H^\dagger H) = 2\dim(H) + \mathcal{O}(1/N^2)$

A Way Forward...

[Rattazzi, Rychkov, Tonni, Vichi '08]:
Crossing Symmetry + Unitarity leads to *bounds* on operator dimensions!

Method then extended to:

- Bounds in $\mathcal{N}=1$ Superconformal Theories
  [DP, Simmons-Duffin '10; Vichi '11]
- Bounds in the presence of global symmetries
  [Rattazzi, Rychkov, Vichi '10; Vichi '11]
- Bounds on various operator product expansion coefficients
  - Scalar 3pt functions [Caracciolo, Rychkov '09]
  - Flavor Symmetry Currents [DP, Simmons-Duffin '10]
  - Stress Tensor $\rightarrow$ Bounds on central charge c
    [DP, Simmons-Duffin '10; Rattazzi, Rychkov, Vichi '10; Vichi '11]

New methods and latest results in [DP, Simmons-Duffin, Vichi '11]

Outline

1. CFT Review
2. Bounds from Crossing Relations
3. Latest Results

CFT Review: Algebra and Primary Operators

The conformal algebra $\mathrm{SO}(4,2)$ contains:

- Translations P^a and rotations M^{ab}
- Dilatations D (scale transformations)
- Special conformal generators K^a (inv. $\rightarrow$ trans. $\rightarrow$ inv.)

$$[K^a, P^b] = 2\eta^{ab} D - 2M^{ab}$$

- *Primary* operators $\mathcal{O}(0)$ are defined by $[K^a, \mathcal{O}(0)] = 0$
- *Descendants* obtained using $[P^a, \mathcal{O}(0)] = \partial^a \mathcal{O}(0)$

CFT Review: Correlation Functions

- Conformal symmetry fixes primary 2pt and 3pt functions in terms of dim's and spins, up to coefficients $\lambda_{\mathcal{O}}$ [Polyakov '70; Osborn, Petkou '93]

$$\langle \mathcal{O}^{a_1..a_\ell}(x_1)\mathcal{O}^{b_1..b_\ell}(x_2)\rangle = \frac{I^{a_1b_1}..I^{a_\ell b_\ell}}{x_{12}^{2\Delta}} \qquad \left[I^{ab} \equiv \eta^{ab} - 2\frac{x_{12}^a x_{12}^b}{x_{12}^2}\right]$$

$$\langle \phi(x_1)\phi(x_2)\mathcal{O}^{a_1..a_\ell}(x_3)\rangle = \lambda_{\mathcal{O}} \frac{Z^{a_1}..Z^{a_\ell}}{x_{12}^{2d-\Delta+\ell}x_{23}^{\Delta-\ell}x_{13}^{\Delta-\ell}} \quad \left[Z^a \equiv \frac{x_{31}^a}{x_{31}^2} - \frac{x_{32}^a}{x_{32}^2}\right]$$

- In *Unitary* CFTs, one also has the bound $\Delta \geq \ell + 2 - \delta_{\ell,0}$ [Mack '77]
  - Requirement that 2pt functions of descendants are ≥ 0
  - Can always work in basis where $\lambda_{\mathcal{O}}$'s are real

- Higher n-pt functions *not* fixed by conformal symmetry alone, but are determined once spectrum and $\lambda_{\mathcal{O}}$'s are known...

CFT Review: Operator Product Expansion

Let ϕ be a scalar primary of dimension d in a 4D CFT:

$$\phi(x)\phi(0) = \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}} C_I(x,\partial)\,\mathcal{O}^I(0) \qquad \text{(OPE)}$$

- Sum runs over *primary* $\mathcal{O}$'s
- $\mathcal{O}^I = \mathcal{O}^{a_1\ldots a_\ell}$ any spin-ℓ Lorentz rep with $\ell = 0, 2, \ldots$
- $C_I(x,\partial)$ fixed by conformal symmetry

CFT Review: Conformal Block Decomposition

Use OPE to evaluate 4-point function [Ferrara, Gatto, Grillo '73; ...]

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

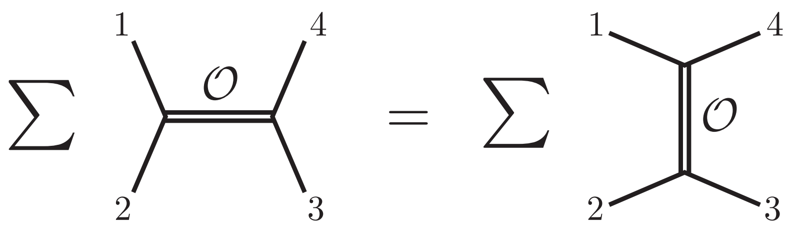
$$= \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 C_I(x_{12},\partial_2)C_J(x_{34},\partial_4)\langle \mathcal{O}^I(x_2)\mathcal{O}^J(x_4)\rangle$$

$$\equiv \frac{1}{x_{12}^{2d}x_{34}^{2d}} \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2\, g_{\Delta,\ell}(u,v)$$

- $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$ conformally-invariant cross ratios.
- $g_{\Delta,\ell}(u,v)$ conformal block ($\Delta = \dim \mathcal{O}$ and $\ell =$ spin of $\mathcal{O}$)
  - Power series expansions known since 70's, now known fully in terms of hypergeometric functions [Dolan, Osborn '00; Dolan, Osborn '03]

CFT Review: Crossing Relations

- $\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$ clearly symmetric under permutations of x_i
- After OPE, symmetry is non-manifest!
- Switching $x_1 \leftrightarrow x_3$ gives the "crossing relation":



$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 g_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^d \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 g_{\Delta,\ell}(v,u)$$

- Other permutations give no new information

CFT Review: Crossing Relations

It is convenient to write this as the sum rule

$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 F_{\Delta,\ell}(u,v) = 0$$

where

$$F_{\Delta,\ell}(u,v) \equiv \frac{v^d g_{\Delta,\ell}(u,v) - u^d g_{\Delta,\ell}(v,u)}{u^d - v^d}.$$

This is a *constraint* on the spectrum of Δ's, ℓ's, and $\lambda_{\mathcal{O}}$'s:

- Important implications for BSM scenarios (once generalized)
- Theoretical gold-mine! Many new insights about CFTs are just waiting to be extracted...

Generalization to Global Symmetries

Now suppose ϕ_i is an $\mathrm{SO}(N)$ fundamental. The OPE is

$$\phi_i \times \phi_j \quad \sim \quad \sum_{S^+} \delta_{ij}\mathcal{O} + \sum_{T^+} \mathcal{O}_{(ij)} + \sum_{A^-} \mathcal{O}_{[ij]},$$

and the 4pt function can be expanded in various tensor structures

$$\begin{aligned} x_{12}^{2d} x_{34}^{2d} \langle \phi_i(x_1)\phi_j(x_2)\phi_k(x_3)\phi_l(x_4)\rangle \\ &= \sum_{S^+} \lambda_{\mathcal{O}}^2 (\delta_{ij}\delta_{kl}) g_{\Delta,\ell}(u,v) \\ &+ \sum_{T^+} \lambda_{\mathcal{O}}^2 \left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{N}\delta_{ij}\delta_{kl} \right) g_{\Delta,\ell}(u,v) \\ &+ \sum_{A^-} \lambda_{\mathcal{O}}^2 \left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} \right) g_{\Delta,\ell}(u,v). \end{aligned}$$

Generalization to Global Symmetries

Symmetry under $x_1 \leftrightarrow x_3$ and $i \leftrightarrow k$ leads to the triple-sum rule:
[Rattazzi, Rychkov, Vichi '10]

$$\sum_{S^+} \lambda_{\mathcal{O}}^2 \begin{pmatrix} 0 \\ F_{\Delta,\ell} \\ H_{\Delta,\ell} \end{pmatrix} + \sum_{T^+} \lambda_{\mathcal{O}}^2 \begin{pmatrix} F_{\Delta,\ell} \\ (1-\frac{2}{N})F_{\Delta,\ell} \\ -(1+\frac{2}{N})H_{\Delta,\ell} \end{pmatrix} + \sum_{A^-} \lambda_{\mathcal{O}}^2 \begin{pmatrix} -F_{\Delta,\ell} \\ F_{\Delta,\ell} \\ -H_{\Delta,\ell} \end{pmatrix} = 0$$

(Here $H_{\Delta,\ell}(u,v)$ is $F_{\Delta,\ell}(u,v)$ with $- \to +$)

- 3 sum rules $\leftrightarrow$ 3 tensor structures

Similar rules for other global symmetries:

- $\mathrm{SU}(N) \to$ 6 sum rules
- $\mathcal{N} = 1$ SCFTs $\to$ 3 sum rules (since $U(1)_R \sim \mathrm{SO}(2)$)
  - $\mathcal{O}$'s in same SUSY multiplet have related λ's: $g_{\Delta,\ell} \to \mathcal{G}_{\Delta,\ell}$ (superconformal blocks) [DP, DSD '10; Fortin, Intriligator, Stergiou '11]

Outline

How Does Crossing Symmetry Lead to CFT Bounds?

Crossing relation for real scalar ϕ:

- Separate out the *unit* operator in $\phi \times \phi \sim 1 + \phi^2 + \ldots$

$$\underbrace{1}_{\text{unit op.}} = \underbrace{\sum \lambda_{\mathcal{O}}^2 F_{\Delta,\ell}(u,v)}_{\text{everything else}},$$

- Make an assumption: all scalars have dimension $\Delta > \Delta_{\mathsf{min}}$
- Search for a linear functional α such that

$$\begin{aligned} \alpha(1) &< 0, \quad \text{and} \\ \alpha(F_{\Delta,\ell}) &\geq 0, \quad \text{for all other } \mathcal{O} \in \phi \times \phi. \end{aligned}$$

- If you find one, the assumption is ruled out!

CFT Bounds

Convenient to phrase search as a convex optimization problem:

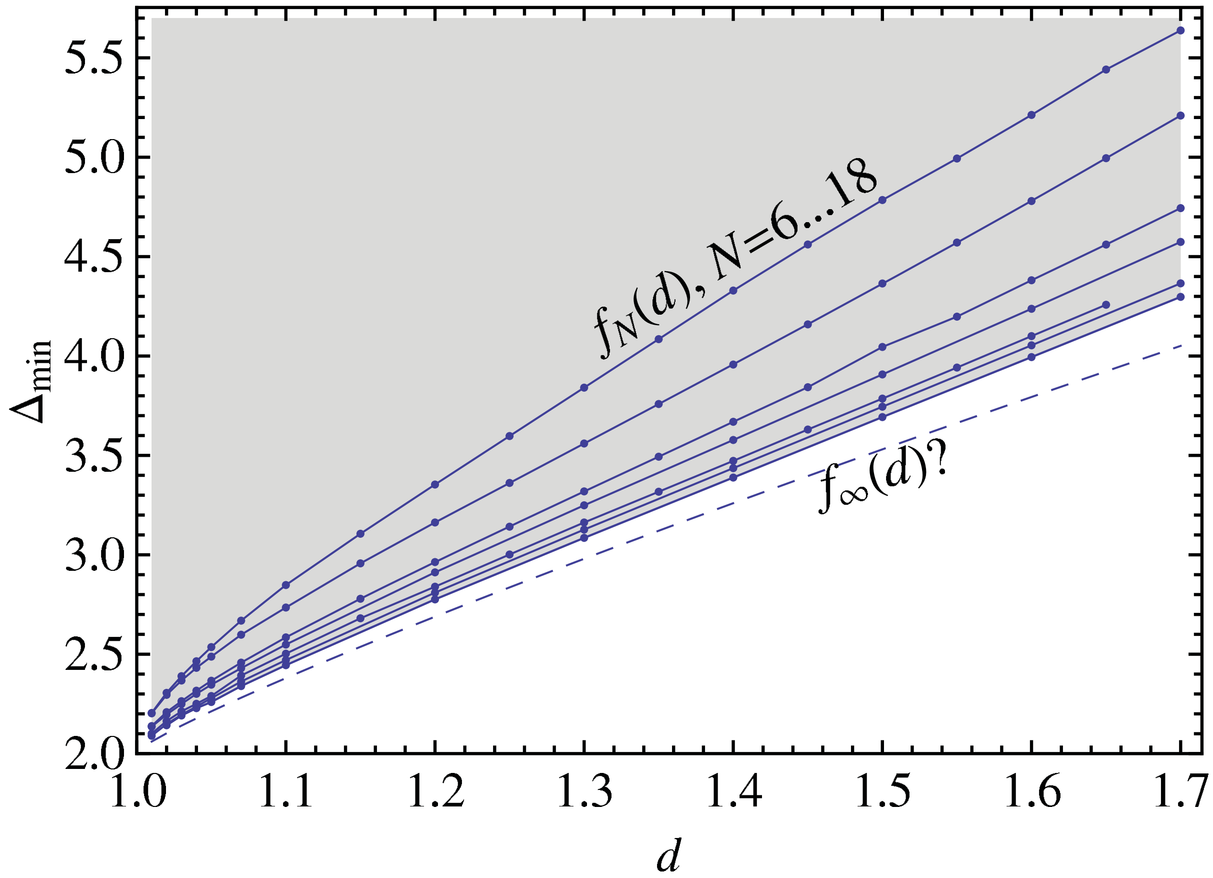
$$\text{Minimize } \alpha(1) \text{ subject to } \alpha(F_{\Delta,\ell}) \geq 0$$

- Adding normalization $\alpha(F_{\Delta_0,\ell_0}) = 1$ gives a bound $\lambda^2_{\mathcal{O}_0} \leq \alpha(1)$
- It would be very interesting to solve this analytically! Hard...
- However, great progress has been made numerically

First Approach: [Rattazzi, Rychkov, Tonni, Vichi '08]

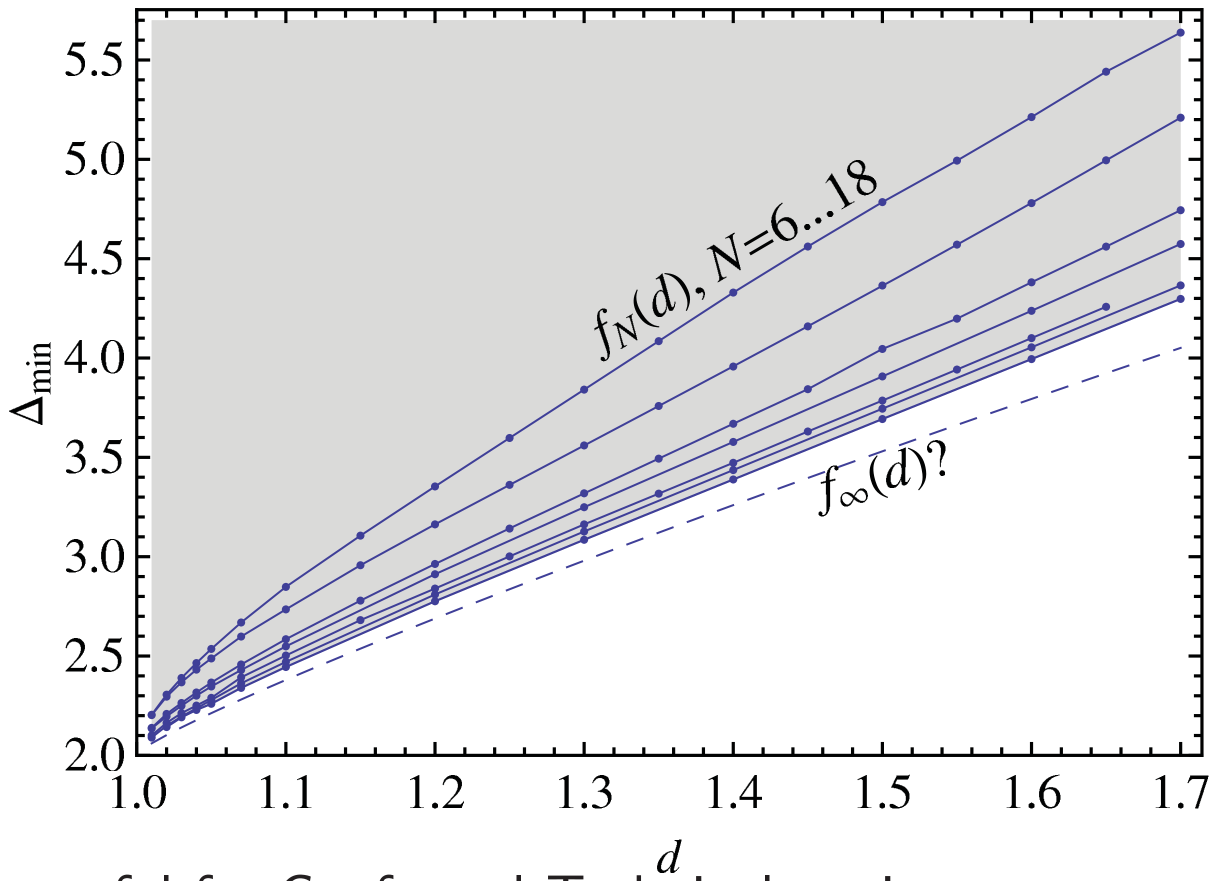
- Impose $\alpha(F_{\Delta_i,\ell_i}) \geq 0$ on a finite lattice $\{(\Delta_i, \ell_i)\}$ (verify positivity on intermediate values later)
- Take α to be linear combinations of $\partial_z^n \partial_{\bar{z}}^m F_{\Delta,\ell}$ at some point
- Implement as a *linear programming* problem that can be solved numerically (e.g., by Mathematica, GLPK, CPLEX, ...)

Bounds on $\dim \phi^2$ (from [Rychkov, Vichi '09])



- Bound on lowest dim scalar in $\phi \times \phi$ OPE, where $d = \dim(\phi)$
- Different lines correspond to increasing space of derivatives ($N = 18 \leftrightarrow 55$-dimensional space)

Bounds on $\dim \phi^2$ (from [Rychkov, Vichi '09])



- Not yet useful for Conformal Technicolor, since $\mathrm{Re}(H_0) \times \mathrm{Re}(H_0) \sim H^\dagger H + H^\dagger \sigma H + \ldots$
  - Need to distinguish between $\mathrm{SU}(2)_W$ representations!
- Linear programming tricky for systems of crossing relations...

Semidefinite Programming

Latest Approach [DP, Simmons-Duffin, Vichi '11]:

- Derivatives of conformal blocks can be arbitrarily-well approximated by positive functions times polynomials:
$$\partial_z^m \partial_{\bar{z}}^n F_{\Delta,\ell} \simeq \chi_\ell(\Delta) P_\ell^{m,n}(\Delta)$$
- A polynomial $P(\Delta)$ is positive over an interval $[0,\infty)$ iff it can be written as $P(\Delta) = f(\Delta) + \Delta g(\Delta)$, where $f(\Delta)$ and $g(\Delta)$ are sums-of-squares of polynomials [Hilbert, 1888]
- A sum-of-squares can be represented by a *positive-semidefinite* matrix A: $f(\Delta) = [\Delta]_d^T A [\Delta]_d$, where $[\Delta]_d^T = (1, \Delta, \ldots, \Delta^d)$

Semidefinite Programming

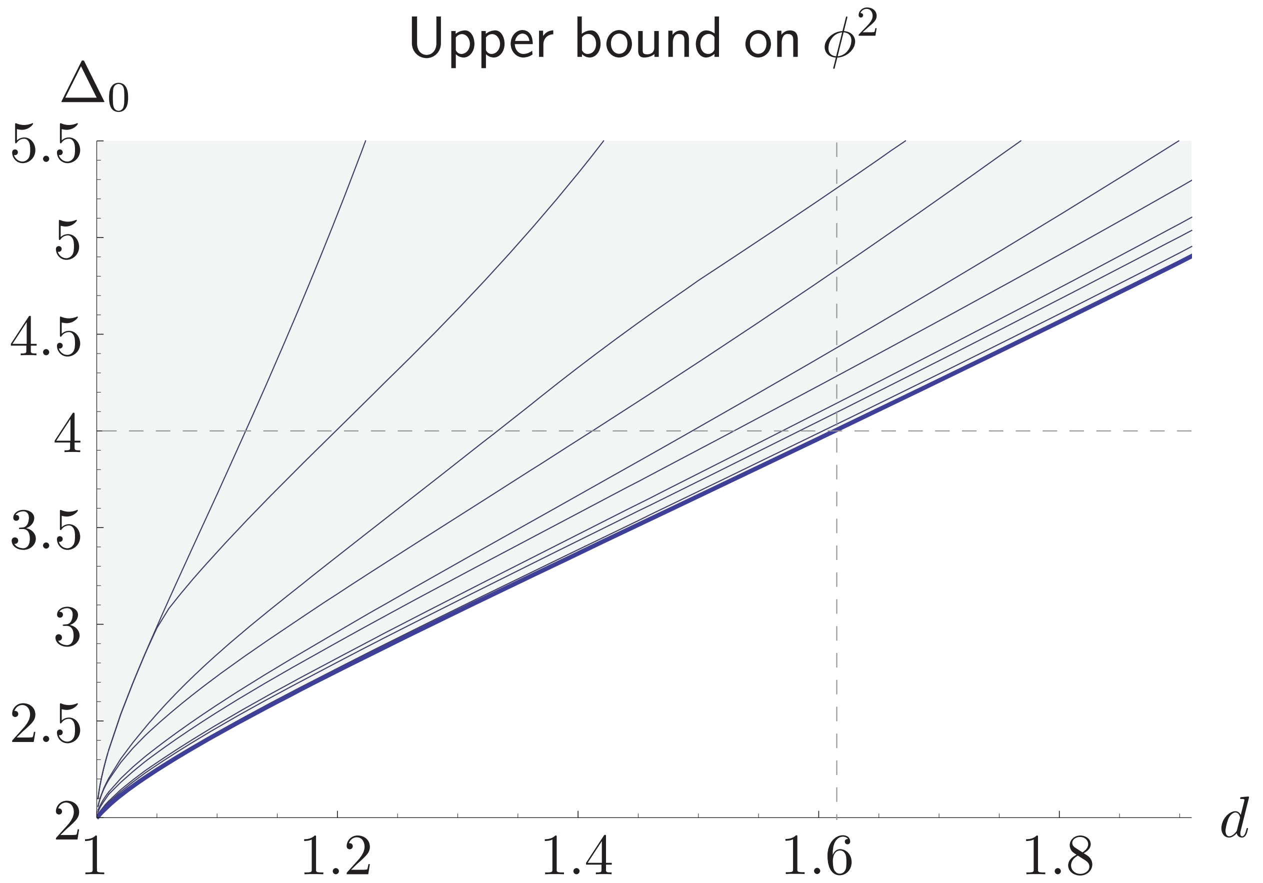
Latest Approach [DP, Simmons-Duffin, Vichi '11]:

- Written in this way, the problem is phrased as a *semidefinite programming* problem, which can be solved by available software packages (we used SDPA-GMP)
- We were able to push bounds w/ global symmetries from a 10-dimensional space of derivatives to a 66-dimensional space
- We ran points in parallel on the $\sim 10,000$ core Odyssey computing cluster at Harvard University

Now for some results...

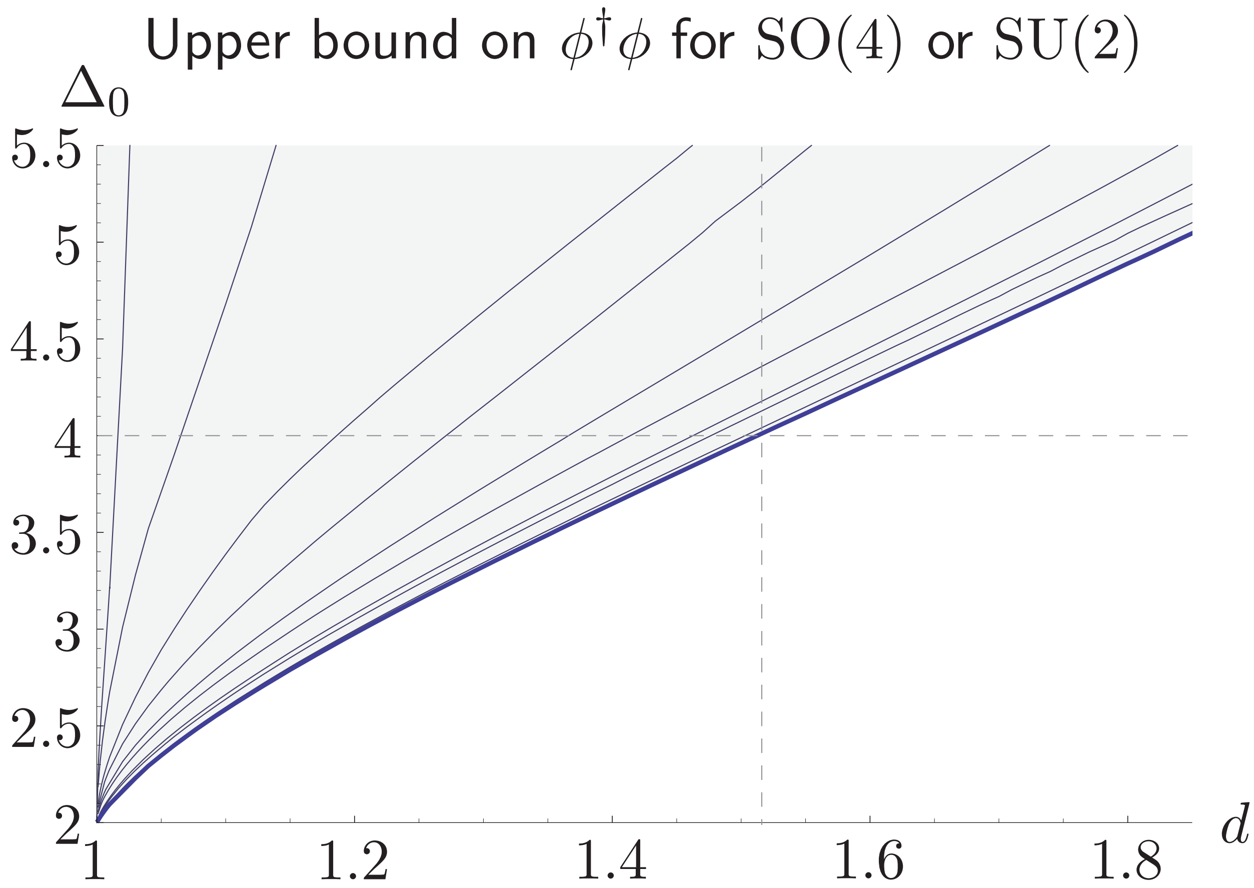
Outline

Singlet Dimension Bounds



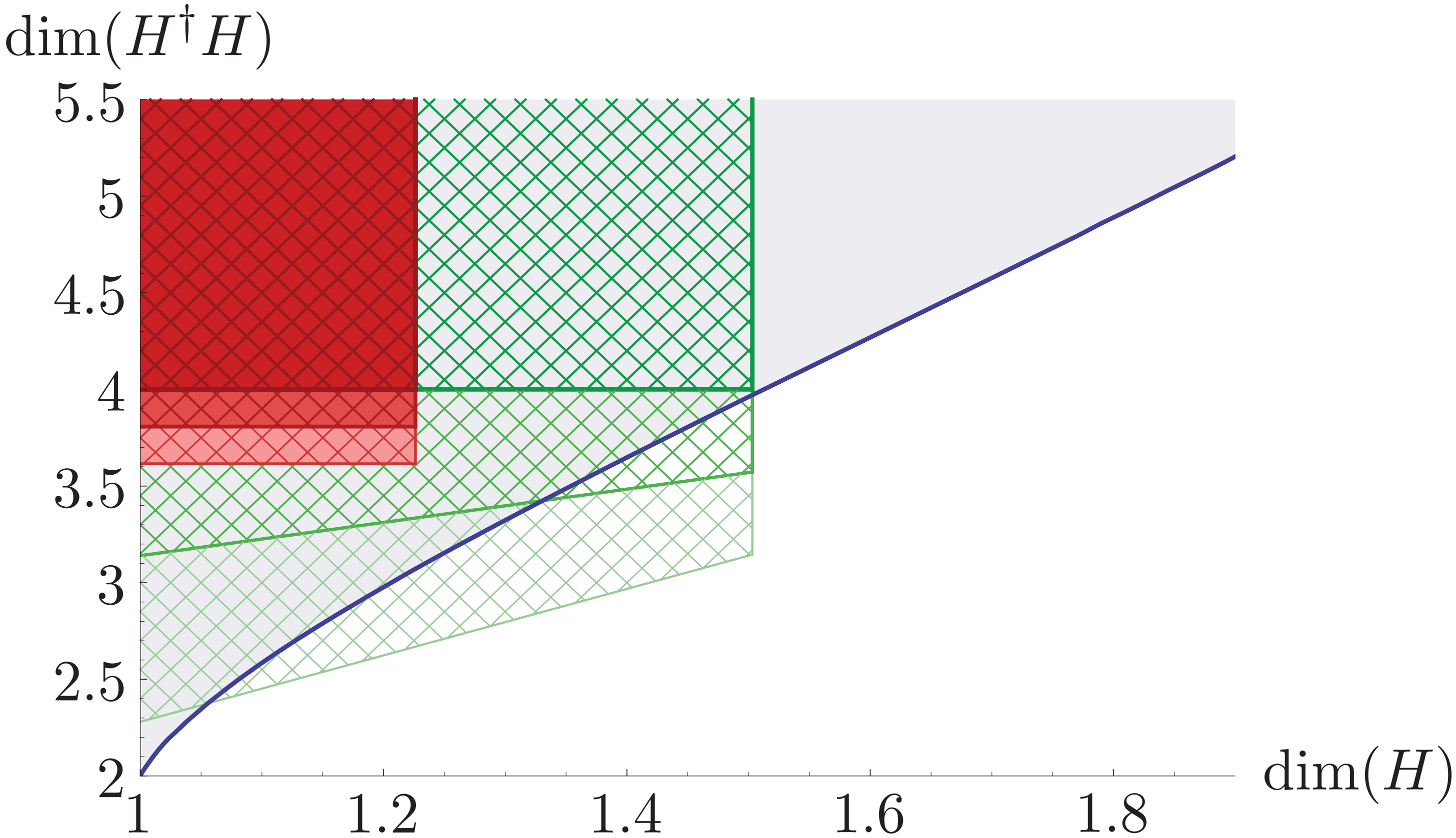
- Bound on lowest dim scalar in $\phi \times \phi$ OPE, where $d = \dim(\phi)$
- Best bound: 66-dimensional space of derivatives

SO(4) or SU(2) Singlet Dimension Bounds



- Lowest dim singlet in $\phi_i^\dagger \times \phi_j$, where ϕ_i is $\mathrm{SU}(2)$ fundamental
- Has implications for Conformal Technicolor [Luty, Okui '04]

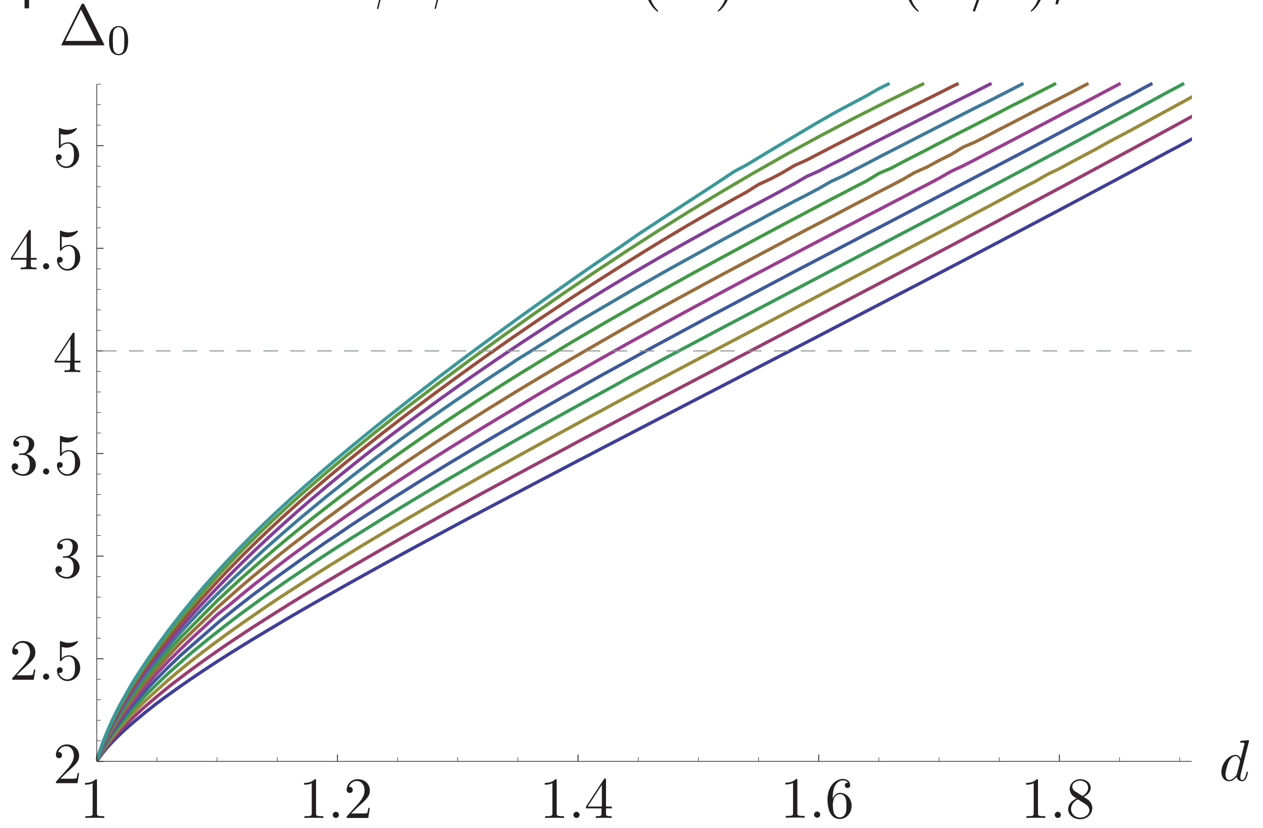
Bounding Conformal Technicolor



- Red: Flavor generic (4-ferm op's have $O(1)$ flavor violation)
- Green: Flavor optimistic (4-ferm op's Yukawa suppressed)
- 3 lines: Stability against perturbation $cH^\dagger H$ with $c \sim (1, 0.1, 0.01)$

SO(N) or SU($N/2$) Singlet Dimension Bounds

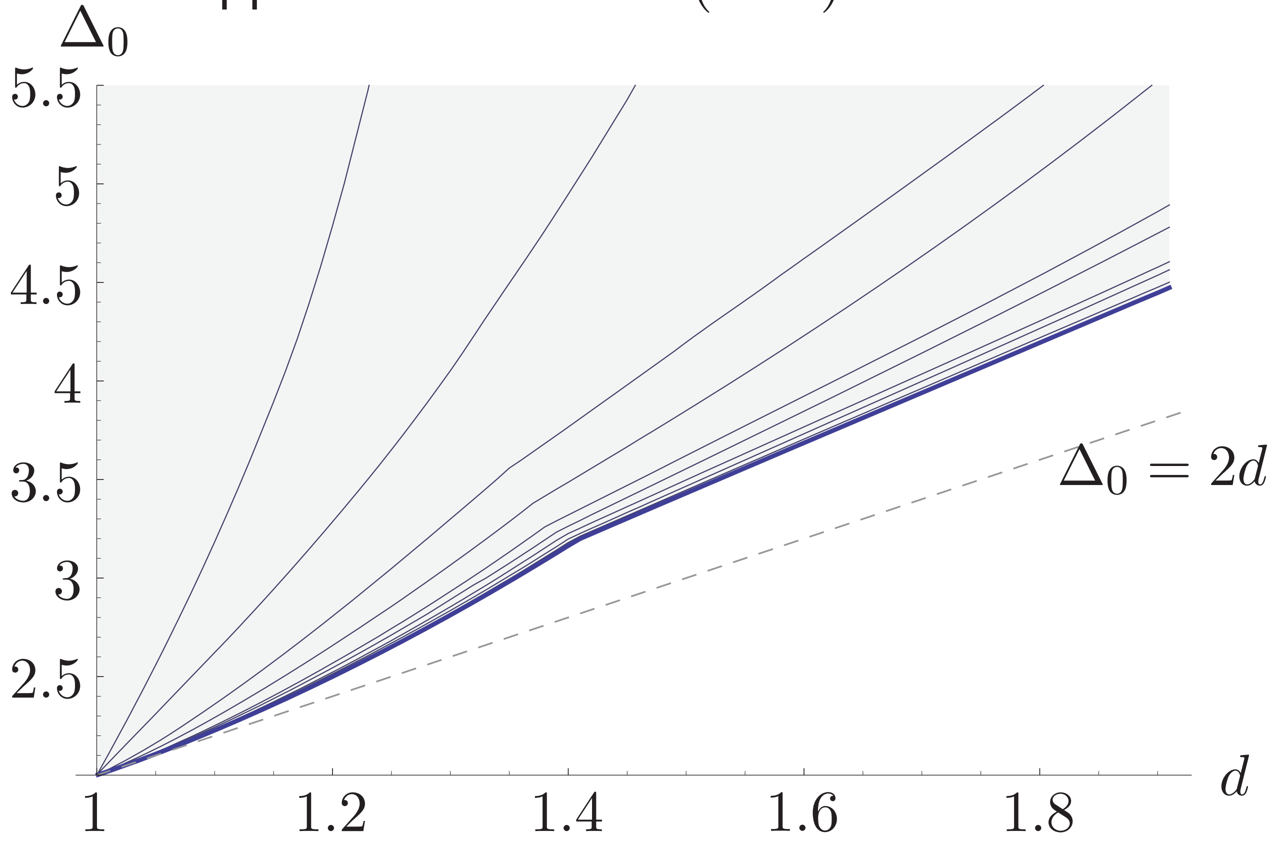
Upper bound on $\phi^\dagger\phi$ for SO(N) or SU($N/2$), $N = 2..15$



- Bounds get weaker as N increases
- SO(N) bounds and SU($N/2$) bounds are identical

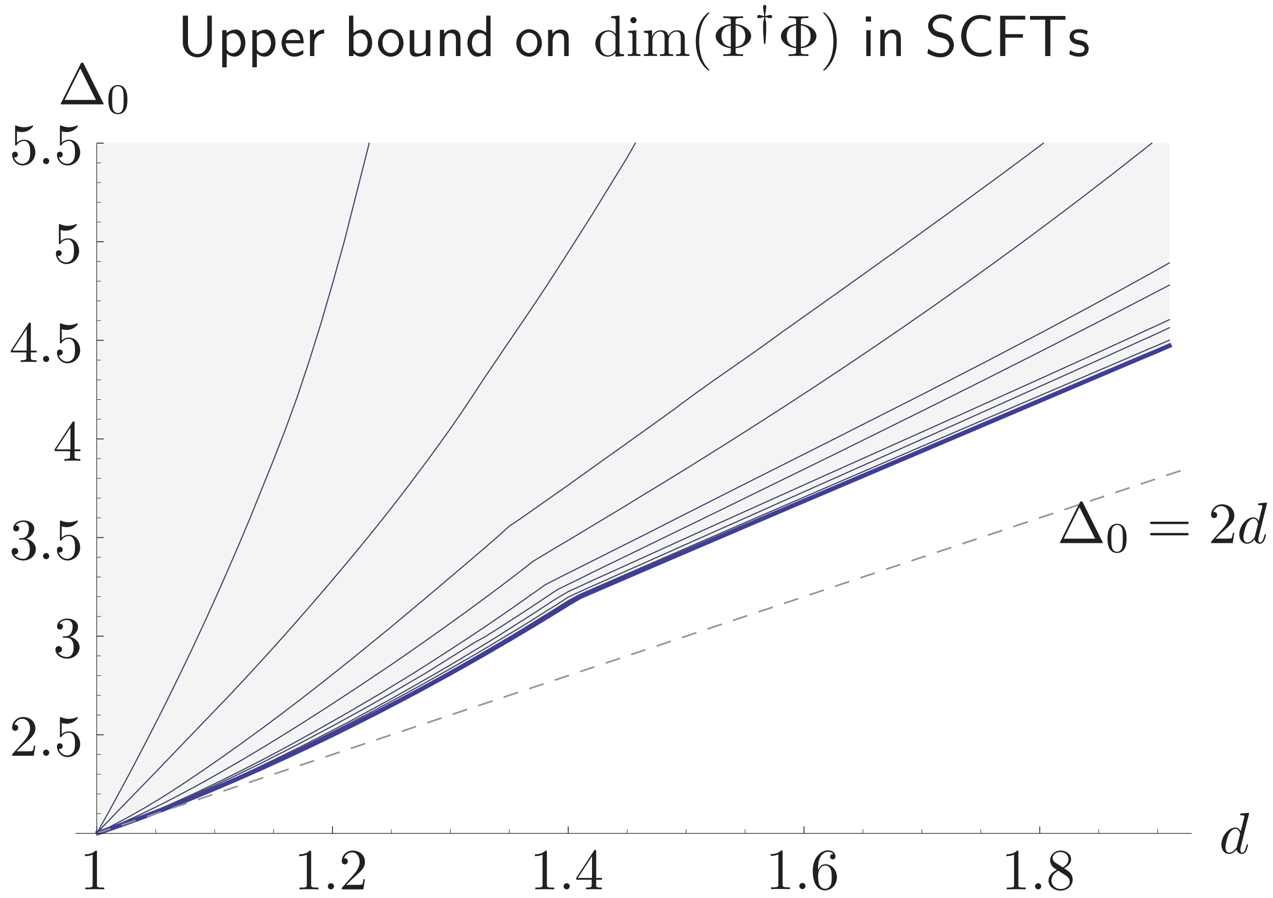
Superconformal Operator Dimension Bounds

Upper bound on $\dim(\Phi^\dagger \Phi)$ in SCFTs



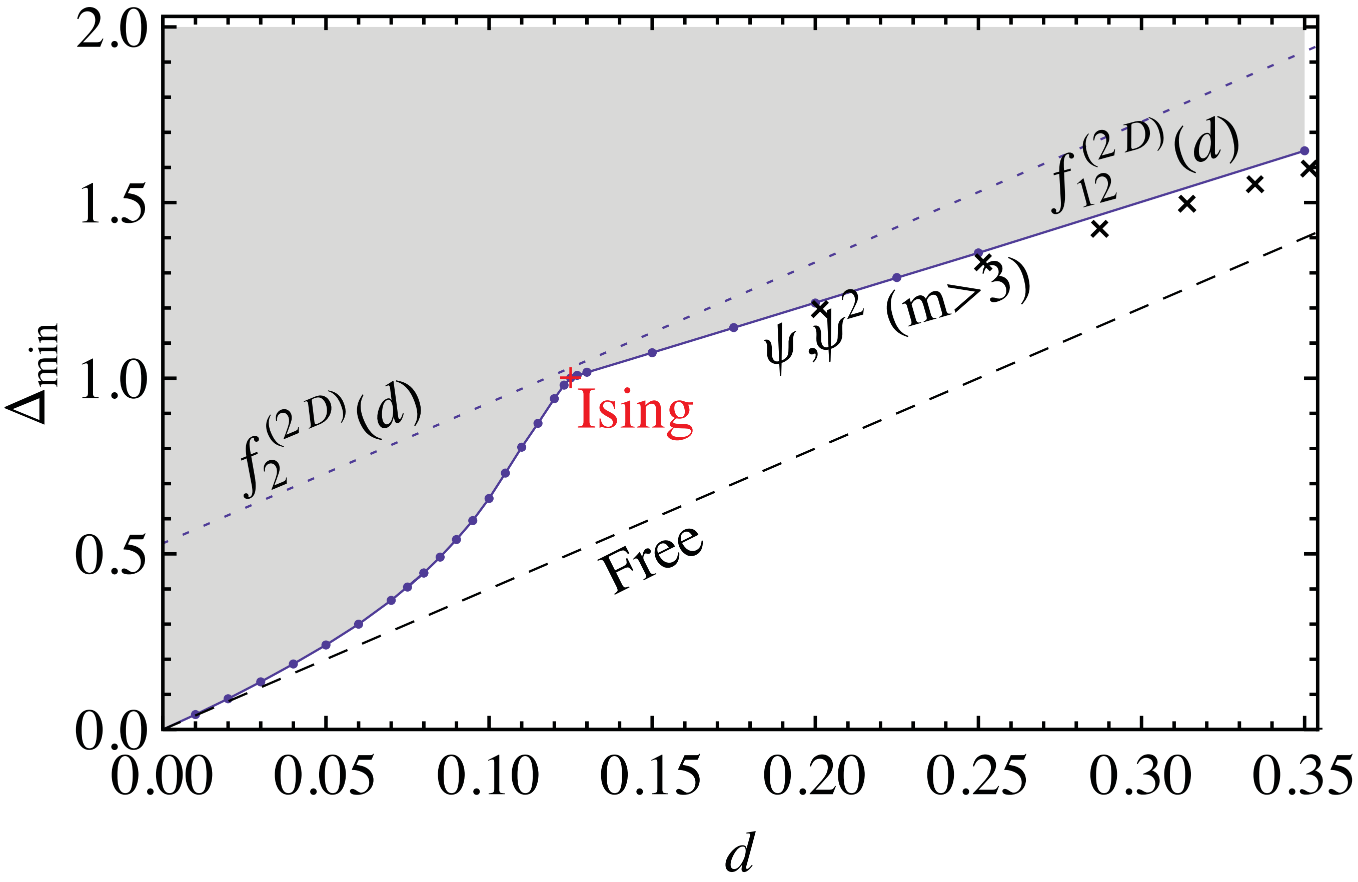
- Bound on lowest dimension scalar in $\Phi \times \Phi^\dagger$ OPE, where Φ is a chiral superconformal primary in an $\mathcal{N} = 1$ SCFT
- Bound appears to asymptote to the line $\Delta_0 = 2d$ near $d \sim 1$

Superconformal Operator Dimension Bounds



- At large-N, constraint on $O(1/N^2)$ corrections to $\dim(\Phi^\dagger\Phi)$
- We also see a *kink* near $d \sim 1.4$, maybe an SCFT lives there?

For Comparison: 2D Dimension Bounds



[Rychkov, Vichi '09]

- Kink at 2D Ising model, exact solution: $\Delta_\sigma = 1/8$, $\Delta_\epsilon = 1$
- Bound saturated by sequence of unitary minimal models

Future Directions

Some future directions for this program:

- Explore bounds in 3D CFTs – see Slava's talk!
  [El-Showk, Paulos, DP, Rychkov, Simmons-Duffin, Vichi, in progress]
- Add assumptions about gaps in spectrum
- Explore the kink in $\Phi^\dagger\Phi$ bound $\rightarrow$ known SCFT or something new?
- Incorporate more operators (e.g., 4pt functions containing ϕ^2)
- 4pt functions of operators with spin
  (for conformal blocks see [Costa, Penedones, DP, Rychkov '11])
- Improve analytic understanding
- AdS dual interpretation?

To Summarize...

We are learning *genuinely new* things about strongly-coupled theories with little or no supersymmetry. Stay tuned!

Backup Slides

CFT Review: Conformal Blocks

Explicit formula [Dolan, Osborn '00]

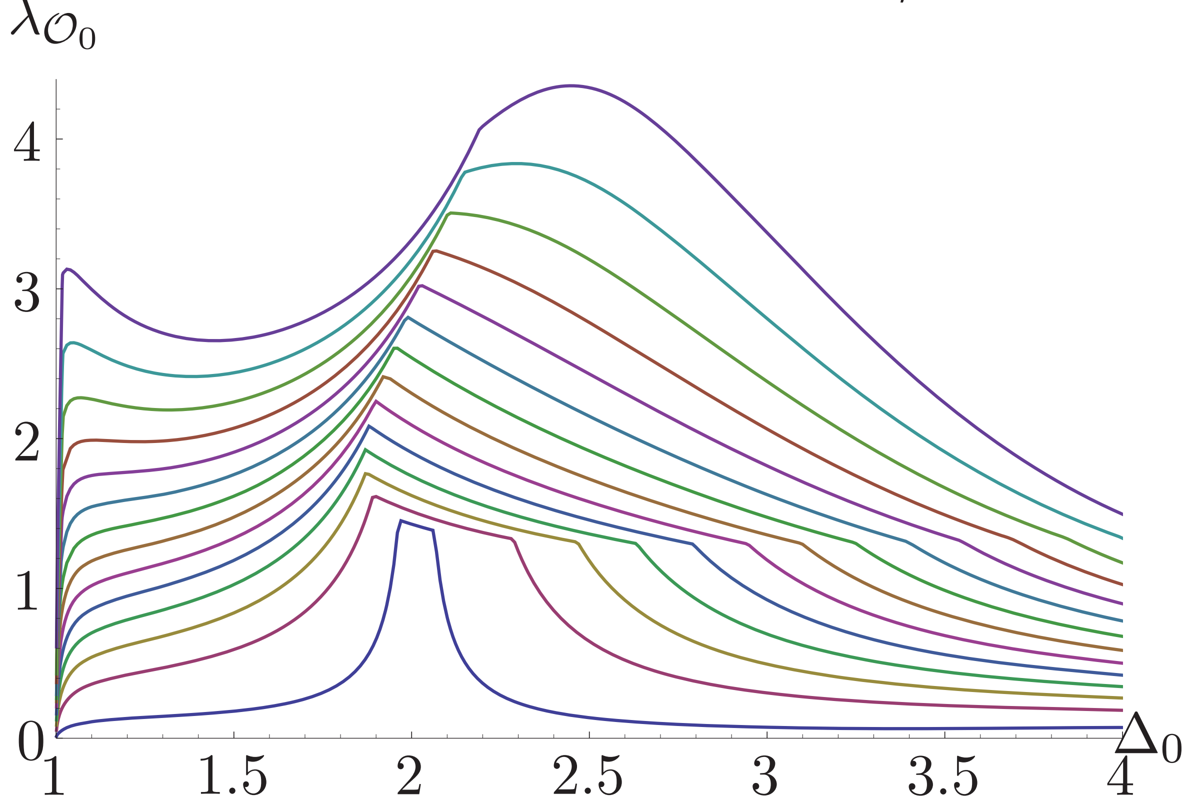
$$
\begin{aligned}
g_{\Delta,l}(u,v) &= \frac{z\bar{z}}{z-\bar{z}}\left[k_{\Delta+l}(z)k_{\Delta-l-2}(\bar{z}) - z \leftrightarrow \bar{z}\right] \\
k_\beta(x) &= x^{\beta/2}{}_2F_1(\beta/2, \beta/2, \beta; x),
\end{aligned}
$$

where $u = z\bar{z}$ and $v = (1-z)(1-\bar{z})$.

- Similar closed-form expressions in other even dimensions, recursion relations known in odd dimensions
- Alternatively can be viewed as eigenfunctions of the quadratic casimir of the conformal group [Dolan, Osborn '03]

Scalar OPE Coefficient Bounds

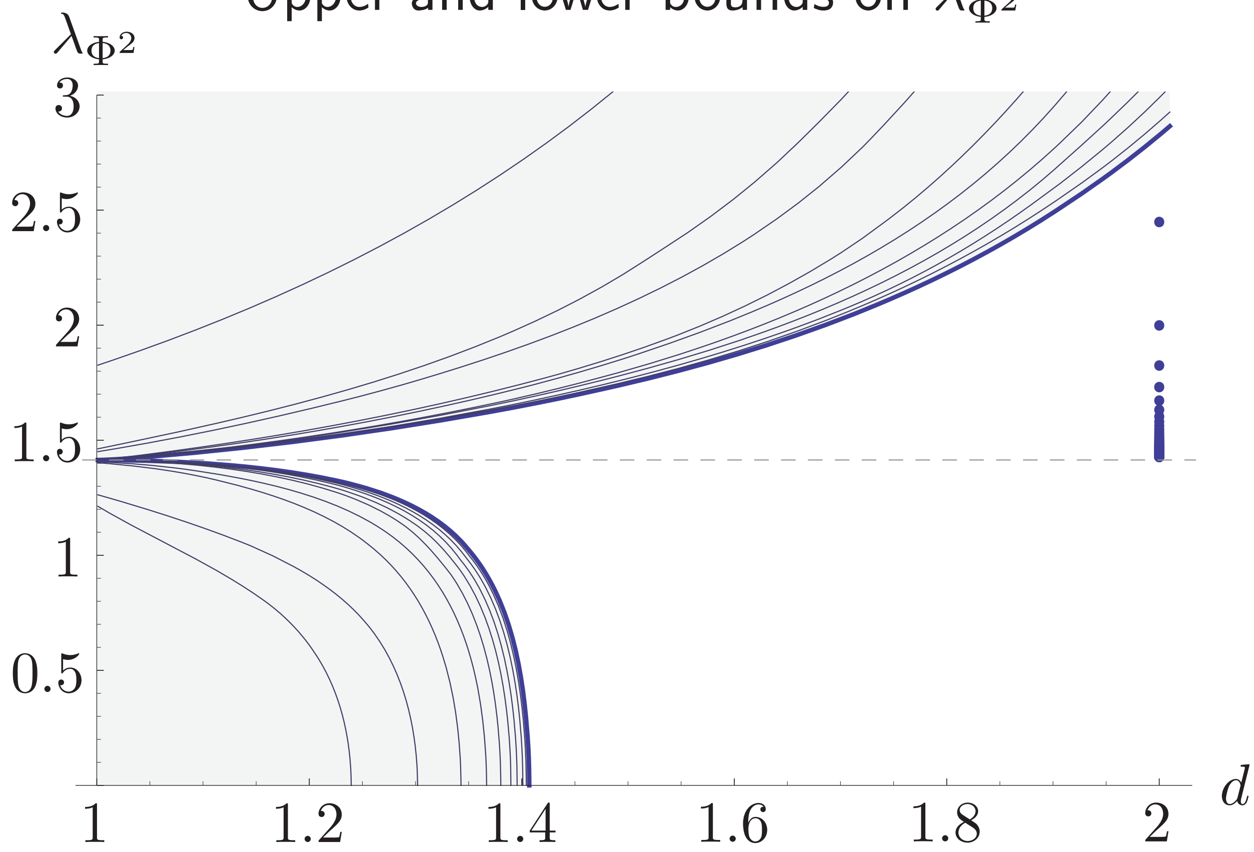
Upper bounds on scalar OPE coefficients, $d = 1.01..1.66$



- Bound on size of scalar OPE coefficient $\phi \times \phi \sim \lambda_{\mathcal{O}_0} \mathcal{O}_0$
- As $d \to 1$ nicely converges to free value, $\lambda_{\mathcal{O}_0} = \sqrt{2}$ at $\Delta_0 = 2$
    - However, distribution of operators with $\lambda < \sqrt{2}$ also allowed?

Upper and Lower Bounds on Φ^2 OPE Coefficient in SCFTs

Upper and lower bounds on λ_{Φ^2}



- Now we consider the OPE $\Phi \times \Phi \sim \Phi^2 + \ldots$, where $\Delta_{\Phi^2} = 2d$
- Scalar descendants of non-chiral operators $\overline{Q}^2\mathcal{O}$ can appear, but unitarity forces $\Delta_{\overline{Q}^2\mathcal{O}} \geq |2d-3|+3$
- *Lower bounds* possible due to gap in dimensions for $d < 3/2$

The Stress Tensor

T^{ab} is a $\Delta = 4, \ell = 2$ operator present in every CFT:

- Ward identity fixes $\langle \phi\phi T \rangle \propto d$
- Only unknown: $\langle TT \rangle \propto c$, the central charge
- In SCFT, T part of $U(1)_R$ current multiplet $(\Delta = 3, \ell = 1)$

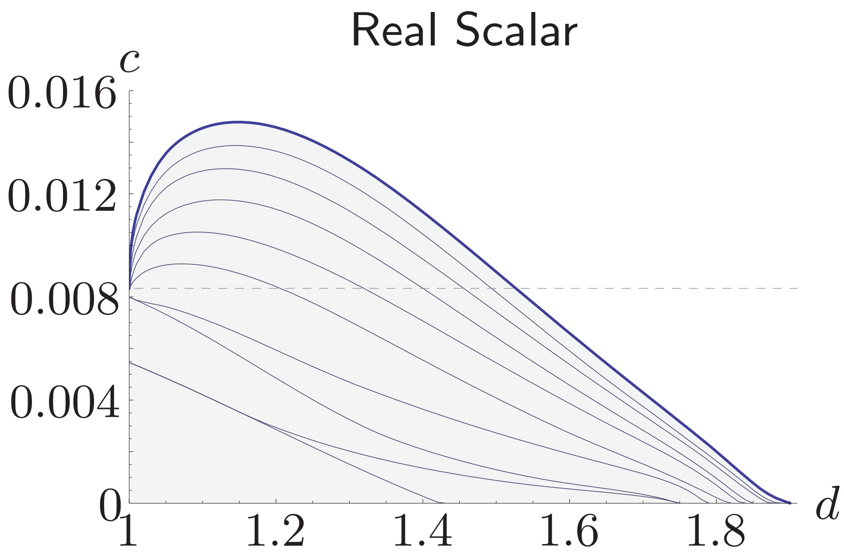
$$\mathcal{J}^a = J_R^a + \theta\sigma_b\bar{\theta}T^{ab} + \ldots$$

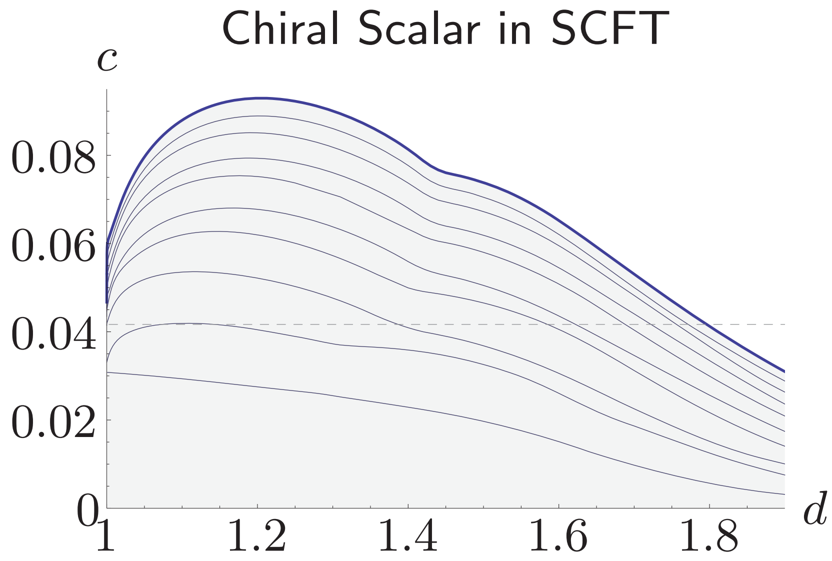
- Conformal block contributions are

$$\langle \phi\phi\phi\phi \rangle \sim \frac{d^2}{360c} g_{4,2} \qquad \text{(general CFTs)}$$

$$\langle \Phi\Phi^\dagger\Phi\Phi^\dagger \rangle \sim \frac{d^2}{72c} \mathcal{G}_{3,1} \qquad \text{(SCFTs)}$$

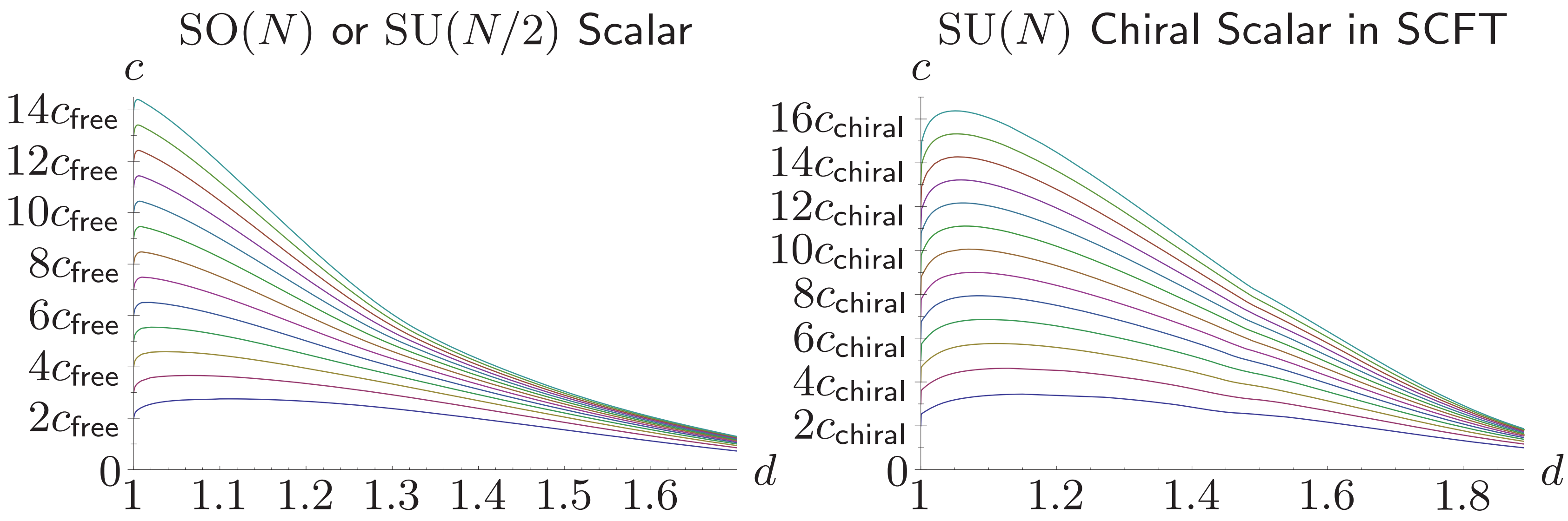
Lower Bounds on c





- Bound smoothly approaches free values as $d \to 1$
  - $c_{\text{free}} = \frac{1}{120}$ (real scalar)
  - $c_{\text{chiral}} = \frac{1}{24}$ (chiral superfield)
- If a CFT contains a $d = 1$ scalar, $c = c_{\text{free}} + c_{\text{int}} \geq c_{\text{free}}$
- In dual AdS$_5$ description, $c \sim R^3 M_P^3$
  - Bound $\to$ Fundamental limit to strength of quantum gravity!

Lower Bounds on c for $SO(N)$ or $SU(N)$, $N = 2..15$



- All lower bounds approach the free values Nc_{free} or Nc_{chiral} as $d \to 1$, growing linearly with N near $d \sim 1$

- Also similar bounds on current 2pt functions: $\langle J^I J^J \rangle \propto \kappa \delta^{IJ}$
  - Bound on strength of bulk gauge couplings in AdS_5!

$\mathcal{N} = 1$ Superconformal Algebra

dim					
$+1$			P_a		
$+1/2$		Q_α		$\overline{Q}_{\dot\alpha}$	
0	$M_{\alpha\beta}$		D, R		$M_{\dot\alpha\dot\beta}$
$-1/2$		S_α		$\overline{S}_{\dot\alpha}$	
-1			$K_a,$		

$$\{Q, \overline{Q}\} = P \qquad\qquad \{S, \overline{S}\} = K$$

- Superconformal primary means $[S, \mathcal{O}(0)] = [\overline{S}, \mathcal{O}(0)] = 0$
- Descendants obtained by acting with $P, Q, \overline{Q}$
- Chiral means $[\overline{Q}, \Phi(0)] = 0$

Superconformal Block Decomposition

Φ: scalar chiral superconformal primary of dimension d in an SCFT

$$\langle \Phi(x_1)\Phi^\dagger(x_2)\Phi(x_3)\Phi^\dagger(x_4)\rangle = \frac{1}{x_{12}^{2d}x_{34}^{2d}} \sum_{\mathcal{O}\in\Phi\times\Phi^\dagger} |\lambda_{\mathcal{O}}|^2 \mathcal{G}_{\Delta,\ell}(u,v)$$

- Sum over s.c. primaries $\mathcal{O}$ with $R=0$ and $\ell = 0,1,2\ldots$
- $x_1 \leftrightarrow x_3$ gives crossing relation only involving $\mathcal{O}\in\Phi\times\Phi^\dagger$
- Additional constraints come from relation to $\Phi\times\Phi$ OPE

Note: $\mathcal{G}_{\Delta,\ell}(u,v)$ is a *finite* sum of conformal blocks, since $\mathcal{O}$ has finite number of descendants that are conformal primaries!

Superconformal Block Derivation

Multiplet built from $\mathcal{O}$ (generically) contains four conformal primaries with vanishing R-charge and definite spin:

name	operator	dim	spin
$\mathcal{O}$	$\mathcal{O}$	Δ	l
J, N	$Q\overline{Q}\mathcal{O} + \#P\mathcal{O}$	$\Delta + 1$	$l+1, l-1$
D	$Q^2\overline{Q}^2\mathcal{O} + \#PQ\overline{Q}\mathcal{O} + \#PP\mathcal{O}$	$\Delta + 2$	l

- Superconformal symmetry fixes coefficients of $\langle \Phi\Phi^\dagger J\rangle, \langle \Phi\Phi^\dagger N\rangle, \langle \Phi\Phi^\dagger D\rangle$ in terms of $\langle \Phi\Phi^\dagger \mathcal{O}\rangle$
- Must also normalize J, N, D to have canonical 2pt functions
- Superconformal block is then a sum of $g_{\Delta,\ell}$'s for $\mathcal{O}, J, N, D$

Superconformal Blocks

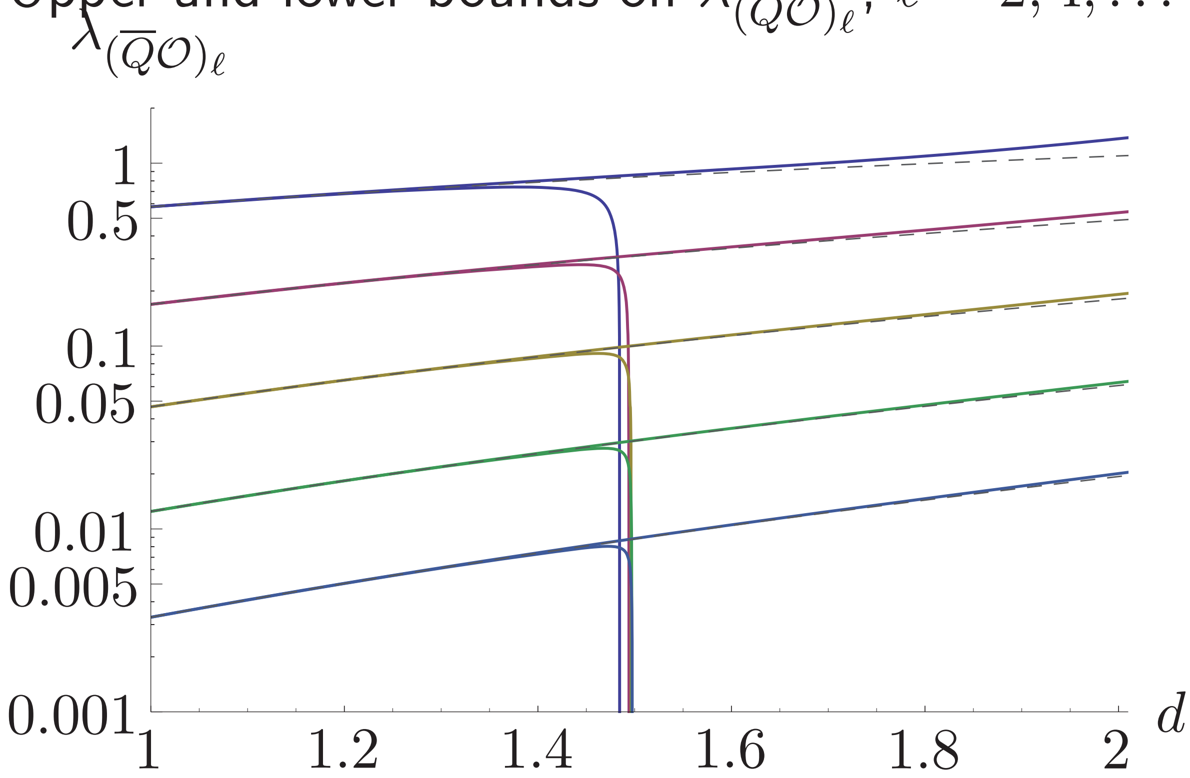
We found, [DP, Simmons-Duffin '10]

$$\mathcal{G}_{\Delta,\ell} = g_{\Delta,\ell} + \frac{(\Delta+\ell)}{4(\Delta+\ell+1)} g_{\Delta+1,\ell+1} + \frac{(\Delta-\ell-2)}{4(\Delta-\ell-1)} g_{\Delta+1,\ell-1} + \frac{(\Delta+\ell)(\Delta-\ell-2)}{16(\Delta+\ell+1)(\Delta-\ell-1)} g_{\Delta+2,\ell}$$

- Unitarity bound $\Delta \geq \ell + 2$ saturated $\rightarrow$ multiplet shortened
- $\mathcal{G}_{\Delta,\ell}$ can also be determined from consistency with $\mathcal{N} = 2$ superconformal blocks computed by [Dolan, Osborn '01]
- Similar results for current 4pt functions recently derived by [Fortin, Intriligator, Stergiou '11]

Higher-Spin Protected Operators in $\Phi \times \Phi$

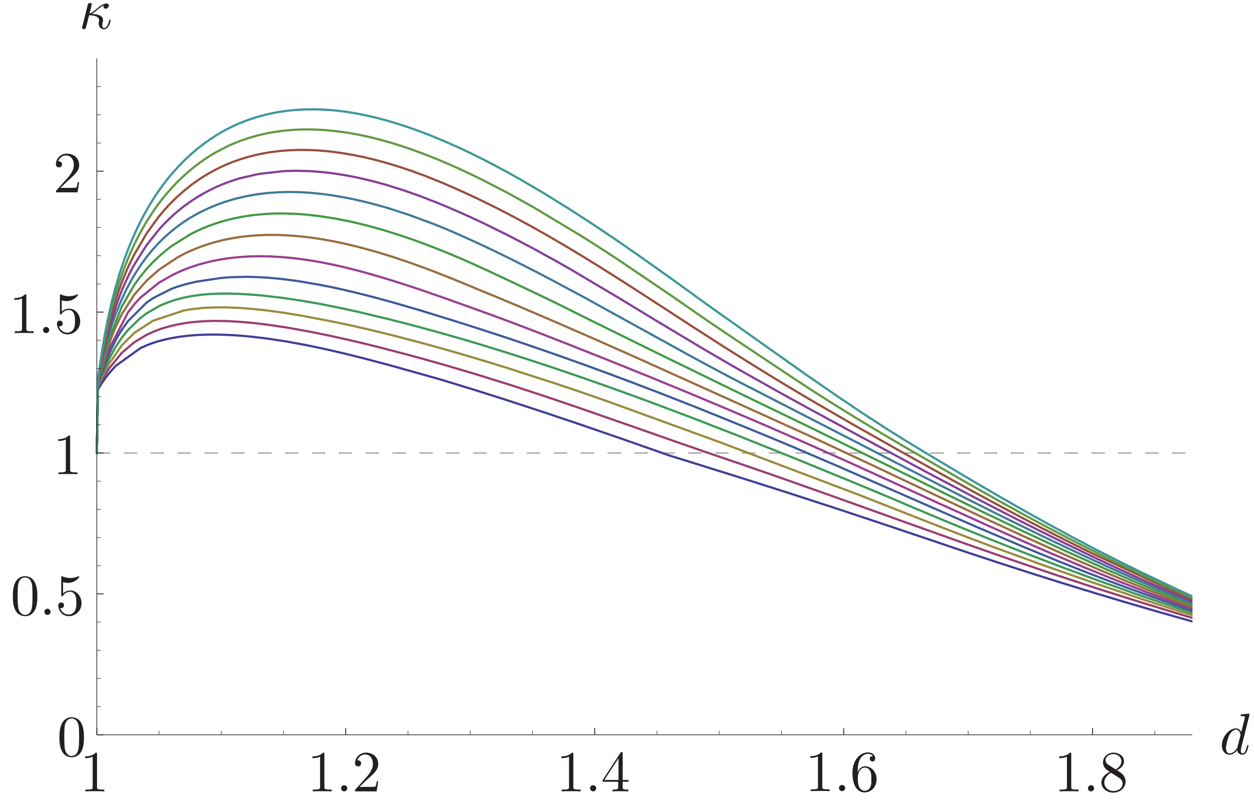
Upper and lower bounds on $\lambda_{(\overline{Q}\mathcal{O})_\ell}$, $\ell = 2, 4, \ldots, 10$



- $\Phi \times \Phi$ OPE also has higher-spin protected operators $(\overline{Q}\mathcal{O})_\ell$
- Gap since $\Delta_{(\overline{Q}\mathcal{O})_\ell} = 2d + \ell$ while $\Delta_{(\overline{Q}^2\mathcal{O})_\ell} \geq |2d-3| + 3 + \ell$
- Dashed lines large-N values...deviations tightly constrained!

Current 2pt Function Bounds in SCFTs

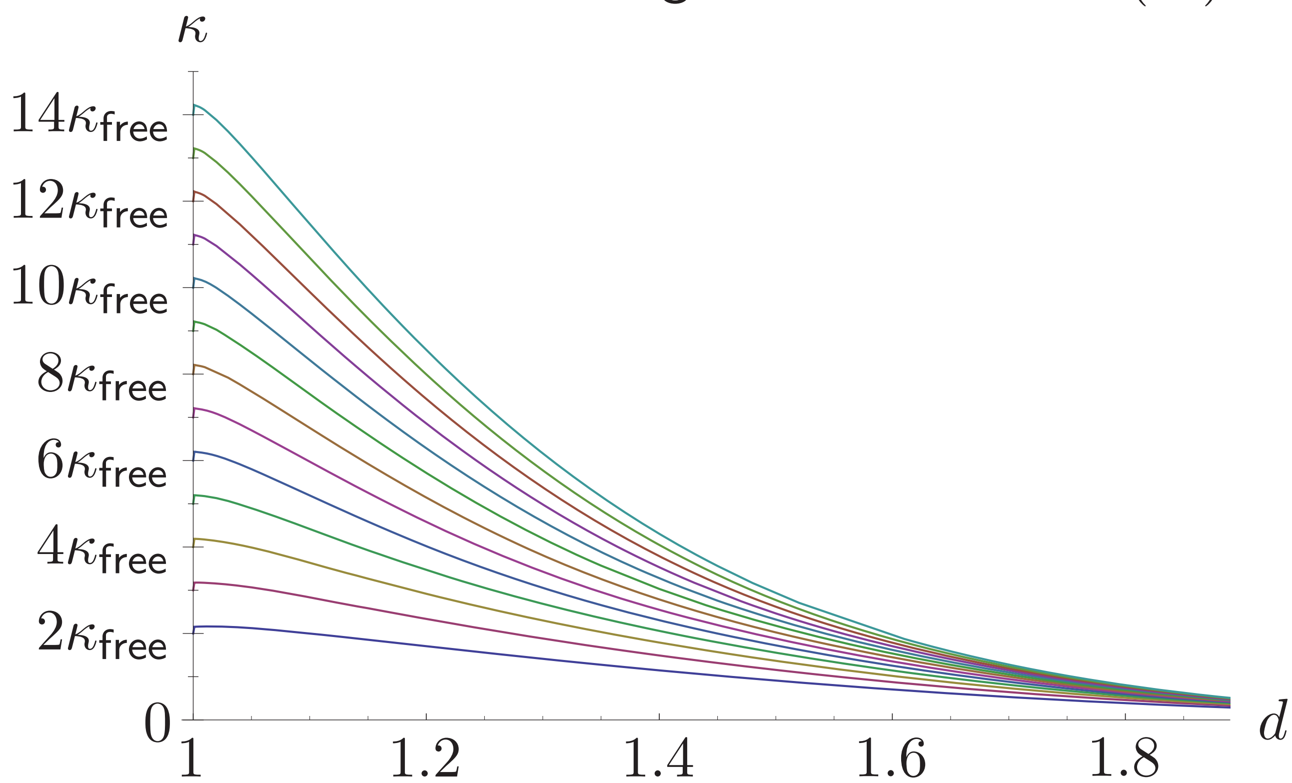
SUSY lower bound on κ for SU(N) adjoint currents, $N = 2..15$



- Lower bounds on coefficient $\langle J^I J^J \rangle \propto \kappa \delta^{IJ}$, if J^I is the adjoint $\mathrm{SU}(N)$ global symmetry current appearing in $\Phi^i \times \Phi^{j\dagger}$

Current 2pt Function Bounds in SCFTs

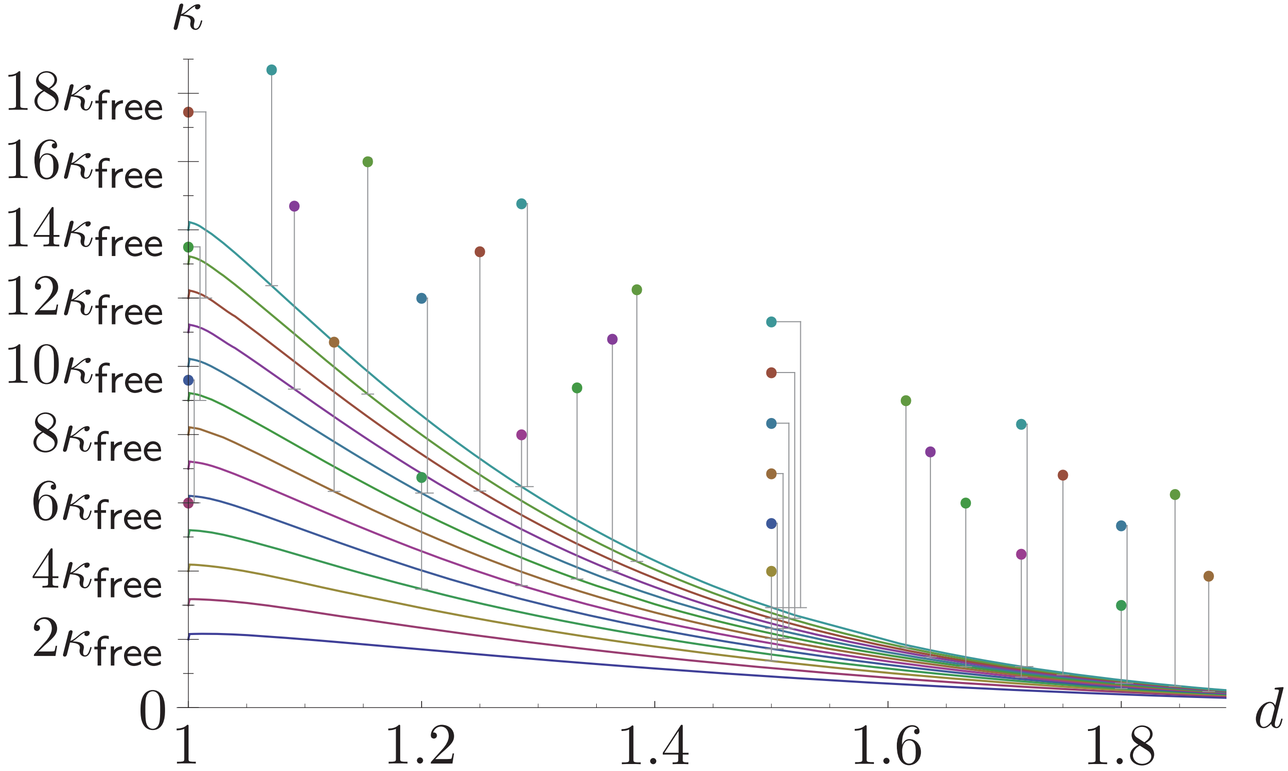
SUSY lower bound on κ for singlet currents of $\mathrm{SU}(N)$, $N = 2..15$



- Bounds on coefficient $\langle J^I J^J \rangle \propto \kappa\delta^{IJ}$, assuming J^I is a *singlet* under the $\mathrm{SU}(N)$ global symmetry
- In SCFTs $\kappa\delta^{IJ} = -3\mathrm{Tr}(F^I F^J R)$ is calculable!

Bounds on Current 2pt Function and Comparison to SQCD

SUSY lower bounds on κ_R using $\mathrm{SU}(N_f)_L$, $N_f = 2..15$



Conformal $\mathrm{SU}(N_c)$ SQCD: $\frac{3}{2}N_c < N_f < 3N_c$, Mesons: $M = Q\tilde{Q}$

- $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$: $M \times M^\dagger \sim J_L + J_R + \ldots$
- Use $\mathrm{SU}(N_f)_L$ crossing relations to bound $\langle J_R J_R \rangle \propto \kappa_R$

Realized values: $d_M = 3 - \frac{3N_c}{N_f}$ and $\kappa_R = \frac{9}{16}\frac{N_c^2}{N_f}$