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# Spin 2 resonances

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# *Spin 2 resonances*

# Motivations

1) They exist

**$a_2(1700)$** $I^G(J^{PC}) = 1^-(2^{++})$

OMITTED FROM SUMMARY TABLE

**$a_2(1700)$ MASS**

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **1732±16 OUR AVERAGE** | | Error includes scale factor of 1.9. | | | | |
| 1737± 5± 7 | | ABE | 04 | BELL | | 10.6 $e^+e^- \to e^+e^-K^+K^-$ |
| 1698±44 | | [1] AMSLER | 02 | CBAR | | 0.9 $\overline{p}p \to \pi^0\eta\eta$ |
| 1660±40 | | ABELE | 99B | CBAR | | 1.94 $\overline{p}p \to \pi^0\eta\eta$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 1675±25 | | ANISOVICH | 09 | RVUE | | 0.0 $\overline{p}p$, $\pi N$ |
| 1722± 9±15 | 18k | [2] SCHEGELSKY | 06 | RVUE | 0 | $\gamma\gamma \to \pi^+\pi^-\pi^0$ |
| 1702± 7 | 80k | [3] UMAN | 06 | E835 | | 5.2 $\overline{p}p \to \eta\eta\pi^0$ |
| 1721±13±44 | 145k | LU | 05 | B852 | | 18 $\pi^-p \to \omega\pi^-\pi^0p$ |
| 1767±14 | 221 | [4] ACCIARRI | 01H | L3 | | $\gamma\gamma \to K^0_SK^0_S$, $E^{ee}_{cm}$= 91, 183–209 GeV |
| ~ 1775 | | [5] GRYGOREV | 99 | SPEC | | 40 $\pi^-p \to K^0_SK^0_Sn$ |
| 1752±21± 4 | | ACCIARRI | 97T | L3 | | $\gamma\gamma \to \pi^+\pi^-\pi^0$ |

[1] T-matrix pole.
[2] From analysis of L3 data at 183–209 GeV.
[3] Statistical error only.
[4] Spin 2 dominant, isospin not determined, could also be $I$=1.
[5] Possibly two $J^P = 2^+$ resonances with isospins 0 and 1.

**$a_2(1700)$ WIDTH**

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **194± 40 OUR AVERAGE** | | Error includes scale factor of 1.6. See the ideogram below. | | | | |
| 151± 22±24 | | ABE | 04 | BELL | | 10.6 $e^+e^- \to e^+e^-K^+K^-$ |
| 265± 55 | | [6] AMSLER | 02 | CBAR | | 0.9 $\overline{p}p \to \pi^0\eta\eta$ |
| 280± 70 | | ABELE | 99B | CBAR | | 1.94 $\overline{p}p \to \pi^0\eta\eta$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $270^{+50}_{-20}$ | | ANISOVICH | 09 | RVUE | | 0.0 $\overline{p}p$, $\pi N$ |
| 336± 20±20 | 18k | [7] SCHEGELSKY | 06 | RVUE | 0 | $\gamma\gamma \to \pi^+\pi^-\pi^0$ |
| 417± 19 | 80k | [8] UMAN | 06 | E835 | | 5.2 $\overline{p}p \to \eta\eta\pi^0$ |
| 279± 49±66 | 145k | LU | 05 | B852 | | 18 $\pi^-p \to \omega\pi^-\pi^0p$ |
| 187± 60 | 221 | [9] ACCIARRI | 01H | L3 | | $\gamma\gamma \to K^0_SK^0_S$, $E^{ee}_{cm}$= 91, 183–209 GeV |
| 150±110±34 | | ACCIARRI | 97T | L3 | | $\gamma\gamma \to \pi^+\pi^-\pi^0$ |

**$f_2(1270)$** $I^G(J^{PC}) = 0^+(2^{++})$

**$f_2(1270)$ MASS**

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **1275.1± 1.2 OUR AVERAGE** | | Error includes scale factor of 1.1. | | | |
| $1262^{+1}_{-2}$ ±8 | | ABLIKIM | 06V | BES2 | $e^+e^- \to J/\psi \to \gamma\pi^+\pi^-$ |
| 1275 ±15 | | ABLIKIM | 05 | BES2 | $J/\psi \to \phi\pi^+\pi^-$ |
| 1283 ± 5 | | ALDE | 98 | GAM4 | 100 $\pi^-p \to \pi^0\pi^0n$ |
| 1278 ± 5 | | [1] BERTIN | 97C | OBLX | 0.0 $\overline{p}p \to \pi^+\pi^-\pi^0$ |
| 1272 ± 8 | 200k | PROKOSHKIN | 94 | GAM2 | 38 $\pi^-p \to \pi^0\pi^0n$ |
| 1269.7± 5.2 | 5730 | AUGUSTIN | 89 | DM2 | $e^+e^- \to 5\pi$ |
| 1283 ± 8 | 400 | [2] ALDE | 87 | GAM4 | 100 $\pi^-p \to 4\pi^0n$ |
| 1274 ± 5 | | [2] AUGUSTIN | 87 | DM2 | $J/\psi \to \gamma\pi^+\pi^-$ |
| 1283 ± 6 | | [3] LONGACRE | 86 | MPS | 22 $\pi^-p \to n2K^0_S$ |
| 1276 ± 7 | | COURAU | 84 | DLCO | $e^+e^- \to e^+e^-\pi^+\pi^-$ |
| 1273.3± 2.3 | | [4] CHABAUD | 83 | ASPK | 17 $\pi^-p$ polarized |
| 1280 ± 4 | | [5] CASON | 82 | STRC | 8 $\pi^+p \to \Delta^{++}\pi^0\pi^0$ |
| 1281 ± 7 | 11600 | GIDAL | 81 | MRK2 | $J/\psi$ decay |
| 1282 ± 5 | | [6] CORDEN | 79 | OMEG | 12–15 $\pi^-p \to n2\pi$ |
| 1269 ± 4 | 10k | APEL | 75 | NICE | 40 $\pi^-p \to n2\pi^0$ |
| 1272 ± 4 | 4600 | ENGLER | 74 | DBC | 6 $\pi^+n \to \pi^+\pi^-p$ |
| 1277 ± 4 | 5300 | FLATTE | 71 | HBC | 7.0 $\pi^+p$ |
| 1273 ± 8 | | [2] STUNTEBECK | 70 | HBC | 8 $\pi^-p$, 5.4 $\pi^+d$ |
| 1265 ± 8 | | BOESEBECK | 68 | HBC | 8 $\pi^+p$ |

**$f_2(1270)$ WIDTH**

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | COMMENT |
|---|---|---|---|---|
| **$185.1^{+2.9}_{-2.4}$ OUR FIT** | | Error includes scale factor of 1.5. | | |
| **$184.2^{+4.0}_{-2.4}$ OUR AVERAGE** | | Error includes scale factor of 1.5. See the ideogram below. | | |

# Motivations

1) They exist $\boxed{m \sim \mathrm{GeV}}$

If there is strongly coupled physics behind the EW scale $\boxed{m \sim \mathrm{TeV}}$

There may even be one with $\boxed{m \sim 10^{-33}\mathrm{eV}}$

2) Self-interactions of the longitudinal polarization of spin-1 at *E>>m* is well known

$$v^2 \mathrm{Tr}[(D_\mu \Sigma)^\dagger D^\mu \Sigma] + \mathcal{O}(p^4)$$

What is the analogue for spin-2?

# Plan of the talk

EFT for a spin-2 resonance with a parametric separation between the mass $m$ and the cutoff $\Lambda$

Massive graviton

$$\Lambda > \mathrm{mm}^{-1}$$

Charged spin-2 resonance coupled to EM

Strong coupling at a low scale

6th ghost-like degree of freedom

Superluminal propagation

## EFT for a massive spin-2

1. Write a massive lagrangian (non-gauge invariant);
2. Introduce the gauge symmetry by adding the Stückelberg fields;
3. For charged particles, introduce the intaraction with a U(1) massless vector by replacing ordinary derivatives with covariant ones;
4. Diagonalize all kinetic terms by field redefinitions and/or covariant gauge fixing
5. Look for the most divergent terms in the Lagrangian
6. Try to remove these terms by adding non-minimal interactions (not always possible)
7. Find the cutoff of the EFT

## Charged Spin-1

Lagrangian of a complex massive spin-1 field

$$L = -\frac{1}{2}\left|\partial_\mu W_\nu - \partial_\nu W_\mu\right|^2 - m^2 W^*_\mu W^\mu$$

Make it gauge invariant by adding a scalar field (Stückelberg) through the substitution

$$W_\mu = V_\mu - \partial_\mu \phi / m \qquad \begin{aligned} \delta V_\mu &= \partial_\mu \epsilon \\ \delta\phi &= m\epsilon \end{aligned}$$

Couple it to a massless *U(1)* via the minimal substitution

$$\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu$$

$$\frac{ie}{2m} F^{\mu\nu} \phi^* (D_\mu V_\nu - D_\nu V_\mu) + \text{c.c.} \qquad - \frac{e^2}{2m^2} F^2_{\mu\nu} \phi^* \phi$$

Strong coupling at $\boxed{\Lambda \sim m/e}$ or new d.o.f

# Charged Spin-1

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We can add another operator $ieF^{\mu\nu} W_\mu^* W_\nu$

Can we cancel the dangerous interaction and raise the cutoff scale? No

## Charged Spin-1

Lagrangian of a complex massive spin-1 field

$$L = -\frac{1}{2}\left|\partial_\mu W_\nu - \partial_\nu W_\mu\right|^2 - m^2 W_\mu^* W^\mu$$

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$$\frac{ie}{2m} F^{\mu\nu} \phi^* (D_\mu V_\nu - D_\nu V_\mu) + \text{c.c.} \qquad - \frac{e^2}{2m^2} F_{\mu\nu}^2 \phi^* \phi$$

We know a UV completion: add a neutral physical Higgs scalar below $\Lambda$

SU(2) broken to U(1) by an adjoint Higgs

# Massive graviton

In flat space, the background metric is $\eta_{\mu\nu}$

$$\mathcal{L} = M_{\rm Pl}^2 (\partial H)^2 + m^2 M_{\rm Pl}^2 (H_{\mu\nu}H^{\mu\nu} - H^2) + \mathcal{O}(H^3)$$

Fierz Pauli mass term. Why?

$$\mathcal{L} = -\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + m^2 \mathrm{Tr}[A_\mu A^\mu] \qquad a(\partial_\mu A^\mu)^2 + b(\partial_\mu A_\nu)^2$$

$$A_\mu \to U A_\mu U^\dagger + U\partial_\mu U^\dagger \qquad U = e^{i\frac{g}{m}\pi}$$

$$(\partial\pi)^2 + \frac{a-b}{m^2}(\partial^2\pi)^2$$

Ghost with mass $m$

$$\frac{1}{16\pi^2}\mathrm{Tr}|D^2 U|^2 \quad \Longrightarrow \quad a - b \sim \frac{g^2}{16\pi^2}$$

$\Lambda \sim 4\pi \frac{m}{g}$

$m$

# Massive graviton

In flat space, the background metric is $\eta_{\mu\nu}$

$$\mathcal{L} = M_{\rm Pl}^2 (\partial H)^2 - m^2 M_{\rm Pl}^2 (H_{\mu\nu} H^{\mu\nu} - H^2) + \mathcal{O}(H^3)$$

Arkani-Hamed, Georgi, Schwartz '02

Reintroduce the gauge symmetry (diff invariance) and 4 Goldstone

$$H_{\mu\nu} = h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu - \partial_\mu A^\alpha \partial_\nu A_\alpha$$

Transform as a covariant tensor under diff $x^\alpha \to x^\alpha + \xi^\alpha$

Provided that $\delta A_\mu = -\xi_\mu$

As before for the spin-1 introduce an extra U(1) gauge symmetry

$$A_\mu = A_\mu + \partial_\mu \phi \qquad \begin{aligned} \delta A_\mu &= -\xi_\mu + \partial_\mu \epsilon \\ \delta \phi &= -\epsilon \end{aligned}$$

$$H = h + \partial A + (\partial A)^2 + \partial A \partial^2 \phi + \partial^2 \phi + (\partial^2 \phi)^2$$

Pathological kinetic term

$(\partial^2 \phi)^2$ absent in FP

## Massive graviton

$$H = h + \partial A + (\partial A)^2 + \partial A \partial^2 \phi + \partial^2 \phi + (\partial^2 \phi)^2$$

The scalar is invariant under the global galilean symmetry

$$\phi \to \phi + c + b^\mu x_\mu$$

$\phi$ acquires a normal 2 derivatives kinetic term only via mixing with $h$

Diagonalize the kinetic operator by doing the field redefinition

$h_{\mu\nu} = \hat{h}_{\mu\nu} - \eta_{\mu\nu}(m_g^2 \phi)$ The kinetic term is $M_{Pl}^2 m_g^4 (\partial \phi)^2$

The mass term contains dim >4 operators

Go to canonical normalization for the fields to read the scales

$$h_{\mu\nu} = \frac{h^c_{\mu\nu}}{M_{\rm Pl}}$$
$$A_\mu = \frac{A^c_\mu}{m M_{\rm Pl}}$$
$$\phi = \frac{\phi^c}{m^2 M_{\rm Pl}}$$

# Massive graviton

$$H = h + \partial A + (\partial A)^2 + \partial A \partial^2 \phi + \partial^2 \phi + (\partial^2 \phi)^2$$

$$\frac{1}{m_g^4 M_{\rm Pl}}(\partial^2 \phi^c)^3 + \frac{1}{m_g^6 M_{\rm Pl}^2}(\partial^2 \phi^c)^4 + \frac{1}{m_g^2 M_{\rm Pl}}\partial^2 \phi^c \partial A^c \partial A^c + \frac{1}{m_g^2 M_{\rm Pl}} h^c (\partial^2 \phi^c)$$

$\Lambda_3 = (m_g^2 M_{\rm Pl})^{1/3}$

$\Lambda_4$

$\Lambda_5 = (m_g^4 M_{\rm Pl})^{1/5}$

$m_g$

Can we add interactions to raise the cutoff?

$$m^2 M_{\rm Pl}^2 (H_{\mu\nu} H^{\mu\nu} - H^2)$$

## Massive graviton

$$H = h + \partial A + (\partial A)^2 + \partial A \partial^2 \phi + \partial^2 \phi + (\partial^2 \phi)^2$$

$$\frac{1}{m_g^4 M_{\mathrm{Pl}}}(\partial^2 \phi^c)^3 + \frac{1}{m_g^6 M_{\mathrm{Pl}}^2}(\partial^2 \phi^c)^4 + \frac{1}{m_g^2 M_{\mathrm{Pl}}}\partial^2 \phi^c \partial A^c \partial A^c + \frac{1}{m_g^2 M_{\mathrm{Pl}}} h^c (\partial^2 \phi^c)$$

$\Lambda_3 = (m_g^2 M_{\mathrm{Pl}})^{1/3}$

$\Lambda_4$

$\Lambda_5 = (m_g^4 M_{\mathrm{Pl}})^{1/5}$

$m_g$

Can we add interactions to raise the cutoff?

$$m^2 M_{\mathrm{Pl}}^2 (H_{\mu\nu} H^{\mu\nu} - H^2 + a_3 H^3 + a_4 H^4 + ...)$$

## Massive graviton

There is a choice of higher order interactions that removes all $(\partial^2\phi)^n$ self-couplings from the action.

Arkani-Hamed, Georgi, Schwartz '02

The final strong coupling scale is $\Lambda_3 = (m_g^2 M_{\rm Pl})^{1/3} \sim (1000\,{\rm Km})^{-1}$

The leading intaractions are $\dfrac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}$

Decoupling limit

$m \to 0 \quad M_{\rm Pl} \to \infty$

$\Lambda_3$ fixed

Is this choice unique? No

For n ≤ D=4 there is a combination $(\partial^2\phi)^n$ that is a total deriv

$$(\Box\phi)^3 - 3\,\Box\phi\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3$$

We can add the combination $\mathcal{L}_3^{\rm TD} = \sqrt{-g}\,\left(3[H][H^2] - [H]^3 - 2[H^3]\right)$

With an arbitrary coeff. without reintroducing $\phi$ self-couplings

Creminellii, Nicolis, Papucci, ET '05

## Massive graviton in the presence of a source

$$(\partial h_c)^2 + (\partial \phi_c)^2 + \frac{(h_c + \phi_c)(\partial^2 \phi_c)^n}{\Lambda_3^{3(n-1)}} + \frac{1}{M_{\rm Pl}}(h_c^{\mu\nu} + \eta^{\mu\nu}\phi_c)T_{\mu\nu}$$

Solution in the presence of a macroscopic source

$$\rho = M\delta^3(r) \qquad h_c \sim \frac{M}{M_{\rm Pl}}\frac{1}{r} \qquad \pi_c \sim \frac{M}{M_{\rm Pl}}\frac{1}{r}$$


Classical non-linearities important $\frac{\partial^2 \pi_c}{\Lambda^3} \sim 1 \Rightarrow r_V \sim \left(\frac{M}{M_{\rm Pl}\Lambda^3}\right)^{\frac{1}{3}} \sim 10^{16}\text{km}$

6th ghost-like dof appears $m^2_{\rm ghost}(r \sim r_V) \sim \frac{1}{r_V^2}$

Creminellii, Nicolis, Papucci, ET '05

## Boulware Deser ghost

Hamiltonian formalism using ADM variables

3D metric on spatial hypersurfaces $g_{ij}$

Lapse $N_j \equiv g_{0j}$

Shift $N \equiv 1/\sqrt{-g^{00}}$

$$\int \sqrt{-g} R$$

the lapse and the shift are non dynamical fields:
no time derivatives → their conjugate momenta vanish

They appear linearly as Lagrange multiplier
Their eom constrain the other dof and conj momenta $g_{ij}, \pi^{ij}$

The Hamiltonian system reduces to 2 indipendent $(q,p)$ pairs

# Boulware Deser ghost

Hamiltonian formalism using ADM variables

3D metric on spatial hypersurfaces $g_{ij}$

Lapse $N_j \equiv g_{0j}$

Shift $N \equiv 1/\sqrt{-g^{00}}$

the lapse and the shift are non dynamical fields:
no time derivatives → their conjugate momenta vanish

$$\int \sqrt{-g}R \; -m^2\sqrt{g}\,F(g,H)$$

They do not appear linearly in general
Their eom determine them but do not constrain other dof

$10-4=6$ Massive graviton + extra dof

# Ghost-free massive gravity and the Galileon

There is a choice of the potential for $H$ such that only the first 3 interactions are non-zero. $\frac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}$

Moreover, the structure of the derivatives on the scalar has no more than 2 time derivatives → no ghost

De Rham, Gabadadze, Tolley '10

Demixing metric and scalar generates the 4 Galileon self-interactions

$$\mathcal{L} = (\partial\phi)^2(\partial^2\phi)^n$$

$$\mathcal{L}^{(2)} = (\partial\pi)^2$$

$$\mathcal{L}^{(3)} = (\partial\pi)^2\Box\pi \qquad \mathcal{L}^{(4)} = (\partial\pi)^2[(\Box\pi)^2 - \partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi]$$

$$\mathcal{L}^{(5)} = (\partial\pi)^2[(\Box\pi)^3 - 3\Box\pi\partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi + 2\partial_\mu\partial_\nu\pi\partial^\nu\partial^\alpha\pi\partial_\alpha\partial^\mu\pi]$$

Nicolis, Rattazzi, ET '09

Galilean invariant up to a total derivative $\qquad \phi \to \phi + c + b^\mu x_\mu$

They have two time-derivative eom

In the full theory in ADM variables, this specific choice of the potential gives an Hamiltonian linear in the shift $N$

Hassan, Rosen, '11

# Ghost-free massive gravity and the Galileon

There is a choice of the potential for $H$ such that only the first 3 $\frac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}$ interactions are non-zero.

Moreover, the structure of the derivatives on the scalar has no more than 2 time derivatives → no ghost

De Rham, Gabadadze, Tolley '10

Demixing metric and scalar generates the 4 Galileon self-interactions

$$\mathcal{L} = (\partial\phi)^2 + \frac{(\partial\phi)^2(\partial^2\phi)^n}{\Lambda_3^{3n}} + \frac{1}{M_{\rm Pl}}\phi T + \frac{1}{M_{\rm Pl}\Lambda_3^3}(\partial\phi)^2 T$$

Non-renormalization theorem

Luty, Porrati, Rattazzi 03

Loops of quantum fields with interactions $\mathcal{L}^{(3)}, \mathcal{L}^{(4)}, \mathcal{L}^{(5)}$ generate terms involving at least 2 derivatives on the external legs.

In particular galilean terms are not renormalized

# The Galileon

$$(\partial\pi_c)^2 + \frac{1}{\Lambda^3}(\partial\pi_c)^2\Box\pi_c + \frac{1}{\Lambda^6}(\partial\pi_c)^2(\partial^2\pi_c)^2 + \frac{1}{\Lambda^9}(\partial\pi_c)^2(\partial^2\pi_c)^3$$
$$+\frac{1}{\Lambda^2}(\partial^2\pi_c)^2 + \frac{1}{\Lambda^5}(\partial^2\pi_c)^3 + \ldots + \frac{1}{M_{\rm Pl}}\pi_c T$$

$$\pi_c \sim \frac{M}{M_{\rm Pl}}\frac{1}{r}$$


Classical non-linearities important $\frac{\partial^2\pi_c}{\Lambda^3} \sim 1 \Rightarrow r_V \sim \left(\frac{M}{M_{\rm Pl}\Lambda^3}\right)^{\frac{1}{3}}$

All the other operators are suppressed by extra powers of $\frac{\partial}{\Lambda}$

# General Relativity

$M_{\rm Pl}^2 \mathcal{R}$ $\qquad$ $\mathcal{R}^2,\ \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \ldots$

$$(\partial h_c)^2 + \frac{h_c}{M_{\rm Pl}}(\partial h_c)^2 + \frac{h_c^2}{M_{\rm Pl}^2}(\partial h_c)^2 + \ldots + \frac{1}{M_{\rm Pl}^2}(\partial^2 h_c)^2 + \frac{h_c}{M_{\rm Pl}^3}(\partial^2 h_c)^2 + \ldots + \frac{1}{M_{\rm Pl}} h_c T$$

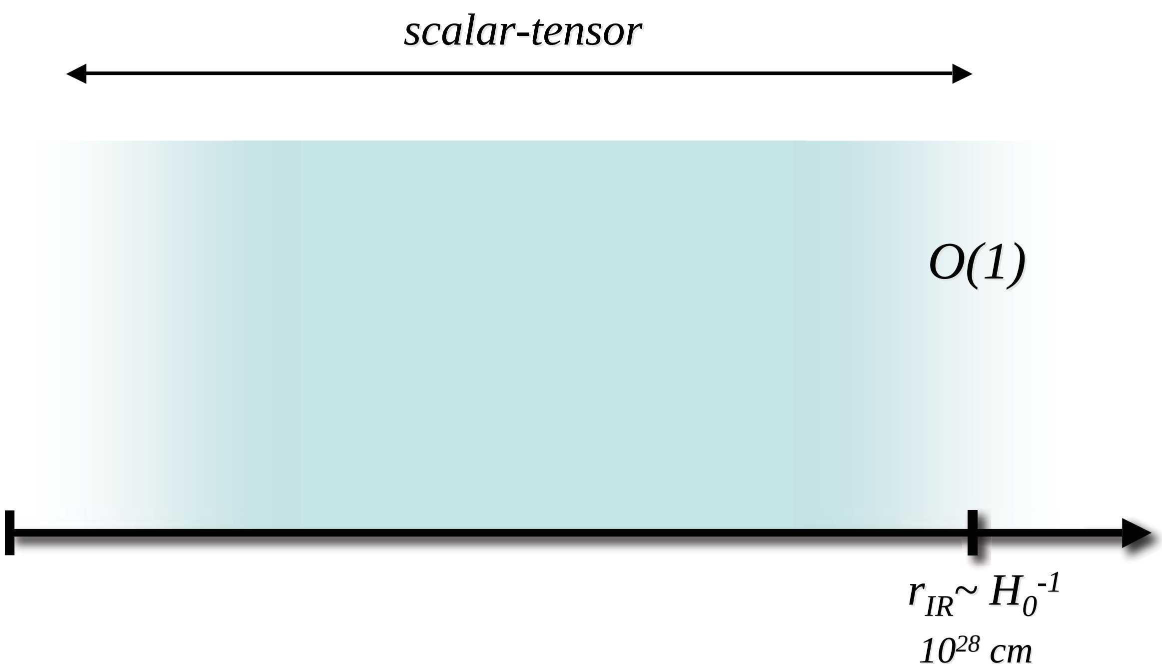
$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h^c_{\mu\nu}}{M_{\rm Pl}}$$

$$\rho = M\delta^3(r) \qquad\qquad h_c \sim \frac{M}{M_{\rm Pl}}\frac{1}{r}$$


Non-linearities become important at a scale $r_S$ where $\frac{h_c}{M_{\rm Pl}} \sim 1 \Rightarrow r_S = \frac{M}{M_{\rm Pl}^2}$

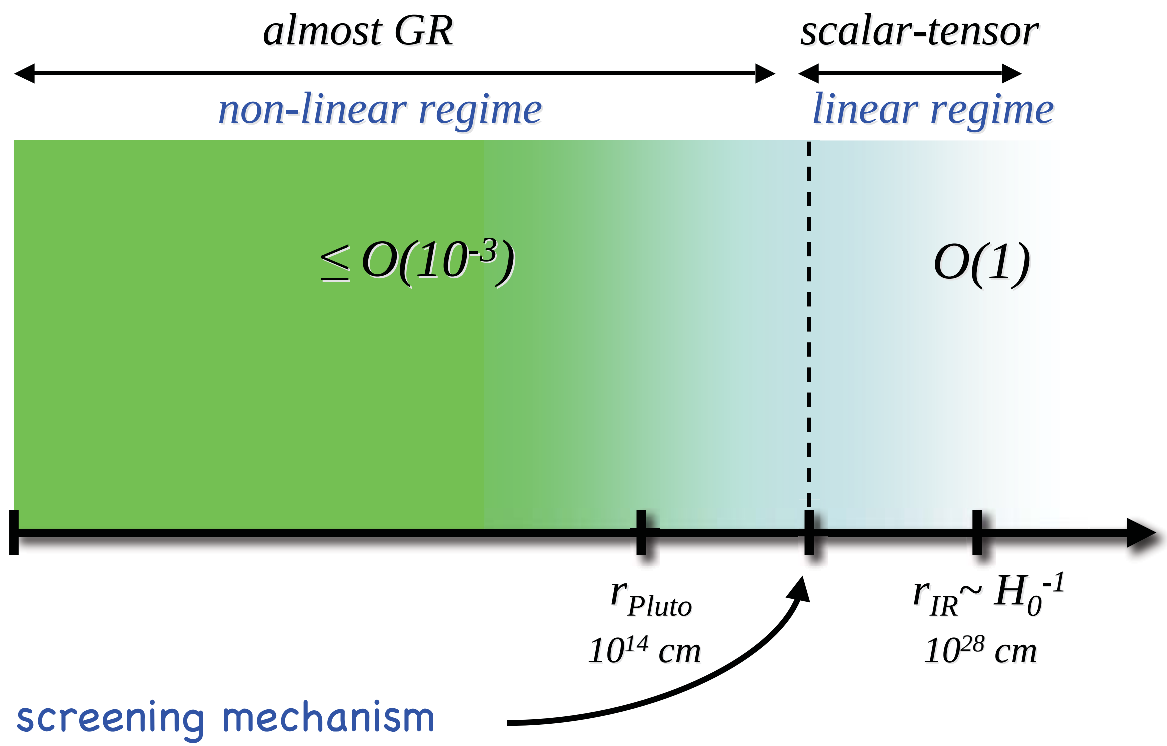
All the other terms are suppressed by extra powers of $\frac{\partial}{M_{Pl}} \sim \frac{1}{r\, M_{\rm Pl}} \ll 1$

We can compute classical non-linearities without knowing the UV compl.




vDVZ discontinuity

Van Dam, Veltman, Zacharov 70




Vainshtein effect: non-linear dynamics suppresses the scalar contribution

Vainshtein 72

## Superluminality

Scalar excitation are luminal around $\phi = 0$

Turn on a localized source that generates a weak stationary field $\phi_0(\vec{x})$

Self-interactions are unimportant to determine the solution, $\nabla^2 \phi_0 \simeq 0$

The quadratic lagrangian for fluctuations around the solution $\phi = \phi_0 + \varphi$

$$(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu \phi_0}{\Lambda_3^3})\, \partial^\mu \varphi \partial^\nu \varphi$$

It is narrower than the Minkowski light-cone in some directions but wider in others

Nicolis, Rattazzi, ET '09

The conclusion relies only on the presence of the cubic Galileon

## Charged massive spin-2

Write the Fierz-Pauli Lagrangian (neglect $h$ self-interactions)
Complexify the fields
Replace ordinary derivatives with covariant ones

Porrati, Rahman '08

$$L = -|D_\mu \tilde{h}_{\nu\rho}|^2 + 2|D_\mu \tilde{h}^{\mu\nu}|^2 + |D_\mu \tilde{h}|^2 - [D_\mu \tilde{h}^{*\mu\nu} D_\nu \tilde{h} + \mathrm{c.c.}] - m^2[\tilde{h}^*_{\mu\nu}\tilde{h}^{\mu\nu} - \tilde{h}^*\tilde{h}]$$

Introduce the Stückerlberg fields $B_\mu$, $\phi$ and the extra-gauge symmetries

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{m} D_\mu \left(B_\nu - \frac{1}{2m} D_\nu \phi\right) + \frac{1}{m} D_\nu \left(B_\mu - \frac{1}{2m} D_\mu \phi\right)$$

$$\begin{aligned} \delta h_{\mu\nu} &= D_\mu \lambda_\nu + D_\nu \lambda_\mu, \\ \delta B_\mu &= D_\mu \lambda - m\lambda_\mu, \\ \delta\phi &= 2m\lambda. \end{aligned}$$

All fields are canonically normalized

The field redefinition $h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\phi$ eliminates the kinetic mixing

# Charged massive spin-2

The intaractions that become strongly coupled are $L_8 + L_7 + L_6 + L_5$

$$L_8 = \frac{e}{m^4}\partial_\mu F^{\mu\nu}[(i/2)\partial_\rho\phi^*\partial^\rho\partial_\nu\phi + \text{c.c.}]$$

$$L_7 = \frac{ie}{m^3}F^{\mu\nu}\left\{2\partial_\mu B^*_\rho\partial^\rho\partial_\nu\phi - \partial_\mu B^*_\nu\Box\phi\right\} + \text{c.c.}$$

$$\boxed{\Lambda \sim \frac{m}{e^{1/4}}}$$

In the limit
$$\begin{aligned} m &\to 0 \\ e &\to 0 \\ m^4/e &\to \text{const} \end{aligned}$$
only the first survives

Can we raise the cutoff? Add a dipole term $ie\alpha F^{\mu\nu}\tilde{h}^*_{\mu\rho}\tilde{h}^\rho{}_\nu$

$$L_8^{(\text{dipole})} = \frac{e}{m^4}\partial_\mu F^{\mu\nu}[-(i\alpha/2)\partial_\rho\phi^*\partial^\rho\partial_\nu\phi + \text{c.c.}]$$

$$L_7^{(\text{dipole})} = -\frac{ie\alpha}{m^3}F^{\mu\nu}\partial_{(\mu}B^*_{\rho)}\partial^\rho\partial_\nu\phi + \text{c.c.}$$

Choose $\alpha = 1$

$$\boxed{\Lambda \sim \frac{m}{e^{1/3}}}$$

$L_7$ cannot be cancelled

Porrati, Rahman '08

# The 6th mode

$$L = -|D_\mu \tilde{h}_{\nu\rho}|^2 + 2|D_\mu \tilde{h}^{\mu\nu}|^2 + |D_\mu \tilde{h}|^2 - [D_\mu \tilde{h}^{*\mu\nu} D_\nu \tilde{h} + \text{c.c.}] - m^2[\tilde{h}^*_{\mu\nu}\tilde{h}^{\mu\nu} - \tilde{h}^*\tilde{h}]$$
$$ie\alpha F^{\mu\nu}\tilde{h}^*_{\mu\rho}\tilde{h}^\rho{}_\nu$$

Write down the equations of motion and combine them

$$m^2\eta^{\mu\nu}(\text{EOM})_{\mu\nu} + D^\mu D^\nu (\text{EOM})_{\mu\nu} = 0$$

$$m^4\tilde{h} = ie(\alpha - 1)F^{\mu\nu}D_\mu D^\rho \tilde{h}_{\rho\nu} + (\frac{\alpha}{2} - 2)e^2 F^{\mu\rho}F_{\rho^\nu}\tilde{h}_{\mu\nu} - \frac{3}{4}e^2 F^{\mu\nu}F_{\mu\nu}\tilde{h}$$

A constraint is turned into a propagating field equation unless $\alpha = 1$

Study propagation in a constant electromagnetic field

Superluminality also for very small values of the EM field invariants

The Velo-Zwanziger causaliy problem

Velo, Zwanziger '69