The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2400-11

Workshop on Strongly Coupled Physics Beyond the Standard Model

25 - 27 January 2012

The a theorem and scale without conformal invariance

Markus A. Luty

U. of California Davis
U.S.A.

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

The a Theorem and Scale without Conformal Invariance

Markus Luty
UC Davis

Based on work with Joe Polchinski and Riccardo Rattazzi
(to appear)

Outline

- Critical review of Komargodski-Schwimmer proof of a theorem

- Coupling of dilaton to QFT in perturbation theory

- Rule out “never-ending” RG flows

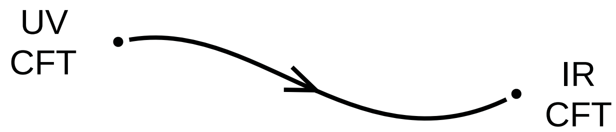
  scale without conformal invariance

  ergodic flows

  ⋮

Main assumption: existence of IR deformation to CFT

The a Theorem



$$\mathcal{L} = \mathcal{L}_{\mathrm{UV}} + \sum_i c_i m^{4-\Delta_i} \mathcal{O}_i \qquad \Delta_i < 4$$

$$T^{\mu}{}_{\mu} = a\, E_4(g) - c\, W^2(g)$$

$$a_{\mathrm{UV}} > a_{\mathrm{IR}}$$

KS Argument

(Komargodski, Schwimmer 2011)

Couple to dilaton

$$m \to m\, e^{-\tau}$$

Wess-Zumino Term

$$\Delta \mathcal{L}_{\mathrm{WZ}} = -a_{\mathrm{WZ}} \left[\tau E_4(g) + \cdots \right] + c_{\mathrm{WZ}}\, \tau\, W^2(g)$$

Anomaly matching

$$a_{\mathrm{IR}} = a_{\mathrm{UV}} + a_{\mathrm{WZ}}$$

Dilaton scattering amplitude

$$\mathcal{A}(\pi\pi \to \pi\pi) = \alpha \frac{s^2}{f^4} + \cdots$$

$$\alpha = a_{\mathrm{WZ}} > 0$$

Important Points

- Dilaton amplitude computed in probe limit
  (no internal dilaton lines)
  ⇒ no need to renormalize/UV complete dilaton theory

- Dilaton decouples in UV and IR by conformal invariance
  ⇒ can prove $\alpha > 0$ in effective theory

Dilaton Effective Theory

Write most general couplings to IR CFT

$$g_{\mu\nu} \mapsto e^{\sigma} g_{\mu\nu} \qquad \tau \mapsto \tau + \sigma$$

$$\hat{g}_{\mu\nu} = e^{-2\tau} g_{\mu\nu} = \text{ invariant}$$

Expand in derivatives:

$$O(\partial^0): \; \mathcal{L}_0 = \sqrt{-\hat{g}}\, m^{4-\Delta'} \Omega^{-\Delta'} \mathcal{O}'$$

$$= \sqrt{-g}\, (m\Omega)^{4-\Delta'} \mathcal{O}'$$

$\mathcal{O}' = 1:$ cosmological constant

Other relevant operators absent or tuned away

Effective Theory (cont'd)

$$O(\partial^2): \quad \mathcal{L}_2 = \sqrt{-\hat{g}}\, \frac{f^2}{12} R(\hat{g})$$

$$= \sqrt{-g}\, \frac{f^2}{2} \left[(\partial\Omega)^2 + \frac{1}{6}\Omega^2 R(g) \right]$$

$\Omega = e^{-\tau}$ $\qquad \Omega = 1 + \frac{\pi}{f}$ $\qquad \pi =$ canonical dilaton field

$f =$ dilaton decay constant

Dilaton effective theory, expansion in $\frac{E}{f}$

Effective Theory (cont'd)

$$O(\partial^2): \quad \mathcal{L}_2' = \sqrt{-\hat{g}}\, m^{2-\Delta} R(\hat{g}) \Omega^{-\Delta'} \mathcal{O}'$$

$$= \sqrt{-g}\, (m\Omega)^{2-\Delta} \left[\Omega^2 R(g) - 6\Omega \Box \Omega \right] \mathcal{O}'$$

0

no dilaton coupling on shell

"Un-improves" $T^{\mu\nu}$

Relevant for $\Delta' < 2$

Other similar terms are always irrelevant

$$\sqrt{-\hat{g}}\, \hat{\nabla}_\mu R(\hat{g}) \mathcal{O}'^\mu$$

$$\sqrt{-\hat{g}}\, R_{\mu\nu}(\hat{g}) \mathcal{O}'^{\mu\nu}$$

$$\vdots$$

Effective Theory (cont'd)

$$O(\partial^4): \quad \mathcal{L}_4 = \sqrt{-\hat{g}}\,\left[E_4(\hat{g}) + W^2(\hat{g}) + R^2(\hat{g})\right]$$

Contains no $\pi\pi$ scattering amplitude

$$\mathcal{L}_{\rm WZ} = \sqrt{-g}\,\left\{-a_{\rm WZ}\left[\tau E_4(g) + \cdots\right] + c_{\rm WZ}\tau W^2(g)\right\}$$

Contains $\pi\pi$ scattering amplitude in flat spacetime

$$\mathcal{L}_{\rm WZ} \to -a_{\rm WZ}\left[4\Omega^{-3}(\partial\Omega)^2\Box\Omega - \underbrace{2\Omega^{-4}(\partial\Omega)^4}_{= \frac{1}{f^4}(\partial\pi)^2 + \cdots}\right]$$

($\Box\Omega \to 0$)

Summary

- Only counterterm for $\pi\pi$ scattering at $O(E^4)$ is $a_{\rm WZ}$

- Dilaton decouples from CFT in IR in flat space despite possible relevant “un-improvement” terms

Anomaly Matching

- Read off anomaly in dilaton-deformed theory from correlation functions

$$\langle \tilde{T}^{\mu\nu} \tilde{T}^{\rho\sigma} \tilde{T}^{\tau\omega} \rangle \leftrightarrow \tilde{a}, \tilde{c}$$

- $\tilde{a}, \tilde{c} =$ constant by conformal Ward identities

- Theory is conformal for any f

  ⇒ $\tilde{a}, \tilde{c} =$ constant order by order in $1/f$

Anomaly Matching (cont'd)

Expand

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \Omega = 1 + \frac{\pi}{f}$$

$$\mathcal{L}_{\text{kin}} \sim f^2 \partial^2 \left[h^2 + h^3 + \cdots \right]$$
$$+ f\, \partial^2 \pi \left[h + h^2 + \cdots \right]$$
$$+ f\, \partial^2 \pi^2 \left[1 + h + \cdots \right]$$

$\Rightarrow$ growing contributions to $\tilde{T}^{\mu\nu}$

h  $h \sim f^2$

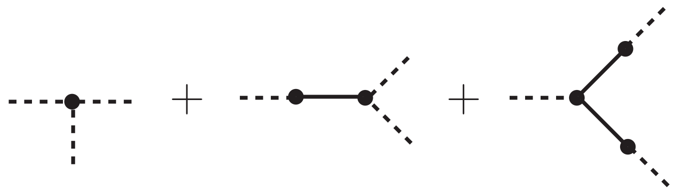
h  $\pi \sim f$

Anomaly Matching (cont'd)

$O(f^2):$

$$\langle \tilde{T}\tilde{T} \rangle =$$

$$\langle \tilde{T}\tilde{T}\tilde{T} \rangle =$$



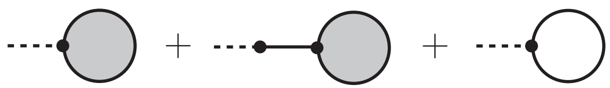
Independent of CFT

No contribution to anomaly

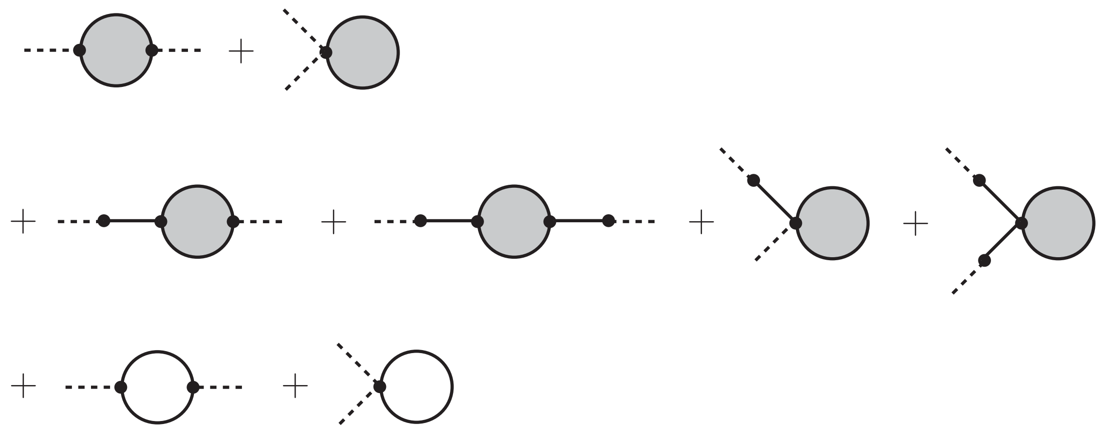
Anomaly Matching (cont'd)

$O(f^0)$:

$\langle \tilde{T} \rangle =$

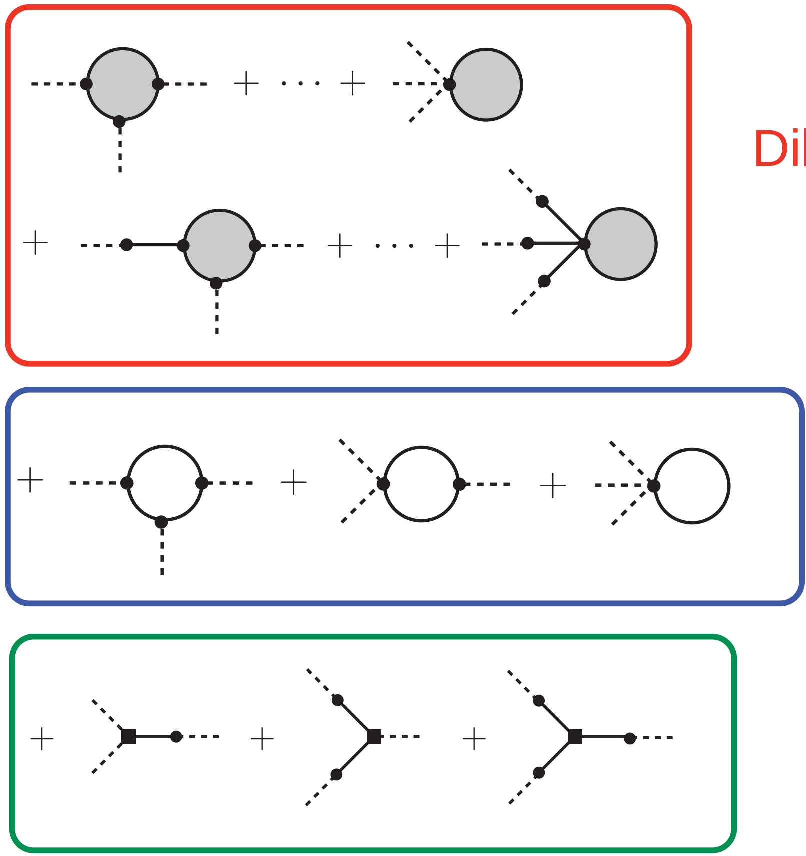


$\langle \tilde{T}\tilde{T} \rangle =$



Anomaly Matching (cont'd)

$\langle \tilde{T}\tilde{T}\tilde{T} \rangle =$



Dilaton in probe limit

Pure dilaton (free scalar)

WZ

Anomaly Matching (cont'd)

Dilaton does not affect CFT anomaly

$$\Rightarrow \tilde{a} = a_{\mathrm{IR}} + a_{\mathrm{WZ}}(\mu \to 0) + a_{\mathrm{dilaton}}$$

Similarly, in UV

$$\tilde{a} = a_{\mathrm{UV}} + a_{\mathrm{WZ}}(\mu \to \infty) + a_{\mathrm{dilaton}}$$

⇒ anomaly matching formula

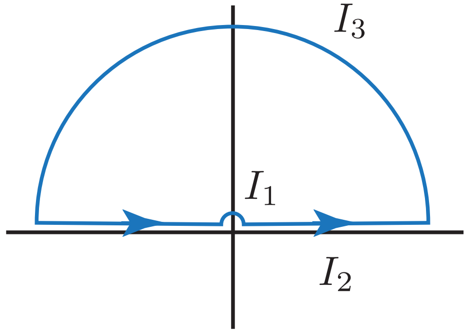
$$a_{\mathrm{UV}} - a_{\mathrm{IR}} = a_{\mathrm{WZ}}(\mu \to 0) - \underbrace{a_{\mathrm{WZ}}(\mu \to \infty)}_{\to\, 0}$$

Dilaton Scattering

$$\mathcal{A}(s) = \lim_{t\to 0} \mathcal{A}(\pi\pi \to \pi\pi)$$
$$= \alpha \frac{s^2}{f^4} + \cdots$$

$$\alpha = a_{\mathrm{WZ}}(\mu \to 0)$$

$$0 = \frac{1}{2\pi i} \oint \frac{ds}{s^3} \mathcal{A}(s)$$
$$= I_1 + I_2 + I_3$$
$$I_1 = -\frac{\alpha}{2f^4}$$

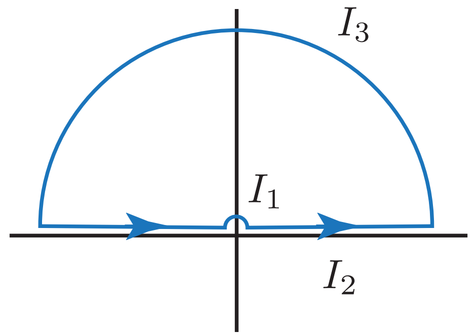


Dilaton Scattering (cont’d)

$$\mathcal{A}(s) = \mathcal{A}(-s)$$

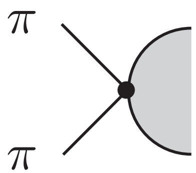
$$\Rightarrow I_2 = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\,\mathcal{A}(s)}{s^3}$$

$$= \frac{1}{\pi}\int_0^\infty ds\, \frac{\sigma(s)}{s^2}$$



$$\sigma = \sigma_{\mathrm{tot}}(\pi\pi \to \mathrm{CFT})$$

Check UV convergence:

$\sigma \sim \Big|$ 

$\Big|^2 \sim \left[\frac{1}{f^2}(m^{4-\Delta})^2\right]^2 s^{\Delta-3}$

Converges for $\Delta < 4$

Dilaton Scattering (cont'd)

- Same argument shows $I_3 = 0$
- Similar argument shows IR convergence for $\Delta' > 4$
- Can extend UV/IR argument to marginally relevant/irrelevant operators (logathmic flows)

$$\Rightarrow \alpha = \frac{2f^4}{\pi} \int_0^\infty ds \, \frac{\sigma(s)}{s^2} > 0$$

$$\Rightarrow a_{\rm UV} - a_{\rm IR} = a_{\rm WZ}(\mu \to 0) = \alpha > 0 \qquad \text{QED}$$

Scale without Conformal

Key point: dilaton does not decouple

$$S^\mu = x^\nu T^\mu{}_\nu + V^\mu$$

$$0 = \partial_\mu S^\mu = \underbrace{T^\mu{}_\mu} + \partial_\mu V^\mu$$

dilaton couples to this

Generalize KS argument to rule out SFTs that can be deformed to CFTs in the IR at arbitrarily low scales

E.g. mass terms for fermions and scalars

Low energy theory contains only U(1) gauge bosons

(Non-abelian gauge fields confine)

The Argument

- Scale invariance:

$$\mathrm{Im}\,\mathcal{A}(s) = C\,\frac{s^2}{f^4} \qquad C = \text{ constant}$$

  Couplings are fixed, wavefunctions run

  Dilaton couplings are flavor-independent

  No counterterm

- $C \neq 0$ if $T^{\mu}{}_{\mu} \neq 0$

The Argument (cont'd)

- Assume theory becomes conformal below Λ_{IR}

$$a_{\mathrm{WZ}}(\mu \to 0) = \int_{\Lambda_{\mathrm{IR}}}^{\Lambda_{\mathrm{UV}}} ds\, \frac{\sigma(s)}{s^2} + \text{ finite as } \Lambda_{\mathrm{IR}} \to 0$$

$$= C \ln \frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}} + \cdots$$

Arbitrarily large for small Λ_{IR} ⇒ contradiction

Perturbation Theory

- WZ term is a dimensionless coupling
  ⇒ logarithmically renormalized

- Interpret positive $a_{\mathrm{WZ}}(\mu \to 0)$ as due to running

- Makes argument above completely precise for perturbative theories

- Rules out more general RG flows that “never settle down”

ϕ^4 Theory

Dimensional regularization

$$d = 4 - \epsilon$$

$$\mathcal{L} = \sqrt{-g}\left\{\tfrac{1}{2}(\partial\phi)^2 + \frac{\xi}{2}\phi^2 R(g) + \tfrac{1}{2}\left(1 - \xi\frac{4(d-1)}{d-2}\right)\phi^2\Omega\Box\Omega - \frac{\lambda_0}{4!}\Omega^{-\epsilon/\delta}\phi^4\right\}$$

0

$$\delta = \frac{d-2}{2}$$

Dilaton coupling is “evanescent”

Finite effects from $\dfrac{1}{\epsilon} \times \epsilon$

ϕ^4 Theory (cont'd)

Write in terms of renormalized couplings:

$$\lambda_0 = \mu^{-\epsilon} F(\lambda(\mu), \epsilon)$$

$$F(\lambda(\mu), \epsilon) = \lambda(\mu) + \sum_{n=1}^{\infty} \frac{c_n(\lambda(\mu))}{\epsilon^n}$$

$$\lambda_0 \Omega^{-\epsilon/\delta} = \mu^{-\epsilon} F(\lambda(\mu\Omega^{1/\delta}), \epsilon)$$ (RG invariance)

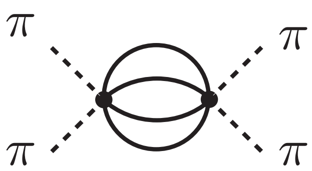
$$\lambda(\mu\Omega^{1/\delta}) = \lambda(\mu) + \frac{d\lambda}{d\ln\mu}\left(\frac{1}{\delta}\ln\Omega\right) + O(\hbar^2)$$

$$\Rightarrow \mathcal{L}_{\text{eff}} = -\frac{\beta(\lambda)}{4!}\ln\Omega\,\phi^4$$

Other couplings are higher order in loop expansion

ϕ^4 Theory (cont'd)

Leading contribution to the running of $a_{\rm WZ}$



$$\sim \hbar^3 \beta^2 \sim \hbar^5$$

Sign fixed by dispersive argument

$$\mu \frac{da_{\rm WZ}}{d\mu} = b\,\beta^2 \qquad\qquad b > 0$$

Generalizations

Generalizes to arbitrary renormalizable couplings

$$\mathcal{L}_{\rm int} = -\frac{\lambda_{abcd}}{4!}\phi^a\phi^b\phi^c\phi^d + y_{aij}\phi^a\psi^i\psi^j + \text{ gauge couplings}$$

$$\mu\frac{da_{\rm WZ}}{d\mu} = \sum_{abcd} b_\lambda(\beta_{abcd})^2 + \sum_{aij} b_y(\beta_{aij})^2 + \sum_A b_g(\beta_A)^2$$

Rules out any perturbative RG flow where

$$I = \int \frac{d\mu}{\mu}\beta^2$$

diverges in IR, for any coupling!

Conclusions

- KS argument is robust
- Existence of IR deformation to CFT rules out many exotic RG flows