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**The measured galaxy correlation function**

R. Durrer

*University of Geneva*

# The measured galaxy correlation function

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**UNIVERSITÉ  
DE GENÈVE**



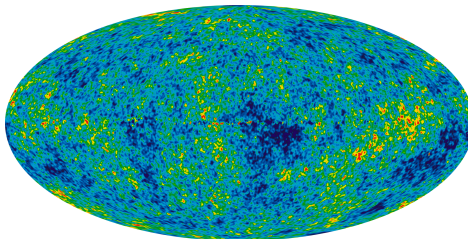
Center for Astroparticle Physics  
GENEVA

ICTP LSS Workshop, Trieste, July 2012

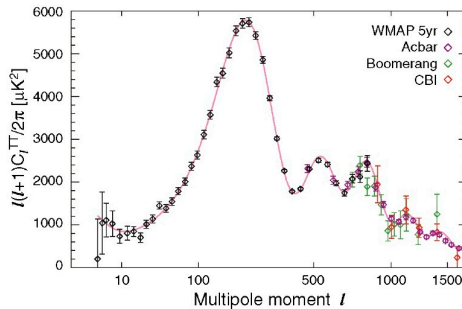
- 1 Introduction
- 2 What are very large scale galaxy catalogs really measuring?
  - Matter fluctuations per redshift bin, volume perturbations
- 3 The angular power spectrum and the correlation function of galaxy density fluctuations
  - The transversal power spectrum
  - The radial power spectrum
- 4 Alcock-Paczyński test
- 5 Conclusions

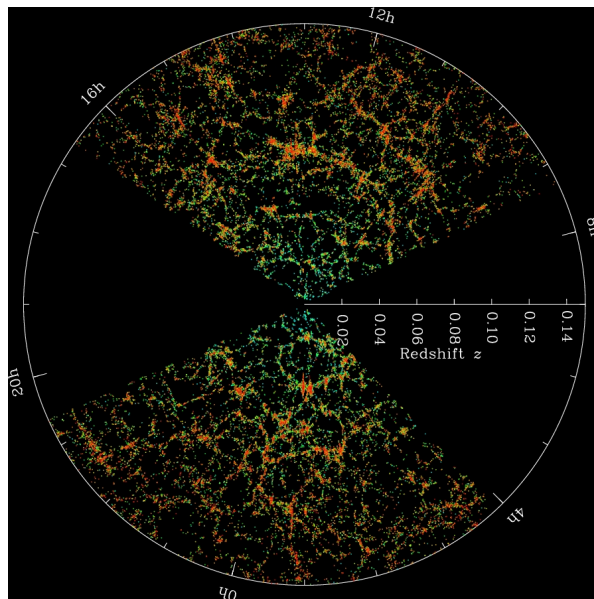
## The CMB

WMAP 7 year CMB sky



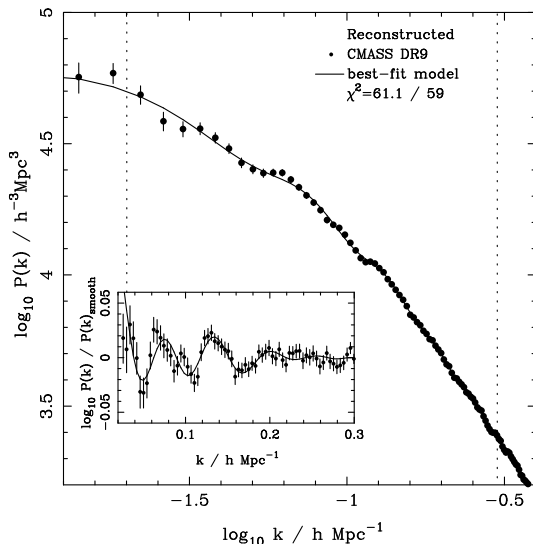
The WMAP Team





M. Blanton and the Sloan Digital Sky Survey Team.

# Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)

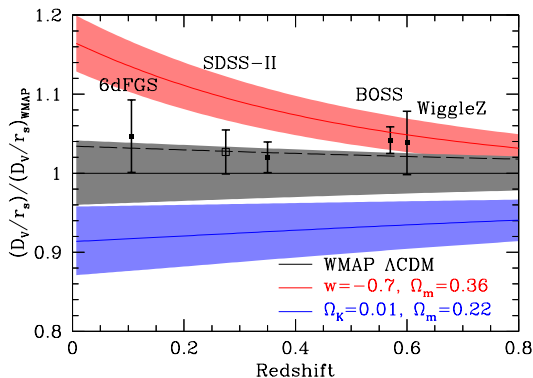


from [Anderson et al. '12](#)

SDSS-III (BOSS)  
power spectrum.

Galaxy surveys  $\simeq$   
matter density fluctuations,  
biasing and redshift space  
distortions.

The observed Universe is well approximated by a  $\Lambda$ CDM model,  
 $\Omega_\Lambda \simeq 0.72$ ,  $\Omega_m = \Omega_{\text{cdm}} + \Omega_b \simeq 0.28$ ,  $\Omega_b \simeq 0.04$ .



from [Anderson et al. '12](#)

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- For small galaxy catalogs, these effects are not very important, but when we go out to  **$z \sim 1$  or more**, they become relevant. Already for SDSS which goes out to  $z \simeq 0.2$  (main catalog) or even  $z \simeq 0.6$  (BOSS).

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- But of course much more for **future surveys like DES, bigBOSS and Euclid**.
- **Whenever we convert a measured redshift or angle into a length scale, we make an assumption about the underlying cosmology.**

# What are very large scale galaxy catalogs really measuring?

Following [C. Bonvin & RD \[arXiv:1105.5080\]](#); [F. Montanari & RD \[arXiv:1206.3545\]](#)  
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$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

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This quantity is directly measurable  $\Rightarrow$  gauge invariant.

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Density fluctuation per redshift bin  $dz$  and per solid angle  $d\Omega$  as  $\delta_z(\mathbf{n}, z)$ .

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n}, z)}{V(\mathbf{n}, z)} - \frac{\bar{N}(z)}{\bar{V}(z)}}{\frac{\bar{N}(z)}{\bar{V}(z)}}$$

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Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.

We consider a photon emitted from a galaxy ( $S$ ), moving in direction  $\mathbf{n}$  into our telescope. The observer ( $O$ ) receives the photon redshifted by a factor

$$1 + z = \frac{(\mathbf{n} \cdot \mathbf{u})_S}{(\mathbf{n} \cdot \mathbf{u})_O} .$$

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To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = - \left[ (\mathbf{n} \cdot \mathbf{V} + \Psi)(\mathbf{n}, z) + \int_0^{r(z)} (\dot{\Phi} + \dot{\Psi}) d\lambda \right]$$

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With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3 \int_0^{r_S} (\dot{\Psi} + \dot{\Phi})(\mathbf{n}, z(\lambda)) d\lambda$$

This quantity is gauge invariant and therefore, in principle, measurable. But when we count numbers of galaxies per solid bangle and per redshift bin, we also have to consider volume perturbations.



# The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

$$\begin{aligned}\Delta(\mathbf{n}, z) = & D_g + \Phi + \Psi + \frac{1}{\mathcal{H}} \left[ \dot{\Phi} + \partial_r(\mathbf{V} \cdot \mathbf{n}) \right] \\ & + \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r(z)\mathcal{H}} \right) \left( \Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r(z)} dr (\dot{\Phi} + \dot{\Psi}) \right) \\ & + \frac{1}{r(z)} \int_0^{r(z)} dr \left[ 2 - \frac{r(z) - r}{r} \Delta_\Omega \right] (\Phi + \Psi).\end{aligned}$$

(C. Bonvin & RD '11)

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(C. Bonvin & RD '11)

# The angular power spectrum of galaxy density fluctuations

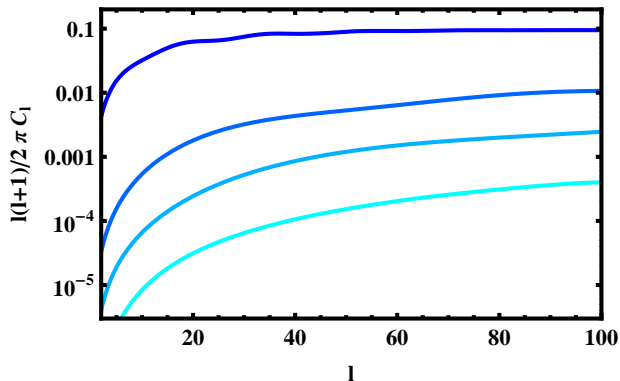
For fixed  $z$ , we can expand  $\Delta(\mathbf{n}, z)$  in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \quad C_{\ell}(z, z') = \langle a_{\ell m}(z) a_{\ell m}^*(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}(z, z') P_{\ell}(\cos \theta)$$

# The transversal power spectrum

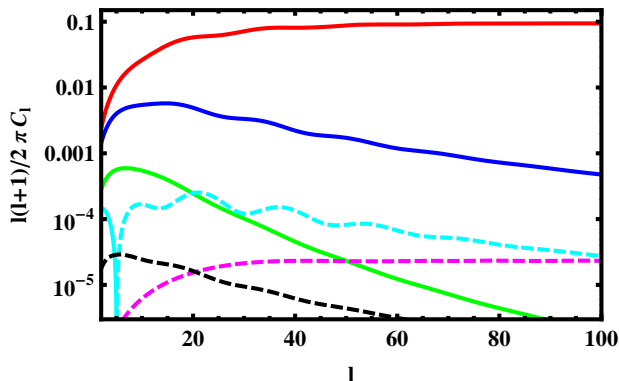
The transverse power spectrum,  $z' = z$  (from Bonvin & RD '11)



From top to bottom  $z = 0.1$ ,  $z = 0.5$ ,  $z = 1$  and  $z = 3$ .

# The transversal power spectrum

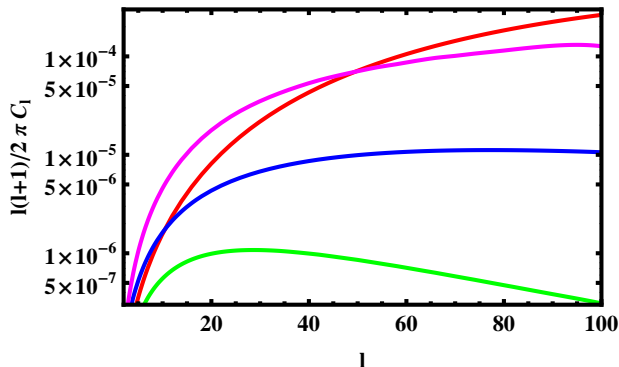
Contributions to the transverse power spectrum at redshift  $z = 0.1$   
(from Bonvin & RD '11)



$C_\ell^{DD}$  (red),  $C_\ell^{zz}$  (green),  $2C_\ell^{Dz}$  (blue),  $C_\ell^{Doppler}$  (cyan),  $C_\ell^{lensing}$  (magenta),  $C_\ell^{grav}$  (black).

# The transversal power spectrum

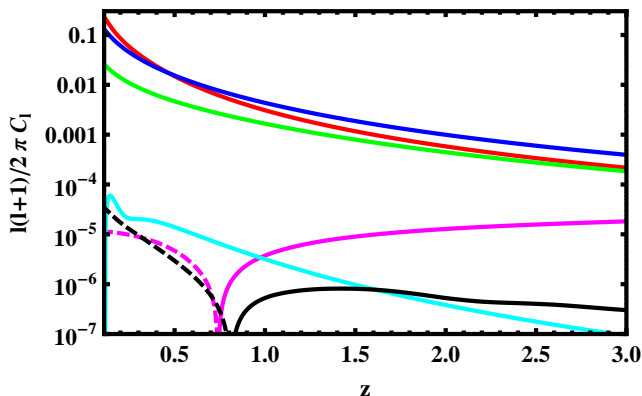
Contributions to the transverse power spectrum at redshift  $z = 3$   
(from Bonvin & RD '11)



$C_l^{DD}$  (red),  $C_l^{zz}$  (green),  $2C_l^{Dz}$  (blue),  $C_l^{lensing}$  (magenta).

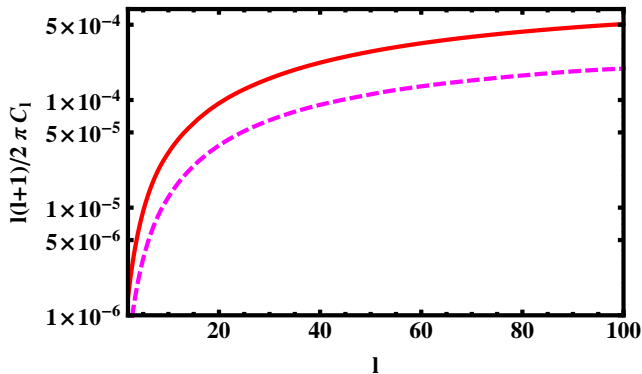
# The transversal power spectrum

Contributions to the transversal power spectrum as function of the redshift,  $\ell = 20$   
(from [Bonvin & RD '11](#))



$C_\ell^{DD}$  (red),  $C_\ell^{zz}$  (green),  $2C_\ell^{Dz}$  (blue),  $C_\ell^{lensing}$  (magenta),  $C_\ell^{Doppler}$  (cyan),  
 $C_\ell^{grav}$  (black).

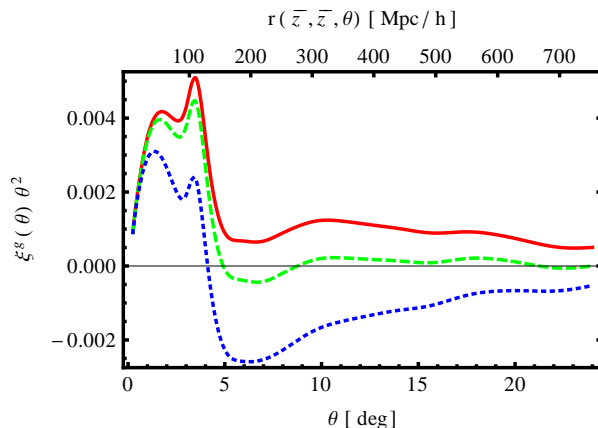
# The transversal power spectrum



$0 < z < 2$   
 $C_\ell^{DD}$  (red),  $C_\ell^{lensing}$  (magenta).



# The transversal correlation function



(from  
Montanari & RD '12)

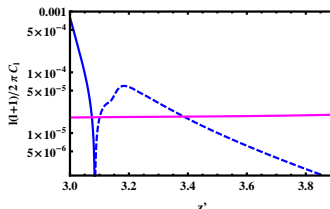
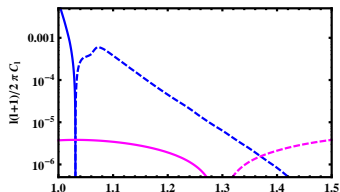
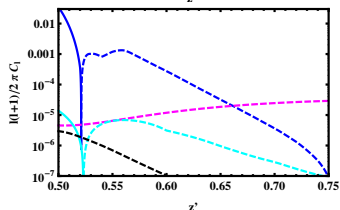
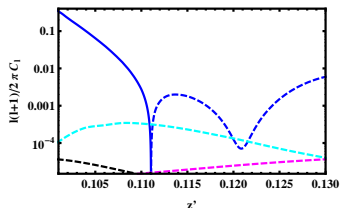
$\theta^2 \xi(\theta, z, z)$

blue  $C_\ell^{DD}$  (real space),

green flat space approximation for redshift space distortions,

red  $C_\ell^{DD}$ ,  $C_\ell^{zz}$  and  $2C_\ell^{Dz}$  (fully positive!).

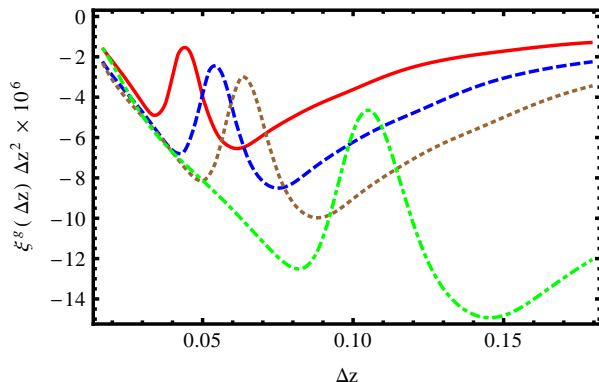
# The radial power spectrum



The radial power spectrum  $C_\ell(z, z')$   
for  $\ell = 20$   
Left, top to bottom:  $z = 0.1, 0.5, 1$ ,  
top right:  $z = 3$

Standard terms (blue),  $C_\ell^{lensing}$  (magenta),  
 $C_\ell^{Doppler}$  (cyan),  $C_\ell^{grav}$  (black),

# The radial correlation function



(from  
Montanari & RD '12)

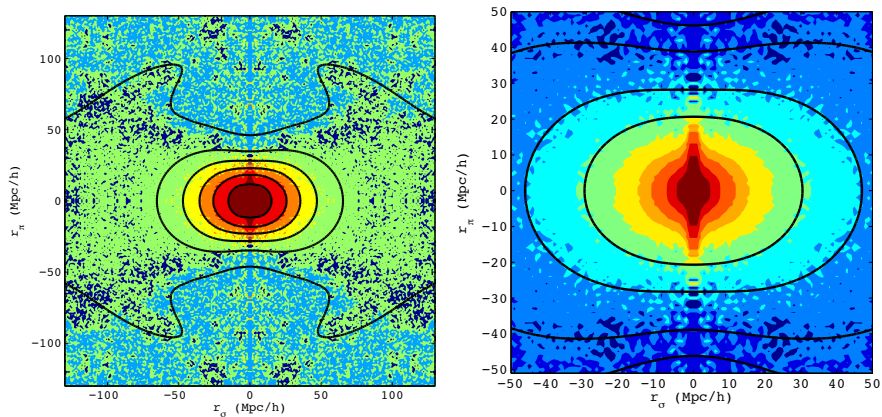
$z = 2$ ,  
 $z = 1$ ,  
 $z = 0.7$ ,  
 $z = 0.3$ .

$$(\Delta z)^2 \xi(\theta, z - \Delta z/2, z + \Delta z/2)$$

Purely negative for  $\Delta z \gtrsim 0.01$ .

# Anisotropic clustering as seen in the BOSS survey

(from Reid et al. '12)



# Example: Alcock-Paczyński test

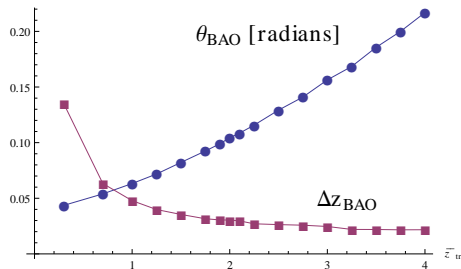
(Alcock & Paczyński '79)

Consider a comoving scale  $L$  in the sky.

Horizontally it is projected to the angle  $\theta_L = \frac{L}{(1+z)D_A(z)}$ .

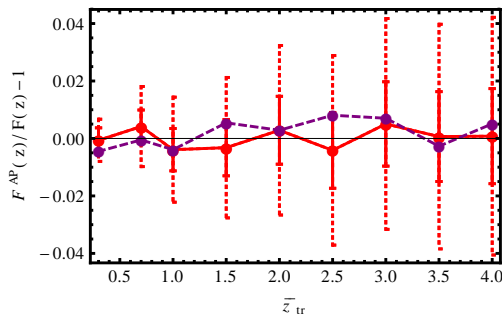
Radially its ends are at a slightly different redshifts,  $\Delta z_L = LH(z)$ .

$$\frac{\Delta z_L}{\theta_L} = (1+z)D_A(z)H(z) = F(z) \equiv \int_0^z \frac{H(z')}{H(z)} dz'$$



# Example: Alcock-Paczyński test

$F(z)^{AP} \equiv \Delta z_L / \theta_L$  measured from the theoretical power spectrum (with Euclid-like redshift accuracies)  $F(z) \equiv \int_0^z \frac{H(z')}{H(z)} dz'$ .



solid errors:  
angular resolution  $0.02^\circ$   
dashed errors:  
angular resolution  $0.05^\circ$

(from Montanari & RD '12)

- So far cosmological LSS data mainly determined  $\xi(r)$ , or equivalently  $P(k)$ . These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{d(z)^2 + d(z')^2 - 2d(z)d(z')\cos\theta}.$$

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