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Redshift space distortions for biased tracers

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Redshift space distortions for biased tracers

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Outline

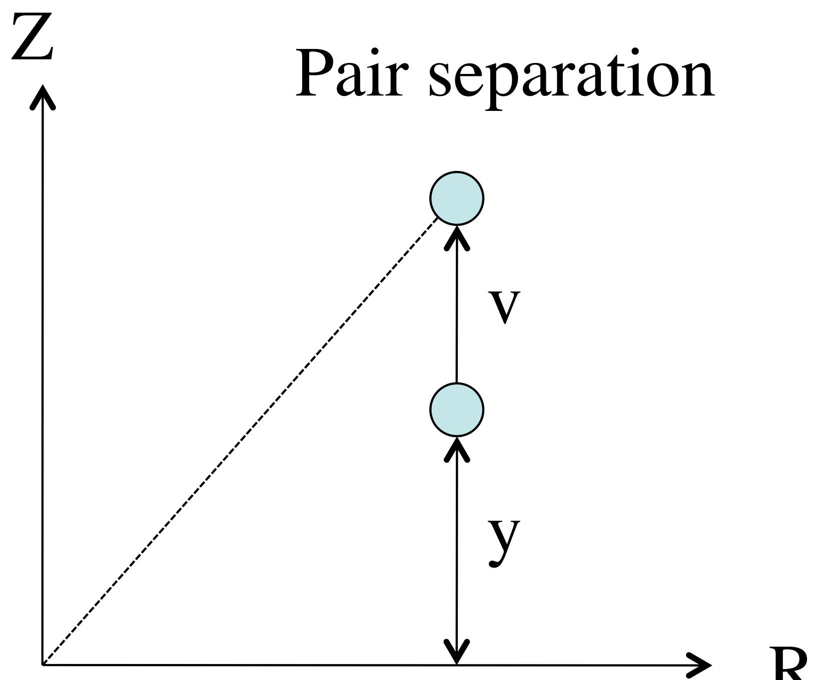
- Introduction
- RSD in configuration space.
  - Difficulties in modeling RSD.
  - Insights from a toy model.
  - Convolution LPT (CLPT)
- Conclusions.

RSD: Why

- What you observe in a redshift survey is the density field in redshift space!
  - A combination of density and velocity fields.
- Tests GI.
  - Structure growth driven by motion of matter and inhibited by expansion.
- Constrains GR.
  - Knowing a(t) and ρ_i, GR provides prediction for growth rate.
  - In combination with lensing measures Φ and Ψ.
- Measures "interesting" numbers.
  - Constrains H(z), DE, m_ν, etc.
- Surveys can make percent level measurements – would like to have theory to compare to!
- Fun problem!

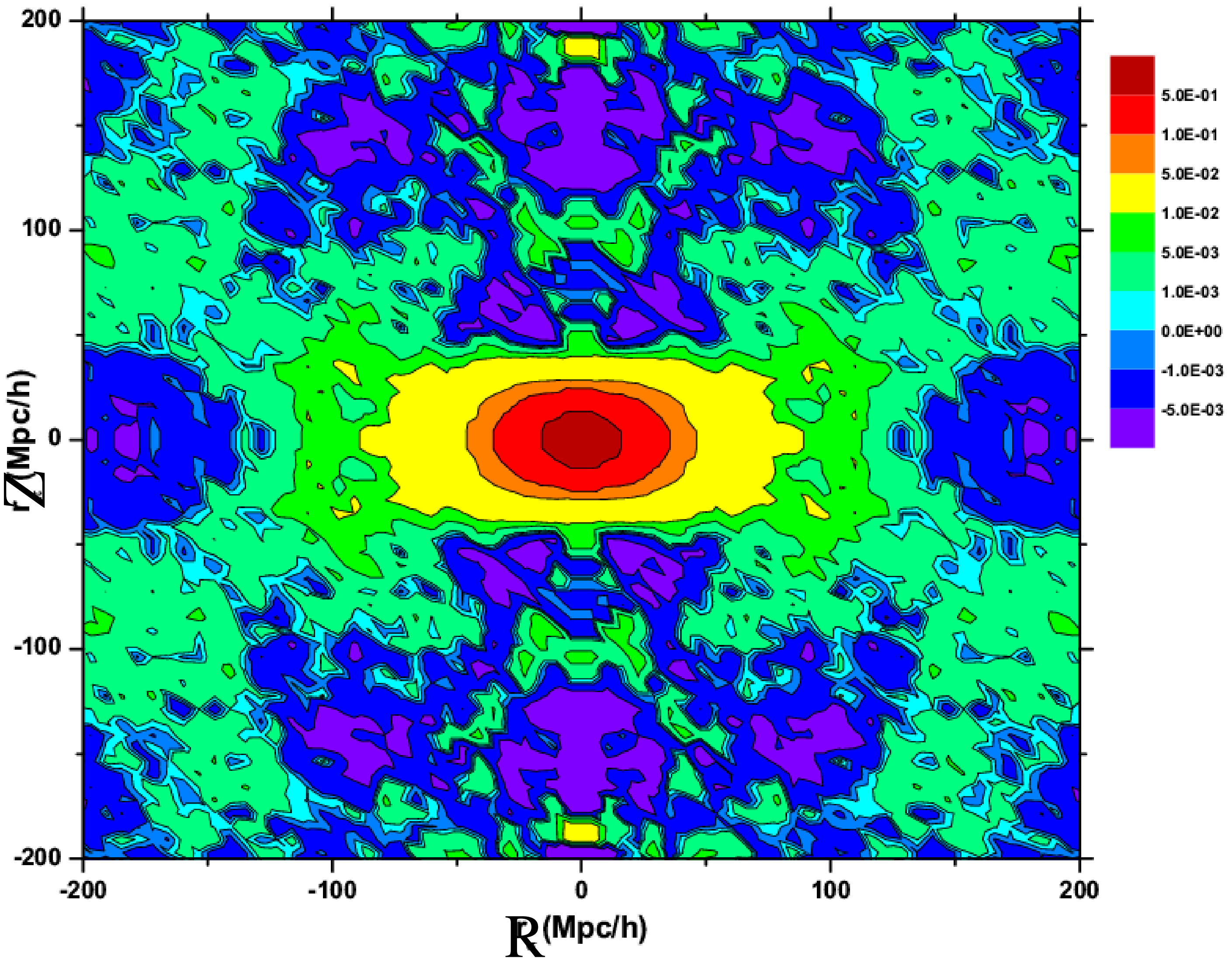
Simplify …

- We will work in the distant observer, plane-parallel approximation(s).
- All velocities will be expressed in units of the Hubble expansion.
  - i.e. in distance units.
- Use polar coordinates.



Two dimensional clustering
(BOSS; Reid++12)

Line-of-sight picks out a preferred direction inducing anisotropy in the 2-point function – measures the growth of structure and tests GR.



In configuration space

- Kaiser's pioneering work was done in Fourier space.
- There are valuable insights to be gained by working in configuration, rather than Fourier, space.
- We begin to see why this is a hard problem …

$$1 + \xi^s(R, Z) = \left\langle \int dy \; (1+\delta_1)(1+\delta_2)\delta^{(D)}(Z - y - v_{12}) \right\rangle$$

$$1 + \xi^s(R, Z) = \left\langle \int dy \; (1+\delta_1)(1+\delta_2) \int \frac{d\kappa}{2\pi} e^{i\kappa(Z-y-v_{12})} \right\rangle$$

- Note all powers of the velocity field enter.

Gaussian limit
(Fisher, 1995, ApJ 448, 494)

- If δ and v are Gaussian can do all of the expectation values.

$$1+\xi^s(R,Z)=\int \frac{dy}{\sqrt{2\pi\sigma_{12}^2(y)}}\exp\left[-\frac{(Z-y)^2}{2\sigma_{12}^2(y)}\right]\times$$

$$\left[1+\xi^r(r)+\frac{y}{r}\frac{(Z-y)v_{12}(r)}{\sigma_{12}^2(y)}-\frac{1}{4}\frac{y^2}{r^2}\frac{v_{12}^2(r)}{\sigma_{12}^2(y)}\left(1-\frac{(Z-y)^2}{\sigma_{12}^2(y)}\right)\right]$$

Expanding around y=Z:

$$\xi^s(R,Z)=\xi^r(s)-\frac{d}{dy}\left[v_{12}(r)\frac{y}{r}\right]\bigg|_{y=Z}+\frac{1}{2}\frac{d^2}{dy^2}\left[\sigma_{12}^2(y)\right]\bigg|_{y=Z}$$

Linear theory: configuration space
(Fisher, 1995, ApJ 448, 494)

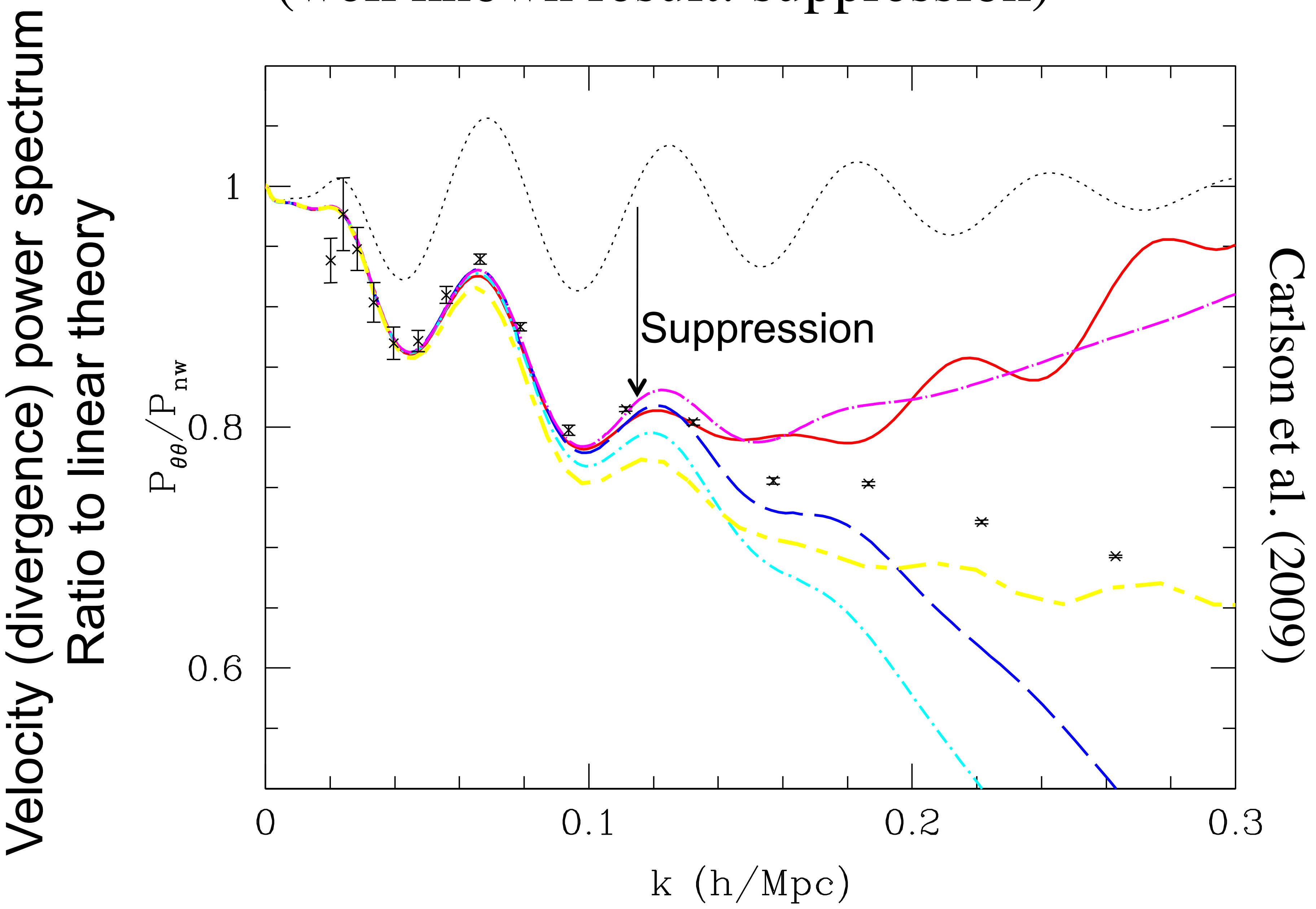
- One can show that this expansion agrees with the Kaiser formula.
- Two important points come out of this way of looking at the problem:
  - Correlation between δ and v leads to v_{12}.
    - Overdensities will fall towards each other.
    - The μ^2 term is a <vδ> correlation as for Kaiser.
  - LOS velocity dispersion is scale- and orientation-dependent.
- ξ^s depends on the 1st and 2nd derivatives of velocity statistics.

Two forms of non-linearity

- Part of the difficulty is that we are dealing with two forms of non-linearity/non-perturbative behavior.
  - The velocity field is non-linear.
  - The mapping from real- to redshift-space is “non-linear”.
- These two forms of non-linearity interact, and can partially cancel.
- They also depend on parameters differently!

Velocity field is nonlinear

(well known result: suppression)



Non-linear mapping?

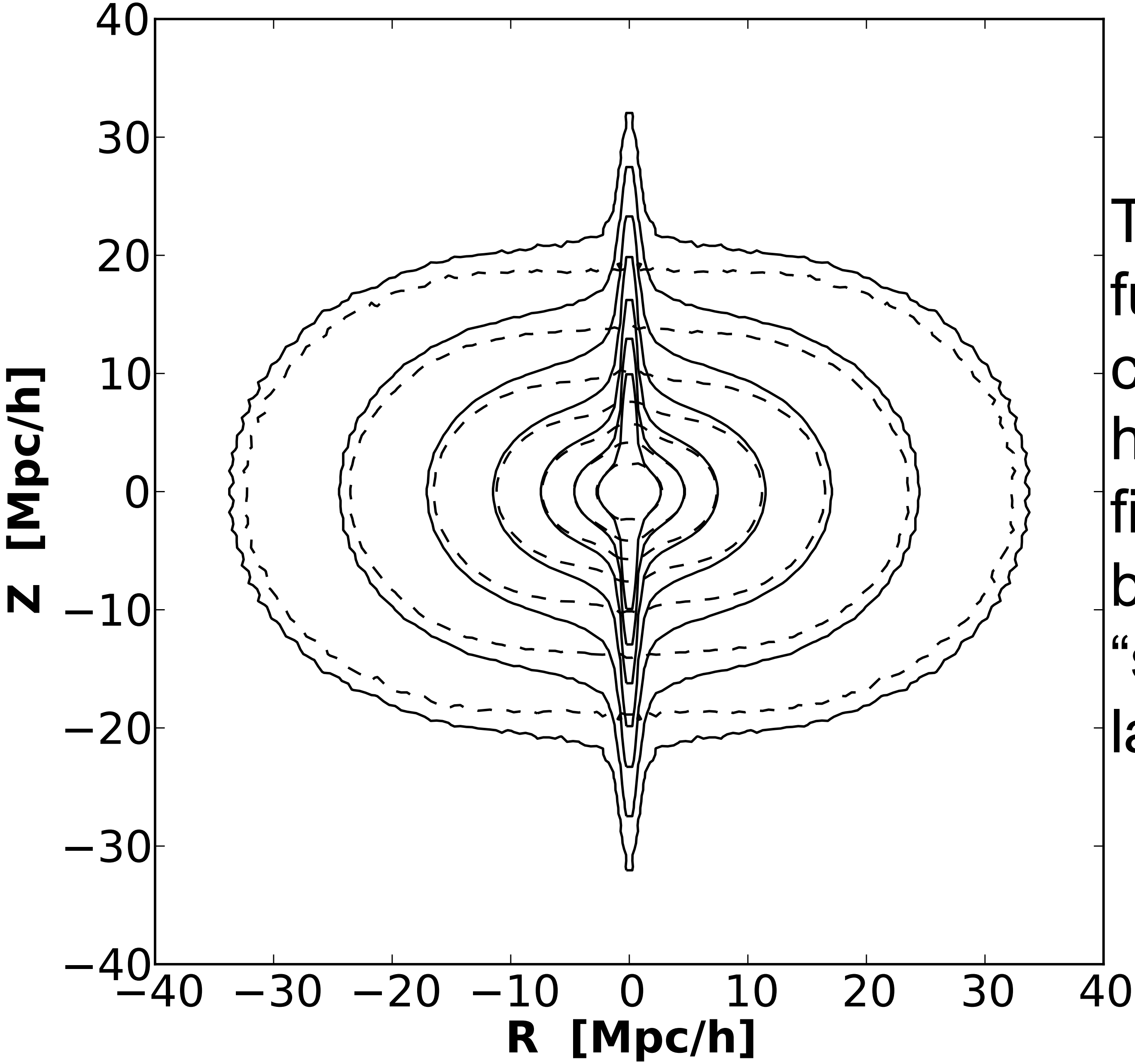
?

Want a fully non-linear “toy model”, like spherical top-hat collapse, to gain some intuition …

A model for the redshift-space clustering of halos

- We would like to develop a model capable of describing the redshift space clustering of halos.
  - This will form the 1st step in a model for galaxies, but it also interesting in its own right.
- The model should try to treat the "non-linear mapping" part of the problem non-perturbatively.
- We will start with a toy model and then add realism/ dynamics …

The correlation function of halos



The correlation function of halo centers doesn't have strong fingers of god, but still has "squashing" at large scales.

Scale-dependent Gaussian streaming model

Let's go back to the exact result for a Gaussian field, a la Fisher:

$$1+\xi^s(R,Z)=\int\frac{dy}{\sqrt{2\pi\sigma_{12}^2(y)}}\exp\left[-\frac{(Z-y)^2}{2\sigma_{12}^2(y)}\right]\times$$
$$\left[1+\xi^r(r)+\frac{y}{r}\frac{(Z-y)v_{12}(r)}{\sigma_{12}^2(y)}-\frac{1}{4}\frac{y^2}{r^2}\frac{v_{12}^2(r)}{\sigma_{12}^2(y)}\left(1-\frac{(Z-y)^2}{\sigma_{12}^2(y)}\right)\right]$$

Looks convolution-like, but with a scale-dependent v_{12} and σ (also, want to resum v_{12} into the exponential …)

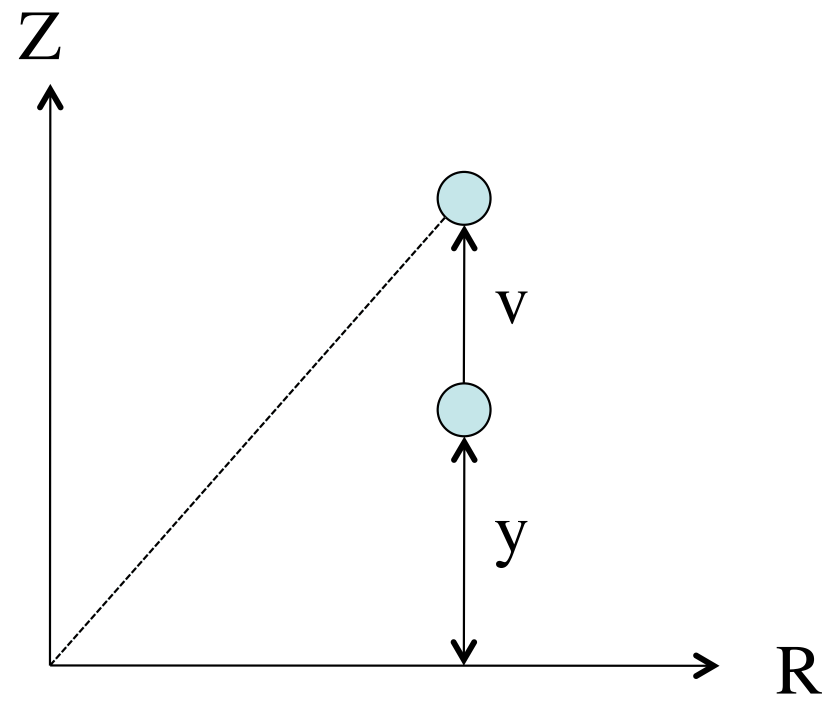
Scale-dependent Gaussian streaming model/ansatz

$$1 + \xi(R, Z) = \int dy \ \left[1 + \xi(r)\right] \mathcal{P}\left(v = Z - y, \mathbf{r}\right)$$

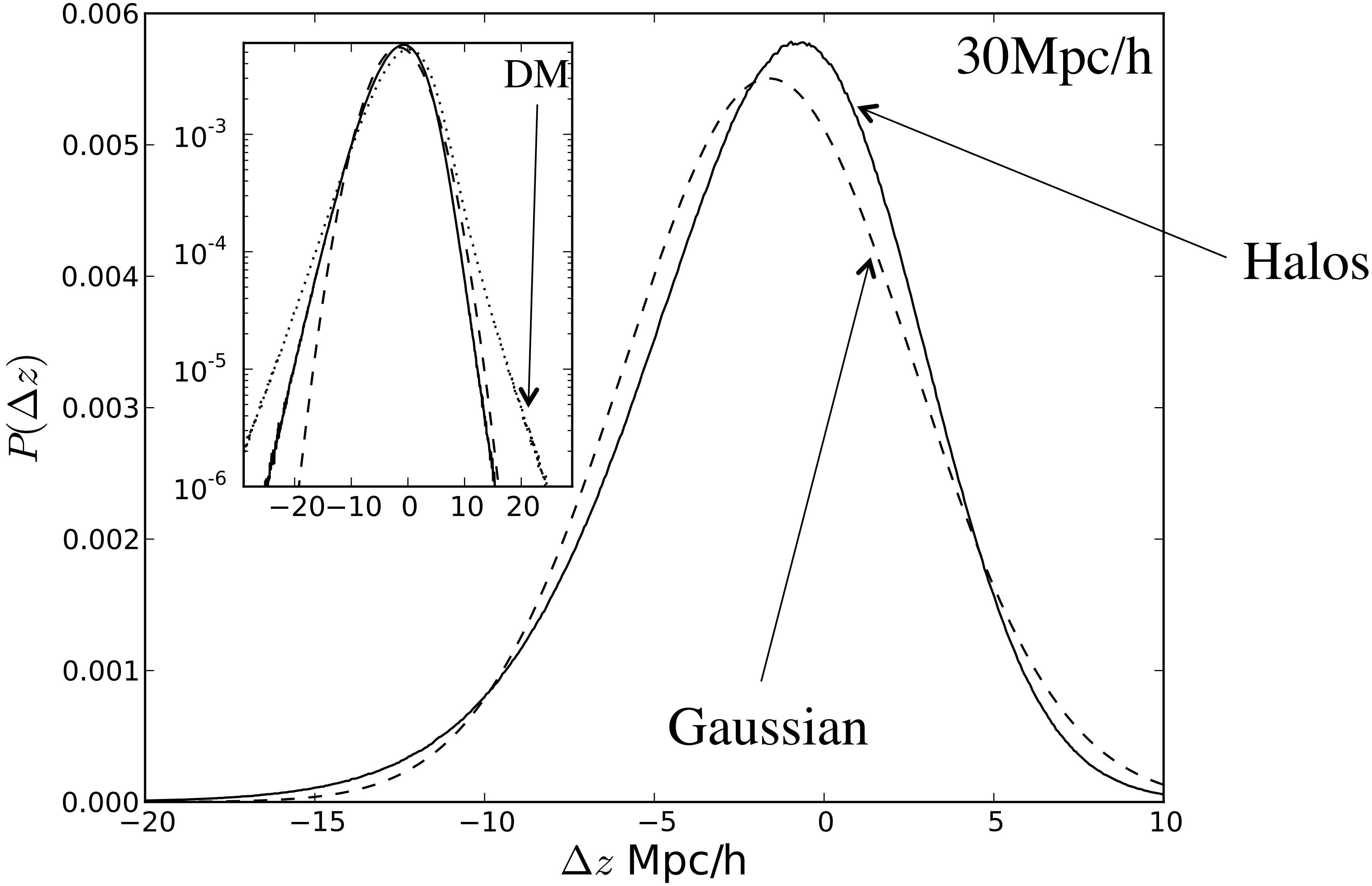
Note: *not* a convolution because of (important!) ***r*** dependence or kernel.

Non-perturbative mapping.

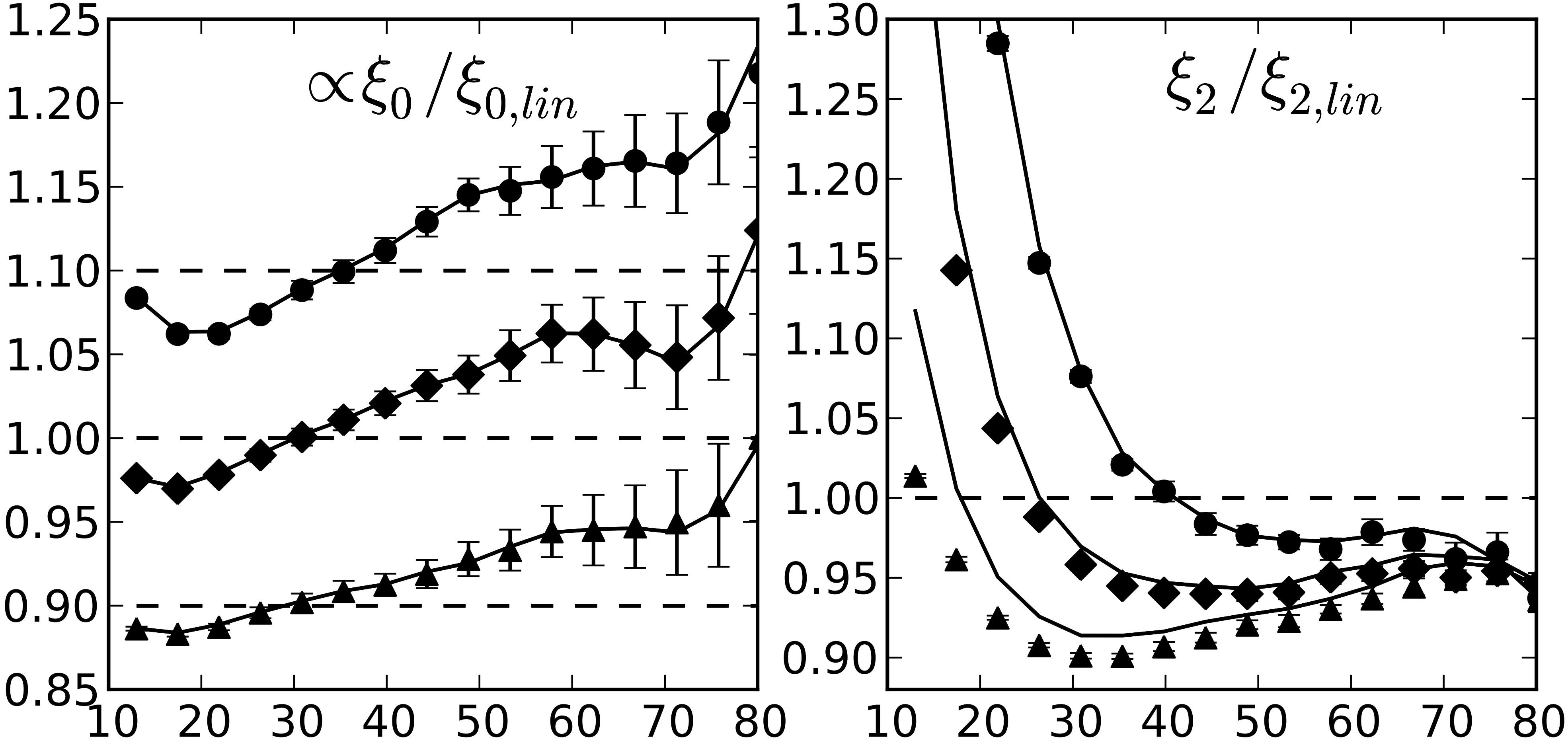
If lowest moments of *P* set by linear theory, agrees at linear order with Kaiser.
Approximate *P* as Gaussian …



Gaussian ansatz

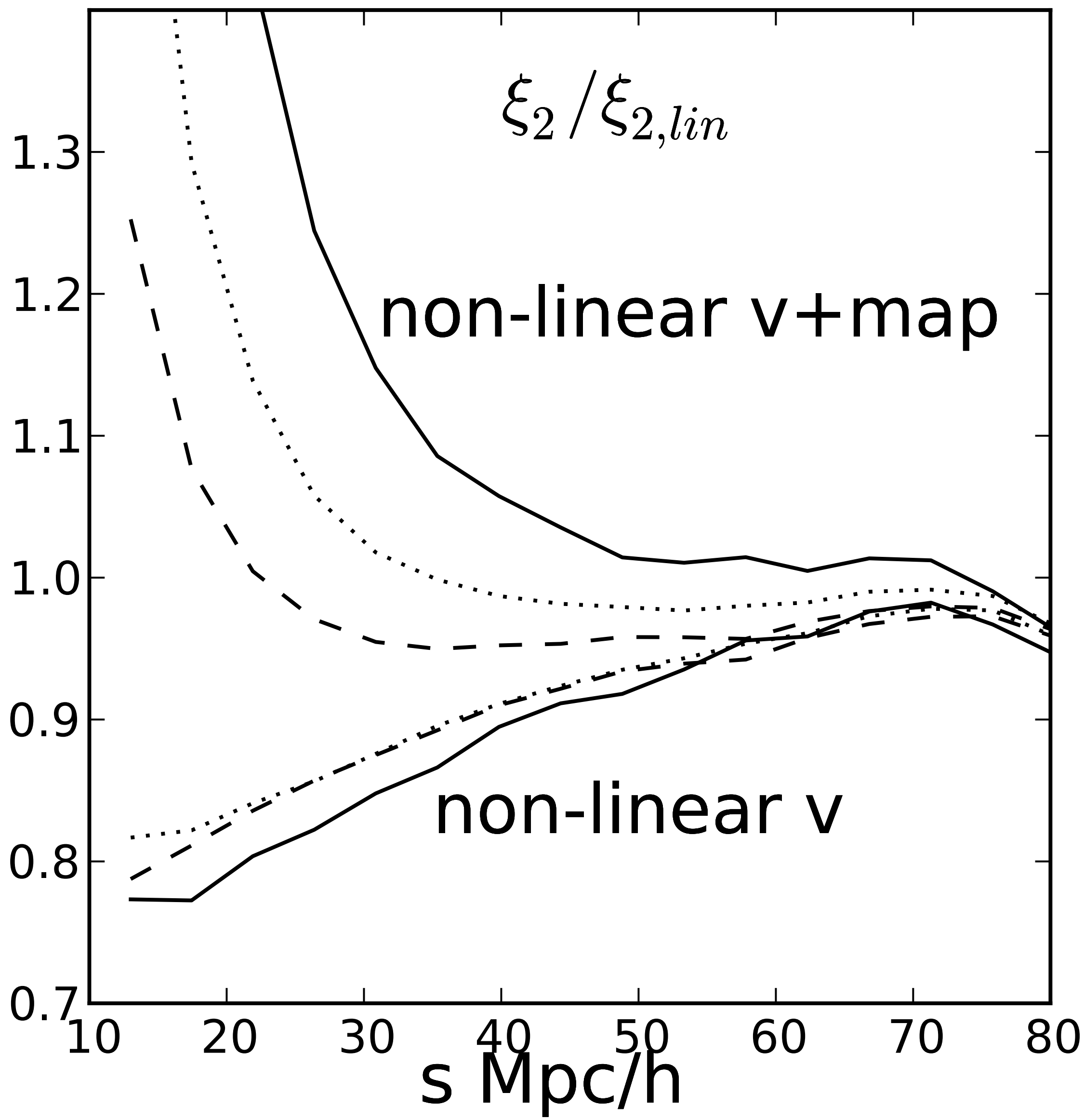


Testing the ansatz



Reid & White (2011)

The mapping



Note, the behavior of the quadrupole is particularly affected by the non-linear mapping. The effect of non-linear velocities is to suppress ξ_2 (by ~10%, significant!). The mapping causes the enhancement. This effect is tracer/ bias dependent!

The "b^3" term?

- One of the more interesting things to come out of this ansatz is the existence of a "b^3" term.
  - Numerically quite important when b~2.
  - Another reason why mass results can be very misleading.
  - But hard to understand (naively) from

$$1+\xi^s(R,Z) = \left\langle \int dy\ (1+\delta_1)(1+\delta_2) \int \frac{d\kappa}{2\pi} e^{i\kappa(Z-y-v_{12})} \right\rangle$$

  - Where does it come from?

Lagrangian perturbation theory

- A different approach to PT.
  - Buchert89, Moutarde++91, Bouchet++92, Catelan95, Hivon++95.
- Relates the current (Eulerian) position of a mass element, $\mathbf{x}$, to its initial (Lagrangian) position, $\mathbf{q}$, through a displacement vector field, $\mathbf{\Psi}$.
- Has been radically extended recently by Matsubara:
  - Matsubara (2008a; PRD, 77, 063530)
  - Matsubara (2008b; PRD, 78, 083519)
- (and is *very* useful for BAO)

Lagrangian perturbation theory

$$\delta(\mathbf{x}) = \int d^3q \; \delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi}) - 1$$

$$\delta(\mathbf{k}) = \int d^3q \; e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1 \right) \; .$$

$$\frac{d^2\mathbf{\Psi}}{dt^2} + 2H\frac{d\mathbf{\Psi}}{dt} = -\nabla_x \phi \left[\mathbf{q} + \mathbf{\Psi}(\mathbf{q})\right]$$

$$\begin{aligned}\mathbf{\Psi}^{(n)}(\mathbf{k}) &= \frac{i}{n!}\int \prod_{i=1}^{n}\left[\frac{d^3k_i}{(2\pi)^3}\right] (2\pi)^3\delta_D\left(\sum_i \mathbf{k}_i - \mathbf{k}\right) \\ &\times \; \mathbf{L}^{(n)}(\mathbf{k}_1, \cdots, \mathbf{k}_n, \mathbf{k})\delta_0(\mathbf{k}_1)\cdots\delta_0(\mathbf{k}_n)\end{aligned}$$

Beyond real-space mass

- One of the more impressive features of Matsubara's LPT approach is that it can gracefully handle both biased tracers and redshift space distortions.
- In redshift space, in the plane-parallel limit,

$$\mathbf{\Psi} \to \mathbf{\Psi} + \frac{\widehat{\mathbf{z}} \cdot \dot{\mathbf{\Psi}}}{H}\, \widehat{z} = R\,\mathbf{\Psi}$$

- In PT $\mathbf{\Psi}^{(n)} \propto D^n \Rightarrow R_{ij}^{(n)} = \delta_{ij} + nf\, \widehat{z}_i \widehat{z}_j$

- For bias local in Lagrangian space:

$$\delta_{\rm obj}(\mathbf{x}) = \int d^3q\ F\left[\delta_L(\mathbf{q})\right] \delta_D(\mathbf{x} - \mathbf{q} - \mathbf{\Psi})$$

- If we assume halos/galaxies form at peaks* of the initial density field ("peaks bias") then explicit expressions exist for the integrals of F that we will need.

*…and assume the peak-background split.

Configuration-space result

- The density of objects can be written:

$$1+\delta_{\rm obj}(\mathbf{x}) = \int d^3q \int \frac{d\lambda}{2\pi}\ \widetilde{F}(\lambda) \int \frac{d^3k}{(2\pi)^3}\ e^{i\{\lambda\delta(\mathbf{q})+\mathbf{k}\cdot[\mathbf{x}-\mathbf{q}-\mathbf{\Psi}]\}}$$

- so the 2-point function is

$$1+\xi_{\rm obj}(\mathbf{r}) = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{q}-\mathbf{r})} \int \frac{d\lambda_1}{(2\pi)} \int \frac{d\lambda_2}{(2\pi)} \widetilde{F}(\lambda_1)\widetilde{F}(\lambda_2) K\left(\mathbf{q},\mathbf{k},\lambda_1,\lambda_2\right)$$

- where we have written

$$K(\mathbf{q},\mathbf{k},\lambda_1,\lambda_2) = \left\langle e^{i\{\lambda_1\delta_1+\lambda_2\delta_2+\mathbf{k}\cdot(\mathbf{\Psi}_1-\mathbf{\Psi}_2)\}} \right\rangle$$

- This is the configuration-space analog of Matsubara's Fourier-space expression.

Example: Zel'dovich

- Let's consider the lowest order expression
  - Zel'dovich approximation.

$$\mathbf{\Psi}(\mathbf{q}) = \mathbf{\Psi}^{(1)}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3}\; e^{i\mathbf{k}\cdot\mathbf{q}}\; \frac{i\mathbf{k}}{k^2}\; \delta_0(\mathbf{k})$$

- Since δ_0 is Gaussian

$$K = \exp\left[-\frac{1}{2}\left(\lambda_1^2 + \lambda_2^2\right)\sigma^2 - \frac{1}{2}A_{ij}k_ik_j - \lambda_1\lambda_2\xi - \left(\lambda_1 + \lambda_2\right)U_ik_i\right]$$

- where we have defined

$$\sigma^2 = \langle \delta^2 \rangle \qquad \xi(\mathbf{q}) = \langle \delta_1\delta_2 \rangle$$
$$A_{ij}(\mathbf{q}) = \langle \Delta_i\Delta_j \rangle \qquad U_i(\mathbf{q}) = \langle \delta\Delta_i \rangle$$

Integrals of P_L times Bessel functions.

- and $\Delta=\Psi_1-\Psi_2$. The matrix A_{ij} can be decomposed into pieces going as δ_{ij} and q_iq_j

Matter & Zel'dovich approximation

$$A_{ij} = \langle \Delta\mathbf{\Psi}_i \Delta\mathbf{\Psi}_j \rangle = B + C = 2\sigma^2\delta_{ij} + C$$

$$\begin{aligned} 1 + \xi^{ZA}(\mathbf{r}) &= \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}|A|^{1/2}}\; e^{-(\mathbf{r}-\mathbf{q})A^{-1}(\mathbf{r}-\mathbf{q})/2} \\ &= \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}|B|^{1/2}}\; e^{-(\mathbf{r}-\mathbf{q})B^{-1}(\mathbf{r}-\mathbf{q})/2} \left[1 + \chi(\mathbf{q})\right] \end{aligned}$$

$$1 + \chi(\mathbf{q}) = \int \frac{d^3p}{(2\pi)^{3/2}|C|^{1/2}} e^{-(\mathbf{q}-\mathbf{p})C^{-1}(\mathbf{q}-\mathbf{p})}$$

Is very similar to the linear theory ξ.

Biased tracers & Zel'dovich

- For biased tracers Taylor expand terms going as ξ and *U* but keep σ and *A* terms exponentiated.
  - Both ξ and *U* vanish as *q*->∞ but σ and *A* do not.
  - Note our result is not simply the FT of Matsubara's expression b/c he keeps only constant piece of *A* exponentiated while we keep all of it.
- Have to plug this into 1+ξ formula, do λ integrals, …

$$1+\xi_X(r) = \int \frac{d^3q}{(2\pi)^{3/2}|A|^{1/2}}\ e^{-\frac{1}{2}(q-r)^T A^{-1}(q-r)}$$
$$\times\left[1-\cdots 2\langle F'\rangle\langle F''\rangle\xi_R U_i g_i+\cdots\right]$$

Peaks bias

- Our final expression contains terms with averages of F’ and F’’ over the density distribution.
- These take the place of “bias” terms
  - b_1 and b_2 in standard perturbation theory*.
- If we assume halos form at the peaks of the initial density field and use the peak-background split we can obtain:

$$b_1 = \frac{\nu^2 - 1}{\delta_c} \quad , \quad b_2 = \frac{\nu^4 - 3\nu^2}{\delta_c^2} \approx b_1^2$$

for large ν

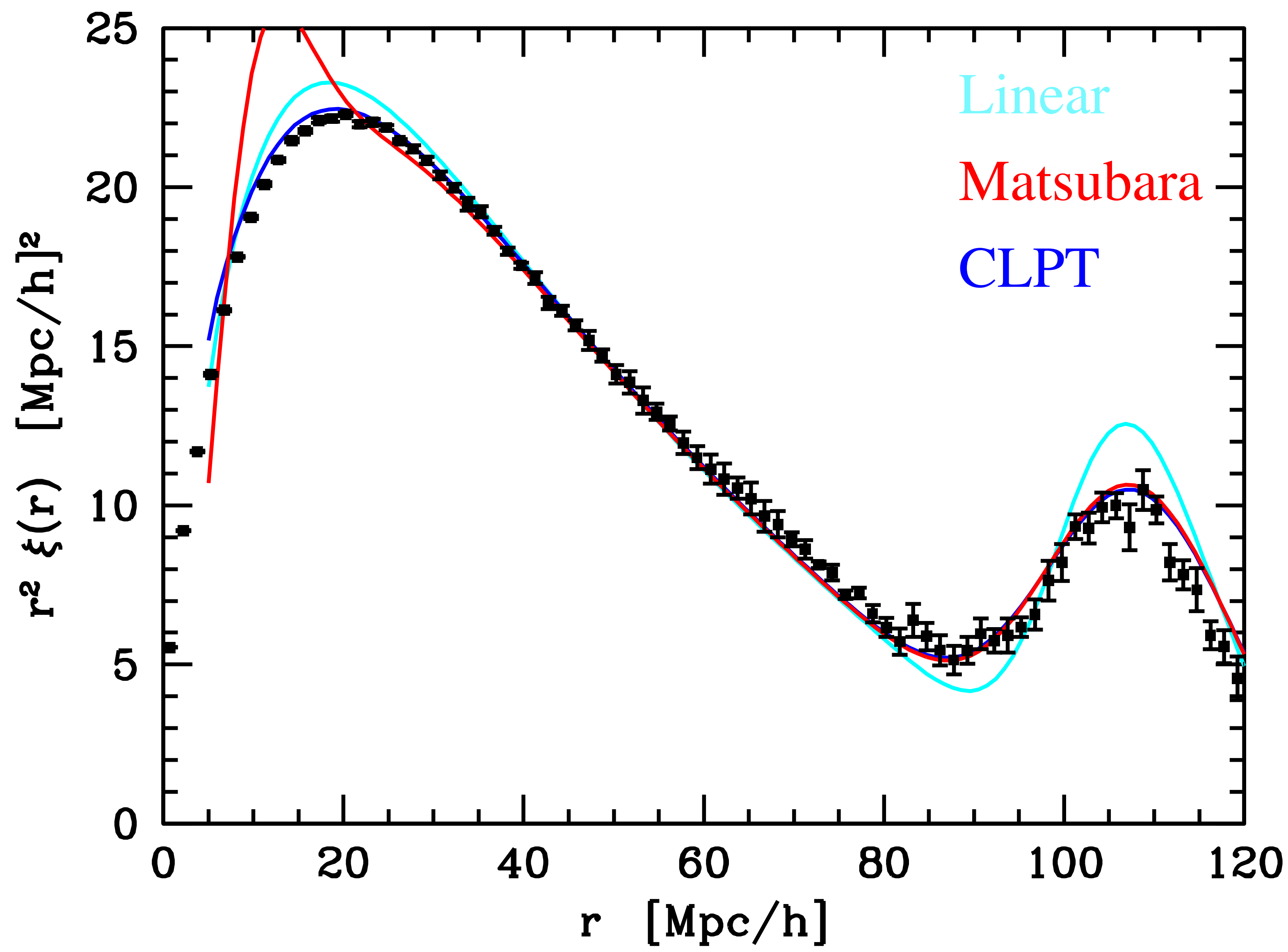
- so <F’><F’’>~b^3.

*but “renormalized”.

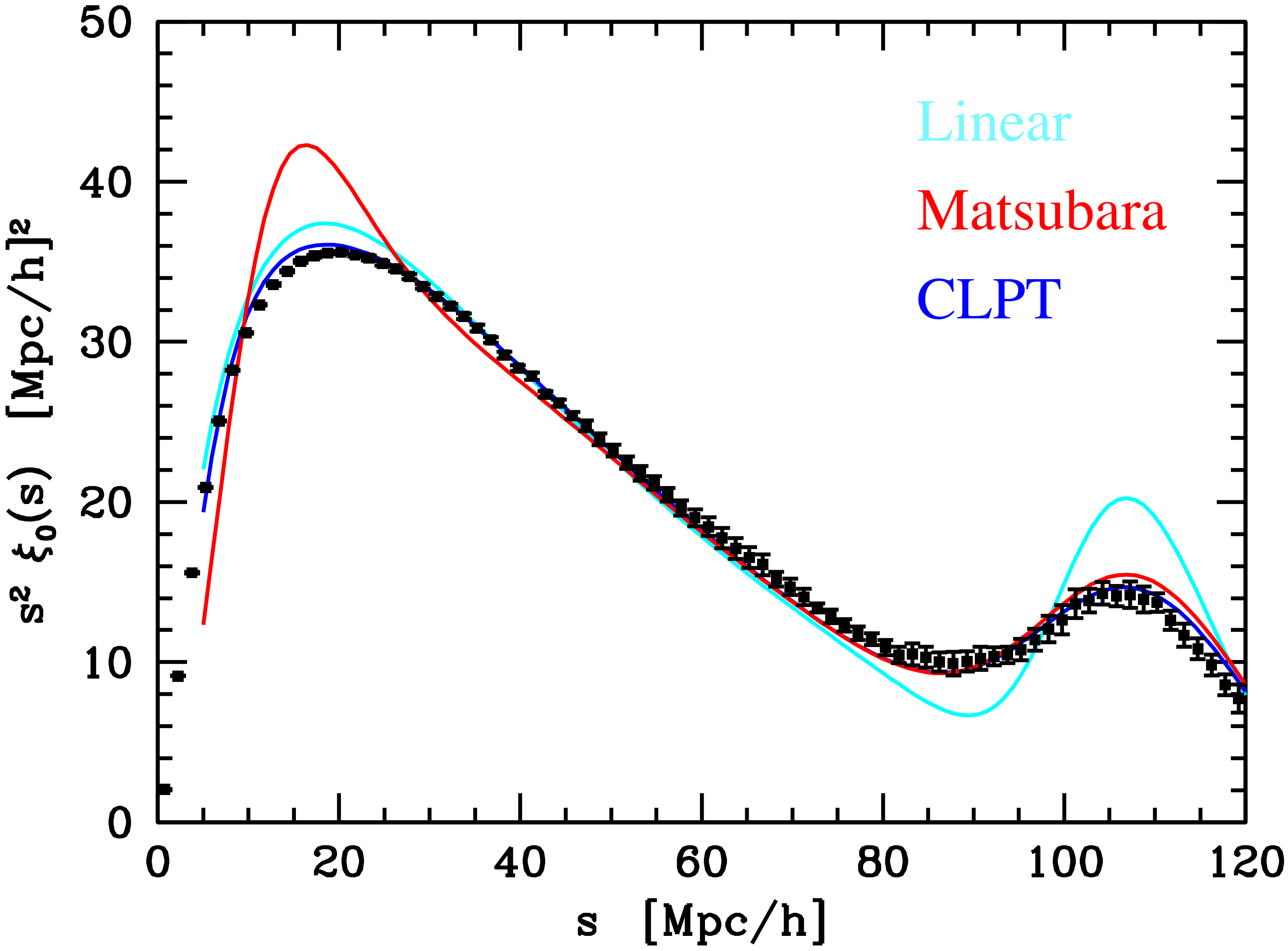
Convolution LPT?

- Can go beyond 1LPT (Zel'dovich) and do perturbative expansion.
- Keep all of <$\Delta\Psi_i \Delta\Psi_j$> (and σ_R) exponentiated.
  - Expand the rest.
  - Do some algebra.
  - Evaluate convolution integral numerically.
  - This is a partial resummation of Matsubara's expression.
- Guarantees we recover the Zel'dovich limit as 0^{th} order CLPT (for the matter).
  - Eulerian and LPT require an ∞ number of terms.
  - Many advantages: as emphasized recently/independently by Tassev & Zaldarriaga

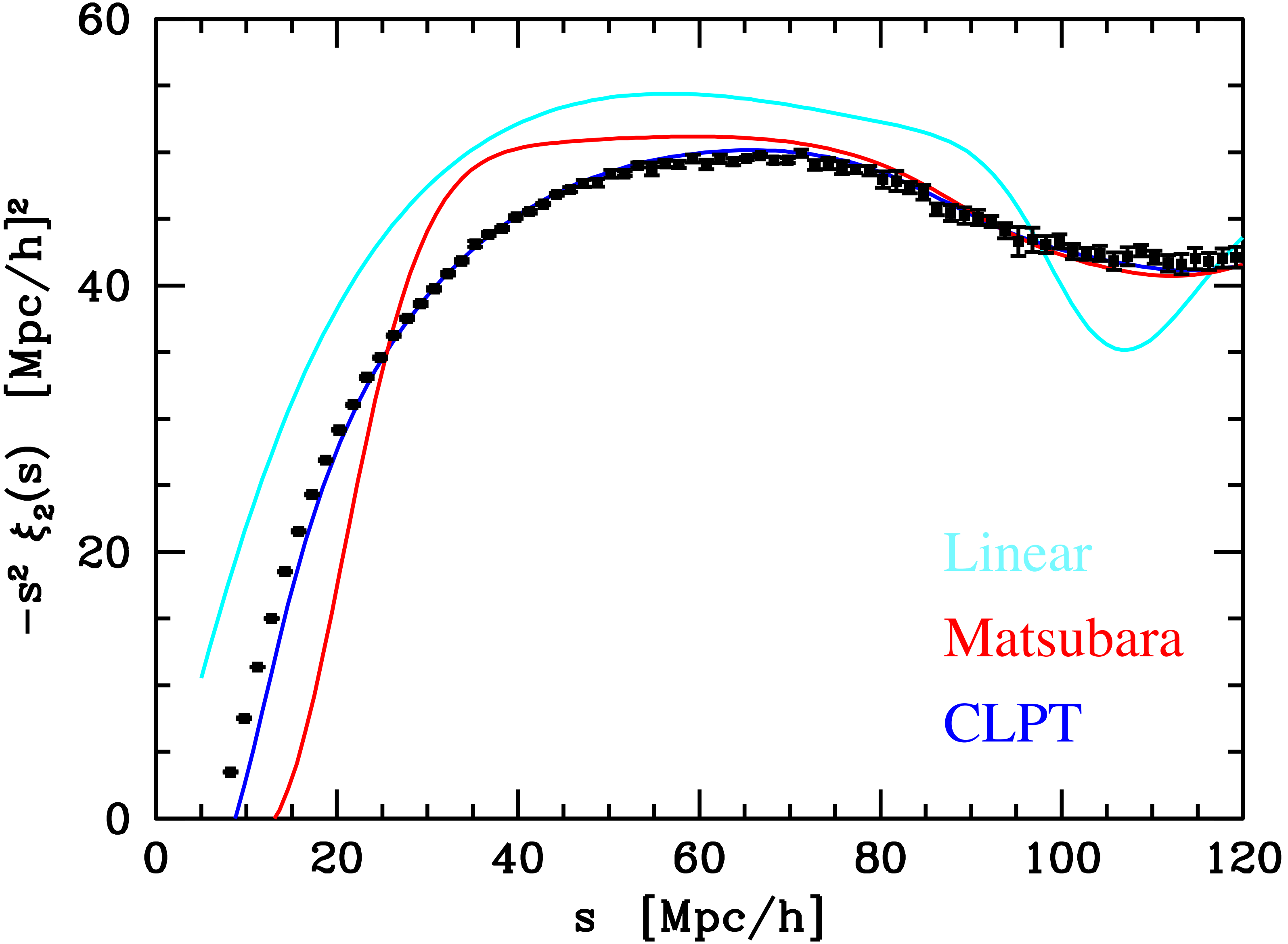
Matter: Real: Monopole



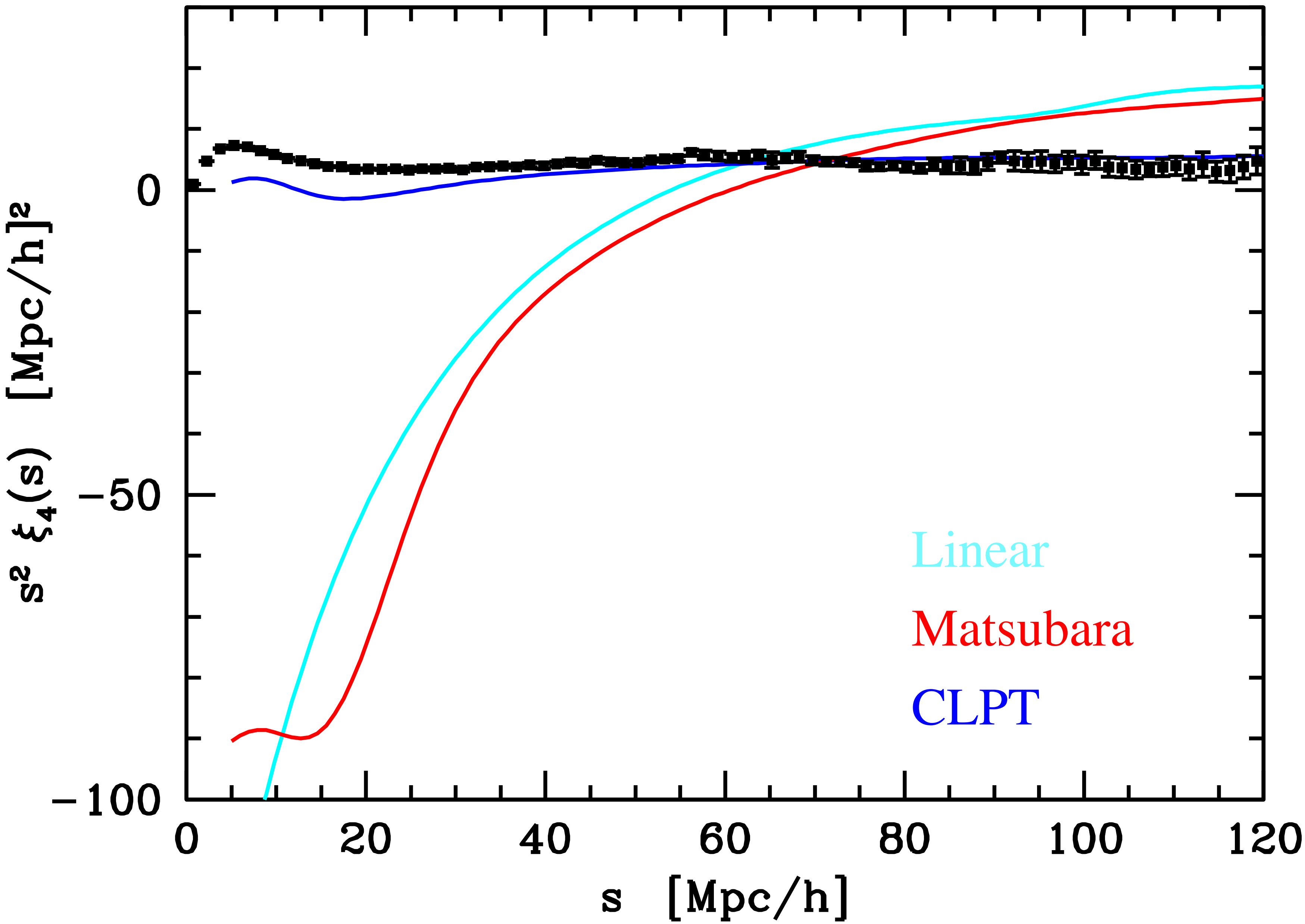
Matter: Red: Monopole



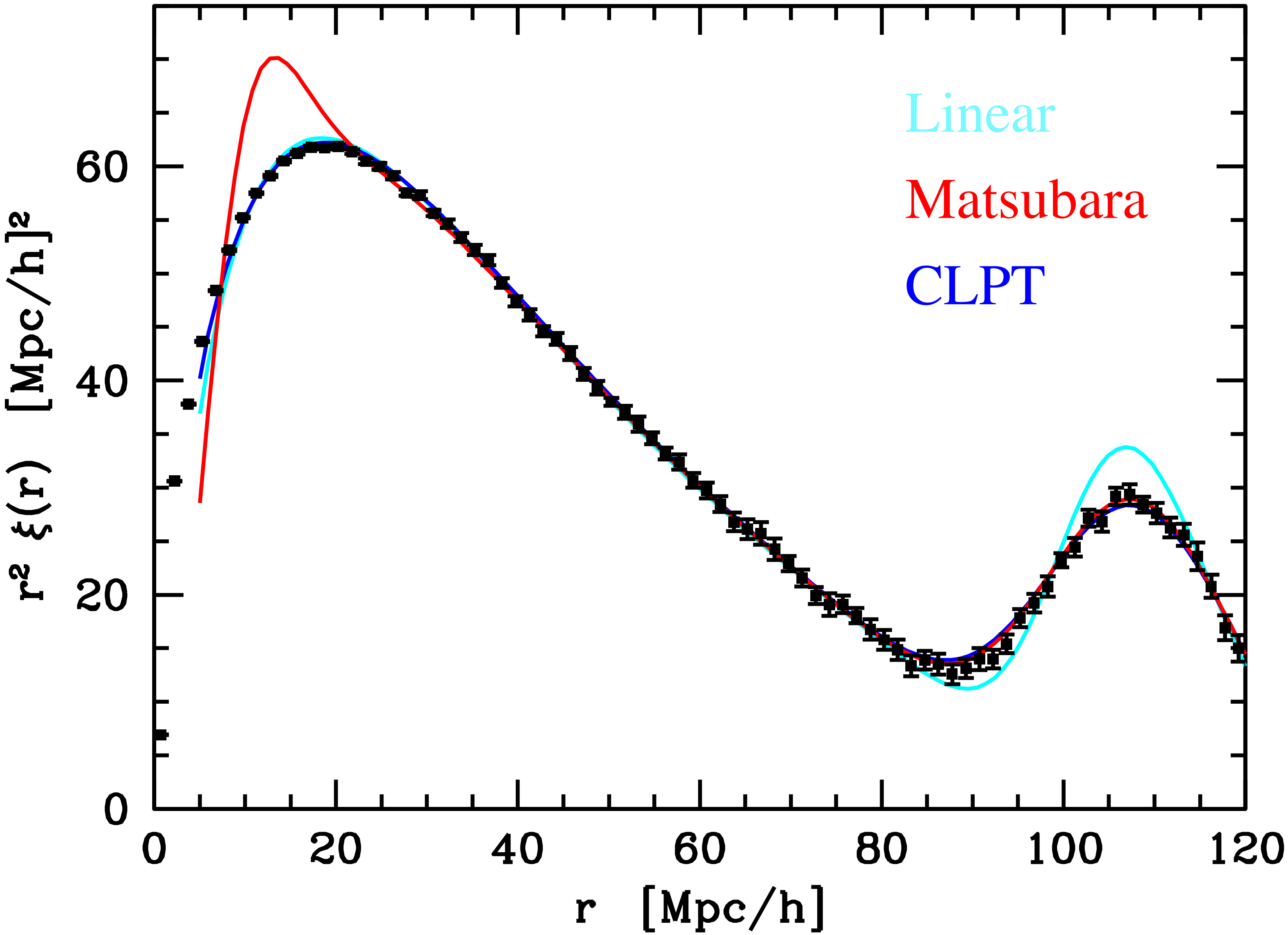
Matter: Quadrupole



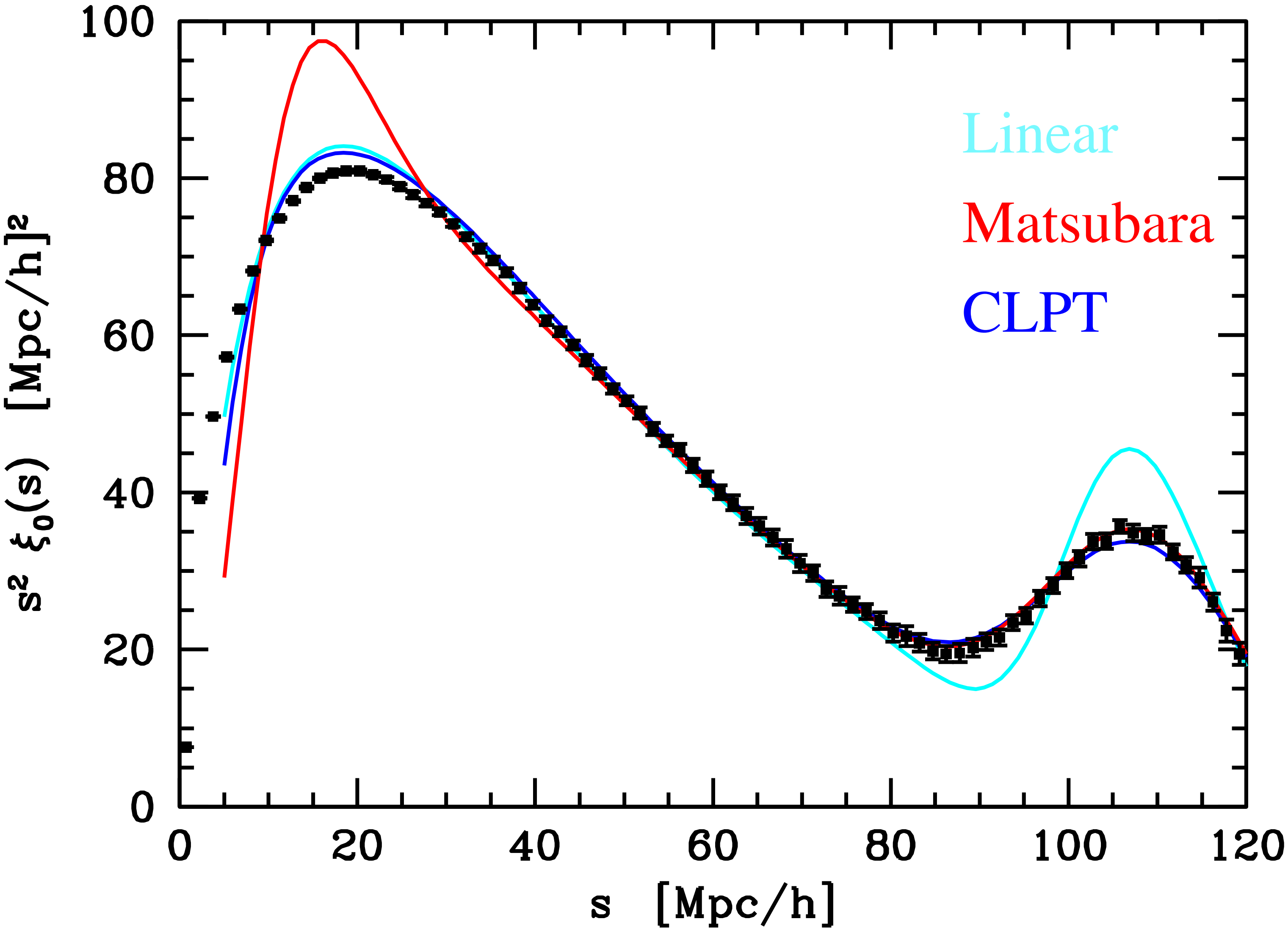
Matter: Hexadecapole



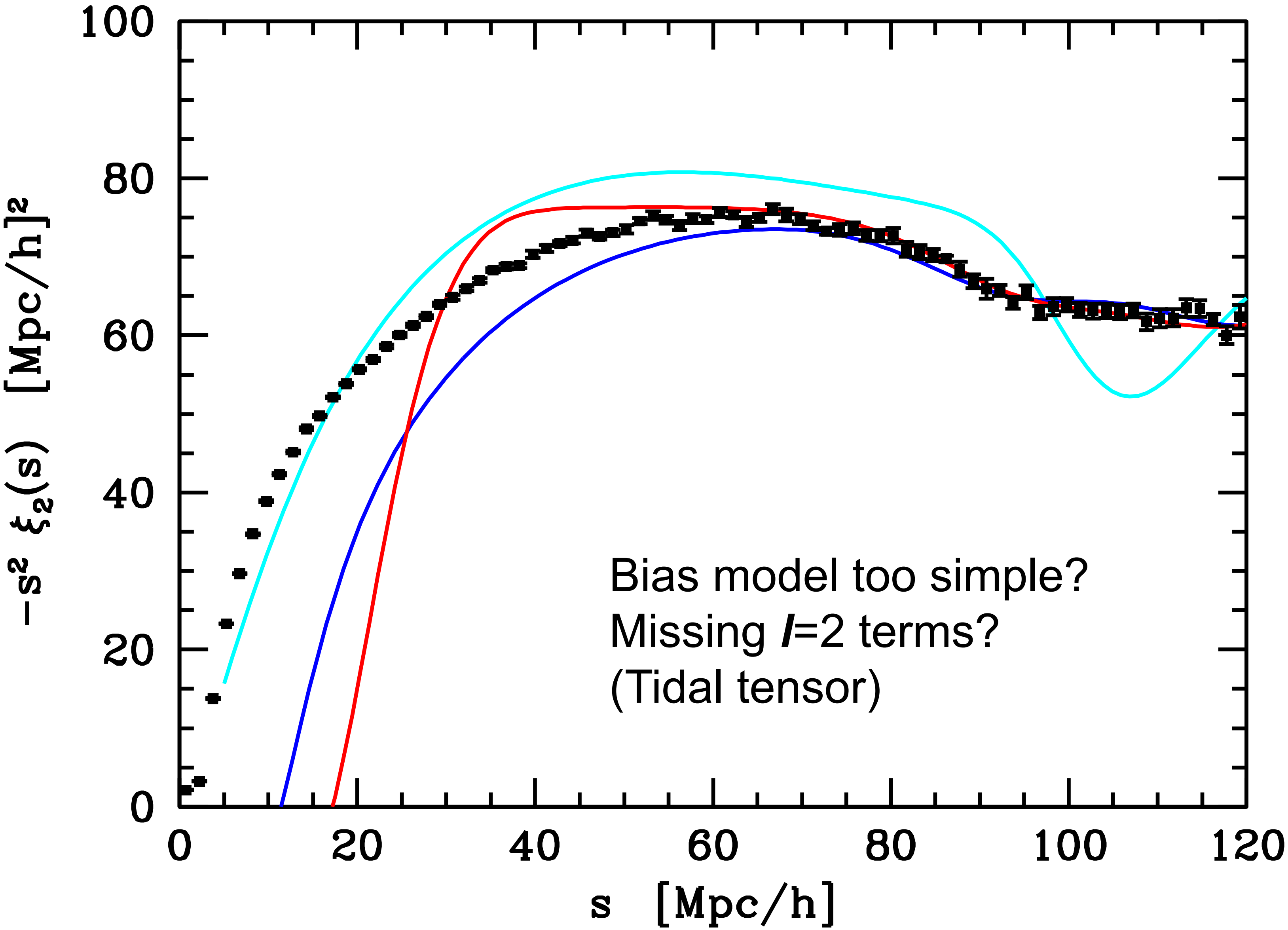
Halos: Real: Monopole



Halos: Red: Monopole



Halos: Quadrupole



A combination of approaches?

$$Z(r,J)=\int d^3q\int\frac{d^3k}{(2\pi)^3}\;e^{ik\cdot(q-r)}\int\frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}\;\tilde{F}(\lambda_1)\tilde{F}(\lambda_2)K(q,k,\lambda_1,\lambda_2,J)$$

$$K=\left\langle e^{i(\lambda_1\delta_1+\lambda_2\delta_2+k\cdot\Delta+J\cdot\dot{\Delta})}\right\rangle$$

$$1+\xi(r)=Z(r,J=0)\equiv Z_0(r),$$

$$v_{12,\alpha}(r)=\left.\frac{\partial Z}{\partial J_\alpha}\right|_{J=0}\equiv Z_{0,\alpha}(r),$$

$$D_{\alpha\beta}(r)=\left.\frac{\partial^2 Z}{\partial J_\alpha\partial J_\beta}\right|_{J=0}\equiv Z_{0,\alpha\beta}(r)$$

… plus streaming model ansatz.

From halos to galaxies

- In principle just another convolution
  - Intra-halo PDF.
- In practice need to model cs, $ss^{(1h)}$ and $ss^{(2h)}$.
- A difficult problem in principle, since have fingers-of-god mixing small and large scales.
  - Our model for ξ falls apart at small scales…
- On quasilinear scales things simplify drastically.
  - Classical FoG unimportant.
  - Remaining effect can be absorbed into a single Gaussian dispersion which can be marginalized over.

Conclusions

- Redshift space distortions arise in a number of contexts in cosmology.
  - Fundamental questions about structure formation.
  - Constraining cosmological parameters.
  - Testing the paradigm.
- Linear theory doesn't work very well.
- Two types of non-linearity.
  - Non-linear dynamics and non-linear maps.
- Bias dependence can be complex.
- We are developing a new formalism for handling the redshift space correlation function of biased tracers.
  - Stay tuned!

The End