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Deriving the scale-dependent bias from the integrated perturbation theory

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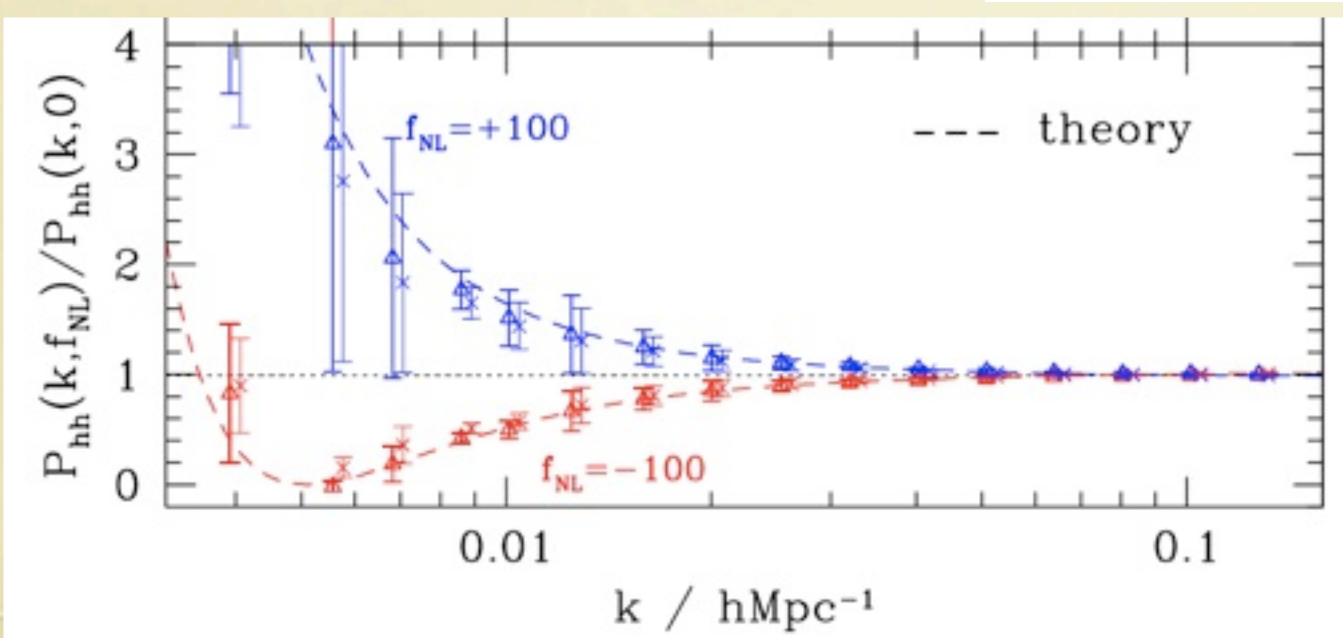
Bias and primordial non-Gaussianity

- Primordial non-Gaussianity
  - A way to observationally discriminates theories of the early universe
- Scale-dependent bias and the primordial non-Gaussianity
  - Strongly scale-dependent bias arises from local-type non-Gaussianity (Dalal et al. 2008; Matarrese & Verde 2008; Slosar et al. 2008)

$$\Phi(\mathbf{r}) = \Phi_L(\mathbf{r}) + f_{\rm NL}\left(\Phi_L^2(\mathbf{r}) - \langle\Phi_L^2(\mathbf{r})\rangle\right)$$

$$\Delta b(M,k) = 3 f_{\rm NL}(b-1)\delta_c \frac{\Omega_m}{k^2 T(k) D(z)}\left(\frac{H_0}{c}\right)^2$$



Desjacques, Seljak & Iliev (2009)

More general formulas

- There are several derivations for the scale-dependent bias with arbitrary non-Gaussianity
  - Halo bias + peak-background split
    - Dalal+ ’08; Slosar+ ’08; Schmidt & Kamionkowski ’10; Desjacques+ ’11, Scoccimarro+ ’11
  - High-peaks limit of threshold regions
    - Matarrese & Verde ’08; Verde & Matarrese ’09; Jeong & Komatsu ’09; Desjacques+ ’11
  - Local bias model
    - McDonald ’08; Taruya+ ’08

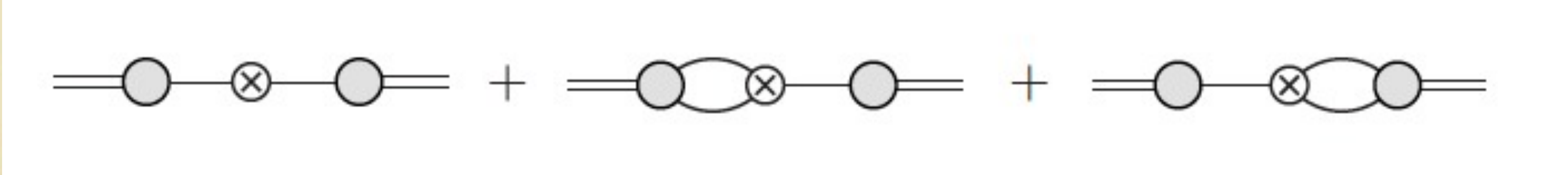
More general formula

- All the above derivations employ different levels of approximations
- Those approximations are not needed in the formalism of “integrated perturbation theory” (iPT) (TM 2011)
- Thus the most general formula of the scale-dependent bias is derived
  - Previous formulas are re-derived from the general formula by taking appropriate limits

integrated Perturbation Theory

- A cosmological perturbation theory in which the (nonlocal) bias is consistently included
  - details are omitted in this talk (see TM 2011 PRD 83 083518)
- Lowest-order power spectrum with nG:

$$
\begin{aligned}
P_{\mathrm{X}}(k) = {} & [\Gamma^{(1)}_{\mathrm{X}}(\boldsymbol{k})]^2 P_{\mathrm{L}}(k) \\
& + \Gamma^{(1)}_{\mathrm{X}}(\boldsymbol{k}) \int \frac{d^3k'}{(2\pi)^3} \Gamma^{(2)}_{\mathrm{X}}(\boldsymbol{k}', \boldsymbol{k} - \boldsymbol{k}') B_{\mathrm{L}}(k, k', |\boldsymbol{k} - \boldsymbol{k}'|) \\
& + \cdots, \qquad (2)
\end{aligned}
$$

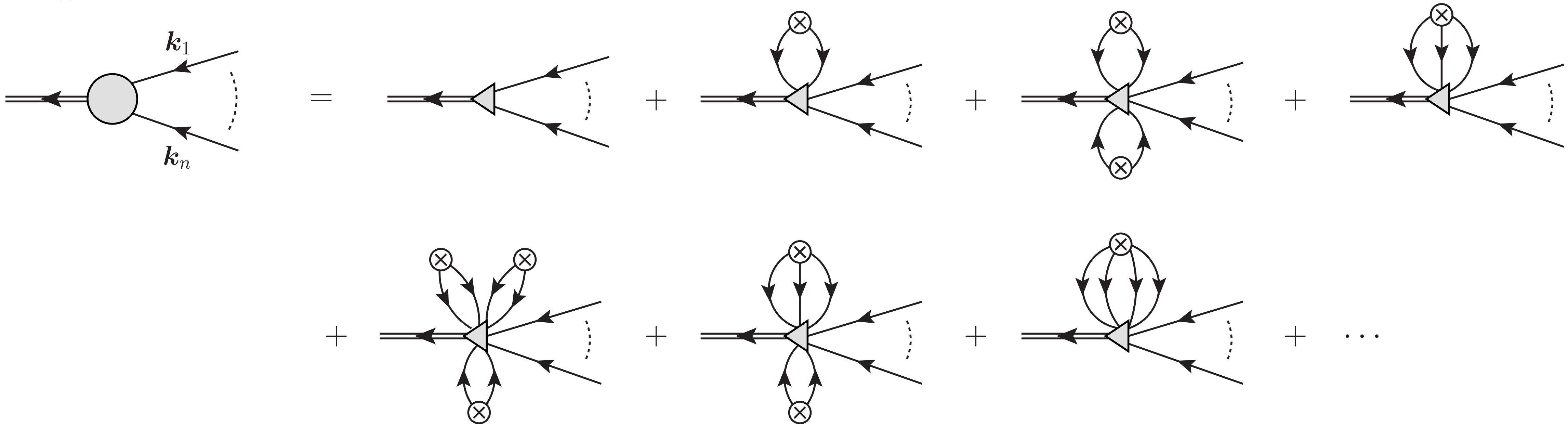


Multipoint propagator with (nonlocal) bias

- Multipoint propagator is an important concept in contemporary PT
  - Bernardeau+ '08,'10, TM '95
- Including (nonlocal) bias in mutipoint propagator (TM '11)

$$\left\langle \frac{\delta^n \delta_X(\boldsymbol{k})}{\delta\delta_L(\boldsymbol{k}_1)\cdots\delta\delta_L(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_D^3(\boldsymbol{k}-\boldsymbol{k}_{1\cdots n})\Gamma_X^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$

$\Gamma_X^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) =$



Renormalized bias functions

- Introduction of the "renormalized bias functions" is essential in iPT

$$b_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left.\frac{\delta^n\delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)}\right|_{\delta_{\mathrm{L}}=0}$$

$$\Rightarrow\ \underset{\downarrow}{c_n^{\mathrm{L}}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n}\int\frac{d^3k'}{(2\pi)^3}\left\langle\frac{\delta^n\boxed{\delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)}\right\rangle$$

"Renormalized bias functions" **Biased field in Lagrangian space**

$$\Gamma_{\mathrm{X}}^{(1)}(\boldsymbol{k}) = 1 + c_1^{\mathrm{L}}(k)$$

$$\begin{aligned}\Gamma_{\mathrm{X}}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) &= F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + \left(1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{{k_1}^2}\right)c_1^{\mathrm{L}}(k_2)\\ &\quad + \left(1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{{k_2}^2}\right)c_1^{\mathrm{L}}(k_1) + c_2^{\mathrm{L}}(\boldsymbol{k}_1,\boldsymbol{k}_2)\end{aligned}$$

(eg.lowest-order)

- dominant term on large scales:

$$\Delta b(k) \approx \frac{1}{2P_{\rm L}(k)} \int \frac{d^3k'}{(2\pi)^3} c_2^{\rm L}(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}') B_{\rm L}(k, k', \boldsymbol{k}-\boldsymbol{k}')$$

- scalings do not depend on details of bias

$$\Delta b^{\rm loc.} \propto k^{-2},$$
$$\Delta b^{\rm eql.} \propto k^{0},$$
$$\Delta b^{\rm fol.} \propto k^{-1},$$
$$\Delta b^{\rm ort.} \propto k^{-1}$$

- amplitudes depend on details of bias

Application to the Halo bias

- Global mass function (a simple model)

$$n(M)MdM = 2\bar{\rho}\left[P_{>\delta_c}(M) - P_{>\delta_c}(M+dM)\right] = -2\bar{\rho}\frac{dP_{>\delta_c}}{dM}dM$$

- Localized mass function

$$n(\boldsymbol{x}, M)MdM = 2\bar{\rho}\left\{\Theta[\delta_M(\boldsymbol{x}) - \delta_c] - \Theta[\delta_{M+dM}(\boldsymbol{x}) - \delta_c]\right\}$$
$$= -2\bar{\rho}\delta_D[\delta_M(\boldsymbol{x}) - \delta_c]\frac{d\delta_M(\boldsymbol{x})}{dM}dM$$

- Derived renormalized bias functions

$$c_n^L(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = \frac{(2\pi)^{3n}}{n(M)}\left\langle \frac{\delta^n n(\boldsymbol{x}=\boldsymbol{0}, M)}{\delta\delta_L(\boldsymbol{k}_1)\cdots\delta\delta_L(\boldsymbol{k}_n)}\right\rangle$$
$$= -\frac{2\bar{\rho}}{n(M)M}\left(-\frac{\partial}{\partial\delta_c}\right)^n \frac{\partial}{\partial M}\left[\underline{\langle\Theta(\delta_M - \delta_c)\rangle}W(k_1R)\cdots W(k_nR)\right]$$

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Renormalized bias functions

- As a result,

$$c_2^{\rm L}(\boldsymbol{k}_1,\boldsymbol{k}_2) = b_2^{\rm L} W(k_1R)W(k_2R) + \frac{\delta_{\rm c} b_1^{\rm L}+1}{\delta_{\rm c}^2}\frac{\partial}{\partial \ln\sigma_M}\left[W(k_1R)W(k_2R)\right]$$

$$= \frac{\delta_{\rm c}^2 b_2^{\rm L} + 2\delta_{\rm c} b_1^{\rm L}+1}{\delta_{\rm c}^2} W(k_1R)W(k_2R)$$

$$+ \frac{\delta_{\rm c} b_1^{\rm L}+1}{\delta_{\rm c}^2}\sigma_M^2 \frac{\partial}{\partial \ln\sigma_M}\left[\frac{W(k_1R)W(k_2R)}{\sigma_M^2}\right]$$

bias nonlocality

- In the case of PS mass fn.: $b_1^{\rm L} = \dfrac{\nu^2-1}{\delta_{\rm c}}, \quad b_2^{\rm L} = \dfrac{\nu^4-3\nu^2}{\delta_{\rm c}^2}$

$(\nu \equiv \delta_{\rm c}/\sigma_M)$

- Cancellation of b2 in c2

$$c_2^{\rm L}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{\delta_{\rm c} b_1^{\rm L}}{\sigma_M^2} W(k_1R)W(k_2R) + \frac{\partial}{\partial \ln\sigma_M}\left[\frac{W(k_1R)W(k_2R)}{\sigma_M^2}\right]$$

Notable findings

- No need to introduce auxiliary Gaussian field (c.f., issue of kernel choice in PBS)
- PBS results are reproduced when PS mass function is assumed

$$\Delta b(k) \approx \frac{1}{2}\delta_c b_1^L \mathcal{I}(k) + \frac{1}{2}\frac{\partial \mathcal{I}(k)}{\partial \ln \sigma_M}.$$

  - (Exactly matches with the results of Desjacques+ '11)

$$\mathcal{I}(k) \equiv \frac{\mathcal{I}_2(k)}{\sigma_M{}^2 P_L(k)} \approx \frac{1}{\sigma_M{}^2 P_L(k)} \int \frac{d^3k'}{(2\pi)^3} W^2(k'R) B_L(k, k', |\boldsymbol{k} - \boldsymbol{k}'|).$$

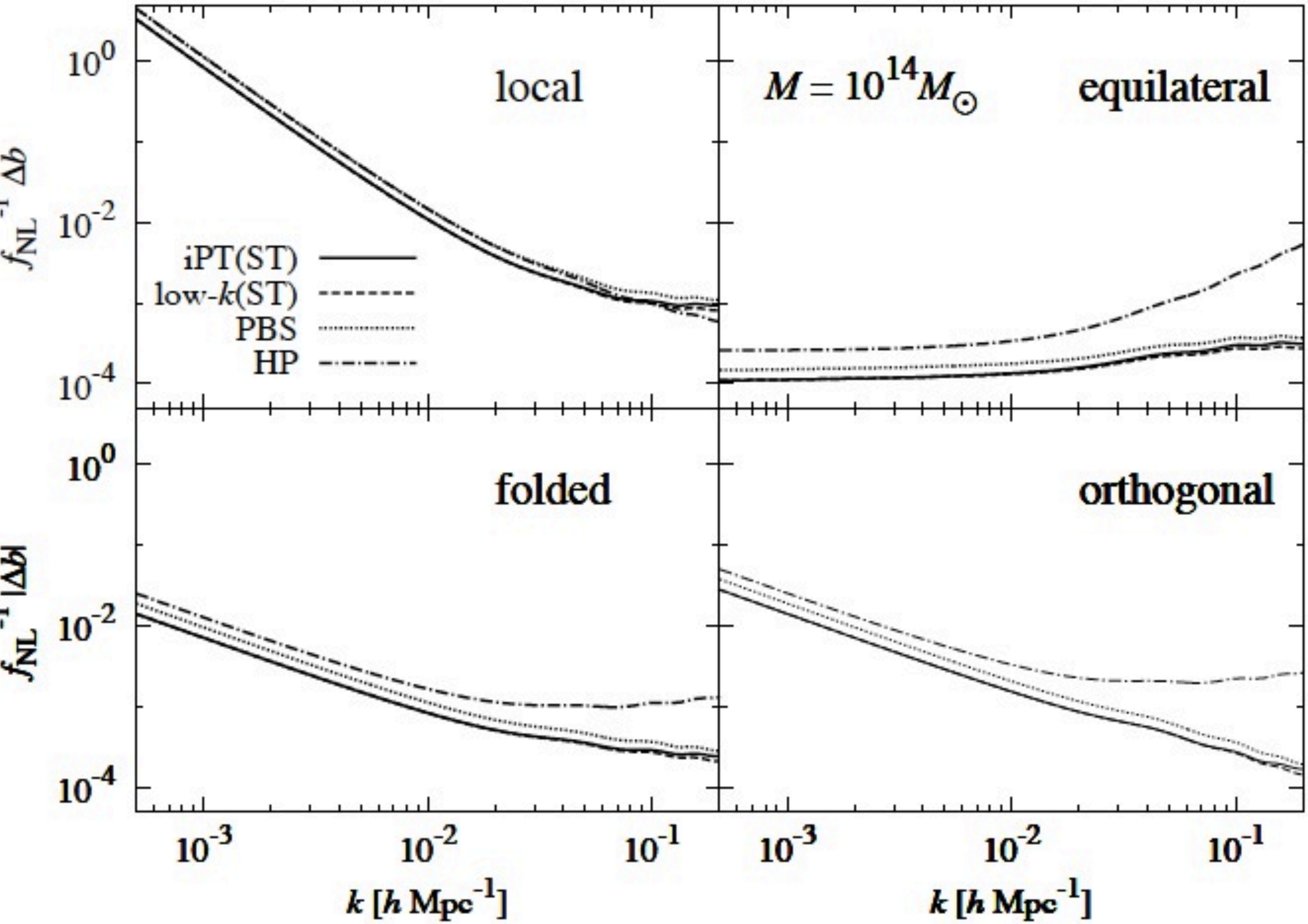
New general formula

- When the mass function is arbitrary, we have a new formula

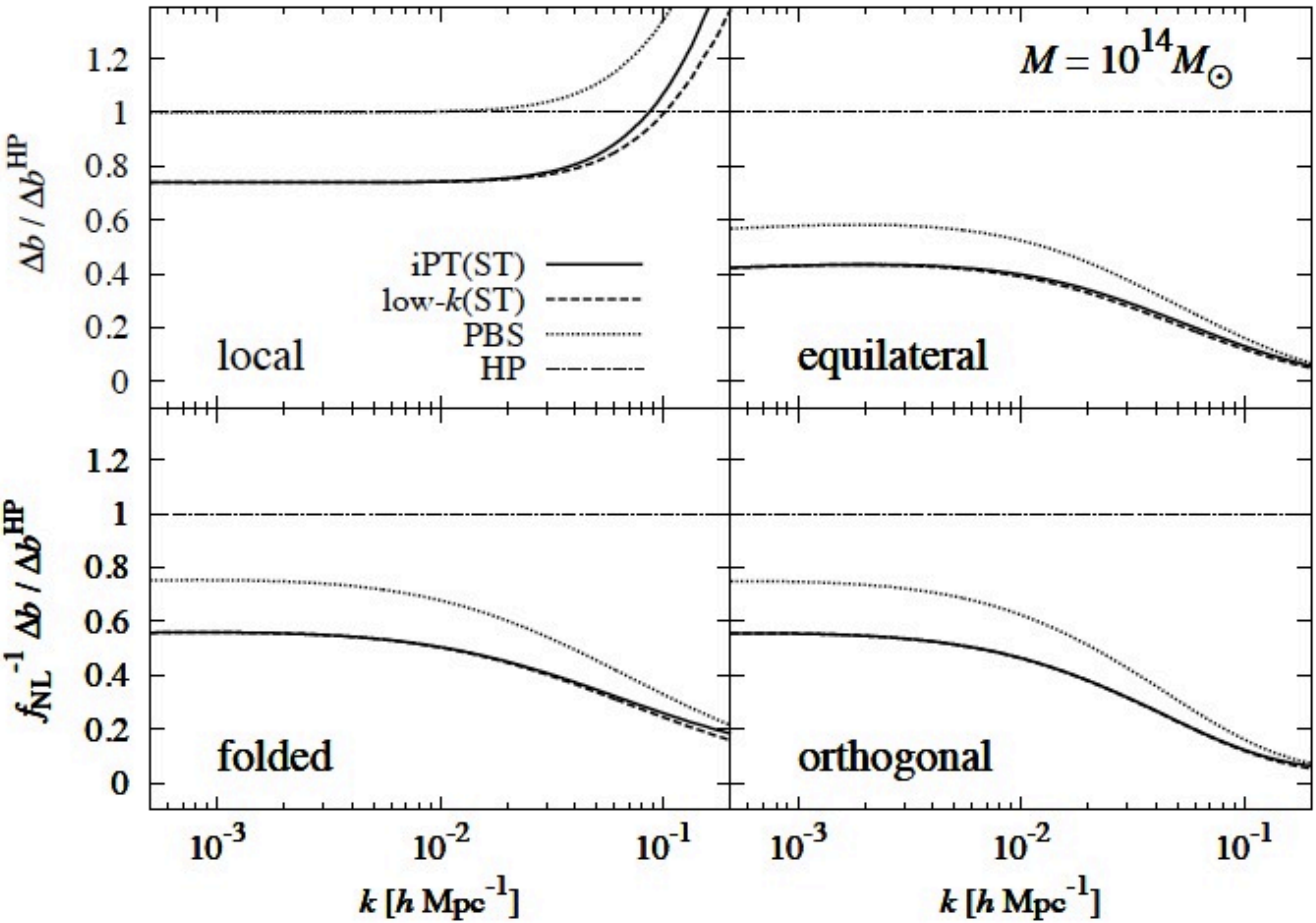
$$\Delta b(k) \approx \frac{{\sigma_M}^2}{2{\delta_c}^2}\left[\left(2 + 2\delta_c b_1^{\mathrm{L}} + {\delta_c}^2 b_2^{\mathrm{L}}\right) \mathcal{I}(k) + \left(1 + \delta_c b_1^{\mathrm{L}}\right) \frac{d\mathcal{I}(k)}{d\ln\sigma_M}\right].$$

  - E.g., Sheth-Tormen mass function:

$$\Delta b(k) \approx \left[\frac{q\delta_c b_1^{\mathrm{L}}}{2} + \frac{1}{\nu^2}\frac{p(q\nu^2 + 2p + 1)}{1 + (q\nu^2)^p}\right]\mathcal{I}(k) + \left[\frac{q}{2} + \frac{1}{\nu^2}\frac{p}{1 + (q\nu^2)^p}\right]\frac{d\mathcal{I}(k)}{d\ln\sigma_M}.$$



local
$M = 10^{14} M_\odot$
equilateral
folded
orthogonal
iPT(ST)
low-k(ST)
PBS
HP
$f_{NL}^{-1}\,\Delta b$
$f_{NL}^{-1}\,|\Delta b|$
10^{0}
10^{-2}
10^{-4}
10^{-3}
10^{-1}
k [h Mpc^{-1}]



$M = 10^{14} M_\odot$
local
equilateral
folded
orthogonal
iPT(ST)
low-k(ST)
PBS
HP
$\Delta b / \Delta b^{\mathrm{HP}}$
$f_{\mathrm{NL}}^{-1}\ \Delta b / \Delta b^{\mathrm{HP}}$
k [h Mpc^{-1}]
k [h Mpc^{-1}]

Redshift-space distortions

- Redshift-space distortions are straightforwardly included in iPT

$$\Delta p_0(k) \approx \left(\frac{f}{3} + b_1\right) Q_2(k), \quad \Delta p_2(k) \approx \frac{2f}{3} Q_2(k),$$

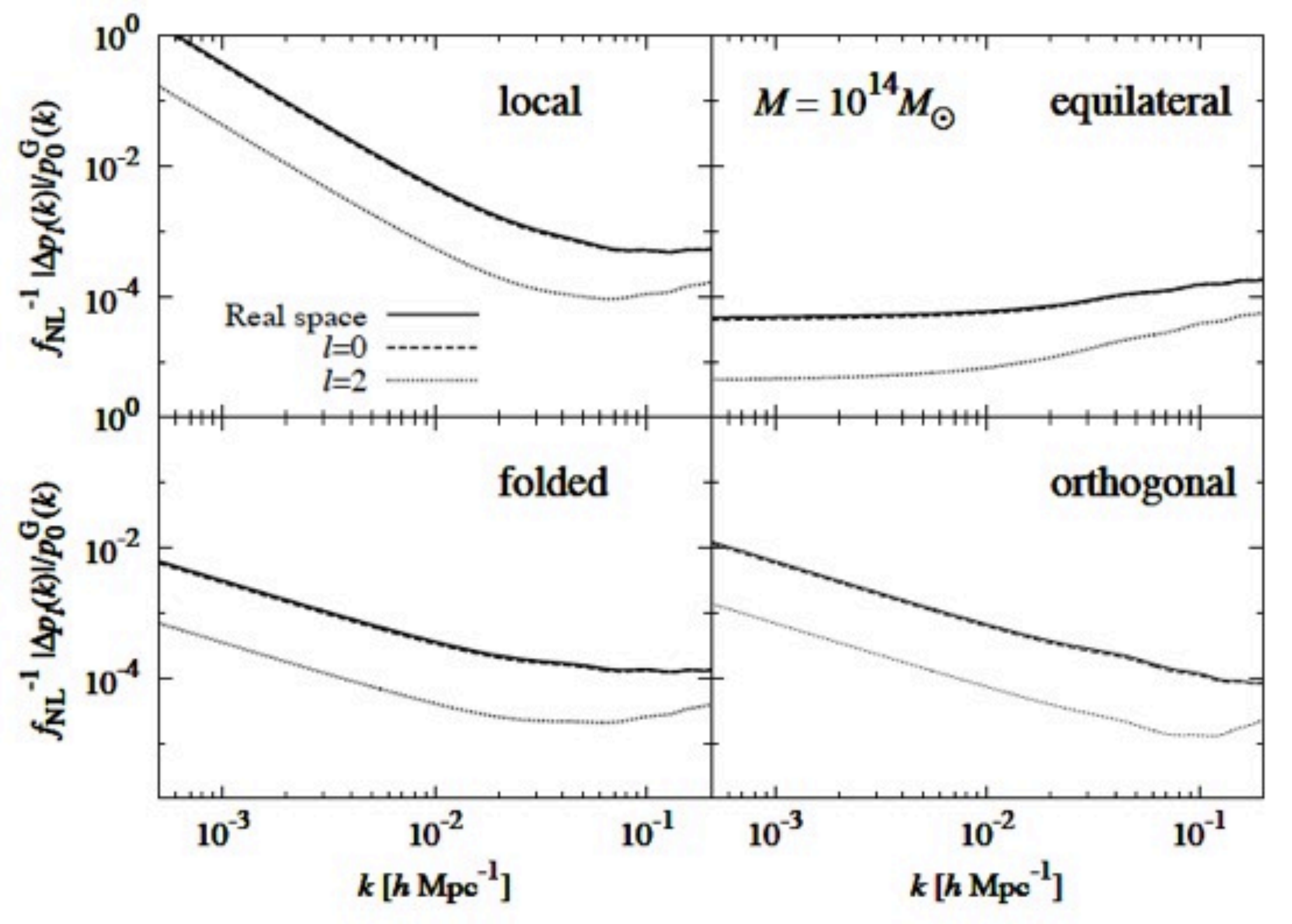
$$\Delta p_4(k), \Delta p_6(k) \ll \Delta p_0(k), \Delta p_2(k),$$

$$\begin{aligned}\Delta p_0(k) = {} & 2\left[\frac{f}{3} + \frac{2f^2}{5} + \frac{f^3}{7} + \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right) b_1(k)\right] R_1(k) \\ & - 2\left[\frac{2f}{21} + \frac{4f^2}{35} + \frac{3f^3}{35} + \left(\frac{2}{7} + \frac{4f}{21} + \frac{2f^2}{15}\right) b_1(k)\right] R_2(k) \\ & + \left[\frac{f}{3} + \frac{f^2}{5} + \left(1 + \frac{f}{3}\right) b_1(k)\right] Q_1(k) \\ & + \left[\frac{f}{3} + b_1(k)\right] Q_2(k),\end{aligned} \tag{147}$$

$$\begin{aligned}\Delta p_2(k) = {} & 4f\left[\frac{1}{3} + \frac{4f}{7} + \frac{5f^2}{21} + \left(\frac{2}{3} + \frac{2f}{7}\right) b_1(k)\right] R_1(k) \\ & - f\left[\frac{8}{21} + \frac{32f}{49} + \frac{11f^2}{21} + \left(\frac{16}{21} + \frac{13f}{21}\right) b_1(k)\right] R_2(k) \\ & + 2f\left[\frac{1}{3} + \frac{2f}{7} + \frac{1}{3} b_1(k)\right] Q_1(k) + \frac{2f}{3} Q_2(k),\end{aligned} \tag{148}$$

$$\begin{aligned}\Delta p_4(k) = {} & 16f^2\left[\frac{2}{35} + \frac{3f}{77} + \frac{1}{35} b_1(k)\right] R_1(k) \\ & - \frac{4f^2}{35}\left[\frac{16}{7} + \frac{26f}{11} + b_1(k)\right] R_2(k) + \frac{8f^2}{35} Q_1(k),\end{aligned} \tag{149}$$

$$\Delta p_6(k) = \frac{32f^3}{231} R_1(k) - \frac{8f^3}{231} R_2(k). \tag{150}$$



Summary

- Applying iPT, the general formula of the scale-dependent bias is derived
  - Previous formulas are re-derived from the new formula by taking appropriate limits
- Formula in redshift space