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Memory matters: random walks to galaxies

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Memory matters: random walks to galaxies

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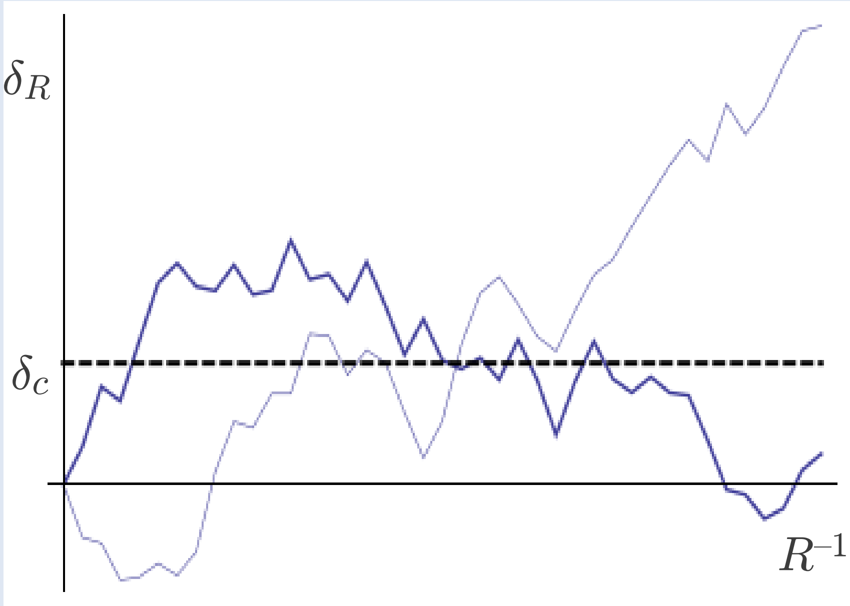
ICTP, July 2012

Excursion set theory

- Halos evolve from “dense enough” patches in the initial matter density distribution
- Mass of each halo is proportional to the volume of its “dense enough” initial patch

$$\delta_R(x) \equiv \frac{1}{V} \int \mathrm{d}^3y\, W\left(\frac{y-x}{R}\right) \delta_{lin}(y)$$

$$s(R) = \langle \delta_R(x) \rangle$$



First crossing distribution

- Abundance $n(M) \longleftrightarrow$ absorption rate of walks at scale $s(R)$

$$f(s) = \frac{\mathrm{d}}{\mathrm{d}s}\int_{B(s)}^{+\infty}\mathrm{d}\delta\, p(\delta; s)$$

Press & Schechter (1974)

- Not any crossing, but FIRST crossing (cloud-in-cloud problem)

Bond et al. (1991)

- Path integral of trajectories with N discrete steps of width ΔS :

$$f(s) = \frac{1}{\Delta S}\int_{-\infty}^{B}\mathrm{d}\delta_1 \cdots \mathrm{d}\delta_{N-1}\int_{B}^{+\infty}\mathrm{d}\delta_N\, p(\delta_1, \ldots, \delta_N)$$

cf. Maggiore & Riotto (2010)

- Strongly correlated walks are less affected by cloud-in-cloud

Paranjape, Lam & Sheth (2011)

First crossing distribution

- Only the last two steps are really important:

$$f(s) \simeq \int_{B'(s)}^{+\infty} \mathrm{d}v\,(v - B')\,p(v, B) \qquad \left[v = \frac{\mathrm{d}\delta}{\mathrm{d}s}\right]$$

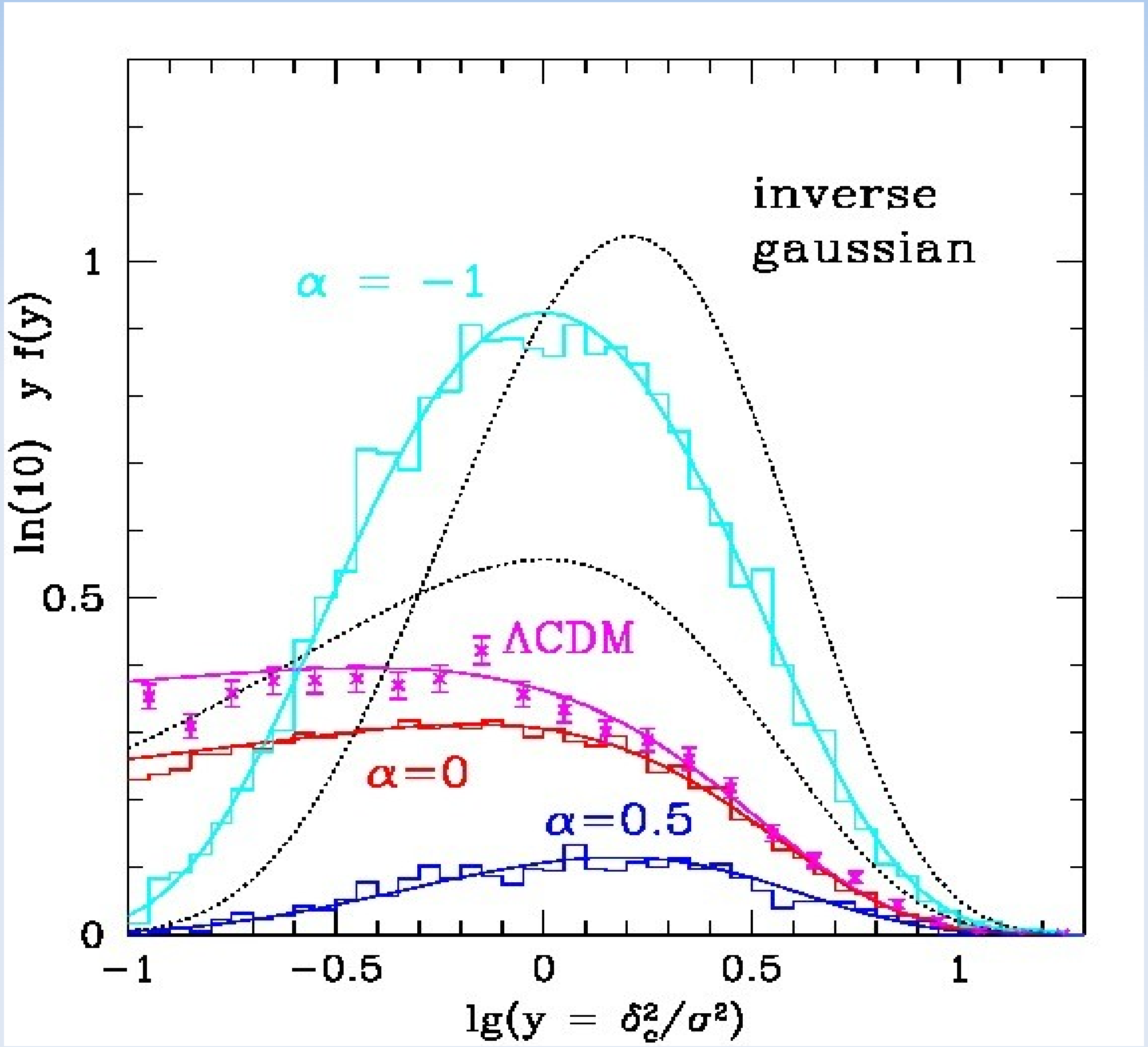
MM & Sheth (2012)

- Correlated Gaussian walks (Gaussian initial conditions and any smooth filter)

$$f(s) = -\beta' \frac{\mathrm{e}^{-\beta^2/2}}{\sqrt{2\pi}} \left[\frac{1 + \mathrm{erf}(\nu_*/\sqrt{2})}{2} + \frac{\mathrm{e}^{-\nu_*^2/2}}{\sqrt{2\pi}\nu_*}\right]$$

$$\beta = B(s)/\sqrt{s} \quad ; \quad \nu_*^2 = \beta'^2/\left\langle(\delta/\sqrt{s})'^2\right\rangle$$

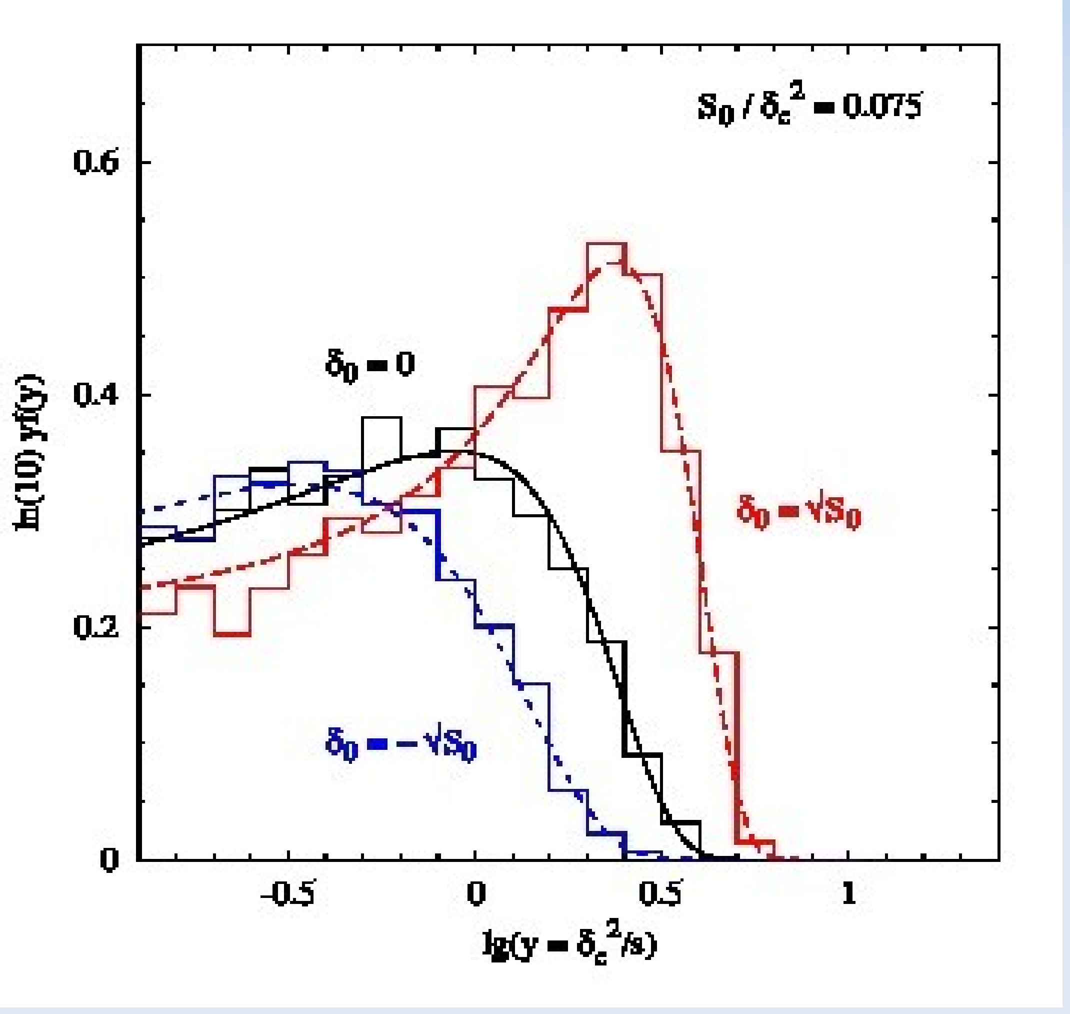
First crossing distribution



MM & Sheth (2012)

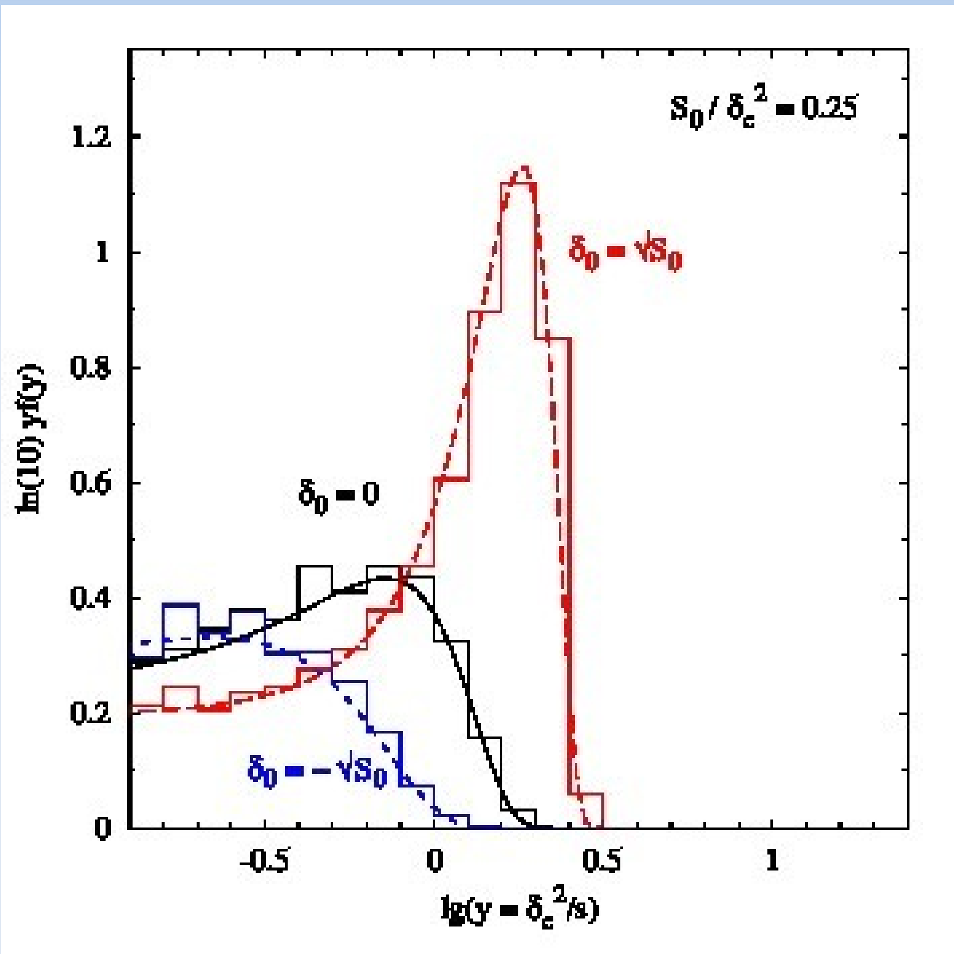
Conditional FC distribution

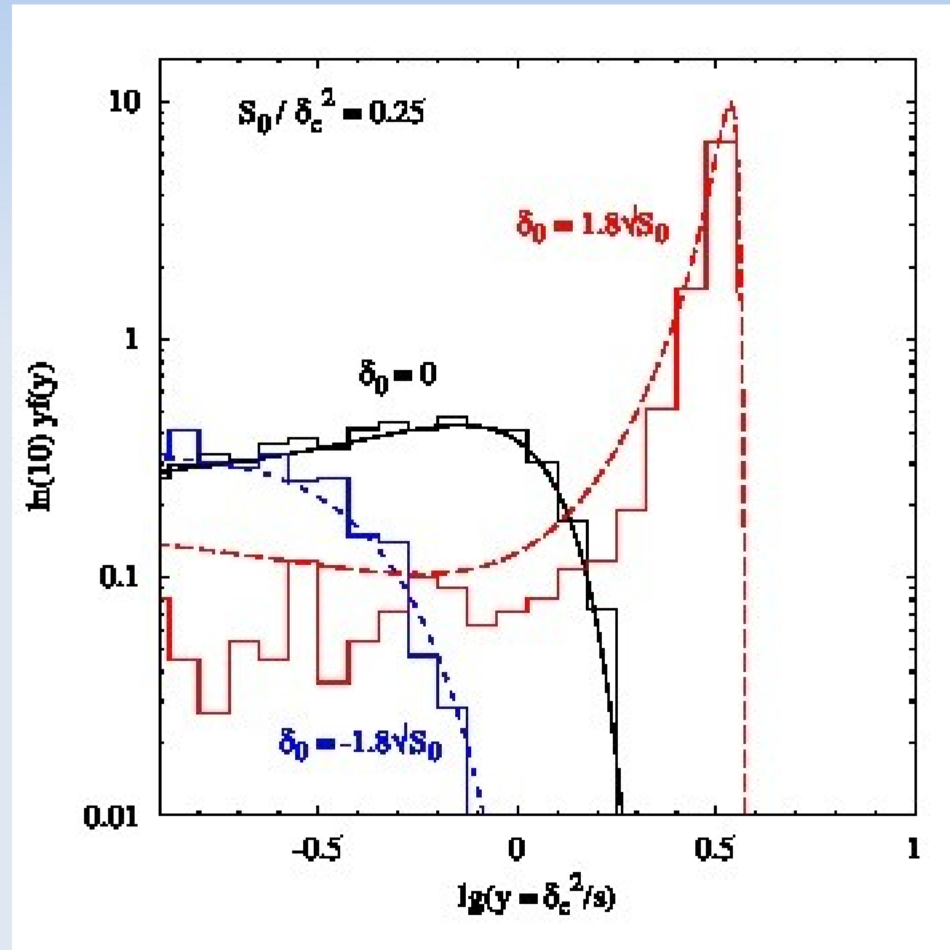
- Same formalism for walks that went through δ_0 at $s(R_0) = s_0$



MM, Paranjape & Sheth (2012)

Conditional FC distribution





MM, Paranjape & Sheth (2012)

Halo Bias

- Suppose the most generic dependence on the matter field:

$$\delta_h(\mathbf{x}) = \sum_{k=1}^{\infty} \frac{1}{k!} \int \mathrm{d}^3 y_1 \ldots \mathrm{d}^3 y_k \, b_k(\mathbf{x}-\mathbf{y}_1, \ldots, \mathbf{x}-\mathbf{y}_k) \delta(\mathbf{y}_1) \ldots \delta(\mathbf{y}_k)$$

e.g. Matsubara (2011)

- Can compute connected correlation functions

$$\langle \delta_h(\mathbf{x}) \delta_0(\mathbf{z}_1) \ldots \delta_0(\mathbf{z}_n) \rangle_c$$

$$= \int \mathrm{d}^3\mathbf{x}_1 \ldots \mathrm{d}^3\mathbf{x}_n \underbrace{\left\langle \frac{\delta^n \delta_h(\mathbf{x})}{\delta\delta(\mathbf{x}_1) \ldots \delta\delta(\mathbf{x}_n)} \right\rangle}_{\text{Bias functions}} \prod_{j=1}^{n} \langle \delta(\mathbf{x}_j) \delta_0(\mathbf{z}_j) \rangle$$

Halo Bias from Excursion Sets

Define the halo random density field:

$$1 + \delta_h(x) = \frac{1}{f(s)\Delta s}\vartheta(B - \delta_1(x)) \cdots \vartheta(B - \delta_{N-1}(x))\,\vartheta(\delta_N(x) - B)$$

$$\langle \delta_h \delta_0^n \rangle_c = \sum_{i_1,\ldots,i_n=1}^{N} \left\langle \frac{\partial^n \delta_h}{\partial \delta_{i_1} \cdots \partial \delta_{i_n}} \right\rangle \langle \delta_{i_1} \delta_0 \rangle \cdots \langle \delta_{i_n} \delta_0 \rangle$$

Halo Bias from Excursion Sets

Two-step approx: $1+\delta_h \simeq \frac{\vartheta(B-\delta_{N-1})\vartheta(\delta_N - B)}{f(s)\Delta s}$

$$\langle \delta_g \delta_0^n \rangle_c = \frac{(-1)^n}{f(s)} \int_0^\infty \mathrm{d}v\, v \left(\langle \delta\delta_0 \rangle \frac{\partial}{\partial B} + \langle \delta'\delta_0 \rangle \frac{\partial}{\partial v} \right)^n p(B,v)$$

MM, Paranjape & Sheth (2012)

For instance:

$$\langle \delta_g \delta_0 \rangle = \langle \delta\delta_0 \rangle\, b_{10}^{(f)} + \langle \delta'\delta_0 \rangle\, 2s\, b_{11}^{(f)}$$

$$\left\langle \delta_g \delta_0^2 \right\rangle_c = \langle \delta\delta_0 \rangle^2\, b_{20}^{(f)} + \langle \delta\delta_0 \rangle \langle \delta'\delta_0 \rangle\, 4s\, b_{21}^{(f)} + \langle \delta'\delta_0 \rangle^2\, 4s^2 b_{22}^{(f)}$$

Bias coefficients

- The highest coefficient looks the same:

$$b_{n0} = \frac{1}{f(s)}\left(-\frac{\partial}{\partial B}\right)^n f(s)$$

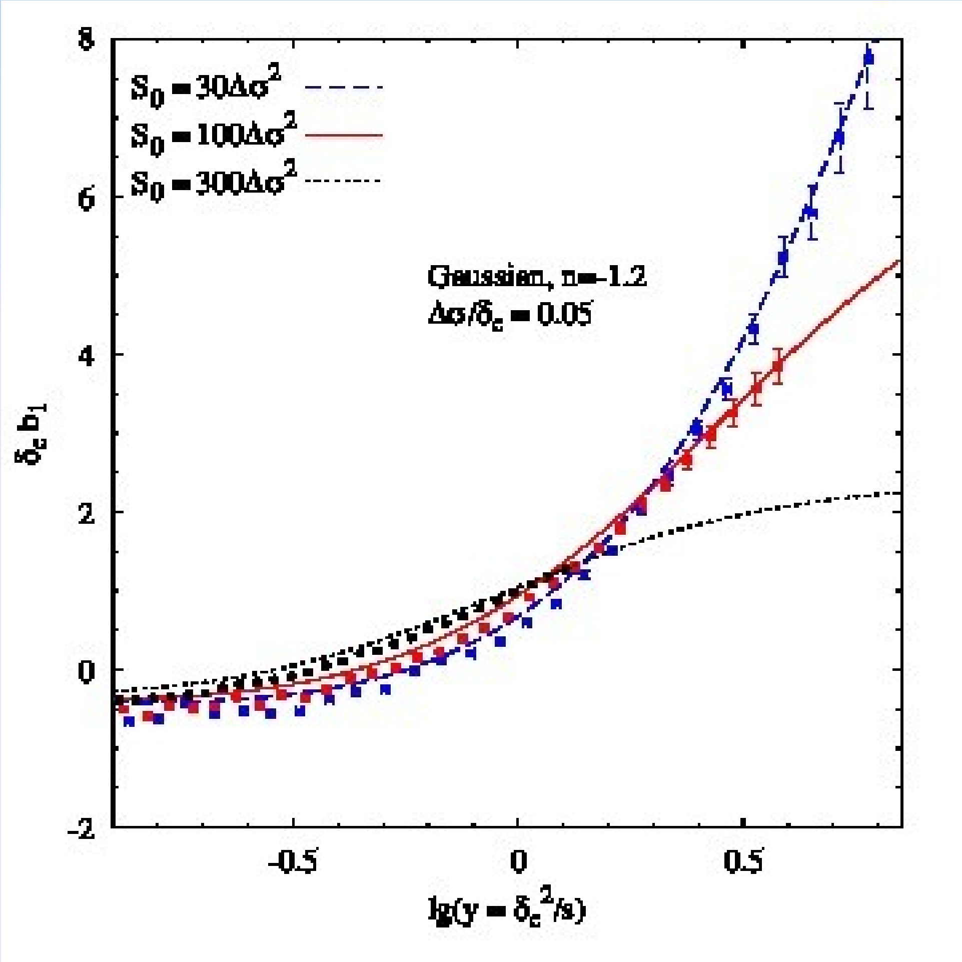
but now $f(s)$ is the result for correlated walks

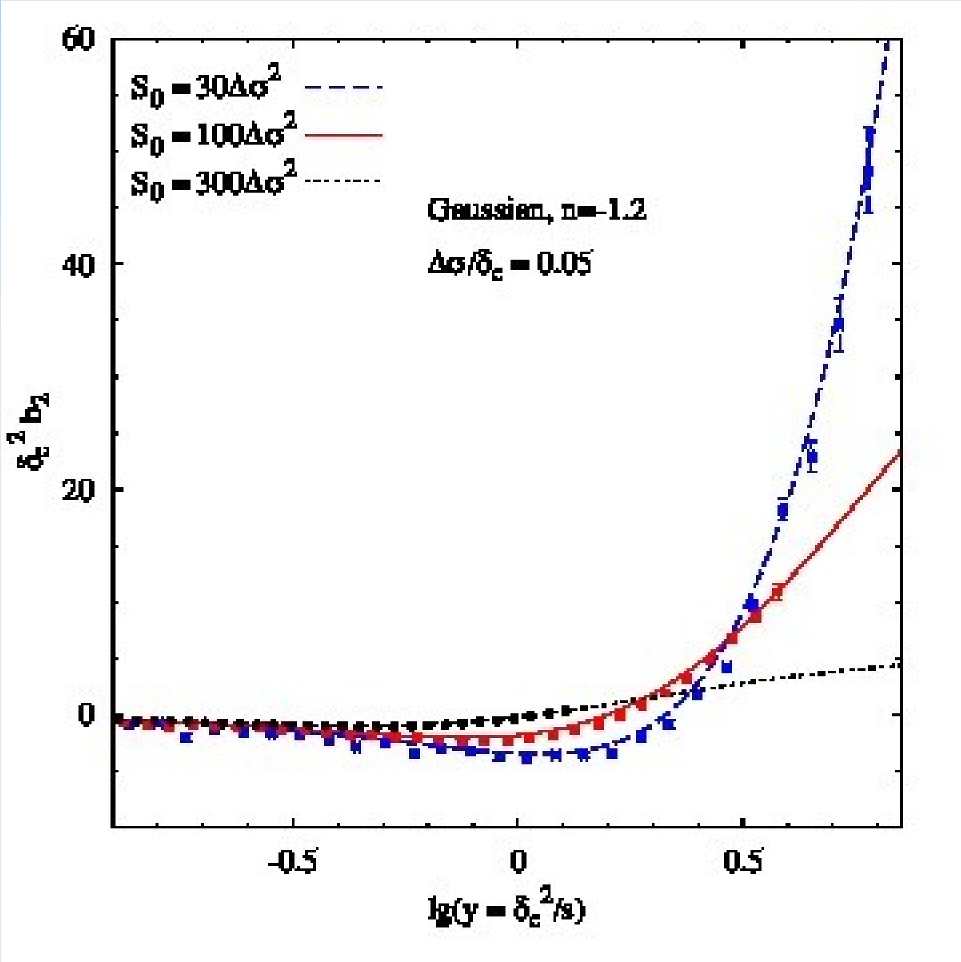
- Moreover, there are non-trivial relations among the coefficients:

$$b_{11}B = \nu^2 - b_{10}B$$
$$b_{21}B^2 = \nu^2(b_{10}B - 1) - b_{20}B^2$$
$$b_{22}B^2 = b_{20}B^2 + \nu^2(\nu^2 - 2b_{10}B + 1)$$

same relations as from peak theory (see Desjacques et al 2010)

Bias coefficients





MM, Paranjape & Sheth (2012)

Fourier Space Bias

- Linear bias:

$$b_1(k) = b_{10}^{(f)} \underbrace{W(kR)}_{\sim 1} + b_{11}^{(f)} \underbrace{2sW'(kR)}_{\sim k^2 R^2}$$

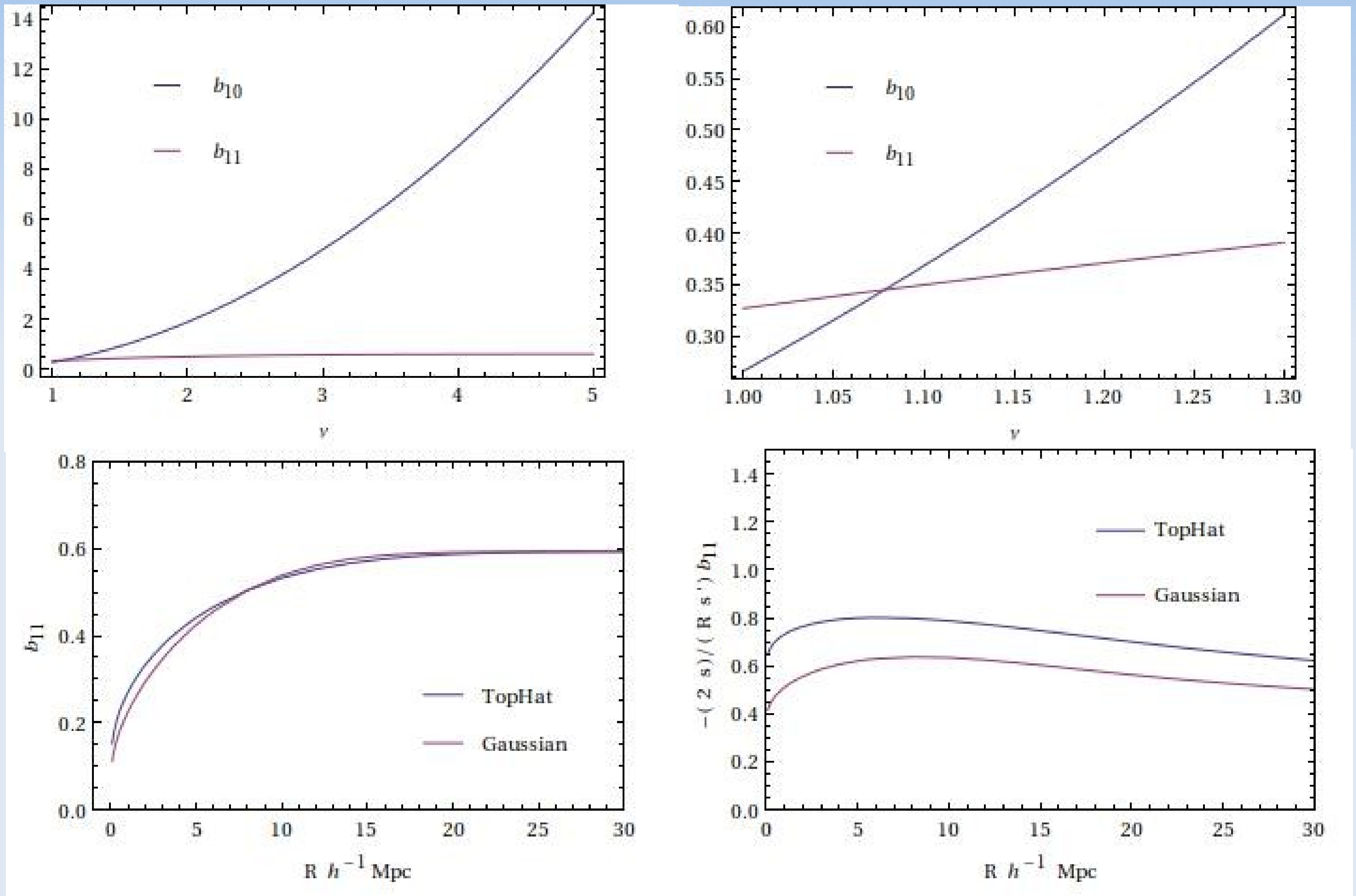
- Quadratic bias:

$$b_2(k_1, k_2) \sim b_{20}^{(f)} + b_{21}^{(f)} (k_1^2 + k_2^2) R^2 + b_{22}^{(f)} k_1^2 \, k_2^2 R^4$$

Gaussian bias is *k*-dependent too!

MM, Paranjape & Sheth (2012)

Fourier Space Bias



Adding non-Gaussianity: MF

Same formalism applies to non-Gaussian initial conditions

$$f(s) = \left[\frac{\mathrm{d}}{\mathrm{d}s}\int_{B(s)}^{\infty}\mathrm{d}\delta\, p(\delta;s)\right]\frac{1+\mathrm{erf}(\nu_*/\sqrt{2})}{2} + p(B;s)\left[\frac{e^{-\nu_*^2/2}}{2\Gamma\sqrt{2\pi s}} + \dots\right]$$

Full NG distribution

same Gaussian factors

Same dependence of f on $p(\delta;s)$ as in the Gaussian case

Adding non-Gaussianity: Bias

- Generic excursion set bias for non-Gaussian walks:

$$\langle\delta_h\delta_0\rangle = \sum_{i=1}^{N}\left\langle\frac{\partial\delta_h}{\partial\delta_i}\right\rangle\langle\delta_i\delta_0\rangle + \frac{1}{2}\sum_{i,j=1}^{N}\left\langle\frac{\partial^2\delta_h}{\partial\delta_i\partial\delta_j}\right\rangle\langle\delta_i\delta_j\delta_0\rangle + \dots$$

- With the two-step approximation:

$$\langle\delta_h\delta_0\rangle \simeq b_{10}^{(f)}\langle\delta\delta_0\rangle + b_{11}^{(f)}\langle\delta'\delta_0\rangle + \frac{1}{2}\left[b_{20}^{(f)}\langle\delta^2\delta_0\rangle + 2\,b_{21}^{(f)}\langle\delta'\delta\delta_0\rangle + b_{22}^{(f)}\langle\delta'^2\delta_0\rangle\right] + \dots$$

Same dependence of b_{ij} on f and $p(\delta)$

⇨ $$\Delta b_1(k) = \frac{2}{k^2 T(k)}\left[s\,b_{20} + b_{21} + \left\langle(\delta'_N)^2\right\rangle b_{22} + \mathcal{O}(k^2)\right]$$

Conclusions

- Accurate approximation of correlated random walks
- Systematic treatment of corrections
- (First?) satisfactory mathematical understanding of the excursion set approach to structure formation
- Straightforward non-perturbative inclusion of NG
- Predictions for bias coefficients, new strategies to measure them in simulations
- To do: check against N-body simulations

  generalisation to excursion set theory of peaks
- Other applications: virus diffusion, stock market, ...

THANKS!