The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA
International Atomic Energy Agency

2419-6

Workshop on Large Scale Structure

30 July - 2 August, 2012

New results and insights into the dynamics of Gravitational Instabilities

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ICTP, LSS workshop, July 2012

New results and insights into the dynamics of Gravitational Instabilities

On-going collaborations with

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T. Nishimichi (IPMU)
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R. Scoccimarro (NYU)
F. Vernizzi (IPhT Saclay)
N. van de Rijt (IPhT Saclay)

Outline

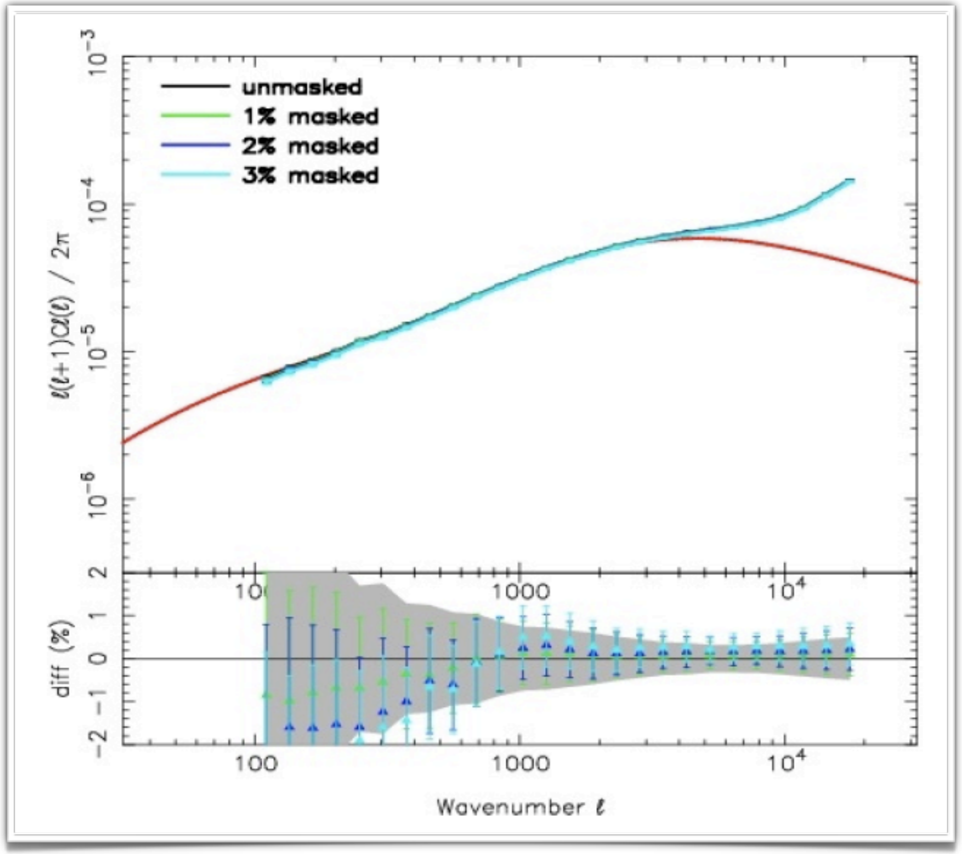
- Motivation and scopes
  - *A field theory reformulation of the dynamical equations*

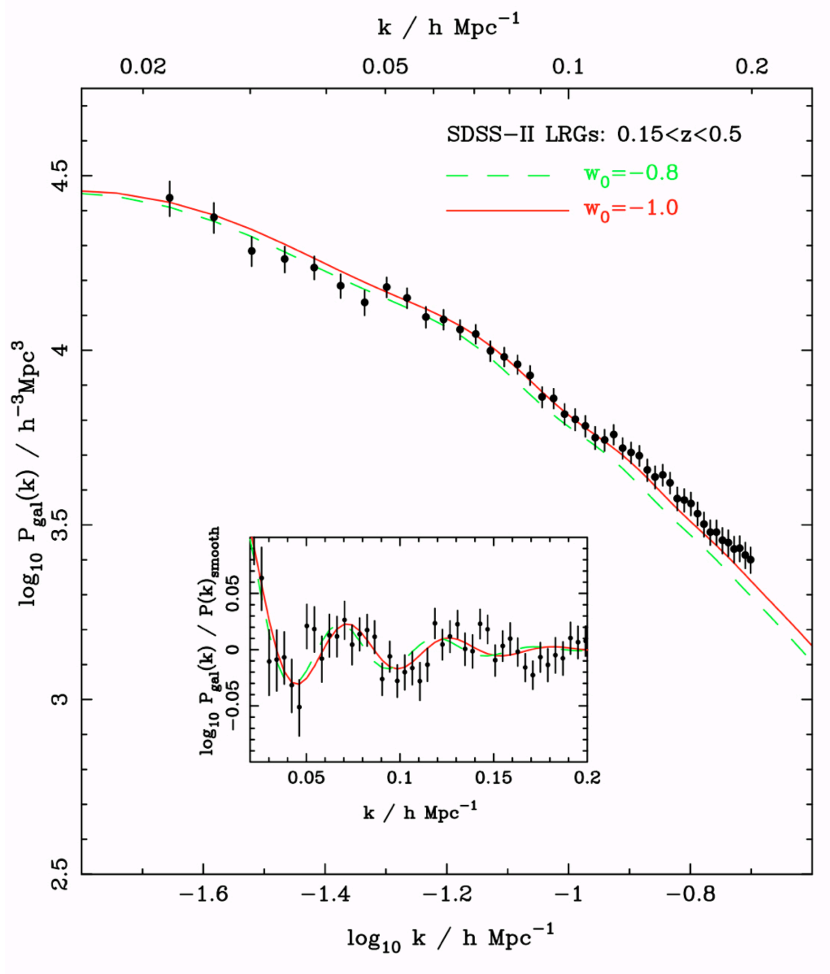
- The **eikonal** approximation
  - *basis*
  - *power spectra*
  - *lessons in the context of the Gamma-expansion*

- Power spectra at **2-loop** order from **`RegPT`** and **`MPTbreeze`** methods
  - *the prescriptions*
  - *a fast algorithm*

Regime of interest

- The transition from linear to quasi-linear regime





How far beyond 0.1 h Mpc^{-1} can we go?

A self-gravitating expanding dust fluid

A reformulation of the theory with a FT like approach

Scoccimarro '97

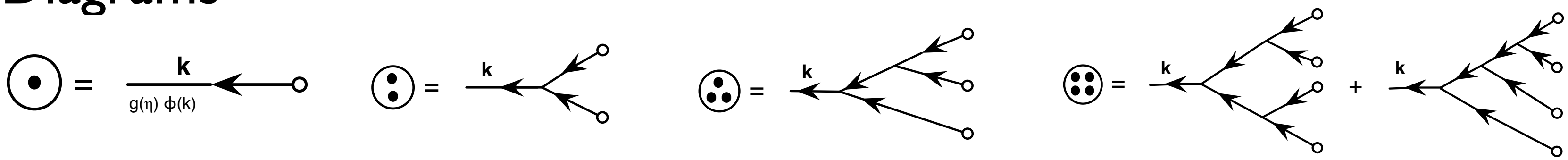
$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

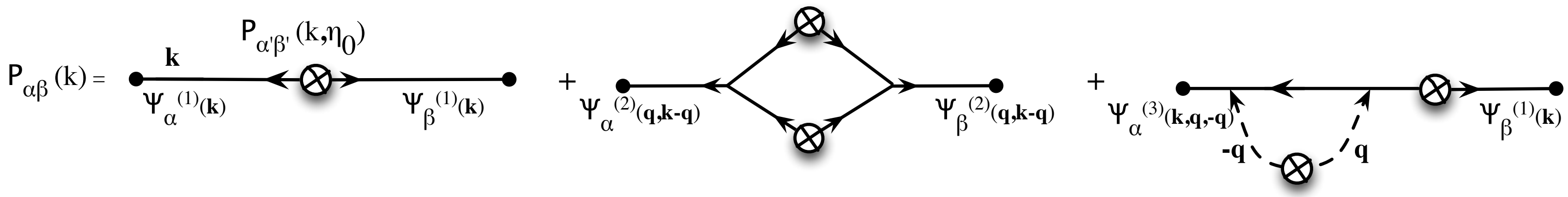
$$\Phi_a(\mathbf{k},\eta)=\begin{pmatrix}\delta(\mathbf{k},\eta)\\ \theta(\mathbf{k},\eta)\end{pmatrix}$$

$$\Phi_a(\mathbf{k},\eta)=g_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta=0)+\int_0^{\eta}d\eta' g_a^{\ b}(\eta-\eta')\gamma_b^{cd}(\mathbf{k}_1,\mathbf{k}_2)\Phi_c(\mathbf{k}_1)\Phi_d(\mathbf{k}_2)$$

doublet *linear propagator* $g_{ab}(\eta)=\frac{e^{\eta}}{5}\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}-\frac{e^{-3\eta/2}}{5}\begin{bmatrix}-2 & 2\\ 3 & -3\end{bmatrix}$

- Diagrams





Note : detailed effects of baryons versus DM can be taken into account (Somogyi & Smith 2010; FB, Van de Rijt, Vernizzi '12) with a 4-component multiplet, for neutrinos it is more complicated...

Methods of Field Theory

Time-flow (renormalization) equations

M. Pietroni '08
Anselmi, Pietroni '12

From the field evolution equation to the multi-spectra evolution equation

The closure theory

Taruya, Hiramatsu, ApJ 2008, 2009

Motion equations for correlators are derived using the Direct-Interaction (DI) approximation in which one separates the field expression in a DI part and a Non-DI part. At leading order in Non-DI >> DI, one gets a closed set of equations,

These equations can more rigorously be derived in a large N expansion.

Valageas P., A&A, 2007

The eikonal approximation

FB, Van de Rijt, Vernizzi 2012

Effective Theory approaches

Pietroni et al '12, or "a la Senatore"

The eikonal approximation

The eikonal approximation :

FB, Van de Rijt, Vernizzi 2011

dynamics :

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{\ bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

Impact of the long-wave modes into the short wave modes (of interest)

Non trivial k dependence!

1. Split the interaction term into 2 parts:

- $k_1 \ll k_2$ or $k_2 \ll k_1$ (soft domain)
- $k_1 \approx k_2$ (hard domain)

2. Compute the first part using simplified form for the vertices

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)-\Xi_a^{\ b}(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{\ bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)|_{\text{hard domain}}$$

$$\Xi_a^{\ b}(\mathbf{k},\eta)=\int \mathrm{d}^3\mathbf{q}\left(\gamma_a^{\ cb}(\mathbf{q},\mathbf{k})+\gamma_a^{\ bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)|_{\text{soft domain}}$$

It leads to a "renormalized" theory that takes into account the long wave modes in a nonlinear manner.

3. Taking ensemble average over Ξ leads to the standard results assuming linear growing modes and Gaussian initial conditions.

The "renormalized" theory at linear order

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)-\Xi_a^{\ b}(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta)=0$$

$$\Xi_a^{\ b}(\mathbf{k},\eta)=\int \mathrm{d}^3\mathbf{q}\left({}^{\text{eik.}}\gamma_a^{\ cb}(\mathbf{q},\mathbf{k})+{}^{\text{eik.}}\gamma_a^{\ bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)\big|_{\text{soft domain}}$$

velocity field component only

What is in this new term ?

A **multi-component** fluid analysis with adiabatic modes and iso-curvature/density modes

$$\Xi_a^b(\mathbf{k},\eta)=\Xi^{(\mathrm{ad})}(\mathbf{k},\eta)\delta_a^b+\Xi_a^{b\,(\nabla)}(\mathbf{k},\eta)$$

diagonal term

non-diagonal term

$$=\int\frac{\mathbf{k}.\mathbf{q}}{q^2}\,\delta_d(\mathbf{q})\mathrm{d}^3\mathbf{q}$$

$$\Xi_a^{b\,(\nabla)}=\Xi^{(\nabla)}\,h_a^b\,,\quad h_a^b\equiv\begin{pmatrix} f_2 & 0 & 0 & 0\\ 0 & f_2 & 0 & 0\\ 0 & 0 & -f_1 & 0\\ 0 & 0 & 0 & -f_1\end{pmatrix}$$

Power spectra in the eikonal approximation

Theorem 1: multi-spectra are independent on the large-scale adiabatic modes (in the eikonal limit)

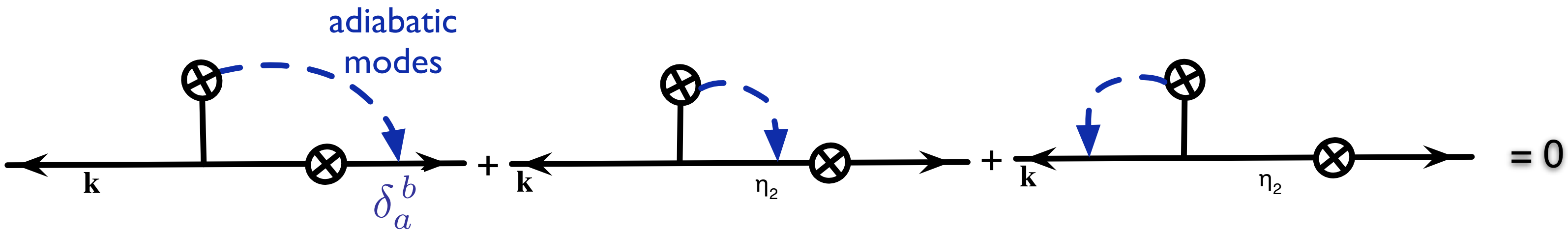
FB, Van de Rijt, Vernizzi, '12 in prep.

This is a direct consequence of the functional dependance on the large-scale adiabatic displacement field.

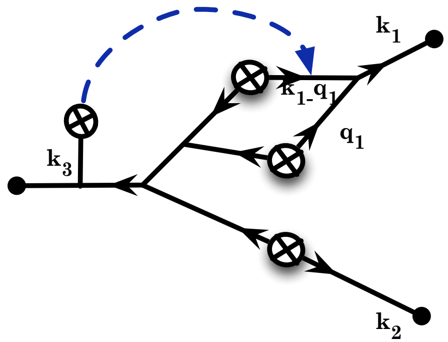
$$\psi_a(\mathbf{k},\eta;\Xi^{\mathrm{adiab.}}) = \xi_a{}^b(\mathbf{k},\eta,\eta_0;\Xi^{\mathrm{adiab.}})\psi_b(\eta_0)$$

$$\xi_a{}^b(\mathbf{k},\eta,\eta_0;\Xi^{\mathrm{adiab.}}) = g_a{}^b(\eta,\eta_0)\exp\left(\mathrm{i}\int_{\eta_0}^{\eta}\mathrm{d}\eta'\ \mathbf{k}.\mathbf{v}^{\mathrm{adiab.}}(\eta')\right)$$

*Theorem 2: multi-spectra are independent on the large-scale adiabatic modes at any order in **standard** Perturbation Theory*



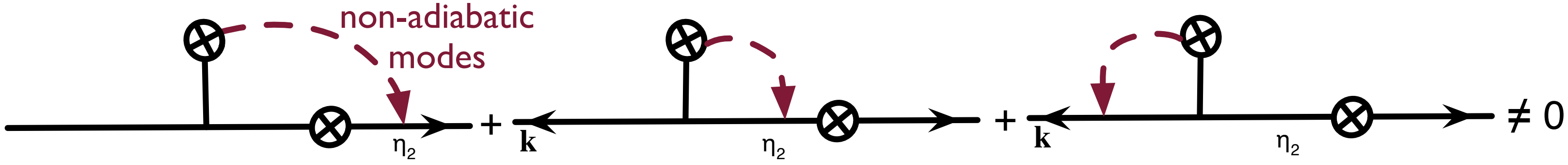
One-loop correction to power spectrum
or ... any poly-spectrum at any loop order



But not necessarily so for all PT schemes...

What is true for adiabatic modes is not true for non-adiabatic modes!

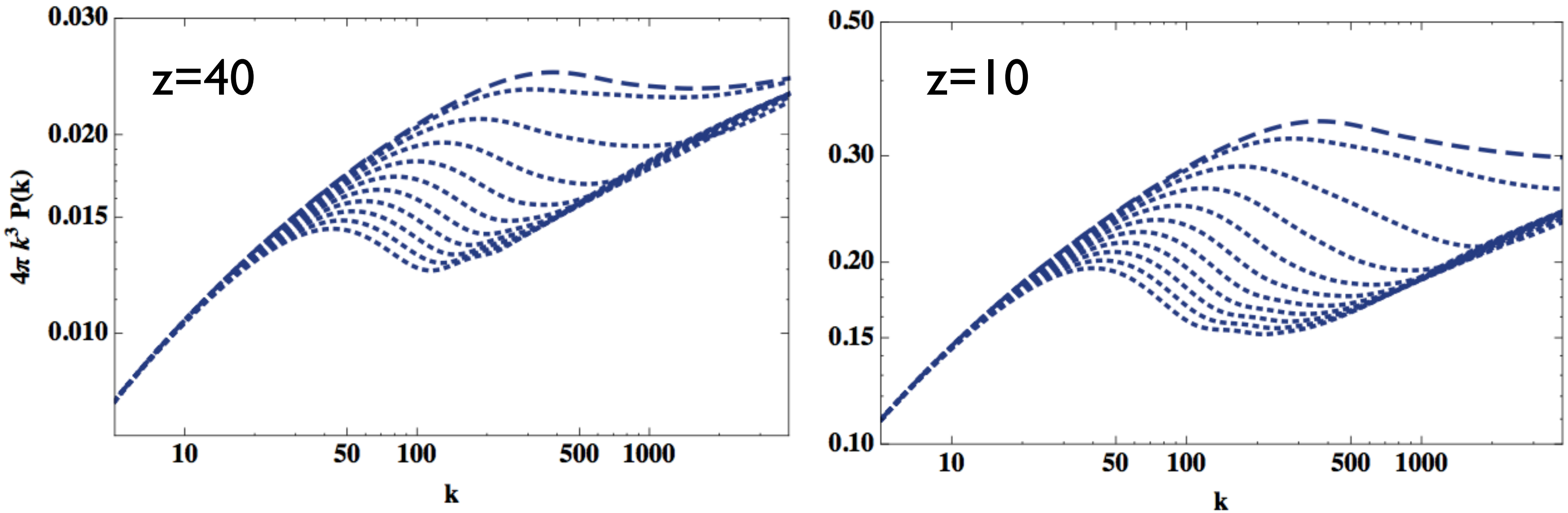
FB, Van de Rijt, Vernizzi, '12 in prep.



Resulting power spectrum in the eikonal limit (beyond one-loop results)

$$P_\delta(\mathbf{k};\Xi^{\rm iso.}) = {\xi_1}^a(\mathbf{k},\eta,\eta_0;\Xi^{\rm iso.})\,{\xi_1}^b(\mathbf{k},\eta,\eta_0;\Xi^{\rm iso.})\,P^{\rm init.}_{ab}(k,\eta_0)$$

modes mainly produced at horizon scale at decoupling



"Relative velocity of dark matter and baryonic fluids and the formation of the first structures", D.Tseliakhovich and C. Hirata, PRD, '10

Bad news for biasing...

Galaxy formation is potentially modulated by large scale velocity modes (at 100-10 Mpc scales).

Dalal, Pen, Seljak '10

Yoo, Dalal, Seljak '11

In general however non-adiabatic modes have very little (totally negligible ?) impact on modes of interest here.

Somogyi & Smith 2010

FB, Van de Rijt, Vernizzi 2011

The Multi-Point Propagator expansion (Gamma expansion)

The key ingredients : the (multipoint) propagators

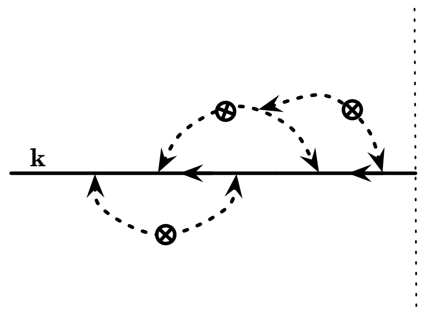
Scoccimarro and Crocce PRD, 2005

Final density / velocity div.

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta \Psi_a(\mathbf{k},\eta)}{\delta \phi_b(\mathbf{k}')} \right\rangle$$

Initial Conditions

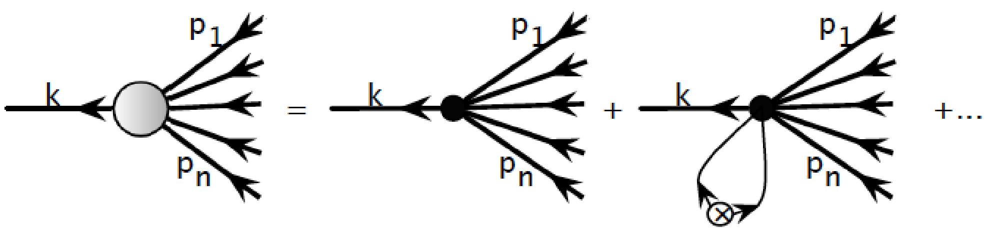
$G_{ab}(k) =$



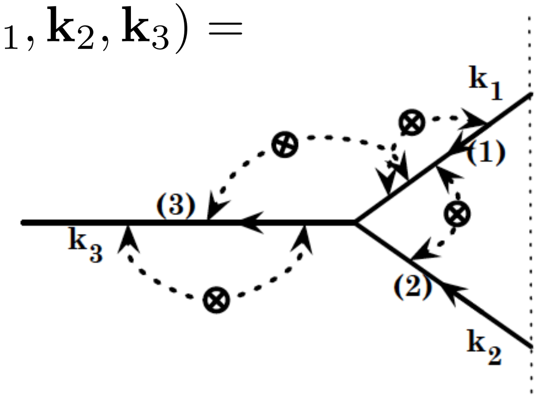
FB, Crocce, Scoccimarro, PRD, 2008

$$\Gamma^{(p)}_{ab_1\ldots b_p}(\mathbf{k}_1,\ldots,\mathbf{k}_p,\eta)\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1\ldots p}) = \frac{1}{p!}\left\langle \frac{\delta^p \Psi_a(\mathbf{k},\eta)}{\delta\phi_{b_1}(\mathbf{k}_1)\ldots\delta\phi_{b_p}(\mathbf{k}_p)} \right\rangle$$

$\Gamma^{(n)}(k,p_1,\ldots,p_n)=$



$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$



- This suggests another scheme: to use the n-point propagators as the building blocks

FB, Crocce, Scoccimarro, PRD, 2008

  - The reconstruction of the power spectrum :

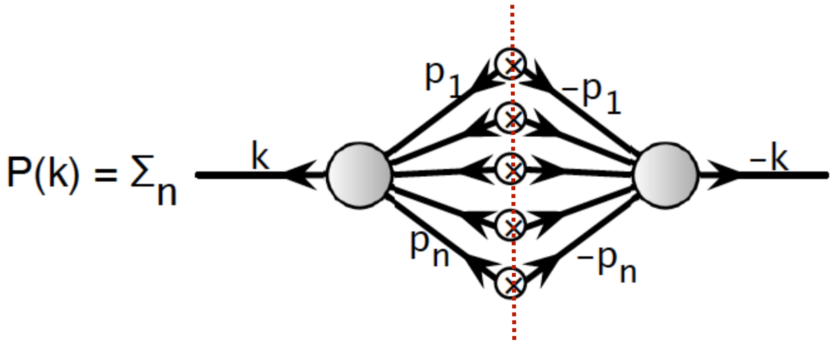
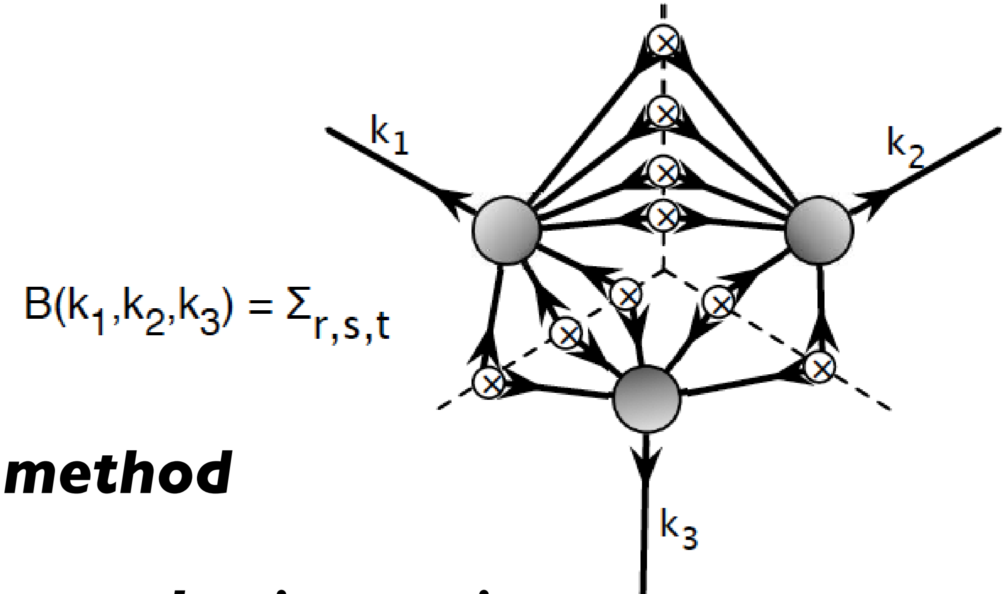


FIG. 3: Reconstruction of the power spectrum out of transfer functions. The crossed circles represent the initial spectrum. The sum runs over the number of internal co ing lines, e.g. the number of such circles. It is to be that each term of this sum is positive.

→ *Sum of positive terms*

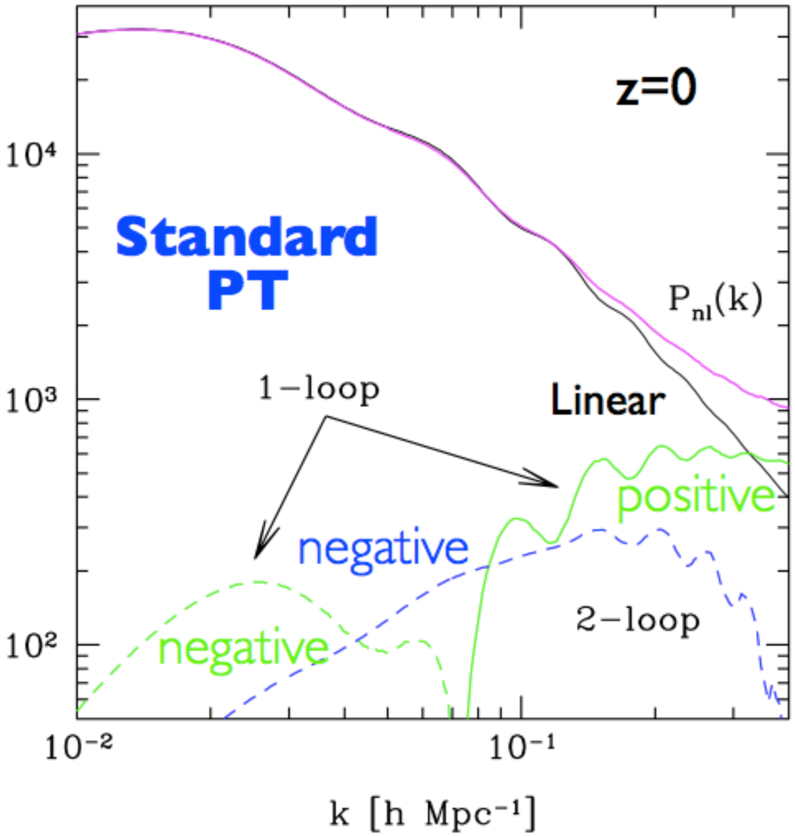
  - Also provide the building blocks for higher order moments...

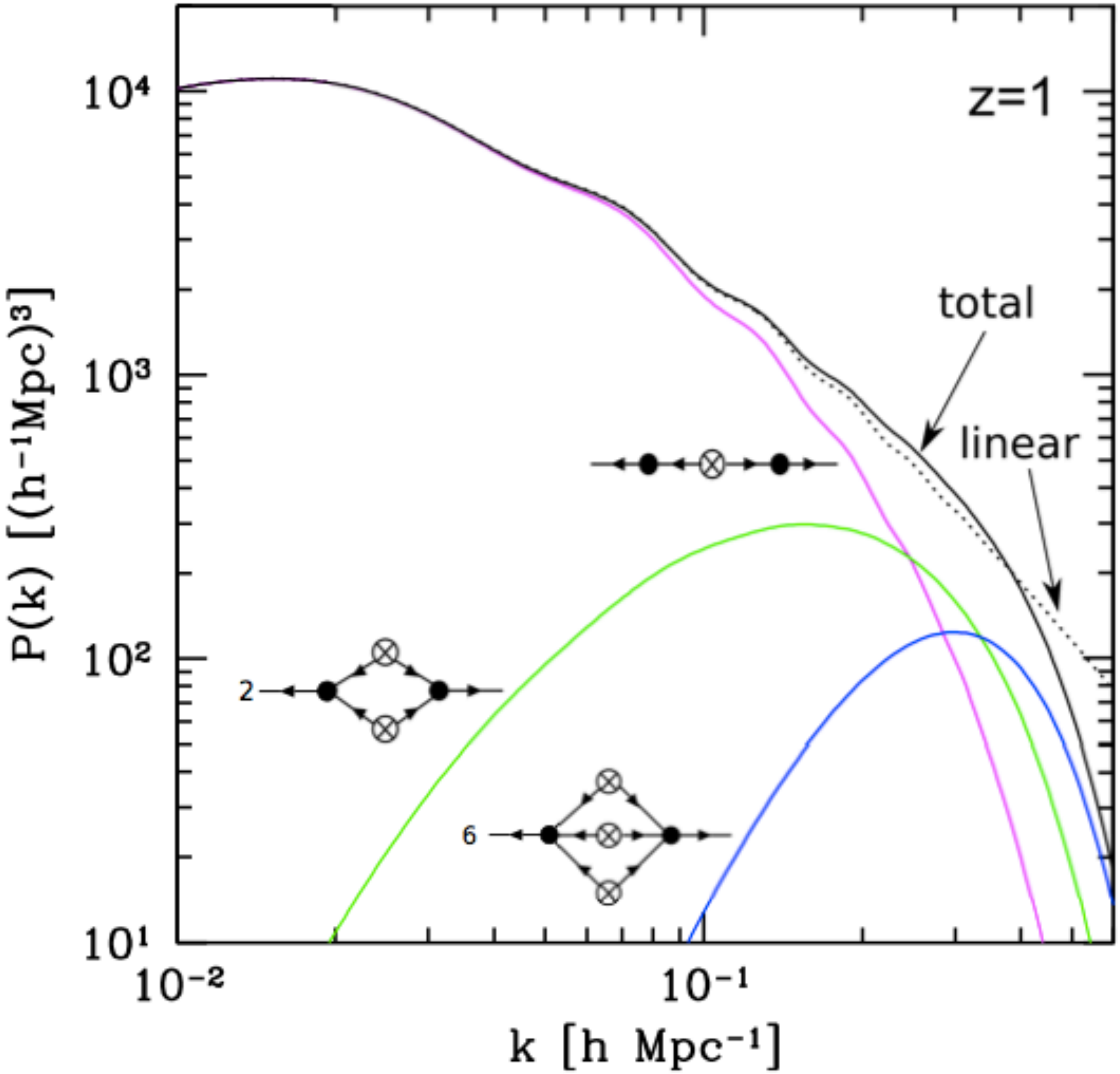


Γ-expansion method

- ***re-organisation(s) of the perturbation series***

Reconstruction of the power spectrum: from sPT to Multi-point propagator reconstruction





The eikonal approximation in the Gamma-expansion context

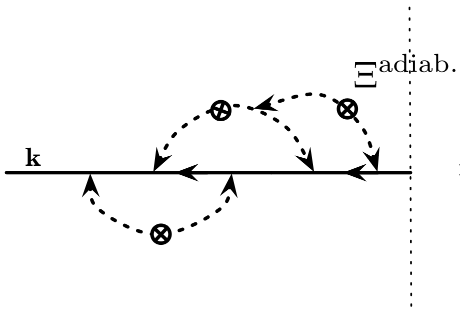
Back with the adiabatic modes. Main outcome is the following :

$$\xi_a{}^b(\mathbf{k},\eta,\eta_0) = g_a{}^b(\eta,\eta_0)\exp\left[\mathrm{i}\mathbf{k}.\mathbf{d}^{\mathrm{adiab.}}(\eta')\right]$$

(adiabatic) displacement field

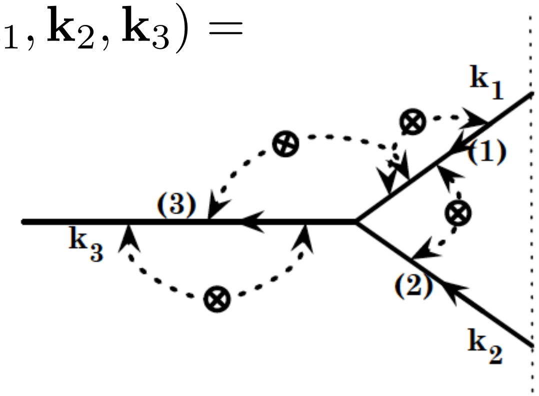
Consequences for propagators

$$G_{ab}(k) =$$



$$= \left\langle \xi_a{}^b(\eta)\right\rangle_\Xi = g_a{}^b(\eta)\exp\left(-\frac{k^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

$$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$$

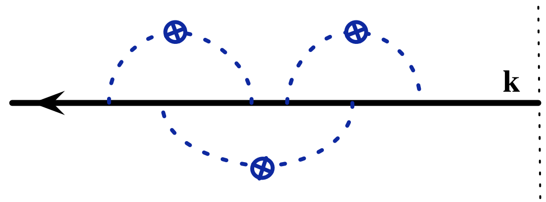


$$= \Gamma^{\mathrm{tree}}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\exp\left(-\frac{k_3^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

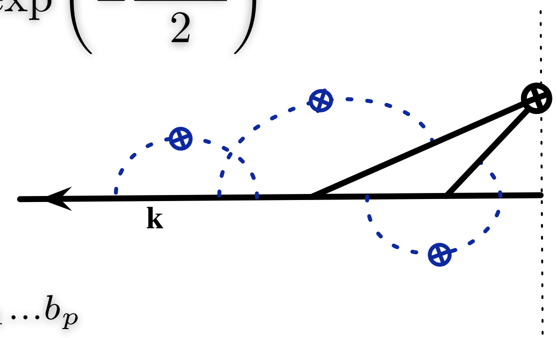
A regularization scheme = how to interpolate between n-loop results and the large-k behavior ?

An ad-hoc solution was provided by Crocce and Scoccimarro (RPT) for the one-point propagator but it cannot be generalized all cases.

- The proposed form is the following



$$
\begin{aligned}
{}^{\mathrm{Reg}}\Gamma^{(p)}{}_a^{\,b_1\ldots b_p} = \quad & {}^{\mathrm{tree}}\Gamma_a^{\,b_1\ldots b_p}\ \exp\left(-\frac{k^2\sigma_d^2}{2}\right) \\
+ \quad & \left[{}^{\mathrm{one-loop}}\Gamma_a^{\,b_1\ldots b_p} + \frac{1}{2}k^2\sigma_d^2\ {}^{\mathrm{tree}}\Gamma_a^{\,b_1\ldots b_p}\right]\exp\left(-\frac{k^2\sigma_d^2}{2}\right) \\
+ \quad & \left[{}^{\mathrm{two-loop}}\Gamma_a^{\,b_1\ldots b_p} + \mathrm{c.t.}\right]\exp\left(-\frac{k^2\sigma_d^2}{2}\right)
\end{aligned}
$$

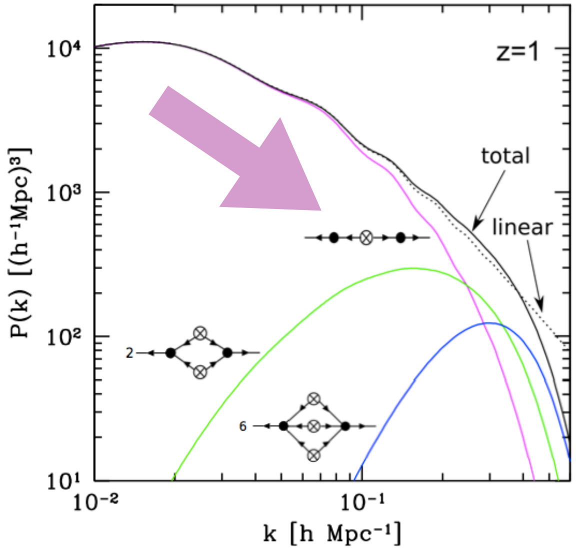


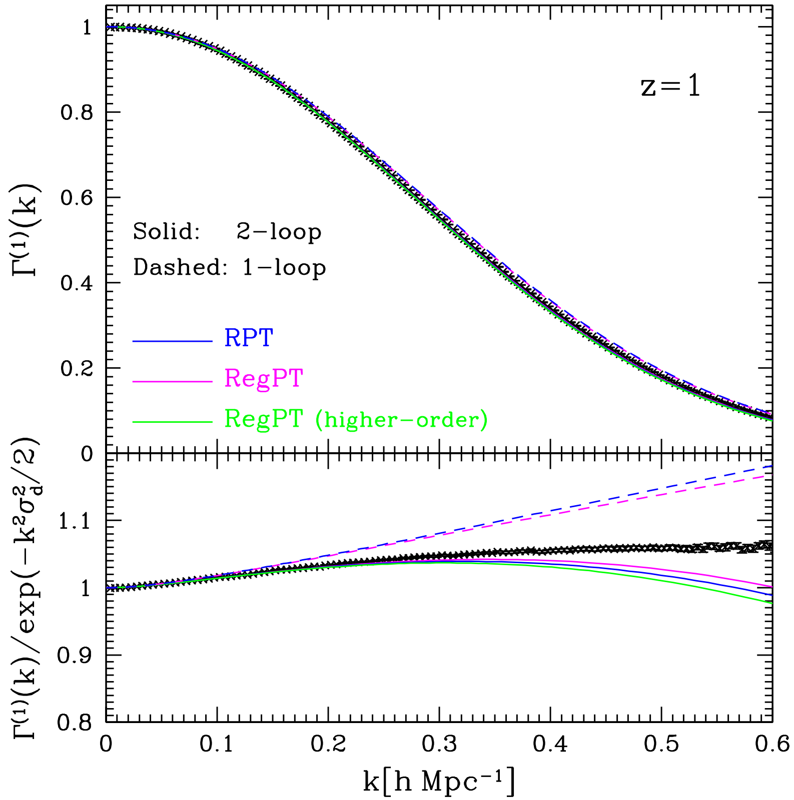
$$
\mathrm{c.t.} = \frac{1}{2}\left(\frac{k^2\sigma_d^2}{2}\right)^2\ {}^{\mathrm{tree}}\Gamma_a^{\,b_1\ldots b_p} + \frac{k^2\sigma_d^2}{2}\ {}^{\mathrm{one-loop}}\Gamma_a^{\,b_1\ldots b_p}
$$

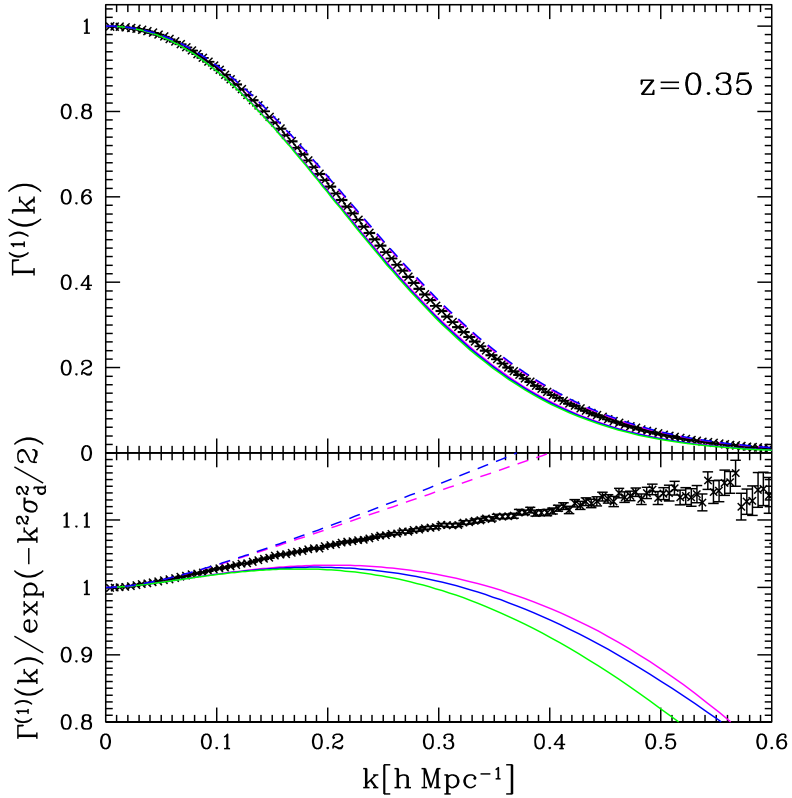
- This is our proposition for regularized propagators: our best guess!

FB, Crocce, Scoccimarro '12

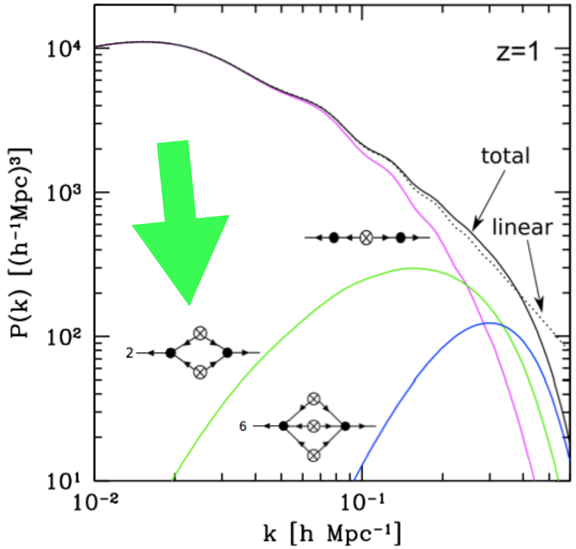
The two-point propagator at 1-loop and 2-loop orders



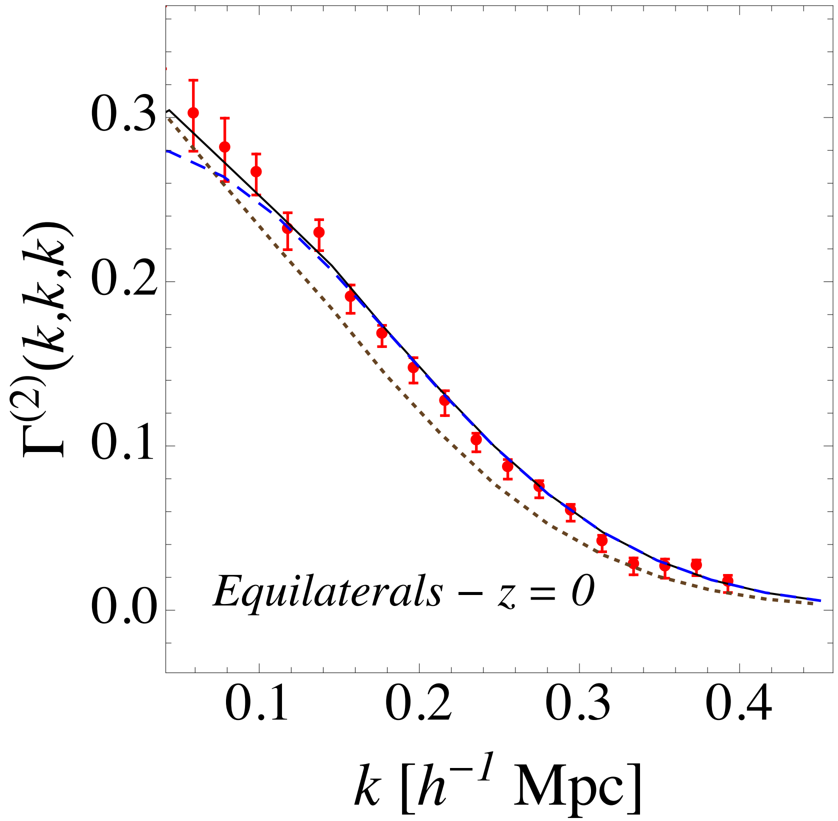


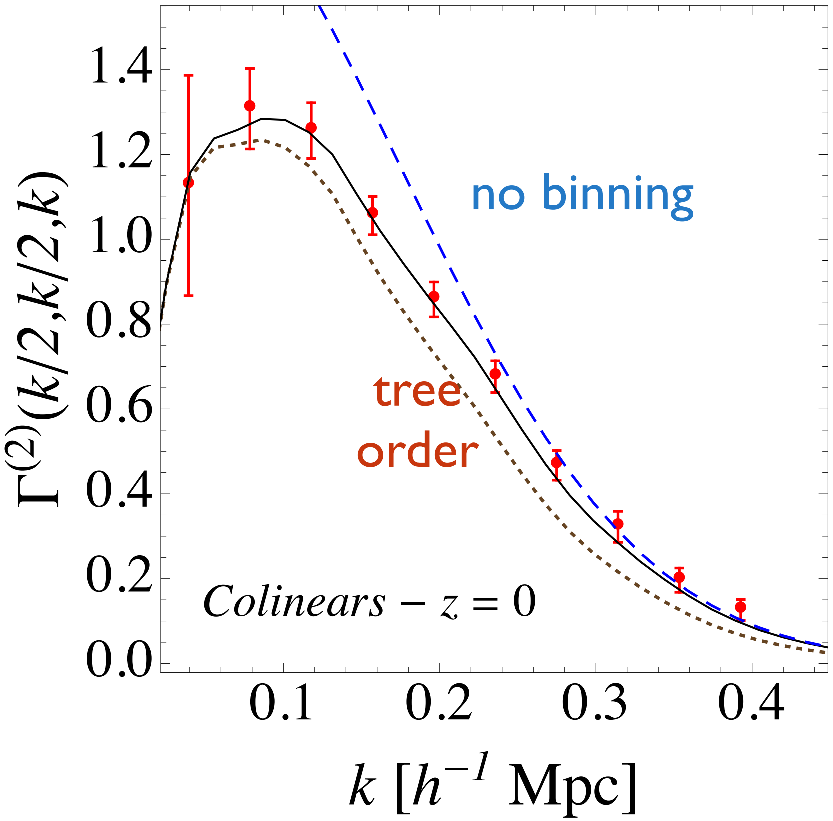


Comparison with numerical simulations at tree and one-loop order for the 3-point propagator



FB, Crocce, Scoccimarro '12





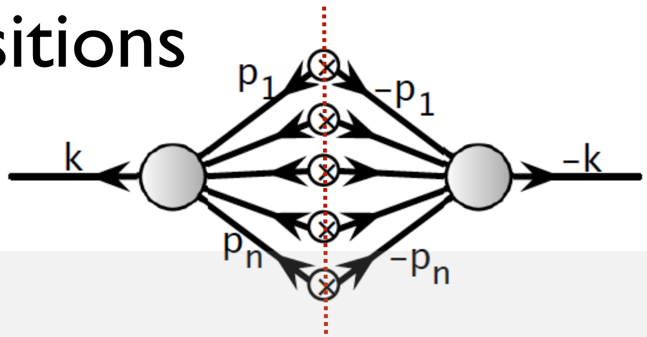
Power spectra in the RegPT and MPTbreeze prescriptions

The RegPT and MPTbreeze propositions

Taruya , FB, Nishimichi, Codis '12 very soon

Crocce, Scocimarro, FB, '12

$P(k) = \Sigma_n$



$$
\begin{aligned}
P_{aa'}(k) = \; & {}^{\mathrm{Reg}}\Gamma^{(1)}{}_a^{\;b}(k)\; {}^{\mathrm{Reg}}\Gamma^{(1)}{}_{a'}^{\;b'}(k)\; P^{\mathrm{ini.}}_{bb'}(k) \\
+ \; & {}^{\mathrm{Reg}}\Gamma^{(2)}{}_a^{\;bc}(\mathbf{k}_1,\mathbf{k}_2)\; {}^{\mathrm{Reg}}\Gamma^{(2)}{}_{a'}^{\;b'c'}(\mathbf{k}_1,\mathbf{k}_2)\; P^{\mathrm{ini.}}_{bb'}(k_1)\; P^{\mathrm{ini.}}_{cc'}(k_2) \\
+ \; & {}^{\mathrm{Reg}}\Gamma^{(3)}{}_a^{\;bcd}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\; {}^{\mathrm{Reg}}\Gamma^{(3)}{}_{a'}^{\;b'c'd'}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\; P^{\mathrm{ini.}}_{bb'}(k_1)\; P^{\mathrm{ini.}}_{cc'}(k_2)\; P^{\mathrm{ini.}}_{dd'}(k_2)
\end{aligned}
$$

MPTbreeze

RegPT

two loop order

one loop order

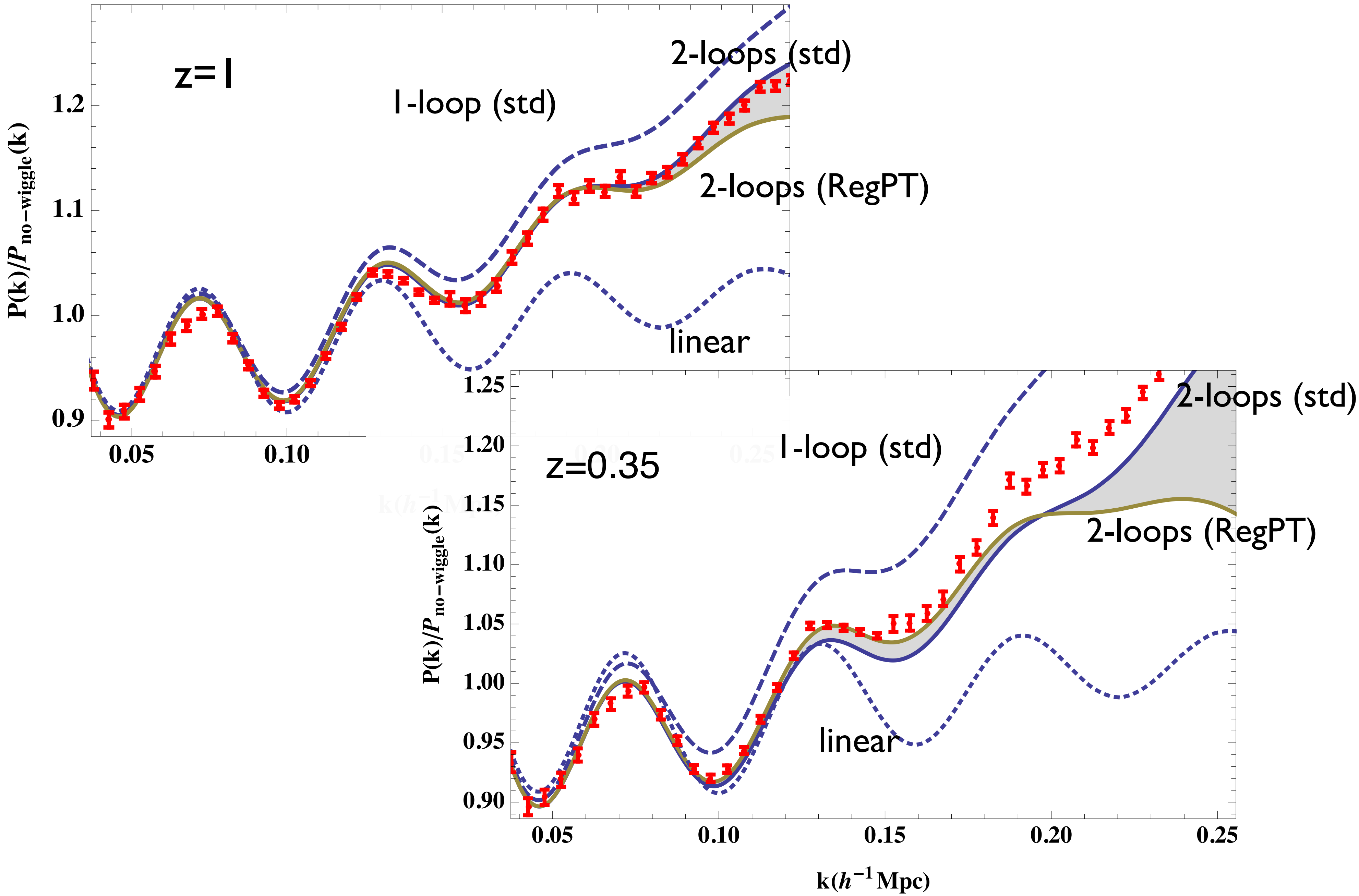
tree order

(effective) one loop order

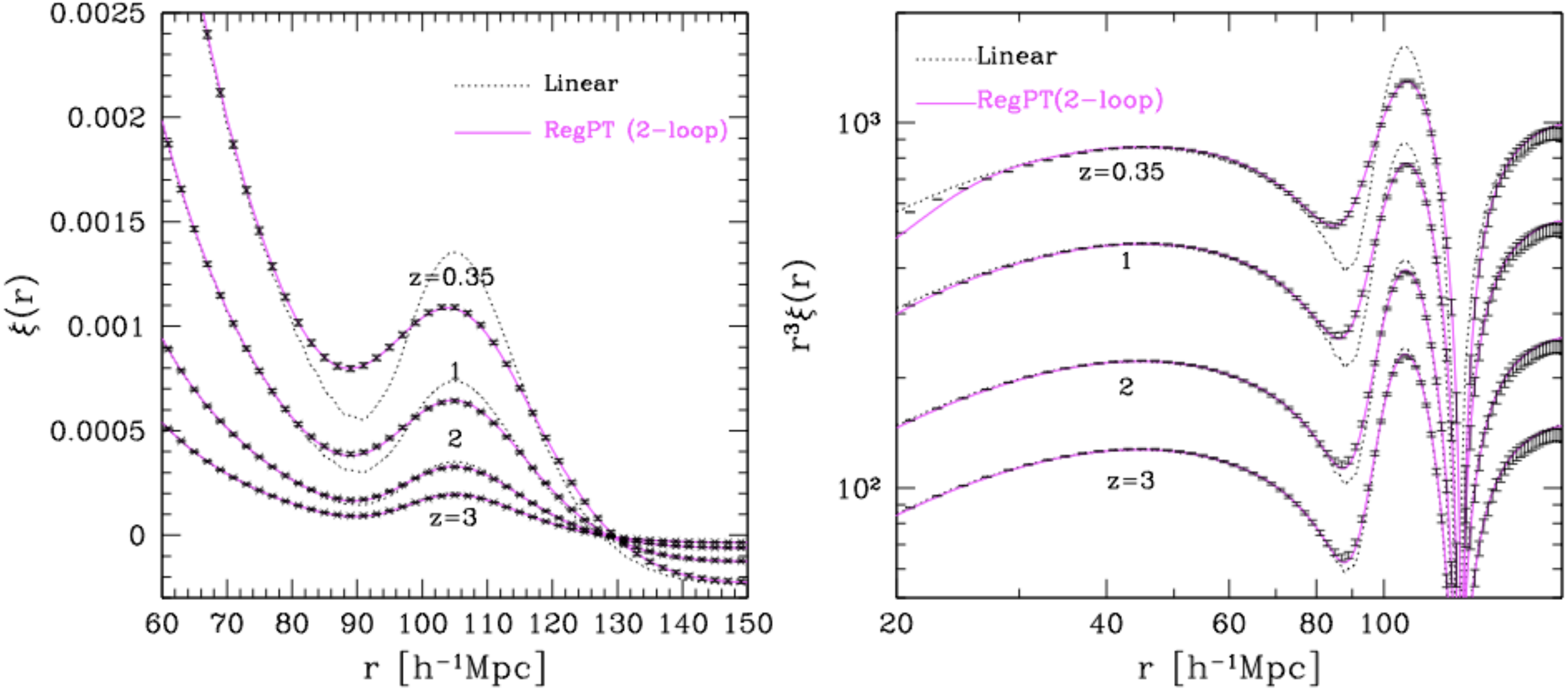
$$
{}^{\mathrm{Reg}}\Gamma^{(p)}{}_a^{\;b_1\dots b_p} = {}^{\mathrm{tree}}\,\Gamma_a^{\;b_1\dots b_p}\;\exp\left(f(k)\right)
$$

$$
\begin{aligned}
{}^{\mathrm{Reg}}\Gamma^{(p)}{}_a^{\;b_1\dots b_p} = \; & {}^{\mathrm{tree}}\Gamma_a^{\;b_1\dots b_p}\;\exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right) \\
& + \left[{}^{\mathrm{one-loop}}\Gamma_a^{\;b_1\dots b_p} + \frac{1}{2}k^2\sigma_d^2\;{}^{\mathrm{tree}}\Gamma_a^{\;b_1\dots b_p}\right]\exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right) \\
& + \left[{}^{\mathrm{two-loop}}\Gamma_a^{\;b_1\dots b_p} + \mathrm{c.t.}\right]\exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right)
\end{aligned}
$$

1-loop and 2-loop corrections



And if you really insist



Accelerated computational method: RegPT -fast

$$
\begin{aligned}
P_{\rm NL}(k) &= \mathcal{F}\left[P_{\rm lin.};k\right] \\
&= \mathcal{F}\left[\alpha P_0;k\right] + \int \mathrm{d}k' \, \frac{\delta\mathcal{F}\left[\alpha P_0;k\right]}{\delta\alpha P_0(k')} \left(P_{\rm lin}(k') - \alpha P_0(k')\right) + \ldots
\end{aligned}
$$

fiducial model (arbitrary normalization)

departure from fiducial model

- Normalization is chosen in order to minimize difference between P(k) and fiducial model.

- Approach is valid for any model where the explicit dependence with linear P(k) can be given.
- Calculations can be made extremely rapid from precomputed functions.
- It leads to the concept of Kernel functions.

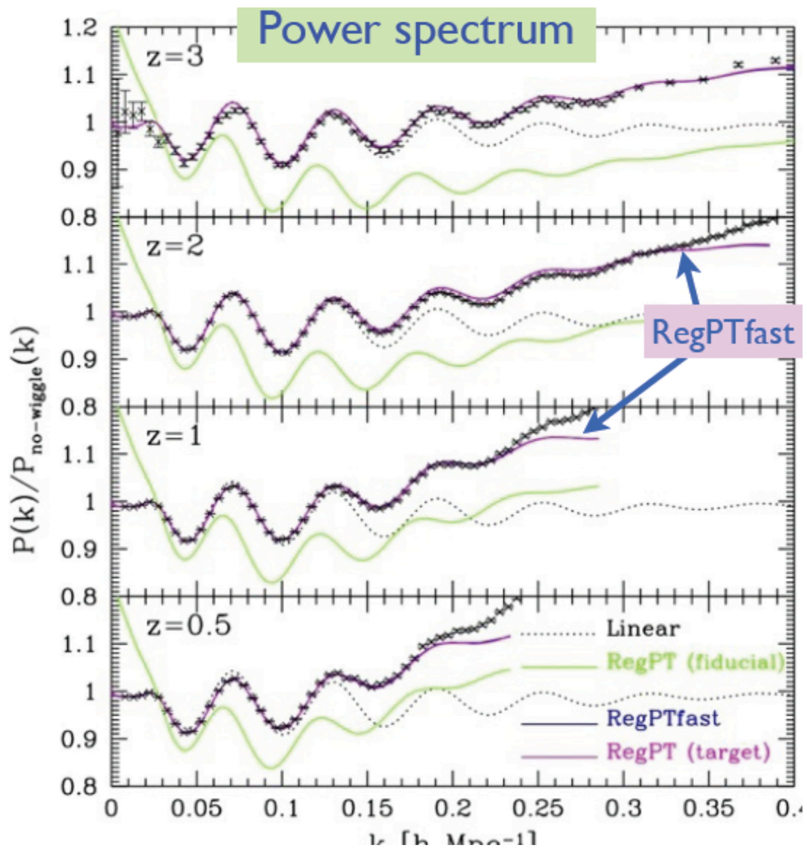
RegPT-fast compared to RegPT direct

Fiducial (wmap3)

$$\Omega_{\rm m} = 0.234$$
$$\Omega_\Lambda = 0.766$$
$$\Omega_{\rm b}/\Omega_{\rm m} = 0.175$$
$$h = 0.734$$
$$\sigma_8 = 0.76$$

Target (wmap5)

$$\Omega_{\rm m} = 0.279$$
$$\Omega_\Lambda = 0.721$$
$$\Omega_{\rm b}/\Omega_{\rm m} = 0.165$$
$$h = 0.701$$
$$\sigma_8 = 0.817$$



Typical time for computation:

For 200 output points in k-space

5-10 min. for RegPT and few secs for RegPT-fast

Discrepancies between RegPT and RegPT-fast are negligible…

Conclusions

The ***eikonal*** approximation is very powerful

- For any fluid content, in particular including dark matter and baryons (new modes appear) *FB, Van de Rijt, Vernizzi 2011*
- The basis for the regularization schemes in which one can incorporate arbitrary order loops *FB et al. 2011*
- Can be used for Non-Gaussian initial conditions *Crocce, Sefusatti, FB, 2010*
  - The Gamma-expansion is still valid.
  - In the large k limit we now have :

$$G(k) \ \rightarrow \ \exp\left[-\sum_{p=2}^{\infty}\frac{\langle(\mathbf{d}.\mathbf{k})^p\rangle_c}{p!}(e^{\eta}-e^{\eta_0})^p\right]$$

- Can be used in Lagrangian coordinates *FB, Valageas 2008*

Two-loop calculations can now be done routinely (and very rapidly)

- Public codes for fast computations of power spectra at 2-loop order are now available. Codes take a few seconds to compute power spectra.

  http://maia.ice.cat/crocce/mptbreeze/
  http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/regpt_code.html

- So far performances are focused on mild values of k for the density field. Theoretical predictions are within 1% accuracy.
- Extensions to velocity components are under construction with the same methods.