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Going beyond the Kaiser redshift-space distortion formula

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Going Beyond the Kaiser Redshift-space Distortion Formula: General Relativistic Effect

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ICTP Large-Scale Structure Workshop, July 30 ~ August 1, 2012

I. INTRODUCTION:

GOING BEYOND THE KAISER FORMULA

Motivation

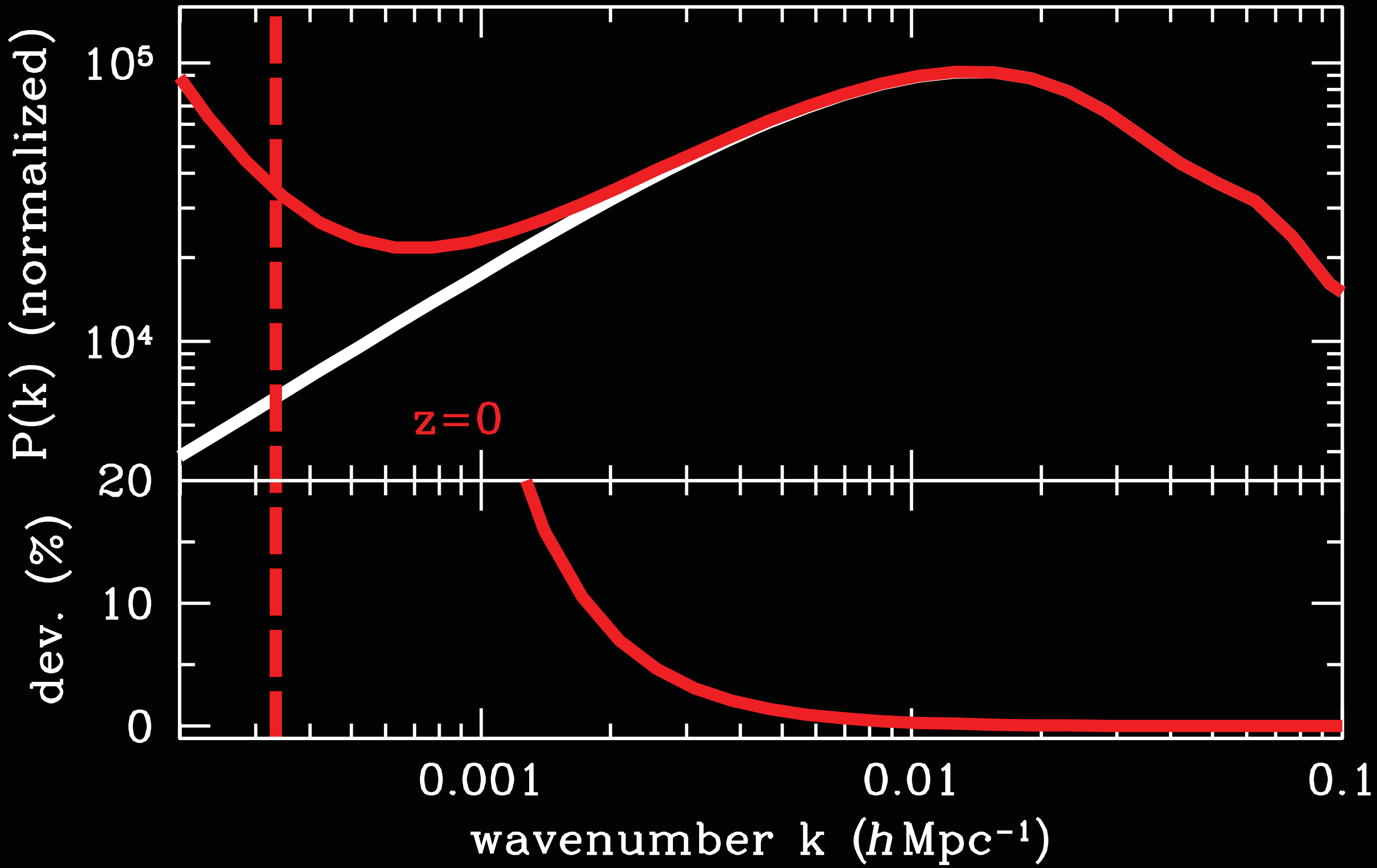
- **recent advances in observational cosmology**
    - **larger sky coverage and higher redshift**
    - **measurements with higher statistical power**

- **theoretical predictions**
    - ***sufficiently accurate* to describe observations*?***
    - **it is *general relativity!***

Relativistic Effect?

- **there are *infinitely* many gauge choices**
- ***order one* effects on horizon scale!**

P_m : Synchronous gauge, P_m : Newtonian gauge



II. BEYOND THE KAISER FORMULA:

GENERAL RELATIVISTIC EFFECT

Observables

- model *observables*, not *unobservable* quantities!
- *observables*: (physical)
  - observed redshift z_{obs}, position $\hat{n} = (\theta, \phi)$
- *unobservables*: (gauge-dependent)
  - redshift z, angular position $\hat{s} = \hat{n} - (\delta\theta, \delta\phi)$
- photon geodesic equation (relativistic)

$$1 + z_{\rm obs} = (1+z)\left[1 + V(z) - V(0) - \psi(z) + \psi(0) - \int_0^r dr' \ (\dot{\psi} - \dot{\phi})\right] .$$

$(\delta r, \ \delta\theta, \ \delta\phi)$

Effects on Galaxy Clustering

- **construct *a galaxy fluctuation field*:**
  - **total number of observed galaxies** $N_{\rm tot}$
  - **observed volume** $dV_{\rm obs}$ **given** $(z_{\rm obs}, \hat{n})$
  - **fluctuation field** $\delta_{\rm obs} = \frac{n_{\rm obs}}{\langle n_{\rm obs} \rangle} - 1$
- **relation to *physical* number density:**
  - **number conservation** $N_{\rm tot} = n_{\rm phy} dV_{\rm phy} = n_{\rm obs} dV_{\rm obs}$
  - **observed number density** $n_{\rm obs} = n_{\rm phy} \frac{dV_{\rm phy}}{dV_{\rm obs}}$

Unified Treatment

- **observable:** $N_{\rm tot} = n_g^{\rm phy}\ dV_{\rm phy} = n_g^{\rm obs}\ dV_{\rm obs}$
- **volume effects:** $\dfrac{dV_{\rm phy}}{dV_{\rm obs}}$
  - **redshift-space distortion:** $\dfrac{\partial z}{\partial z_{\rm obs}} \simeq \dfrac{1+z}{H}\dfrac{\partial V}{\partial r}$
  - **lensing magnification:** $\dfrac{\partial \Omega}{\partial \Omega_{\rm obs}}\dfrac{\partial f}{\partial f_{\rm obs}} \simeq \dfrac{1}{\mu^2} = 1 - 4\ \kappa$
- **source effects:**
  - **magnification bias:** $n_g^{\rm obs}(f_{\rm obs}) \simeq n_g^{\rm phy}(f_{\rm obs}/\mu)$
- ***complete description of different effects***
  - **holds in Newtonian & GR descriptions**

Yoo, PRD, 2009

Physical Volume in 4D

- **unified treatment:** $N_{\rm tot}=n_g^{\rm phy}\;dV_{\rm phy}=n_g^{\rm obs}\;dV_{\rm obs}$
- **integral of 3-form in 4D spacetime manifold:**
  - **observables** $z_{\rm obs},\;\theta_{\rm obs},\;\phi_{\rm obs}$
  - **photon geodesic path** $x^a(\lambda)=\bar x^a(\lambda)+\delta x^a(\lambda)$
  - **Sachs-Wolfe and gravitational lensing effects**
  - **distortion in local Lorentz frame**
  - ***manifestly gauge-invariant***

$$N_{\rm tot}=\int\sqrt{-g}\;n_{\rm phy}\;\varepsilon_{abcd}\;u^d\;\frac{\partial x^a}{\partial z_{\rm obs}}\frac{\partial x^b}{\partial\theta_{\rm obs}}\frac{\partial x^c}{\partial\phi_{\rm obs}}\;dz_{\rm obs}\;d\theta_{\rm obs}\;d\phi_{\rm obs}$$

Yoo, Fitzpatrick, Zaldarriaga, PRD, 2009

Levi-Civita symbol ε_{abcd} **,** **comoving velocity** u^a

Galaxy Bias

- **galaxy formation**
  - **described in a local coordinate (proper time)**

$$n_{\rm phy} = F[\rho_m] \quad \rightarrow \quad n_{\rm phy} = F[\rho_m, t_p] \ ,$$

- **time slicing in observation** $1 + z_{\rm obs} = (1+z)(1+\delta z)$
  - ***observed redshift*** **defines simultaneity**
  - **measured at observed redshift**

$$n_{\rm phy} = \bar n_{\rm phy}(z_{\rm obs})[1 + b\ \delta_m^{(v)} - e\ \delta z^{(v)}] \qquad e = \frac{d\ln \bar n_{\rm phy}}{d\ln(1+z)}$$

Baldauf et al. 2011, Bonvin & Durrer 2011, Bruni et al. 2012

Challinor & Lewis 2011, Jeong, Schmidt, Hirata 2012, Yoo et al. 2012

Relativistic Formula

- ***standard Kaiser*** **formula:** $\delta_g = b\,\delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} = b\,\delta_m + f\mu_k^2\delta_m$
- ***general relativistic*** **formula:**

$$\delta_g = b\,\delta_m^{(v)} - e\,\delta z^{(v)} + \alpha_\chi + 2\,\varphi_\chi + V - C_{\alpha\beta}\,e^\alpha e^\beta$$
$$+3\,\delta z_\chi + 2\,\frac{\delta\mathcal{R}}{r} - H\frac{\partial}{\partial z}\left(\frac{\delta z_\chi}{\mathcal{H}}\right) - 5p\,\delta\mathcal{D}_L - 2\,\mathcal{K}\,,$$

it can be computed in ***any gauges!***

Yoo, Fitzpatrick, Zaldarriaga 2009

Yoo 2010

Bonvin & Durrer 2011, Challinor & Lewis 2011

Jeong, Schmidt, Hirata 2012, Yoo, Hamaus, Seljak, Zaldarriaga 2012

What are They?

$$\delta_g = b\,\delta_m^{(v)} - e\,\delta z^{(v)} + \alpha_\chi + 2\,\varphi_\chi + V - C_{\alpha\beta}\,e^\alpha e^\beta$$
$$+3\,\delta z_\chi + 2\,\frac{\delta\mathcal{R}}{r} - H\frac{\partial}{\partial z}\left(\frac{\delta z_\chi}{\mathcal{H}}\right) - 5p\,\delta\mathcal{D}_L - 2\,\mathcal{K}\,,$$

- ***conservation*** **of the number of galaxies**
- ***source*** **effect:** $e\,\delta z^{(v)}\,,\quad 5p\,\delta\mathcal{D}_L$
- ***volume*** **effect:** $\dfrac{dV_{\rm phy}}{dV_{\rm obs}}\,,\quad dV_{\rm obs} = \dfrac{r^2(z_{\rm obs})}{H(z_{\rm obs})}\,dz_{\rm obs}\,d\Omega_{\rm obs}$
- **distortions:**
  - **redshift:** $1+z_{\rm obs} = (1+z)(1+\delta z)\,,\quad \delta z = V + \phi + \int_0^r d\tilde r\;2\phi'$
  - **radial position:** $\delta\mathcal{R} = -\dfrac{\delta z}{\mathcal{H}} - \int_0^r d\tilde r\;2\phi$
  - **angular position:** $\mathcal{K}$
  - **luminosity distance:** $\delta\mathcal{D}_L\qquad D_L(z_{\rm obs}) = \bar D_L(z_{\rm obs})(1+\delta\mathcal{D}_L)$

Newtonian Correspondence

- **Newtonian correspondence *beyond the horizon***
  - ***no horizon* in Newtonian dynamics**
- **identical dynamical equations up to 2nd order**
  - **matter density → comoving gauge** $\delta_m \to \delta_m^{(v)}$
  - **velocity → Newtonian gauge** $v \to v_\chi = v_N$
  - **potential → Newtonian gauge** $\phi \to \phi_\chi = \phi_N$
  - ***flat* universe with *pressureless* medium**

- **horizon simulations**

Hwang, Noh 1999,2005, Chisari & Zaldarriaga 2011

Kaiser Equation

- connection to the *general relativistic* formula:
  - selection function: $\alpha \equiv \frac{d\ln r^2 \bar{n}_g}{d\ln r} = 2 + \frac{rH}{1+z}(e-3)$

    α *unobservable*, e *observable*

    $$e = \frac{d\ln \bar{n}_{\rm phy}}{d\ln(1+z)}$$

  - line-of-sight velocity: $\mathcal{V} \equiv \frac{1+z}{H} V \simeq \frac{1+z}{H}\,\delta z_\chi$

    $$1+z = (1+\bar{z})(1+\delta z)$$

  - redshift-space distance: $s \equiv \int_0^z \frac{dz}{H} = r + \frac{1+z}{H}\delta z \simeq r + \mathcal{V}$

  - full Kaiser formula: $n_z(s)\; d^3s = n_r(r)\; d^3r$

$$\delta_z = b\,\delta_m - \left(\frac{\partial}{\partial r} + \frac{\alpha}{r}\right)\mathcal{V} = b\,\delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} - e\,V + 2\,V - \frac{2V}{\mathcal{H}r} + \frac{1+z}{H}\frac{dH}{dz}\,V$$

Yoo, Hamaus, Seljak, Zaldarriaga 2012

Connection to the Kaiser

- connection to the *general relativistic* formula:
  - Newtonian correspondence is required
  - velocity is reproduced, *if* luminosity fluctuation is accounted: *important correction (missing)*
  - gravitational redshift-space distortion is cancelled
  - potential contribution is *purely* relativistic
  - validity of the Newtonian theory on horizon scales can only be judged retroactively

Yoo, Hamaus, Seljak, Zaldarriaga 2012

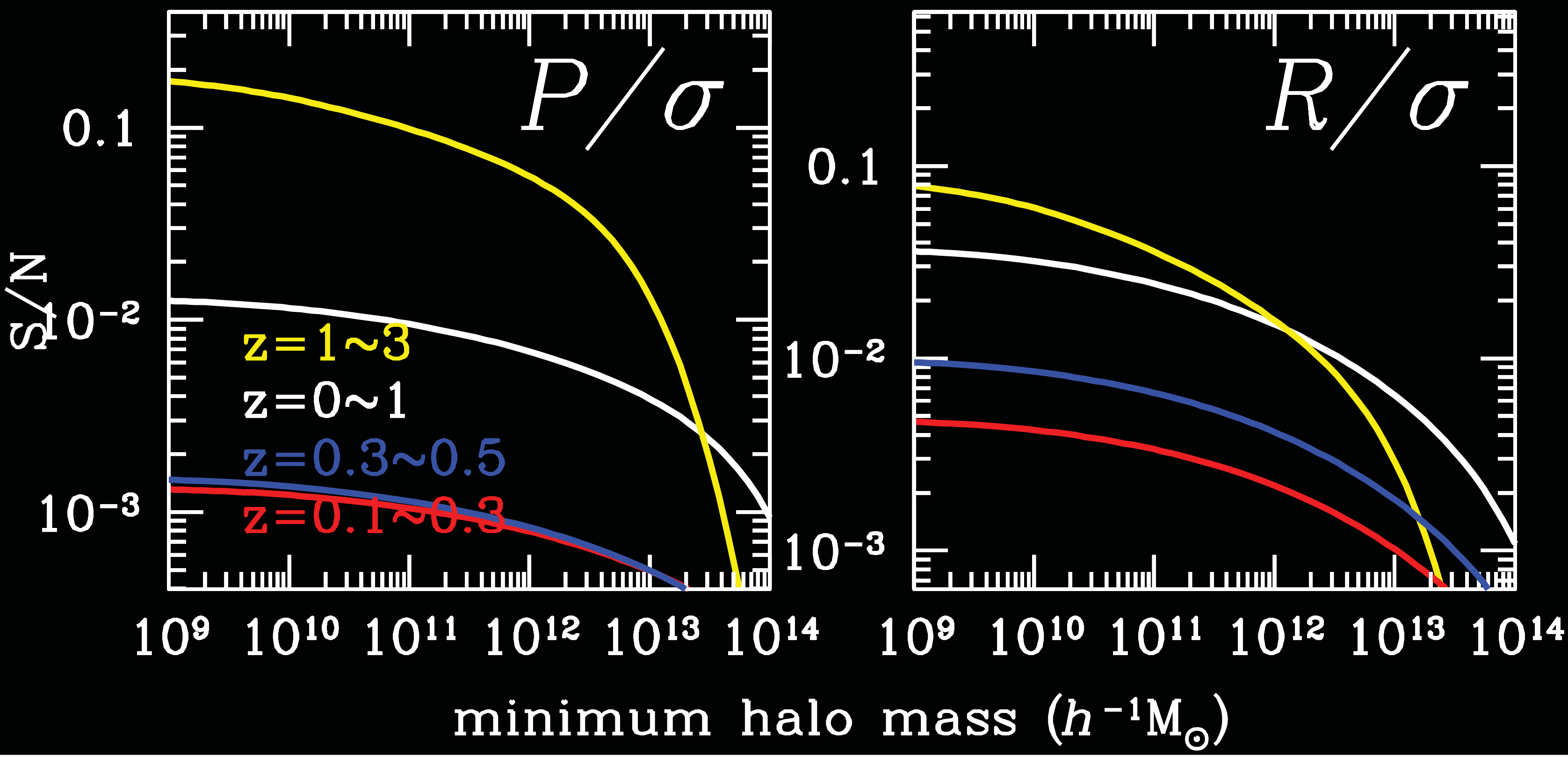
III. RESULTS

Traditional Analysis

- **gravitational potential *P* and velocity *R* terms**
- **corrections: *negligible***

$$\delta_g = \delta_{\rm Newt} + \left[\frac{\mathcal{P}}{(k/\mathcal{H})^2} - i\mu_k \frac{\mathcal{R}}{k/\mathcal{H}}\right]\delta_m$$

Yoo, Hamaus, Seljak, Zaldarriaga 2012



Breakthrough

- **multi-tracer method: eliminate cosmic variance**

Seljak 2009, Hamaus, Seljak, Desjacques 2011

$$\delta_1 = b_1 \delta_m \ , \qquad \mathrm{var}(\delta_1) = b_1^2 \sigma_m^2$$
$$\delta_2 = b_2 \delta_m \ , \qquad \mathrm{var}(\delta_2) = b_2^2 \sigma_m^2$$

$$\frac{\delta_1}{\delta_2} = \frac{b_1}{b_2} \ , \quad \mathrm{var}\left(\frac{\delta_1}{\delta_2}\right) = 0$$

- **optimal weighting:**
  - **reduce stochasticity between halos and dark matter**

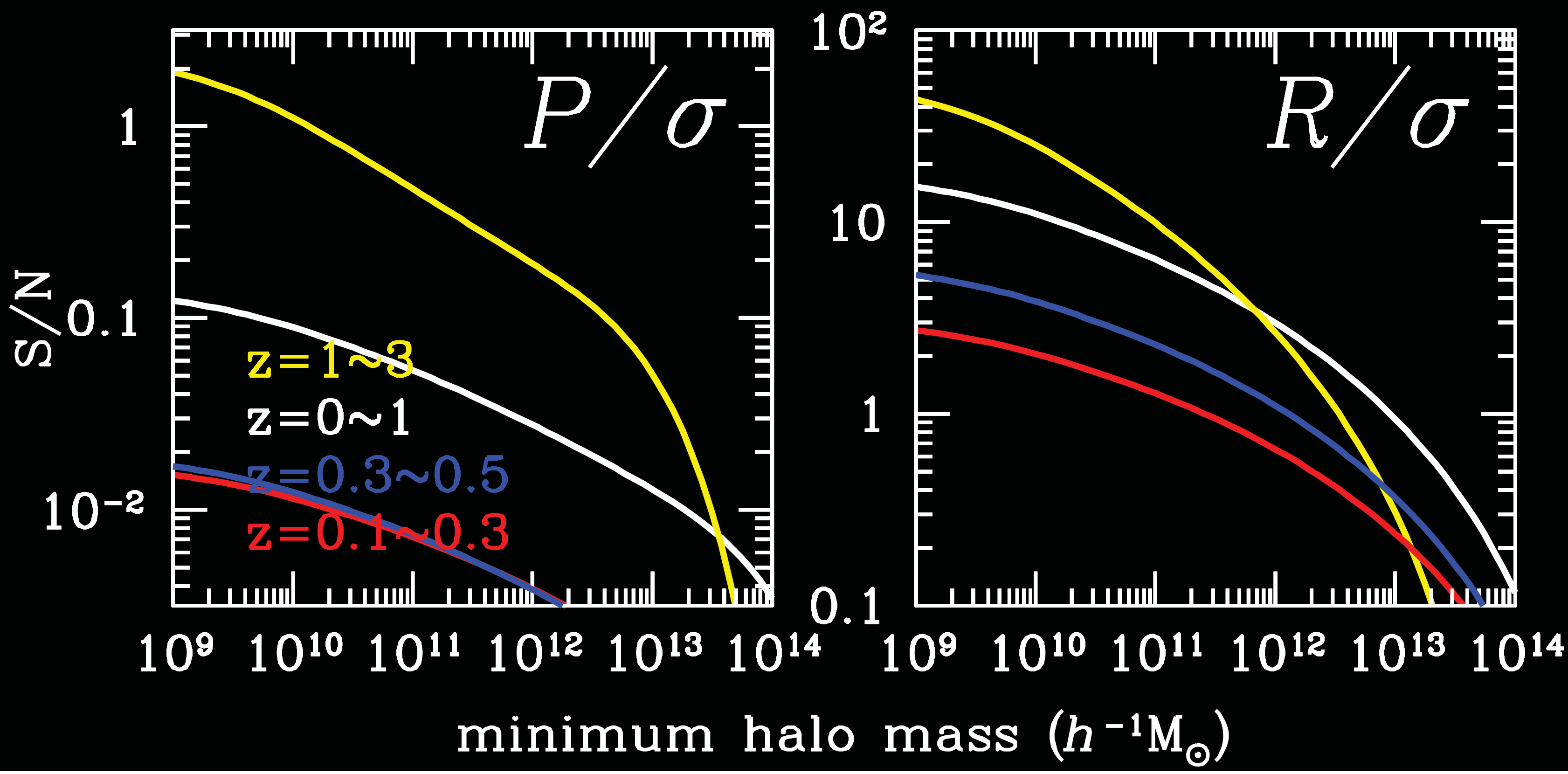
Seljak, Hamaus, Desjacques 2009, Hamaus et al. 2010

Hamaus, Seljak, Desjacques 2012

Measuring GR Effects

- optimal weighting, multiple samples
- corrections: *measurable!*

Yoo, Hamaus, Seljak, Zaldarriaga 2012



Wide Angle Effect?

- **What is "*wide angle*" effect?**
  - **deviation from the distant observer approximation**

$$\mu_1 = \hat{x}_1 \cdot \hat{k} \ , \quad \mu_2 = \hat{x}_2 \cdot \hat{k} \ , \quad \mu = \hat{x} \cdot \hat{k} \qquad \text{vs} \qquad \hat{x}_1 = \hat{x}_2$$

- **In relativistic context: P & R *not wide angle effect***

$$\delta_g = \delta_{\rm Newt} + \left[\frac{\mathcal{P}}{(k/\mathcal{H})^2} - i\mu_k \frac{\mathcal{R}}{k/\mathcal{H}}\right]\delta_m \qquad \delta_{\rm Newt} = b\ \delta_m + f\mu_k^2\delta_m$$

  - ***5p* is also missing in literature**

$$\mathcal{R} = f\left[e - \frac{1+z}{H}\frac{dH}{dz} + (5p-2)\left(1 - \frac{1}{\mathcal{H}r}\right)\right]$$

$$\mathcal{P} = ef - \frac{3}{2}\Omega_m(z)\left[e + f - \frac{1+z}{H}\frac{dH}{dz} + (5p-2)\left(2 - \frac{1}{\mathcal{H}r}\right)\right]$$

Impact on Correlation?

- "*wide angle*" **effect in correlation function?**

$$\delta_g = \delta_{\rm Newt} + \left[\frac{\mathcal{P}}{(k/\mathcal{H})^2} - i\mu_k \frac{\mathcal{R}}{k/\mathcal{H}}\right]\delta_m \qquad \delta_{\rm Newt} = b\ \delta_m + f\mu_k^2\delta_m$$

$$\mathcal{R} = f\left[e - \frac{1+z}{H}\frac{dH}{dz} + (5p-2)\left(1 - \frac{1}{\mathcal{H}r}\right)\right]$$

$$\mathcal{P} = ef - \frac{3}{2}\Omega_m(z)\left[e + f - \frac{1+z}{H}\frac{dH}{dz} + (5p-2)\left(2 - \frac{1}{\mathcal{H}r}\right)\right]$$

- ***wide-angle*** **vs** ***distant-observer*** **correlation functions**

$$\langle\delta_1\delta_2\rangle = \int \frac{d^3k}{(2\pi)^3}\ e^{i\vec{k}\cdot\vec{x}}\ \left(b + f\mu_1^2 + \frac{\mathcal{P}}{(k/\mathcal{H})^2} - i\mu_1\frac{\mathcal{R}}{k/\mathcal{H}}\right) \cdot$$
$$\times\left(b + f\mu_2^2 + \frac{\mathcal{P}}{(k/\mathcal{H})^2} + i\mu_2\frac{\mathcal{R}}{k/\mathcal{H}}\right)P_m(k)$$

$$\langle\delta_1\delta_2\rangle = \int \frac{d^3k}{(2\pi)^3}\ e^{i\vec{k}\cdot\vec{x}}\left|\left(b + f\mu^2 + \frac{\mathcal{P}}{(k/\mathcal{H})^2}\right)^2 + \mu^2\left(\frac{\mathcal{R}}{k/\mathcal{H}}\right)^2\right| P_m(k)$$

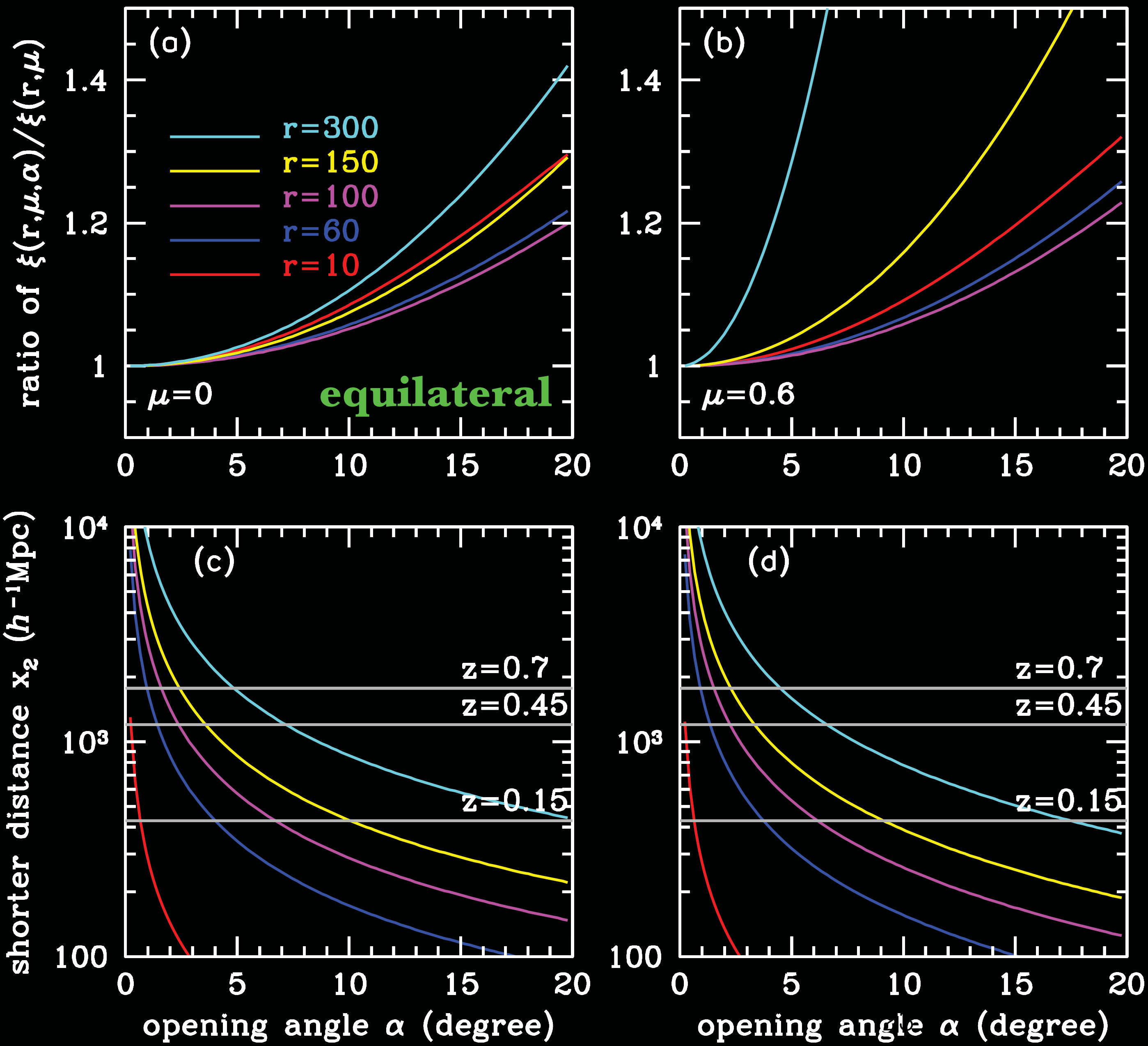
Wide Angle Correlation

- **wide angle correlation function:**

  Szalay, Matsubara, Landy 1998, Szapudi 2004, Papai & Szapudi 2008
  - ***no*** **gravitational potential contribution (*P-term*)**
  - ***5p=0*** **(special case): no luminosity fluctuation**

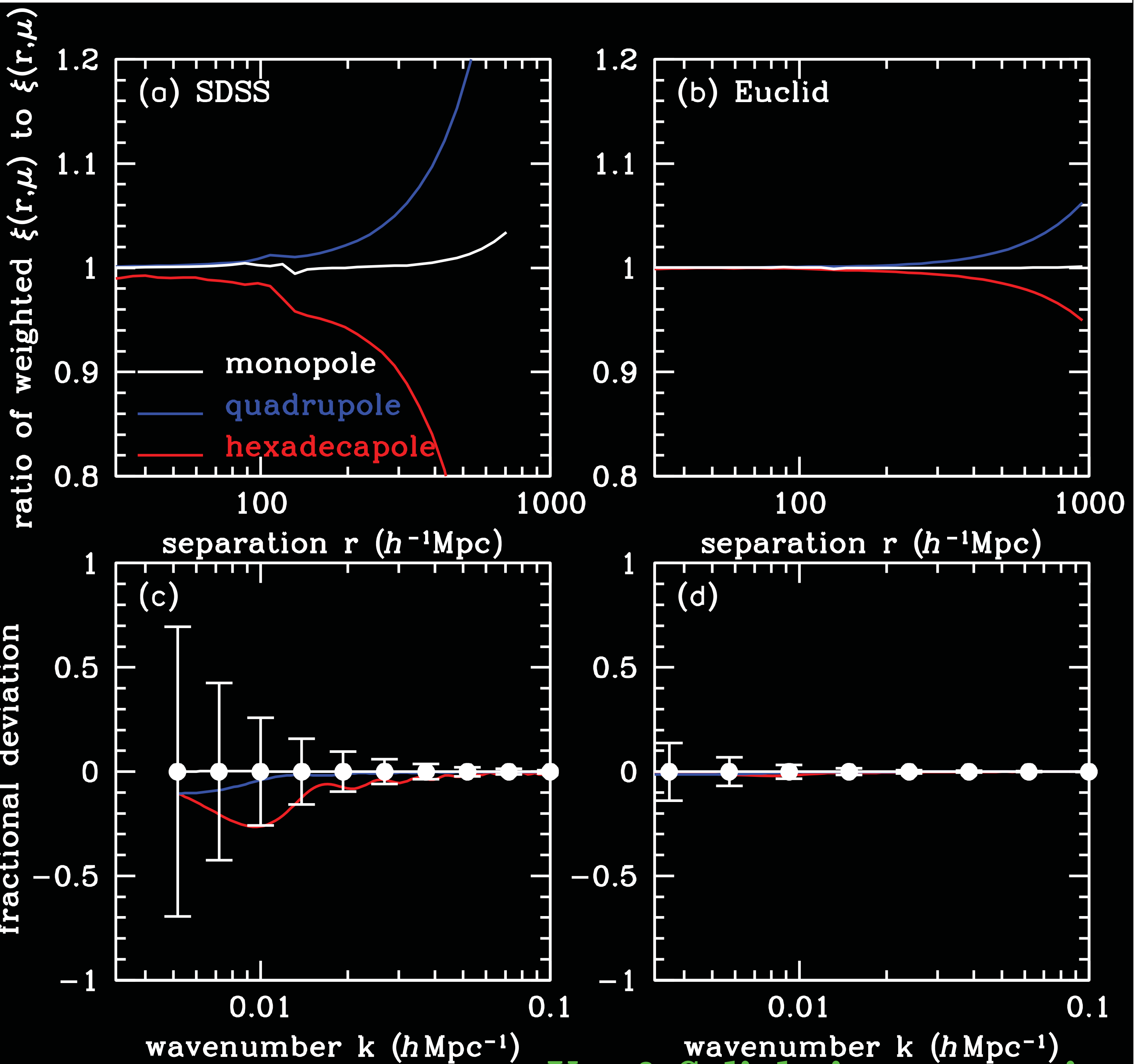
- ***negligible*** **in traditional analysis**

- **deviation: largely from velocity R, not from "*wide angle*"**
- **R ~ *1/r* due to volume effect (r: distance to galaxies)**
- **number of pairs is ~ volume**
- ***no wide-angle galaxy pairs***



Yoo & Seljak, in preparation

- **correlation:** ***excess*** **from the mean**
- **average over all pairs (opening angle) given shape μ & r**
- ***uncertainties in monopole and larger for quadrupole***



Yoo & Seljak, in preparation

Take Home Message

- with single tracer:
  - velocity & potential: practically *negligible*
  - *no* relativistic effect, or wide-angle effect

- with multi-tracer:
  - velocity & potential: potentially *measurable*
  - *tests* of general relativity on horizon scales

Yoo, Hamaus, Seljak, Zaldarriaga 2012

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