The Abdus Salam
International Centre for Theoretical Physics

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Educational, Scientific and
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**2419-10**

**Workshop on Large Scale Structure**

***30 July - 2 August, 2012***

**Halo Clustering Beyond the Local Bias Model**

T. Baldauf
*University of Zurich*

# Halo Clustering beyond the Local Bias Model

Tobias Baldauf

with Vincent Desjacques, Nico Hamaus, Pat McDonald, Uroš Seljak and Robert E. Smith

Institute for Theoretical Physics
University of Zurich

Workshop on Large Scale Structure
ICTP Trieste
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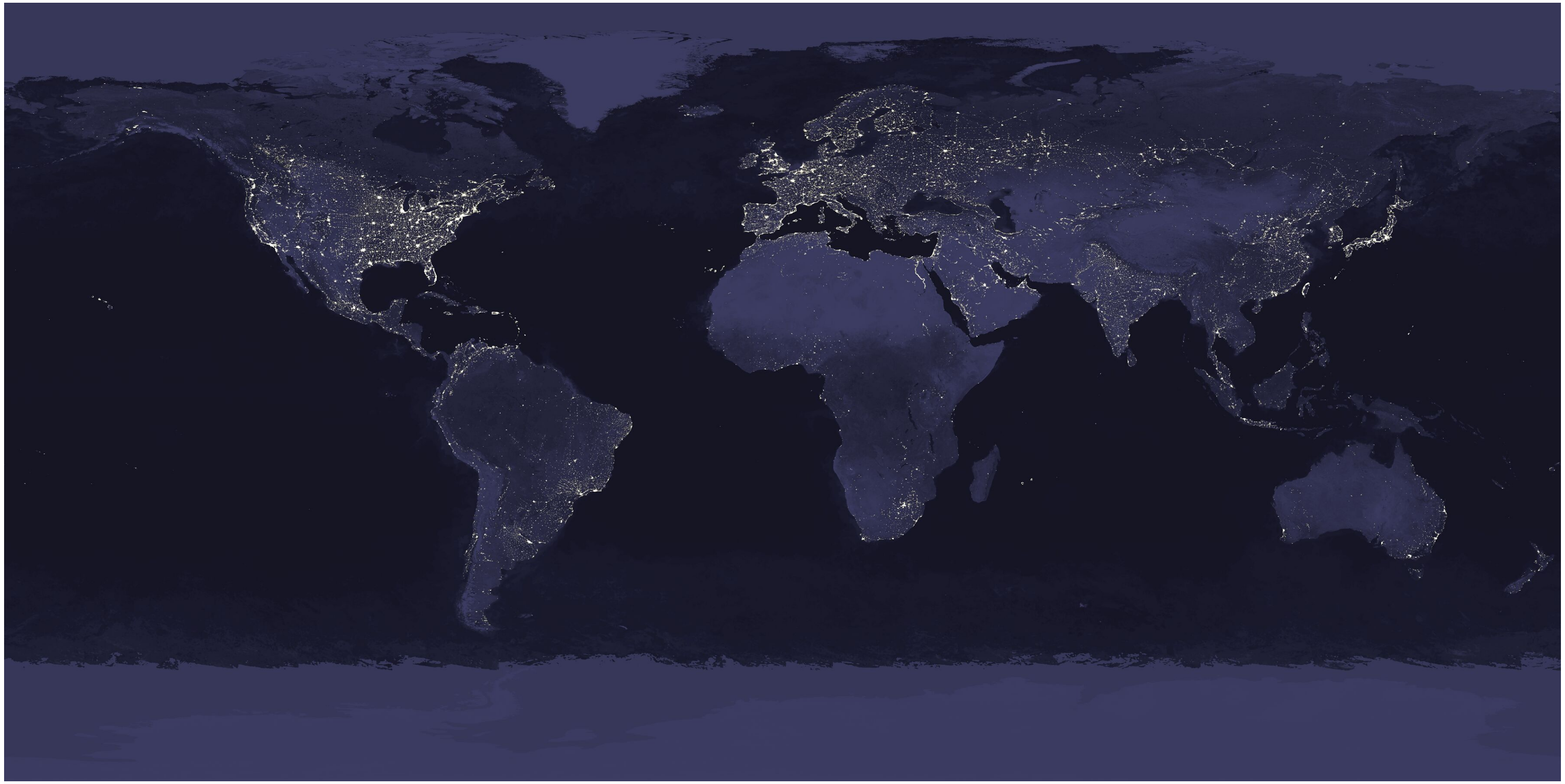
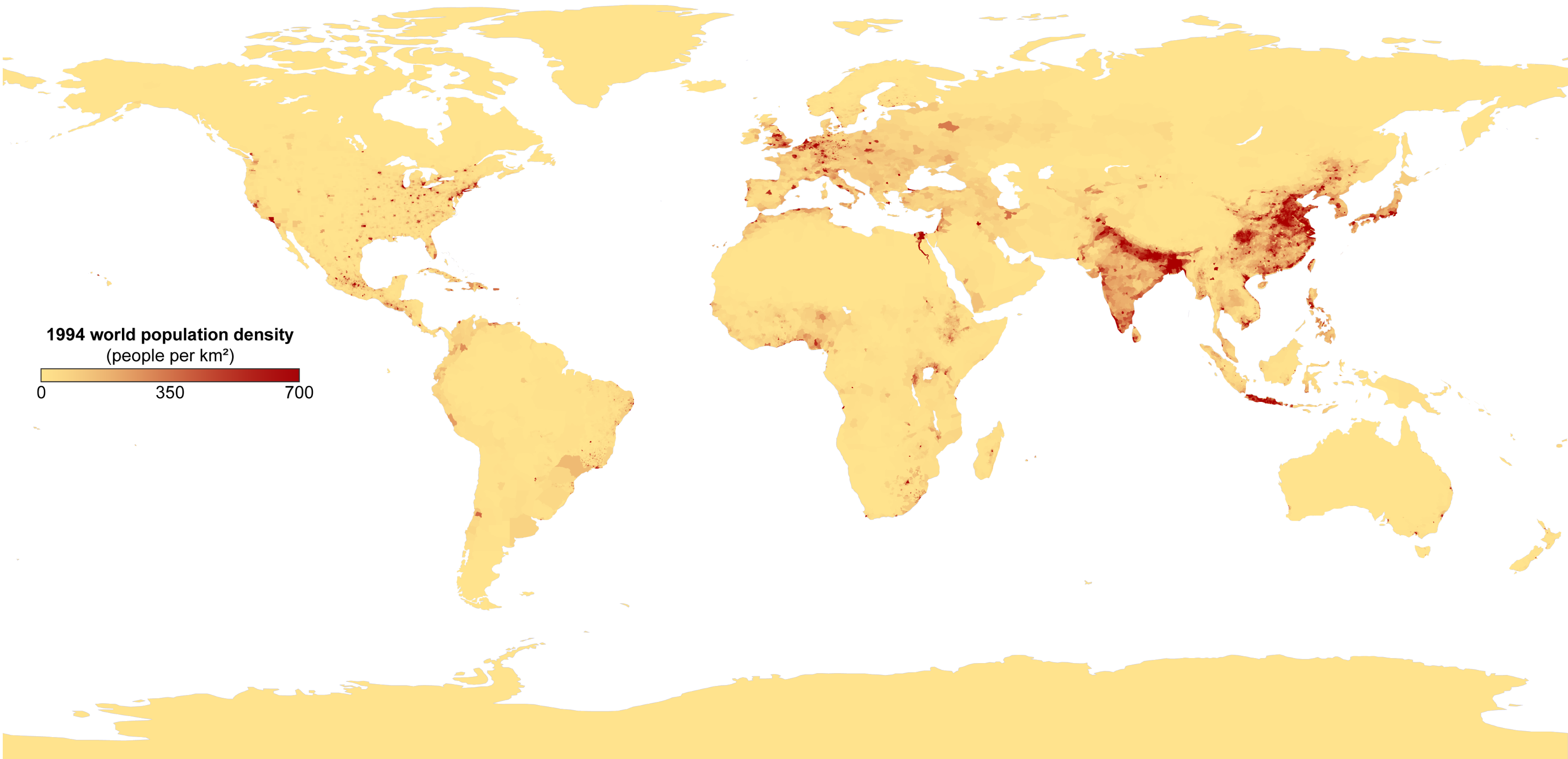
# Light traces Population Density



# Light traces ~~Population Density~~ Money!



# Local Eulerian Bias in Various Statistics

$$\delta_\mathrm{h}(\mathbf{x}) = b_1^{(\mathrm{E})}\delta_\mathrm{m}(\mathbf{x}) + \frac{1}{2!}b_2^{(\mathrm{E})}\delta_\mathrm{m}(\mathbf{x})^2 + \dots$$

## Auto Power Spectrum

$$P_\mathrm{hh}(k) = b_1^2 P_\mathrm{mm}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2}b_2^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

$$P_\mathrm{hm}(k) = b_1 P_\mathrm{mm}(k) + \frac{1}{2}b_2 I_{12}(k)$$

## Cross Bispectrum

$$B_\mathrm{mmh}(k_1, k_2, k_3) = b_1 B_\mathrm{mmm}(k_1, k_2, k_3) + b_2 P(k_1)P(k_2)$$

# Local Eulerian Bias in Various Statistics

$$\delta_{\mathrm{h}}(\mathbf{x}) = b_1^{(\mathrm{E})}\delta_{\mathrm{m}}(\mathbf{x}) + \frac{1}{2!}b_2^{(\mathrm{E})}\delta_{\mathrm{m}}(\mathbf{x})^2 + \dots$$

## Auto Power Spectrum

$$P_{\mathrm{hh}}(k) = {b_1}^2 P_{\mathrm{mm}}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2}{b_2}^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

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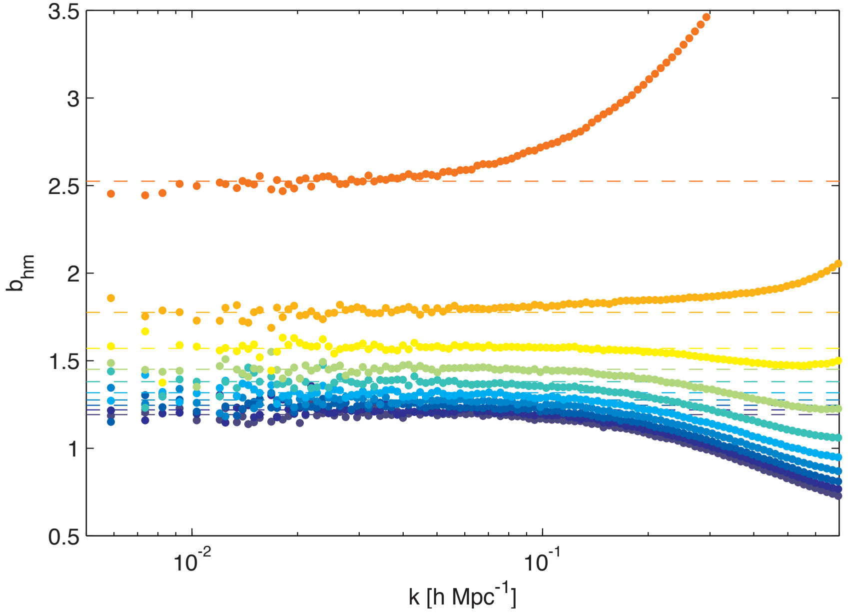
$$B_{\mathrm{mmh}}(k_1, k_2, k_3) = b_1 B_{\mathrm{mmm}}(k_1, k_2, k_3) + b_2 P(k_1)P(k_2)$$

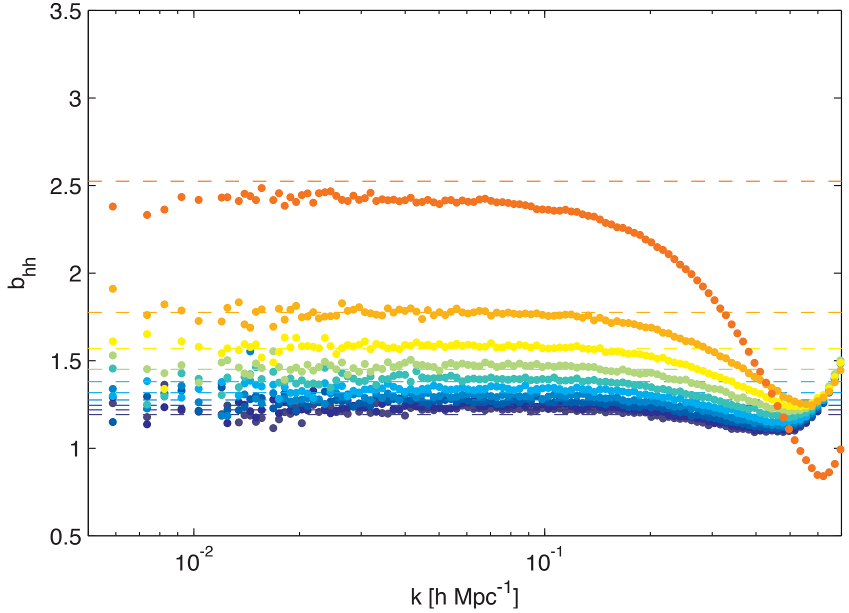
$\Rightarrow$ Bias parameters should be consistent! (at least in low-$k$ regime)

# Are the biases Consistent?

## Bias from Cross Power




## Bias from Auto Power



# The Local Bias Model and its Extensions

## The original Local Model[1]

$$\delta_{\rm h}\left(\mathbf{x},\eta\right) = \mathcal{F}\left[\delta(\mathbf{x}',\eta)\right] \approx b_1\delta(\mathbf{x},\eta) + \frac{1}{2!}b_2\delta^2(\mathbf{x},\eta) + \dots$$

## Tidal Terms allowed by Symmetry[2]

$$s_{ij}(\mathbf{x},\eta) = \left[\frac{\partial_i\partial_j}{\partial^2} - \frac{1}{3}\delta_{ij}^{(\mathrm{K})}\right]\delta(\mathbf{x},\eta)$$

$$\delta_{\rm h}\left(\mathbf{x},\eta\right) = \mathcal{F}\left[\delta(\mathbf{x}',\eta)\right] \approx b_1\delta(\mathbf{x},\eta) + \frac{1}{2!}b_2\delta^2(\mathbf{x},\eta) + b_{s^2}\underbrace{s_{ij}(\mathbf{x})s^{ij}(\mathbf{x})}_{s^2(\mathbf{x})} + \dots$$

[1] [Fry & Gaztanaga 1993]
[2] [McDonald & Roy 2009]

# Coevolution of Haloes and Dark Matter

## Assumptions

- initial bias is local $\delta_{\mathrm{h,i}}(\mathbf{q}) = b_1^{(\mathrm{L})}\delta_{\mathrm{m,i}}(\mathbf{q}) + b_2^{(\mathrm{L})}\delta^2_{\mathrm{m,i}}(\mathbf{q})$
- haloes flow with the dark matter $\mathbf{v}_{\mathrm{h}} = \mathbf{v}_{\mathrm{m}}$ (no velocity bias)

## Coevolution

- mapping from Lagrangian to Eulerian space $\mathbf{x}(\mathbf{q}) = \mathbf{q} + \mathbf{\Psi}(\mathbf{q})$

$$\begin{aligned}\delta_{\mathrm{h}}(\mathbf{x},\eta) = &\left(1 + b_1^{(\mathrm{L})}(\eta)\right)\left({}^{(1)}\delta(\mathbf{x},\eta) + {}^{(2)}\delta(\mathbf{x},\eta)\right) \\ &+ \left(\frac{4}{21}b_1^{(\mathrm{L})}(\eta) + \frac{1}{2}b_2^{(\mathrm{L})}(\eta)\right){}^{(1)}\delta^2(\mathbf{x},\eta) - \frac{2}{7}b_1^{(\mathrm{L})}(\eta)s^2(\mathbf{x},\eta),\end{aligned}$$

# Coevolution of Haloes and Dark Matter

## Assumptions

- initial bias is local $\delta_{\mathrm{h,i}}(\mathbf{q}) = b_1^{(\mathrm{L})}\delta_{\mathrm{m,i}}(\mathbf{q}) + b_2^{(\mathrm{L})}\delta^2_{\mathrm{m,i}}(\mathbf{q})$
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## Coevolution

- mapping from Lagrangian to Eulerian space $\mathbf{x}(\mathbf{q}) = \mathbf{q} + \mathbf{\Psi}(\mathbf{q})$

$$\begin{aligned}\delta_\mathrm{h}(\mathbf{x},\eta) =& b_1^{(\mathrm{E})}(\eta)\left({}^{(1)}\delta(\mathbf{x},\eta) + {}^{(2)}\delta(\mathbf{x},\eta)\right) \\ &+ \frac{1}{2}b_2^{(\mathrm{E})}(\eta){}^{(1)}\delta^2(\mathbf{x},\eta) \underbrace{-\frac{2}{7}b_1^{(\mathrm{L})}(\eta)}_{b_{s^2}} s^2(\mathbf{x},\eta),\end{aligned}$$

# Imprint on the Bispectrum[3]

## Unsymmetrized Cross Bispectrum

$$B_{\mathrm{mmh}}^{(\mathrm{unsym})}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\ (2\pi)^3\delta^{(\mathrm{D})}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) = \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta_{\mathrm{h}}(\mathbf{k}_3)\rangle\,,$$

## $2^{\mathrm{nd}}$ order Bias + Tidal Terms

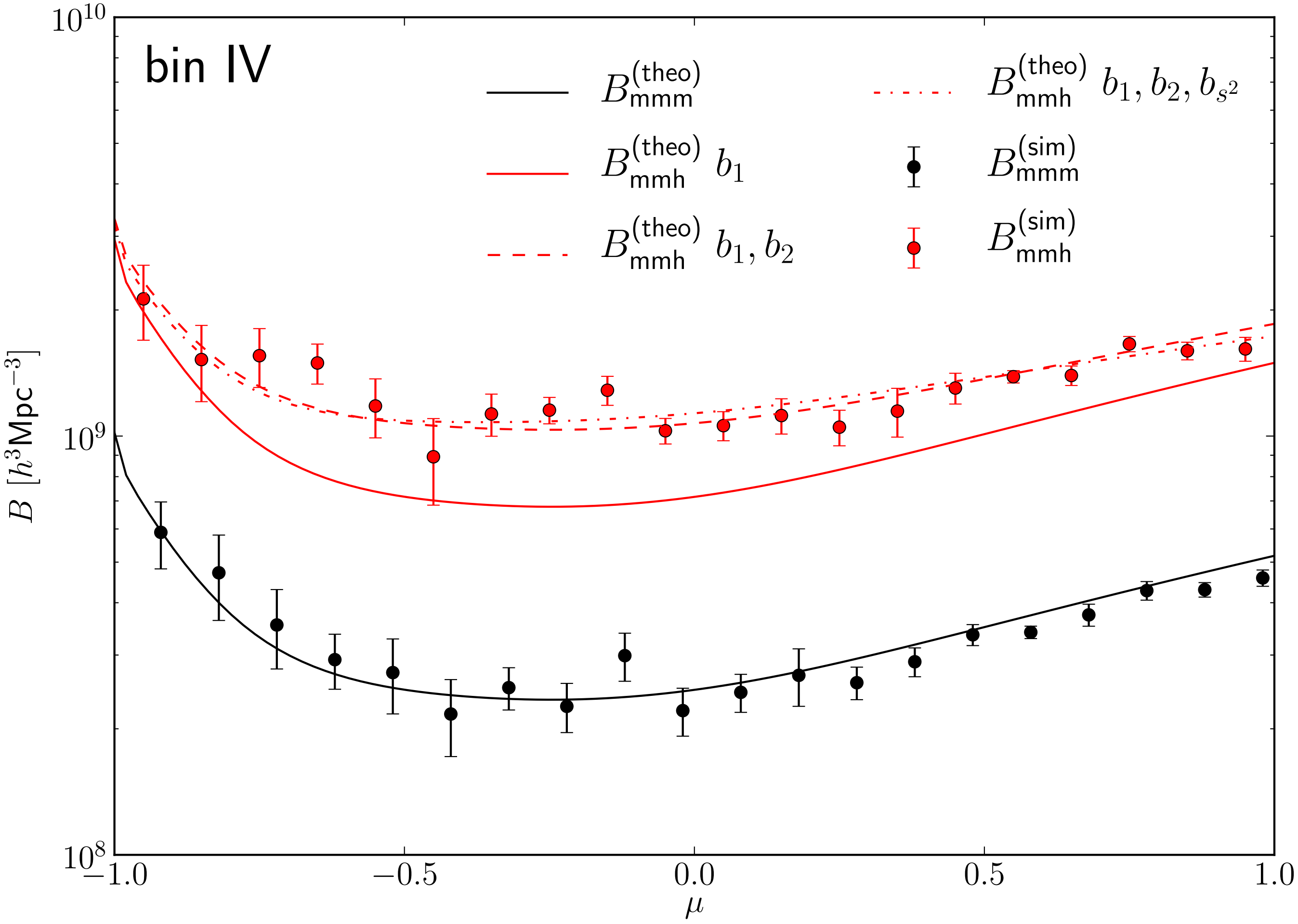
$$B_{\mathrm{mmh}}^{(\mathrm{unsym})}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) - b_1 B_{\mathrm{mmm}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2P(k_1)P(k_2)\left[b_2 + b_{s^2}\left(\mu^2 - \frac{1}{3}\right)\right].$$

## Isolate Angular Dependence

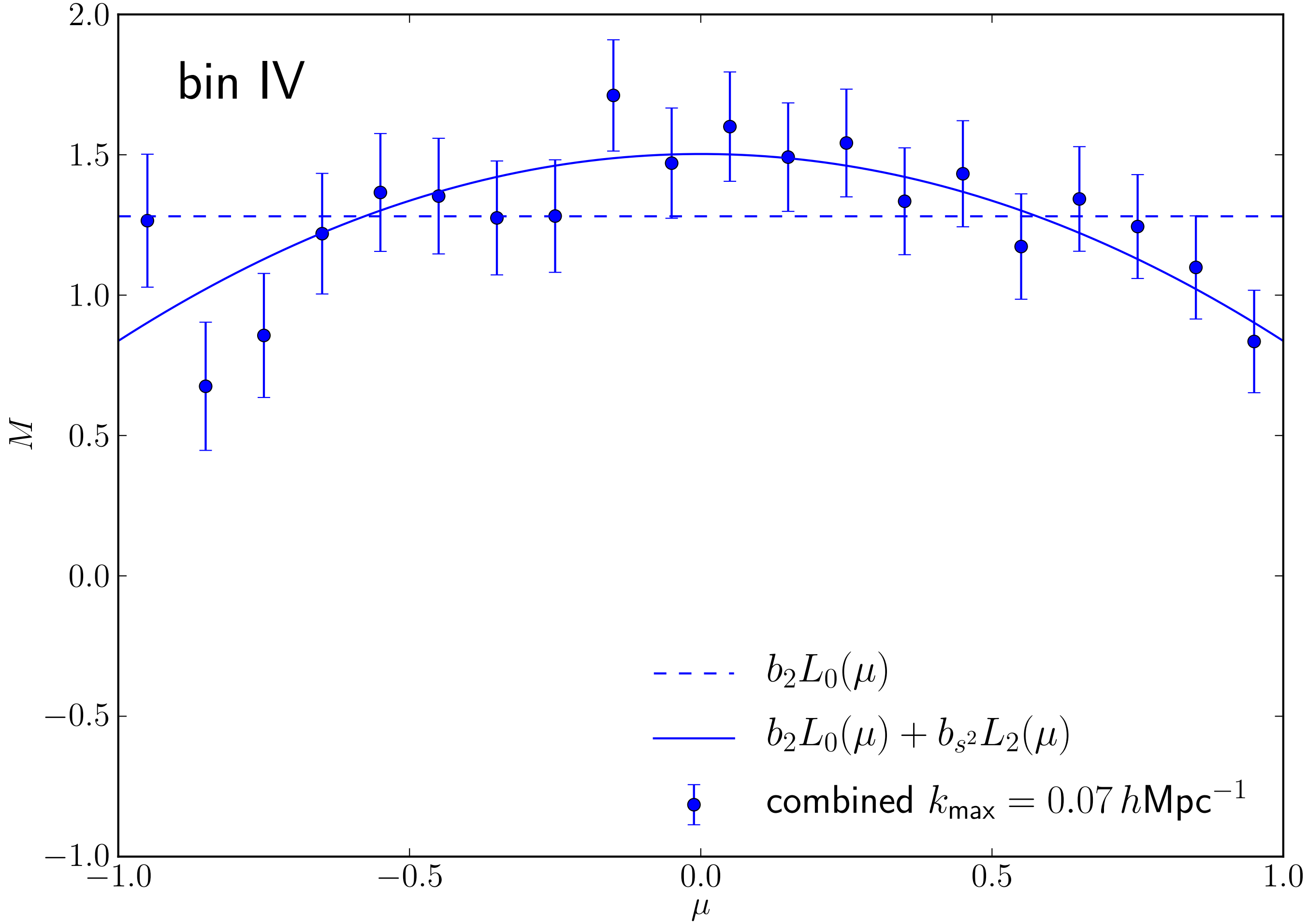
$$M(\mu) = \frac{B_{\mathrm{mmh}}^{(\mathrm{unsym})}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) - b_1 B_{\mathrm{mmm}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{2P(k_1)P(k_2)} = b_2 L_0(\mu) + b_{s^2} L_2(\mu).$$

[3] see also [Kwan, Scoccimarro & Sheth 2012]

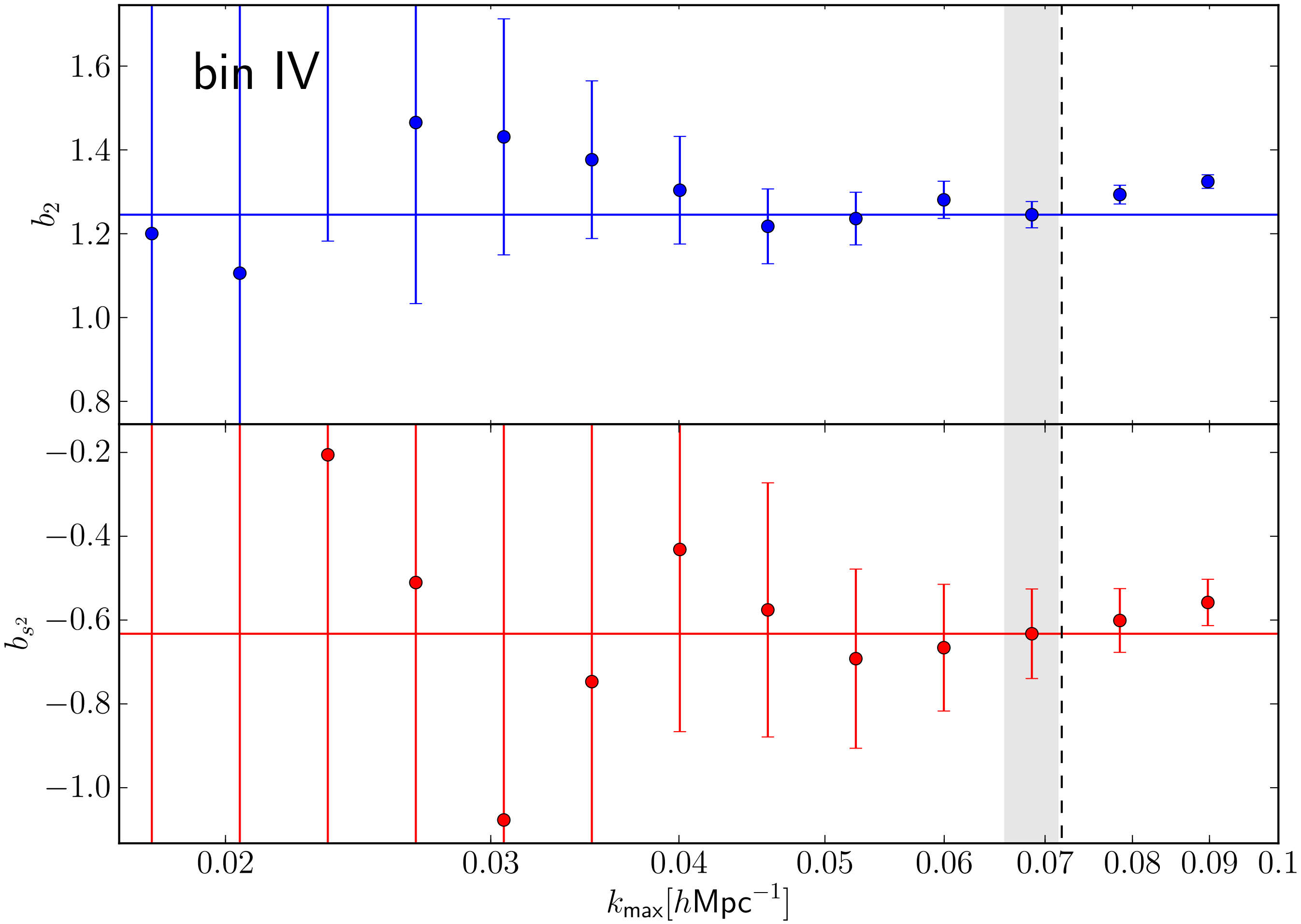
# Imprint on the Bispectrum



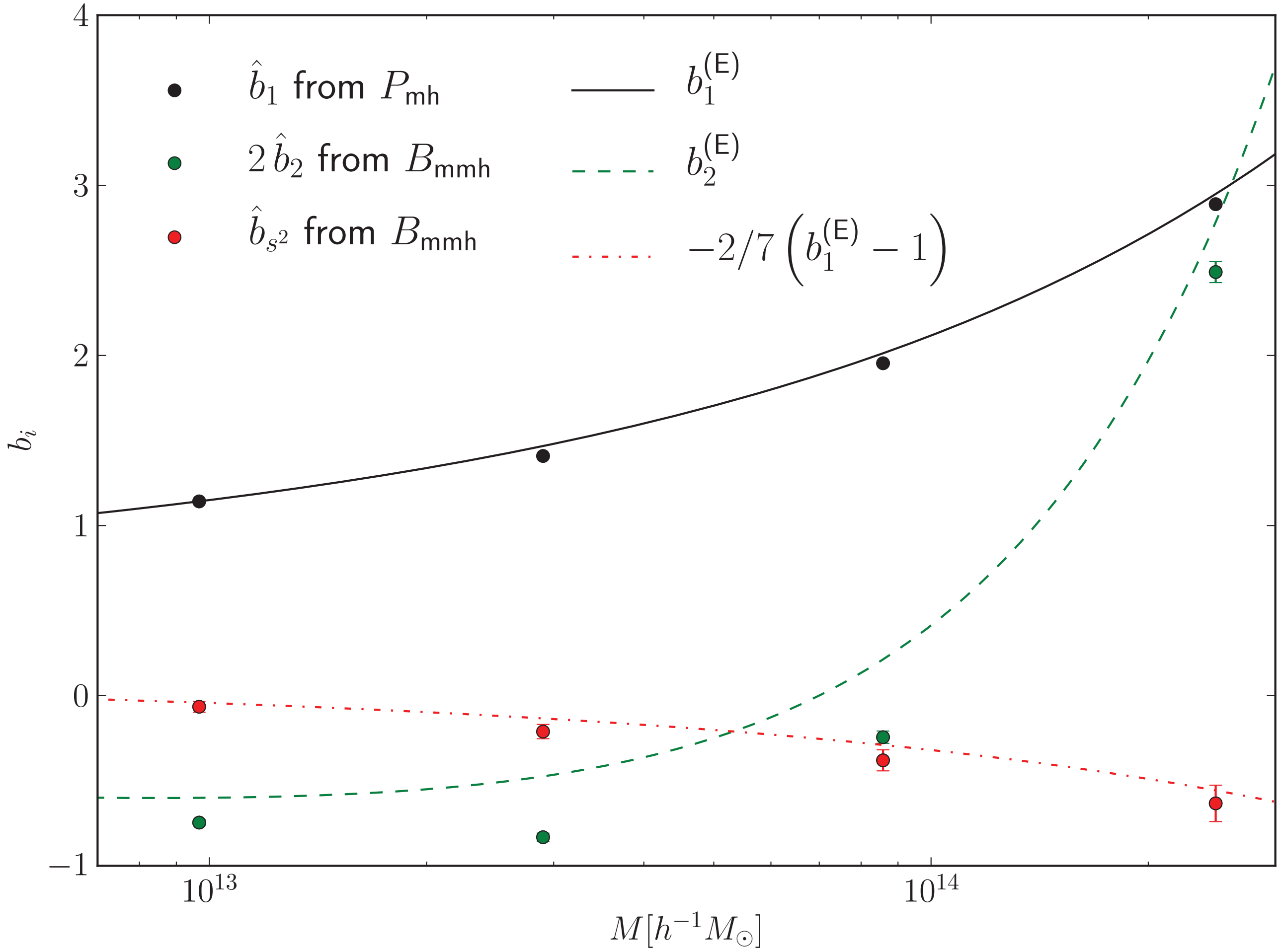
# Imprint on the Bispectrum



# Imprint on the Bispectrum



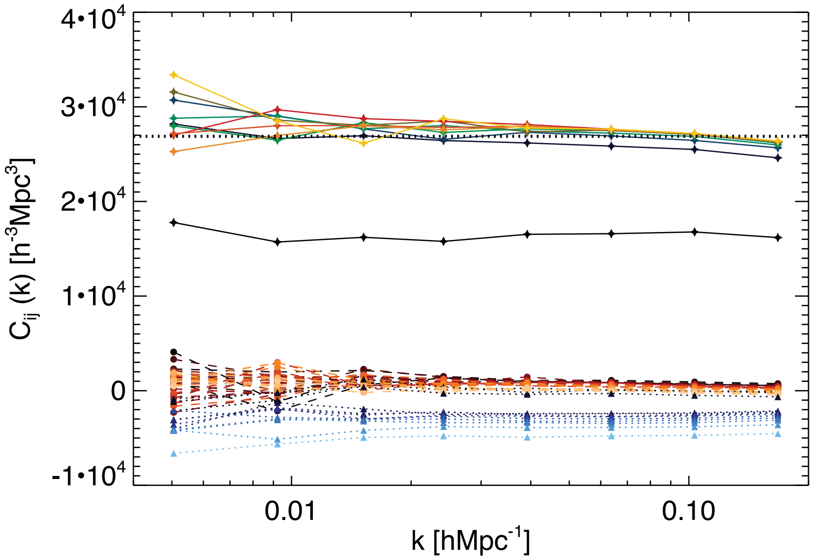
# Imprint on the Bispectrum



# Motivation & Approach

## Motivation

- sub-Poissonian shotnoise in [Hamaus et al. 2009]
- discrepancies between $\hat{b}_{\text{hh}}$ and $\hat{b}_{\text{hm}}$ [Okumura et al. 2012]
- presence of large scale corrections in the perturbative local bias model

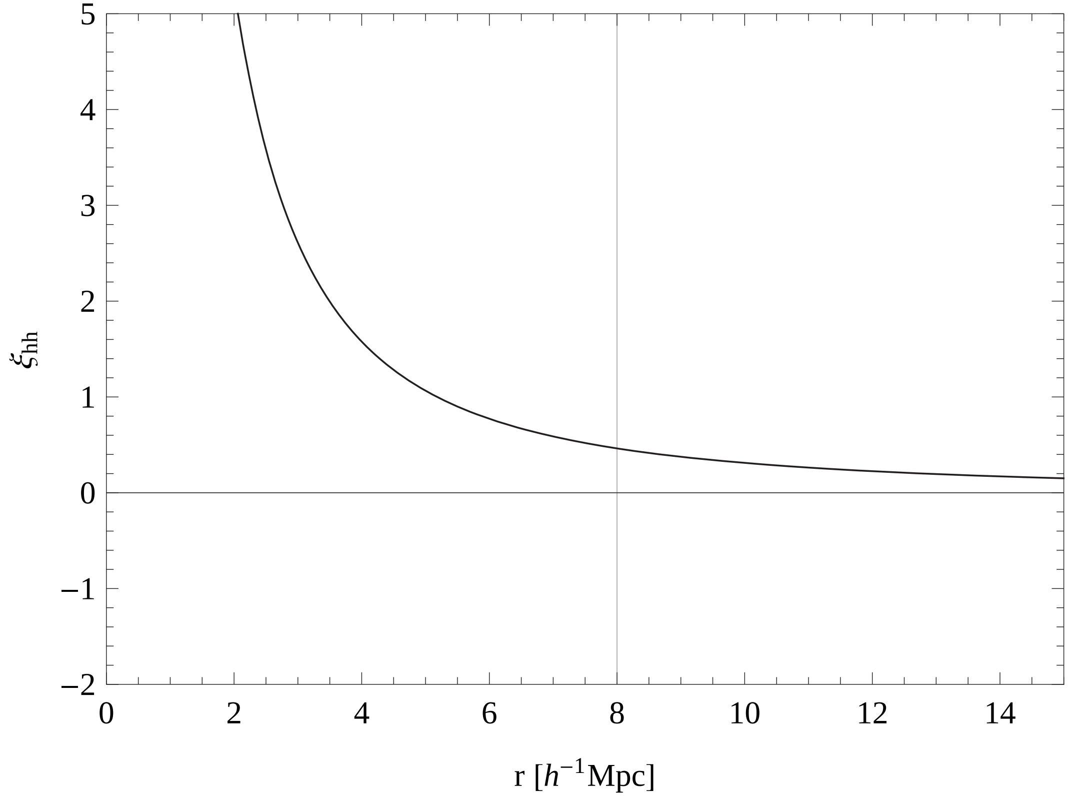



[Hamaus et al. 2009]

## Approach

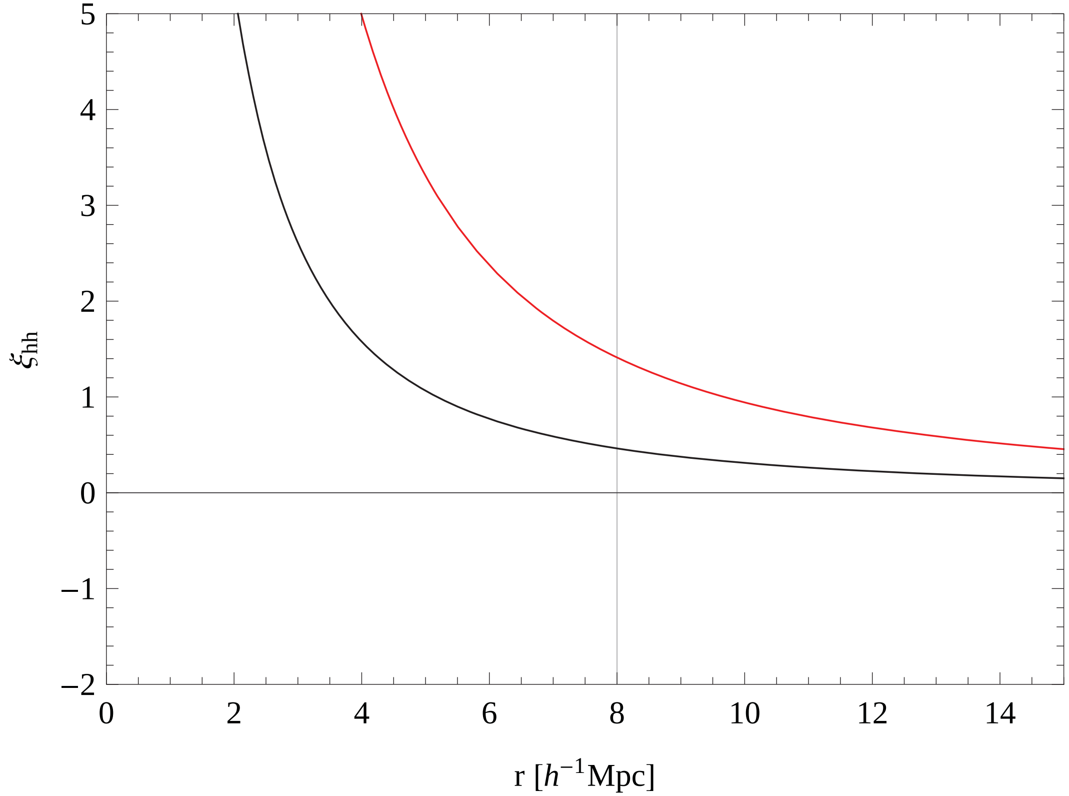
- noise: all deviations from linear bias power spectrum in the $k \to 0$ limit
- small scale correlation function $\leftrightarrow$ large scale power spectrum

# Phenomenology of the Small Scale Correlation Function



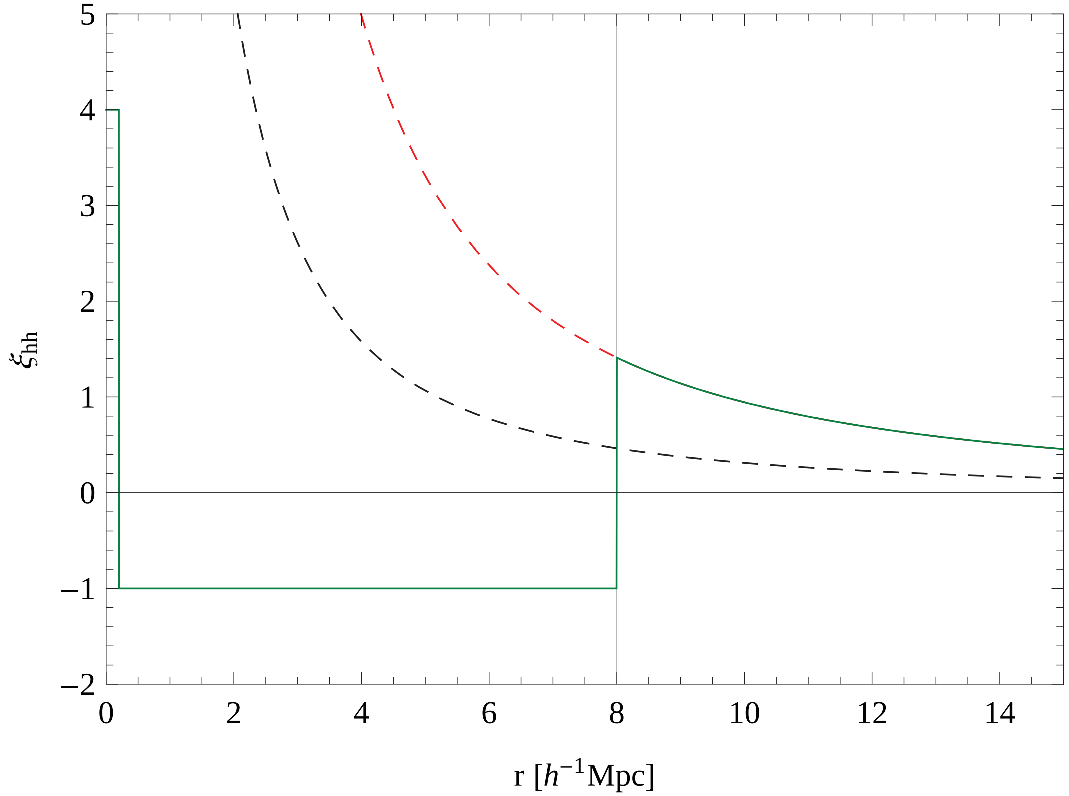

$$\xi_{\rm hh} = \text{linear bias}$$

# Phenomenology of the Small Scale Correlation Function



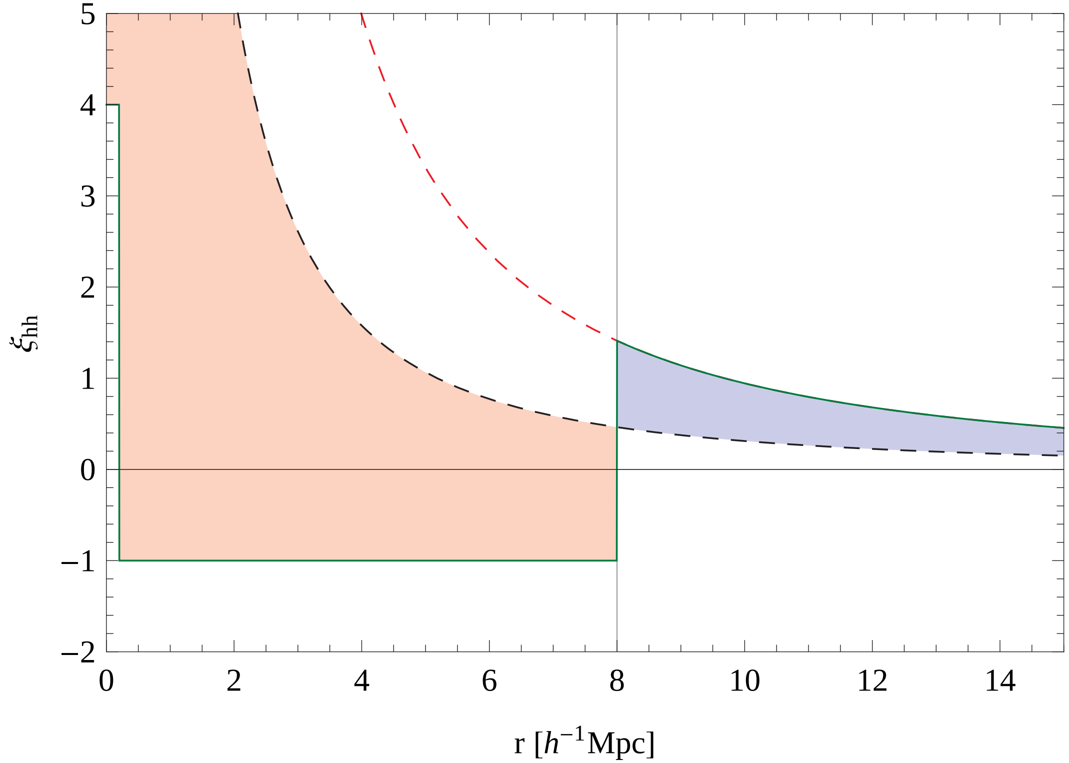

$$\xi_{hh} = \text{linear bias} + \text{non-linear bias}$$

# Phenomenology of the Small Scale Correlation Function




$$\xi_{\rm hh} = \text{linear bias} + \text{non-linear bias} - \text{exclusion} + \text{shot noise}$$

# Phenomenology of the Small Scale Correlation Function




$P_{\rm hh}$ = linear bias+shot noise+non-linear bias correction−exclusion correction

# Power Spectrum of Discrete Tracers

## Density Perturbation - real-space

$$\delta^{(\mathrm{d})}(\mathbf{r}) = \frac{n(\mathbf{r})}{\bar{n}} - 1 = \frac{1}{\bar{n}} \sum_i \delta^{(\mathrm{D})}(\mathbf{r} - \mathbf{r}_i) - 1,$$

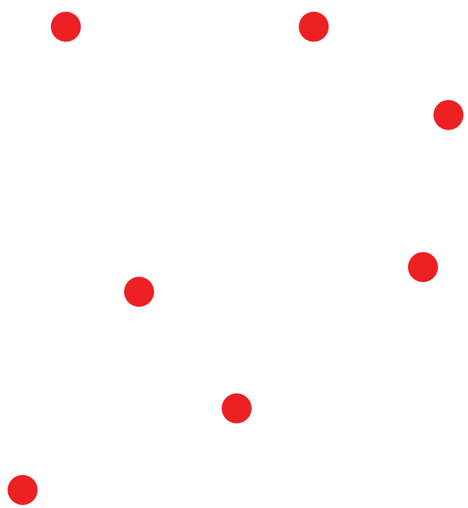
## Density Perturbation - *k*-space

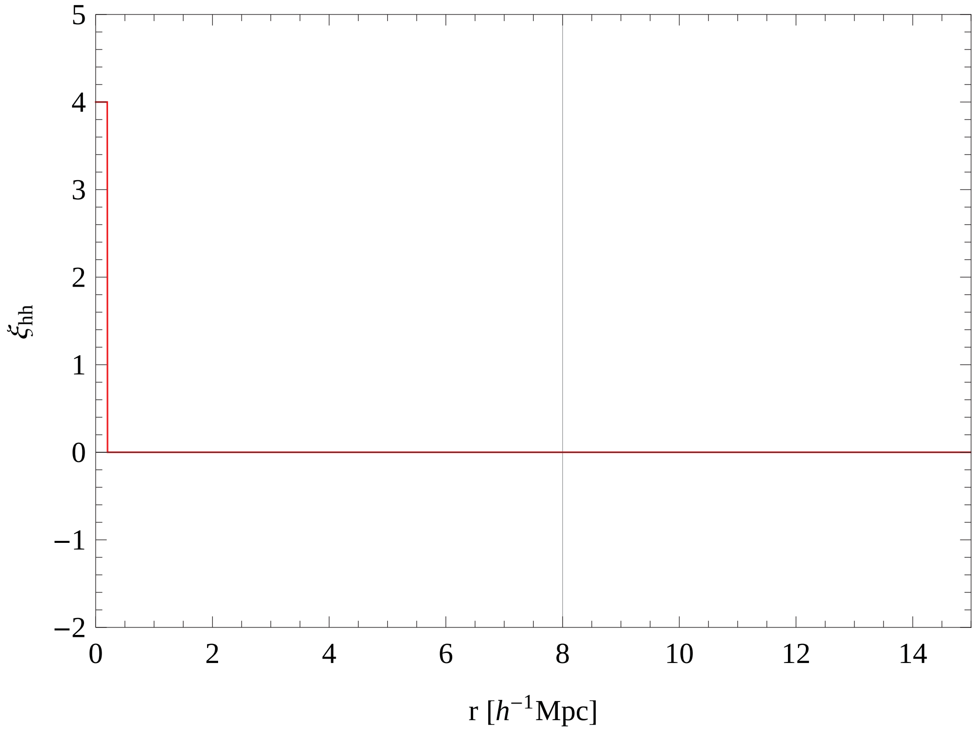
$$\delta^{(\mathrm{d})}(\mathbf{k}) = \int \mathrm{d}^3 r \exp\left[\mathrm{i}\mathbf{k} \cdot \mathbf{r}\right] \delta(\mathbf{r}) = \frac{1}{\bar{n}} \sum_i \exp\left[\mathrm{i}\mathbf{k} \cdot \mathbf{r}_i\right] - V \delta^{(\mathrm{K})}_{\mathbf{k},\mathbf{0}}.$$

## Power Spectrum

$$P^{(\mathrm{d})}(\mathbf{k}) = \frac{1}{\bar{n}} + \frac{V}{N^2} \sum_{i \neq j} \left\langle \exp\left[i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)\right] \right\rangle - V \delta^{(\mathrm{K})}_{\mathbf{k},\mathbf{0}}$$

# Toy Model: Random Sample with Exclusion





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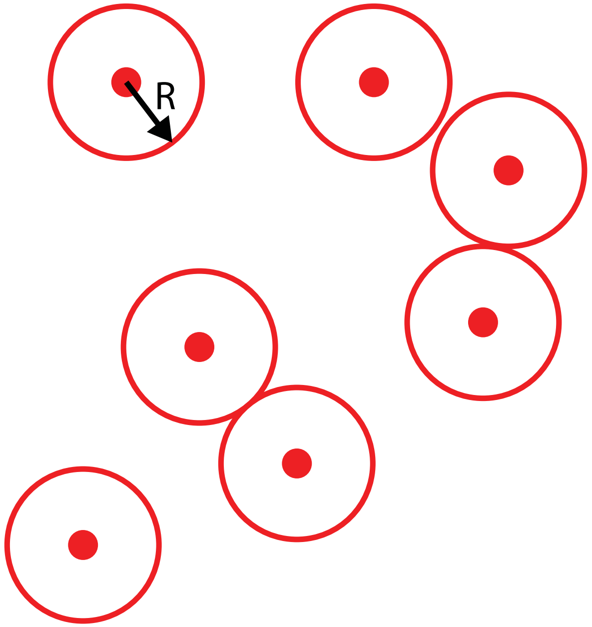


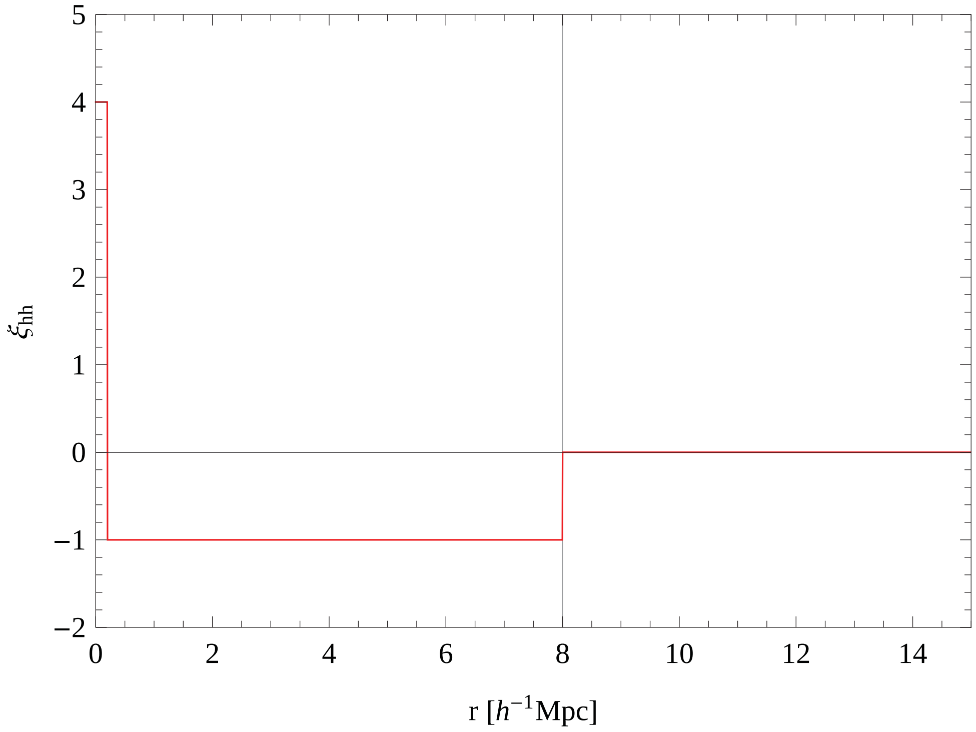

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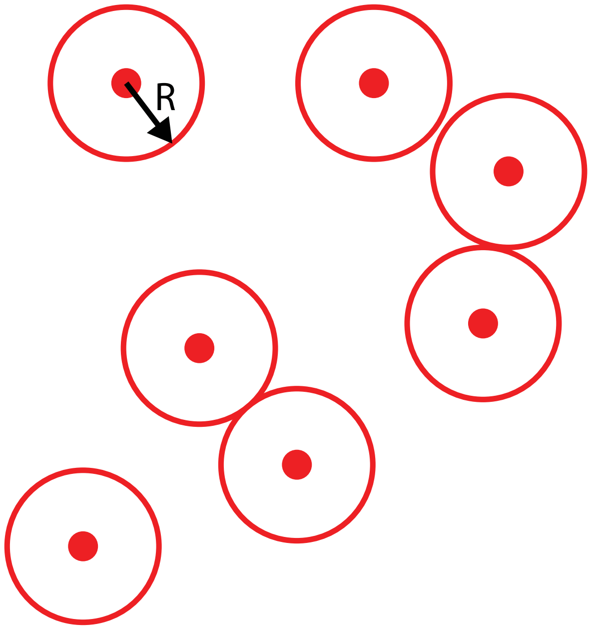
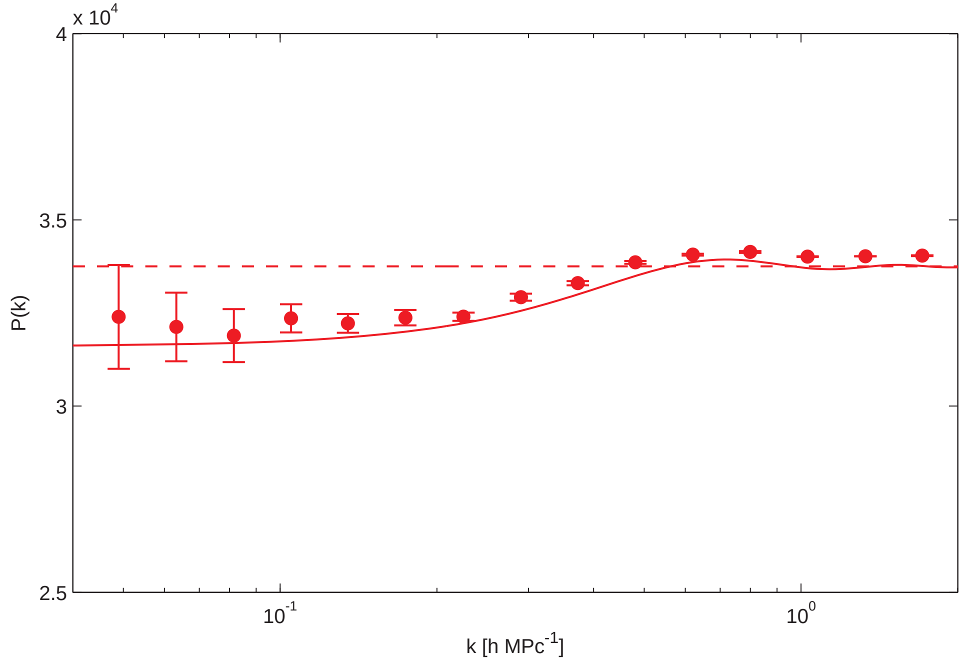
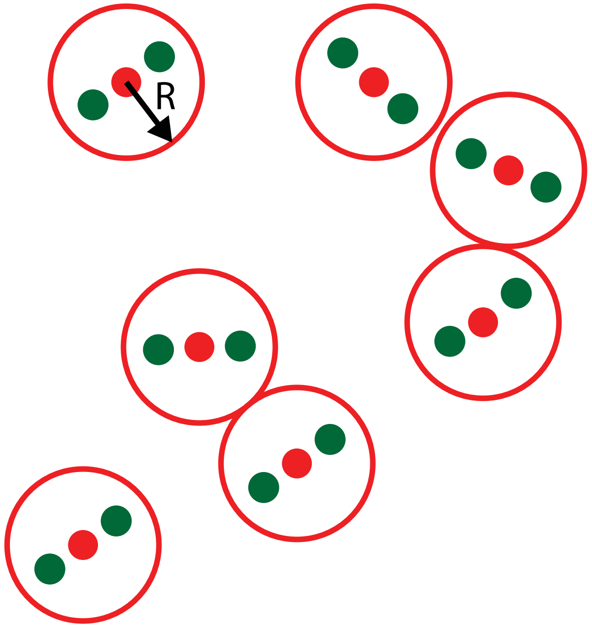
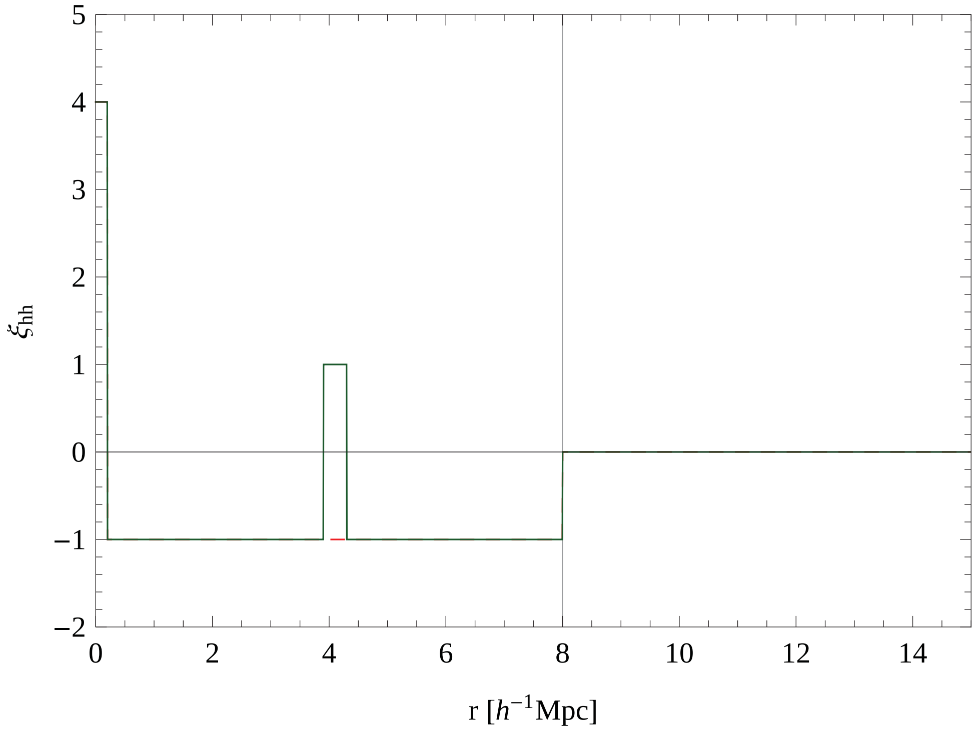


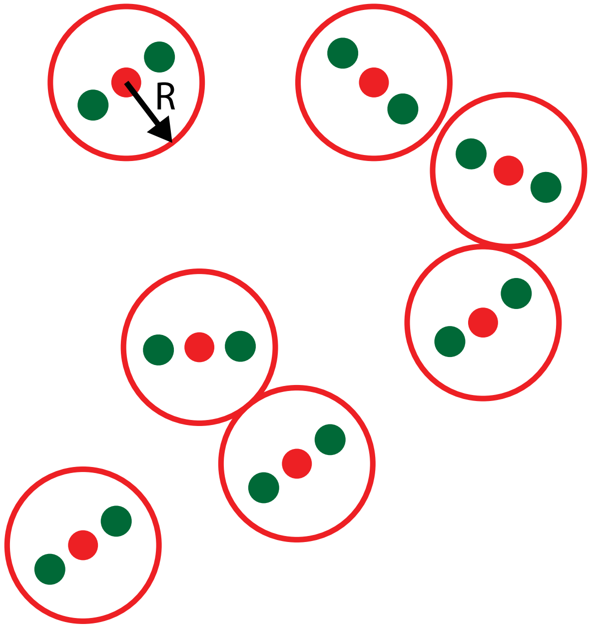

$$P_{\text{hh}}(k) = \frac{1}{\bar{n}} - \frac{4\pi R^3}{3} W_R(k)$$

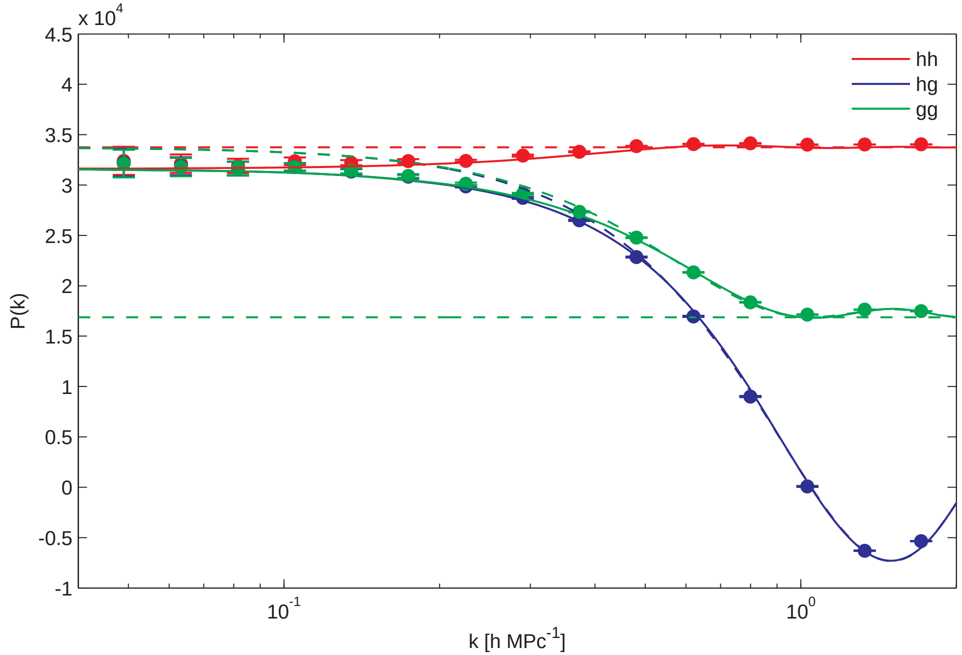
# Toy Model: Random Sample with Exclusion



# Toy Model: Random Sample with Exclusion




$$P_{\mathrm{gg}}(k) = \frac{1}{\bar{n}_{\mathrm{g}}}\left(1 + (N_{\mathrm{g,h}} - 1)\frac{\sin(kR_{\mathrm{g}})}{kR_{\mathrm{g}}}\right) - \frac{4\pi R^3}{3} W_R(k)$$

# Small Scale Exclusion

## Random Sample

$$\xi_{\mathrm{hh}}^{(\mathrm{d})}(r) = \begin{cases} -1 & r < R \\ 0 & r \geq R \end{cases} \qquad P_{\mathrm{hh}}^{(\mathrm{d})}(k) = \frac{1}{\bar{n}} - V_{\mathrm{excl}} W_R(k)$$

## Correlated Sample

$$\xi_{\mathrm{hh}}^{(\mathrm{d})}(r) = \begin{cases} -1 & r < R \\ \xi_{\mathrm{hh}}^{(\mathrm{c})}(r) & r \geq R \end{cases} \qquad P_{\mathrm{hh}}^{(\mathrm{d})}(k) = \frac{1}{\bar{n}} - V_{\mathrm{excl}} W_R(k)$$

$$+P_{\mathrm{hh}}^{(\mathrm{c})}(k) - V_{\mathrm{excl}}[P_{\mathrm{hh}}^{(\mathrm{c})} * W_R](k)$$

$$W_R(k) = 3\frac{\sin(kR) - kR\cos(kR)}{(kR)^3}$$

# Halo Power Spectra in the Local Bias Model[4]

## Halo-Halo

$$
\begin{aligned}
P_{\text{hh}}^{(\text{c})}(k) =& b_{1,\text{E}}^2 P_{\text{1-loop}}(k) \\
&+ b_{1,\text{E}} b_{2,\text{E}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k}-\mathbf{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k}-\mathbf{q}|) \\
&+ \frac{1}{2} b_{2,\text{E}}^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k}-\mathbf{q}|)
\end{aligned}
$$

## Halo-Matter

$$
\begin{aligned}
P_{\text{hm}}^{(\text{c})}(k, \eta_i) =& b_{1,\text{E}} P_{\text{1-loop}}(k) \\
&+ b_{2,\text{E}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k}-\mathbf{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k}-\mathbf{q}|)
\end{aligned}
$$

[4] [McDonald 2006]

# Halo Power Spectra in the Local Bias Model[4]

## Halo-Halo

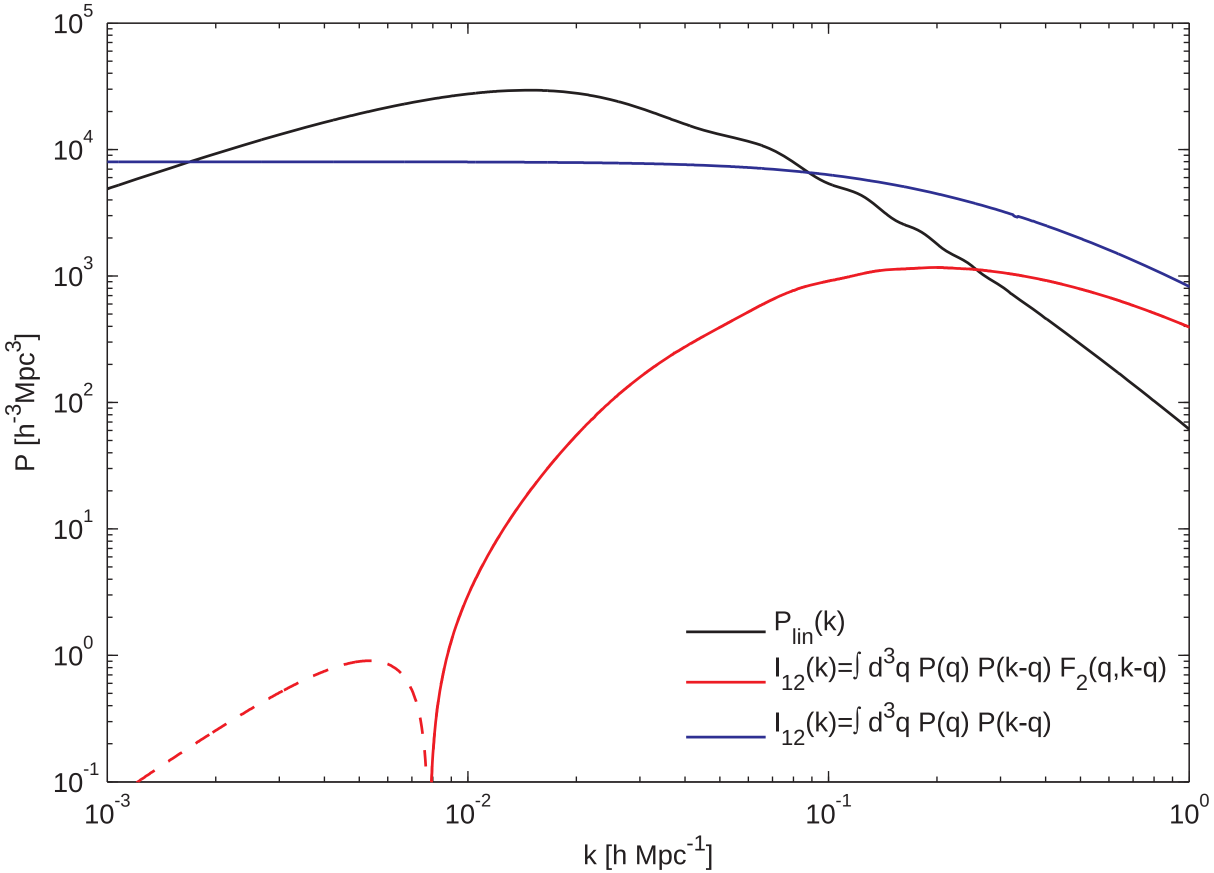
$$P^{(c)}_{\mathrm{hh,i}}(k) = b_{1,\mathrm{L}}^2 P_{\mathrm{lin,i}}(k) + \cancel{b_{1,\mathrm{L}} b_{2,\mathrm{L}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_2(\mathbf{q},\mathbf{k}-\mathbf{q}) P_{\mathrm{lin,i}}(q) P_{\mathrm{lin,i}}(|\mathbf{k}-\mathbf{q}|)} + \frac{1}{2} b_{2,\mathrm{L}}^2 \underbrace{\int \frac{\mathrm{d}^3 q}{(2\pi)^3} P_{\mathrm{lin,i}}(q) P_{\mathrm{lin,i}}(|\mathbf{k}-\mathbf{q}|)}_{I_{22}(k)}$$

## Halo-Matter

$$P^{(c)}_{\mathrm{hm,i}}(k) = b_{1,\mathrm{L}} P_{\mathrm{lin,i}}(k) + \cancel{b_{2,\mathrm{L}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_2(\mathbf{q},\mathbf{k}-\mathbf{q}) P_{\mathrm{lin,i}}(q) P_{\mathrm{lin,i}}(|\mathbf{k}-\mathbf{q}|)}$$

[4] [McDonald 2006]

# Scale Dependence of the Terms



# Halo Power Spectra Including Exclusion

Perturbation Theory + Exclusion

$$P^{(\mathrm{d})}_{\mathrm{hh,i}}(k) = \frac{1}{\bar{n}} + b_1^2 P_{\mathrm{lin,i}}(k) + \frac{1}{2} b_2^2 I_{22}(k)$$
$$- V_{\mathrm{excl}} W_R(k) - b_1^2 V_{\mathrm{excl}} [P_{\mathrm{lin,i}} * W_R](k) - \frac{1}{2} b_2^2 V_{\mathrm{excl}} [I_{22} * W_R](k)$$

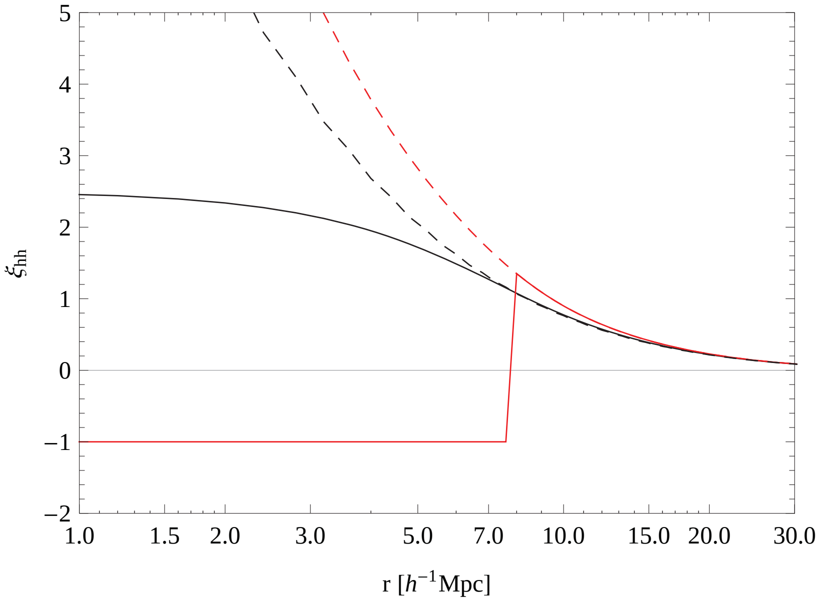
$k \to 0$ limit

$$P^{(\mathrm{d})}_{\mathrm{hh,i}}(k \to 0) = \frac{1}{\bar{n}} + \frac{1}{2} b_2^2 \int_R^\infty \mathrm{d}^3 r \, \xi^2_{\mathrm{lin,i}}(r) - V_{\mathrm{excl}} - b_1^2 \int_0^R \mathrm{d}^3 r \, \xi_{\mathrm{lin,i}}(r)$$

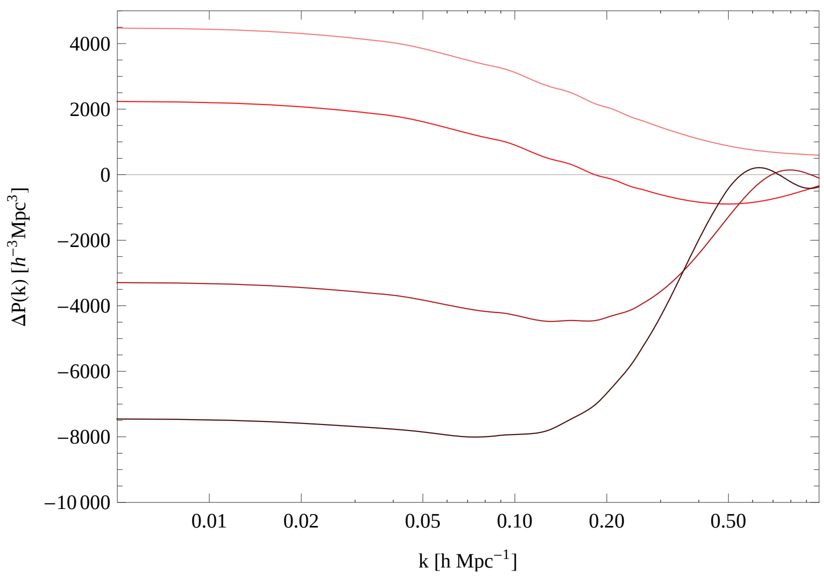
# Correlation of Thresholded Regions[5]

$$1+\xi_{\rm tr}(r)=\frac{1}{n_{\rm tr}^2}\int_{\nu_{\rm min}}^{\nu_{\rm max}}\mathrm{d}\nu_1\int_{\nu_{\rm min}}^{\nu_{\rm max}}\mathrm{d}\nu_2\,\frac{1}{(2\pi)^3\sqrt{1-\xi_0^2(r)/\sigma_0^4}}\exp\left[-\frac{1}{2}\frac{\nu_1^2+\nu_2^2-2\nu_1\nu_2\xi_0(r)/\sigma_0^2}{1-\xi_0^2(r)/\sigma_0^4}\right]$$

## Correlation Function




## Power Spectrum Correction

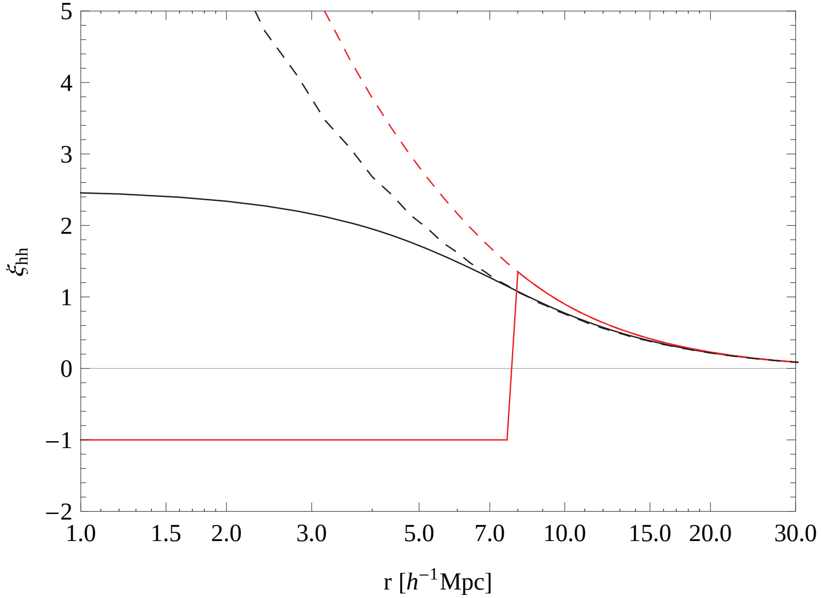



5 [Kaiser 1984, Beltran & Durrer 2011]

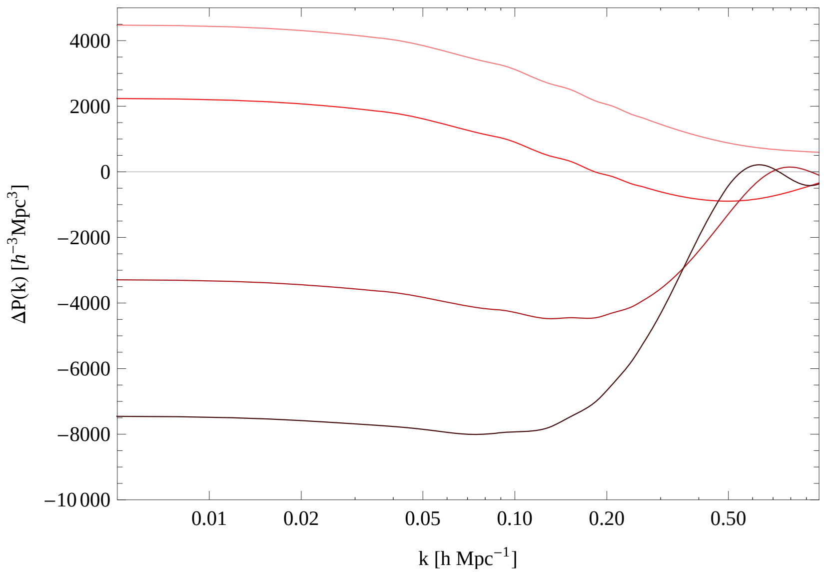
# Correlation of Thresholded Regions[5]

$$\Delta P_{\rm tr}(k) = {\rm FT}[\xi_{\rm tr}](k) - b^2 P_{\rm lin}(k)$$

## Correlation Function




## Power Spectrum Correction




[5] [Kaiser 1984, Beltran & Durrer 2011]

# Estimating Exclusion in Lagrangian Space

## Why should the noise be the same in Lagrangian and Eulerian Space?

- Peebles: mass and momentum conservation $\rightarrow$ gravity can only generate $k^2$ terms in $\delta(\mathbf{k})$
- low $k$-limit $P_{22}(k) \propto k^4$ and $P_{13}(k) \propto k^2 P(k)$

## Methodology

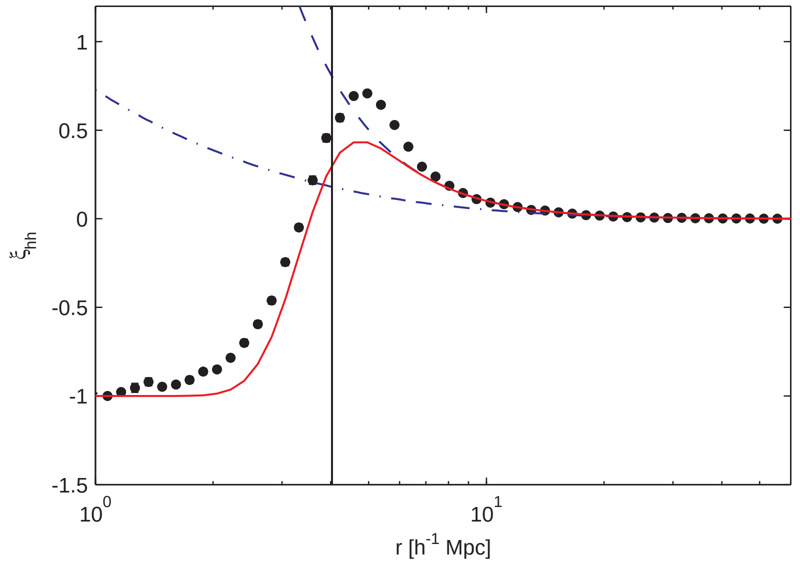
- 10 equal number density mass bins
- trace back the particles that form halo at $z_\mathrm{f} = 0$ to initital conditions $z_\mathrm{i} = 50$

# Correlation Function from Simulations

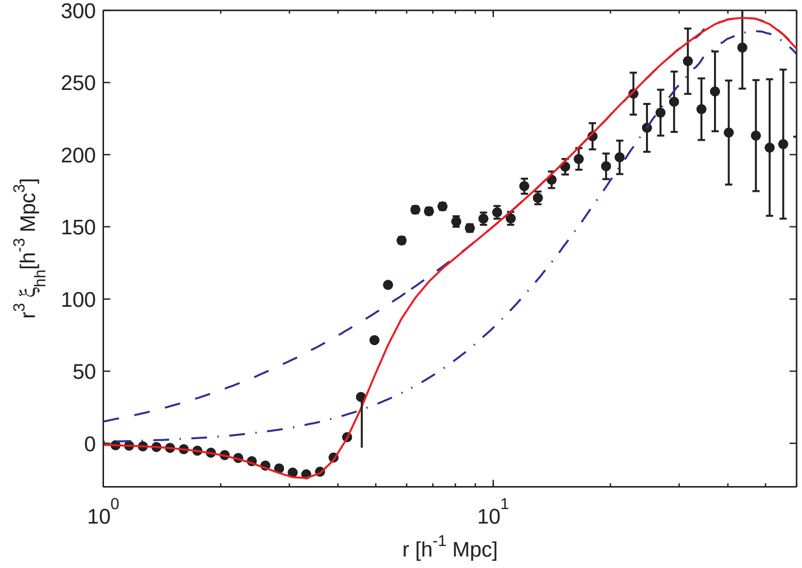
## Local Lagrangian Bias

$$\xi_{\mathrm{hh}}^{(\mathrm{c})}(r) = b_1^2 \xi_{\mathrm{mm,lin}}(r) + \frac{1}{2} b_2^2 \xi_{\mathrm{mm,lin}}^2(r) + \dots$$

### Initial Conditions - $\xi_{\mathrm{hh}}(r)$




### Initial Conditions - $r^3 \xi_{\mathrm{hh}}(r)$

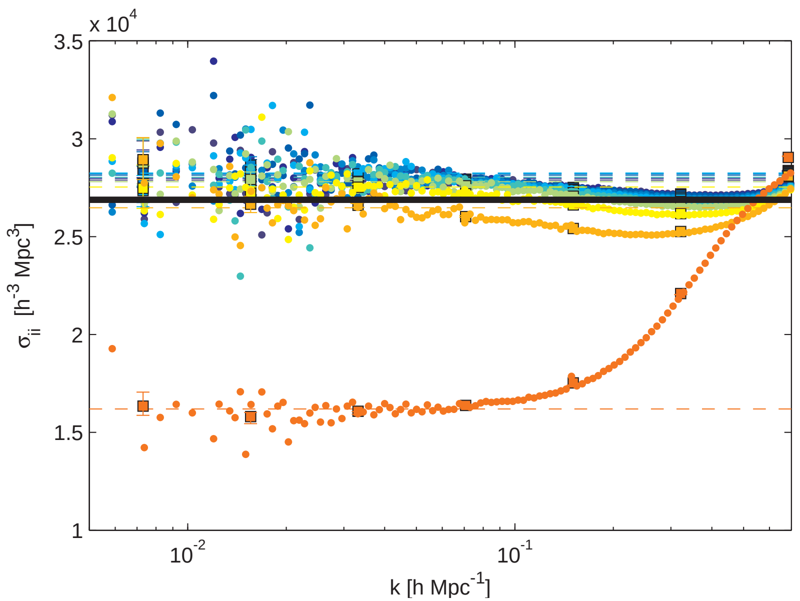


# SN Matrix from Simulations

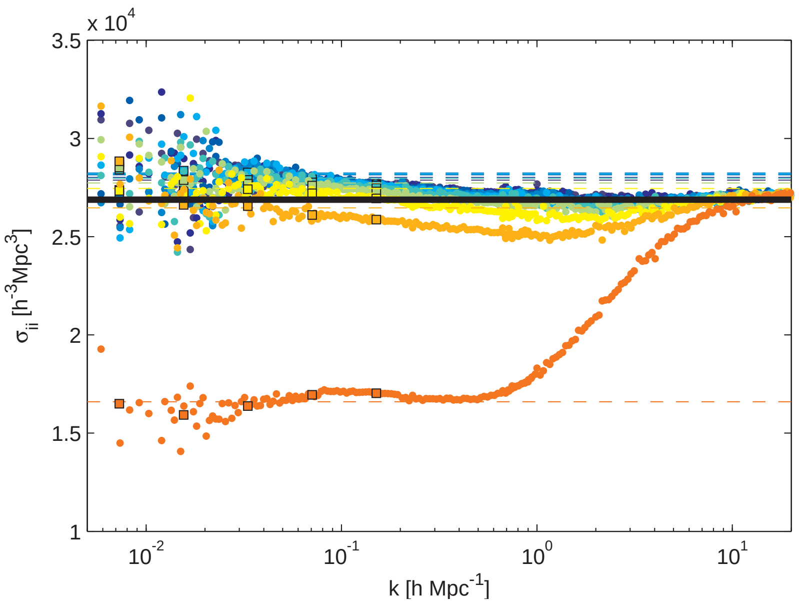
## Shotnoise Matrix

$$\sigma_{ij}(k) = \langle(\delta_i - b_{1,i}\delta)(\delta_j - b_{1,j}\delta)\rangle$$
$$= P_{ij}(k) - b_{1,i}P_{j\delta}(k) - b_{1,j}P_{i\delta}(k) + b_{1,i}b_{1,j}P_{\text{mm}}(k)$$

Initial Conditions $z_{\text{i}} = 50$




Final Distribution $z_{\text{f}} = 0$

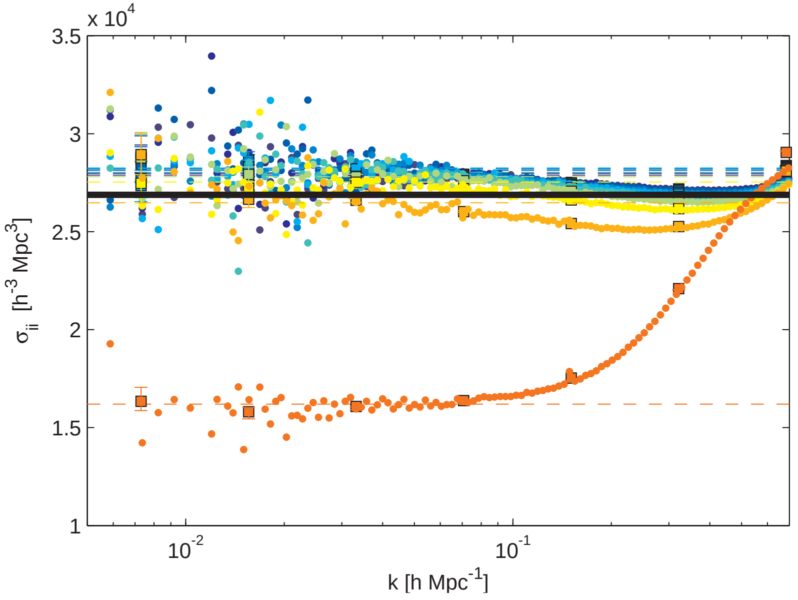


# SN Matrix from Simulations

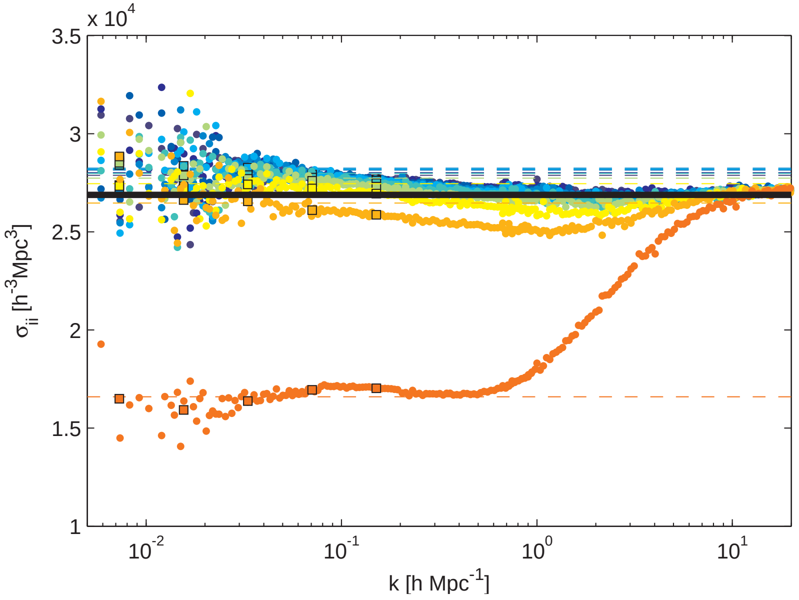
## Shotnoise Matrix

$$\sigma_{ii}(k) = P_{ii}(k) - 2b_{1,i}P_{i\delta}(k) + b_{1,i}^2 P_{\mathrm{mm}}(k)$$

### Initial Conditions $z_{\mathrm{i}} = 50$




### Final Distribution $z_{\mathrm{f}} = 0$



# SN Matrix from Simulations and Theory

## Shotnoise Matrix

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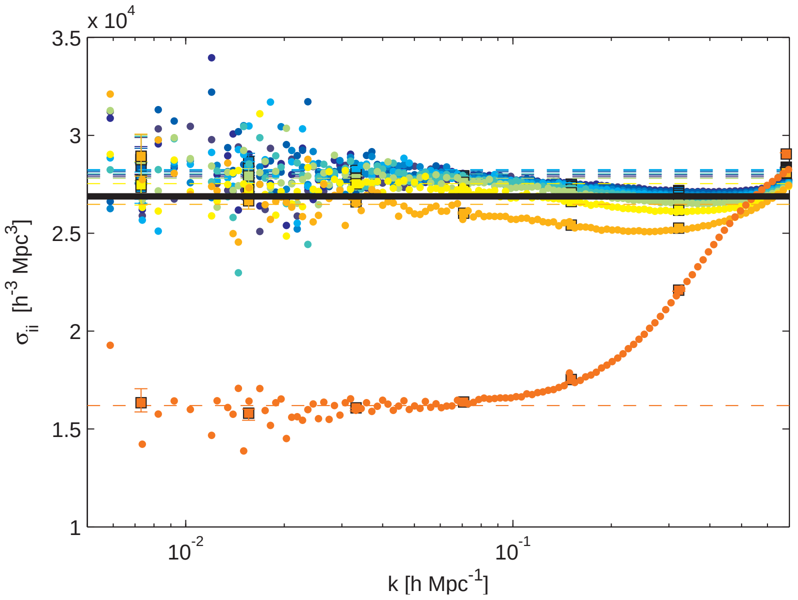
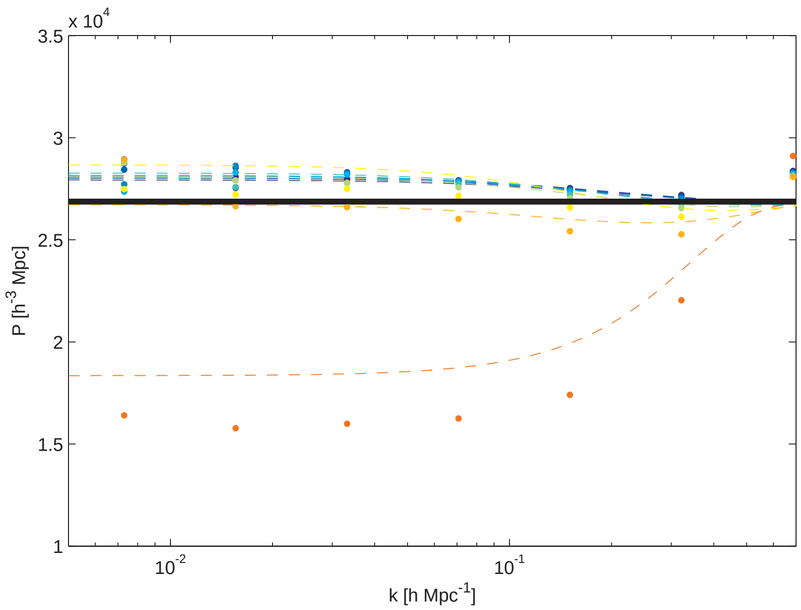
### Initial Conditions - Simulation




### Initial Conditions - Model



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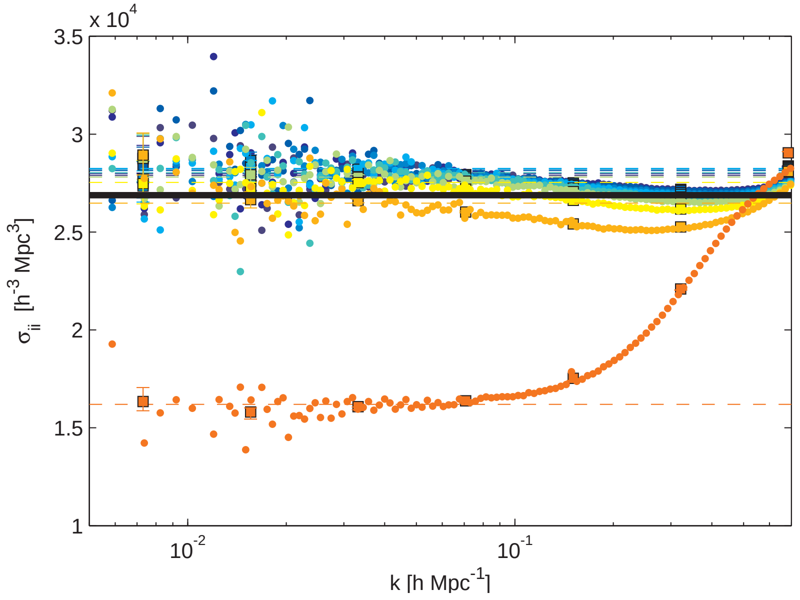
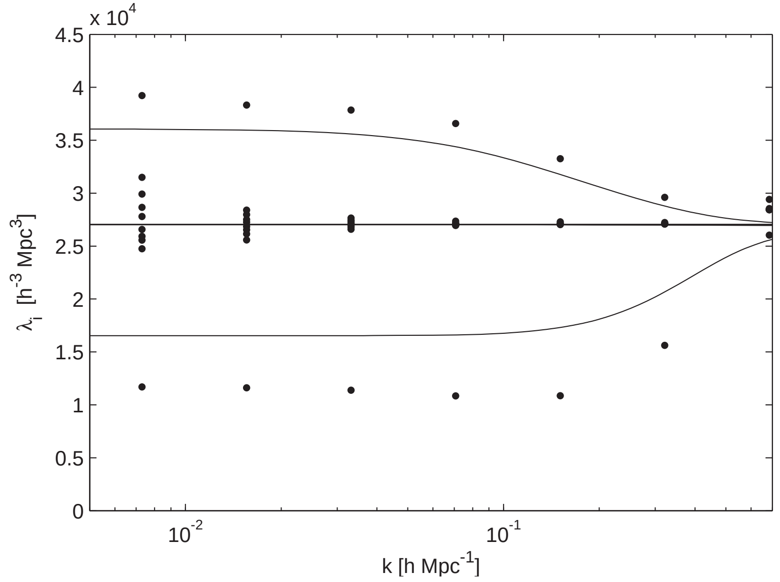


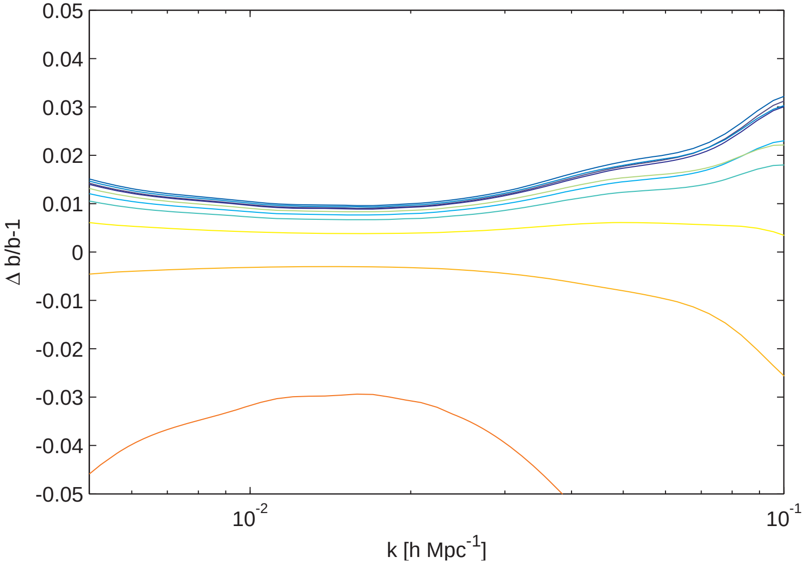

### Initial Conditions - Model



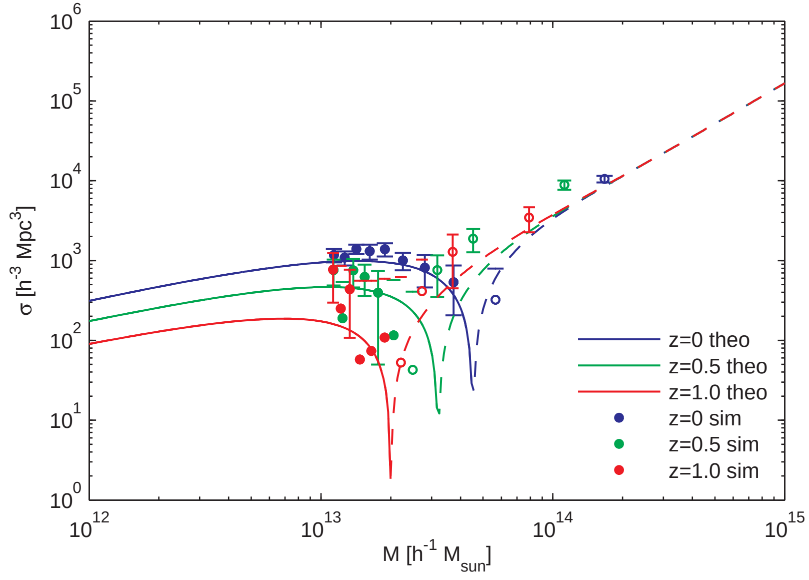
# Effect on Bias and Redshift Dependence

$$\hat{P}_{\mathrm{hh}}(k) - \frac{1}{\bar{n}} = \Delta P_{\mathrm{hh}}(k) + b_1^2 P_{\mathrm{mm,lin}}(k) = \hat{b}_{1,\mathrm{hh}}^2 \hat{P}_{\mathrm{mm}}$$

## Error on $\hat{b}_{\mathrm{hh}}$




## Mass & Redshift Dependence



# Summary & Outlook

## Findings

- non-local terms in the late time halo bias at $2^{\mathrm{nd}}$ and $3^{\mathrm{th}}$ order
- noise component is not white and different from $1/\bar{n}$
- amplitude of noise component is unaffected by evolution in $k \to 0$ and $k \to \infty$ limits
- qualitative explanation of the effects based on a model of $\xi_{\mathrm{hh}}$
  - positive correction from non-linear clustering of haloes
  - negative correction from exclusion

## Open Questions

- non-Local bias in the ICs (peak bias, velocity bias, tidal terms)
- accurate model for small scale correlation function & exclusion
- combine with perturbation theory for late time full $P_{\mathrm{hh}}$