Gamma Matrices and Wick Contractions

Gamma Matrices and Lorentz Algebra

We can implement a representation of the Lorentz algebra

$$\left[\mathcal{J}^{\mu\nu}\,,\,\mathcal{J}^{\alpha\beta}\right] = i\left(\eta^{\nu\beta}\,\mathcal{J}^{\mu\alpha} + \eta^{\mu\alpha}\,\mathcal{J}^{\nu\beta} - \eta^{\mu\beta}\,\mathcal{J}^{\nu\alpha} - \eta^{\nu\alpha}\,\mathcal{J}^{\mu\beta}\right)$$

using γ matrices $J^{\alpha\beta} = \eta\left(\gamma^\alpha\,\gamma^\beta - \gamma^\beta\,\gamma^\alpha\right)$. Check this statement and fix η.

- **Solution**

Wick Contractions and Graphs

Consider the outcomes of the Wick contractions in the lecture (i.e. the 4-pt function of four Tr[y.ϕ y.ϕ] (the **simple** one) and of four Tr[y.ϕ y.ϕ y.ϕ] (the **complicated** one). For example, we have

```
simple = 4 ℕ^4 (G[1, 4]^2 G[2, 3]^2 y[1,4]^2 y[2,3]^2 +
       G[1, 3]^2 G[2, 4]^2 y[1,3]^2 y[2,4]^2 + G[1, 2]^2 G[3, 4]^2 y[1,2]^2 y[3,4]^2) +
   16 ℕ^2 (G[1, 2] G[1, 4] G[2, 3] G[3, 4] y[1,2] y[1,4] y[2,3] y[3,4] +
       G[1, 3] G[2, 4] y[1,3] y[2,4] (G[1, 4] G[2, 3] y[1,4] y[2,3] + G[1, 2] G[3, 4] y[1,2] y[3,4]));
```

• Insert **complicated**.

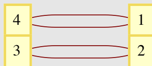
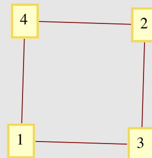
• Plot them (ignoring double line notation). Hint: First create a function that prepares the expression to be plotted. For example

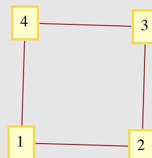
```
PrepareToPlot@simple
```

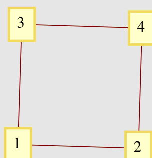
```
4 ℕ^4 y[1,4]^2 y[2,3]^2 ToPlot[G[1, 4]^2 G[2, 3]^2] +
 16 ℕ^2 y[1,3] y[1,4] y[2,3] y[2,4] ToPlot[G[1, 3] G[1, 4] G[2, 3] G[2, 4]] +
 4 ℕ^4 y[1,3]^2 y[2,4]^2 ToPlot[G[1, 3]^2 G[2, 4]^2] +
 16 ℕ^2 y[1,2] y[1,4] y[2,3] y[3,4] ToPlot[G[1, 2] G[1, 4] G[2, 3] G[3, 4]] +
 16 ℕ^2 y[1,2] y[1,3] y[2,4] y[3,4] ToPlot[G[1, 2] G[1, 3] G[2, 4] G[3, 4]] +
 4 ℕ^4 y[1,2]^2 y[3,4]^2 ToPlot[G[1, 2]^2 G[3, 4]^2]
```

• Note that this is very similar to what we did to identify monomials to be Wick contracted. Now it should be easy to transform each **ToPlot** into a nice Graph. For that check **GraphPlot**. To prepare an input to **GraphPlot**, replacements like `G[a_, b_]^n_. :> Table[a -> b, {n}]` might be useful. At the end, you should get something like

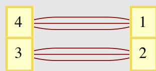
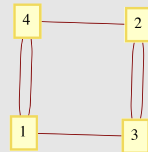
```
plot@simple
```

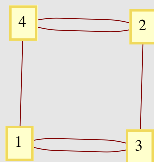
$4\,\mathbb{N}^4$  $y_{1,4}^2\, y_{2,3}^2 + 16\,\mathbb{N}^2$  $y_{1,3}\, y_{1,4}\, y_{2,3}\, y_{2,4} +$

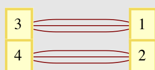
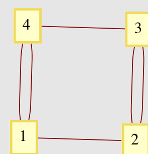
$4\,\mathbb{N}^4$  $y_{1,3}^2\, y_{2,4}^2 + 16\,\mathbb{N}^2$  $y_{1,2}\, y_{1,4}\, y_{2,3}\, y_{3,4} +$

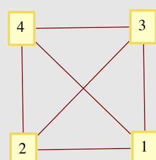
$16\,\mathbb{N}^2$  $y_{1,2}\, y_{1,3}\, y_{2,4}\, y_{3,4} + 4\,\mathbb{N}^4$  $y_{1,2}^2\, y_{3,4}^2$

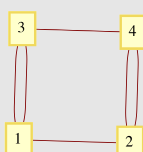
```
plot@complicated
```

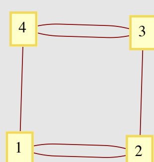
$9\left(\mathbb{N}+\mathbb{N}^3\right)^2$  $y_{1,4}^3\, y_{2,3}^3 + 81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,3}\, y_{1,4}^2\, y_{2,3}^2\, y_{2,4} +$

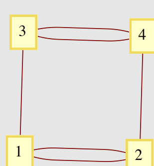
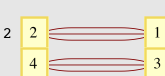
$81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,3}^2\, y_{1,4}\, y_{2,3}\, y_{2,4}^2 +$

$9\left(\mathbb{N}+\mathbb{N}^3\right)^2$  $y_{1,3}^3\, y_{2,4}^3 + 81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,2}\, y_{1,4}^2\, y_{2,3}^2\, y_{3,4} +$

$162\,\mathbb{N}^2\left(7+\mathbb{N}^2\right)$  $y_{1,2}\, y_{1,3}\, y_{1,4}\, y_{2,3}\, y_{2,4}\, y_{3,4} +$

$81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,2}\, y_{1,3}^2\, y_{2,4}^2\, y_{3,4} +$

$81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,2}^2\, y_{1,4}\, y_{2,3}\, y_{3,4}^2 +$

$81\,\mathbb{N}^2\left(3+\mathbb{N}^2\right)$  $y_{1,2}^2\, y_{1,3}\, y_{2,4}\, y_{3,4}^2 + 9\left(\mathbb{N}+\mathbb{N}^3\right)^2$  $y_{1,2}^3\, y_{3,4}^3$

▫ **Solution**