The Abdus Salam
International Centre
for Theoretical Physics

2443-2

Winter College on Optics: Trends in Laser Development and Multidisciplinary Applications to Science and Industry

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Edge emitting semiconductor lasers

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Edge emitting semiconductor lasers

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POLITECNICO DI TORINO

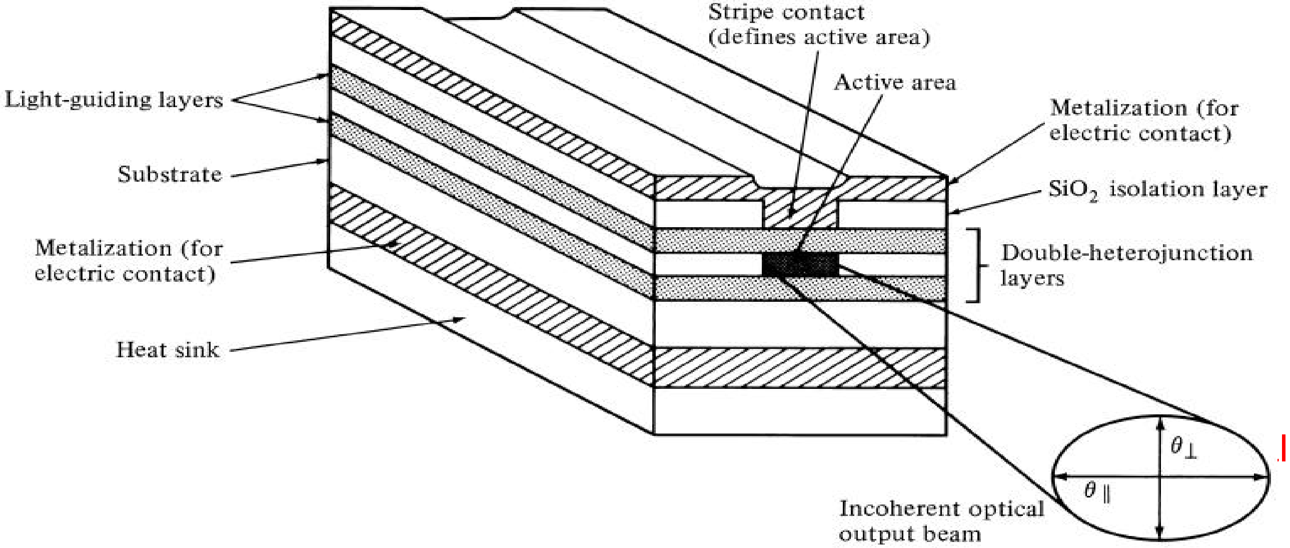
- Suggested book
- L.A.Coldren, et al., Diode Lasers and Photonic Integrated Circuits, J.Wiley, 1995
- Or the new edition

OUTLINE

- Introduction
- Semiconductor Laser ingredients
- Rate equation analysis
- DBR tunable lasers and DFB
- Laser dynamic modeling with FDTW
- Examples of mode locked lasers

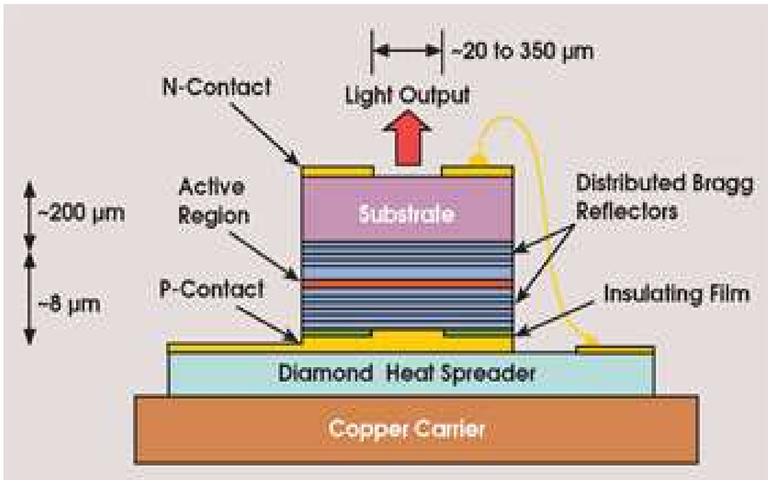
Laser diodes structures

Edge emitting laser diode



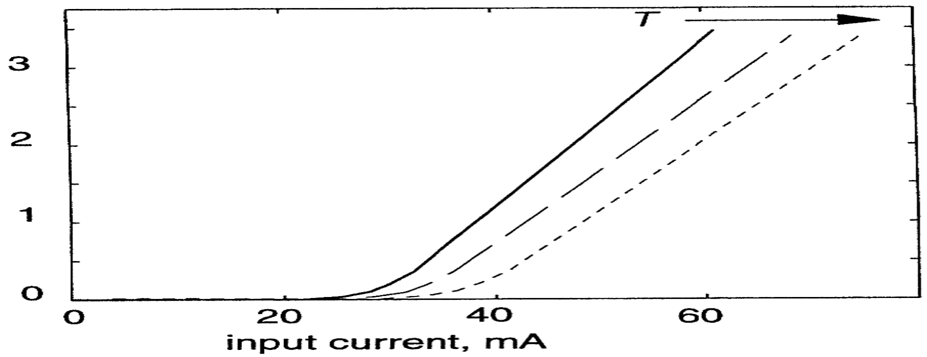
The optical cavity

Vertical cavity surface emitting laser (VCSEL)



Laser characteristics

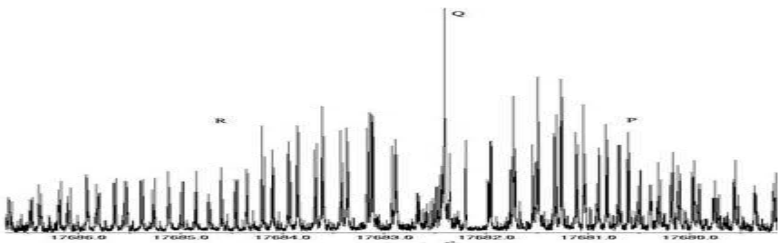
Output power mW

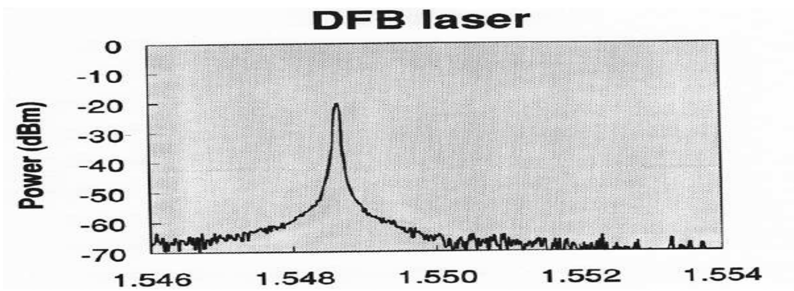


Typical output power/ current curve for a semiconductor laser

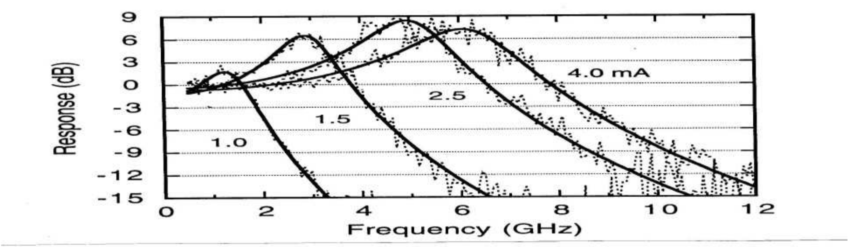
Power spectrum

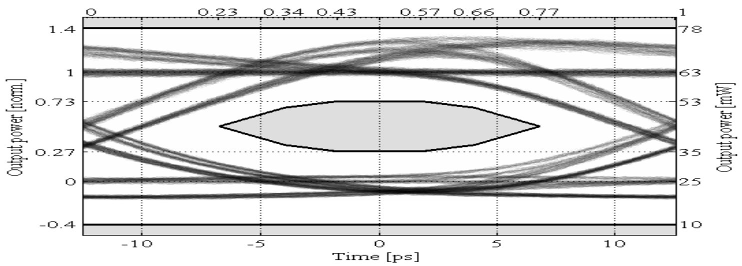
Fabry Perot laser





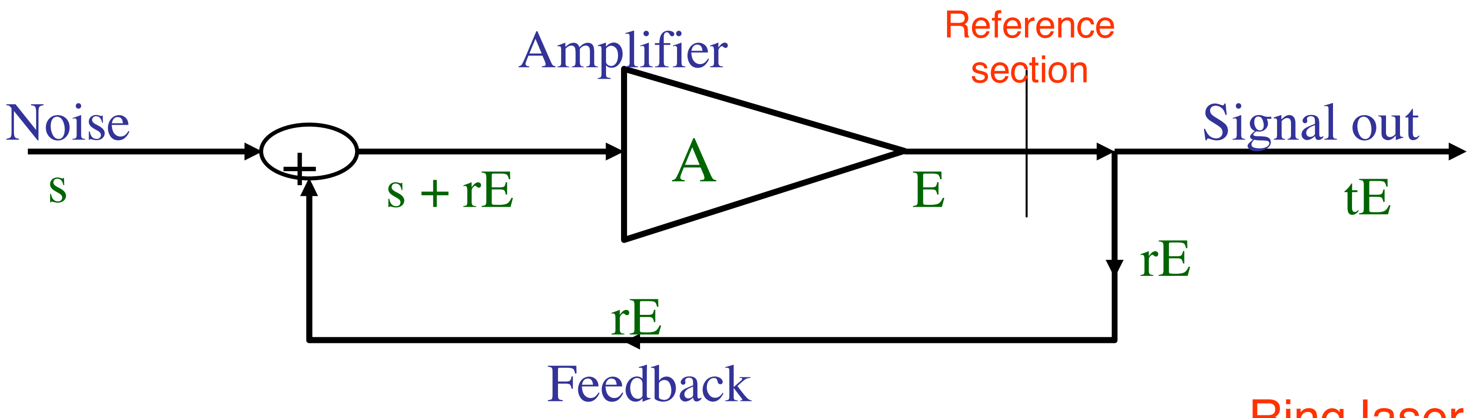
Power spectrum for a multimode and single mode laser





Dynamic characteristics: small signal modulation & eye diagram

The electronic oscillator



Ring laser

<u>Condition for stationary solutions</u>

$$A\,(s + rE) = E$$

$$\Downarrow$$

$$E = \frac{H\,s}{1 - G}, \qquad G = rA = \text{Loop gain}$$

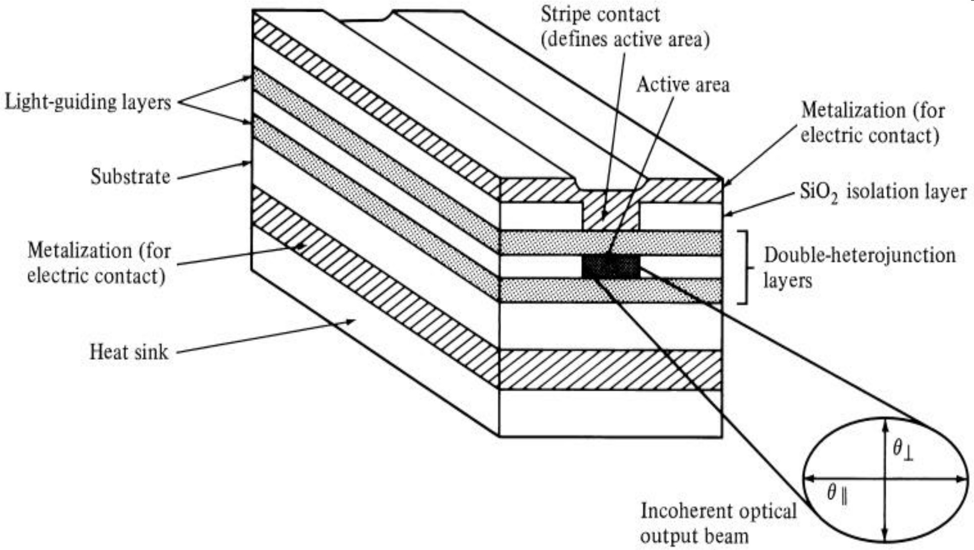
<u>Oscillation condition:</u>

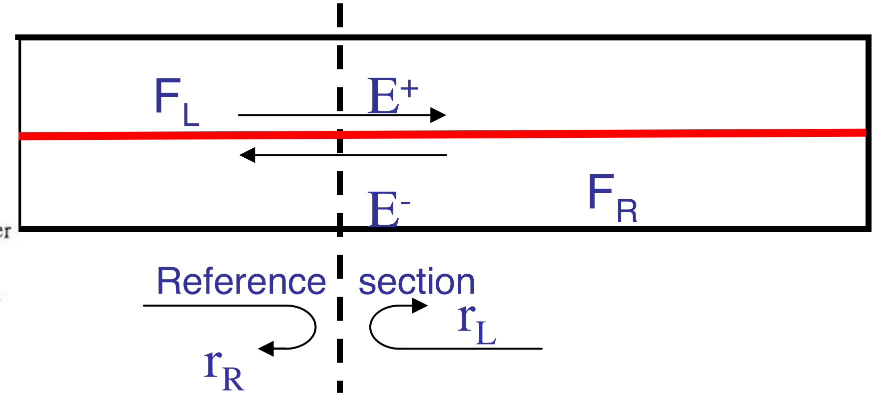
$$G = 1$$

Numerator: noise term

Denominator: loop gain characteristics

Edge emitting or VCSEL laser cavity





Field reflectivities

$$\left.\begin{aligned} E^- &= r_R E^+ + F_R \\ E^+ &= r_L E^- + F_L \end{aligned}\right\} \Rightarrow$$

Field equation

$$\mathbf{E}^+ = \frac{\mathbf{r}_L \mathbf{F}_R + \mathbf{F}_L}{1 - \mathbf{r}_L \mathbf{r}_R}$$

Numerator: noise term

Denominator: active cavity characteristics

<u>Oscillation condition:</u>

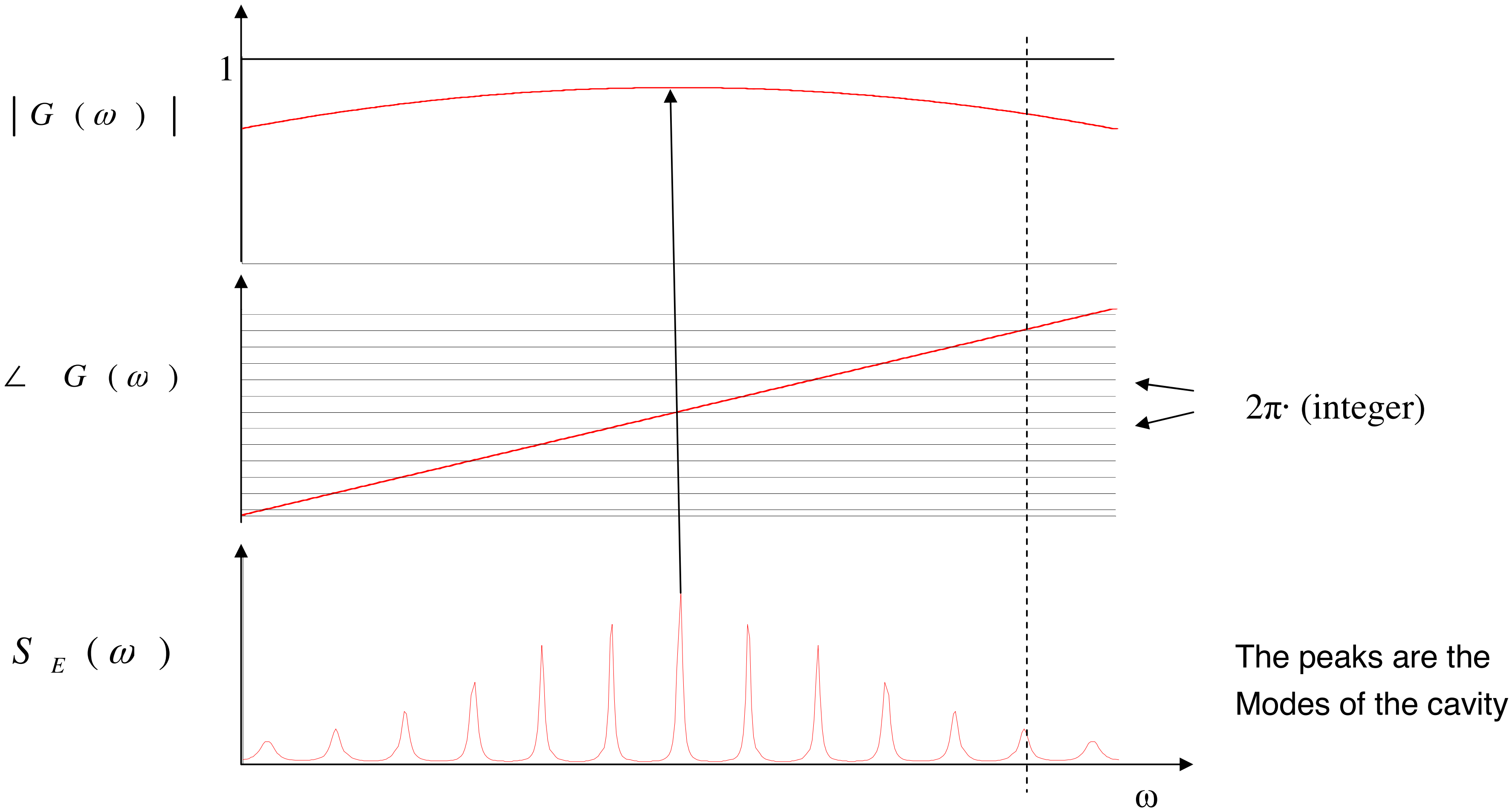
$$r_L r_R = 1 = G$$

$\mathbf{r}_{L,R}$ can be >/< 1 depending on the structure

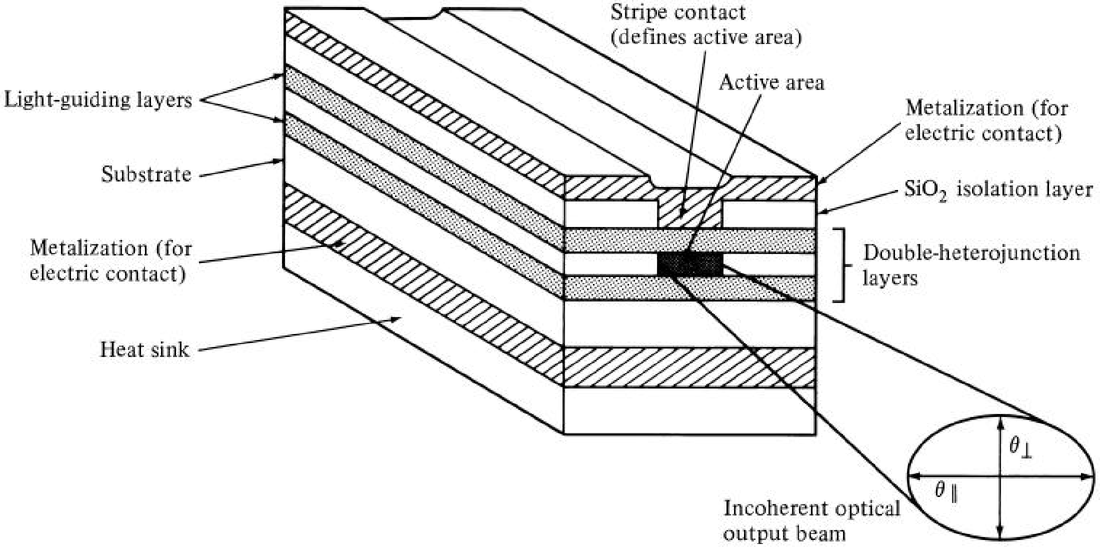
Power spectrum

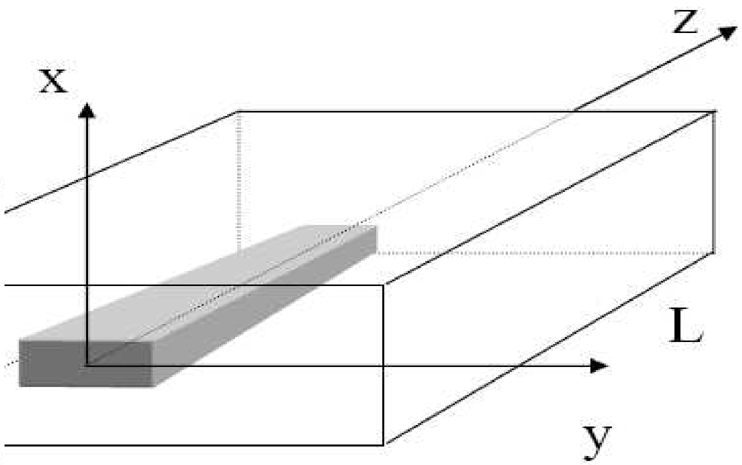
<u>Power spectrum:</u>

$$S_E(\omega) \propto |E(\omega)|^2 = \frac{|Hs|^2}{|1 - G(\omega)|^2}$$



Laser ingredients

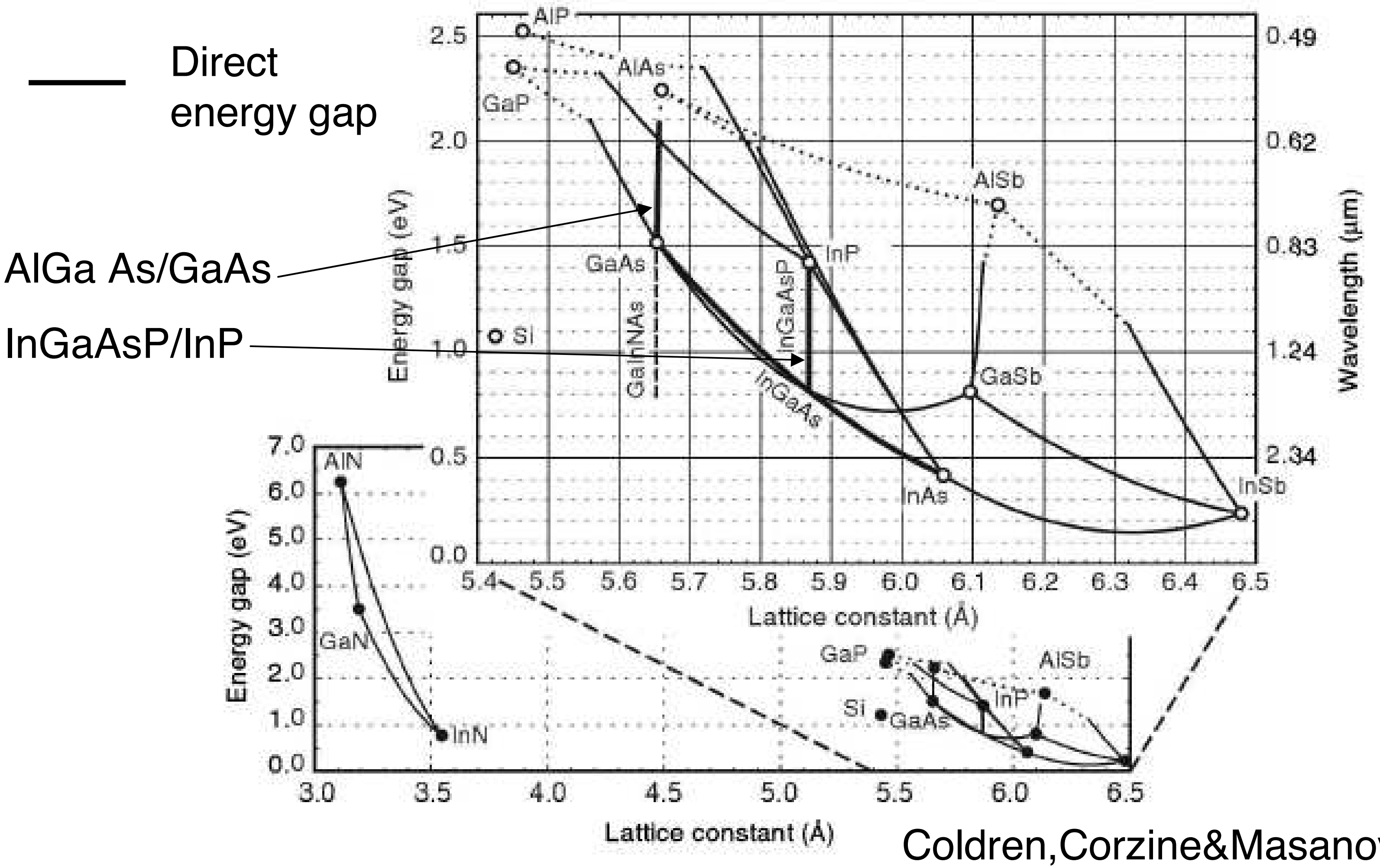




- The longitudinal optical cavity
- The semiconductor optical waveguide to confine the field in the transversal direction (avoid diffraction) and to confine the carriers
- Active semiconductor material: photon amplification and noise by e-h recombination process

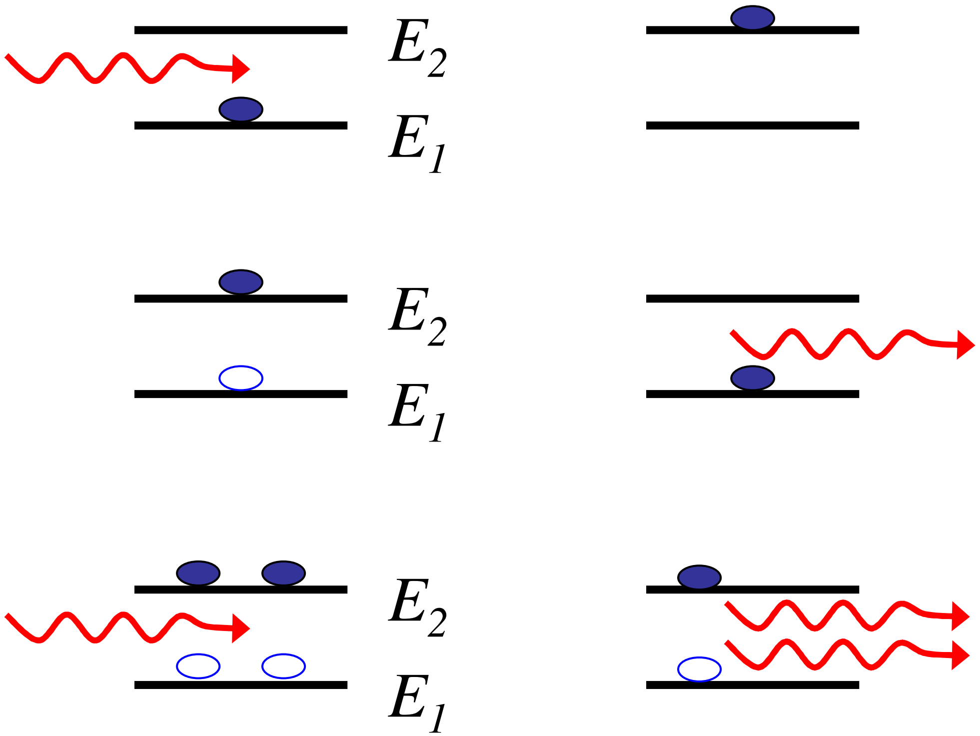
The semiconductor materials

The semiconductor laser structure is realized by epitaxial growth of material typically with the same lattice constant and with different energy gap



Coldren,Corzine&Masanovic

Radiative transitions in direct E_g semiconductors



Absorption
- photodiode

Spontaneous emission
- LED

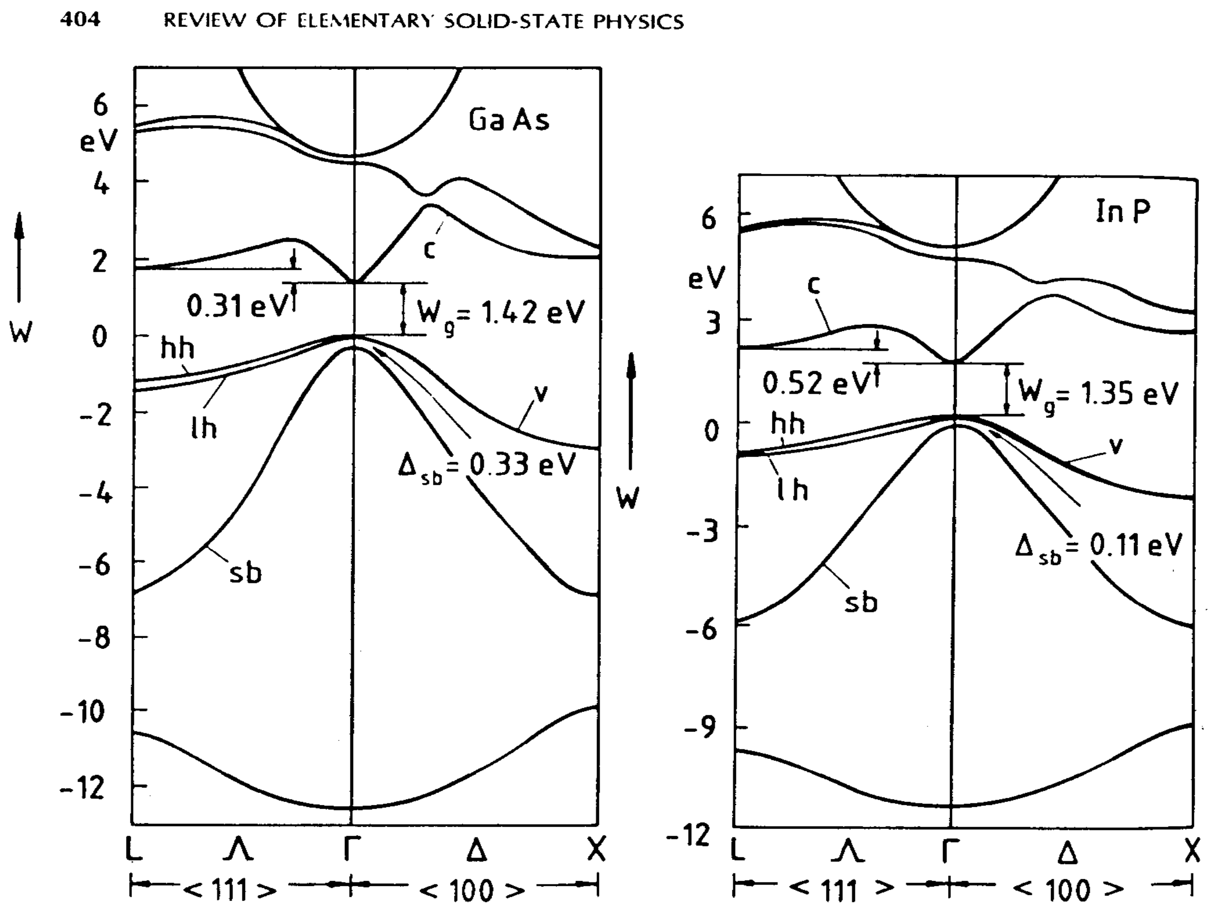
Stimulated emission
- amplifier
- laser

Direct Energy gap material

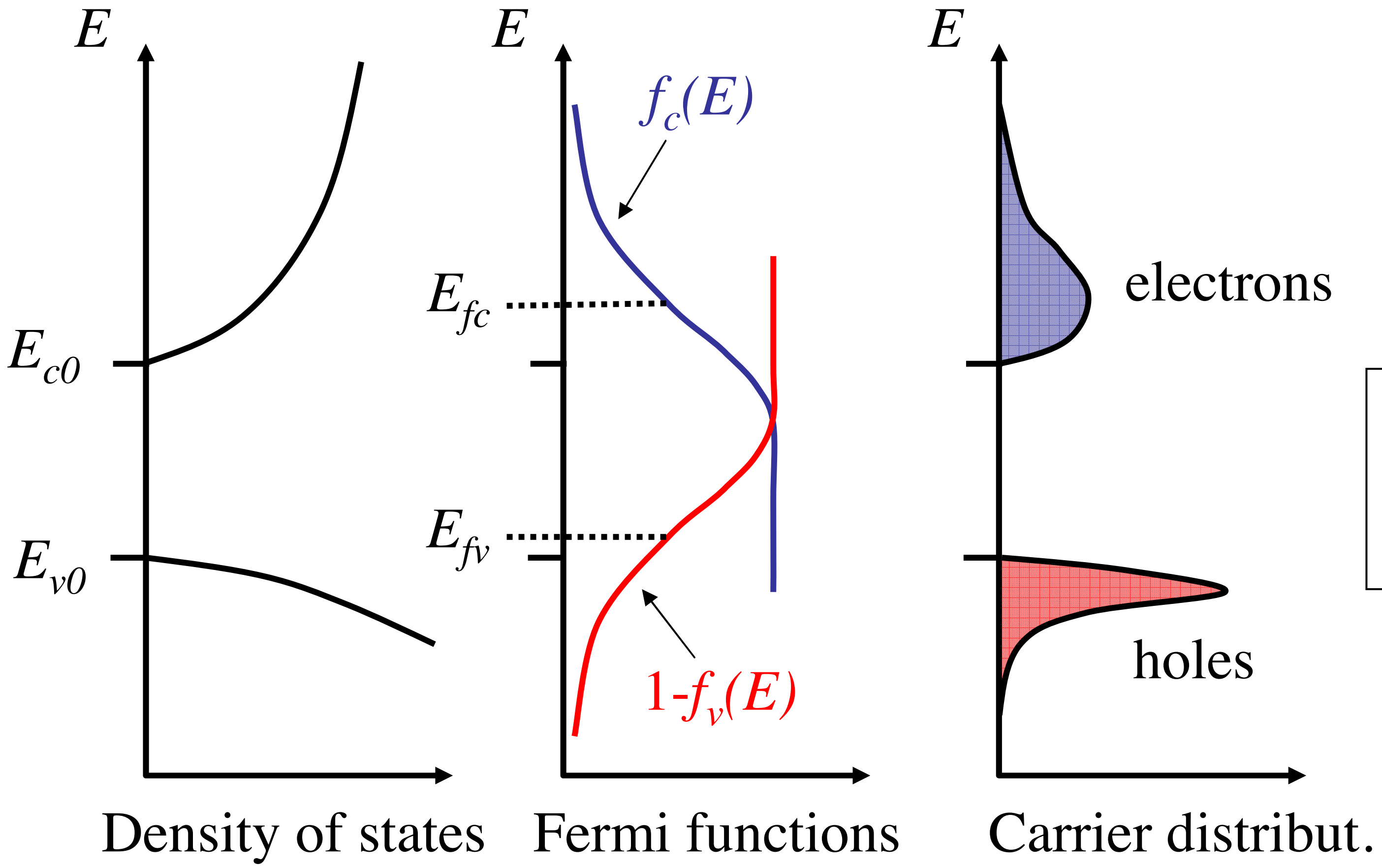
Photon energy: $\hbar\omega = E_2 - E_1$

Gain -> stimulated emission > absorption

Band structure for GaAs and InP



Carrier distributions under injection



Charge neutrality

$$N = P$$

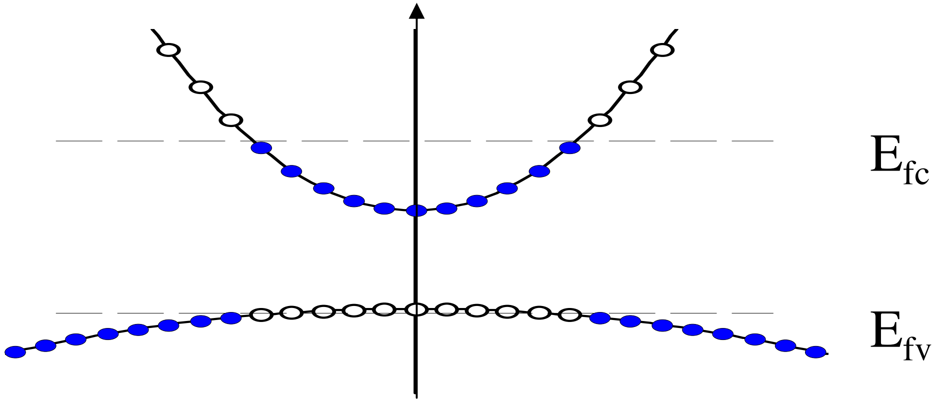
Density of cb-electrons: $\mathbf{N(E_f) = \int_{E_{c0}}^{\infty} \rho_c(E) f(E, E_f) dE}$

Density of vb-holes: $\mathbf{P(E_f) = \int_{-\infty}^{E_{v0}} \rho_v(E)(1 - f(E, E_f)) dE}$

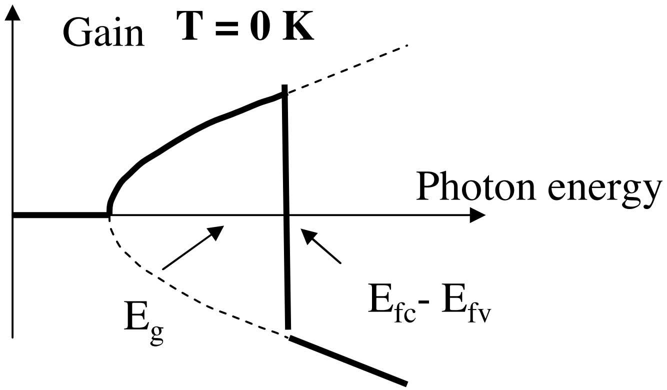
Optical gain in semiconductors

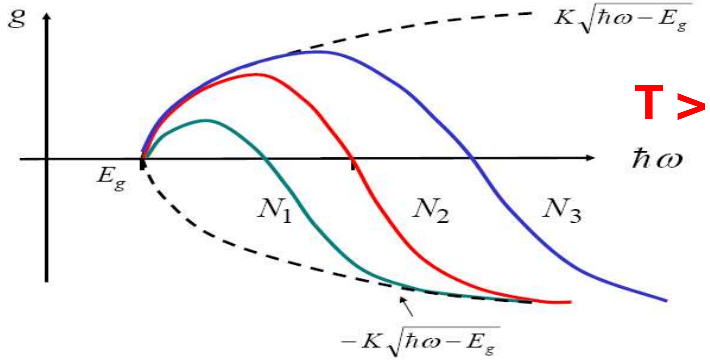
Optical gain requires **stimulated emission > stimulated absorption**

Can be fulfilled for <u>quasi-thermal equilibrium</u>



We have gain when $E_{fc} - E_{fv} > E_g$ Bernard-Duraffour inversion condition

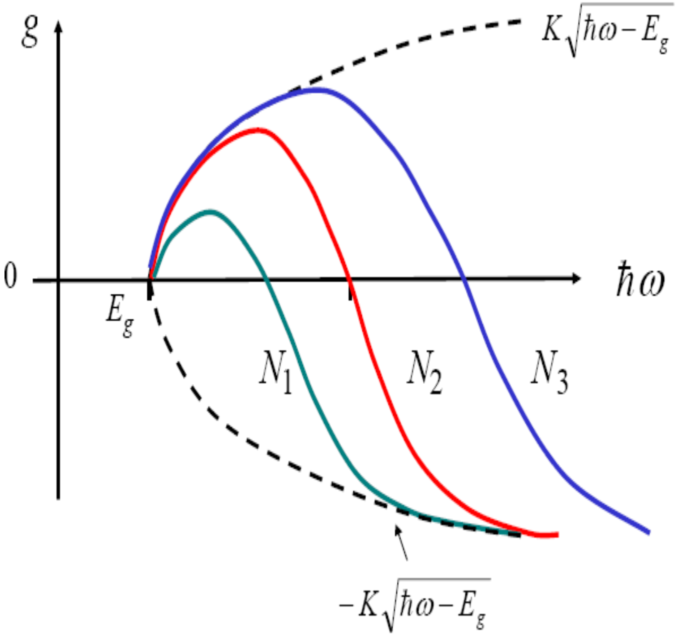




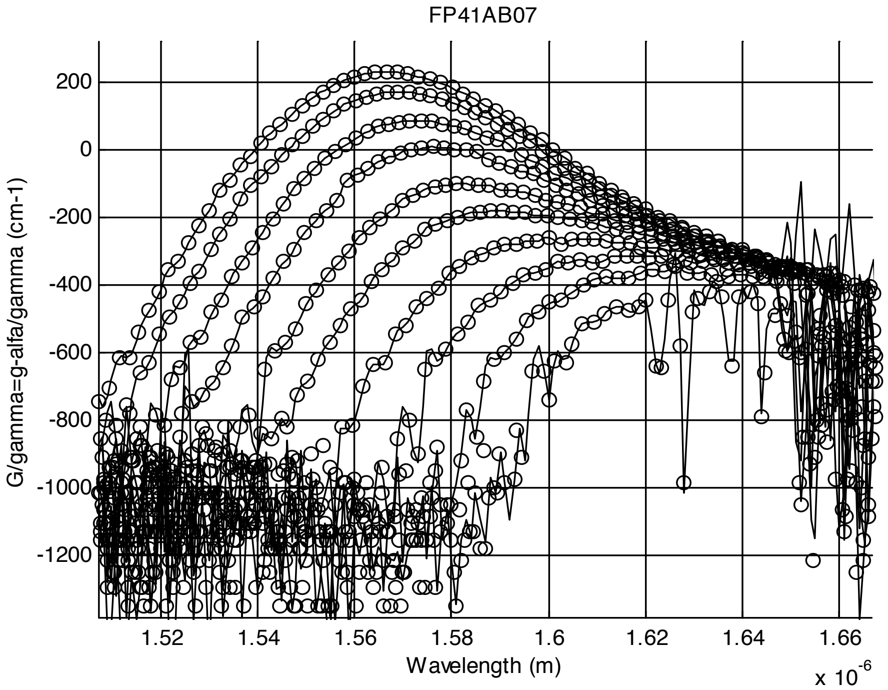
T > 0 K

Bulk gain function

$$\mathbf{g} \propto \boldsymbol{\rho}_{\mathbf{r}} \, (\mathbf{f}_2 - \mathbf{f}_1)$$

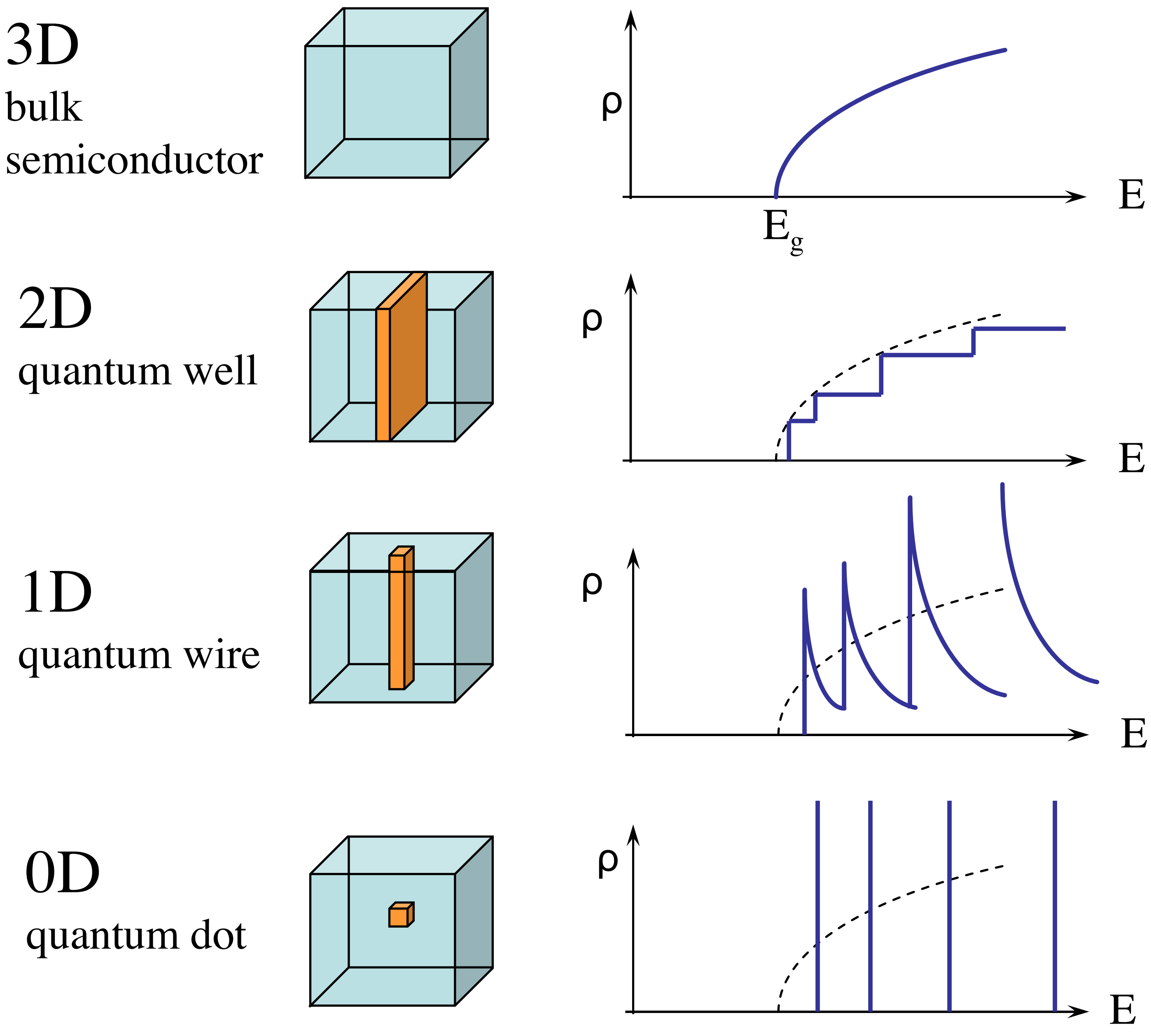


Measured gain



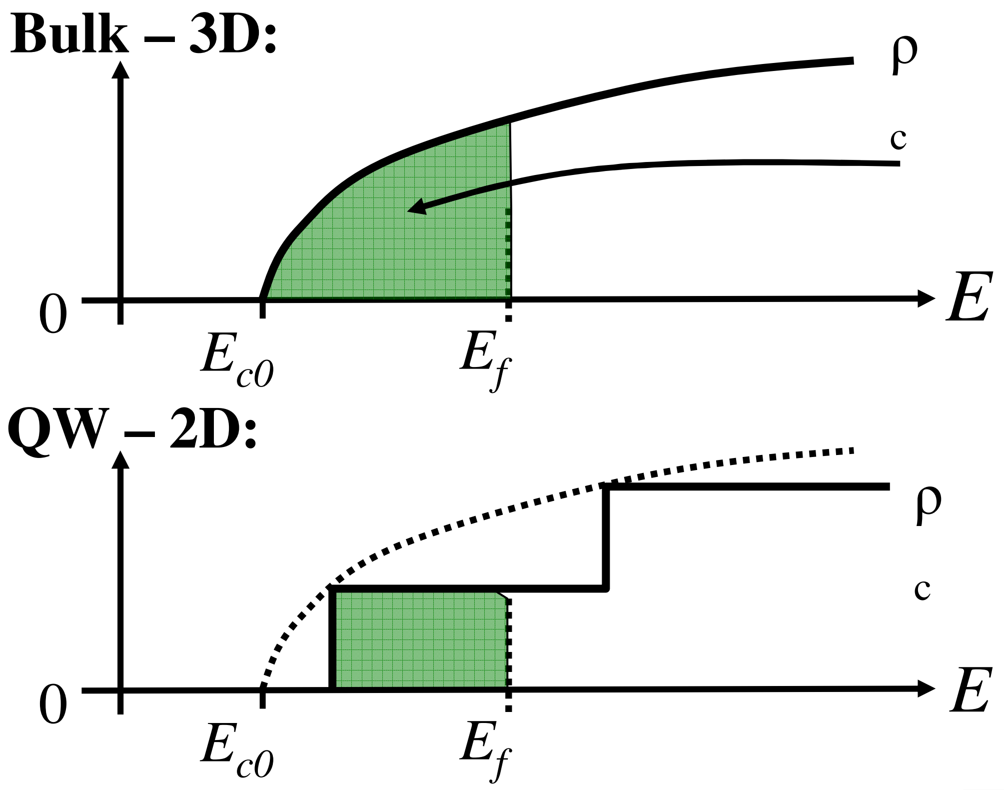
Same considerations applies for the others semiconductor materials

Density of States semiconductor materials



WHY LOWER DENSITY OF STATE ARE BETTER ?

Carriers filling – bulk vs. QW



Carrier distribution at $T=0$ K

$$\boxed{N = \int \rho(E) f(E; E_f) dE}$$

Let compare 3D and 2D material at the same E_f

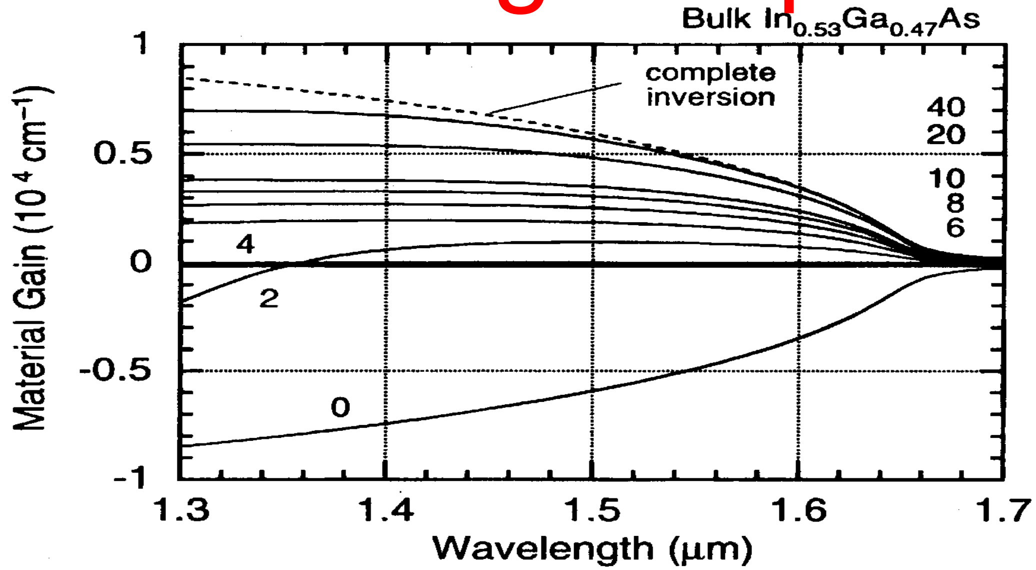
$$\text{Fixed } E_f : \mathbf{N}_{\mathbf{2D}} < \mathbf{N}_{\mathbf{3D}} \Rightarrow \left.\frac{\partial \mathbf{E}_\mathbf{f}}{\partial \mathbf{N}}\right|_{\mathbf{2D}} > \left.\frac{\partial \mathbf{E}_\mathbf{f}}{\partial \mathbf{N}}\right|_{\mathbf{3D}}$$

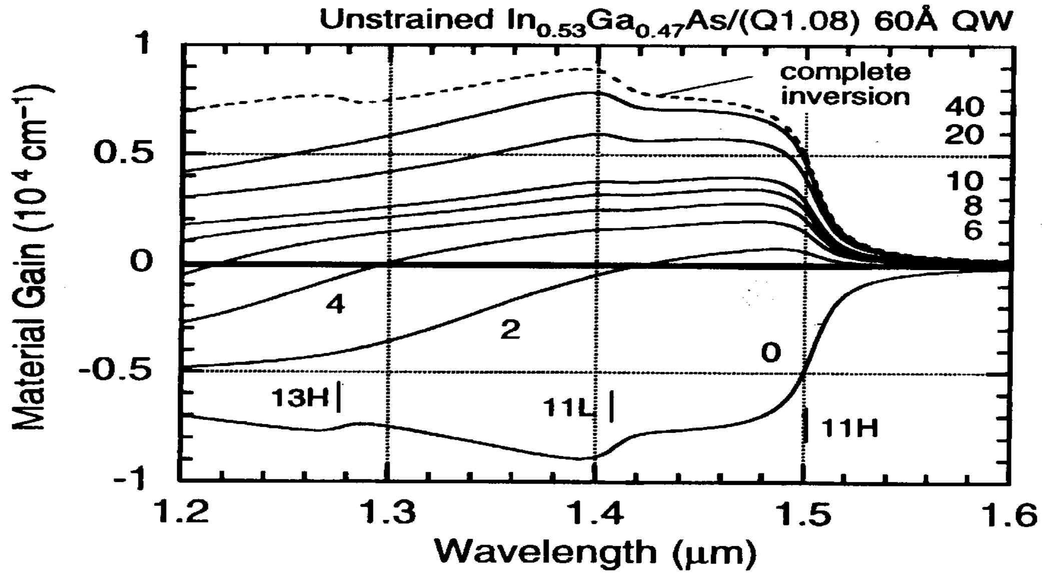
Easier to invert population
Less current

$$\text{Diff. gain} : \left.\frac{\partial \mathbf{g}}{\partial \mathbf{N}}\right|_{\mathbf{2D}} > \left.\frac{\partial \mathbf{g}}{\partial \mathbf{N}}\right|_{\mathbf{3D}}$$

Higher gain sensitivity to carrier variation -> higher modulation bandwidth with QW!

Calculated gain spectra

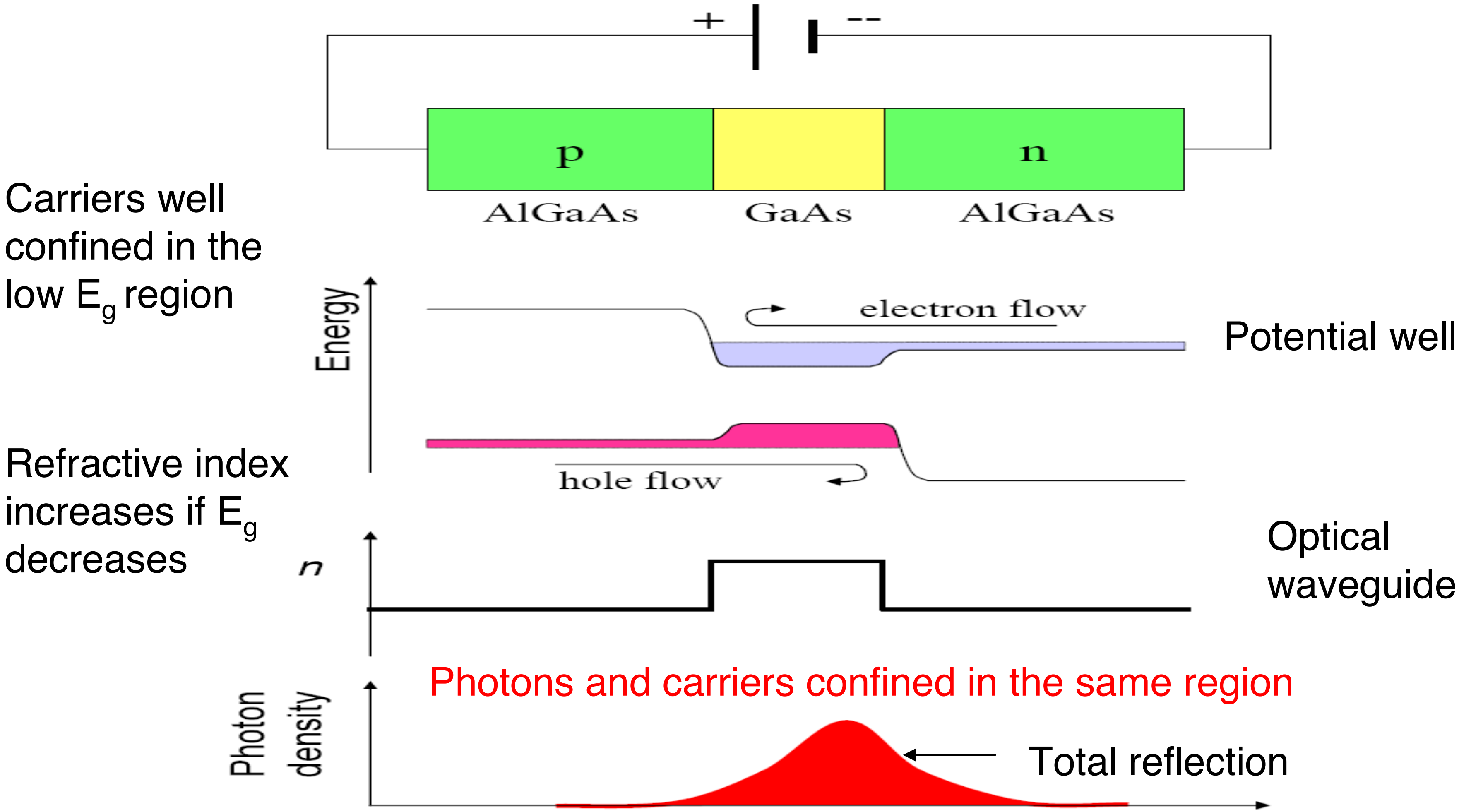




ColdrenCorzine , p.165

The semiconductor waveguide: 1D

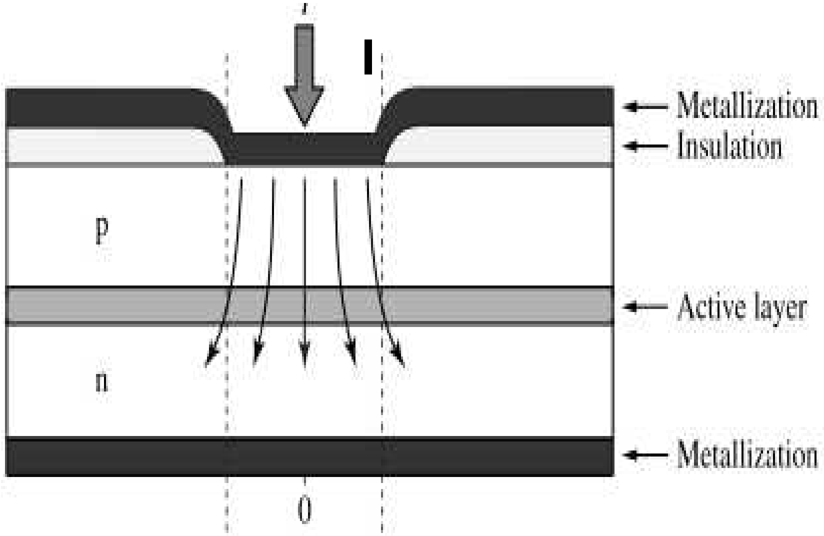
- Should be able to **maximize the interaction by carriers (e&h) and photons** by confining them in the same as small as possible region
- This is obtained by the realization of a double heterostructure diode



Semiconductor waveguides : 2D confinement of current, carriers and photons

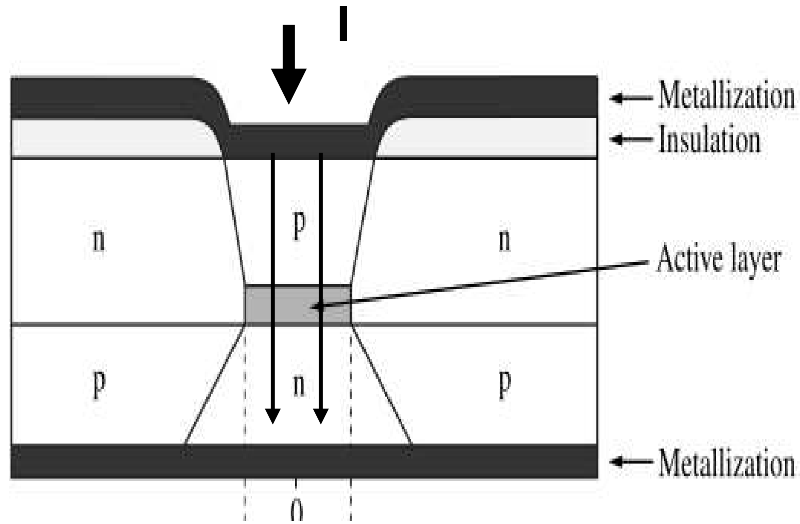
Important to realize in the lateral direction structures able to control not only the carrier and photon confinement but also the current flow.

Gain guided



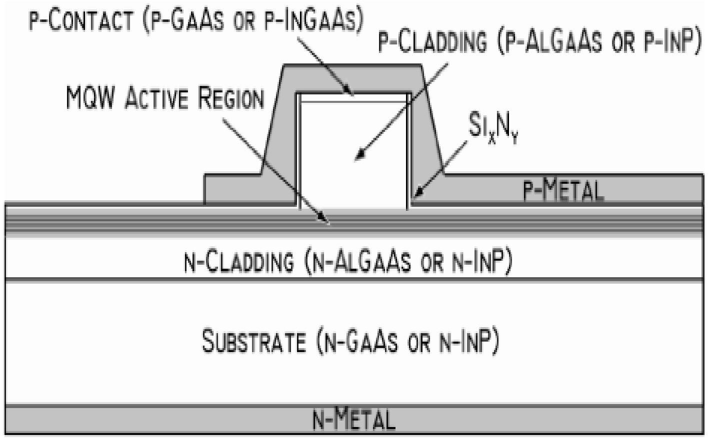
Current spreading: no control
Lateral carrier diffusion in the active layer
Lateral guiding only due to carriers -> gain non uniformity

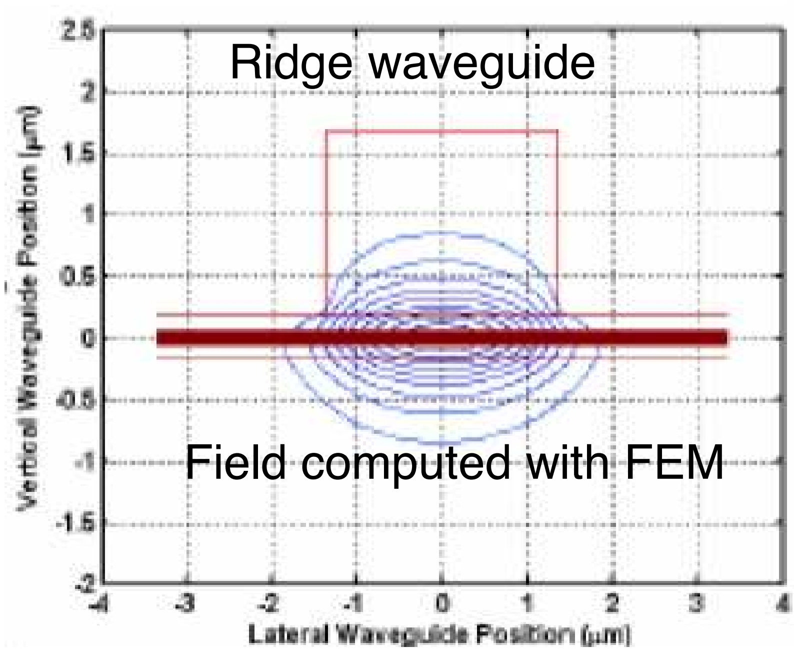
Index guided



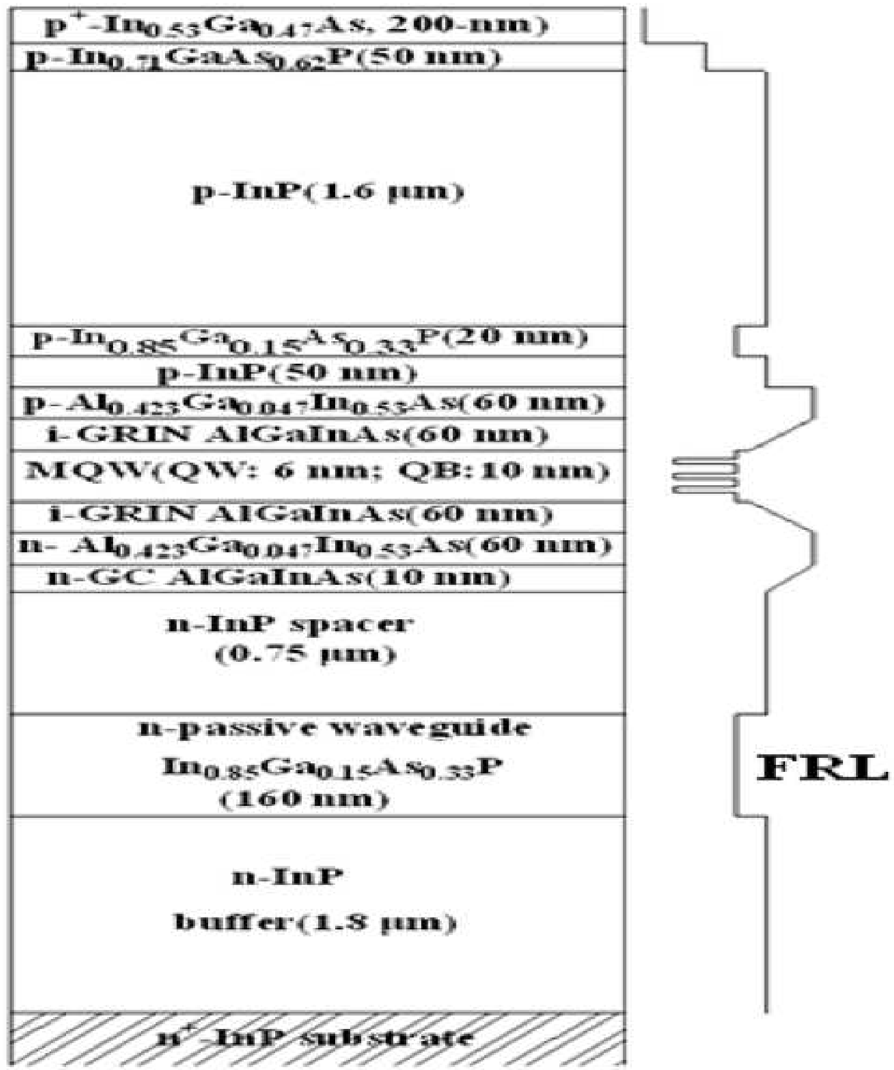
Current laterally localized by multi-junctions
No lateral carrier diffusion in the active layer: higher carrier density N
Lateral guiding due to refractive index change: higher photon density

Examples of advanced semiconductor waveguides (UGlasgow)

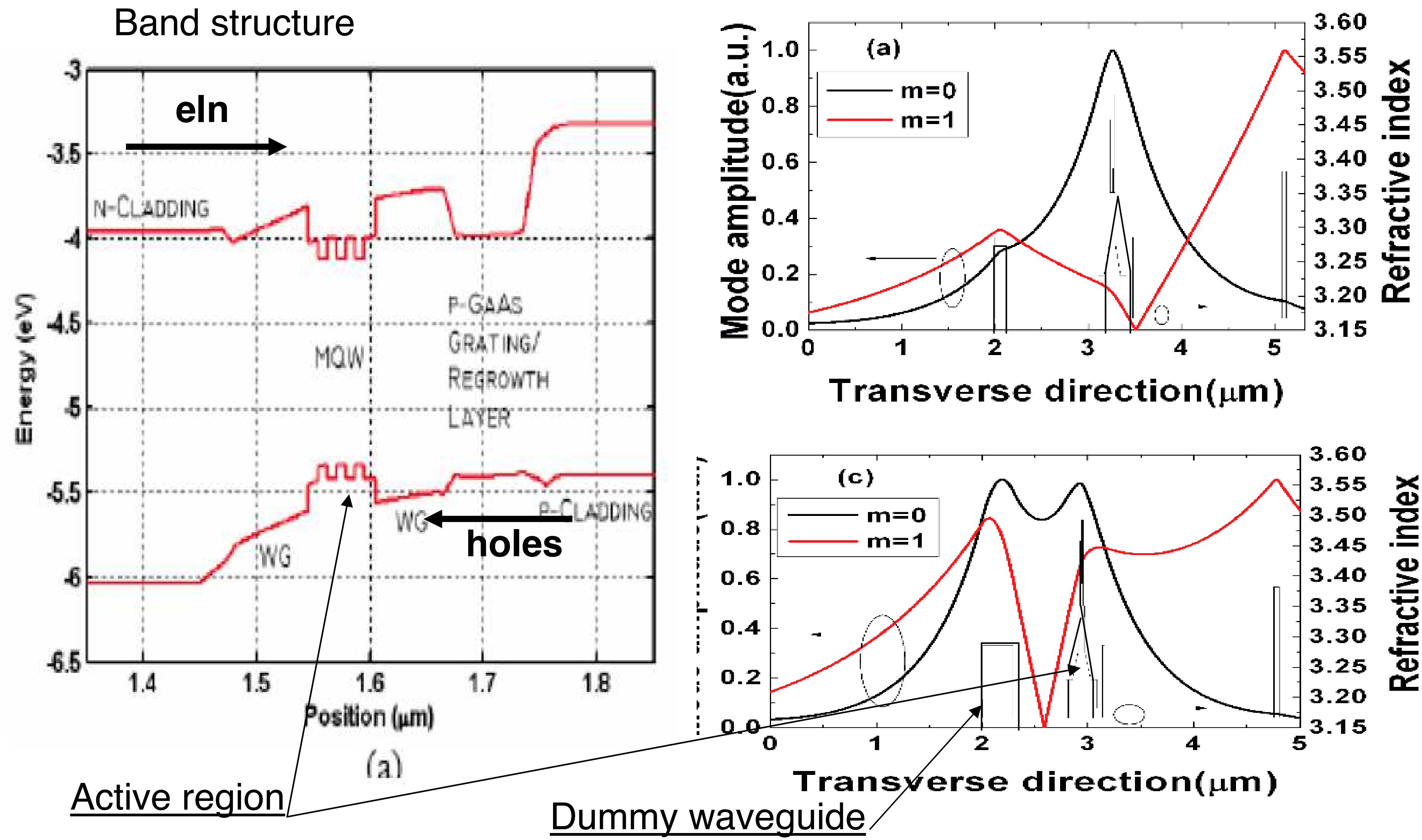




The heterostructure



MQW waveguide with vertical field optimized for optical waveguide coupling (UGlasgow)



Semiconductor waveguide analysis

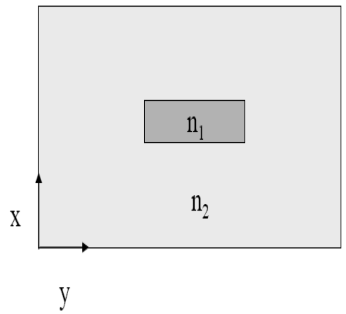
- The semiconductor waveguide can be analyzed as lossless dielectric waveguide with a perturbation due to the presence of losses (scattering, doping,..) and of the carriers in the active region

- The electromagnetic waveguide modes of the lossless structure are computed than the other effects are added perturbatively assuming that the field distribution remain unchanged and the perturbations only affect the modes propagation constant

- The two steps of the analysis consist in:
  - evaluate the modal fields and their propagation constants
  - estimate the contribution of the perturbation (losses, gain, etc.) to the propagation constant variation

Lossless Wave guides: approximate analysis

Wave equation: $$\nabla^2 \tilde{\mathbf{E}} + k_0^2 \varepsilon(x,y)\tilde{\mathbf{E}} = 0$$



For weakly guiding optical waveguides

Quasi-TE modes: $$\tilde{\mathbf{E}} \approx \mathbf{e}_y U(x,y) Z(z)$$

-TM modes $$\mathbf{H} \approx \mathbf{h}_y U(x,y) Z(z)$$

Applying the separation of variables

Transverse mode equation:

$$\nabla_t^2 \mathbf{U(x,y)} + k_0^2 \varepsilon(x,y)\mathbf{U(x,y)} = \beta^2 \mathbf{U(x,y)}$$

$$\equiv \beta^2 = k_0^2 \overline{n}^2$$

where β is the mode propagation constant and $\overline{\mathbf{n}}$ is the mode effective index

Longitudinal mode equation: $$\frac{\partial^2}{\partial z^2} Z(z) + \beta^2 Z(z) = 0$$

$$Z(z) \propto \exp[\mp \mathbf{j}(\boldsymbol{\beta} \mathbf{z})]$$

Propagation constant perturbation correction

For ε of the form $\tilde{\varepsilon} = n_w^2 + \Delta\tilde{\varepsilon}$ $n^2{}_w$ -. Lossless waveguide

From perturbation theory $$2\beta\Delta\tilde{\beta} = k_0^2 \frac{\int \Delta\tilde{\varepsilon}\,|U|^2\,dxdy}{\int |U|^2\,dxdy}$$

$$\tilde{\beta} = \beta + \Delta\tilde{\beta} \cong k_0\overline{n} + k_0\Gamma_{xy}\,\Delta\tilde{n}^{(gain)} - j\tfrac{1}{2}\alpha_i$$

where $$\boxed{\Gamma_{xy} = \frac{\int_{active\ region} |U|^2\,dxdy}{\int |U|^2\,dxdy}}$$ is the **waveguide confinement factor** when assuming $\chi^{(gain)}$ constant in the active region

and $$\alpha_i = \frac{\int (\alpha(x,y))|U(x,y)|^2\,dxdy}{\int |U(x,y)|^2\,dxdy}$$ are the internal losses

Propagation constant, group velocity and LEF

β is usually expressed as:

$$\beta \cong k_0 \bar{n} + j\tfrac{1}{2}(\Gamma_{xy} g - \alpha_i)$$

Expanding the propagation constant around a reference point (ω_r, N_r) we obtain:

$$\beta \approx \beta(\omega_r, N_r) + \frac{\partial \beta}{\partial \omega}(\omega - \omega_r) + \frac{\partial \beta}{\partial N}(N - N_r)\,.$$

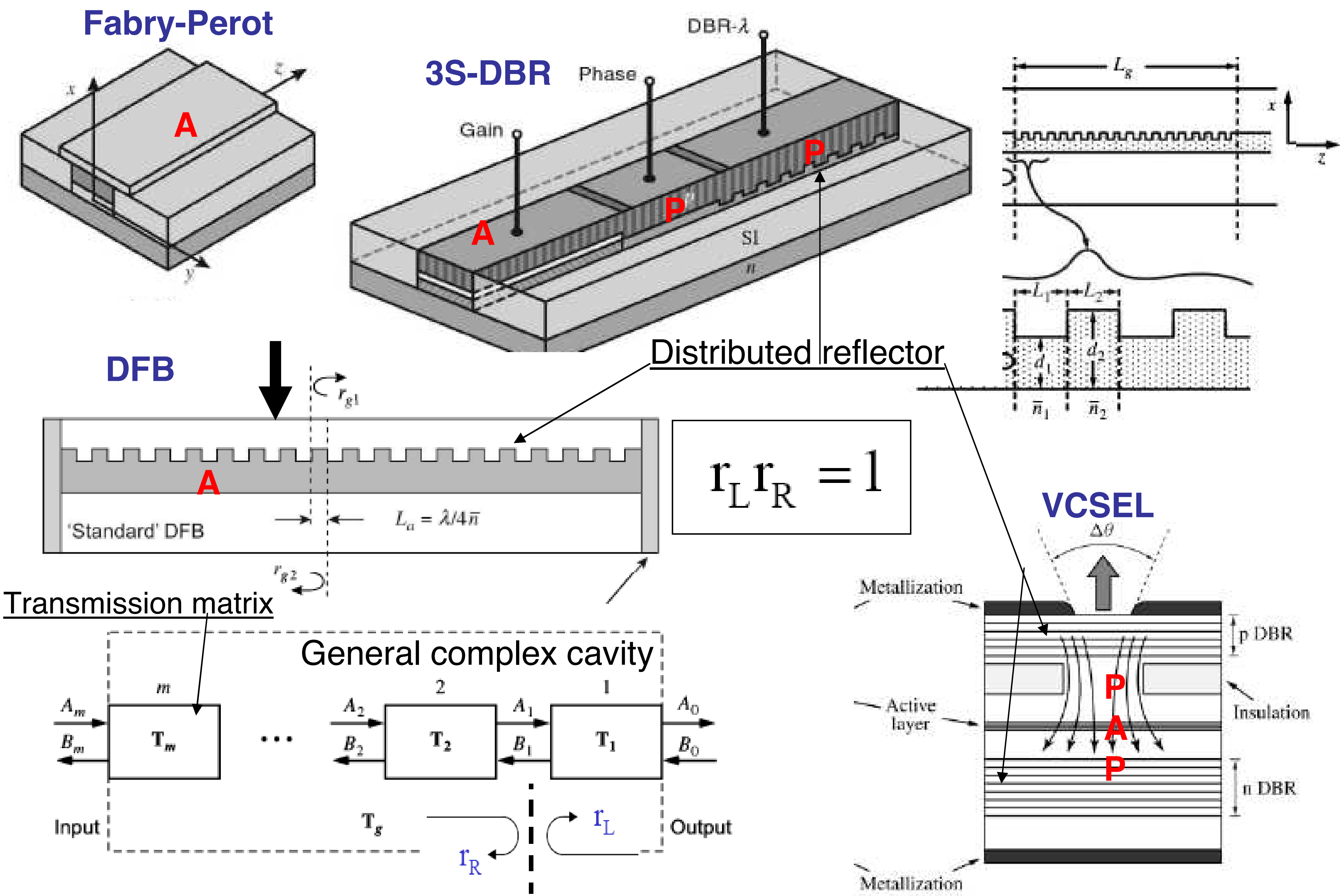
where $$\frac{\partial \beta}{\partial \omega} \cong \frac{1}{v_g}$$

and $$\frac{\partial \beta}{\partial N} \cong k_0 \Gamma_{xy} \frac{\partial}{\partial N} \Delta \tilde{n}^{(gain)} = j\tfrac{1}{2}(1 + j\alpha)\Gamma_{xy} \frac{\partial g}{\partial N}\,.$$

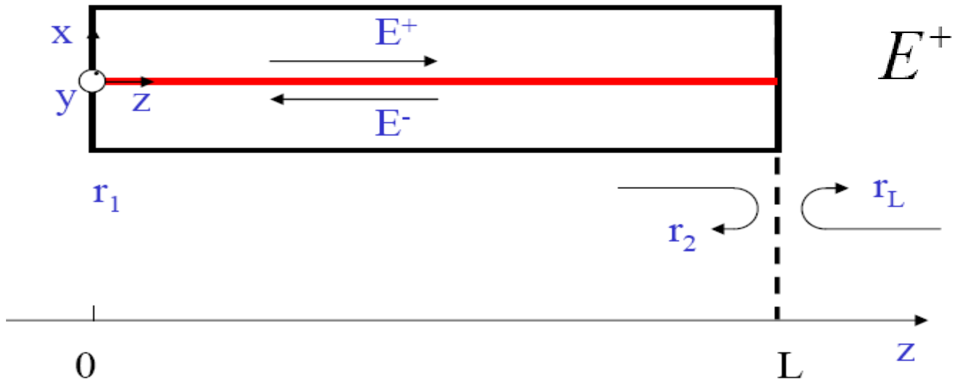
where $$\boxed{\alpha \equiv - \frac{\partial \operatorname{Re} \tilde{n}}{\partial N} \Big/ \frac{\partial \operatorname{Im} \tilde{n}}{\partial N} = -2k_0 \frac{\frac{\partial n}{\partial N}}{\frac{\partial g}{\partial N}}}$$

Where α is the active material **linewidth enhancement factor -> LEF -> α_H**

The optical cavities



The Fabry Perot cavity



$$E^{+} \propto e^{-j\beta z}, \qquad E^{-} \propto e^{j\beta z}$$

$$E^{-}(L) = r_2\, E^{+}(L)$$

$$r_L = e^{-j\beta L} r_1\, e^{-j\beta L} = r_1\, e^{-j2\beta L}$$

Oscillation condition

$$r_2\, r_L = 1 = r_1\, r_2\, e^{-j2\beta L}$$

Gain condition

$$\Gamma g = \alpha_i - \frac{1}{L}\ln(r_1 r_2) \quad = \alpha_m$$

Phase condition

$$2\frac{\omega n}{c} L = 2\pi m \quad , m \text{ integer}$$

$$\alpha_m \equiv -\frac{1}{L}\ln(r_1 r_2): \quad \text{Mirror loss}$$

$$\frac{1}{\tau_p} \equiv v_g(\alpha_i + \alpha_m): \quad \text{Loss rate per second}$$

Field power spectrum:

$$S_E(\omega) \propto \left|E^{+}(L)\right|^2 = \frac{\left|F_L(\omega)\right|^2}{\left|1 - r_1\, r_2\, e^{-2j\beta L}\right|^2}$$

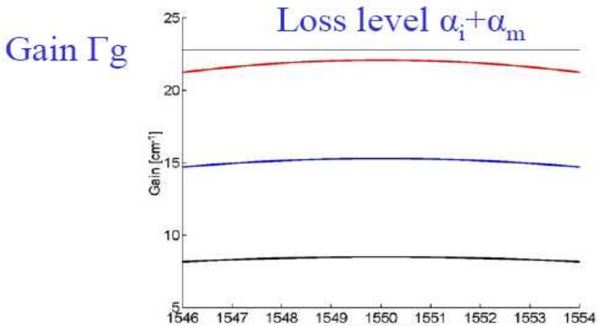
FP– Field power spectrum below threshold

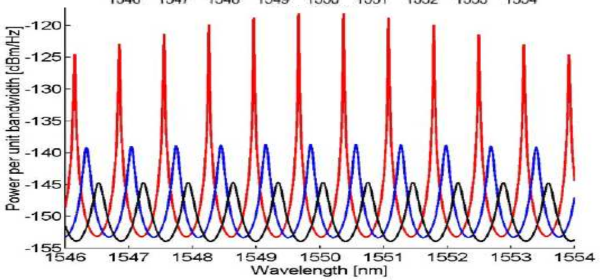
Field power spectrum:

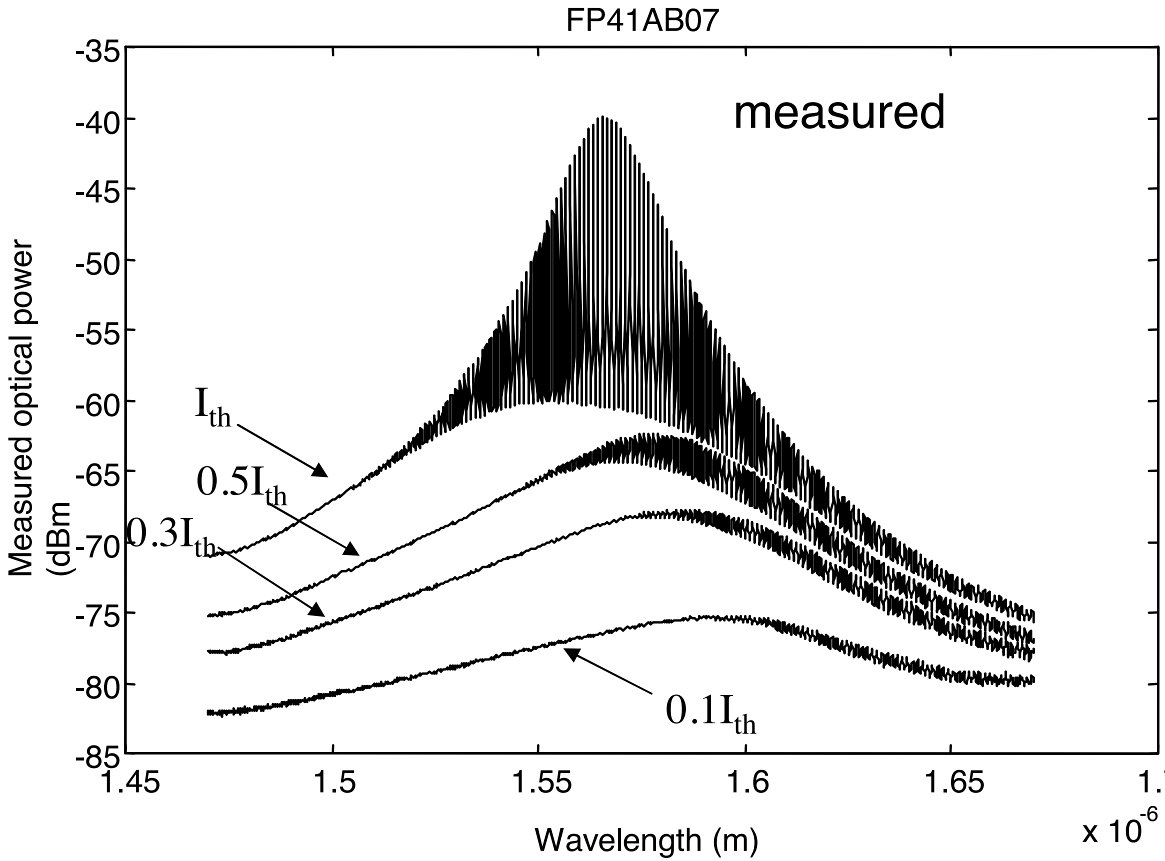
$$S_E(\omega) \propto \left|E^+(L)\right|^2 = \frac{\left|F_L(\omega)\right|^2}{\left|1 - r_1\, r_2\, e^{-2j\beta L}\right|^2}$$

$$\beta \cong k_0 n + j\tfrac{1}{2}[\Gamma(1 + j\alpha_H)g - \alpha_i]$$

The difference in the peaks is due only to difference in the gain function



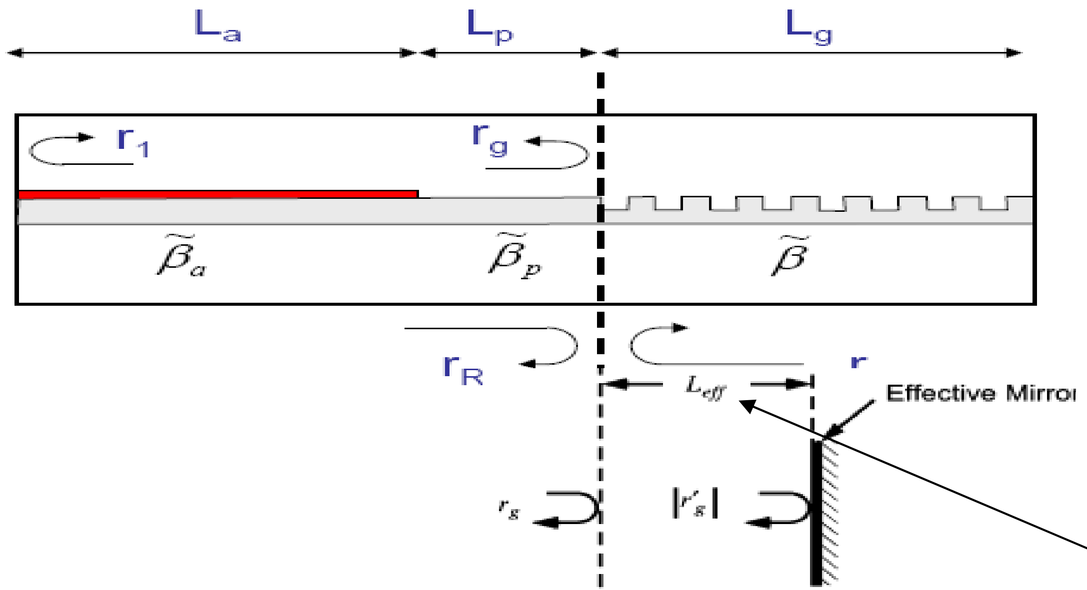


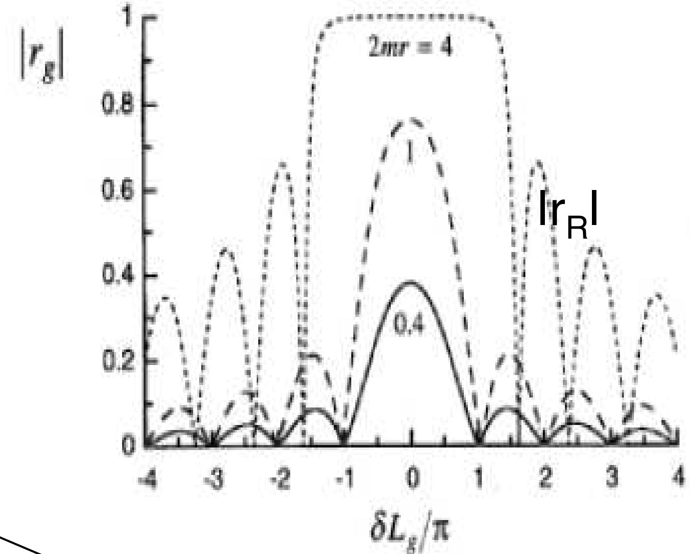


Injection ->
carrier density increase -> gain increase-> peaks increase
Peaks shift -> LEF ≠ 0 -> refractive index decrease

Peaks separation-> Free Spectral Range $\Delta\nu = \dfrac{v_g}{2L}$

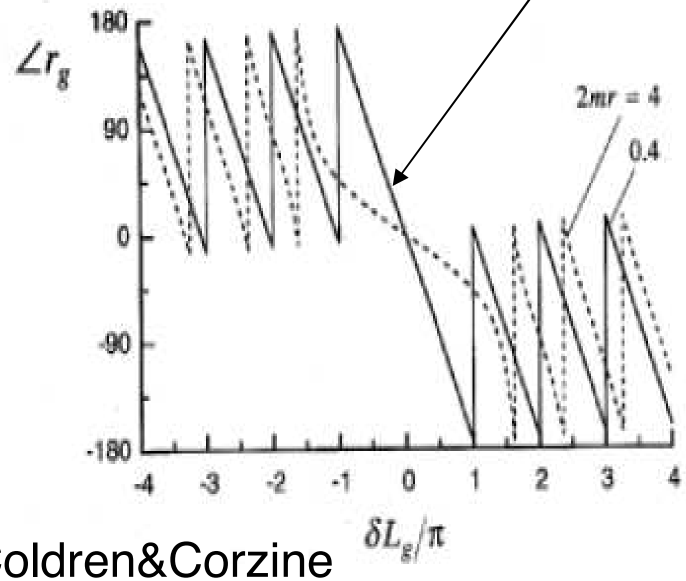
The DBR cavity





Effective grating length

Bragg grating reflectivity →



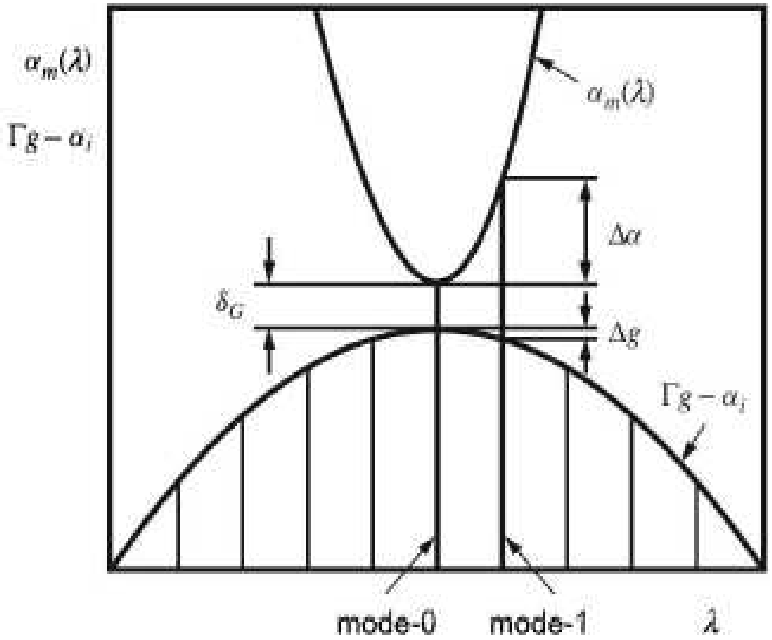
Oscillation condition

$$1 = r_R\, r_L = r_g\, r_1\, e^{-2j(\tilde{\beta}_a L_a + \tilde{\beta}_p L_p)}$$

$$= |\, r_g \,|\, r_1\, e^{\Gamma_{xy} g L_a - \alpha_{ai} L_a - \alpha_{pi} L_p}\, e^{-2j(\beta_a L_a + \beta_p L_p - \frac{1}{2}\phi)}$$

Coldren&Corzine

DBR resonance condition and gain margin



Coldren&Corzine

In FP &DBR cavities **Δg** is the same

In FP cavity Δα=0 (constant mirror loss)   Weak mode selectivity

In DBR cavity Δα>> Δg strong mode selectivity -> Stronger peaks difference
SLM-> Single Longitudinal Mode

How to correlate previous results with the current injection: **the rate equations** (RE)

- The variables are:
  - The average photon density in the active region: N_P
  - The average carrier density in the active region: N
  - The injected current: I
- The RE can be written:
  - In each sections of the cavity after its longitudinal discretization
  - Averaging the variables in all the cavity
- The first case is more accurate (DFB) and the other more simple numerically and reasonably accurate
- We consider now the second case

Carrier balance in the active region

$$\frac{dN}{dt} = G_{gen} - R_{rec}$$

$$G_{gen} = \underbrace{\frac{I}{q}}_{\substack{\text{electrons} \\ \text{per second}}} \times \underbrace{\eta_i}_{\substack{\text{internal} \\ \text{q-efficiency}}} \times \underbrace{\frac{1}{V}}_{\substack{\text{..per} \\ \text{volume}}}$$

$$R_{rec} = R_{sp} + R_{nr} + R_{st}$$

$$R(N) = R_{sp} + R_{nr} \cong AN + BN^2 + CN^3 \cong \frac{N}{\tau_s}$$

Rate equation for carrier density

$$\frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau_s} - R_{st}$$

Rate of stimulated emission: R_{st}

For a homogeneous gain material the gain g is

$$\frac{dN_p}{dz} = g\,N_p$$

$v_g N_p A$ = number of photons injected into the volume per second

Total number of generated photons

$$\mathbf{v_g\left(N_{pout} - N_{pin}\right)A} = v_g A\, dN_p = v_g A\, g\, N_p\, dz = \mathbf{v_g g N_p V}$$

$$\boxed{R_{st} = v_g\, g\, N_p}$$

g depends on (N, λ); the value to be used is that at the λ of operation of the laser -> g(N, λ_{op}):

Semiconductor gain approximations

For bulk material

$$g \approx g_{peak} \approx a(N - N_{tr})$$

For QW material

$$\mathbf{g(N,N_p) = \frac{g_0}{1+\varepsilon N_p} \ln\left(\frac{N+N_s}{N_{tr}+N_s}\right)}$$

$$\mathbf{g(N,N_p) \cong \frac{g_0}{1+\varepsilon N_p} \ln\left(\frac{N}{N_{tr}}\right)}$$

Gain saturation

The Rate Equation for the carriers density is:

$$\mathbf{\frac{dN}{dt} = \eta_i \frac{I}{qV} - R(N) - v_g g N_p}$$

Rate equation for the **lasing mode** average photon density in the active region

The effects to be considered are:
-The stimulated emission
-The spontaneous emission coupled into the lasing mode
-The losses in the cavity: intrinsic and mirrors

For what concern the **Stimulated emission** we can write the relation between the total rate of stimulated emission (**$R_{st}V$**), the variation of the total number of carriers (**NV)** and the variation of the total number of photons **N_{Ptot}**

$$R_{st}V = -\frac{d(NV)}{dt} = \frac{d(N_{Ptotal})}{dt}$$

$$\mathbf{N_P} = \frac{\mathbf{N_{Ptotal}}}{\mathbf{V_P}}$$

Been relevant for the carrier rate equation the Photon density **N_P** we can define:

where **V_P is the "volume"** occupied by the **photons**

$$\frac{\mathbf{d(N_P)}}{\mathbf{dt}} = R_{st}\frac{V}{V_P} = v_g g\left(\frac{V}{V_P}\right)N_P = \mathbf{v_g(\Gamma g)N_P}$$

Γ g is the cavity average modal gain

Losses and Spontaneous emission

Average Cavity Losses:

characterized by **photon decay rate** ($\tau_p = 1/(v_g\, \alpha_T)$) where ($\alpha_T$) are the cavity modal losses: **intrinsic** (α_i) and **mirror** ($\alpha_m = -\ln(|r1\ r2|)/L$)

Average Spontaneous Emission density coupled to the cavity mode.

Can be obtained from the total spontaneous emission ($R_{sp}\, V$) normalized respect to the volume occupied by the photon (V_p) and taking into account that only the small part (β_{sp}) is coupled into the spectral interval of the lasing mode:

$\beta_{sp}\, R_{sp}\, V / V_p = \Gamma\, R'_{sp}$

The rate equation for the average photon density in the active region is:

$$\frac{dN_p}{dt} = \left(\Gamma v_g g - \frac{1}{\tau_p} \right) N_p + \Gamma R'_{sp}$$

RE summary:

Rate Equations for the total number of carriers and photons

$$\frac{dN_T}{dt} = \eta_i \frac{I}{q} - R(N)V - v_g \Gamma g N_{pT}$$

$$\frac{dN_{pT}}{dt} = \left(\Gamma v_g g - \frac{1}{\tau_p} \right) N_{pT} + R'_{sp} V$$

Static solution (see Coldren Corzine)

$$P_0 \cong h\nu \, F \frac{\alpha_m}{\langle \alpha_i \rangle + \alpha_m} \frac{N_p V_p}{\tau_p}$$ or $$N_p = \frac{\Gamma \beta_{sp} R_{sp}}{1/\tau_p - \Gamma v_g g(N)}$$

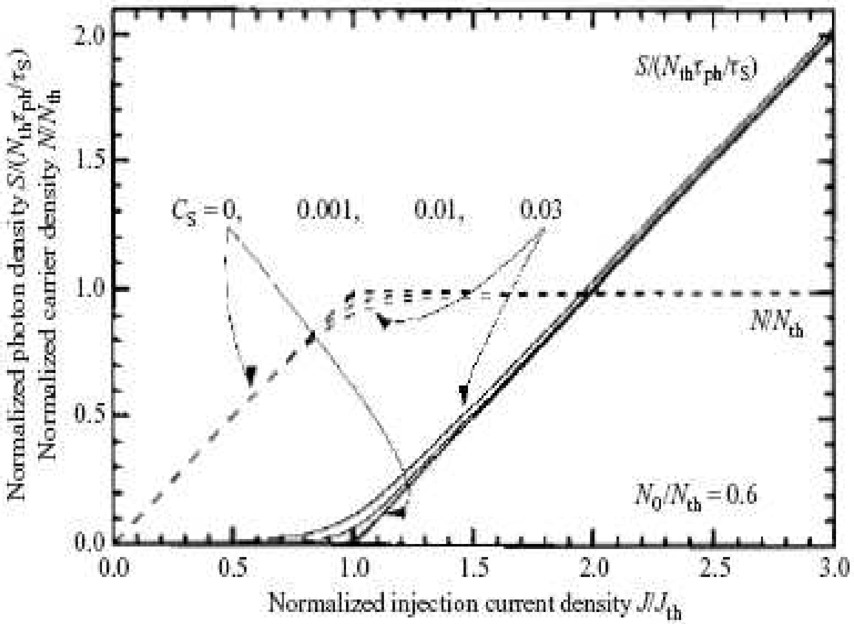
Multimode Rate Equations

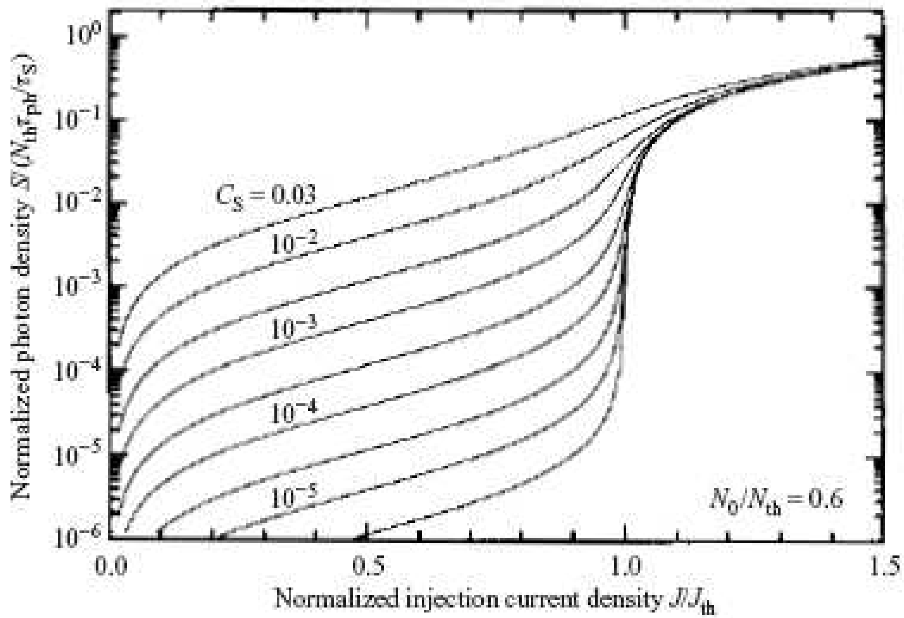
$$\frac{dN}{dt} = \eta_i \frac{I}{qV} - (R_{sp} + R_{n.r}) - \sum_m v_{gm} g_m N_{pm}$$

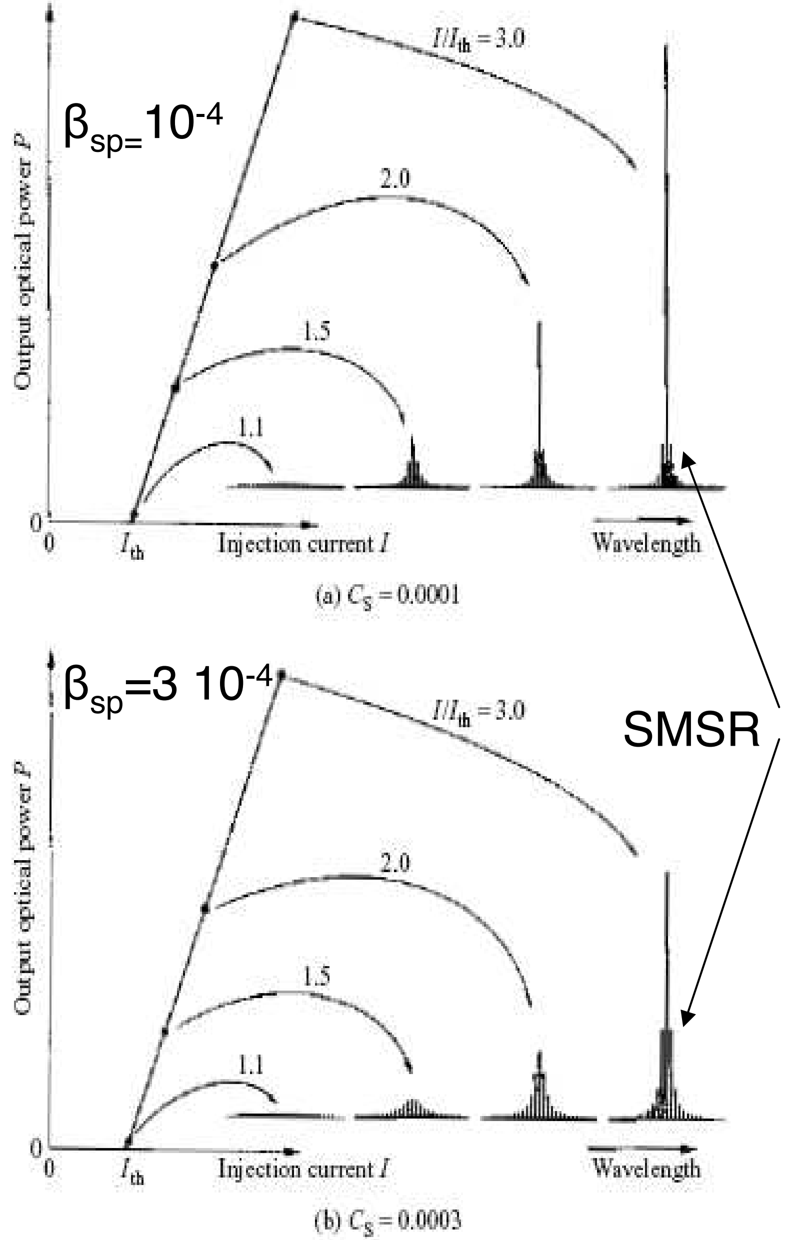
$$\frac{dN_{pm}}{dt} = \left(\Gamma_m v_{gm} g_m - \frac{1}{\tau_{pm}} \right) N_{pm} + \Gamma_m R'_{spm}$$

$$N_{pm} = \frac{\Gamma \beta_{sp} R_{spm}}{1/\tau_{pm} - \Gamma v_g g_m (N, N_{pm})}$$

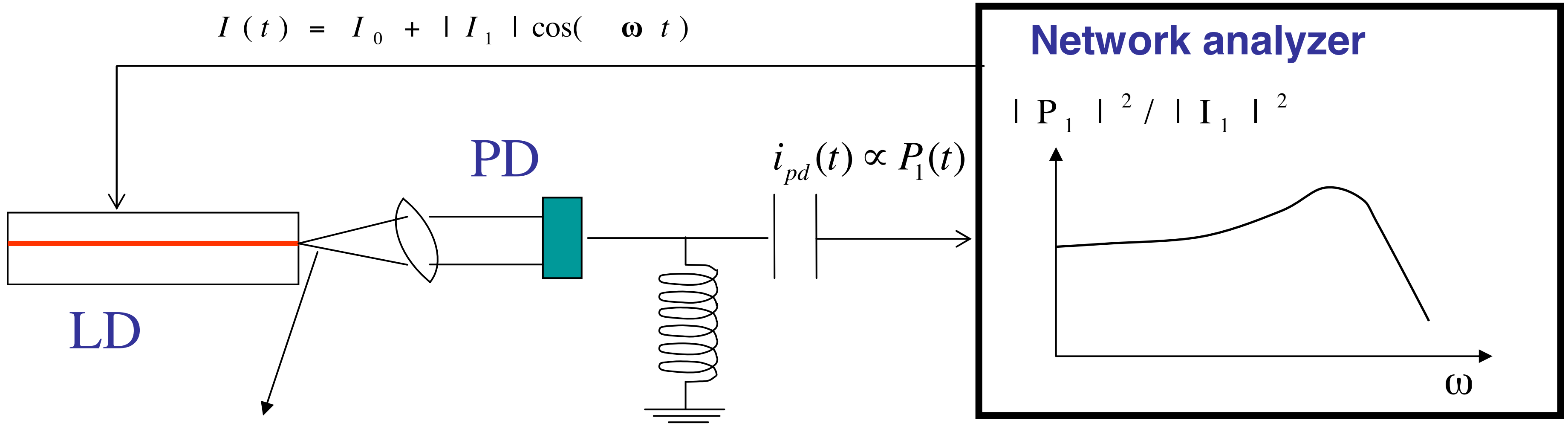
FP lasers P-I & Spontaneous emission factor effect







Measurement of modulation response



$$P(t) = P_0 + P_1(t)$$

$$P_1(t) = \mathrm{Re}[P_1(\omega) e^{j\omega t}] = |P_1(\omega)| \cos(\omega t + \theta_p)$$

Usually is measured the so called **electrical modulation response**

$$\left| \frac{H(\omega)}{H(\omega = 0)} \right|^2_{dB} = 20 \; Log \left| \frac{P_1(\omega)}{P_1(\omega = 0)} \right|$$

Small-signal modulation response

From the RE linearization (see Coldren & Corzine) and assuming a small harmonic current excitation over the bias, the system of differential equation can be solved analytically and the impulse response function is obtained:

$$\mathbf{H}(\omega) = \frac{P_1(\omega)}{P_1(\omega = 0)} = \frac{\omega_R^2}{\Delta} = \frac{\boldsymbol{\omega}_{\mathbf{R}}^2}{\boldsymbol{\omega}_{\mathbf{R}}^2 - \boldsymbol{\omega}^2 + \mathbf{j}\boldsymbol{\omega}\gamma}$$

whose parameters are:

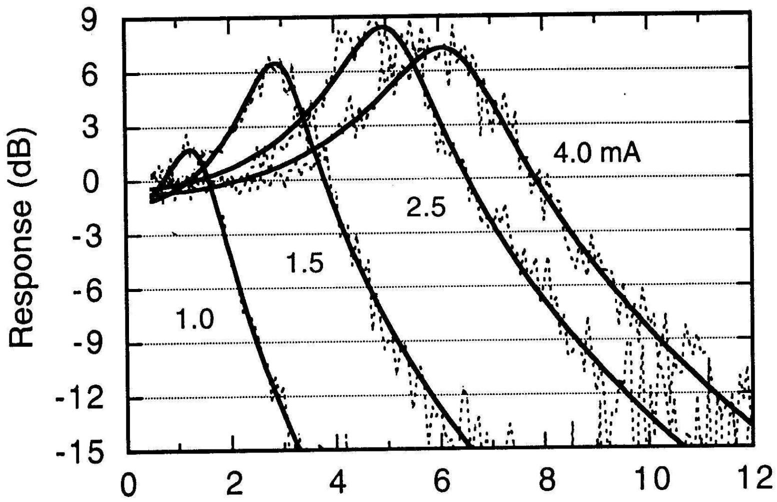
Relaxation oscillation frequency

$$\boldsymbol{\omega}_{\mathbf{R}}^2 \equiv \gamma_{NP}\gamma_{PN} + \gamma_{NN}\gamma_{PP} \approx \frac{\mathbf{v}_{\mathbf{g}}\mathbf{aN}_{\mathbf{p}}}{\boldsymbol{\tau}_{\mathbf{p}}},$$

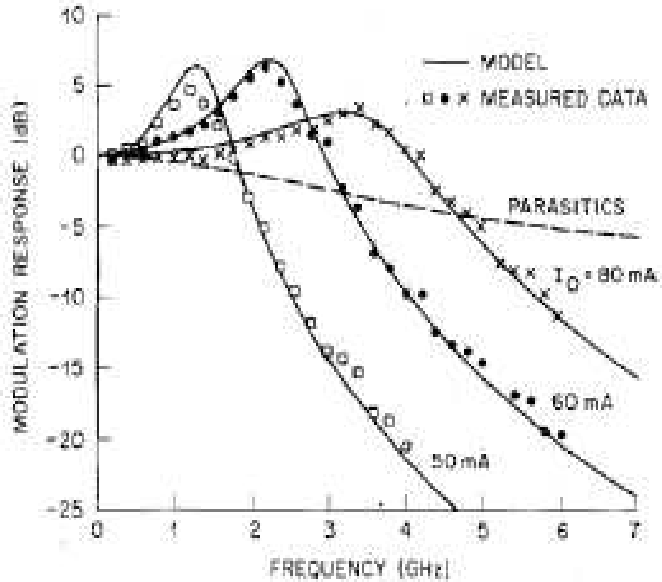
Damping rate $\boldsymbol{\gamma} \equiv \gamma_{NN} + \gamma_{PP} = \mathbf{Kf}_{\mathbf{R}}^2 + \boldsymbol{\gamma}_0$

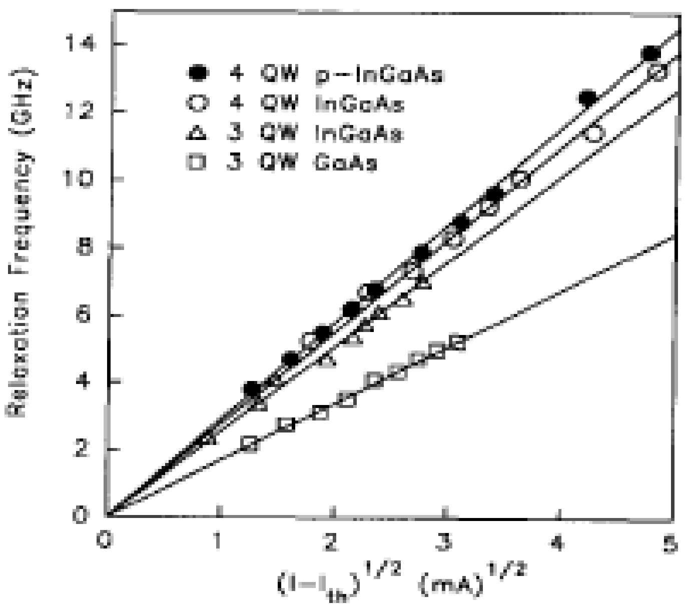
and $f_{3dBmax} = 2\pi\sqrt{2}/K$ $\quad K = 4\pi^2\tau_p\left[1 + \frac{\Gamma a_p}{a}\right],$

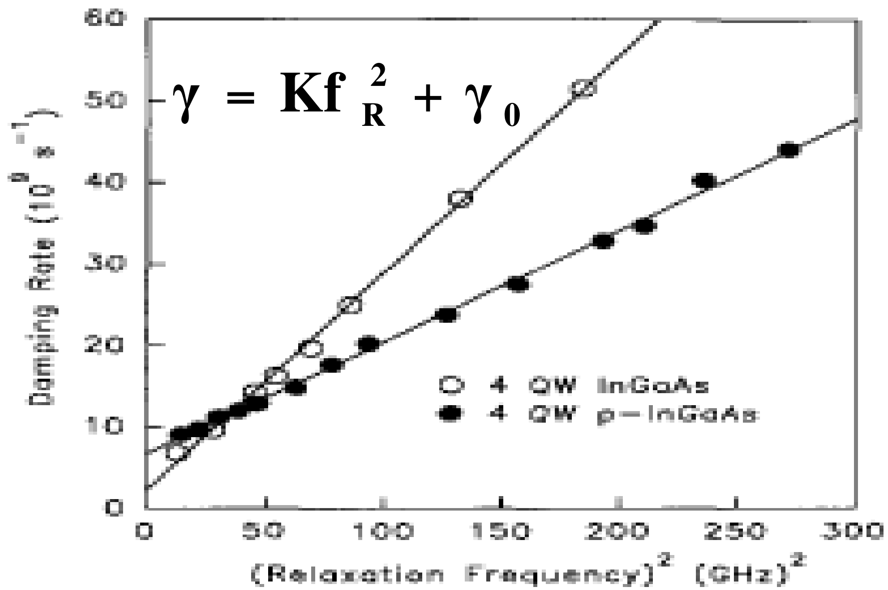
Experimental results small signal modulation



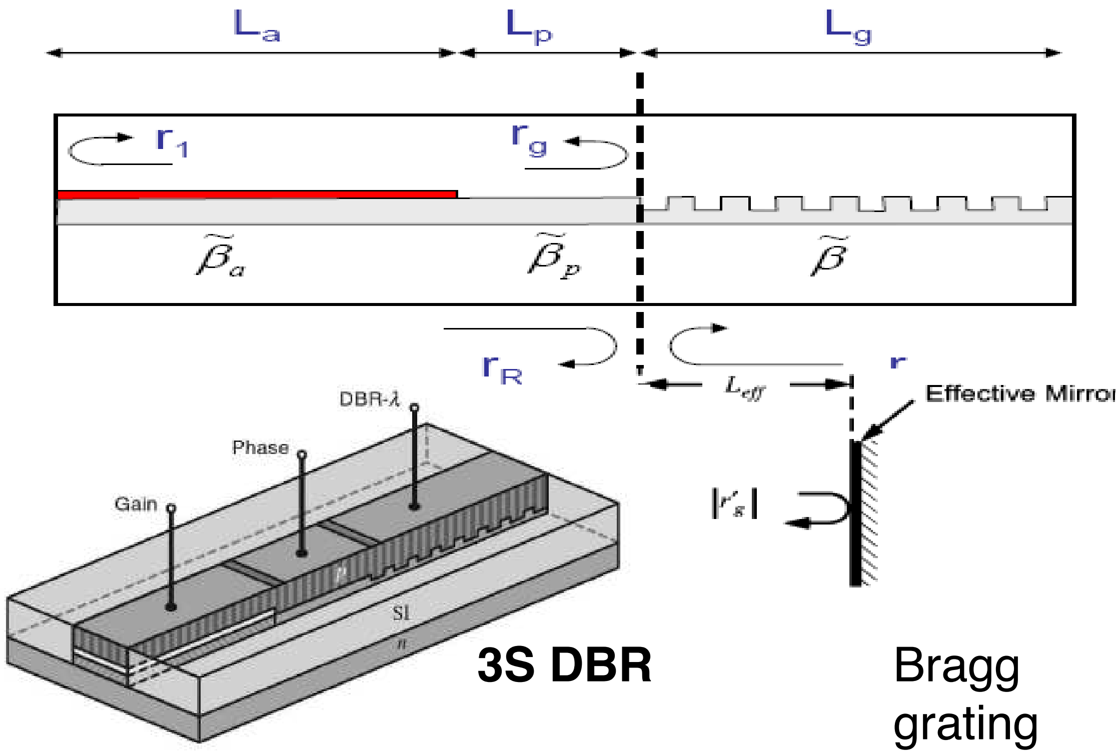
Coldren &Corzine



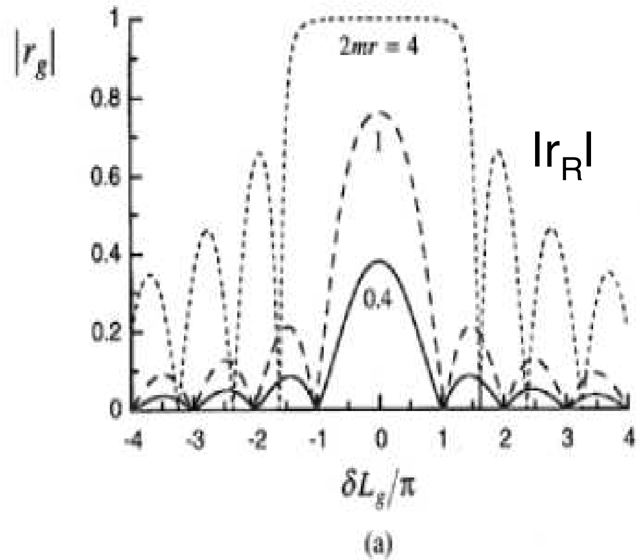


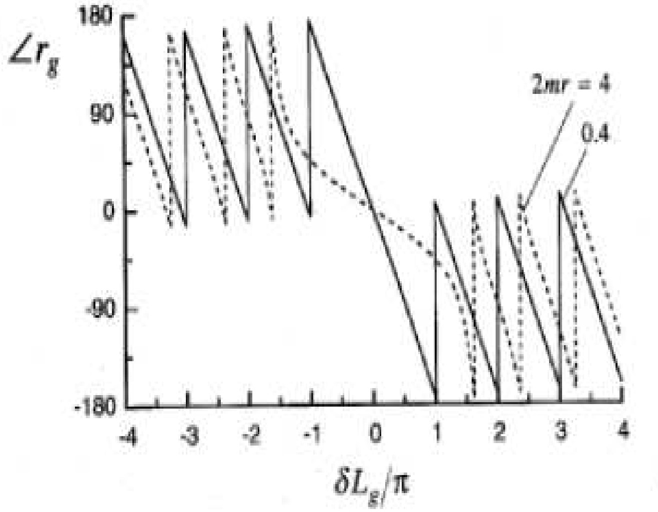


The DBR laser: tunability



Bragg grating reflectivity →





Oscillation condition

$$1 = r_R r_L = r_g r_1 e^{-2j(\tilde{\beta}_a L_a + \tilde{\beta}_p L_p)}$$

$$= |r_g| r_1 e^{\Gamma_{xy} g L_a - \alpha_{ai} L_a - \alpha_{pi} L_p} e^{-2j(\beta_a L_a + \beta_p L_p - \frac{1}{2}\phi)}$$

Tuning of a DBR laser

Wavelength of mode m:

$$m\lambda_m \approx 2\,(n_a L_a + n_p L_p + n_{DBR} L_{eff})$$

Relative change $\Delta\lambda_m$ due to changes in refractive index n_a, n_p, n_{DBR}

$$\frac{\Delta\lambda_m}{\lambda_m} = \frac{\Delta n_a L_a + \Delta n_p L_p + \Delta n_{DBR} L_{eff}}{n_a L_a + n_p L_p + n_{DBR} L_{eff}}$$

Coldren&Corzine

Index change due to current injection:

For active section the carrier density is clamped $\Delta n_a \approx 0$

For passive sections

$$\frac{\eta_i I_j}{qV_j} = R(N_j)\,, \qquad j = p, DBR$$

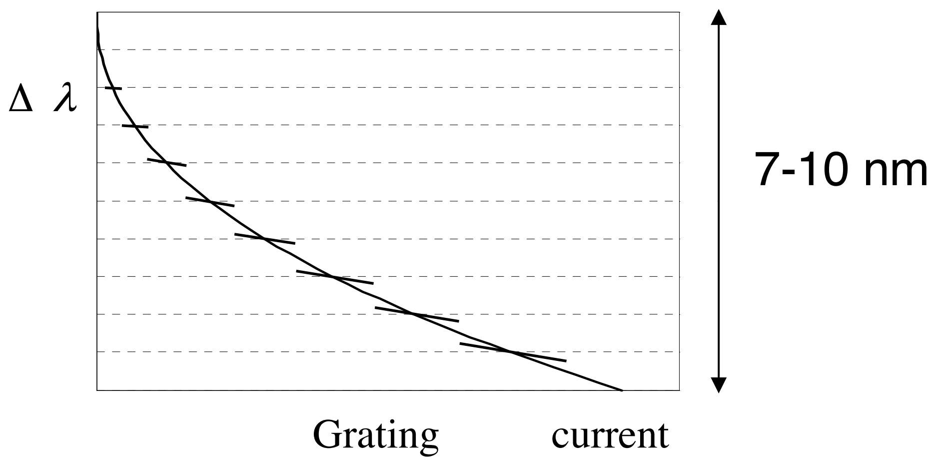
$$\Delta n_j = \frac{\partial n}{\partial N} N(I_j)\,, \qquad j = p, DBR$$

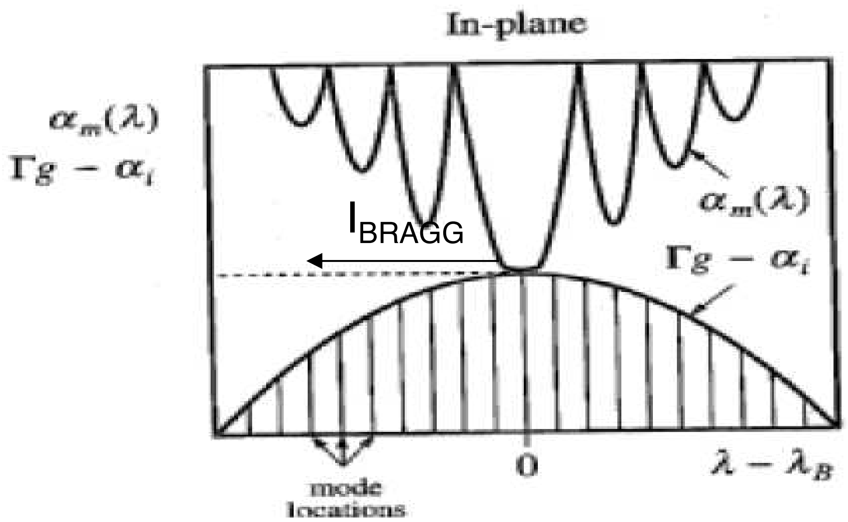
Tuning of the DBR grating

Relative change of the Bragg wavelength:

$$\lambda_{Bragg} = 2 n_{DBR} \Lambda$$

$$\frac{\Delta \lambda_{Bragg}}{\lambda_{Bragg}} = \frac{\Delta n_{DBR}}{n_{DBR}} = \frac{1}{n_{DBR}} \frac{\partial n_{DBR}}{\partial N} \Delta N \left(I_{DBR} \right)$$



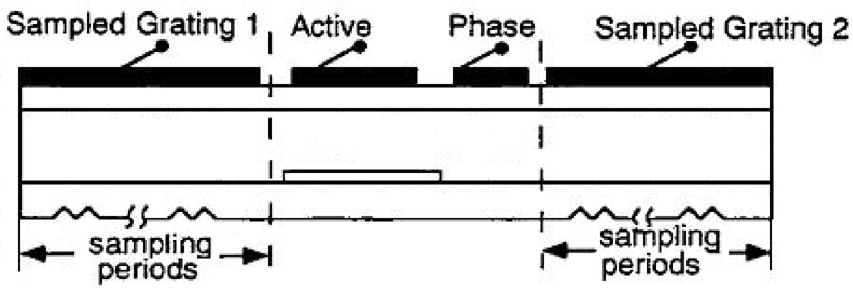


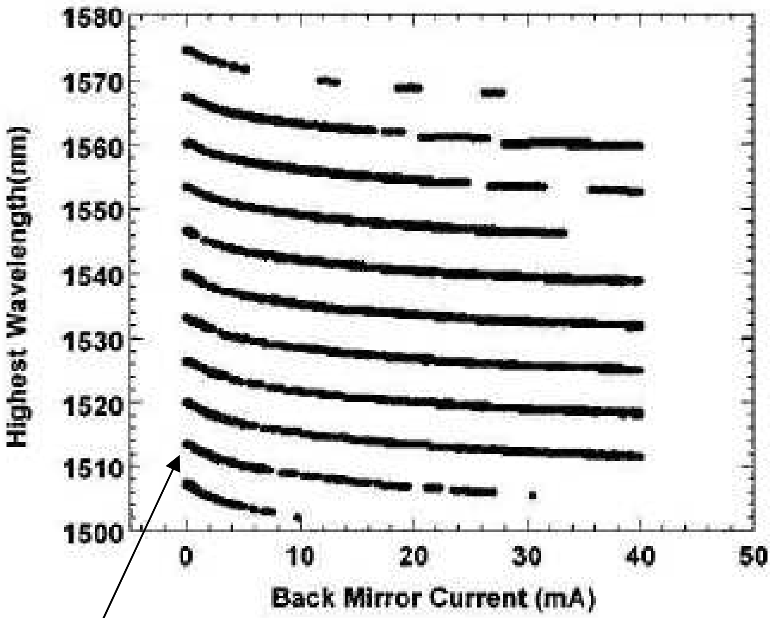
Discontinuous line for grating current injection only

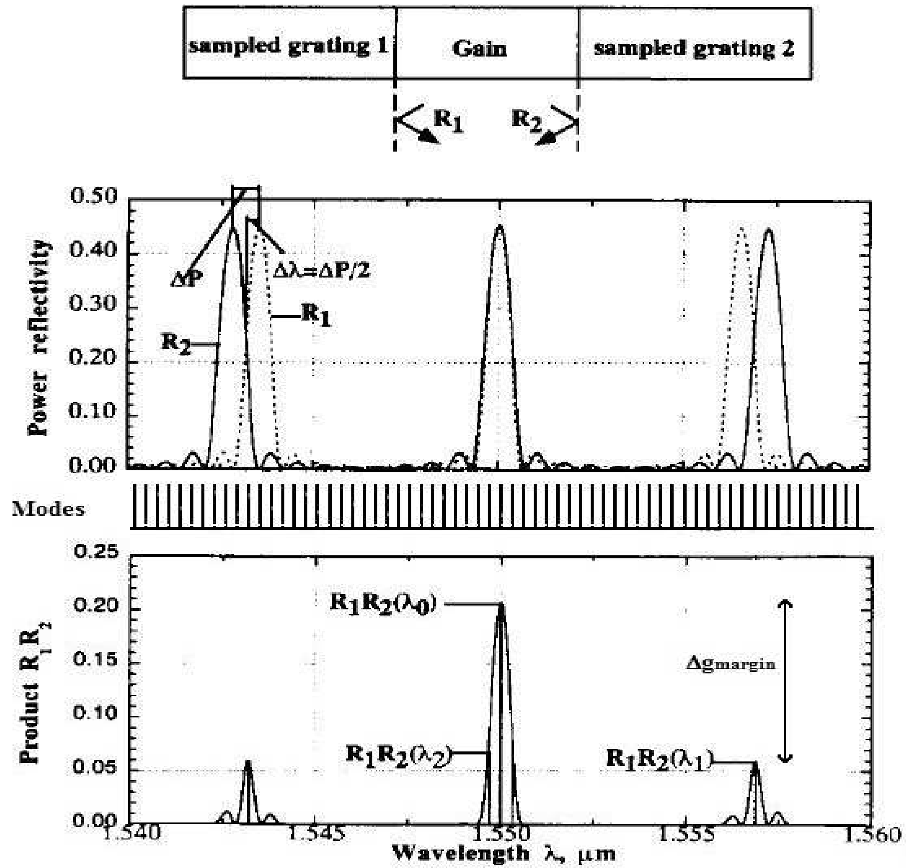
Continuous line with proper grating and phase section currents injection: synchronous shift of modes and Bragg wavelength

DBR with wide tunability:SG DBR

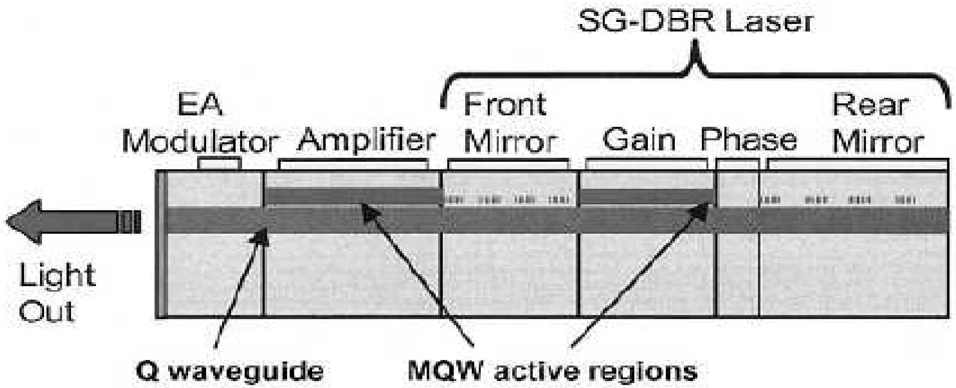
(UCSB)





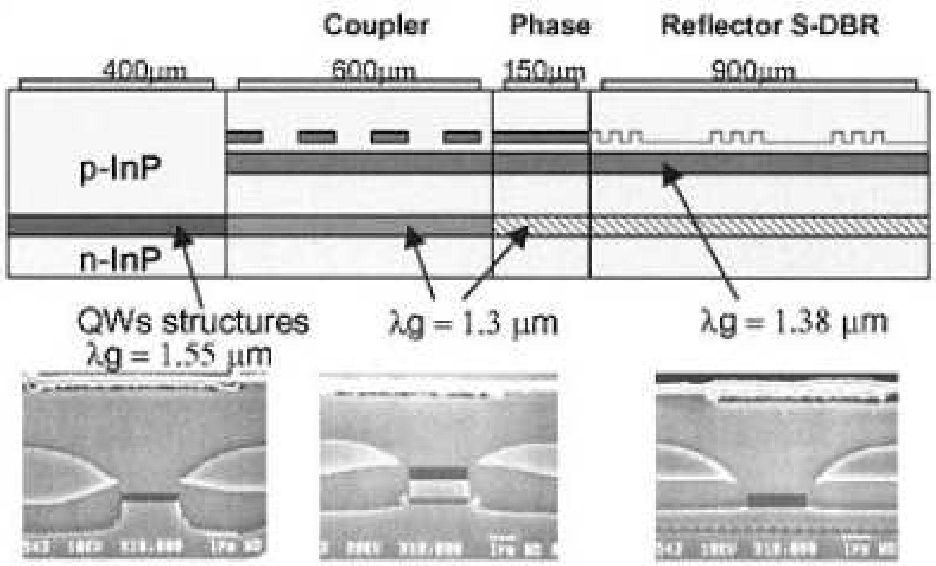


Change in the aligned peaks

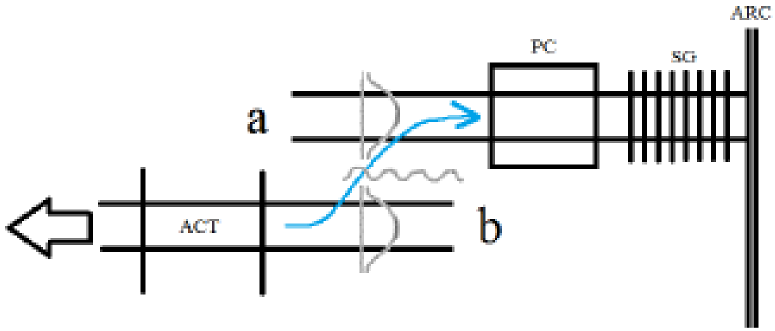


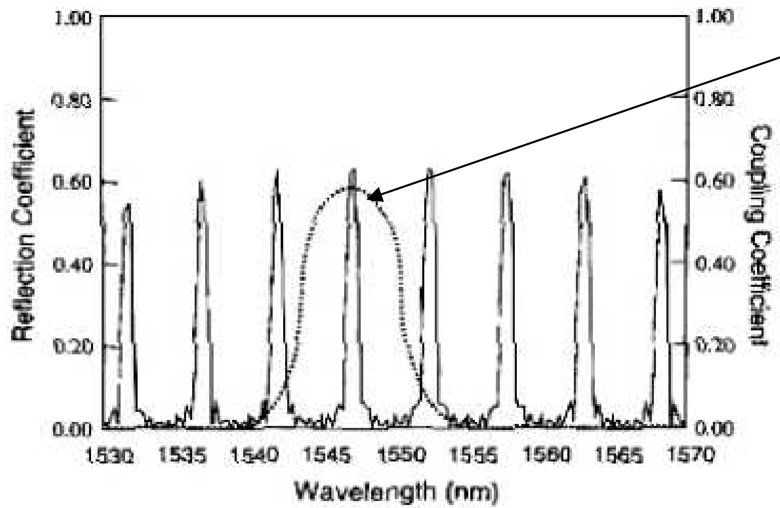
Coldren & Corzine

DBR with wide tunability:GCSR



KTH



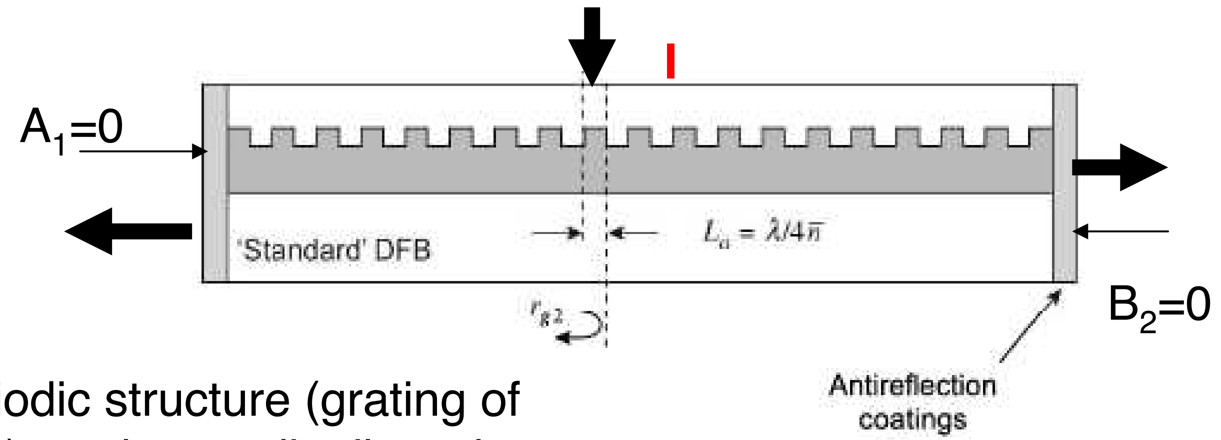


Selection of the peak by current injection in the coupler

Overlap of the GACC coupler bandwidth with the SG mirror spectrum

Respect to SG-DBR
Similar tunability
More output power
Reduced spectral characteristics

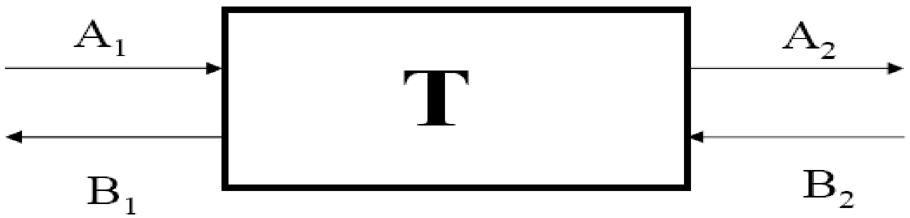
The DFB laser



The periodic structure (grating of period Λ) produce a distributed coupling between the forward (A) and backward (B) propagating waves represented by →

$$\begin{cases} \dfrac{\mathrm{d}A(\omega,z)}{\mathrm{d}z} = -\mathrm{j}\left(\tilde{\beta}-\beta_0\right)A(\omega,z) - \mathrm{j}kB(\omega,z) \\ \dfrac{\mathrm{d}B(\omega,z)}{\mathrm{d}z} = +\mathrm{j}\left(\tilde{\beta}^*-\beta_0\right)B(\omega,z) + \mathrm{j}kA(\omega,z) \end{cases}$$

k the coupling coefficient and $\beta_0 = 2\pi/\Lambda$



$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

For lasing: $A_1 = B_2 = 0$

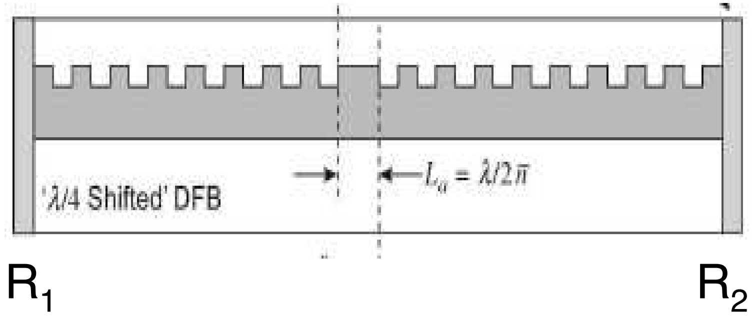
$$0 = \mathbf{T}_{11} A_2$$

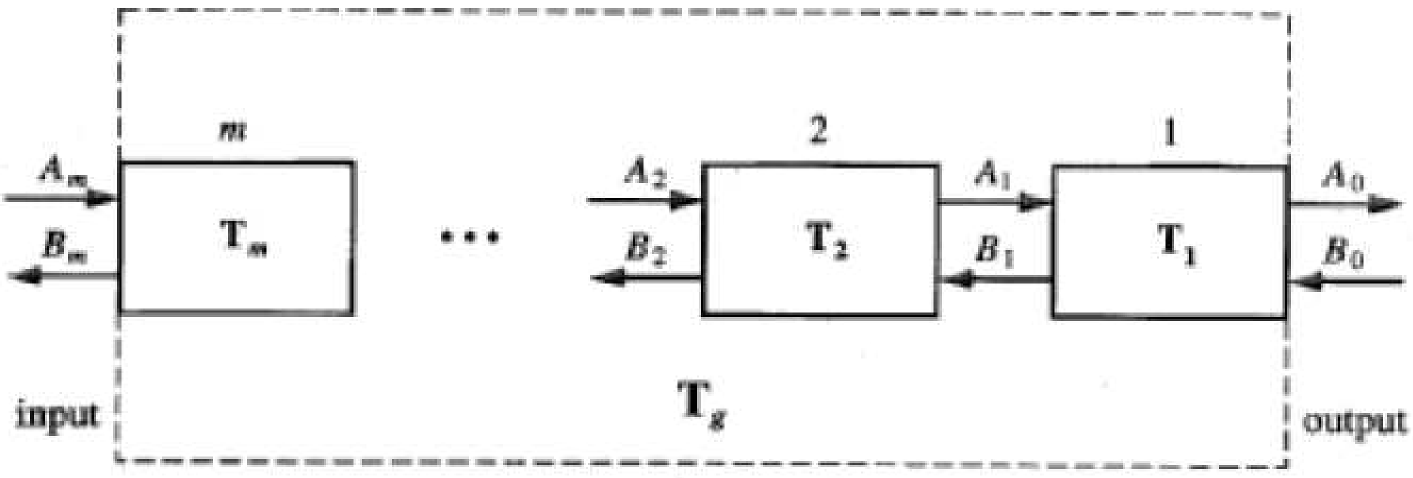
$$B_1 = T_{21} A_2$$

Lasing condition **$T_{11}=0$**

The DFB laser - II

Similarly for more complex cavities cascading the transmission matrix of each element





$$\begin{pmatrix} A_m \\ B_m \end{pmatrix} = \mathbf{T} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}, \qquad \mathbf{T} = \mathbf{T}_m \cdots \mathbf{T}_2\, \mathbf{T}_1$$

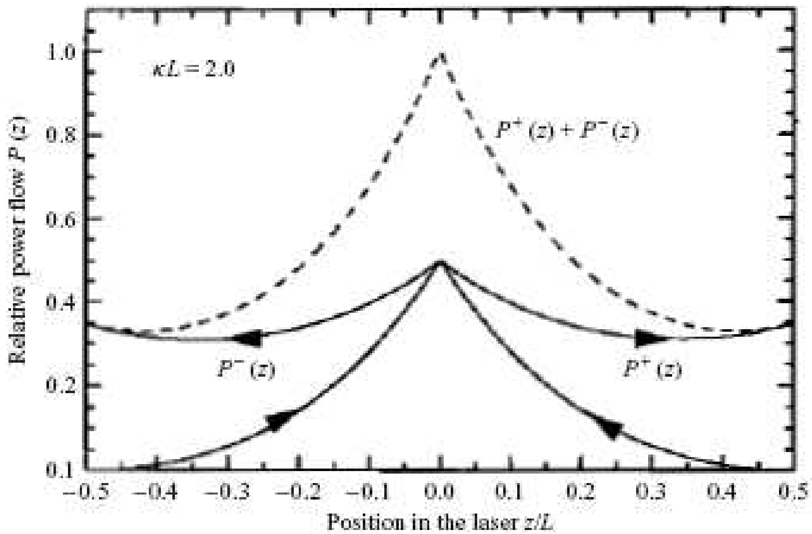
$$0 = \mathbf{T}_{11} A_0$$

$$B_m = T_{21} A_0$$

DFB above threshold

QWS-DFB

The **non uniform field distribution along the cavity** significantly changes the β parameter of the propagation equation that depends on the local power density



$$\begin{cases} \dfrac{\mathrm{d}A(z)}{\mathrm{d}z} = -\mathrm{j}\left(\tilde{\boldsymbol{\beta}}(\mathbf{N},\mathbf{N_p}) - \beta_0\right)A(z) - \mathrm{j}kB(z) \\ \dfrac{\mathrm{d}B(z)}{\mathrm{d}z} = +\mathrm{j}\left(\tilde{\boldsymbol{\beta}}^*(\mathbf{N},\mathbf{N_p}) - \beta_0\right)B(z) + \mathrm{j}kA(z) \end{cases}$$

A longitudinal segmentation of the cavity is needed both for the static and dynamic analysis.

For the dynamic analysis the previous equation can be transformed in a time domain equation using a Fourier transform technique and linearizing the frequency dependence of the propagation constant β.

$$\tilde{\beta}(\omega,N) = \frac{\omega_0}{\mathrm{c}}\tilde{n}_{eff}(\omega_0,N) + \left.\frac{\mathrm{d}\beta(\omega,N)}{\mathrm{d}\omega}\right|_{\omega=\omega_0}(\omega-\omega_0)$$

DFB dynamic propagation equations

after Fourier transform

$$\begin{cases} \left[\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial z}\right] a(z,t) = \left\{\frac{\Gamma g(N) - \alpha}{2} - j\frac{\omega_0}{c}\left[n_{eff}(N) - n_{eff,0}\right]\right\} a(z,t) - jkb(z,t) \\ \left[\frac{\partial}{\partial z} - \frac{1}{v_g}\frac{\partial}{\partial z}\right] b(z,t) = \left\{\frac{\Gamma g(N) - \alpha}{2} + j\frac{\omega_0}{c}\left[n_{eff}(N) - n_{eff,0}\right]\right\} b(z,t) + jka(z,t) \end{cases}$$

$$\tilde{n}_{eff}(\omega_0, N) = \frac{\omega_0}{c} n_{eff}(N) + j\left[\frac{\Gamma g(\omega_0, N) - \alpha}{2}\right]$$

Boundary conditions

$$a(0,t) = \sqrt{R_0}\, b(0,t)$$

$$b(L,t) = \sqrt{R_L}\, a(L,t)$$

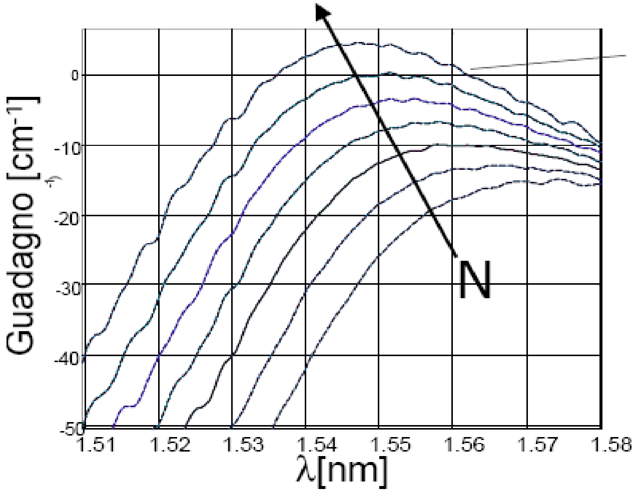
Carrier rate equation

$$\frac{d}{dt} N(z,t) = \frac{I(t)}{eV} - \left[AN(z,t) + BN^2(z,t) + CN^3(z,t)\right] - \frac{v_g g(N) S(z,t)}{1 + \varepsilon S}$$

$$S(z,t) = \left|a(z,t)^2\right| + \left|b(z,t)^2\right|$$

How to include the physical effects

The gain:



The dependence with λ can be included in the time domain using a numerical filter

$$\begin{cases} f_{z,t} = (1-A)\tilde{f}_{z,t} + A\tilde{f}_{z,t-\Delta t} \\ r_{z,t} = (1-A)\tilde{r}_{z,t} + A\tilde{r}_{z,t-\Delta t} \end{cases}$$

$$H(\omega) = \frac{1-A}{1-Ae^{-j\omega\Delta t}}$$

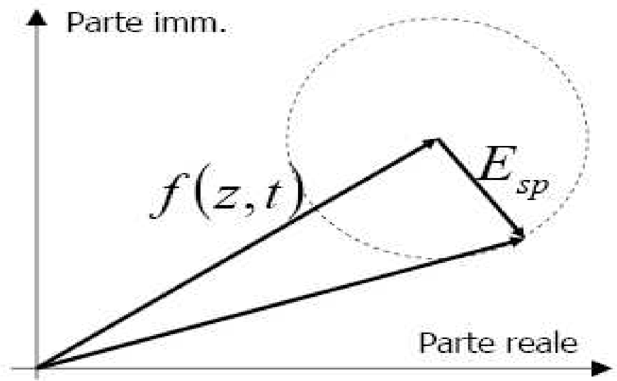
The refractive index variation with the carriers

$$n_{eff}(z,t) = n_{eff,0} - \Delta n\left[N(z,t) - N_0\right] \qquad \Delta n = \alpha_{LEF}\frac{\lambda_0}{2\pi}\frac{\partial g(N(z,t),t)}{\partial N(z,t)}$$

The spontaneous emission in each section

$$E_{sp} = \sqrt{\beta_{sp} B N(z,t)^2 \Delta t}$$

with a random phase



The dynamic propagation equations

The previous equations are very general and can be used to study the characteristics not only of a DFB but also of a quite general guided wave optoelectronic component when assuming:

- the **parameters** describing the propagation may change in the longitudinal direction considering both active, passive, with and without grating sections

- different **terminal boundary conditions** (with reflection or not)

- **static or dynamic excitation**.

In practice all the structures discussed before and many others as:

- Lasers for pulse generation: Q-and gain switched, **mode locked, ..**

- **SOA** : semiconductor optical amplifiers

- **SLED**: super luminescent LED based on waveguide configuration

- etc…

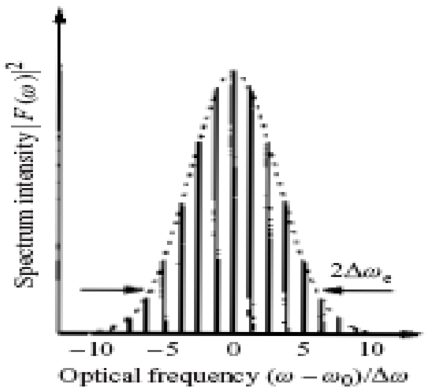
Integrated semiconductor mode locked (ML) lasers for short pulse generation

A short pulse is generated when the longitudinal modes of a laser are locked in phase .

From Fourier analysis the pulse duration depends on the number of locked modes and the repetition rate from the mode frequency separation.

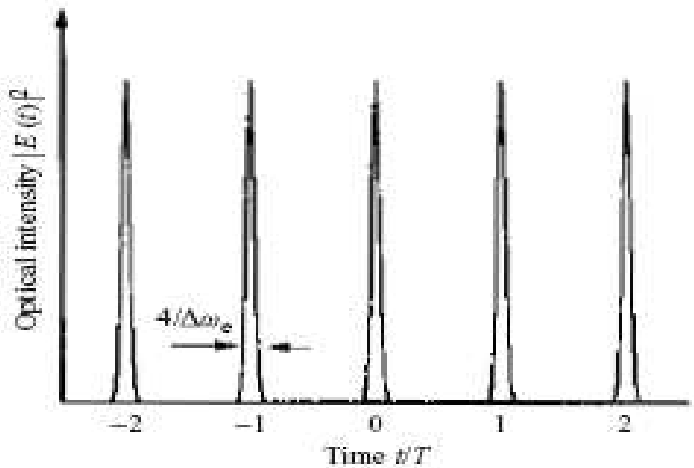
Ex. if

$$F(\omega) = \sum_m \exp\left[-\left(\frac{m\Delta\omega}{\Delta\omega_e}\right)^2\right]\delta(\omega - \omega_0 - m\,\Delta\omega)$$



In the time domain:

$$E(t) = \frac{1}{2\pi^{1/2}}\frac{\Delta\omega_e}{\Delta\omega}\sum_n \exp\left[-\left(\frac{\Delta\omega_e}{2}\right)^2 (t - nT)^2\right]\exp(i\omega_0 t)$$



where the repetition period is

$$T = \frac{2\pi}{\Delta\omega} = \frac{2LN_g}{c}$$

How mode locking take place

The **carrier pulsation** **at a frequency close to the cavity modes FSR helps the power transfer and the phase locking between the cavity modes**.

How Mode Locking can be obtained:

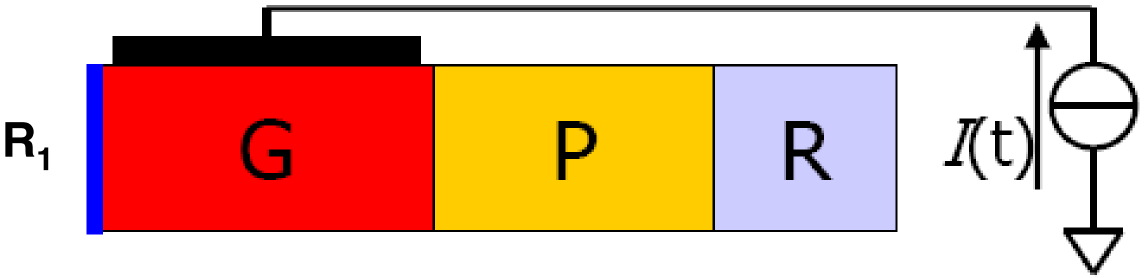
- by current modulation at the FSR frequency: **ACTIVE ML**

- by a self induced modulation in a 2 sections lasers: **PASSIVE ML**

- by a combination of active and passive ML: **HYBRID ML**

- by self mode locking due to carrier modulation induced by mode beating

Active ML



G: gain section

P: passive section

R: reflector

$$I(t) = I_{bias} + I_m \sin(2\pi f_m t)$$

$$f_m \approx FSR$$

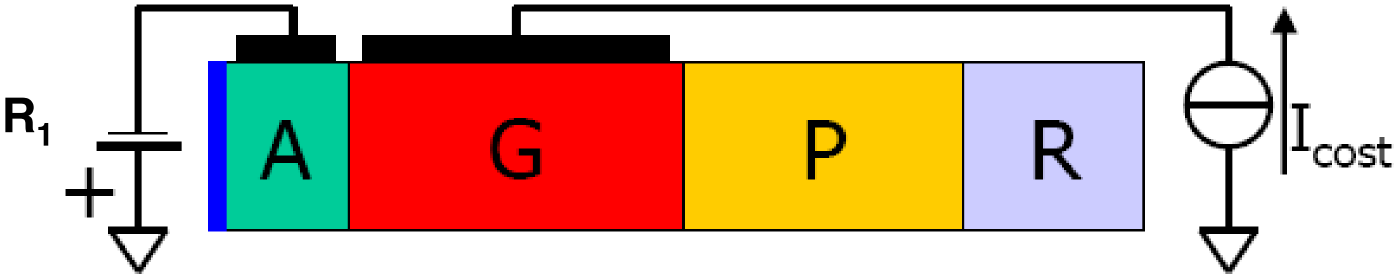
The **active section** allow the lasing

The **passive section** can be used to set the FSR and to modify it by current tuning

The reflector if is a grating it allows to define and tune the pulse wavelength by current injection

The repetition rate is precisely defined by the modulation frequency when around the FSR

Passive ML



The **active (G) section** allow the cavity to lase and contributes with its saturation to the ML pulse formation
The **saturable absorber (A)** reverse biased contribute to the pulse formation with its fast recovery
The **passive section (P)** to set the FSR and to modify it by current tuning
The **reflector (R)** if a grating allow to define and tune the pulse wavelength

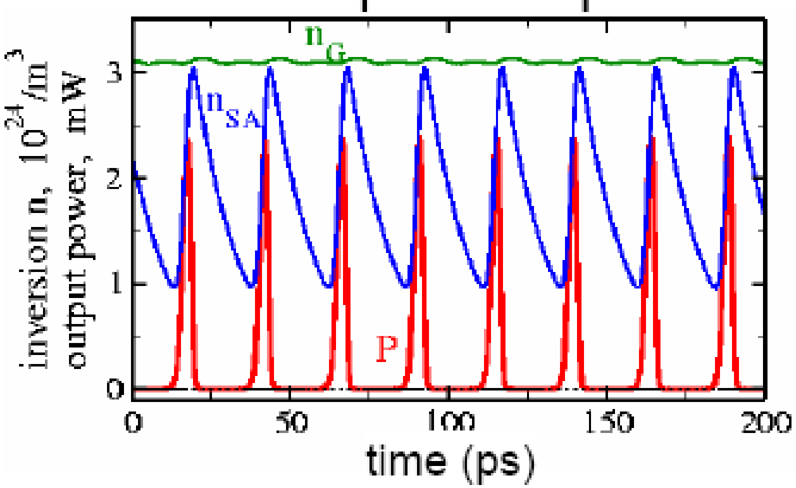
The repetition rate is reasonably stable around the cavity FSR

Mode locking take place if the cavity operates in the condition:

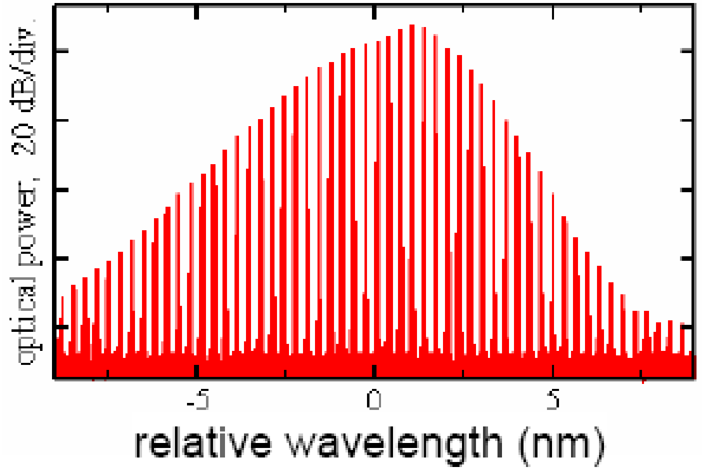
$$\left.\frac{\mathrm{d}g}{\mathrm{d}N}\right|_{A} > \left.\frac{\mathrm{d}g}{\mathrm{d}N}\right|_{G}$$

Simulation results for passive ML

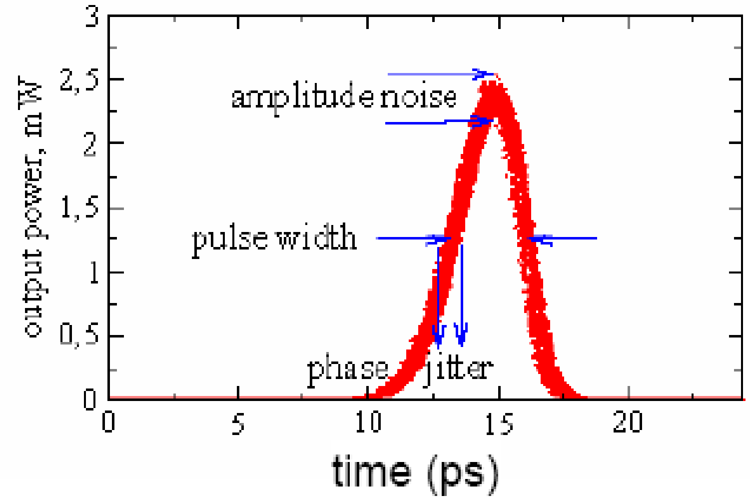
transient pulse sequence



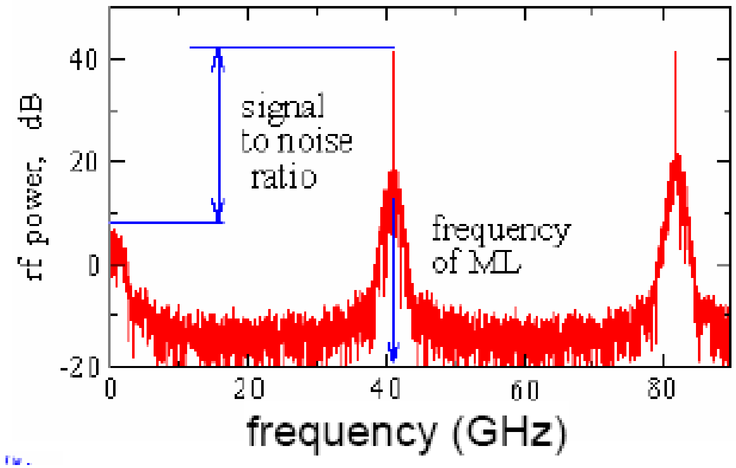
optical spectra: many locked modes



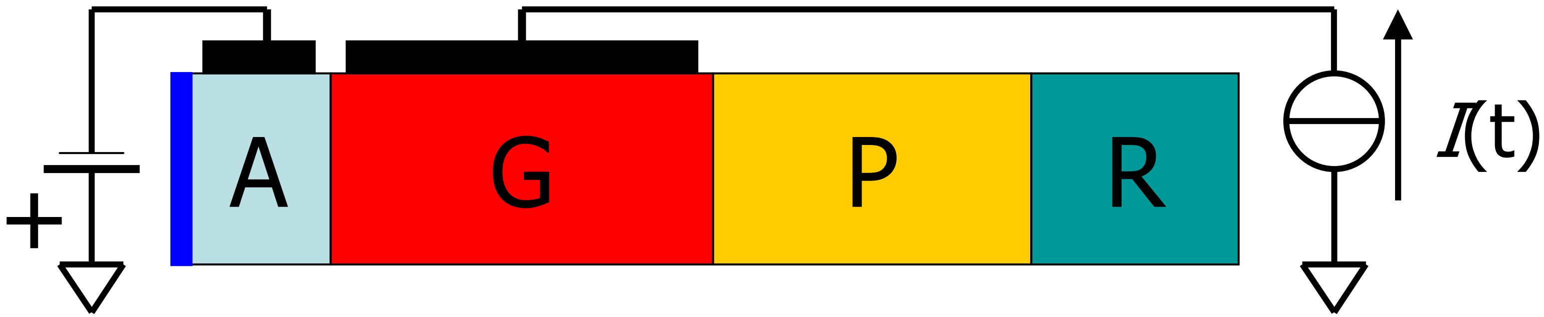
sampled pulses (eye)



RF spectrum: SNR



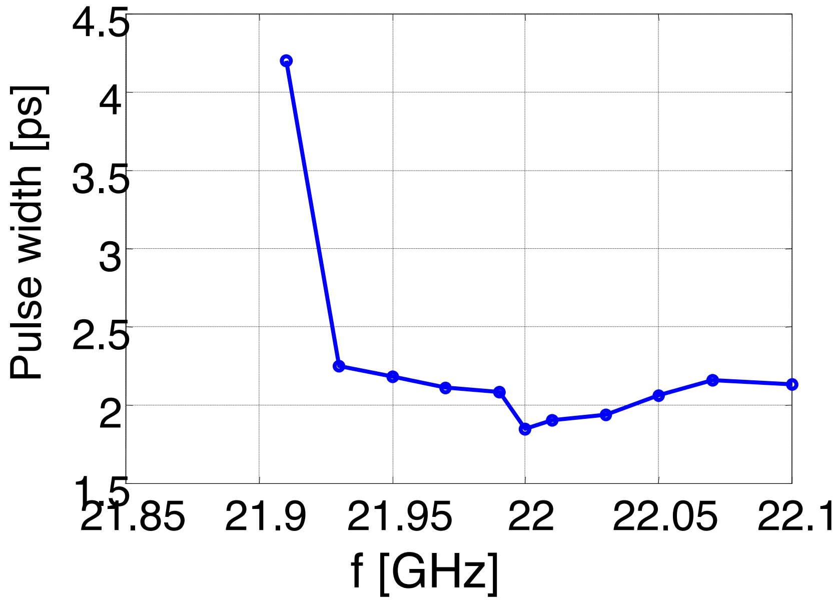
Hybrid ML



Has the advantage of:

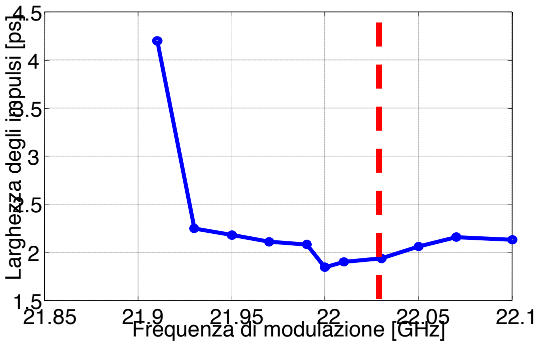
-**repetition rate stability and accuracy of active ML** when it operates around the cavity FSR.

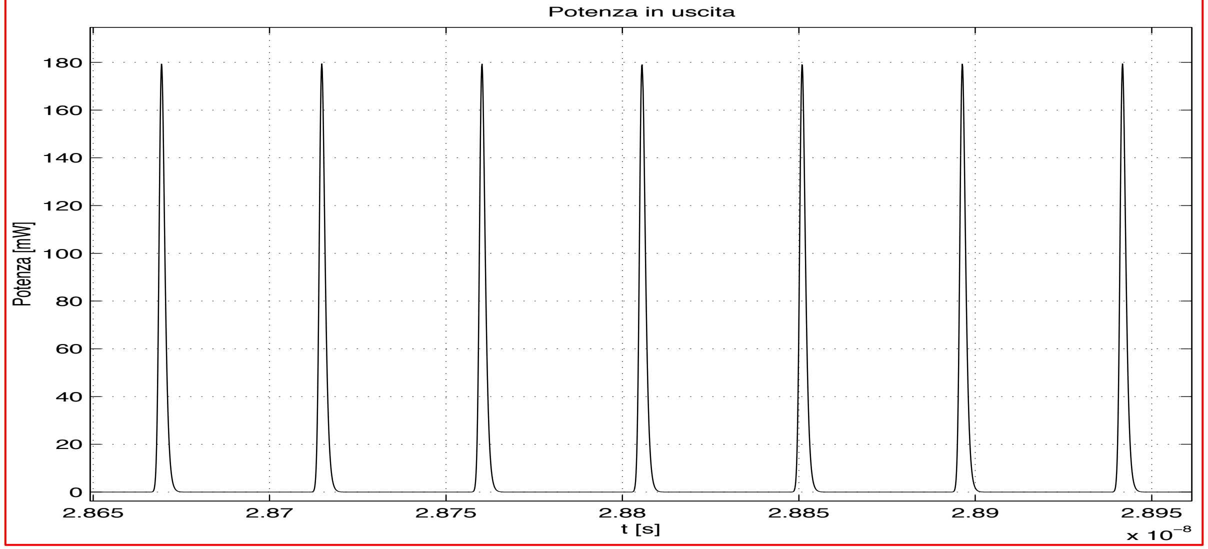
- it requires a **smaller modulation current**

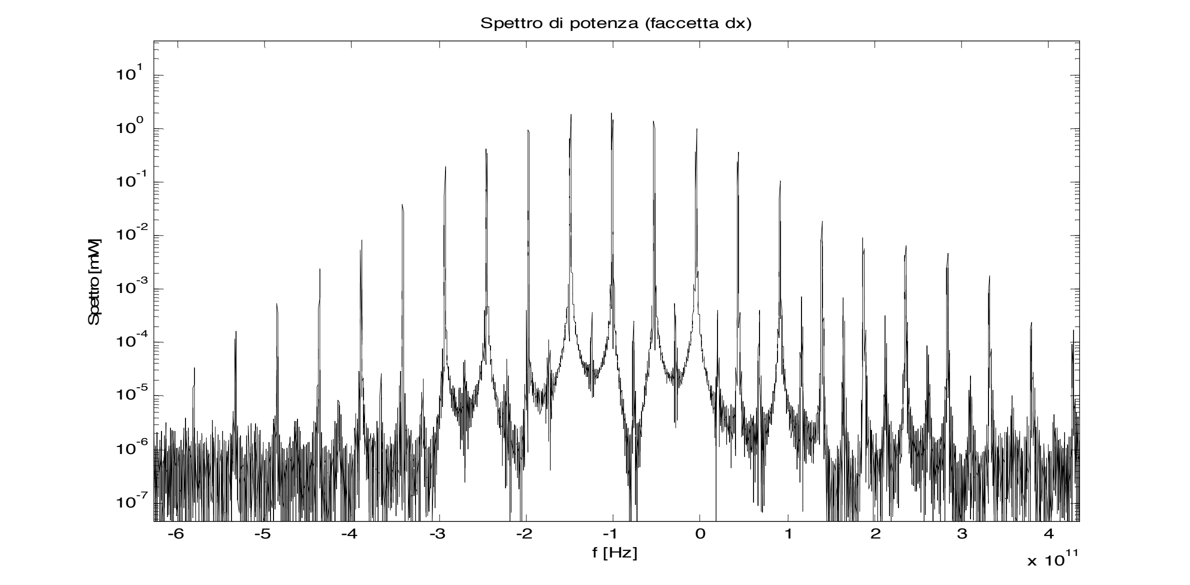


Ex.1 of Active ML simulation results

- Modulazion at **22.03GHz**
- Pulse width FWHM 1.74ps

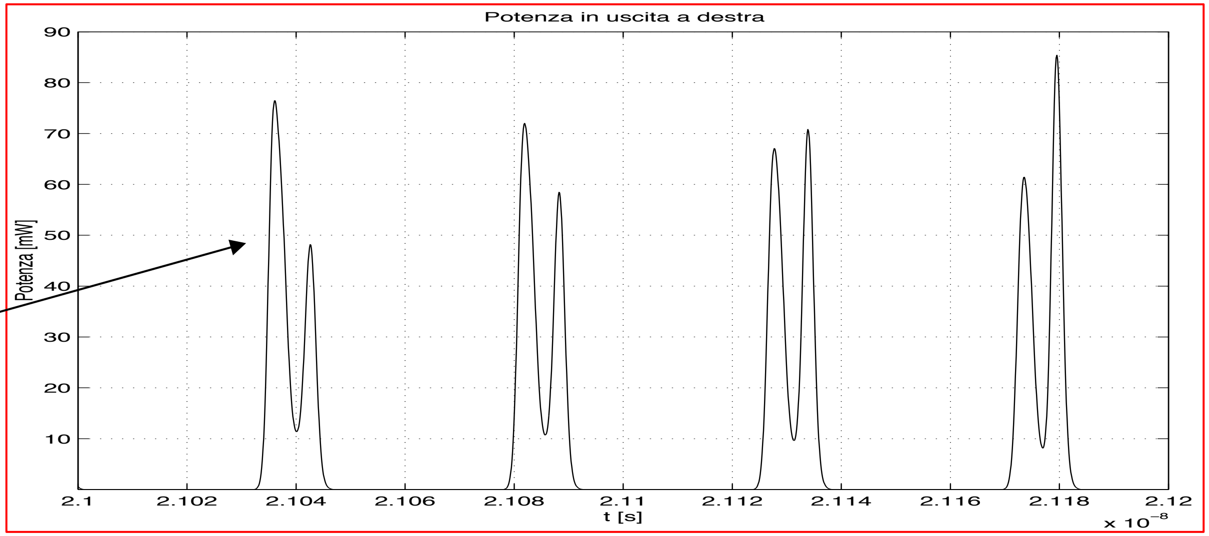


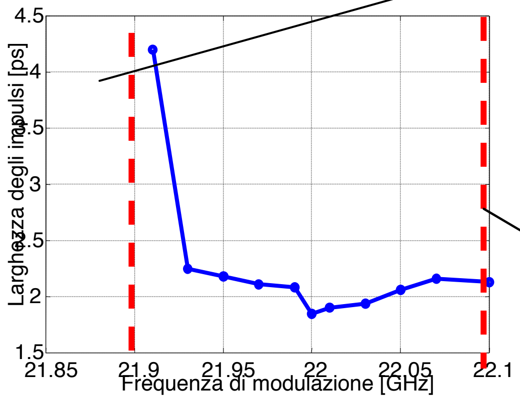


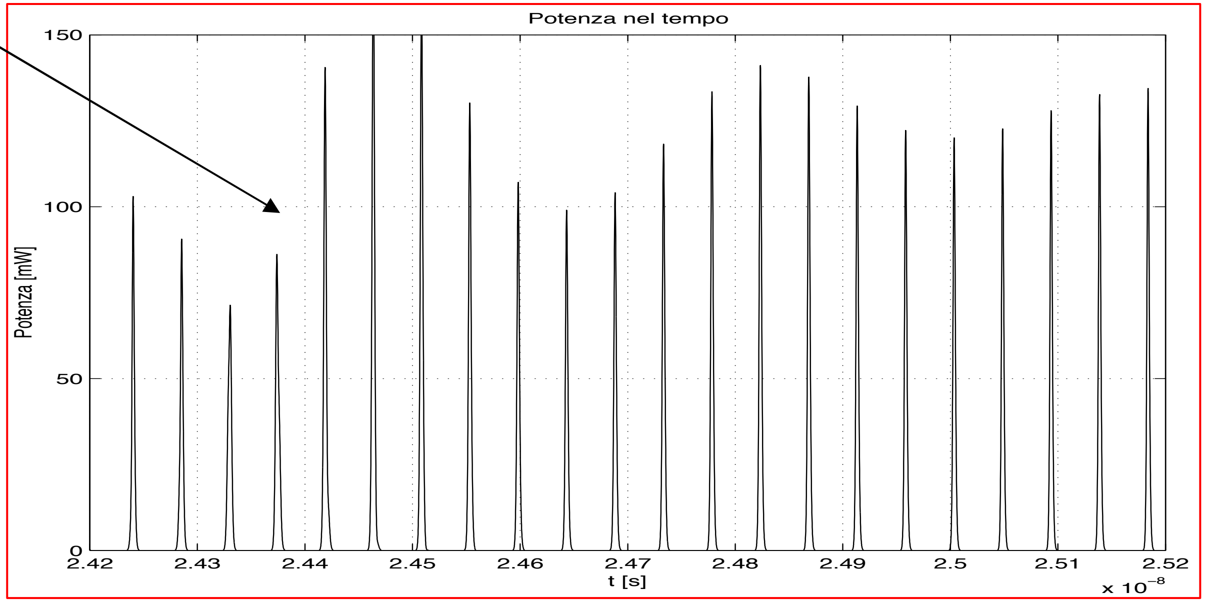


Ex.2 of Active ML simulation results

- Modulazion at **21.9 GHz**
- Multiple peaks



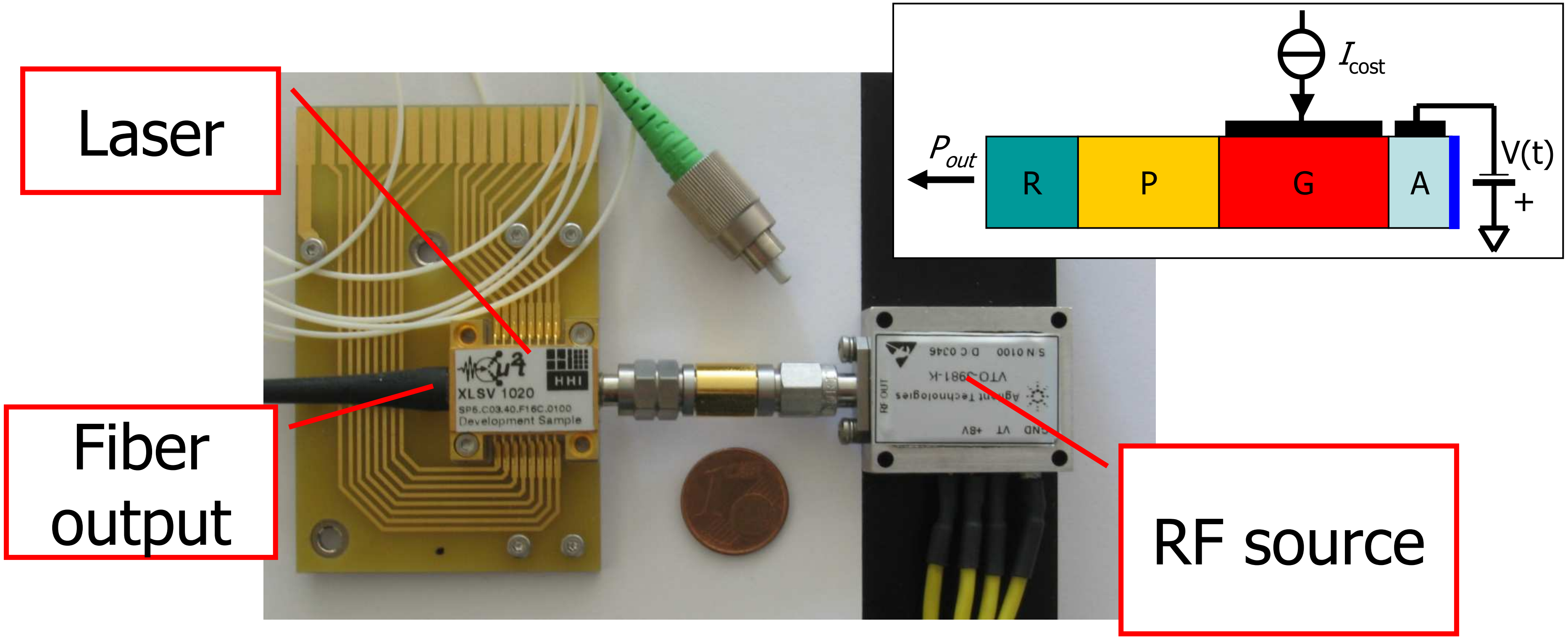




- Modulazion at **22.1GHz**
- Narrow peaks with strong modulation

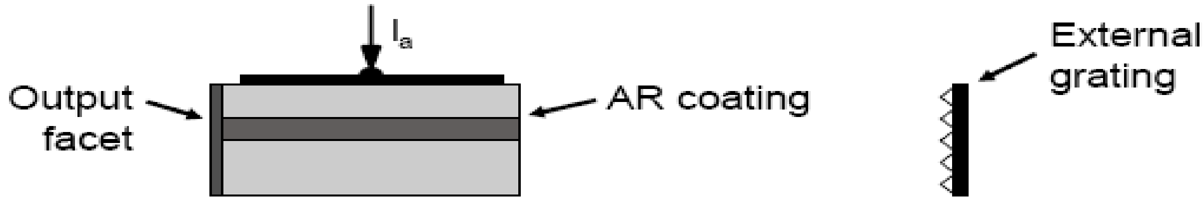
Example of a realized device

- Hybrid ML laser with a saturable absorber modulated at 40GHz
- H. Hertz Institute Berlin



Examples of self ML

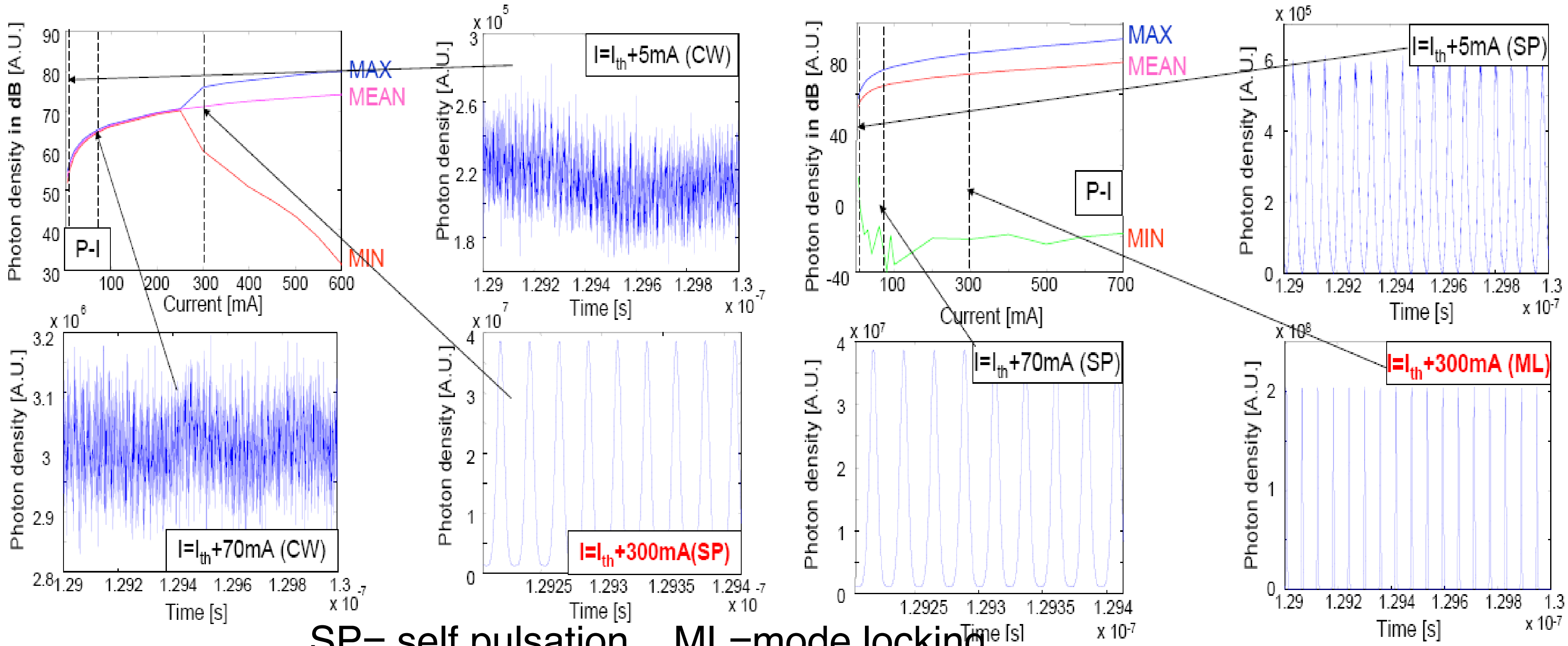
Easy to be found in long cavity or external cavity lasers with high α_{LEF} -> 4-6



P-I characteristic (I)

Equivalent external length in air: 700μm

Equivalent external length in air: 6000μm



SP= self pulsation ML=mode locking