The Abdus Salam International Centre for Theoretical Physics

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Winter College on Optics: Trends in Laser Development and Multidisciplinary Applications to Science and Industry

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General introduction to solid-state lasers

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Trends in laser development and multidisciplinary applications to science and industry

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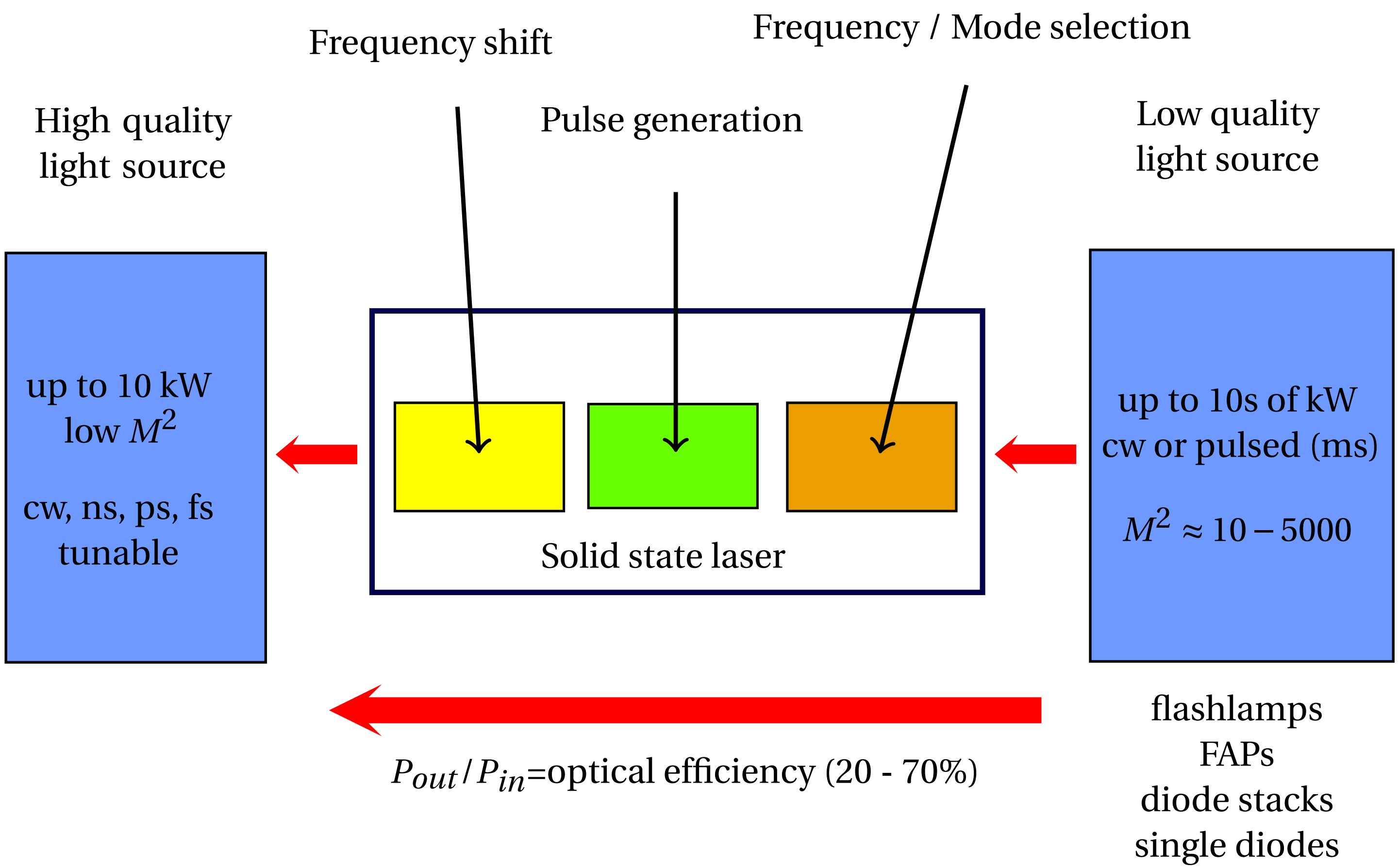
Outline

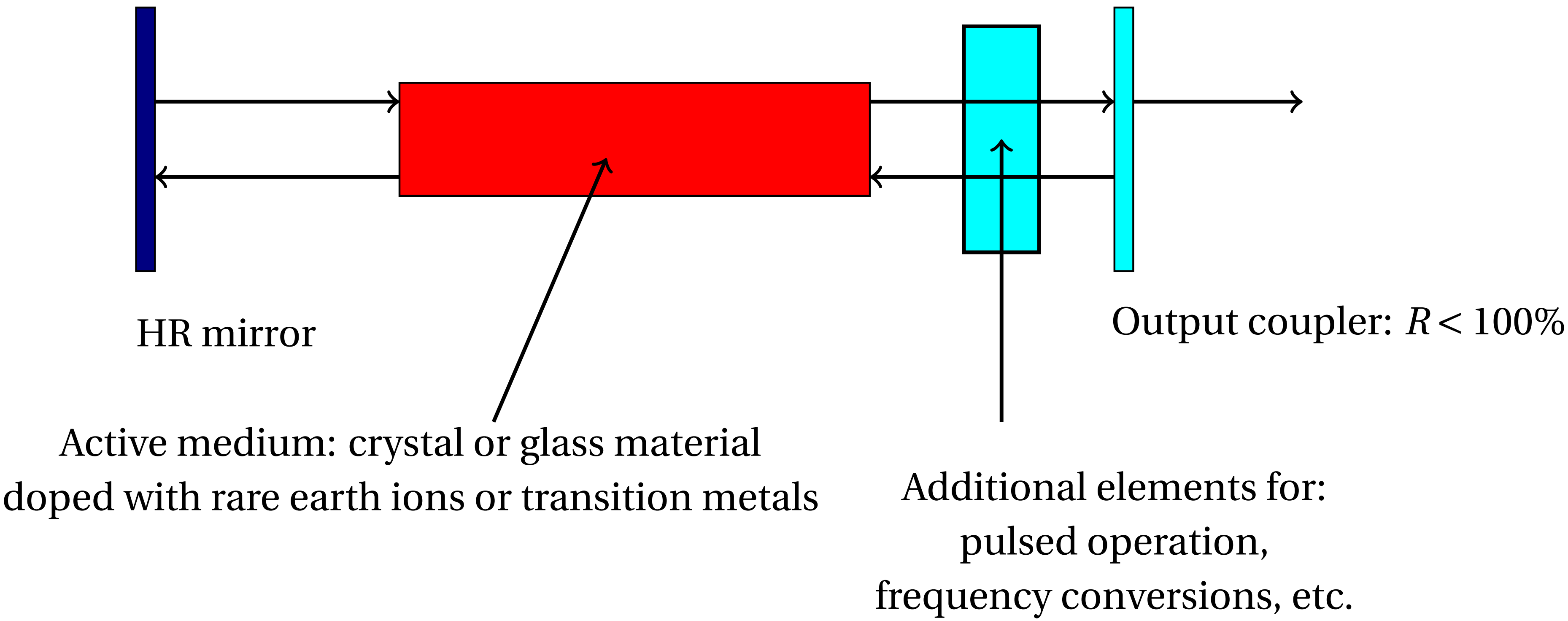
- Introduction
- Doping, inversion, gain, power model, CW operation
- Materials
- Geometries (pump schemes, resonators, active media shapes)
- Fiber lasers
- Pulsed operation: Q-switching
- Pulsed operation: mode-locking
- Non linear optics: harmonics generation

Introduction (I)

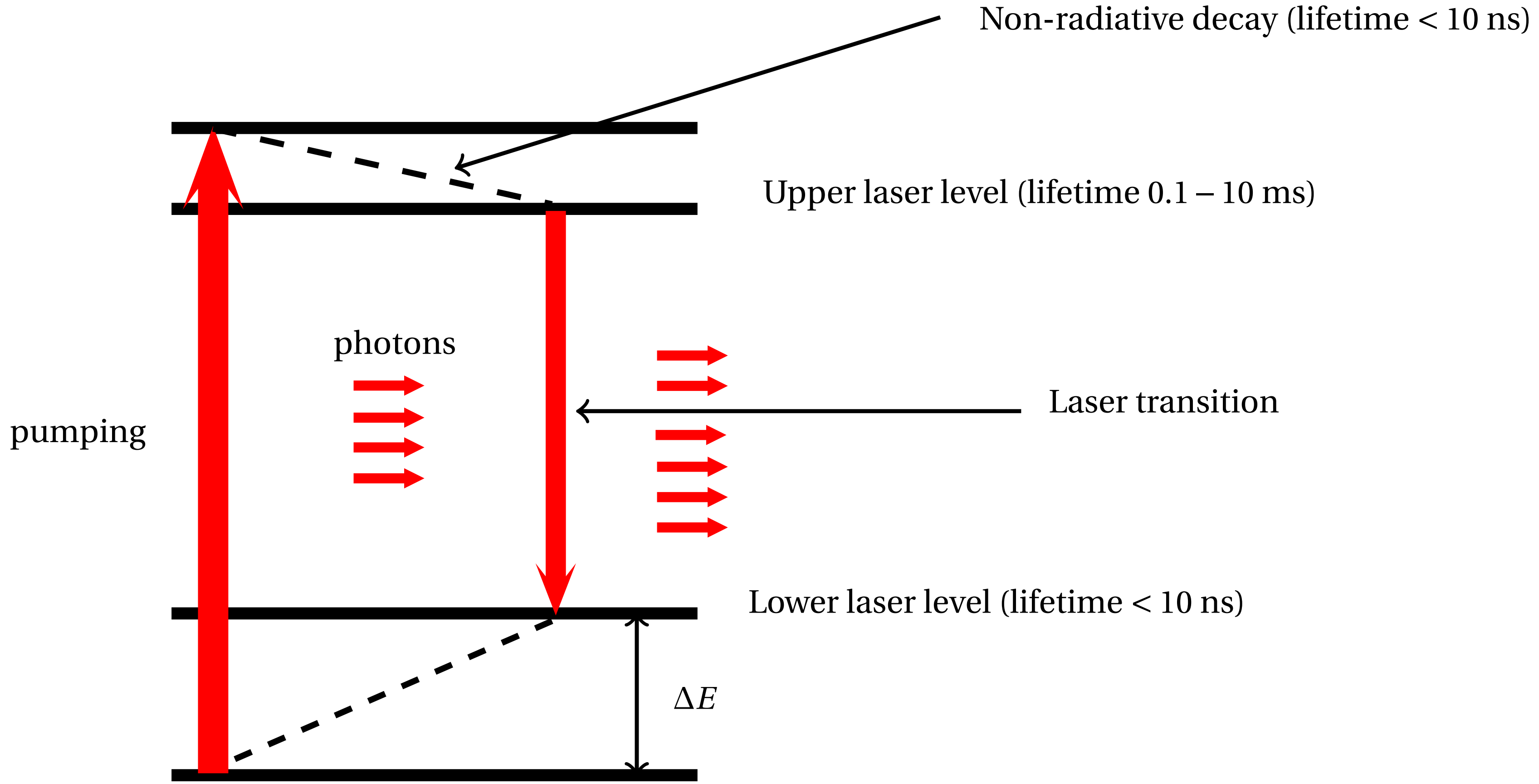


A solid state laser is a transformer box for light

Introduction (II)



Energy level diagram of active ions in solid-state lasers



- $\Delta E > 0.1$ eV → Four-level system (Nd:YAG, Nd:YVO_4, Ti:Sapphire)
- $\Delta E < 0.01$ eV → Three-level system (Ruby)
- 0.02 eV $< \Delta E < 0.1$ eV → Quasi-three level system (Yb:YAG)

Solid state lasers dopants and hosts

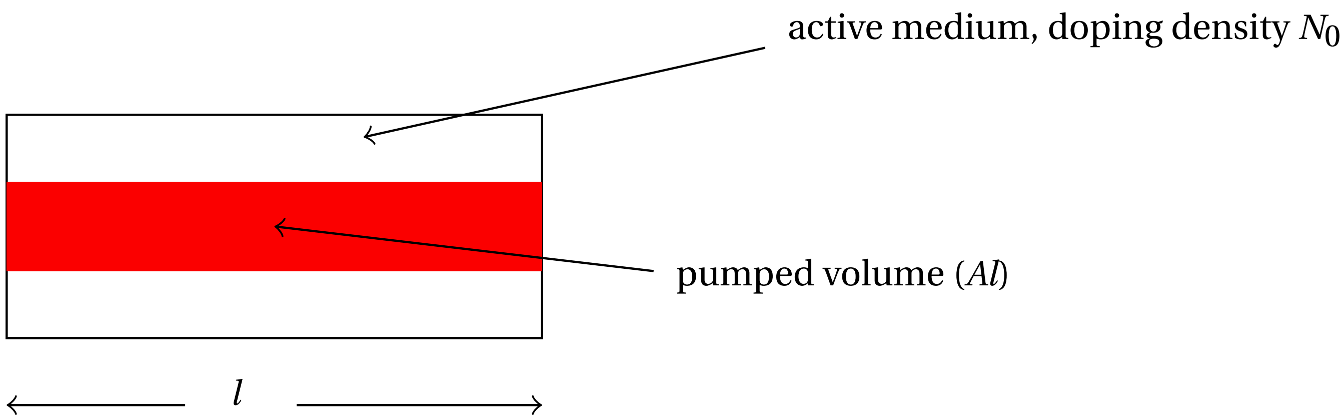
Dopants:

- Rare Earths or Lanthanides (narrow emission lines): Nd, Yb, Er, Tm, Gd, Pm, Sm, . . .
- Transition Metals (tunable lasers): Ti, Cr, . . .

Hosts:

- Oxides: $Y_3Al_5O_{12}$ (YAG), YVO_4 (Yttrium Vanadate), Al_2O_3 (Sapphire), . . .
- Fluorides: $LiYF_4$ (YLF), $LiSrAlF_6$ (LISAF), . . .
- Glass: phosphate and silicate

Pumping



pump rate

optical pump power

$$\boxed{W} \cdot N_0 A l \cdot h\nu_P = \eta_{abs} \cdot \boxed{P_{pump}}$$

number of dopant ions

absorption efficiency

ENERGY STORED IN TERMS OF POPULATION INVERSION:

$$W \cdot N_0 A l \cdot h\nu_L = h\nu_L / h\nu_P \cdot \eta_{abs} P_{pump} = \lambda_P / \lambda_L \cdot \eta_{abs} P_{pump}$$

quantum efficiency

Population inversion

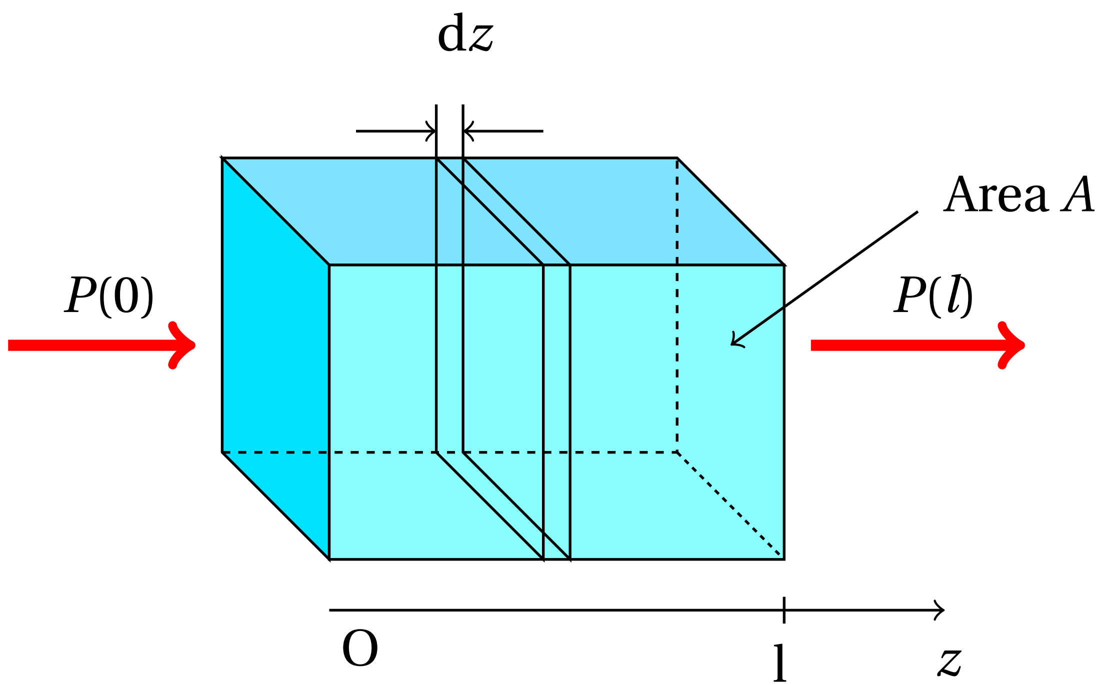
Population inversion: $\boxed{\Delta N = N_2 - N_1}$

$$\frac{\mathrm{d}\Delta N}{\mathrm{d}t} = W(N_0 - \Delta N) - \frac{\Delta N}{\tau} \quad \rightarrow \quad \tau \text{ is the upper-state lifetime}$$

$$\Delta N \approx W\tau N_0 (1 - \exp(-t/\tau)) \approx W\tau N_0 \quad \text{for} \quad t \gg \tau$$

STORED ENERGY: $E_S = \Delta N \cdot Al \cdot h\nu_L \approx W N_0 \tau \cdot Al \cdot h\nu_L \quad (t \gg \tau)$

Gain



$$\mathrm{d}P = \Delta N \cdot A\mathrm{d}z \cdot \sigma / A \cdot P$$

The interaction probability between photons and ions defines an **emission cross-section** σ

$$P(L)/P(0) = G_0 = \exp(\sigma \cdot \Delta N \cdot l) = \exp(g_0 \cdot l)$$

$g_0 = \sigma \cdot \Delta N$ is the **small signal gain coefficient**, $g_0 l$ is the small signal gain

Losses and gain saturation in a laser amplifier

Laser Medium

Intensity I_0 → $g_0\,l,\ \alpha_0\,l$ → Intensity $I_1 = I_0 \exp(g_0\,l - \alpha_0\,l)$

Absorption cross-section: σ_A → Attenuation: $P(l) = P(0)\exp(-N_0\sigma_A l) = \exp(-\alpha_0\,l)$

SMALL SIGNAL GAIN FACTOR: $G_0 = \exp(g_0\,l)$
LOSS FACTOR: $V = \exp(-\alpha_0\,l)$

By rewriting the power available as population inversion and stored energy, we get:

$g_0\,lAI_S = \lambda_P/\lambda_L \cdot \eta_{abs} P_{pump}$ and $E_S = g_0\,lAI_S\tau = \lambda_P/\lambda_L \cdot \eta_{abs} P_{pump}\tau$

where

$I_S = h\nu_L/(\sigma\tau)$ is called saturation intensity

By extracting power from a laser medium, population inversion and gain has to decrease

SATURATED GAIN: $gl = g_0\,l/(1 + I/I_S)$
GAIN FACTOR: $G = \exp(gl)$

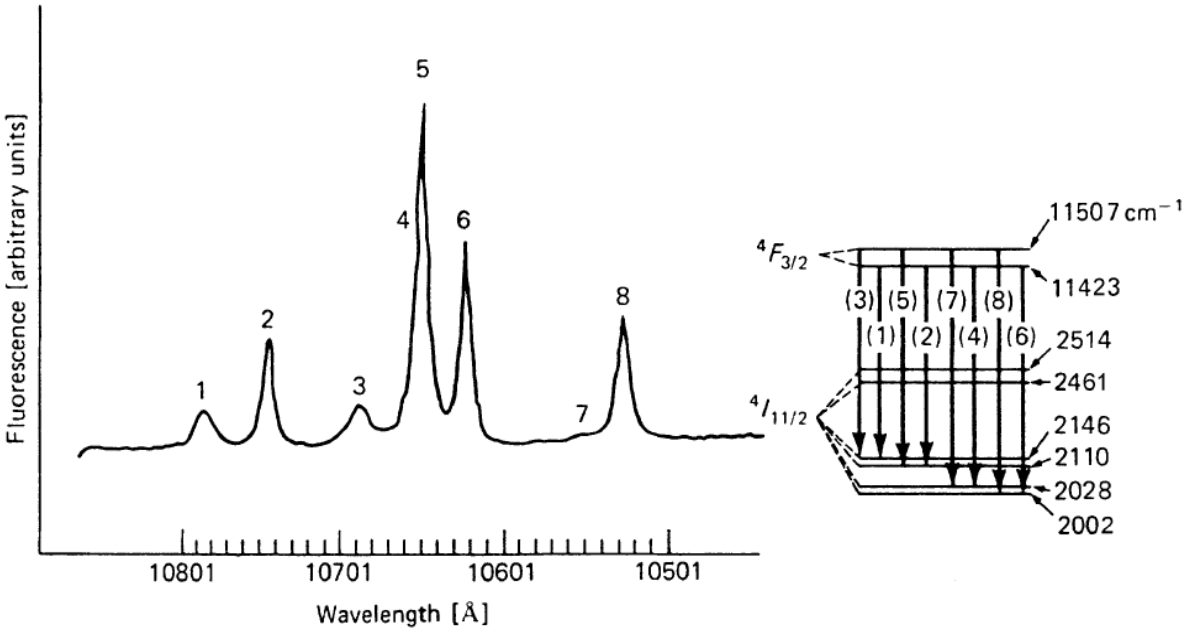
Some examples

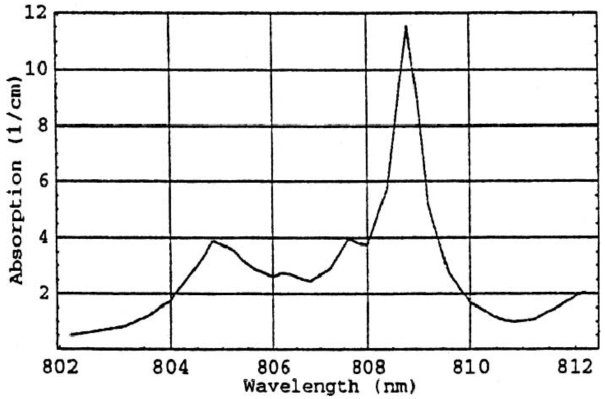
Laser medium	λ_L [μm]	σ [10^{-19} cm^{-2}]	τ [μs]	I_S [kW/cm^2]
Nd:YAG	1.064	4.1	230	2.00
Nd:YVO_4	1.064	15.0	100	1.26
Nd:YLF	1.047	1.8	480	2.15
Yb:YAG	1.03	0.21	970	9.50
Yb:KYW	1.03	0.3	300	21.58
Cr:LiSAF	0.85	0.5	67	70.25
Ti:Sapphire	0.79	2.8	3.2	160
Yb: SiO_2	1.03	0.08	800	30.5

High $\sigma\tau$ product → low saturation intensity → high gain and low laser threshold

Long upper-state lifetime τ → high energy storage

Nd:YAG absorption and emission spectra

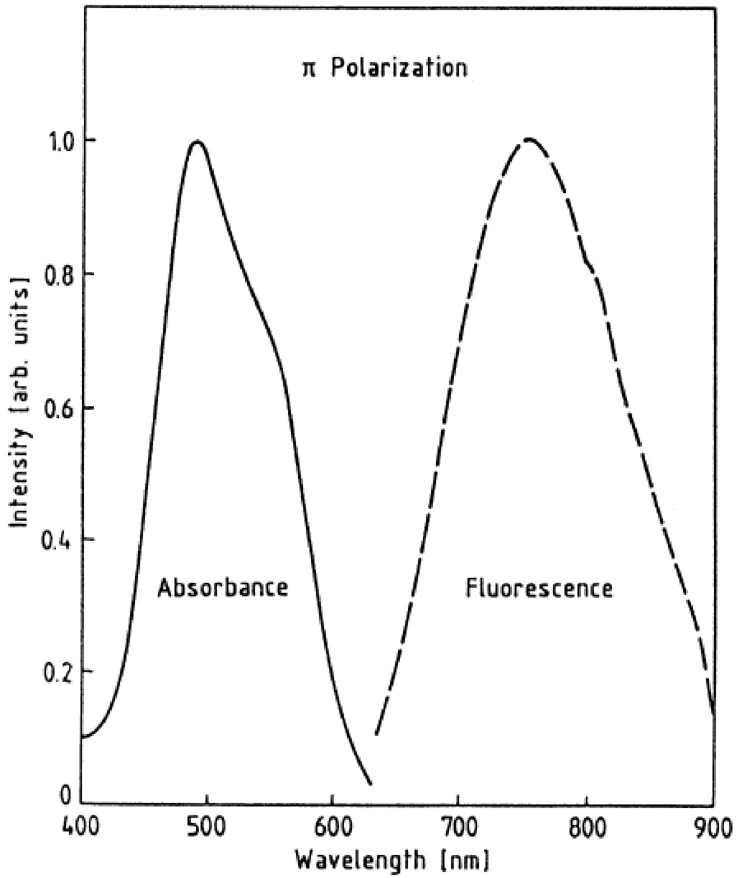




Nd:YAG emission (left) and absorption spectra (right)

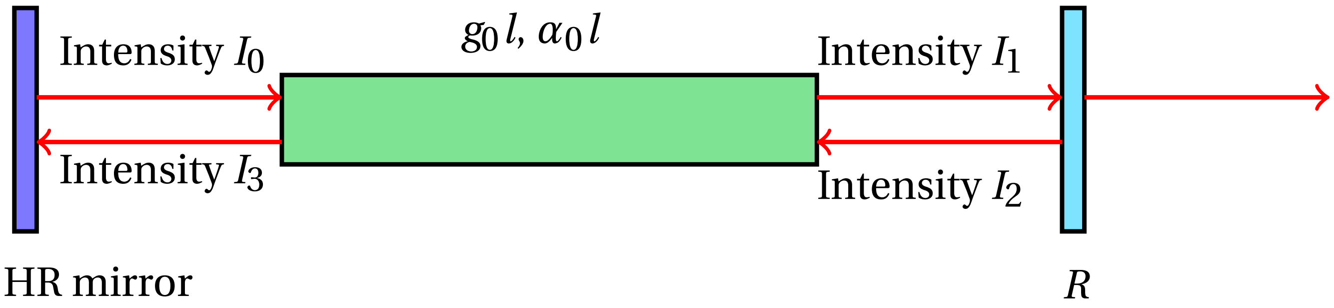
W. Koechner, SOLID-STATE LASER ENGINEERING, Sixth Ed.

Ti:Sapphire emission and absorption spectra



Ti:Sapphire emission and absorption spectra
W. Koechner, SOLID-STATE LASER ENGINEERING, Sixth Ed.

Round-trip in a laser resonator



I is the average intensity in the resonator

Gain factor: $G = \exp[g_0 l / (1 + I/I_S)]$

Loss factor: $V_S = \Gamma \exp(-\alpha_0 l)$

Γ is the single-pass diffraction loss

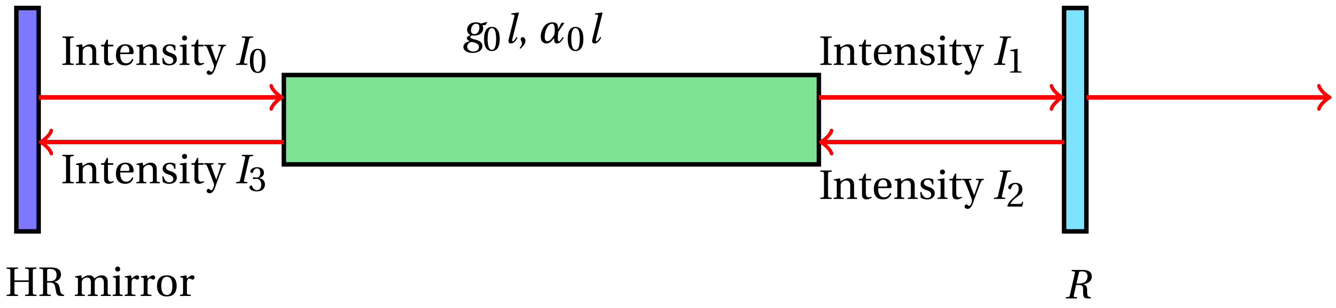
Round-trip steps

- $I_1 = G V_S I_0$
- $I_2 = R I_1 = R G V_S I_0$
- $I_3 = G V_S I_2 = \boxed{G^2 V_S^2 R} \cdot I_0$

Lasing condition: $I_3 = I_0$

$$G^2 V_S^2 R = 1$$

Laser output power (I)



$$G^2 V_S^2 R = 1 \quad \rightarrow \quad \exp[2g_0 l/(1+I/I_S) - 2\alpha_0 l] R\Gamma^2 = 1$$

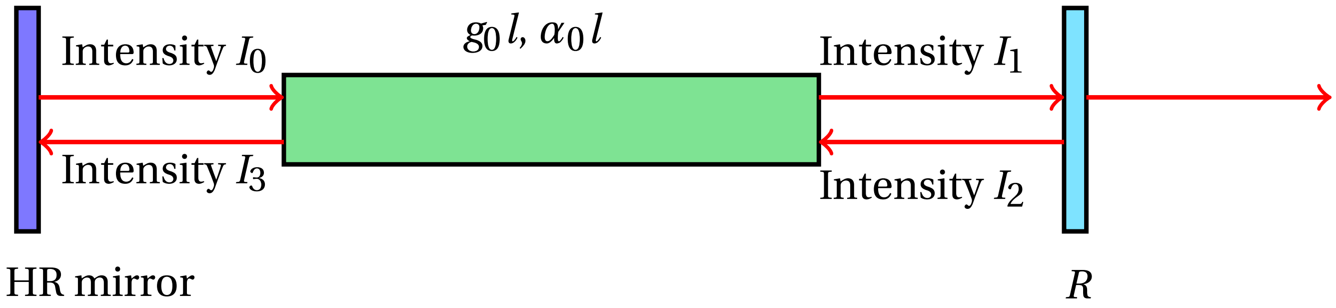
$$g_0 l/(1+I/I_S) - \alpha_0 l = -\ln(\Gamma\sqrt{R}) \quad \rightarrow \quad g_0 l - \alpha_0 l(1+I/I_S) = -\ln(\Gamma\sqrt{R}) \cdot (1+I/I_S)$$

$$I = I_S \frac{1}{|\ln(\sqrt{R}V_S)|} \cdot [g_0 l - |\ln(\sqrt{R}V_S)|]$$

The average intensity I is the sum of the two counter-propagating intensities
Assuming that both waves carry the same intensity (low output coupling approximation):

$$P_{out} = A_B I_1 (1-R) = A_B I/2 \cdot (1-R) = A_B I_S \frac{1-R}{|2\ln(\sqrt{R}V_S)|} \cdot [g_0 l - |\ln(\sqrt{R}V_S)|]$$

Laser output power (II)



By taking into account the expression for the small signal gain:

$$g_0 l A I_S = \lambda_P/\lambda_L \cdot \eta_{abs} P_{pump}$$

and assuming a constant mode beam area A_B, we get:

$$P_{out} = \frac{A_B}{A} \cdot \frac{1-R}{2|\ln(\sqrt{R}V_S)|} \cdot [\lambda_P/\lambda_L \cdot \eta_{abs} P_{pump} - A I_S |\ln(\sqrt{R}V_S)|] = \eta \cdot (P_{pump} - P_{pump,th})$$

Threshold pump power

$$P_{pump,th} = \frac{A I_S}{\lambda_P/\lambda_L \cdot \eta_{abs}} \cdot |\ln(\sqrt{R}V_S)|$$

Slope efficiency

$$\eta = \frac{A_B}{A} \cdot \frac{1-R}{2|\ln(\sqrt{R}V_S)|} \cdot \lambda_P/\lambda_L \cdot \eta_{abs}$$

Optimum output coupling

The output coupling ratio R can be chosen for maximum output power:

Optimum output coupling:

$$\ln R_{opt} = -2\alpha_0 l \left[\sqrt{\frac{g_0 l}{\alpha_0 l}} - 1\right]$$

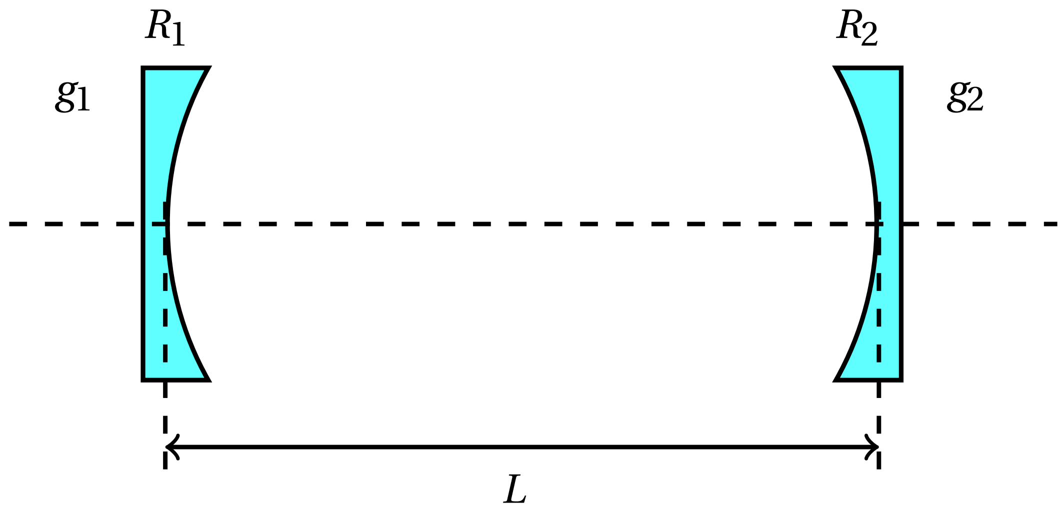
Maximum optical efficiency:

$$\eta_{opt,max} = \frac{P_{out,max}}{P_{pump}} =$$

$$= \lambda_P/\lambda_L \cdot \eta_{abs} \cdot \frac{\alpha_0 l}{g_0 l} \cdot \left[\sqrt{\frac{g_0 l}{\alpha_0 l}} - 1\right]^2$$

The optimum output coupling depends on the small signal gain and losses

Laser resonators



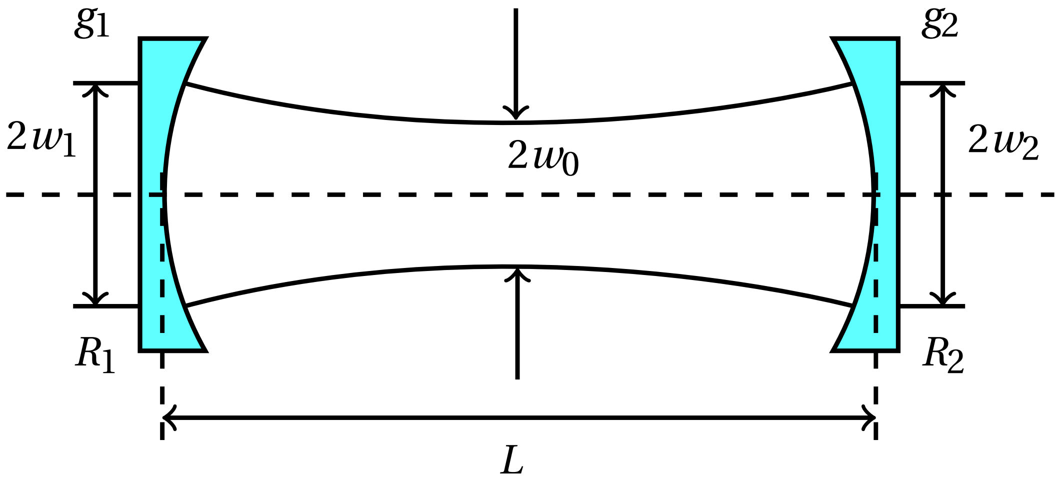
The mode size A_B in the previous equations depends on the resonator design.
We consider resonators with two mirrors. Actual resonator might be way more complicated!
Mirrors have their own radius of curvature ($R > 0$ for convex mirrors)
We define g-parameters as follows:

$$g_i = 1 - \frac{L}{R_i} \quad i = 1,2$$

$\rightarrow$ L is the resonator length

Resonators define eigenmodes, E-field distributions which retain their profile after every round-trip

Stable resonators



Condition for resonator stability: $0 \le g_1 g_2 \le 1$

Beam radius of fundamental mode on mirrors 1,2: $w_i^2 = \dfrac{\lambda_L L}{\pi} \sqrt{\dfrac{g_j}{g_i(1 - g_1 g_2)}}$ $\quad i, j = 1, 2; i \neq j$

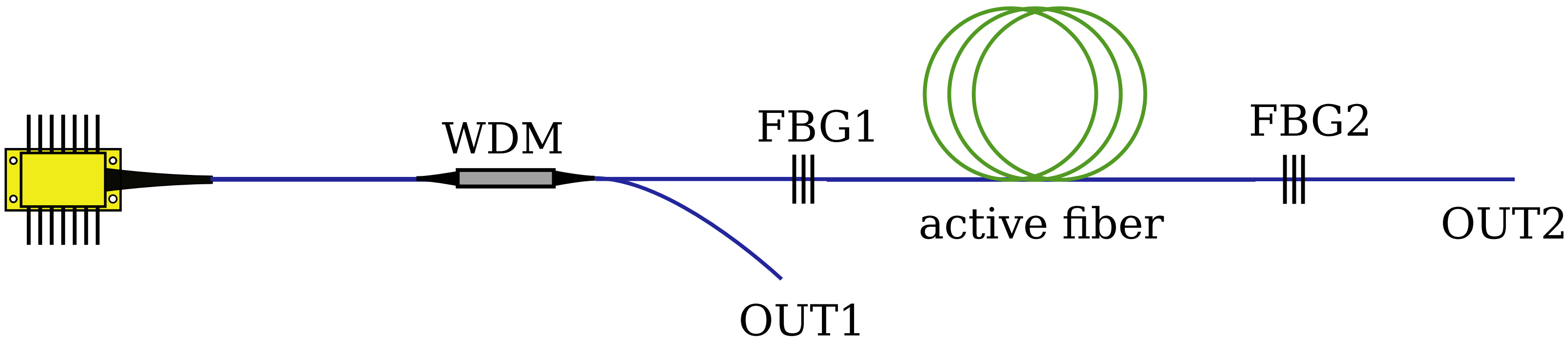
Beam waist in the resonator: $w_0^2 = \dfrac{\lambda_L L}{\pi} \dfrac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{|g_1 + g_2 - 2 g_1 g_2|}$

In multimode operation the beam area is $\approx M^2$ larger than the fundamental mode area

Pumping schemes and active media geometries

- OPTICAL PUMPING:
  - flash-lamps or arc-lamps:
    - very high pump power, immune to current or voltage spikes ☺
    - limited lifetime, low wall-plug efficiency (a few percents), low brightness ☹
  - diode-pumping (end-pumping or side-pumping):
    - high wall-plug efficiency, narrow bandwidth, high beam quality (allowing end-pumping and precise mode-matching), long lifetime, compactness ☺
    - higher cost per watt of pump power ☹
- ACTIVE MEDIA GEOMETRIES:
  - Rod: side-pumping or end-pumping (Nd:YAG)
  - Slab: side-pumping (Nd:YVO_4, Nd:YAG)
  - Thin-Disk: optimum for heat dissipation, requires high pump absorption and high doping concentration (Yb:YAG)
  - Fiber: optimum for heat dissipation and beam confinement

Fiber lasers

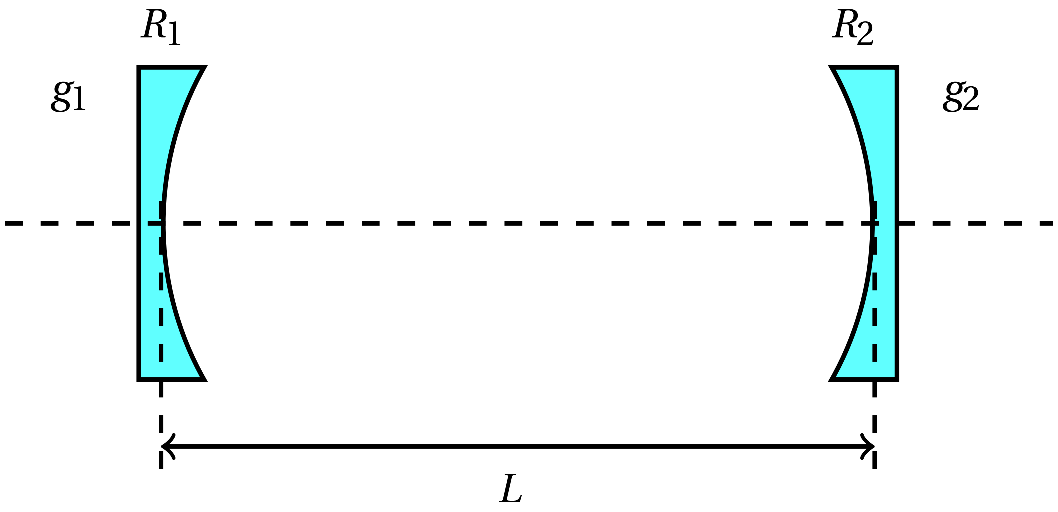


- Resonator layout significantly simplified
- Mode area and beam quality determined by the propagation condition inside of the optical fiber
- The pump power needs to be coupled into the resonator through WDMs or pump combiners
- The mirrors can be realized inside of the fiber (Fiber Bragg Gratings, FBG)
- Easier management of thermal problems than in bulk materials
- High brightness

Pulsed operation

- CW operation allows to achieve output power in the order of 10 kW (fiber lasers) or few kW (bulk materials)
- Lasers can also operate in pulsed regimes (e.g. by using a pulsed pump source)
- Q-switching and mode-locking allows to redistribute the stored energy to get high peak powers and pulse energies
- High peak powers and pulse energies are required in a number of applications (material processing and non-linear processes)
- Q-switching allows to generate ns-long pulses with energies up to 100's of mJ and repetition rates from a few Hz to a few 100's of kHz (peak power in the order of kWs)
- Mode-locking operation allows to get μJ-level ps-long or fs-long pulses (ultrafast pulses) with repetition rates ranging from 10's of MHz up to a 10's of GHz (peak power in the order of MWs)

Q-factor



$$T_R = \frac{2L}{c}$$ → round-trip time

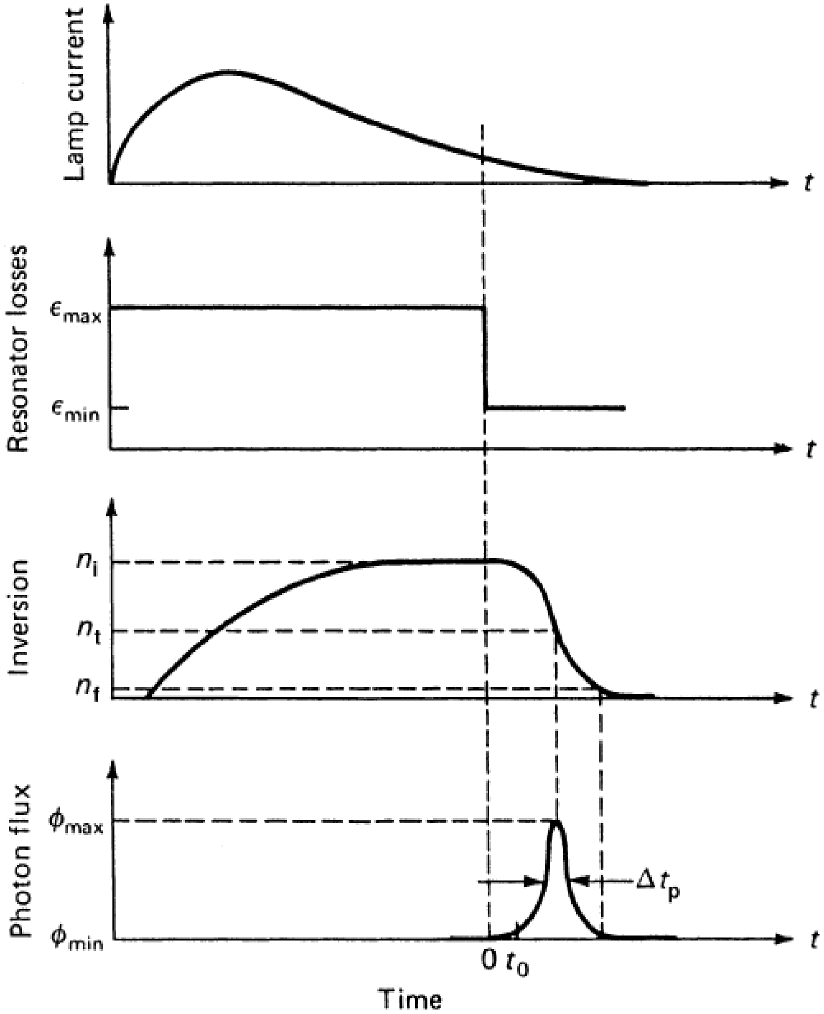
$$\epsilon = 2\alpha_0 l - \ln R$$ → fractional loss per round-trip

$$\tau_c = \frac{T_R}{\epsilon}$$ → average lifetime of photons in the resonator

We define a Q-factor as ratio between the stored energy E_{st}
and the dissipated energy E_d in an optical cycle $T_0 = \nu_0^{-1} = \lambda_L / c$:

$$Q = 2\pi \frac{E_{st}}{E_d} = 2\pi \frac{E_{st}}{E_{st} \cdot [1 - \exp(-T_0/\tau_c)]} \approx \frac{2\pi\tau_c}{T_0} = 2\pi\nu_0\tau_c$$ → the lower the losses the higher Q

Q-switching principle of operation



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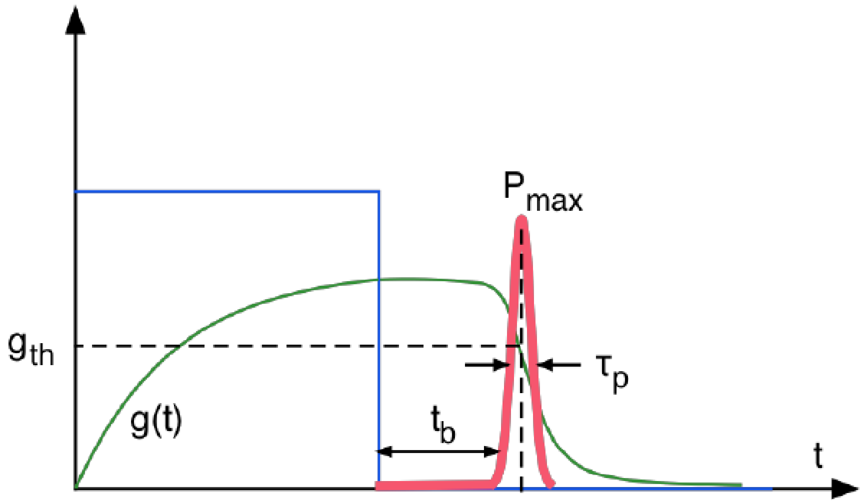
- Initially Q is low for energy storing by pumping over a time interval $\approx \tau$
- A much higher population inversion than the threshold value for high Q is achieved
- The Q level is suddenly increased and the energy is released in the form of a giant pulse
- Due to the high gain established, the energy is released in a short pulse
- The peak power exceeds by several orders of magnitude the power of CW lasers in the same pumping conditions

Q-switching pulse parameters



Peak power $\rightarrow$ $\boxed{P_{peak} \approx \frac{h\nu_L}{\sigma} A_B \cdot \frac{\ln(1/R)}{T_R} g_i}$

Pulse energy $\rightarrow$ $\boxed{E_P \approx \frac{h\nu_L}{\sigma} A_B \cdot \frac{\ln(1/R)}{\epsilon} g_i}$

Pulse duration $\rightarrow$ $\boxed{\tau_P \approx \frac{E_P}{P_{peak}} \approx \frac{T_R}{\epsilon} = \tau_c}$

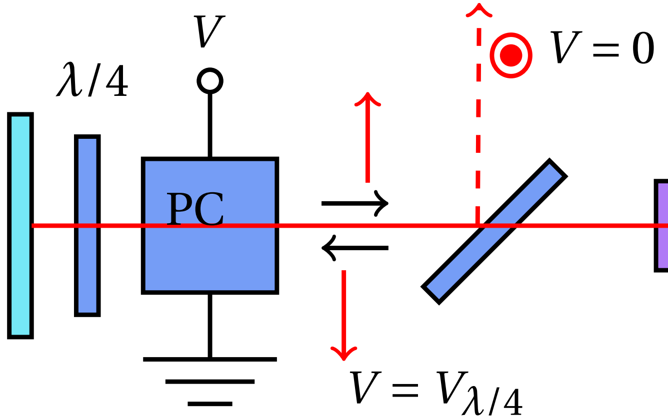
Build-up time $\rightarrow$ $\boxed{t_b \approx \frac{20\tau_c}{g_i/g_{th} - 1}}$

g_i is the initial gain before the switching time
g_{th} is the laser threshold gain value for high Q value

- The pulse energy is proportional to the saturation fluence $h\nu_L/\sigma$
- Comparing Q-switching and CW operation at the same pump power: $P_{QS}/P_{CW} \approx \tau/T_R$
- For Q-switching operation $\boxed{\tau \gg T_R}$ is required

Q-switching techniques

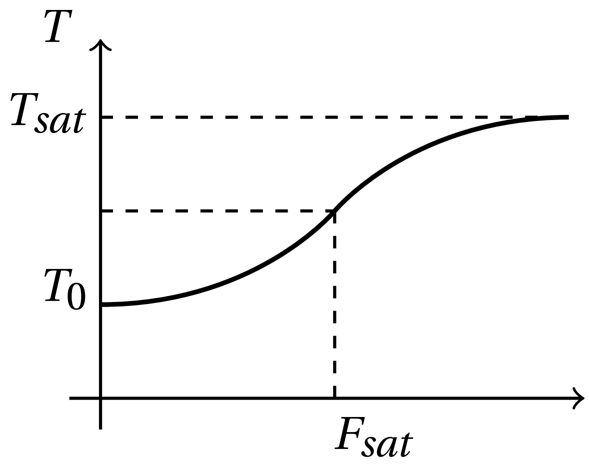
Active Q-switching



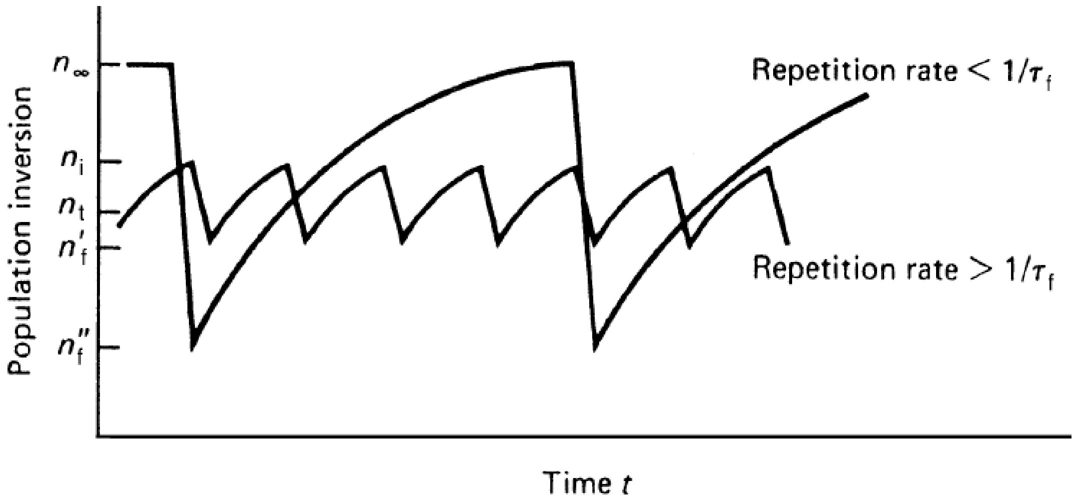
- Electro-optic or acousto-optic switches might me used
- Electro-optic switches are faster, acousto-optic switches more suited for repetition rates > 1 kHz
- Electro-optic switches (figure) include a waveplate, a Pockels cell (PC) and a polarizer

Passive Q-switching

- A saturable absorber is a non-linear device with transmission increasing with the incident intensity
- The absorber has to saturate before than gain: $\frac{E/A_A}{F_{SA}} > \frac{E/A_L}{F_{SL}}$



Repetitive Q-switching (I)



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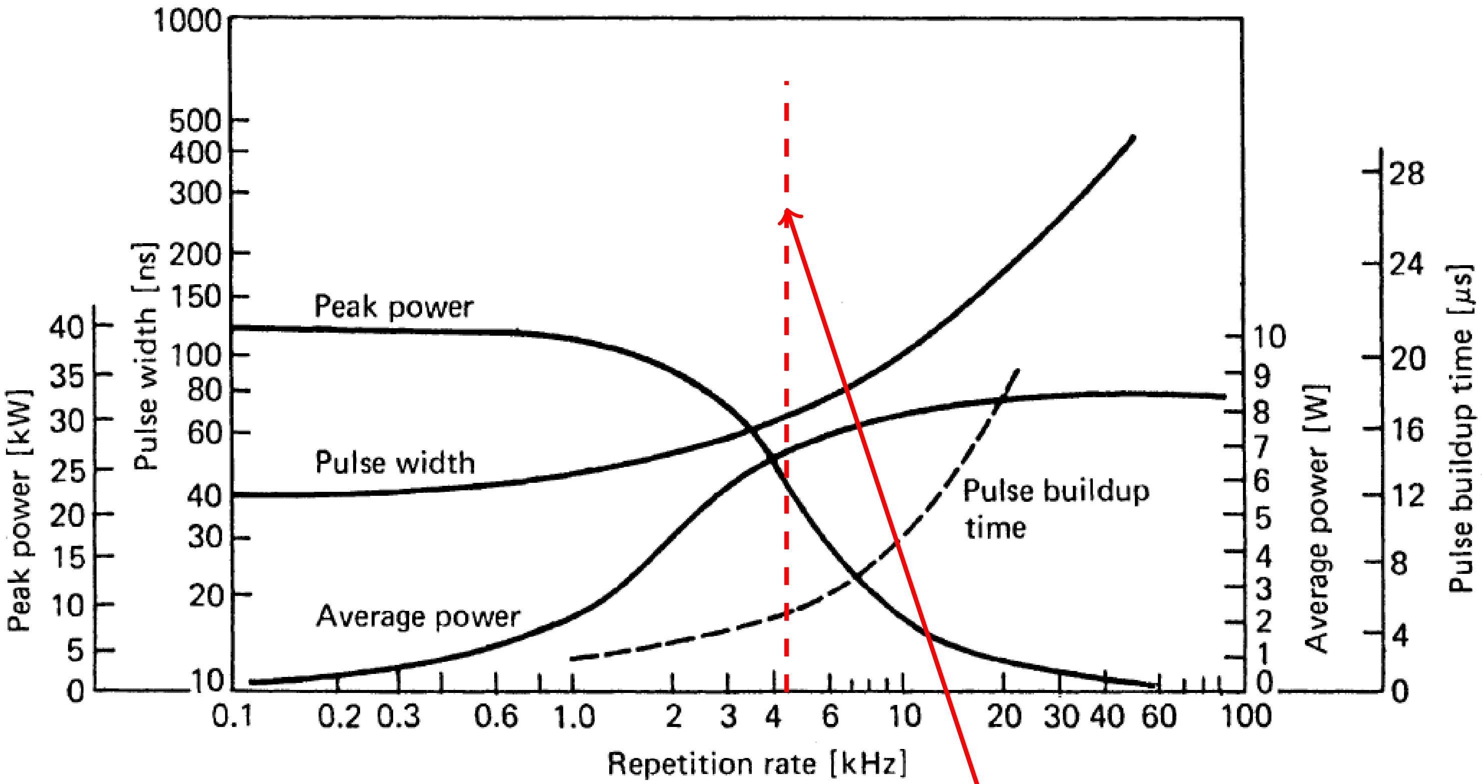
- The upper-state lifetime sets a critical frequency $1/\tau_f$
- If the repetition rate is in the order of $1/\tau_f$ the initial gain at the switching time is clamped at a lower value because population inversion can not recover completely
- High τ_f is required for energy storage while low τ_f is required for operation at high repetition rates

Repetitive Q-switching (II)



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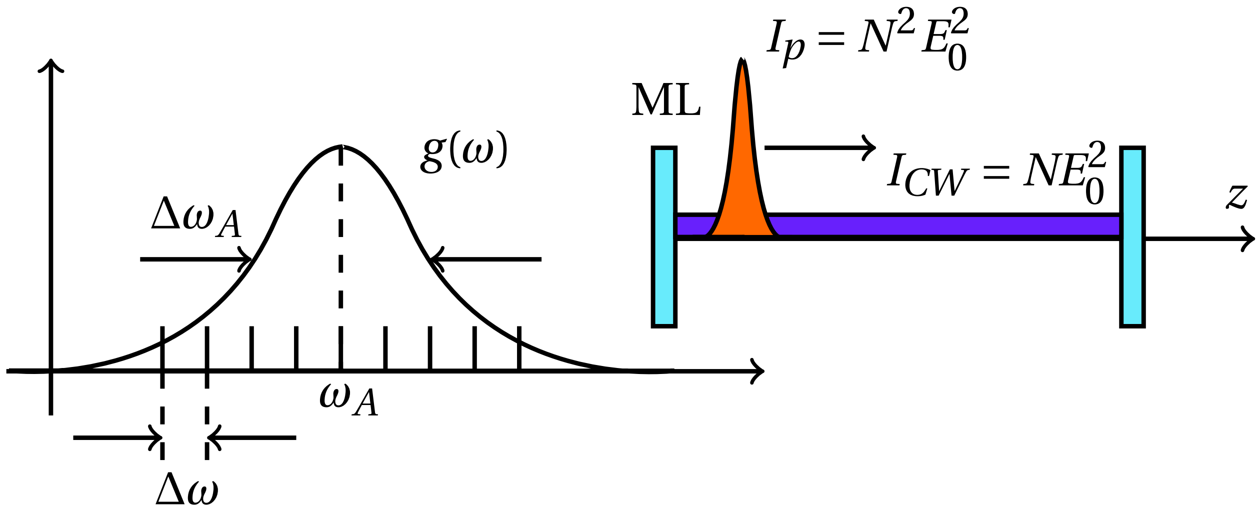
For Nd:YAG $\tau_f = 230\ \mu s \quad \rightarrow \quad \boxed{1/\tau_f \approx 4.3\ \text{kHz}}$

Mode-locking: principle of operation

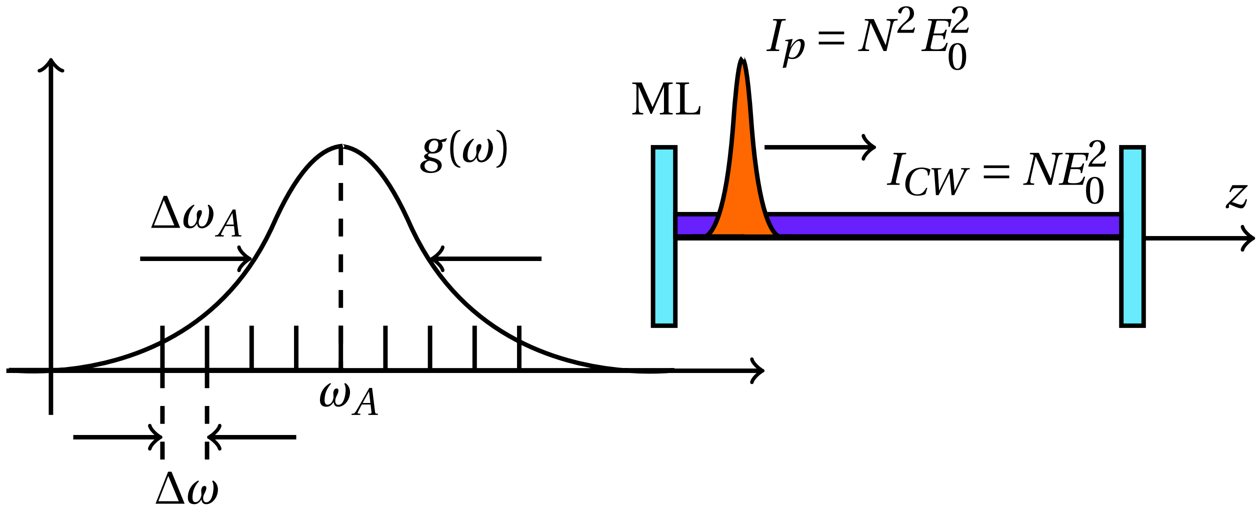


- A resonator with length L creates a set of equally spaced resonance frequencies

  $\Delta\omega = \dfrac{c}{2L}$ $\rightarrow$ frequency separation between two adjacent modes

- The mode-locking regime is based on the inset of a precise phase relationship between adjacent longitudinal modes: repetition rate $\rightarrow$ $f_R = c/2L$
- Pulses are created as a consequence of constructive interference between phase-locked modes
- Higher number of phase-locked modes means larger pulse spectra and shorter pulse durations and higher peak powers: minimum pulse duration related to $\Delta\omega_A$

Mode-locking techniques



- Mode-locking operation might be achieved with:
  - ACTIVE TECHNIQUES employing electro-optic or acousto-optic modulators
  - PASSIVE TECHNIQUES employing saturable absorbers: semiconductor saturable absorber mirrors (SESAM) or Kerr-effect
- Passive techniques allow shorter pulse durations
- Pulse durations range from 10's of picosecond down to a few femtoseconds
- For pulse duration below 1 ps group velocity dispersion has to be taken into account

Non-linear optics

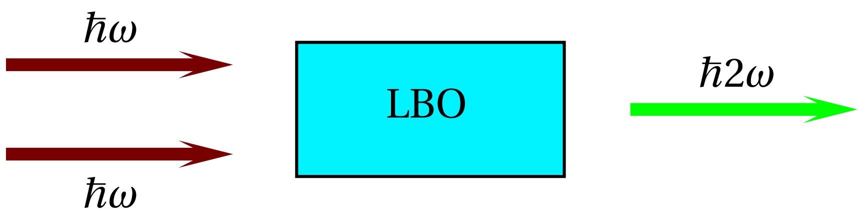
$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = -\frac{1}{\epsilon_0}\frac{\partial^2 P}{\partial t^2}$$

The incident electric field E creates in the medium a polarization P. For weak fields: $P = \epsilon_0 \chi^{(1)} E$

In general: $P = \epsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots\right) = P^{(L)} + P^{(NL)}$

$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$
$0.01 - 5$	$\sim 10^{-12}$ m/V	$\sim 10^{-20}$ m^2/V^2

QUANTUM MECHANICAL DESCRIPTION (SHG):



ENERGY CONSERVATION:

$$\hbar \cdot \omega + \hbar \cdot \omega = \hbar \cdot 2\omega$$

MOMENTUM CONSERVATION:

$$\hbar \cdot \mathbf{k} + \hbar \cdot \mathbf{k} = \hbar \cdot \mathbf{2k}$$

$$\frac{\hbar\omega}{c} n(\omega) + \frac{\hbar\omega}{c} n(\omega) = \frac{\hbar \cdot 2\omega}{c} n(2\omega) \rightarrow n(\omega) = n(2\omega)$$

Polarized light is required!

Generation of new frequencies

Second order nonlinearity ($\chi^{(2)}$)

- Second harmonic generation
- Sum frequency generation
- Parametric amplification

Third order nonlinearity ($\chi^{(3)}$)

- Third harmonic generation
- Sum and difference frequency generation
- Kerr effect

- Due to the typical values of $\chi^{(2)}$ the required optical intensities for efficient frequency conversion are $\gtrsim 100\ \mathrm{MW/cm^2}$
- New frequencies can be easily generated with Q-switched and mode-locked lasers
- SHG (second harmonic generation) and THG (third harmonic generation) can be achieved also in CW lasers by intra-cavity generation
- Due to the typically low values for $\chi^{(3)}$ it is usually more convenient to generate third order effects by cascading two second order processes

Further reading

- *W. Koechner*, SOLID-STATE LASER ENGINEERING, Sixth Ed., Springer
- *A. E. Siegman*, LASERS, University Science Books