General introduction to solid-state lasers

L. Carra'

Bright Solutions, Pavia
Italy
General introduction to solid-state lasers
Trends in laser development and multidisciplinary applications

to science and industry

Dr. Luca Carrà

Bright Solutions srl - Pavia - Italy

February 5, 2013
Outline

- Introduction
- Doping, inversion, gain, power model, CW operation
- Materials
- Geometries (pump schemes, resonators, active media shapes)
- Fiber lasers
- Pulsed operation: Q-switching
- Pulsed operation: mode-locking
- Non linear optics: harmonics generation

http://www.brightsolutions.it
Introduction (I)

High quality light source

- up to 10 kW
- low $M^2$
- cw, ns, ps, fs tunable

Frequency shift

Frequency / Mode selection

Pulse generation

Low quality light source

- up to 10s of kW
- cw or pulsed (ms)
- $M^2 \approx 10 - 5000$

Solid state laser

- $P_{out}/P_{in}$ = optical efficiency (20 - 70%)

- flashlamps
- FAPs
- diode stacks
- single diodes

A solid state laser is a transformer box for light
Introduction (II)

HR mirror

Active medium: crystal or glass material doped with rare earth ions or transition metals

Output coupler: $R < 100\%$

Additional elements for: pulsed operation, frequency conversions, etc.

http://www.brightsolutions.it
Energy level diagram of active ions in solid-state lasers

- Non-radiative decay (lifetime < 10 ns)
- Upper laser level (lifetime 0.1 – 10 ms)
- Lower laser level (lifetime < 10 ns)

- $\Delta E > 0.1 \text{ eV} \rightarrow$ Four-level system (Nd:YAG, Nd:YVO$_4$, Ti:Sapphire)
- $\Delta E < 0.01 \text{ eV} \rightarrow$ Three-level system (Ruby)
- $0.02 \text{ eV} < \Delta E < 0.1 \text{ eV} \rightarrow$ Quasi-three level system (Yb:YAG)

http://www.brightsolutions.it
Solid state lasers dopants and hosts

**Dopants:**
- **Rare Earths or Lanthanides** (narrow emission lines): Nd, Yb, Er, Tm, Gd, Pm, Sm, . . .
- **Transition Metals** (tunable lasers): Ti, Cr, . . .

**Hosts:**
- **Oxides**: $\text{Y}_3\text{Al}_5\text{O}_{12}$ (YAG), $\text{YVO}_4$ (Yttrium Vanadate), $\text{Al}_2\text{O}_3$ (Sapphire), . . .
- **Fluorides**: $\text{LiYF}_4$ (YLF), $\text{LiSrAlF}_6$ (LISAF), . . .
- **Glass**: phosphate and silicate
Pumping

active medium, doping density $N_0$

pumped volume ($Al$)

pump rate

optical pump power

number of dopant ions

absorption efficiency

ENERGY STORED IN TERMS OF POPULATION INVERSION:

$$W \cdot N_0Al \cdot h\nu_P = \eta_{abs} \cdot P_{pump}$$

$$W \cdot N_0Al \cdot h\nu_L = h\nu_L / h\nu_P \cdot \eta_{abs} P_{pump} = \lambda_P / \lambda_L \cdot \eta_{abs} P_{pump}$$

quantum efficiency
Population inversion: $\Delta N = N_2 - N_1$

$$\frac{d\Delta N}{dt} = W(N_0 - \Delta N) - \frac{\Delta N}{\tau} \rightarrow \tau \text{ is the upper-state lifetime}$$

$$\Delta N \approx W\tau N_0 (1 - \exp(-t/\tau)) \approx W\tau N_0 \quad \text{for} \quad t \gg \tau$$

Stored energy: $E_S = \Delta N \cdot A_l \cdot h\nu_L \approx W N_0 \tau \cdot A_l \cdot h\nu_L \quad (t \gg \tau)$
The interaction probability between photons and ions defines an **emission cross-section** $\sigma$

$$dP = \Delta N \cdot A dz \cdot \sigma / A \cdot P$$

$$P(L) / P(0) = G_0 = \exp(\sigma \cdot \Delta N \cdot l) = \exp(g_0 \cdot l)$$

$g_0 = \sigma \cdot \Delta N$ is the **small signal gain coefficient**, $g_0 \cdot l$ is the small signal gain
Losses and gain saturation in a laser amplifier

Laser Medium

Intensity $I_0$ → $g_0 l, \alpha_0 l$ → Intensity $I_1 = I_0 \exp(g_0 l - \alpha_0 l)$

Absorption cross-section: $\sigma_A$ → Attenuation: $P(l) = P(0) \exp(-N_0 \sigma A l) = \exp(-\alpha_0 l)$

**SMALL SIGNAL GAIN FACTOR:** $G_0 = \exp(g_0 l)$

**LOSS FACTOR:** $V = \exp(-\alpha_0 l)$

By rewriting the power available as population inversion and stored energy, we get:

$$g_0 l A I_S = \frac{\lambda_p}{\lambda L \cdot \eta_{abs} P_{pump}}$$

and

$$E_S = g_0 l A I_S \tau = \frac{\lambda_p}{\lambda L \cdot \eta_{abs} P_{pump} \tau}$$

where

$$I_S = h\nu_L/(\sigma \tau)$$ is called **saturation intensity**

By extracting power from a laser medium, population inversion and gain has to decrease

**SATURATED GAIN:** $g l = g_0 l/(1 + I/I_S)$

**GAIN FACTOR:** $G = \exp(g l)$

http://www.brightsolutions.it
### Some examples

<table>
<thead>
<tr>
<th>Laser Medium</th>
<th>( \lambda_L [\mu m] )</th>
<th>( \sigma [10^{-19} \text{ cm}^{-2}] )</th>
<th>( \tau [\mu s] )</th>
<th>( I_S [\text{kW/cm}^2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nd:YAG</td>
<td>1.064</td>
<td>4.1</td>
<td>230</td>
<td>2.00</td>
</tr>
<tr>
<td>Nd:YVO(_4)</td>
<td>1.064</td>
<td>15.0</td>
<td>100</td>
<td>1.26</td>
</tr>
<tr>
<td>Nd:YLF</td>
<td>1.047</td>
<td>1.8</td>
<td>480</td>
<td>2.15</td>
</tr>
<tr>
<td>Yb:YAG</td>
<td>1.03</td>
<td>0.21</td>
<td>970</td>
<td>9.50</td>
</tr>
<tr>
<td>Yb:KYW</td>
<td>1.03</td>
<td>0.3</td>
<td>300</td>
<td>21.58</td>
</tr>
<tr>
<td>Cr:LiSAF</td>
<td>0.85</td>
<td>0.5</td>
<td>67</td>
<td>70.25</td>
</tr>
<tr>
<td>Ti:Sapphire</td>
<td>0.79</td>
<td>2.8</td>
<td>3.2</td>
<td>160</td>
</tr>
<tr>
<td>Yb: SiO(_2)</td>
<td>1.03</td>
<td>0.08</td>
<td>800</td>
<td>30.5</td>
</tr>
</tbody>
</table>

High \( \sigma \tau \) product \( \rightarrow \) low saturation intensity \( \rightarrow \) high gain and low laser threshold

Long upper-state lifetime \( \tau \) \( \rightarrow \) high energy storage
Nd:YAG absorption and emission spectra

Nd:YAG emission (left) and absorption spectra (right)

*W. Koechner*, **SOLID-STATE LASER ENGINEERING**, Sixth Ed.
Ti:Sapphire emission and absorption spectra

W. Koechner, SOLID-STATE LASER ENGINEERING, Sixth Ed.
Round-trip in a laser resonator

Intensity $I_0$

$g_0 l, \alpha_0 l$

Intensity $I_1$

Intensity $I_3$

HR mirror

Intensity $I_2$

$R$

$I$ is the average intensity in the resonator

Gain factor: $G = \exp[g_0 l/(1 + I/I_S)]$

Loss factor: $V_S = \Gamma \exp(-\alpha_0 l)$

$\Gamma$ is the single-pass diffraction loss

Round-trip steps

$I_1 = GV_SI_0$

$I_2 = RI_1 = RGV_SI_0$

$I_3 = GV_SI_2 = G^2 V_S^2 R \cdot I_0$

Lasing condition: $I_3 = I_0$

$G^2 V_S^2 R = 1$

http://www.brightsolutions.it
The average intensity $I$ is the sum of the two counter-propagating intensities
Assuming that both waves carry the same intensity (low output coupling approximation):

$$P_{out} = AB I_1 (1 - R) = AB I / 2 \cdot (1 - R) = AB I_S \frac{1 - R}{2 \ln(\sqrt{RV_S})} \cdot [g_0 l - |\ln(\sqrt{RV_S})|]$$
Laser output power (II)

By taking into account the expression for the small signal gain:

$$g_0 l A I_S = \frac{\lambda_P}{\lambda_L \cdot \eta_{abs}} P_{pump}$$

and assuming a constant mode beam area $A_B$, we get:

$$P_{out} = \frac{A_B}{A} \cdot \frac{1 - R}{2|\ln(\sqrt{RV_S})|} \cdot [\frac{\lambda_P}{\lambda_L \cdot \eta_{abs}} P_{pump} - A I_S |\ln(\sqrt{RV_S})|] = \eta \cdot (P_{pump} - P_{pump,th})$$

Threshold pump power

$$P_{pump,th} = \frac{A I_S}{\frac{\lambda_P}{\lambda_L \cdot \eta_{abs}}} \cdot |\ln(\sqrt{RV_S})|$$

Slope efficiency

$$\eta = \frac{A_B}{A} \cdot \frac{1 - R}{2|\ln(\sqrt{RV_S})|} \cdot \frac{\lambda_P}{\lambda_L \cdot \eta_{abs}}$$
Optimum output coupling

The output coupling ratio \( R \) can be chosen for maximum output power:

**Optimum output coupling:**

\[
\ln R_{opt} = -2\alpha_0 l \left[ \sqrt{\frac{g_0 l}{\alpha_0 l}} - 1 \right]
\]

**Maximum optical efficiency:**

\[
\eta_{opt,max} = \frac{P_{out,max}}{P_{pump}} = \lambda_P / \lambda_L \cdot \eta_{abs} \cdot \frac{\alpha_0 l}{g_0 l} \cdot \left[ \sqrt{\frac{g_0 l}{\alpha_0 l}} - 1 \right]^2
\]

The optimum output coupling depends on the small signal gain and losses
The mode size $A_B$ in the previous equations depends on the resonator design. We consider resonators with two mirrors. Actual resonator might be way more complicated! Mirrors have their own radius of curvature ($R > 0$ for convex mirrors) We define $g$-parameters as follows:

$$g_i = 1 - \frac{L}{R_i} \quad i = 1, 2$$

$L$ is the resonator length

Resonators define eigenmodes, E-field distributions which retain their profile after every round-trip.
Stable resonators

\[ 0 \leq g_1 g_2 \leq 1 \]

Beam radius of fundamental mode on mirrors 1,2:

\[ w_i^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g_j}{g_i(1 - g_1 g_2)}} \quad i, j = 1, 2; i \neq j \]

Beam waist in the resonator:

\[ w_0^2 = \frac{\lambda L}{\pi} \frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{|g_1 + g_2 - 2g_1 g_2|} \]

In multimode operation the beam area is \( \approx M^2 \) larger than the fundamental mode area

http://www.brightsolutions.it
Pumping schemes and active media geometries

**Optical Pumping:**
- Flash-lamps or arc-lamps:
  - Very high pump power, immune to current or voltage spikes 😊
  - Limited lifetime, low wall-plug efficiency (a few percent), low brightness 😐
- Diode-pumping (end-pumping or side-pumping):
  - High wall-plug efficiency, narrow bandwidth, high beam quality (allowing end-pumping and precise mode-matching), long lifetime, compactness 😊
  - Higher cost per watt of pump power 😐

**Active Media Geometries:**
- Rod: side-pumping or end-pumping (Nd:YAG)
- Slab: side-pumping (Nd:YVO₄, Nd:YAG)
- Thin-Disk: optimum for heat dissipation, requires high pump absorption and high doping concentration (Yb:YAG)
- Fiber: optimum for heat dissipation and beam confinement
Fiber lasers

- Resonator layout significantly simplified
- Mode area and beam quality determined by the propagation condition inside of the optical fiber
- The pump power needs to be coupled into the resonator through WDMs or pump combiners
- The mirrors can be realized inside of the fiber (Fiber Bragg Gratings, FBG)
- Easier management of thermal problems than in bulk materials
- High brightness
Pulsed operation

- CW operation allows to achieve output power in the order of 10 kW (fiber lasers) or few kW (bulk materials)
- Lasers can also operate in pulsed regimes (e.g. by using a pulsed pump source)
- Q-switching and mode-locking allows to redistribute the stored energy to get high peak powers and pulse energies
- High peak powers and pulse energies are required in a number of applications (material processing and non-linear processes)
- Q-switching allows to generate ns-long pulses with energies up to 100’s of mJ and repetition rates from a few Hz to a few 100’s of kHz (peak power in the order of kWs)
- Mode-locking operation allows to get µJ-level ps-long or fs-long pulses (ultrafast pulses) with repetition rates ranging from 10’s of MHz up to a 10’s of GHz (peak power in the order of MWs)
We define a Q-factor as ratio between the stored energy $E_{st}$ and the dissipated energy $E_d$ in an optical cycle $T_0 = \nu_0^{-1} = \lambda / c$:

$$Q = \frac{2\pi}{E_d} = \frac{2\pi}{E_{st} \cdot \left[1 - \exp\left(-T_0 / \tau_c\right)\right]} \approx \frac{2\pi \tau_c}{T_0} = 2\pi \nu_0 \tau_c$$

$\rightarrow$ the lower the losses the higher Q.
Q-switching principle of operation

- Initially $Q$ is low for energy storing by pumping over a time interval $\approx \tau$.
- A much higher population inversion than the threshold value for high $Q$ is achieved.
- The $Q$ level is suddenly increased and the energy is released in the form of a giant pulse.
- Due to the high gain established, the energy is released in a short pulse.
- The peak power exceeds by several orders of magnitude the power of CW lasers in the same pumping conditions.

*W. Koechner, Solid-State Laser Engineering, Sixth Ed.*
Q-switching pulse parameters

- **Peak power** → \( P_{\text{peak}} \approx \frac{h \nu L}{\sigma} A_B \cdot \frac{\ln(1/R)}{T_R} g_i \)
- **Pulse energy** → \( E_P \approx \frac{h \nu L}{\sigma} A_B \cdot \frac{\ln(1/R)}{\epsilon} g_i \)
- **Pulse duration** → \( \tau_P \approx \frac{E_P}{P_{\text{peak}}} \approx \frac{T_R}{\epsilon} = \tau_c \)
- **Build-up time** → \( t_b \approx \frac{20 \tau_c}{g_i/g_{th} - 1} \)

- \( g_i \) is the initial gain before the switching time
- \( g_{th} \) is the laser threshold gain value for high Q value

- The pulse energy is proportional to the saturation fluence \( h \nu L/\sigma \)
- Comparing Q-switching and CW operation at the same pump power: \( P_{QS}/P_{CW} \approx \tau / T_R \)
- For Q-switching operation \( \tau \gg T_R \) is required
**Q-switching techniques**

**ACTIVE Q-SWITCHING**

- Electro-optic or acousto-optic switches might be used.
- Electro-optic switches are faster, acousto-optic switches more suited for repetition rates $> 1 \text{ kHz}$.
- Electro-optic switches (figure) include a waveplate, a Pockels cell (PC) and a polarizer.

\[ V = 0 \]
\[ V = V_{\lambda/4} \]

**PASSIVE Q-SWITCHING**

- A saturable absorber is a non-linear device with transmission increasing with the incident intensity.
- The absorber has to saturate before than gain: $\frac{E/A_A}{F_{SA}} > \frac{E/A_L}{F_{SL}}$.

\[ T \]
\[ T_{sat} \]
\[ T_0 \]
\[ F_{sat} \]

**Figure:**
- The figure shows a schematic of a Q-switching system, including a waveplate, a Pockels cell (PC), and a polarizer. The voltage $V = 0$ and $V = V_{\lambda/4}$ are applied to the system, affecting the transmission and gain.

[http://www.brightsolutions.it](http://www.brightsolutions.it)
The upper-state lifetime sets a critical frequency $1/\tau_f$

If the repetition rate is in the order of $1/\tau_f$ the initial gain at the switching time is clamped at a lower value because population inversion can not recover completely.

High $\tau_f$ is required for energy storage while low $\tau_f$ is required for operation at high repetition rates.
For Nd:YAG $\tau_f = 230 \, \mu s \rightarrow 1/\tau_f \approx 4.3 \, \text{kHz}$
Mode-locking: principle of operation

- A resonator with length $L$ creates a set of equally spaced resonance frequencies
  \[ \Delta \omega = \frac{c}{2L} \rightarrow \text{frequency separation between two adjacent modes} \]

- The mode-locking regime is based on the inset of a precise phase relationship between adjacent longitudinal modes: repetition rate \[ f_R = \frac{c}{2L} \]

- Pulses are created as a consequence of constructive interference between phase-locked modes

- Higher number of phase-locked modes means larger pulse spectra and shorter pulse durations and higher peak powers: minimum pulse duration related to $\Delta \omega_A$
Mode-locking techniques

Mode-locking operation might be achieved with:

- **ACTIVE TECHNIQUES** employing electro-optic or acousto-optic modulators
- **PASSIVE TECHNIQUES** employing saturable absorbers: semiconductor saturable absorber mirrors (SESAM) or Kerr-effect

Passive techniques allow shorter pulse durations

- Pulse durations range from 10’s of picosecond down to a few femtoseconds
- For pulse duration below 1 ps group velocity dispersion has to be taken into account
Non-linear optics

\[
\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\frac{1}{\varepsilon_0} \frac{\partial^2 P}{\partial t^2}
\]

The incident electric field \( E \) creates in the medium a polarization \( P \). For weak fields:

\[
P = \varepsilon_0 \chi^{(1)} E
\]

In general:

\[
P = \varepsilon_0 \left( \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right) = P^{(L)} + P^{(NL)}
\]

<table>
<thead>
<tr>
<th>( \chi^{(1)} )</th>
<th>( \chi^{(2)} )</th>
<th>( \chi^{(3)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 - 5 m/V</td>
<td>( \sim 10^{-12} ) m/V</td>
<td>( \sim 10^{-20} ) m²/V²</td>
</tr>
</tbody>
</table>

**Quantum mechanical description (SHG):**

<table>
<thead>
<tr>
<th>Energy conservation:</th>
<th>( \hbar \cdot \omega + \hbar \cdot \omega = \hbar \cdot 2\omega )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Momentum conservation:</th>
<th>( \hbar \cdot \mathbf{k} + \hbar \cdot \mathbf{k} = \hbar \cdot 2\mathbf{k} )</th>
</tr>
</thead>
</table>

\[
\frac{\hbar \omega}{c} n(\omega) + \frac{\hbar \omega}{c} n(\omega) = \frac{\hbar \cdot 2\omega}{c} n(2\omega) \rightarrow n(\omega) = n(2\omega)
\]

Polarized light is required!

http://www.brightsolutions.it
Generation of new frequencies

Second order nonlinearity ($\chi^{(2)}$)
- Second harmonic generation
- Sum frequency generation
- Parametric amplification

Third order nonlinearity ($\chi^{(3)}$)
- Third harmonic generation
- Sum and difference frequency generation
- Kerr effect

Due to the typical values of $\chi^{(2)}$ the required optical intensities for efficient frequency conversion are $\gtrsim 100 \text{ MW/cm}^2$

New frequencies can be easily generated with Q-switched and mode-locked lasers

SHG (second harmonic generation) and THG (third harmonic generation) can be achieved also in CW lasers by intra-cavity generation

Due to the typically low values for $\chi^{(3)}$ it is usually more convenient to generate third order effects by cascading two second order processes
Further reading
