

# **THE SIMILARITY CONCEPT OF PREVEDELLO FOR THE ANALYSIS OF WATER INFILTRATION INTO SOILS**

Klaus Reichardt<sup>1</sup>, Donald R. Nielsen<sup>2</sup>

<sup>1</sup> Soil Physics Laboratory, Center for Nuclear Energy in Agriculture (CENA), University of São Paulo (USP), Piracicaba, SP, Brazil. E-mail: klaus@cena.usp.br

<sup>2</sup> Professor at UCD, Davis, California, USA.

The illustration shown below for the infiltration process serves as an introduction for the understanding of Prevedello's similarity concept:

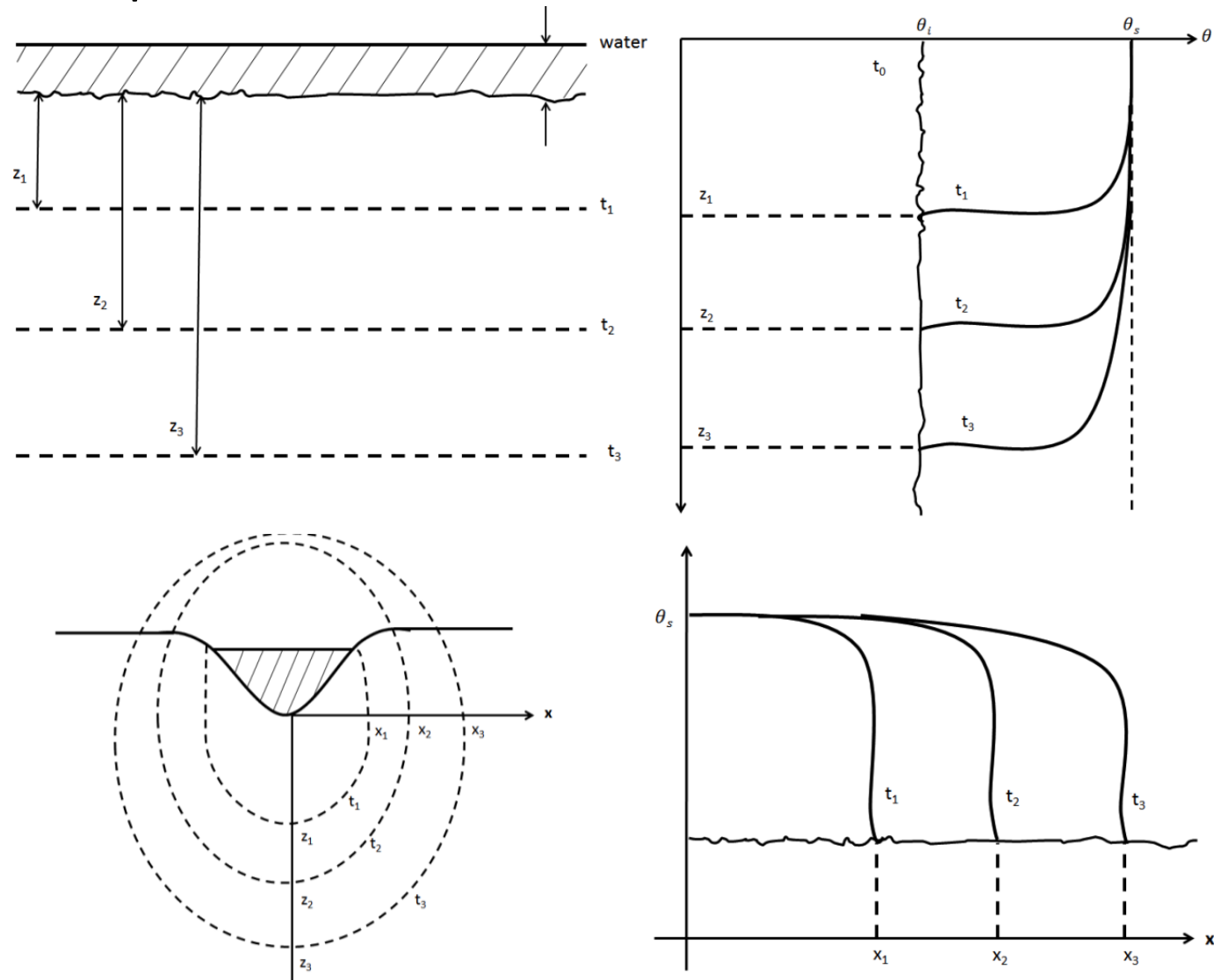


Figure 000. Shallow ponding and furrow infiltrations.

# Green and Ampt (1911)

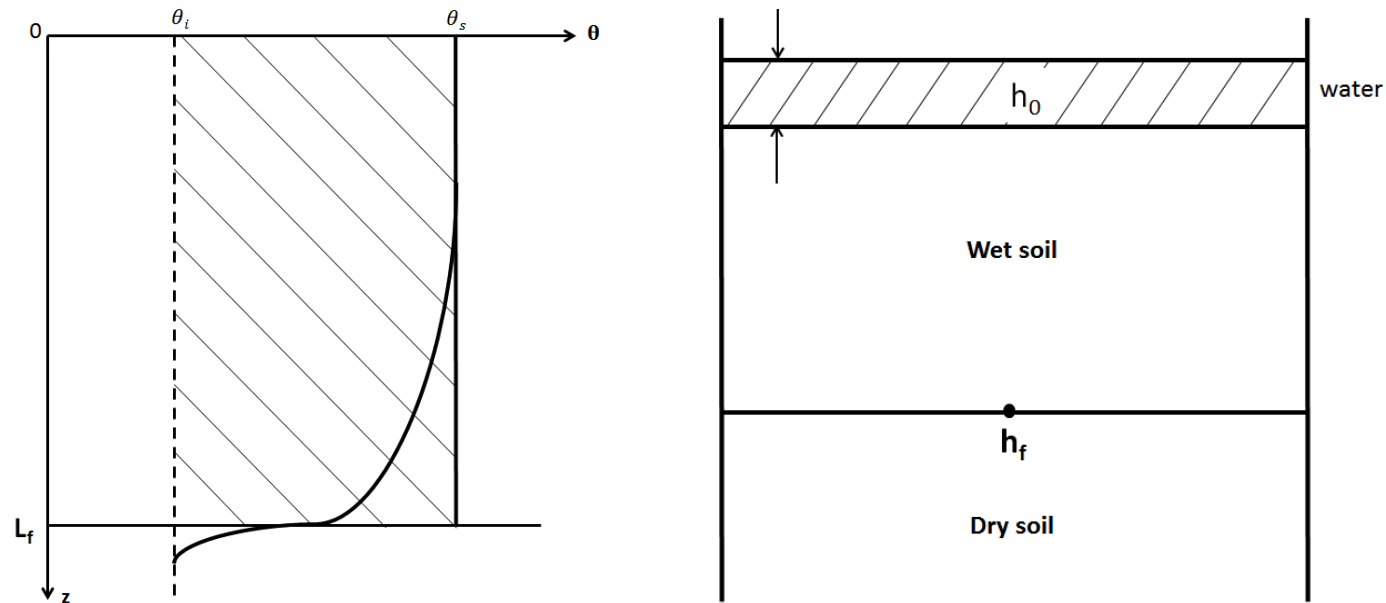


Figure 00. Green and Ampt assumptions.

$$i = \frac{dI}{dt} = -K_s \frac{h_0 + L_f(t)}{L_f(t)}$$

where  $i$  = infiltration rate,  $I$  = cumulative infiltration,  $K_s$  = saturated hydraulic conductivity.

$$I = [2K_s (h_0 - h_f) \Delta\theta]^{1/2} * t^{1/2}$$

where  $\Delta\theta = (\theta_s - \theta_i)$

Based on the fact that **shapes of the soil water retention curves and of the infiltration soil water content profiles are very similar, i.e., one being a mirror image of the other**

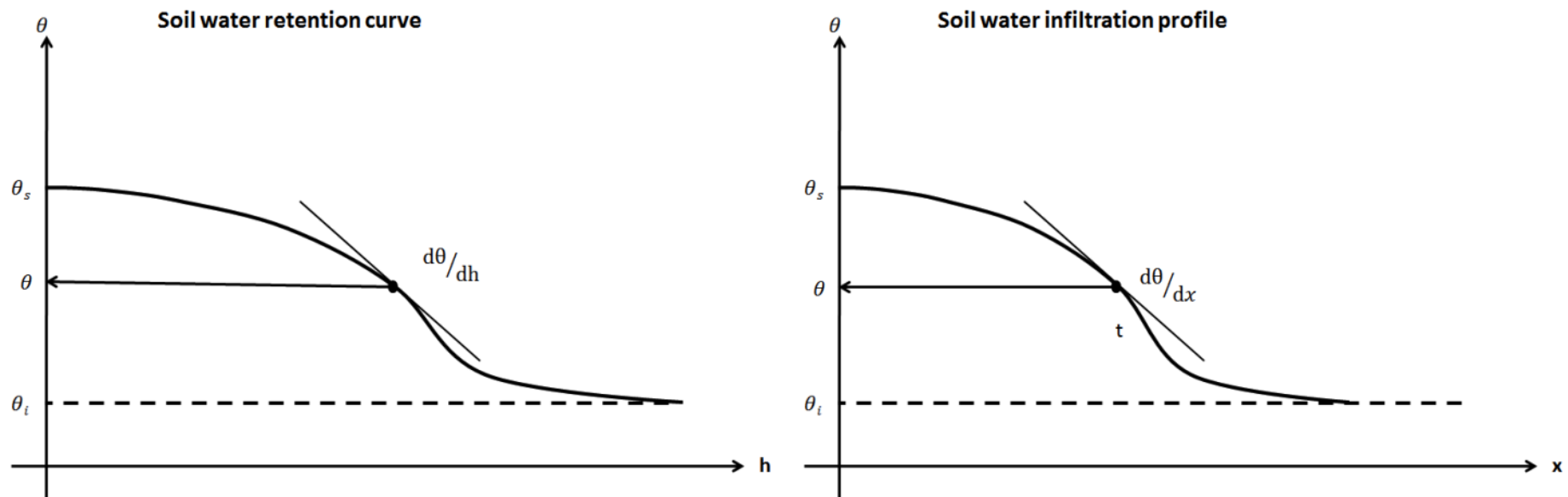


Figure 0. Schematic explanation of the similarity (mirror) hypothesis of Prevedello.

## Prevedello et al (2008)

Their similarity hypothesis illustrated in Figure 1A relates the soil water retention function  $\Theta(h)$  to an extended Boltzmann transform function  $\theta(\lambda^N)$  by the equation

$$\lambda^N \frac{d\theta}{d\lambda^N} = h \frac{d\theta}{dh} \quad [1]$$

where they selected  $N = 2$  and used experimental data of a marine sand.

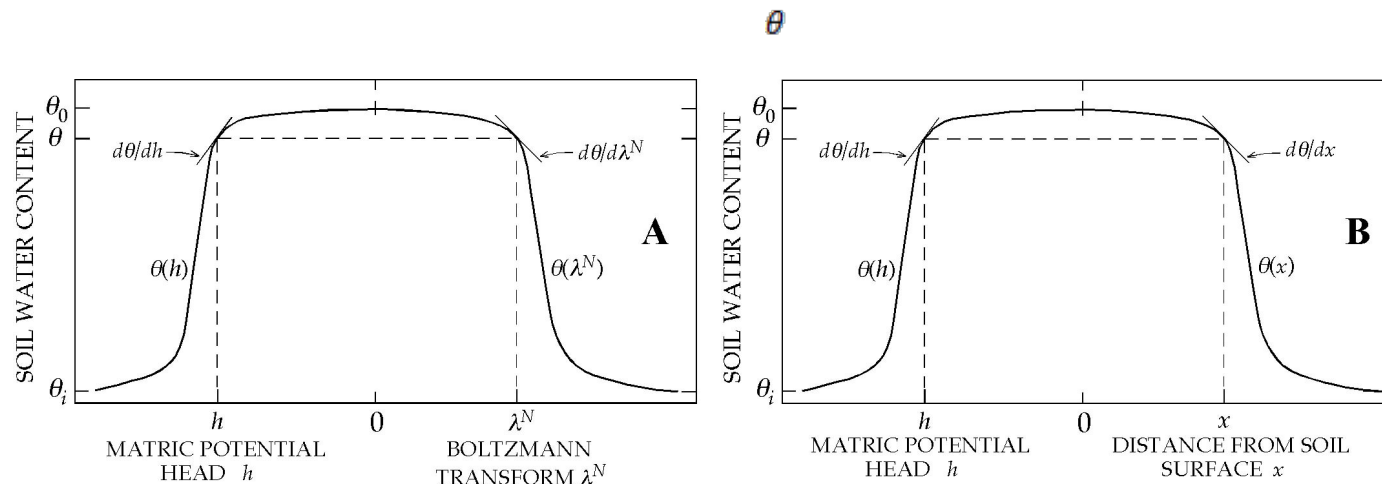


Figure 1. Diagrams of similarity hypotheses relating the soil water retention function  $\Theta(h)$  to a generalized Boltzmann transform  $\theta(\lambda^N)$  and to the distribution of  $\theta$  in the soil profile.  $\theta_i$  is the initial soil water content throughout the soil and  $\theta_0$  the water content at the soil surface for  $t > 0$ .

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ K(h) \frac{\partial h}{\partial x} \right]$$

$$\frac{d\lambda^2}{\lambda^2} = \frac{dh}{h}$$

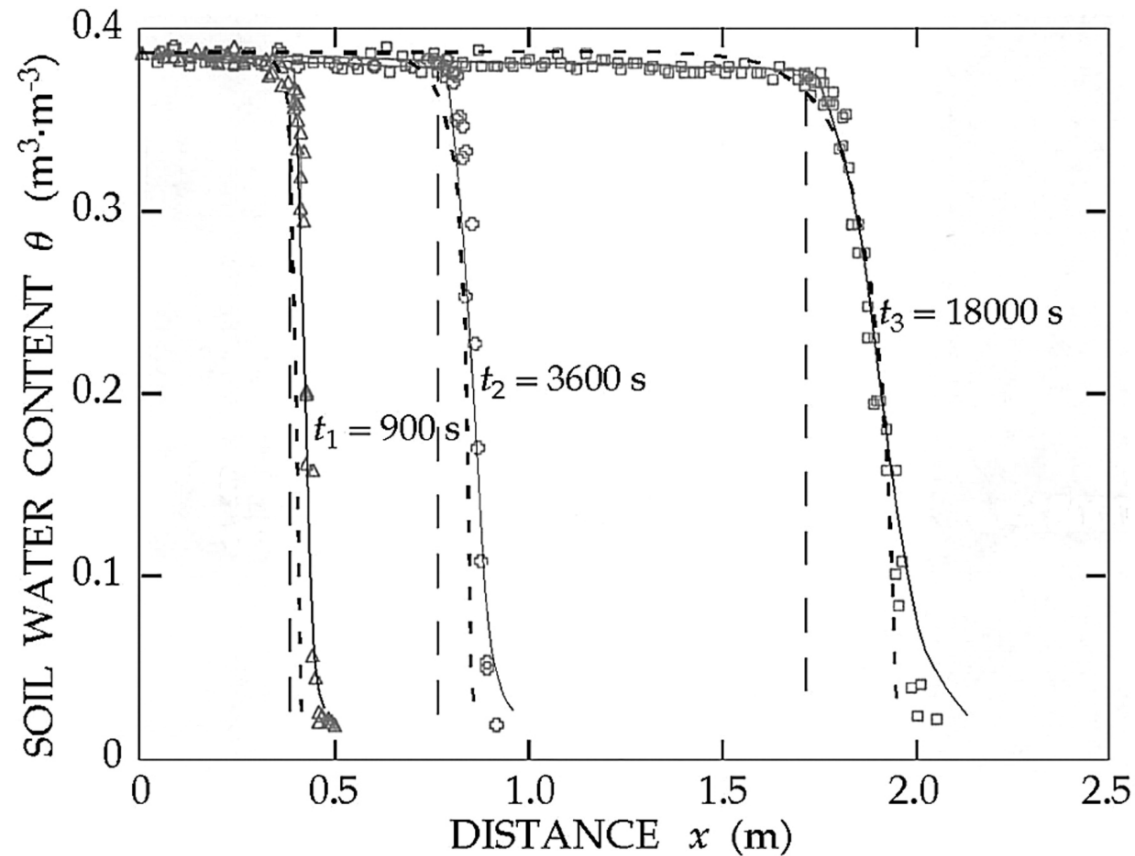
$$x(h, t) = \sqrt{\frac{-2(K_0 - K_i)ht}{(\theta_0 - \theta_i)}}$$

that satisfies the assumption that

$$x \frac{\partial \theta}{\partial x} = h \frac{d\theta}{dh}$$

without invoking a Boltzmann transform.

$$I(t) = \sqrt{\frac{2(K_0 - K_i)}{(\theta_0 - \theta_i)}} \int_{\theta_i}^{\theta_0} |h|^{1/2} d\theta \cdot t^{1/2}$$



**FIG. 5.** Soil water content profiles showing experimental data points and solid lines estimated by [Eq. \[22\]](#) and [\[30\]](#) using the soil parameters given in [Table 1](#) for horizontal infiltration times  $t_1$ ,  $t_2$ , and  $t_3$ . Broken curved lines are numerical solutions of the Richards equation using measured values of hydraulic properties. Broken vertical lines indicate the wetting front from Green and Ampt [Eq. \[27\]](#).

Prevedello, C.L.; Loyola, J.M.T.; Reichardt, k.; Nielsen, D.R. New analytic solution of Boltzmann transform for horizontal water infiltration into sand. **Vadose Zone Journal**, v.7, n.4, 2008.

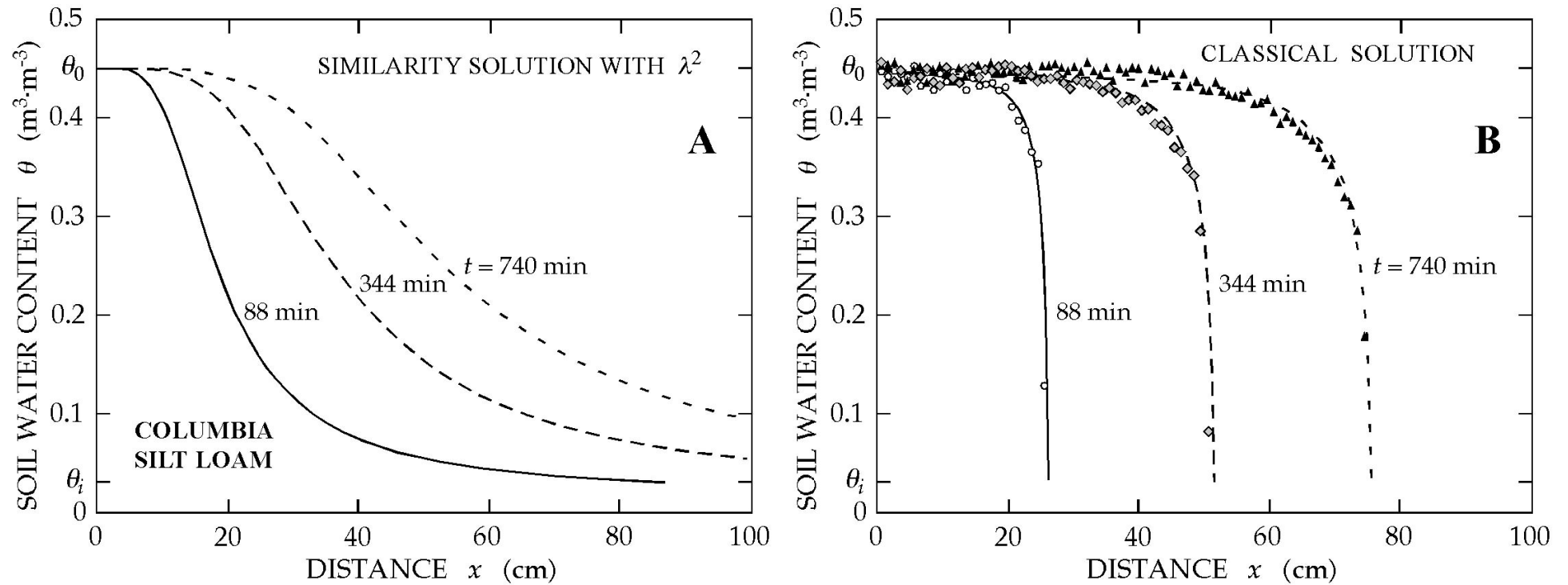


Figure 13. Soil water content distributions within Columbia silt loam from the similarity solution satisfying the extended Boltzmann transform having exponent  $N = 2$ , and those measured and calculated from classical solution of Richards' equation



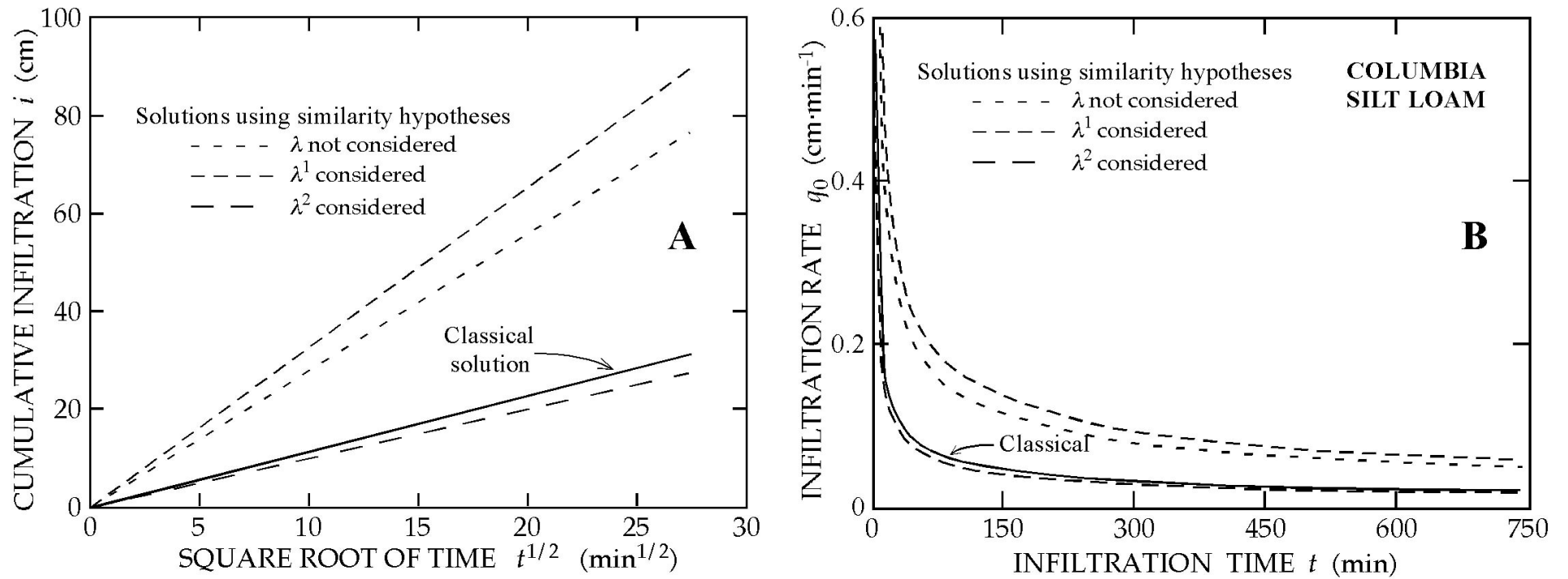


Figure 14. Infiltration rate into the surface of Columbia silt loam as a function of time from the similarity solution satisfying the extended Boltzmann transform having exponent  $N = 2$ , and that from the classical solution of Richards' equation.

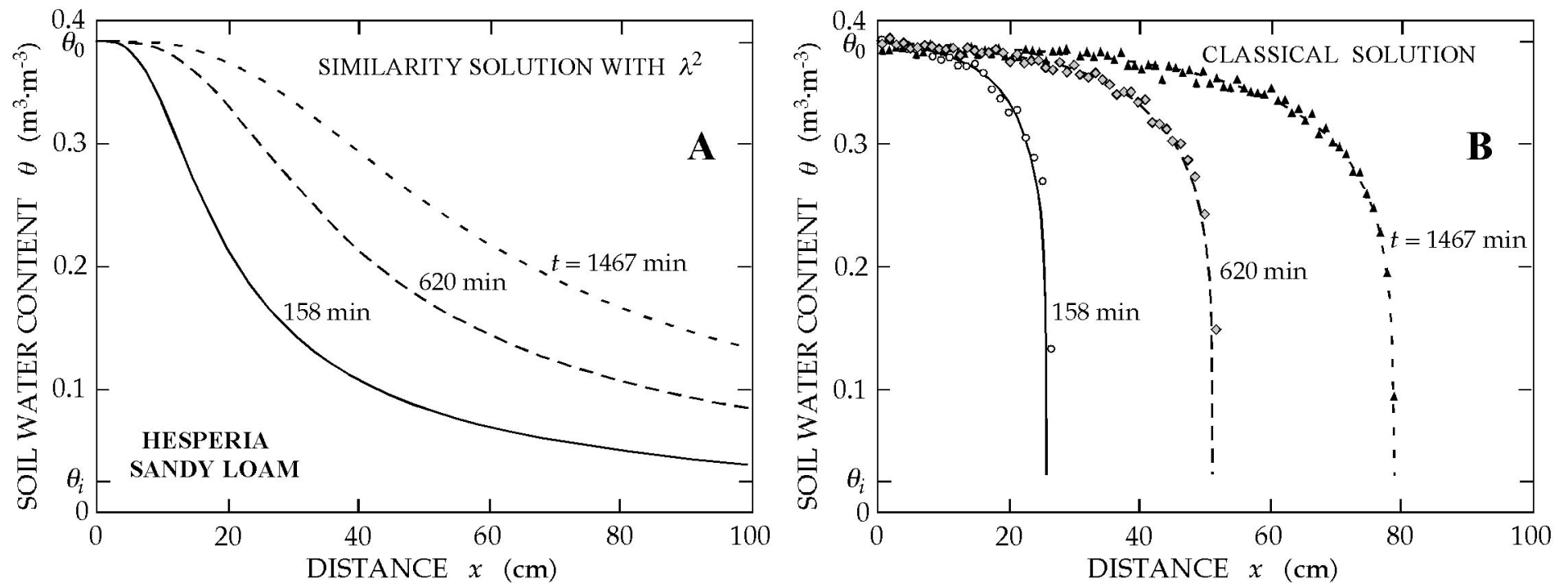


Figure 15. Soil water content distributions within Hesperia sandy loam from the similarity solution satisfying the extended transform having exponent  $N = 2$ , and those measured and calculated from classical solution of Richards' equation.

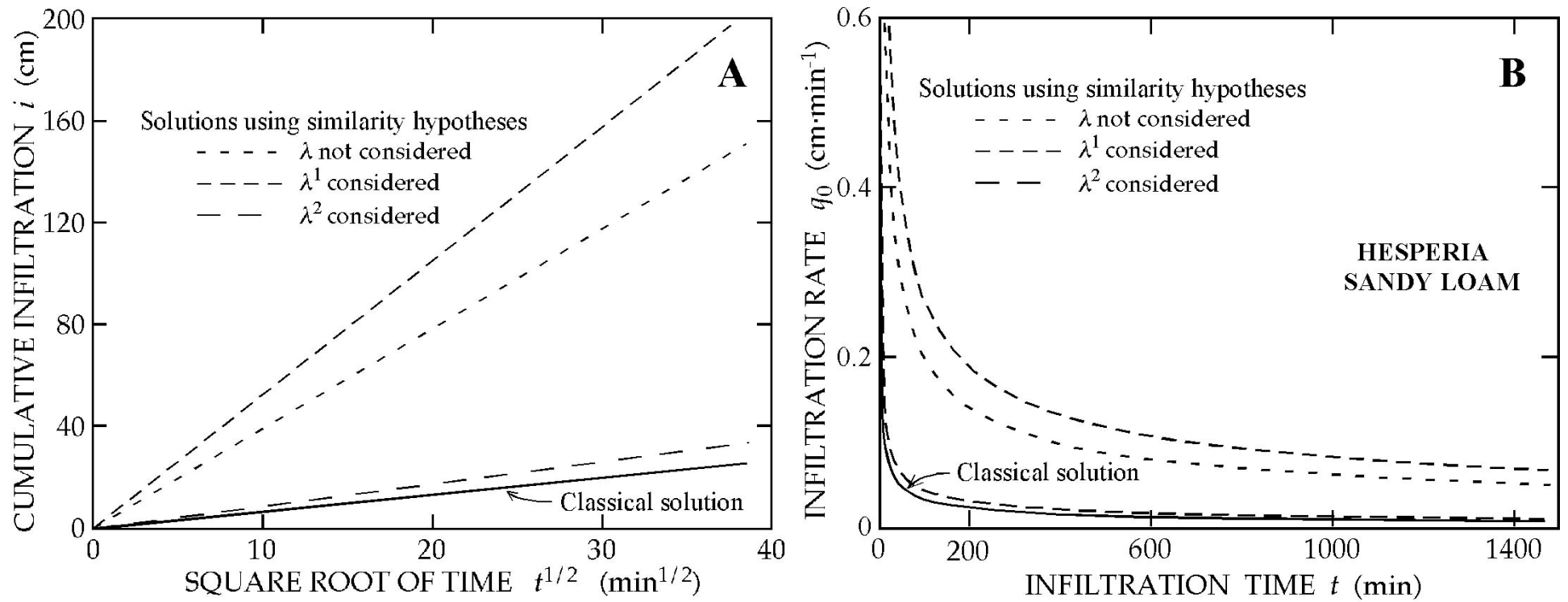


Figure 16. Infiltration rate into the surface of Hesperia sandy loam as a function of time from the similarity solution satisfying the extended Boltzmann transform having exponent  $N = 2$ , and that from the classical solution of Richards' equation.

## **PRESENT OUTLOOK**

Without a comprehensive analysis of any additional infiltration experiments on a small or large number of natural-occurring soils each made with the aid of experimentally verified soil hydraulic properties, we tacitly predict that such research shall verify the beneficial utility of the innovative "Prevedello similarity solution" of Richards' equation. In other words, the empirical parameter identified as the matric potential head at the wetting front that is critical in the Green and Ampt analysis no longer needs to be ascertained or approximated. We anticipate that for any homogeneous soil, the horizontal infiltration process subject to the initial and boundary conditions of this presentation shall be adequately predicted by Eqs. [33]. [39] and [40].