The Abdus Salam
International Centre
for Theoretical Physics

2445-05

Advanced Workshop on Nanomechanics

9 - 13 September 2013

Nanomechanical Qubits

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Nanomechanical Qubits

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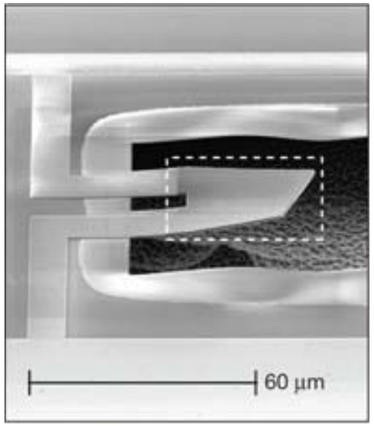
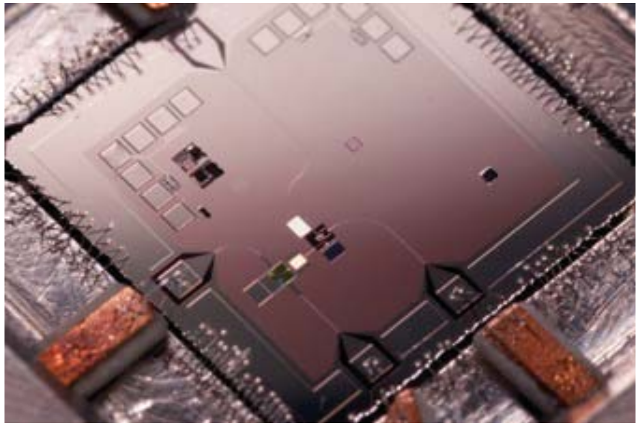
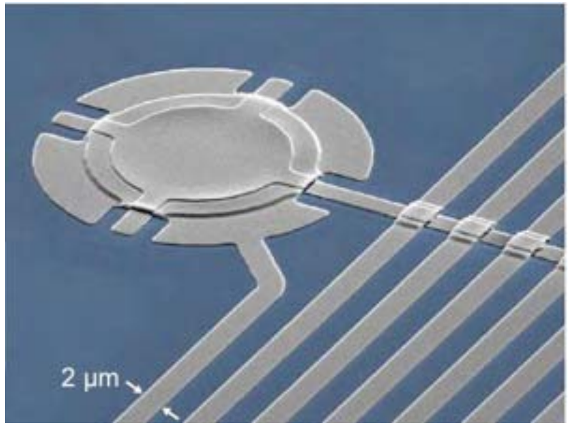
Technische Universität München
www.ph.tum.de/quantumdynamics

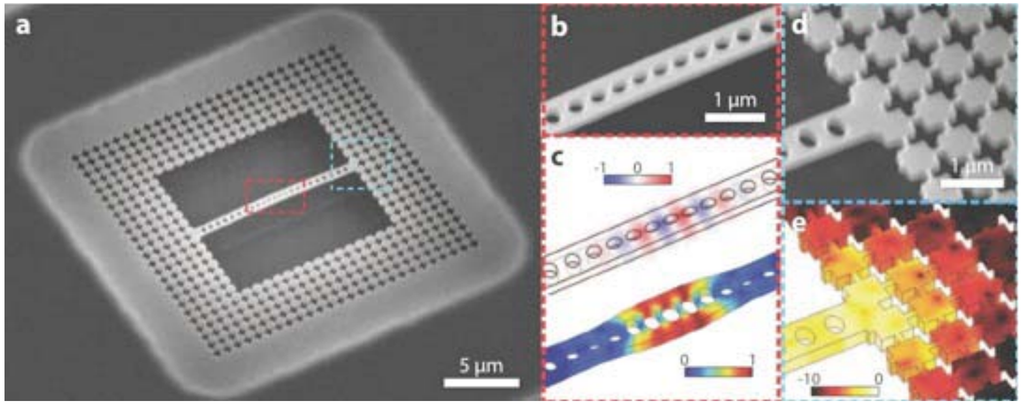
Frontiers of Nanomechanics
Trieste, September 2013

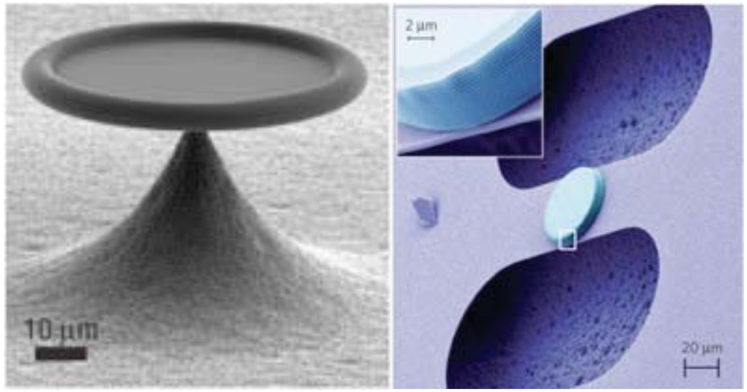
Ground State Cooling







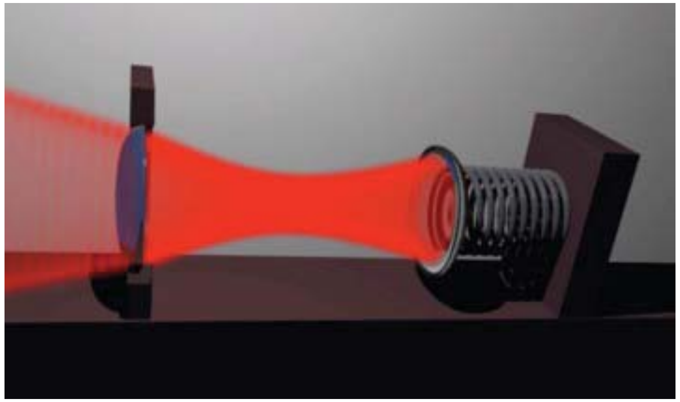




A.D. O'Connell et al., Nature **464**, 697 (2010).
J.D. Teufel et al., Nature **475**, 359 (2011).
J. Chan et al., Nature **478**, 89 (2011).
Aspelmeyer, Kippenberg, Marquardt, arXiv:1303.0733 (2013)

ground state cooling realized → quantum regime

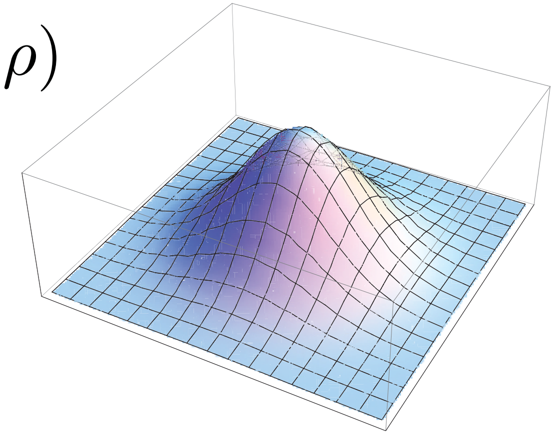
Classical vs Nonclassical States



$$\dot{\rho} = -i\,[H, \rho] + \frac{\gamma}{2}\mathcal{D}(\rho)$$

at most quadratic

in $a^\dagger$, a, X and P



➡ states with Gaussian Wigner function stay Gaussian
➡ asymptotic states have Gaussian Wigner function

➡ can only generate classical states
➡ need nonlinearity to generate genuine quantum states

coupling to external nonlinearity
nonlinear coupling to photons

Jacobs, Phys. Rev. Lett. **99**, 117203 (2007).
Rabl, Phys. Rev. Lett. **107**, 063601 (2011)
Nunnenkamp et al., Phys. Rev. Lett. **107**, 063602 (2011)

here we consider a nonlinearity in the mechanical oscillator

Linear and Nonlinear Quantum Systems

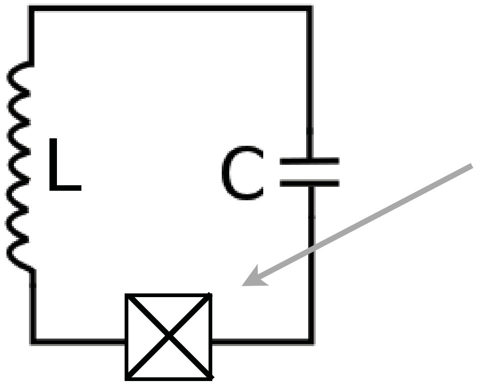
$$H = \frac{P^2}{2m} + \frac{m}{2}\omega X^2 \qquad \frac{d^2}{dt^2}X = -\omega^2 X \qquad \frac{d^2}{dt^2}\langle X\rangle = -\omega^2\langle X\rangle$$

$$H = \frac{P^2}{2m} + \frac{m}{2}\omega X^2 + \lambda X^4 \qquad \frac{d^2}{dt^2}X = -\omega^2 X - 4\frac{\lambda}{m}X^3$$

$$\langle X^3\rangle \neq \langle X\rangle^3$$

e.g. electrical circuits:

$$I \propto \Phi$$



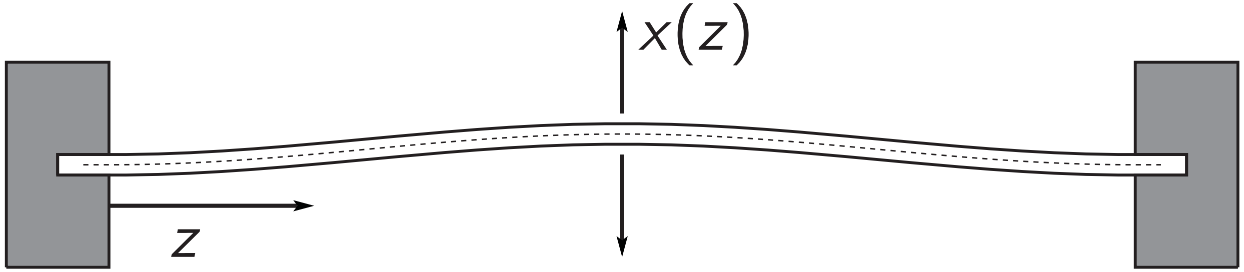
Josephson junction

$$I \propto \sin\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

Outline

- nonlinear elasticity in thin beams
- phonon Fock states
- nanomechanical qubits

Elasticity of Thin Beams



bending energy

$$V_b = \frac{\mathcal{F}\kappa^2}{2}\int_0^L dz \left(\frac{d^2x}{dz^2}\right)^2$$

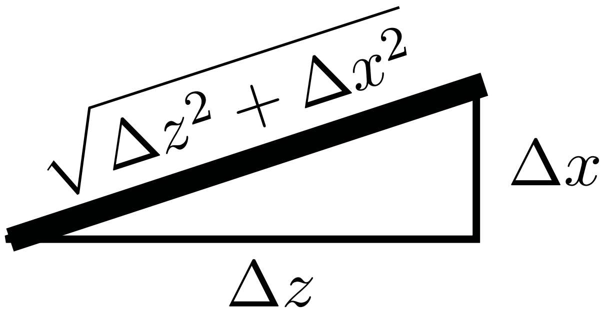
→ harmonic potential

$$V_b \propto X^2$$

stretching energy

$$V_s = \frac{\mathcal{F}}{2}\frac{(L_t - L)^2}{L} \approx \frac{\mathcal{F}}{8L}\left[\int_0^L dz \left(\frac{dx}{dz}\right)^2\right]^2$$

$$V_s \propto X^4$$



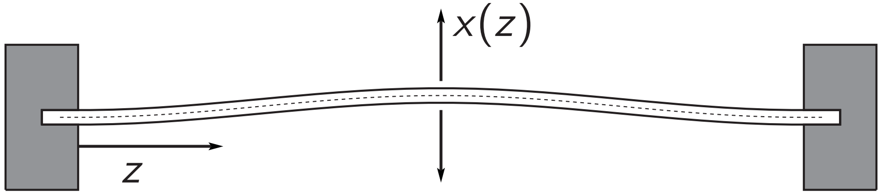
$$\sqrt{\Delta z^2 + \Delta x^2} - \Delta z \approx \frac{\Delta x^2}{2\Delta z}$$

→ nonlinear potential

Elasticity of Thin Beams

in normal modes

$$x(z) = \sum_n \Phi_n(z) X_n(t)$$



for one mode only

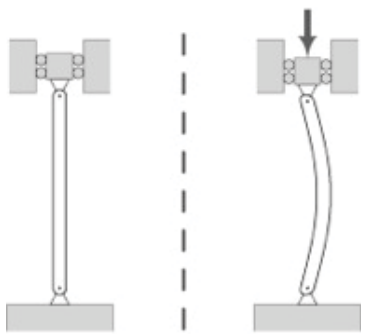
$$H = \frac{P^2}{2m} + \frac{m}{2}\tilde{\omega}_m X^2 + \tilde{\lambda} X^4$$

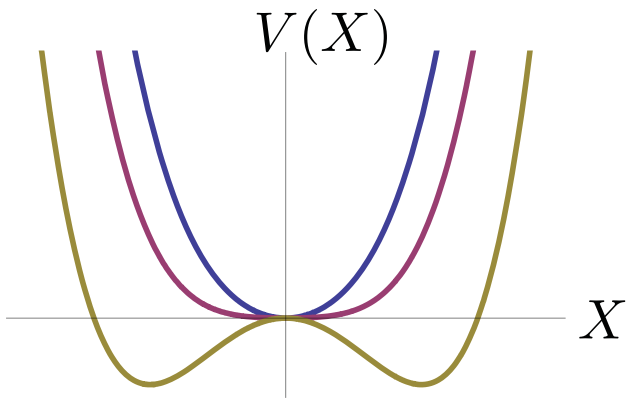
diamond bar of
500nm x 20nm x 10nm

$$\tilde{\omega}_m \sim 2\mathrm{GHz}$$
$$\hbar/(4m^2\tilde{\omega}_m^2)\,\tilde{\lambda} \sim 3\mathrm{Hz}$$

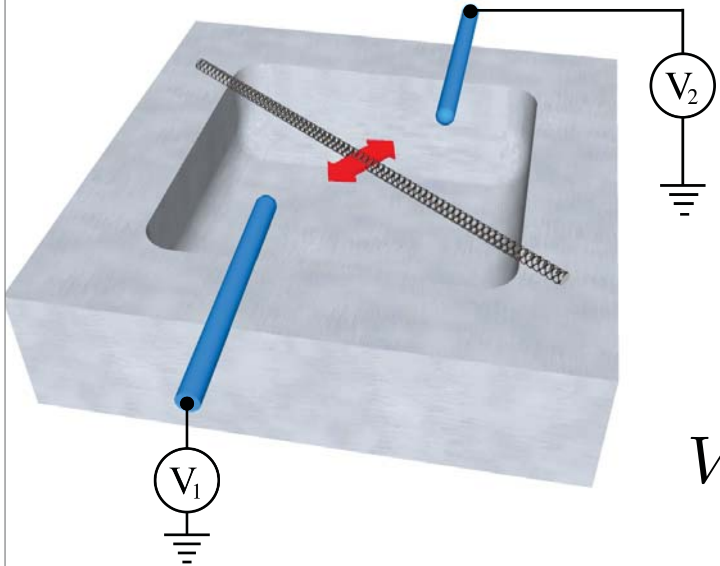
very small

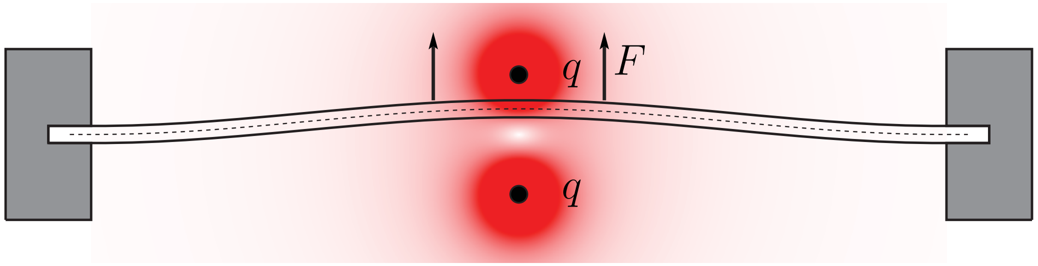
compressive strain
→ buckling





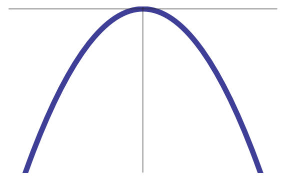
Applying Electrostatic Fields





inhomogeneous electrostatic field

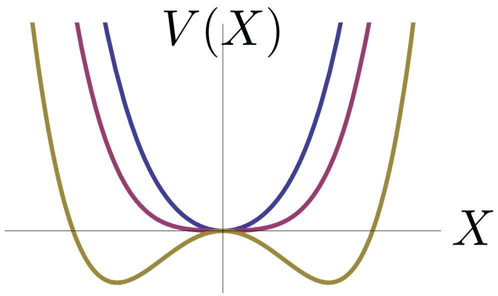
$$V_e = -\frac{\alpha}{2}\int dV\,|E|^2 \approx V_{e,0} + \xi_1 X - \xi_2 X^2$$



$\xi_1 \propto V_1 - V_2$ shift equilibrium $\tilde{X}_0 \rightarrow X_0$

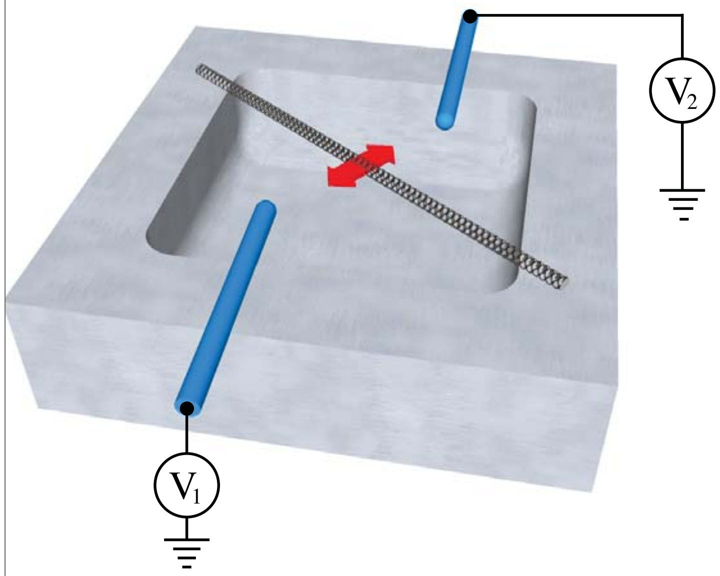
$\xi_2 \propto V_1 + V_2$ soften frequency $\tilde{\omega}_m \rightarrow \omega_m < \tilde{\omega}_m$

$$H = \frac{P^2}{2m} + \frac{m}{2}\tilde{\omega}_m X^2 + \tilde{\lambda} X^4$$



precise control and tuneability

Nonlinearity per Phonon



dynamics of mechanical oscillator:

$$H = \frac{P^2}{2m} + \frac{m}{2}\tilde{\omega}_m X^2 + \tilde{\lambda} X^4$$

if we decrease the harmonic oscillation frequency:

$$\tilde{\omega}_m \rightarrow \omega_m < \tilde{\omega}_m$$

$$X = \sqrt{\frac{\hbar}{2m\omega_m}}\,(b + b^\dagger)$$

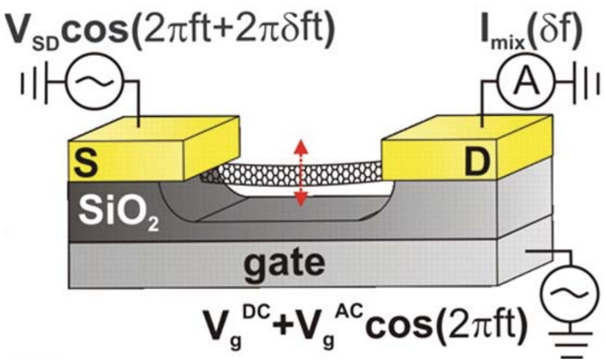
$$\tilde{\lambda} X^4 \approx \lambda\, b^\dagger b^\dagger b b$$

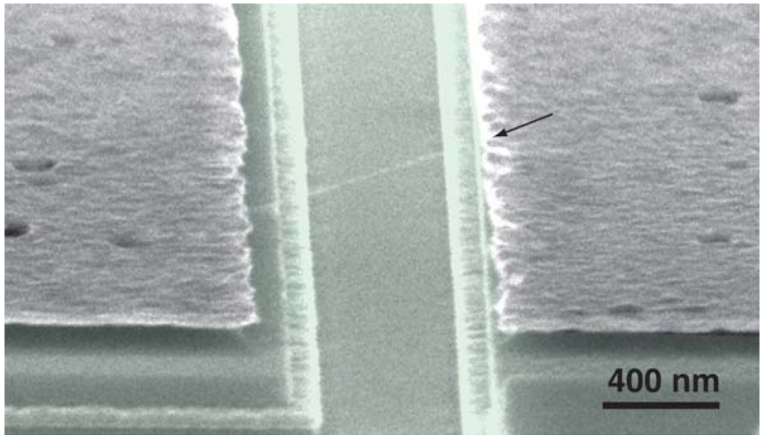
$$\lambda \propto \frac{1}{\omega_m^2}$$

decrease of harmonic frequency enhances nonlinearity per phonon

Carbon Nanotubes

What is a good candidate?



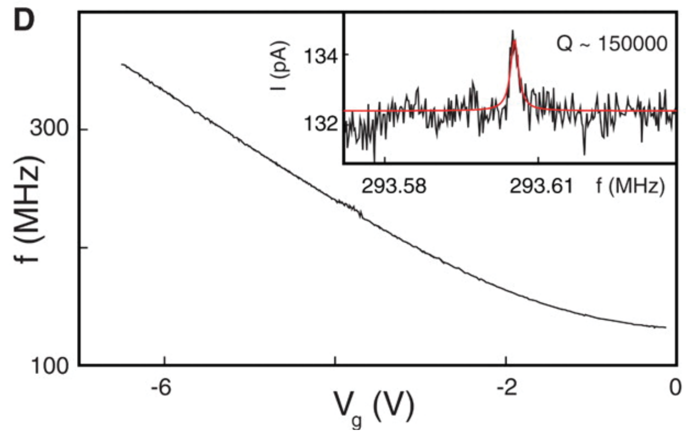


Steele et al., *Science* **325,** 1103 (2009),
Lassagne et al., *Science* **325,** 1107 (2009),
Schneider et al., Scientific Reports **2**, 599 (2012)

nonlinearity without softening:

$\lambda_0 \propto 1/(mR^2)$

$\rightarrow$ low mass m and transverse dimension R

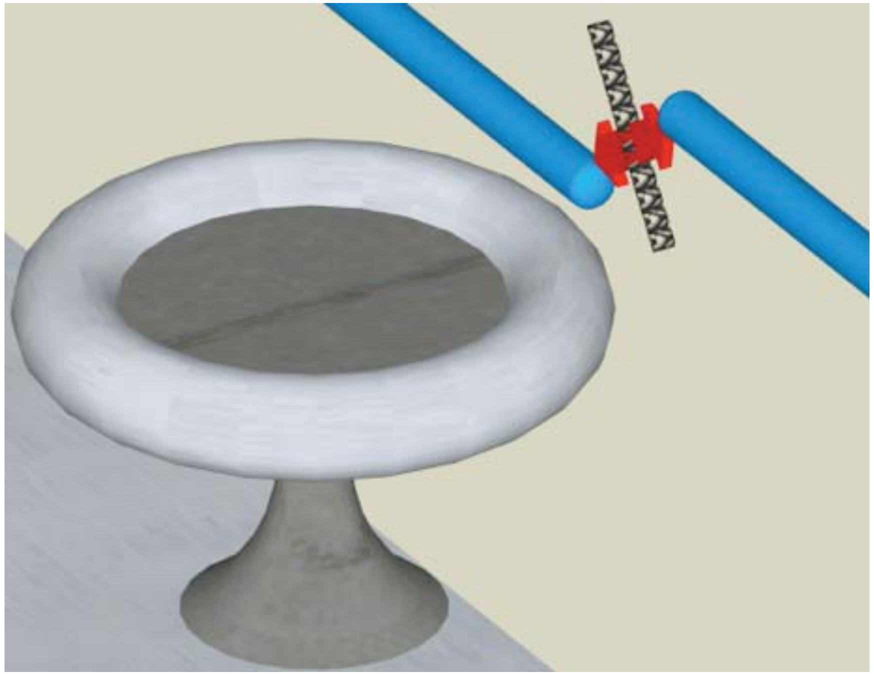


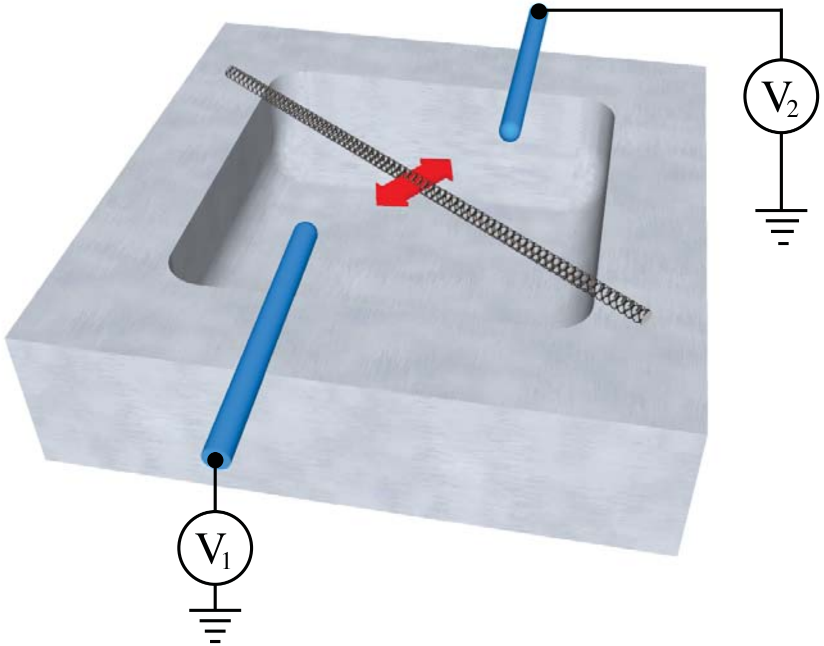
$f \times Q \sim 10^{14}$

Outline

- nonlinear elasticity in thin beams
- phonon Fock states
- nanomechanical qubits

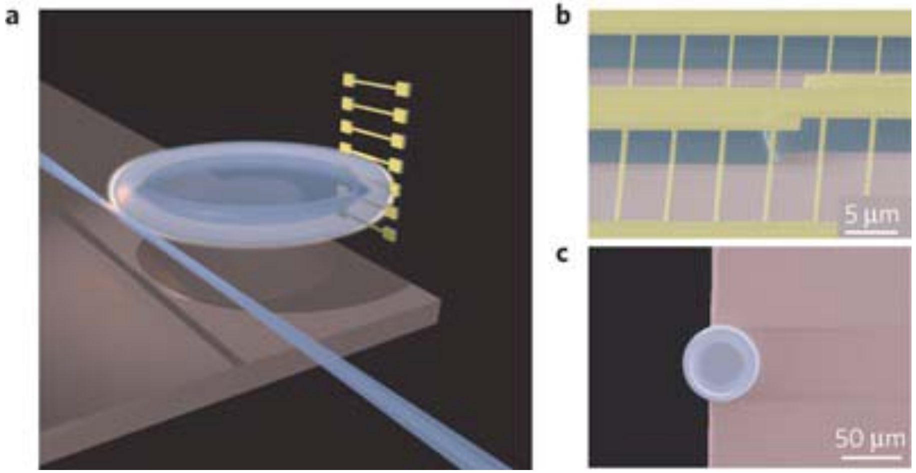
Optomechanics with Softened Nanobeam

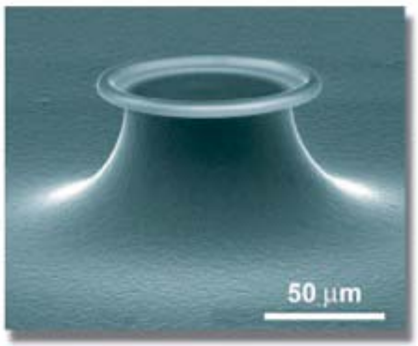




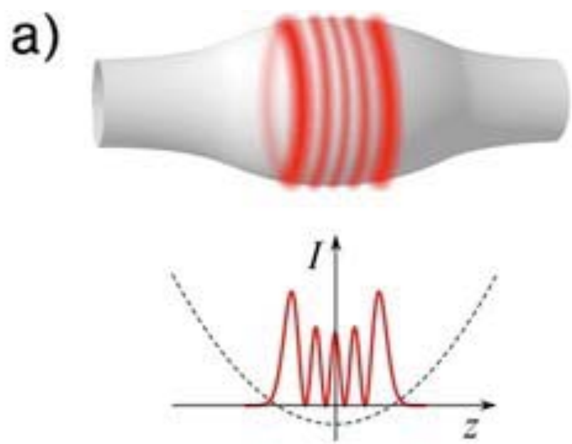
$$H = \Delta a^\dagger a + \frac{\Omega}{2}\left(a^\dagger + a\right) + G a^\dagger a\left(b^\dagger + b\right) + \omega_m b^\dagger b + \frac{\lambda}{2}(b^\dagger + b)^4$$

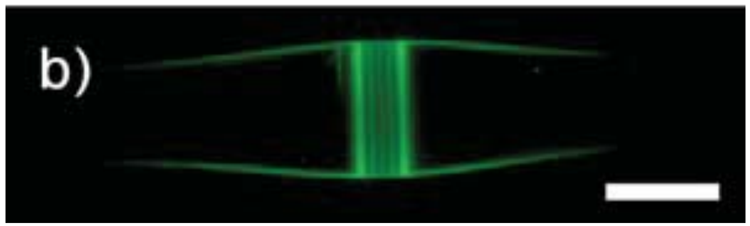
Mechanical Oscillators in Evanescent Fields





$$m \sim 10^{-12}\,\mathrm{g}$$
$$\omega_m \sim 60\,\mathrm{MHz}$$
$$Q_m \sim 10^5$$
$$\mathcal{F} \sim 2.3 \times 10^5$$





$$\mathcal{F} \sim 10^6$$

G. Anetsberger et al., Nature Physics **5**, 909 - 914 (2009)
Quirin P. Unterreithmeier, Eva M. Weig & Jörg P. Kotthaus, Nature **458**, 1001 (2009)

M. Pöllinger, D. O'Shea, F. Warken, and A. Rauschenbeutel, Phys. Rev. Lett. **103**, 053901 (2009)

Radiation Pressure Cooling

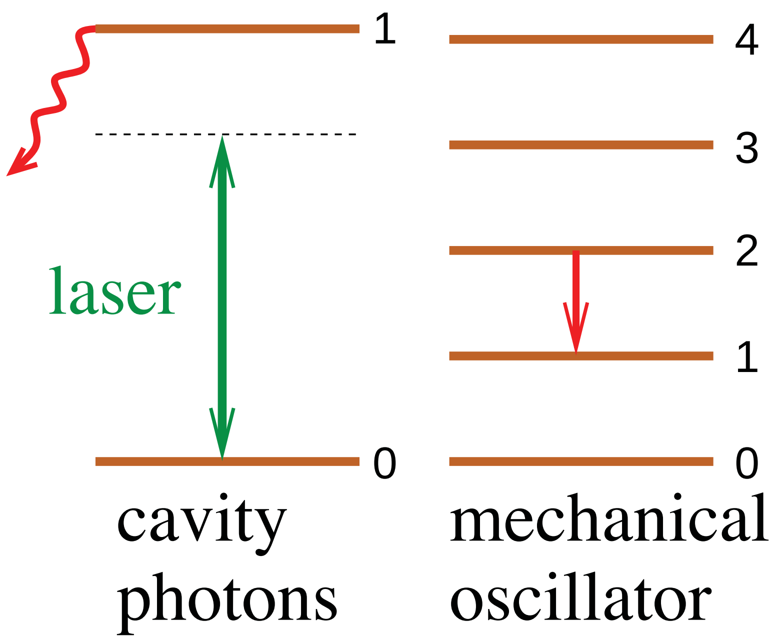
cavity decay can be used to extract energy from mechanical oscillator

for small nonlinearity λ

ground state cooling:

$$\kappa < \omega_m$$

resolved sideband regime



I. Wilson-Rae et al., Phys. Rev. Lett. **99**, 093901 (2007)
F. Marquardt et al., Phys. Rev. Lett. **99**, 093902 (2007)

J.D. Teufel et al., Nature **475**, 359 (2011)
J. Chan et al., Nature **478**, 89 (2011)

Sideband Splitting

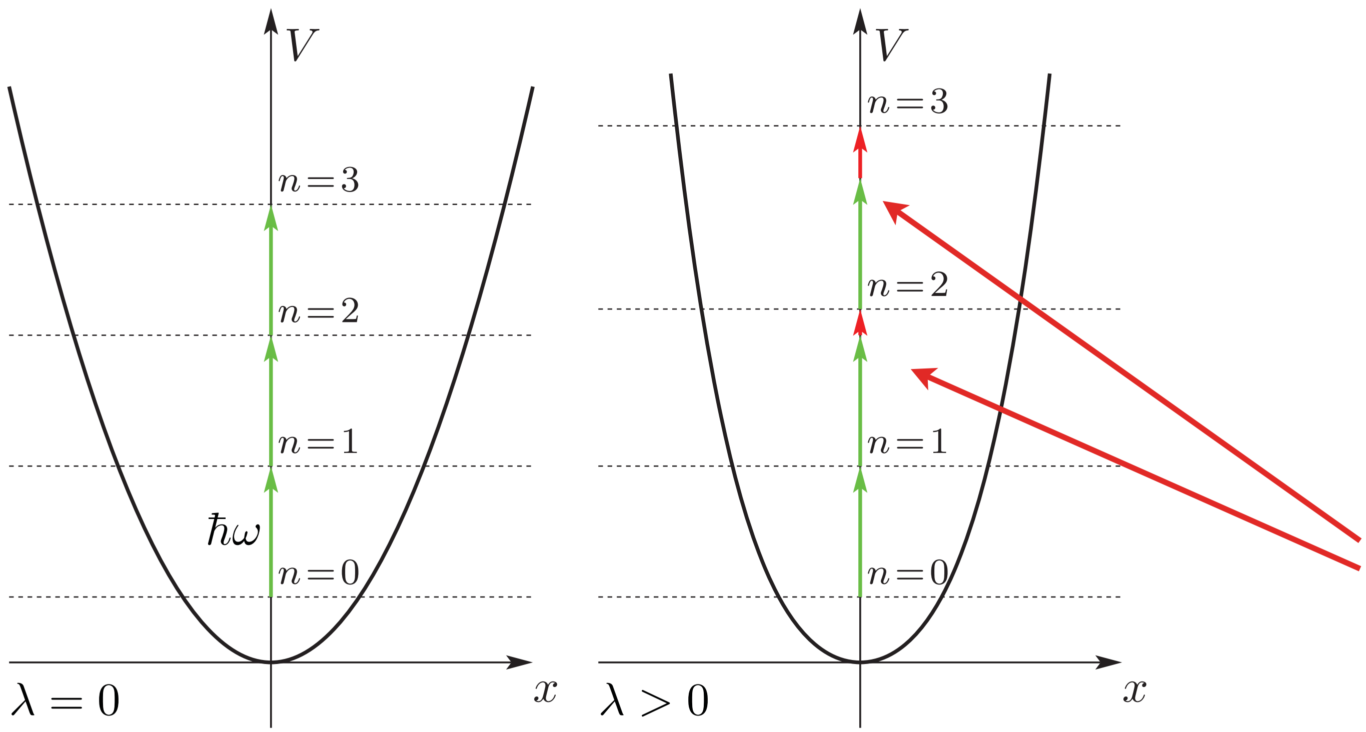
$$X = \sqrt{\frac{\hbar}{2m\omega_m}}\,(b + b^\dagger)$$

$$\tilde{\lambda} X^4 \approx \lambda\, b^\dagger b^\dagger b b$$

$$\lambda \propto \frac{1}{\omega_m^2}$$

Hamiltonian of mechanical oscillator diagonal in phonon Fock basis

$$\omega_n = \omega_m n + \lambda n(n-1)$$



can reach

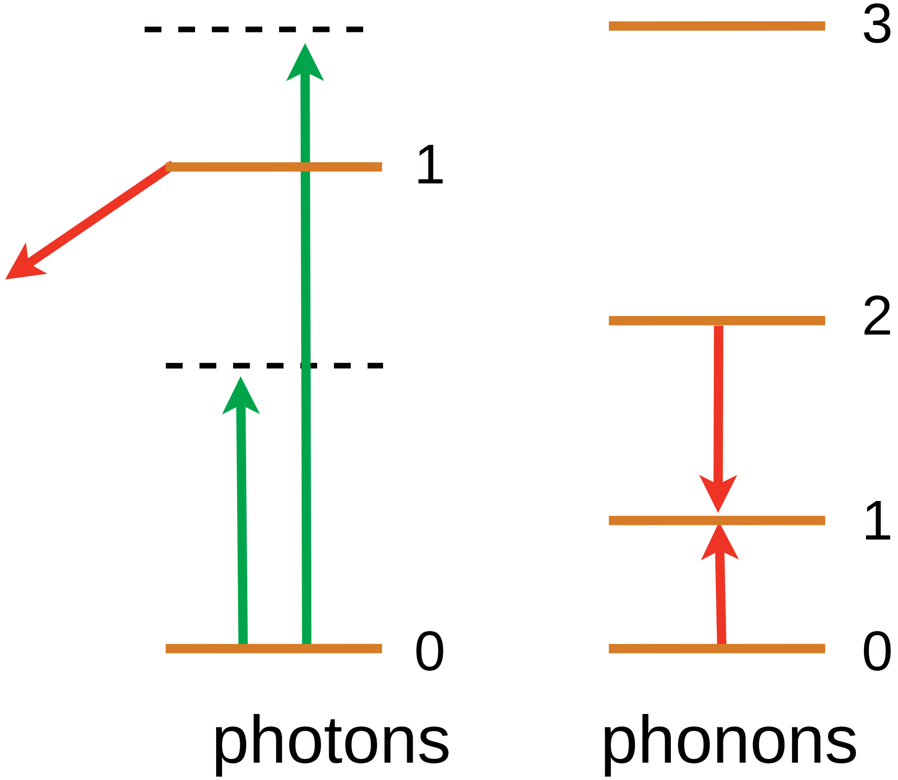
$$\lambda \sim \kappa$$

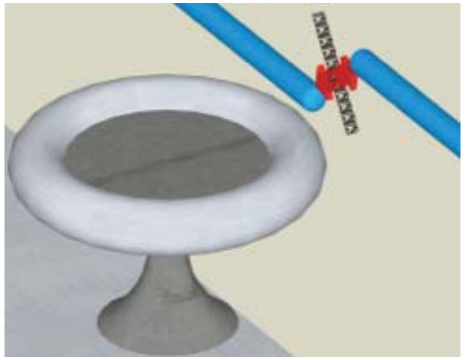
individually addressable by lasers

Driving Split Sidebands

$$E_{m,n} = \omega_m n + \lambda n(n-1)$$

$$\lambda \sim \kappa$$





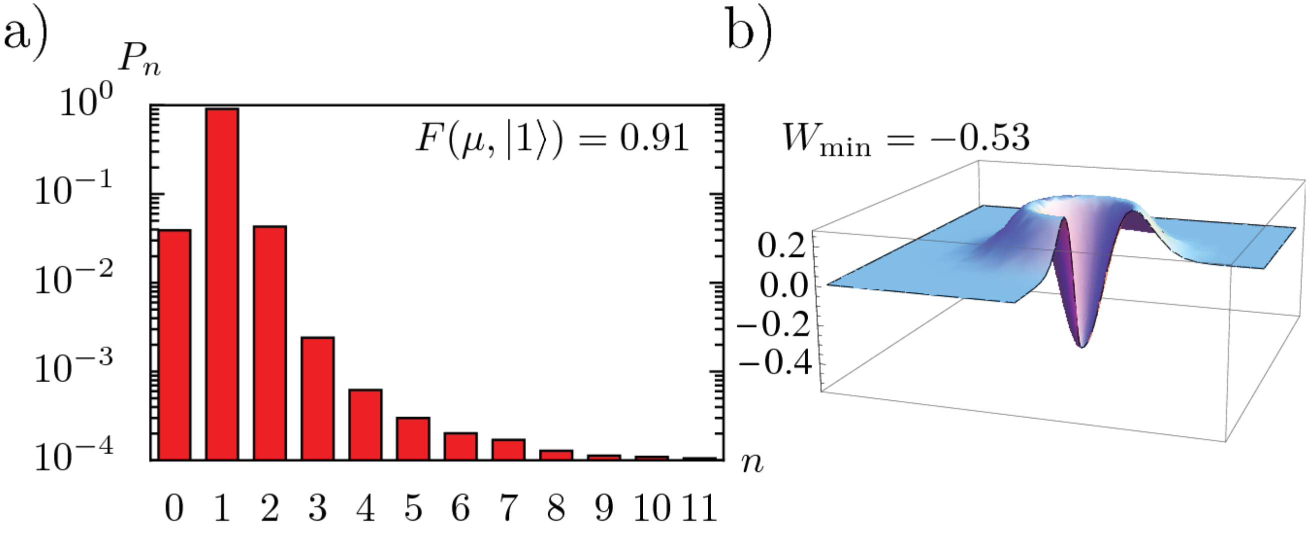
non-classical steady state

$\rightarrow$ approaching phonon Fock states

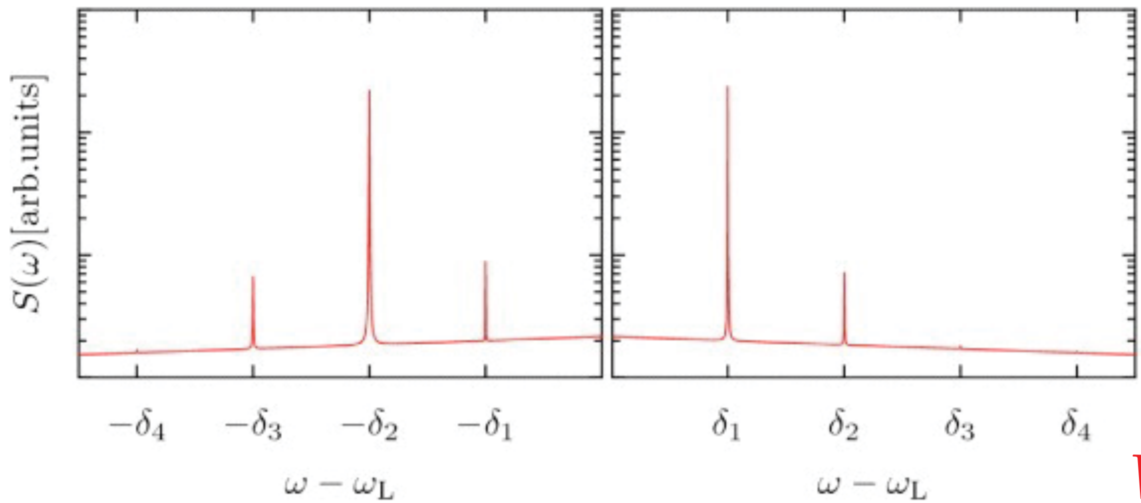
$\rightarrow$ negative Wigner function

S. Rips, M. Kiffner, I. Wilson-Rae and M.J. Hartmann, New J. Phys. **14**, 023042 (2012)

Steady Phonon Fock State



$$
\begin{aligned}
L &= 1\,\mu\mathrm{m} \\
R &= 0.4\,\mathrm{nm} \\
\omega_m &= 2\pi \times 3.45\,\mathrm{MHz} \\
\lambda &= 2\pi \times 209\,\mathrm{kHz} \\
\kappa &= 2\pi \times 53\,\mathrm{kHz} \\
T &= 20\,\mathrm{mK} \\
Q_m &= 5 \times 10^6 \\
g_j &= 2\pi \times 21\,\mathrm{kHz}
\end{aligned}
$$

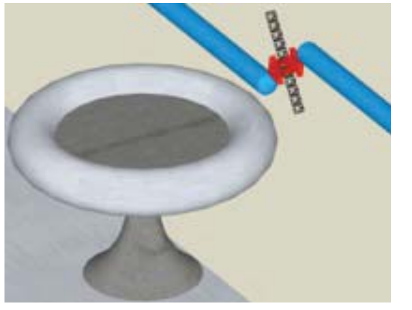


$$\frac{S(\omega_L + \delta_n)}{S(\omega_L - \delta_n)} \approx \frac{P_n}{P_{n-1}}$$

$$W(0,0) = \frac{2}{\pi} \sum_n (-1)^n P_n$$

S. Rips, M. Kiffner, I. Wilson-Rae and M.J. Hartmann, New J. Phys. **14**, 023042 (2012)

Hamiltonian and Damping



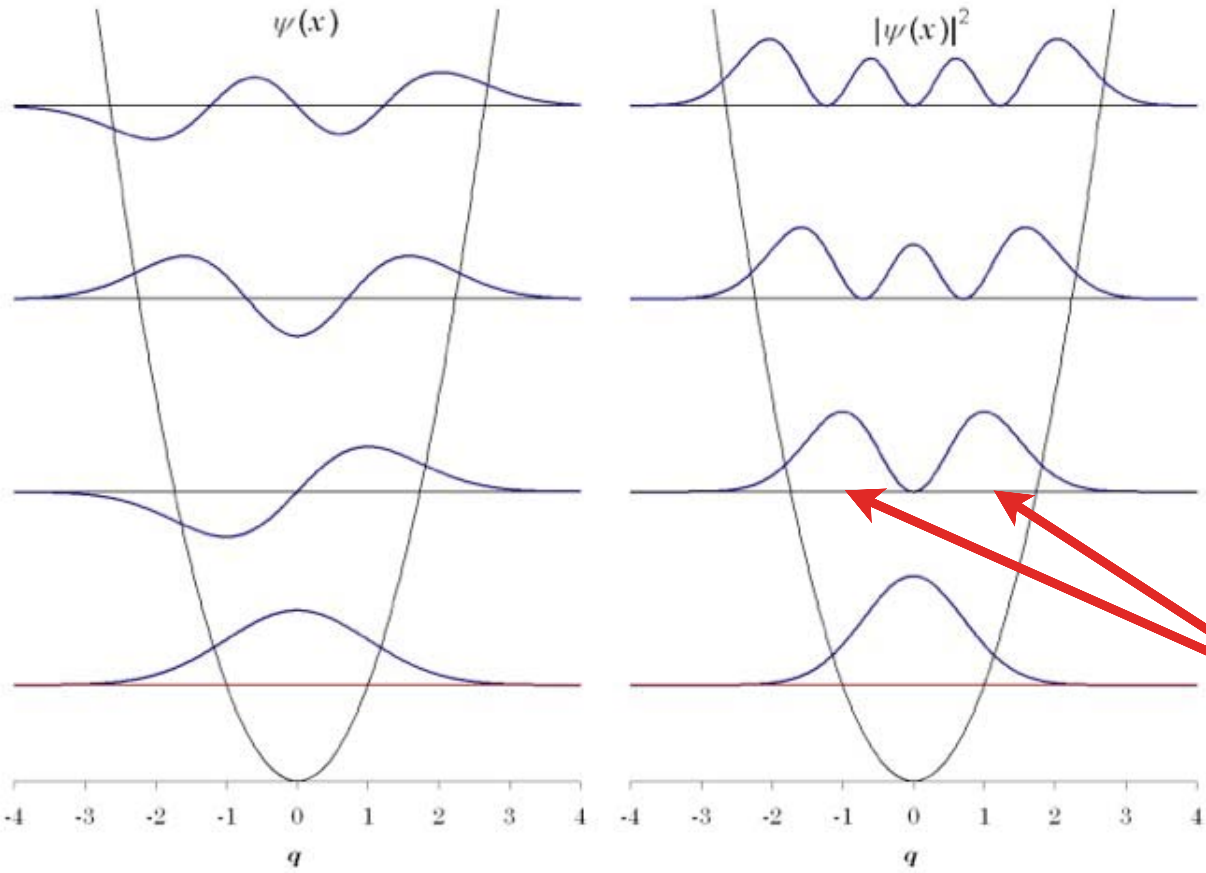
$$\dot{\rho} = -i\,[H, \rho] + \sum_j \frac{\kappa}{2}\mathcal{D}_{\mathrm{c},j}(\rho) + \frac{\gamma_m}{2}\mathcal{D}_{\mathrm{m}}(\rho)$$

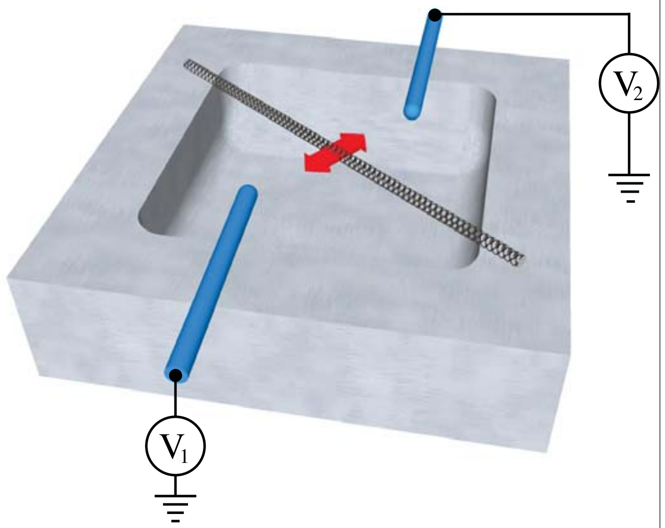
$$\begin{aligned} H &= \sum_j \left[-\Delta_j a_j^\dagger a_j + \left(\frac{\Omega_j^*}{2} a_j + \frac{\Omega_j}{2} a_j^\dagger\right) + G_j a_j^\dagger a_j \left(b^\dagger + b\right)\right] \\ &+ \omega_m b^\dagger b + \lambda b^\dagger b^\dagger b b \end{aligned}$$

$$a_j = \langle a_j \rangle + \delta a_j \qquad G_j a_j^\dagger a_j (b^\dagger + b) \to G_j \langle a_j \rangle^* \, \delta a_j \, b^\dagger + \mathrm{H.c.}$$

photons in cavity field fluctuations decay much faster than they are created $\Rightarrow$ adiabatic elimination of photons in δa_j

Position Representation



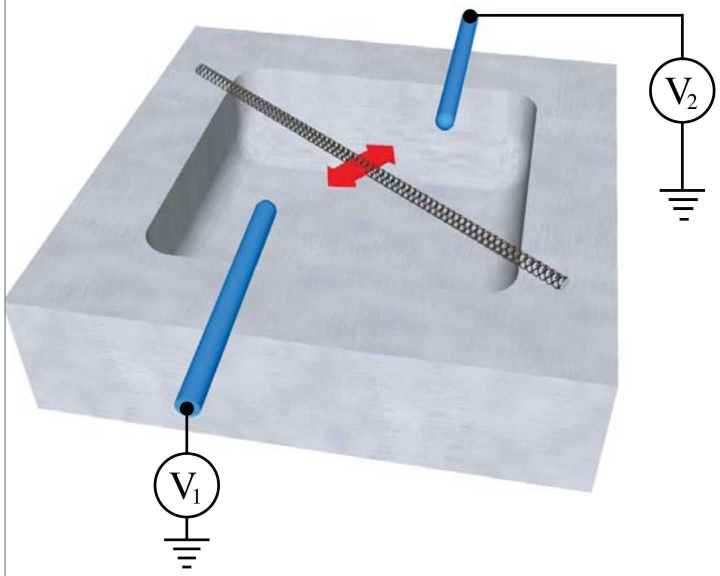


n=1 state: nanotube is in a superposition of two different positions

Outline

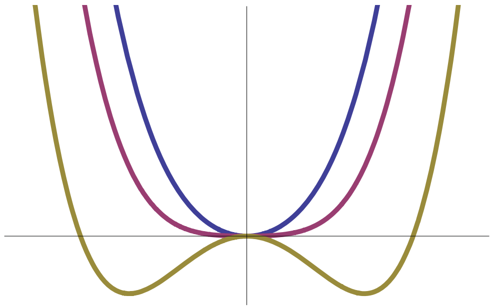
- nonlinear elasticity in thin beams
- phonon Fock states
- nanomechanical qubits

Nanomechanical Qubits



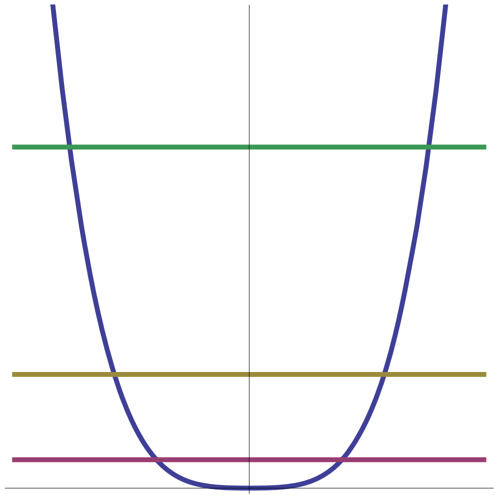
$$V_e = -\frac{\alpha}{2}\int dV\,|E|^2$$
$$\approx V_{e,0}(t) + V_e^{xy}(t)\,X - V_e^z(t)\,X^2$$

V_e^z enhances nonlinearity

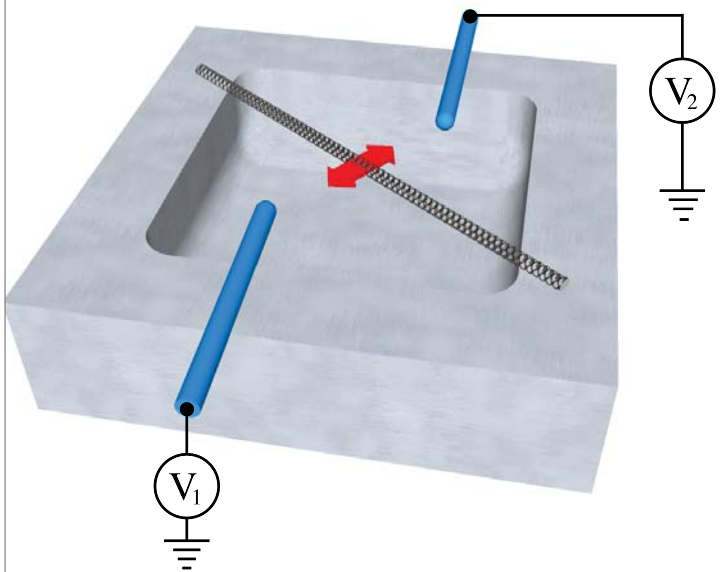


$$H = \frac{P^2}{2m} + \frac{m}{2}\tilde{\omega}_m X^2 + \tilde{\lambda} X^4$$

make so nonlinear that 2nd excited state becomes unreachable $\rightarrow$ **qubit** $\Rrightarrow$



Local Operations

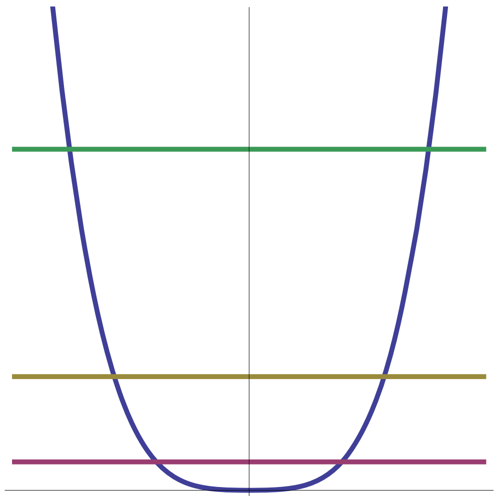


$$V_e = -\frac{\alpha}{2}\int dV\,|E|^2$$
$$\approx V_{e,0}(t) + V_e^{xy}(t)\,X - V_e^z(t)\,X^2$$

$$V_e^z \to V_{e,0}^z + V_{e,1}^z(t)$$

$V_{e,0}^z \to \omega_1 - \omega_0$

$V_{e,1}^z(t) \to \sigma^z$ - rotation



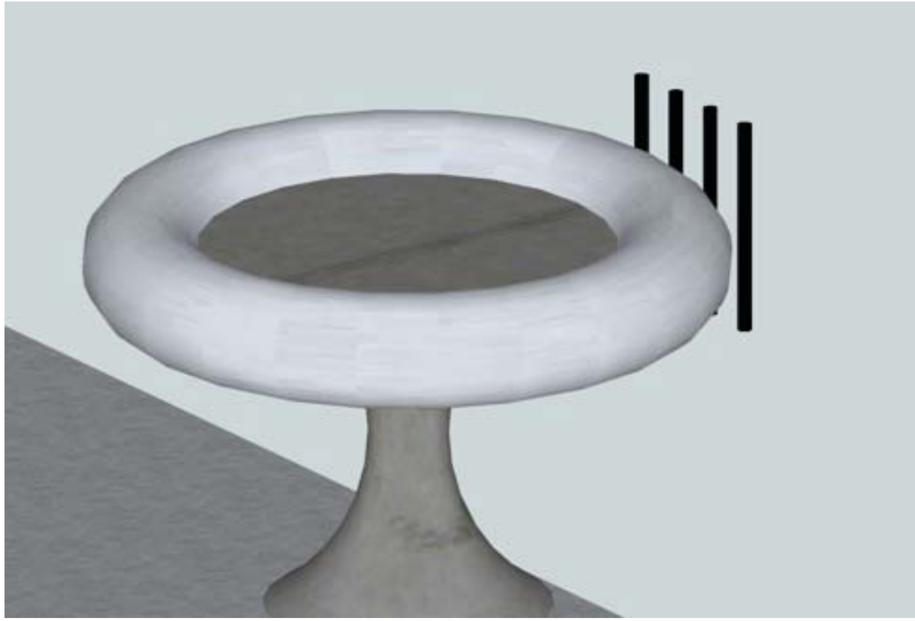
$$V_e^{xy} \propto \cos[(\omega_1 - \omega_0)t + \theta]$$

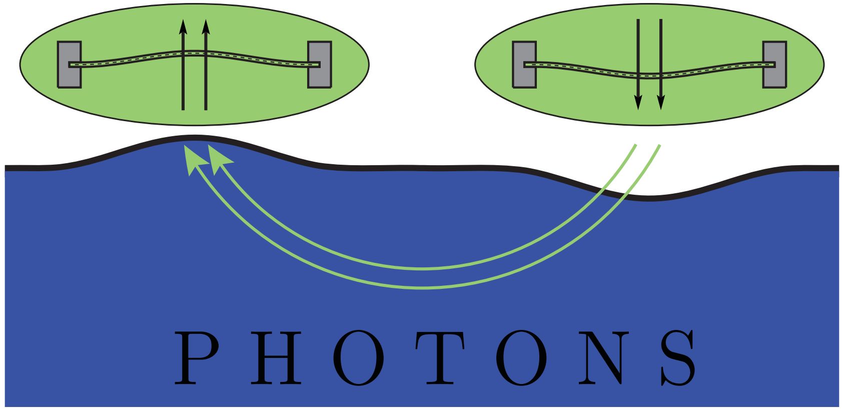
$\downarrow$

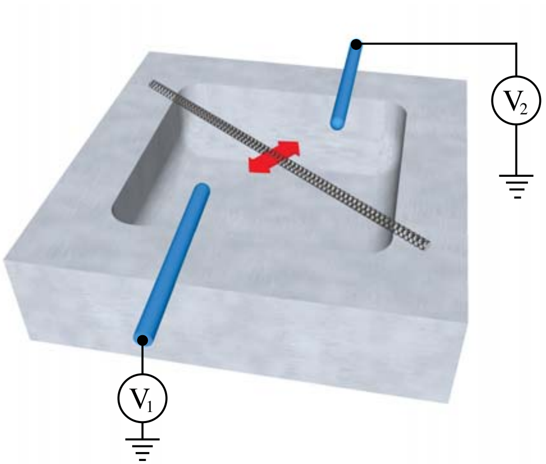
$\cos(\theta)\,\sigma^x + \sin(\theta)\,\sigma^y$ - rotation

$$X \to X_{10}|1\rangle\langle 0| + X_{10}^*|0\rangle\langle 1|$$

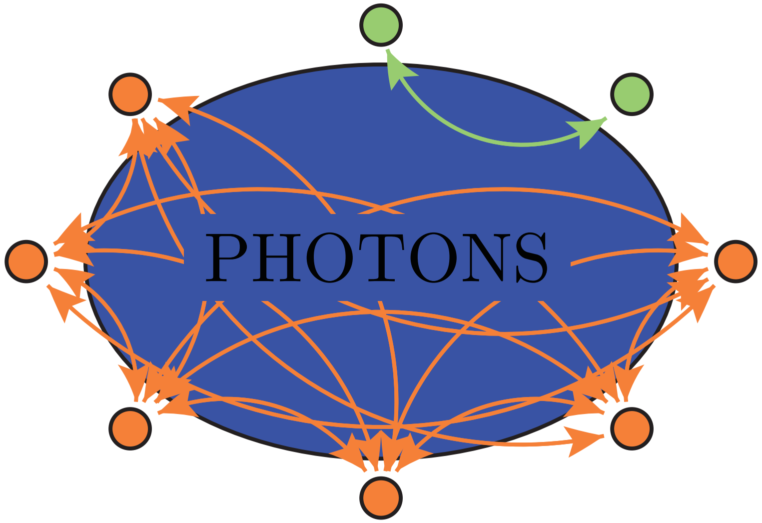
Two-Qubit Gates



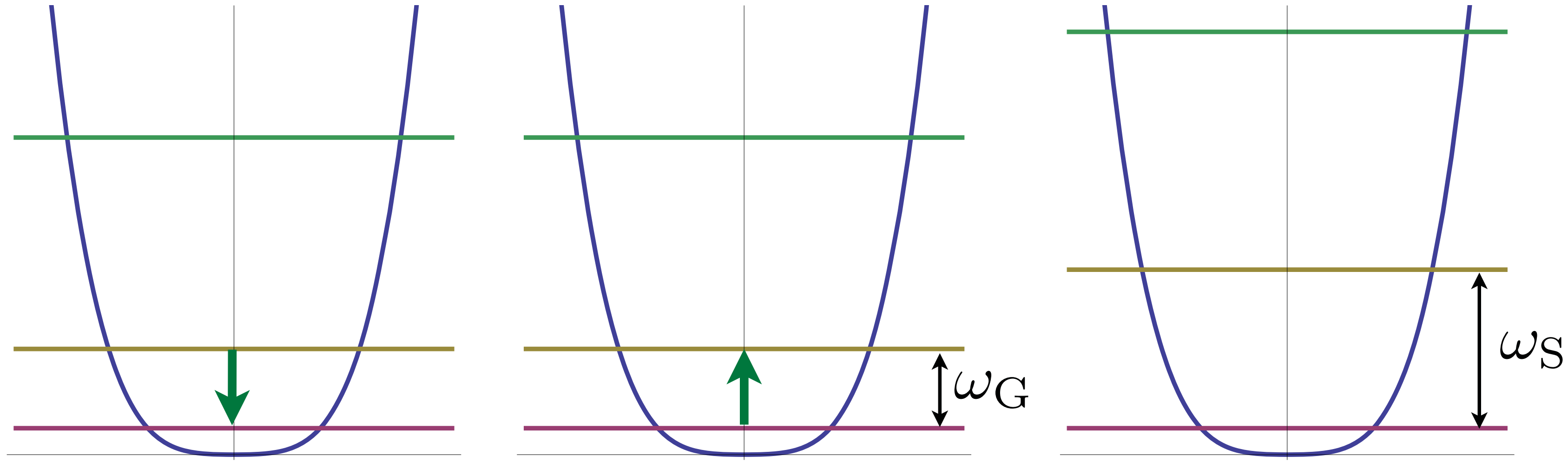




tune each qubit individually to different transition frequency



Two-Qubit Gates



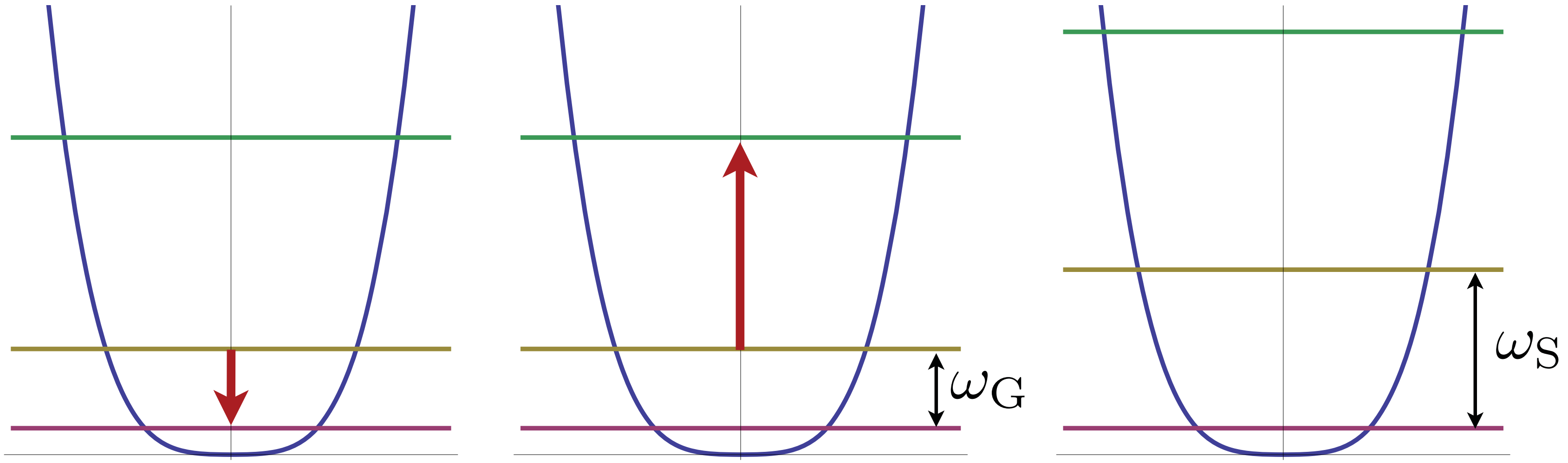
$$H = \Delta a^\dagger a + g\frac{|\alpha|}{\sqrt{2}}\sum_j X_j + gX_c \sum_j X_j$$

$$+ \sum_j \left[\omega_m b_j^\dagger b_j + 2\lambda X_j^4 + V_j^{xy}(t) X_j + V_j^z(t) X_j^2\right]$$

$$X_j = (b_j + b_j^\dagger)/\sqrt{2} \qquad X_c = (a + a^\dagger)/\sqrt{2}$$

$$a \to \alpha + a$$

Two-Qubit Gates



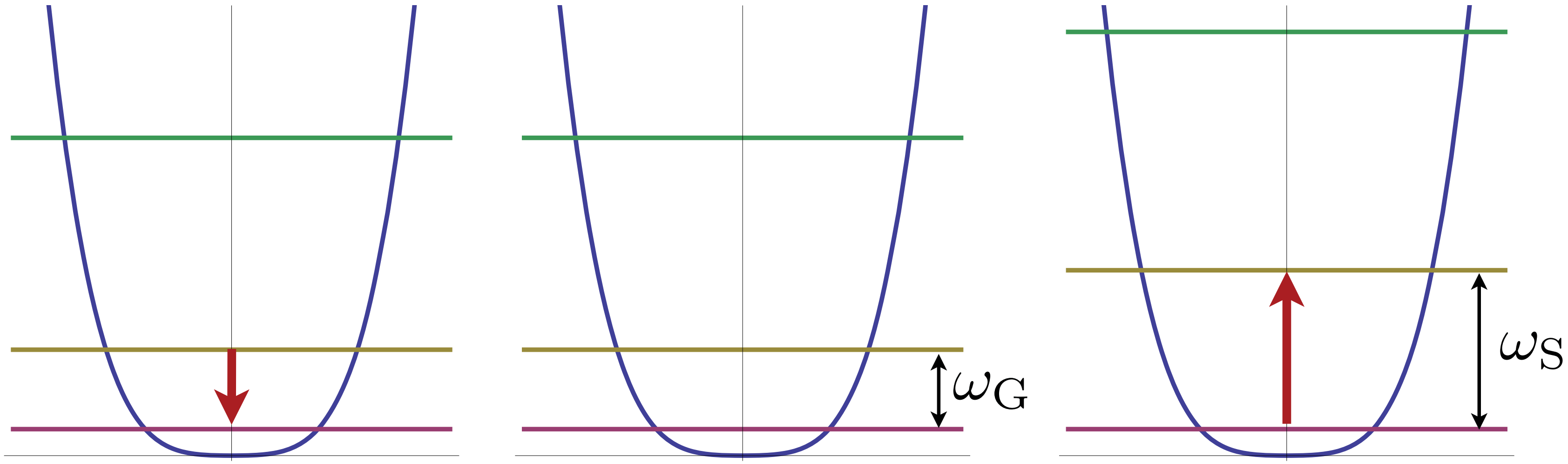
$$H = \Delta a^\dagger a + g\frac{|\alpha|}{\sqrt{2}}\sum_j X_j + g X_{\mathrm{c}} \sum_j X_j$$

$$+ \sum_j \left[\omega_m b_j^\dagger b_j + 2\lambda X_j^4 + V_j^{xy}(t) X_j + V_j^z(t) X_j^2\right]$$

$$X_j = (b_j + b_j^\dagger)/\sqrt{2} \qquad X_{\mathrm{c}} = (a + a^\dagger)/\sqrt{2}$$

$$a \rightarrow \alpha + a$$

Two-Qubit Gates



$$
\begin{aligned}
H &= \Delta a^\dagger a + g\frac{|\alpha|}{\sqrt{2}}\sum_j X_j + gX_{\rm c}\sum_j X_j \\
&+ \sum_j \left[\omega_m b_j^\dagger b_j + 2\lambda X_j^4 + V_j^{xy}(t)X_j + V_j^z(t)X_j^2\right]
\end{aligned}
$$

$$X_j = (b_j + b_j^\dagger)/\sqrt{2} \qquad X_{\rm c} = (a + a^\dagger)/\sqrt{2}$$

$$a \rightarrow \alpha + a$$

Two-Qubit Gates

gate operation only on qubits 1 and 2:

adiabatic elimination of photon mode → $H_{\mathrm{eff}} \approx H_{\mathrm{G}}(t) + H_{\mathrm{S}}$

$\Delta \gg \omega_{\mathrm{G}}, \omega_{\mathrm{S}}$

$$H_{\mathrm{G}}(\Delta) = -\frac{g^2 X_{\mathrm{G}}^2}{\Delta} \left(\sigma_1^{01}\,\sigma_2^{10} + \mathrm{H.c.}\right) - \sum_{i=1}^{2} H_{\mathrm{G},i}$$

$$H_{\mathrm{S}}(\Delta) = -\frac{g^2 X_{\mathrm{S}}^2}{\Delta} \sum_{i \neq j > 2} \left(\sigma_i^{01}\,\sigma_j^{10} + \mathrm{H.c.}\right) - \sum_{i>2} H_{\mathrm{S},i}$$

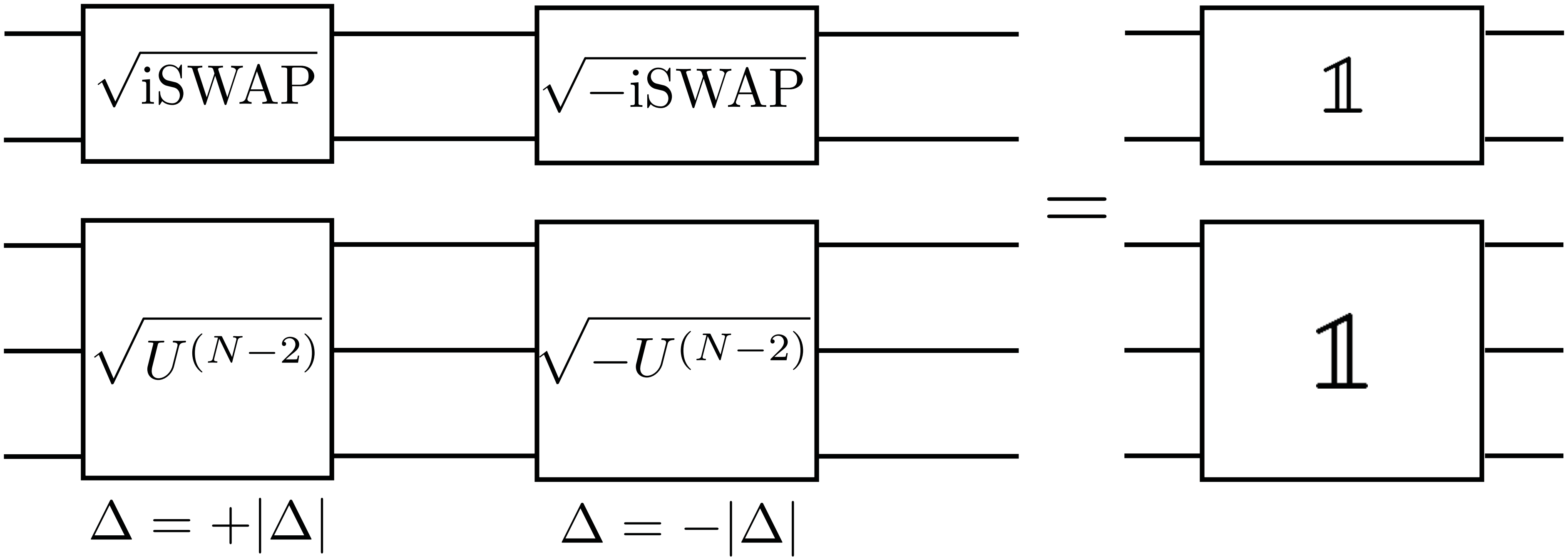
$$H_{\mathrm{G}}(-\Delta) = -H_{\mathrm{G}}(\Delta)$$

$$H_{\mathrm{S}}(-\Delta) = -H_{\mathrm{S}}(\Delta)$$

iSWAP on Two Qubits

$$\text{iSWAP} \equiv \begin{bmatrix} 1 & & & \\ & 0 & i & \\ & i & 0 & \\ & & & 1 \end{bmatrix}$$

$$H_{\text{eff}} \approx H_{\text{G}} + H_{\text{S}}$$
$$H_{\text{G}}(-\Delta) = -H_{\text{G}}(\Delta)$$
$$H_{\text{S}}(-\Delta) = -H_{\text{S}}(\Delta)$$



iSWAP on Two Qubits

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$$H_{\mathrm{eff}} \approx H_{\mathrm{G}} + H_{\mathrm{S}}$$
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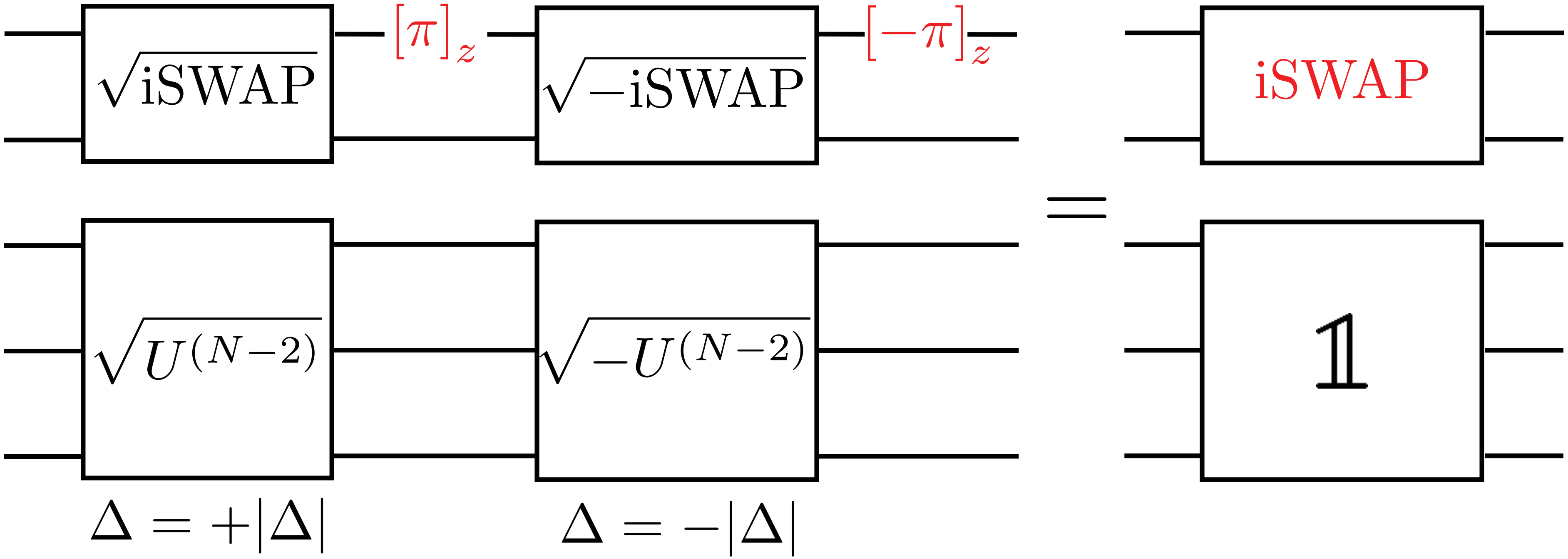


Gate Errors

$$H = \Delta a^\dagger a + g(a + a^\dagger) \sum_j X_j + \sum_j \left[\omega_m b_j^\dagger b_j + 2\lambda X_j^4 + V_j^{xy}(t) X_j + V_j^z(t) X_j^2 \right]$$

$$X_j = \frac{b_j + b_j^\dagger}{\sqrt{2}}$$

$$a \longrightarrow \langle a \rangle + a$$

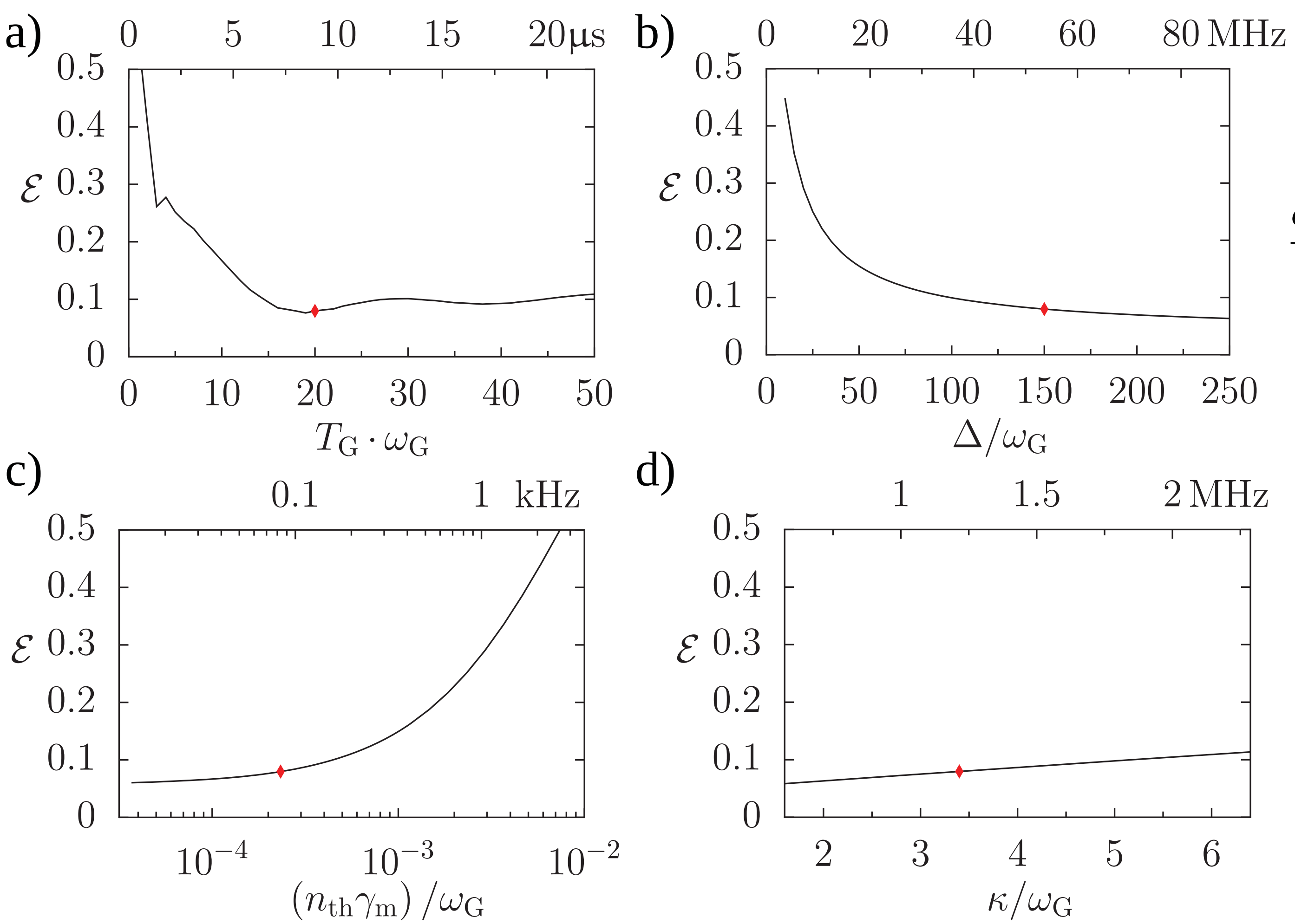
$$\dot{\varrho} = -i\,[H, \varrho] + \frac{\gamma_m}{2} \sum_j [\overline{n}\,\mathcal{D}_{\uparrow,j}(b_j) + (\overline{n}+1)\mathcal{D}_{\downarrow,j}(b_j)] + \frac{\kappa}{2}\mathcal{D}_{\downarrow,c}(a)$$

gate error:

initial states $|\phi\rangle = \frac{|j, k, \ldots\rangle + |0, 0, \ldots\rangle}{\sqrt{2}}$ $\quad j, k = 0, 1$

$$f = \mathrm{Tr}\sqrt{\sqrt{\varrho}\,\sigma\sqrt{\varrho}} \qquad \mathcal{E} = 1 - \overline{f}\Big|_{\mathrm{all}\,|\phi\rangle}$$

Gate Performance



$$
\begin{aligned}
L &= 3\,\mu\text{m} \\
R &= 0.4\,\text{nm} \\
\frac{\omega_G}{2\pi} &= 357\,\text{kHz} \\
\frac{\delta_{21}-\delta_{10}}{2\pi} &= 109\,\text{kHz} \\
\frac{g}{2\pi} &= 2.16\,\text{MHz} \\
P_{in} &= 1.1\,\text{W} \\
\frac{\Delta}{2\pi} &= 53.6\,\text{MHz} \\
T_G &= 8.9\,\mu\text{s} \\
T &= 20\,\text{mK} \\
Q &= 5\cdot 10^6 \\
\frac{\kappa}{2\pi} &= 1.22\text{MHz} \\
E_e &= 76\,\text{V}/\mu\text{m}
\end{aligned}
$$

gate fidelity > 92%

S. Rips and M.J. Hartmann, Phys. Rev. Lett. **110**, 120503 (2013)

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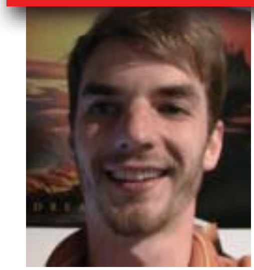

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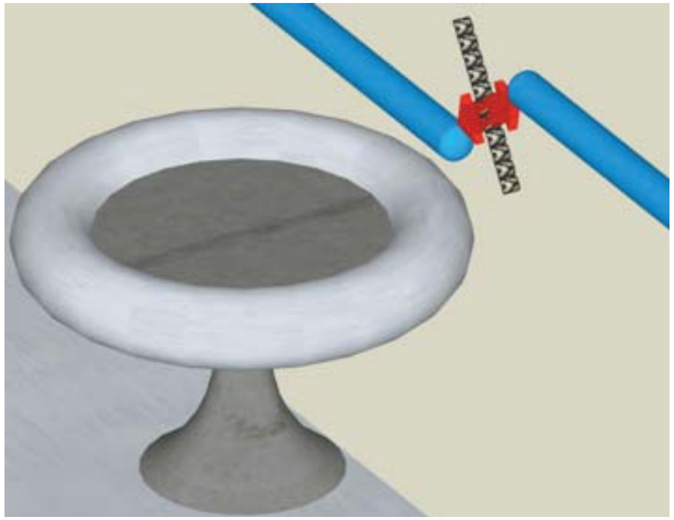
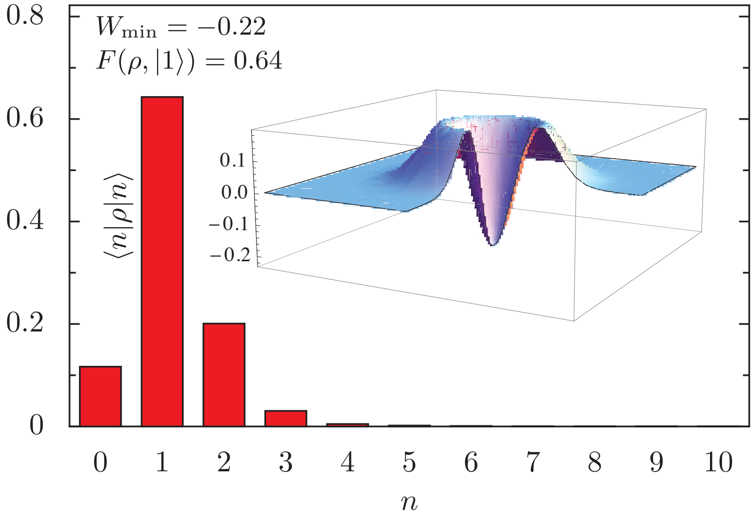
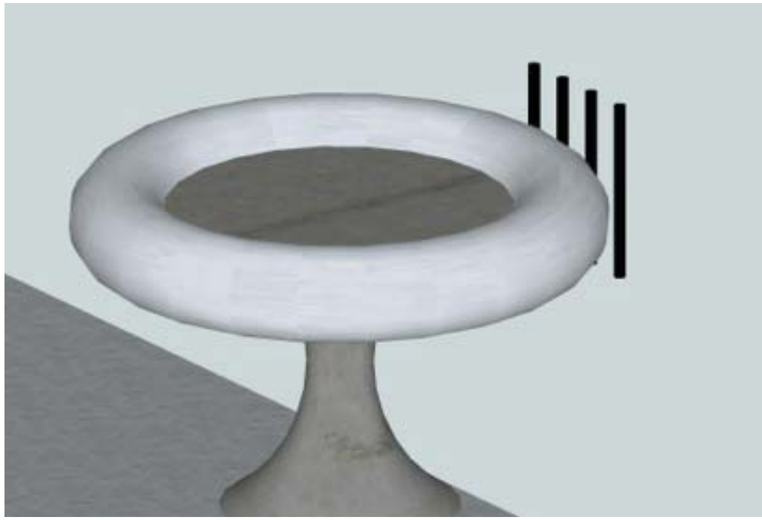
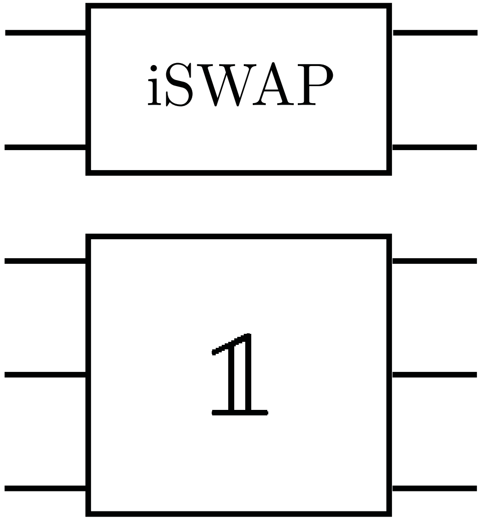
Thank you for listening.









New J. Phys. **14**, 023042

arXiv:1206.0147

Phys. Rev. Lett. **110**, 120503