2445-01

Advanced Workshop on Nanomechanics

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Coherent manipulation and phonon lasing in electromechanical resonators

Hiroshi Yamaguchi
NTT Basic Research Laboratories


Coherent manipulation and phonon lasing in electromechanical resonators
H. Yamaguchi, I. Mahboob, and H. Okamoto
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1. **Introduction**

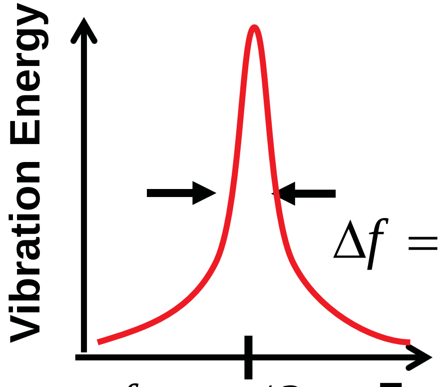
2. **Parametric mechanical resonators**
   - GaAs-based piezoelectric resonators

3. **Two-modes coupling**
   - Stress-driven inter-modal coupling
   - Strong-coupling and Electromechanically-induced transparency

4. **Three-modes coupling and all-mechanical phonon lasing**
   - Basic concepts of phonon lasing
   - Noise-excited sharp mechanical oscillation and threshold properties

5. **Coherent phonon manipulation in paired mechanical resonators**
   - Intermodal parametric coupling of two paired resonators
   - Coherent control of mechanical oscillation
   - Higher order parametric mixing

Micro/Nanomechanical Resonators

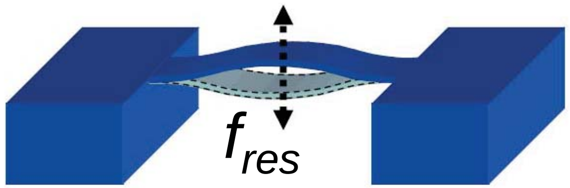


$$\Delta f = \frac{f_0}{Q}$$

$$f_0 = \omega_0 / 2\pi$$

f_0 : 100Hz -1GHz, Q: 10^3 -10^6

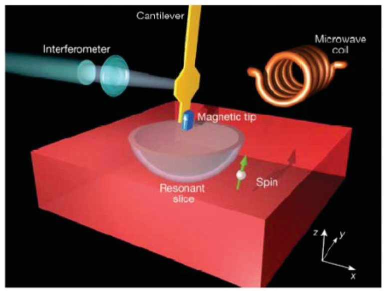
Equation of motion



Beam resonator

$$m\frac{d^2x}{dt^2} + \frac{m\omega_0}{Q}\frac{dx}{dt} + m\omega_0{}^2 x = F\cos\omega t$$

$$x(t) = \frac{F}{\sqrt{(\omega_0{}^2 - \omega^2)^2 + \omega_0{}^2\omega^2/Q^2}}\cos(\omega t - \theta)$$



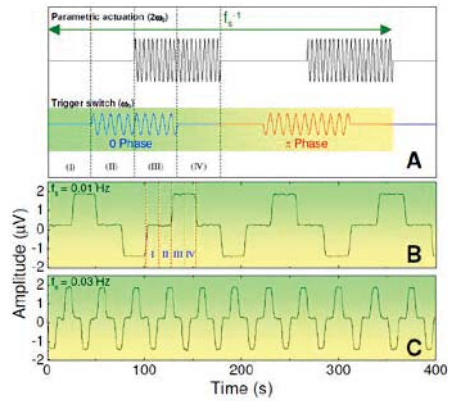
D. Rugar et al. Nature

Ultrasensitive Force/Mass Detection

- *Displacement detection up to femtometer scale (UCSB)*
- *Zeptogram mass sensing (Caltech)*
- *Single spin sensing (IBM)*

Signal porcessing

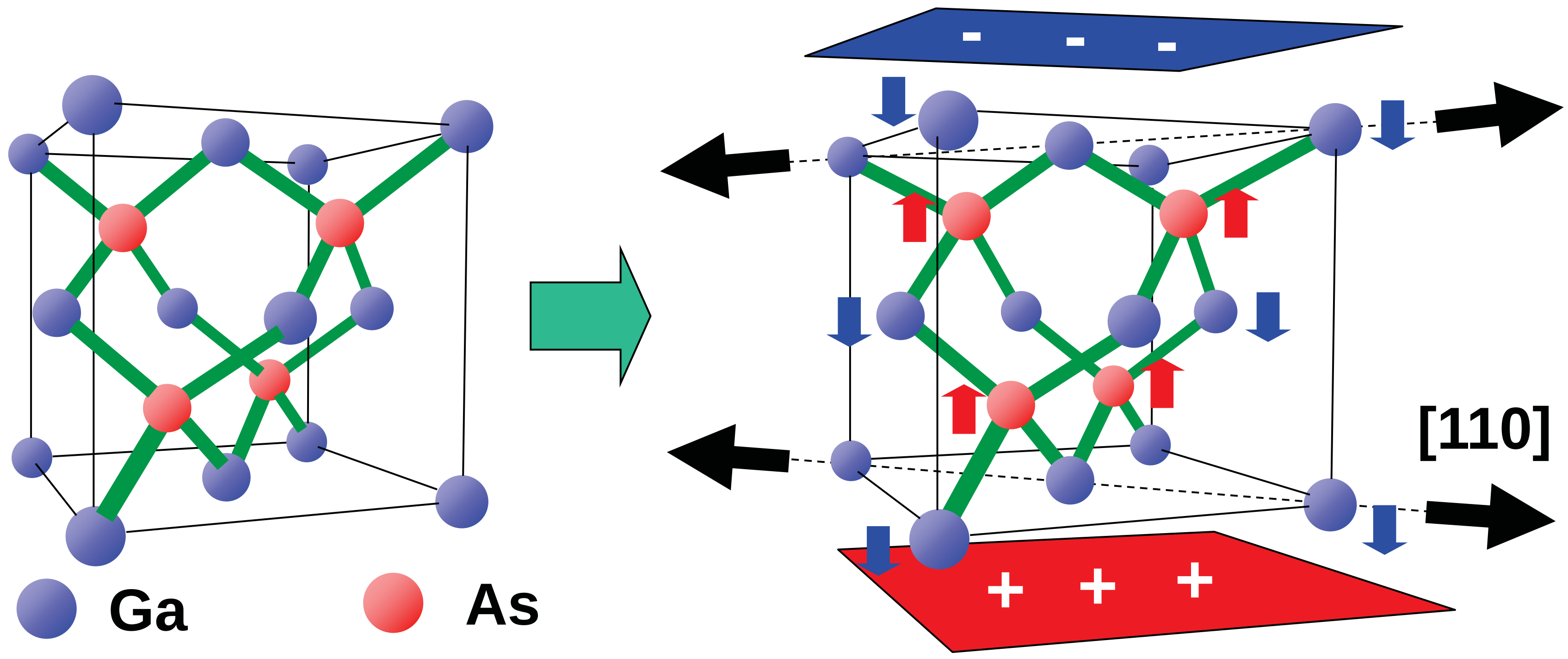
- *Bistability in nonlinear Duffing resonators (Boston, APL 2004)*
- *Mechanical Parametron Computers (NTT, Nature Nano. 2008)*



Piezoelectric effect in Zinc Blende structure

NTT

Stress-voltage transduction



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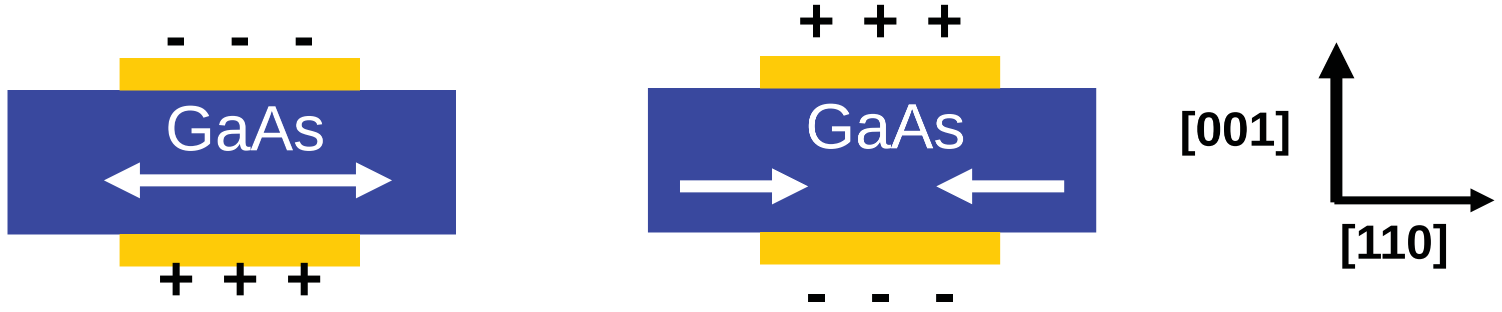
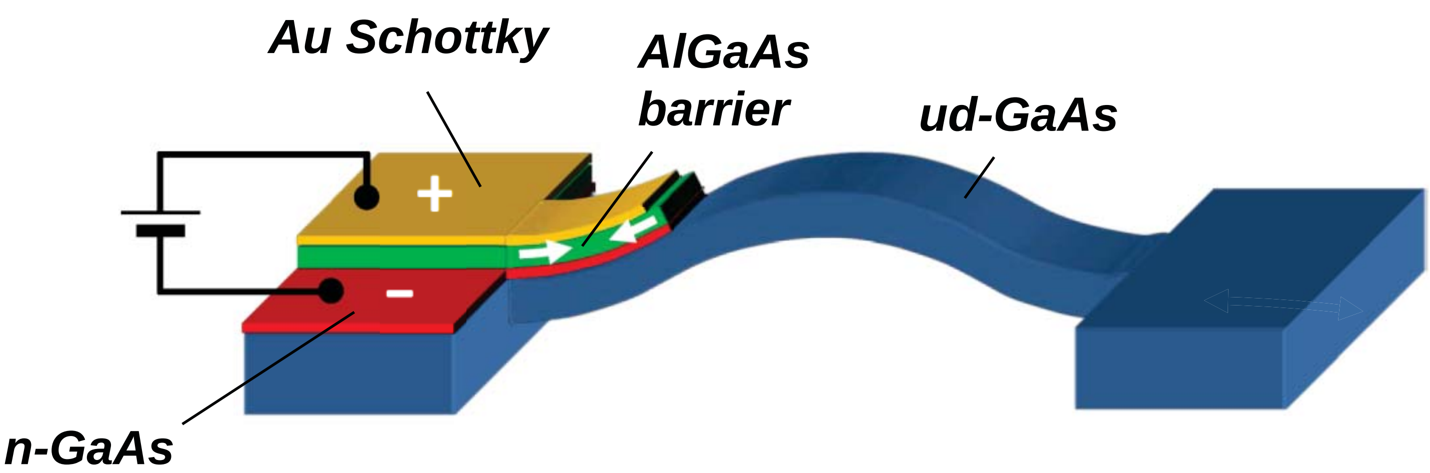


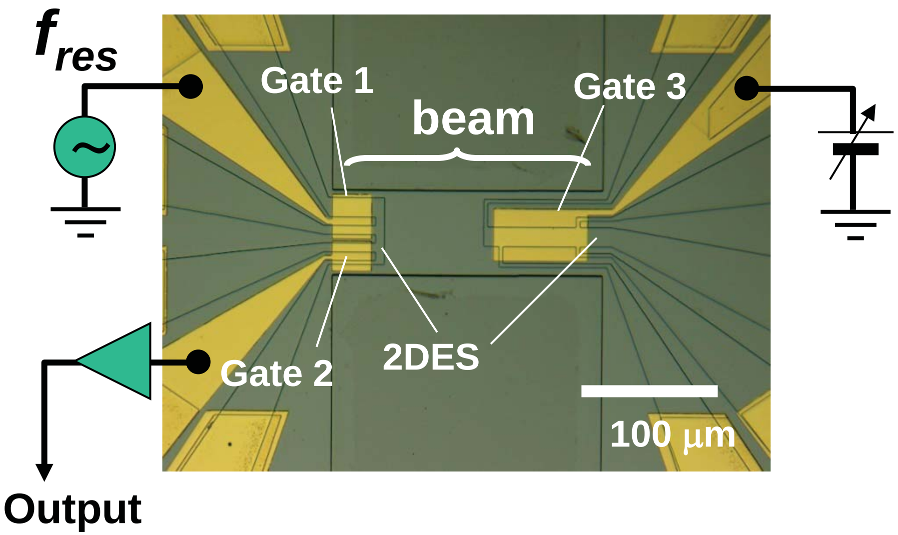


Electrical actuation, detection and frequency control

S.C. Masmanidis, et al. Science 317, 780 (2007)
I. Mahboob and H. Y., Nature Nanotechnology 3, 275 (2008)

Piezoelectric mechanical resonator

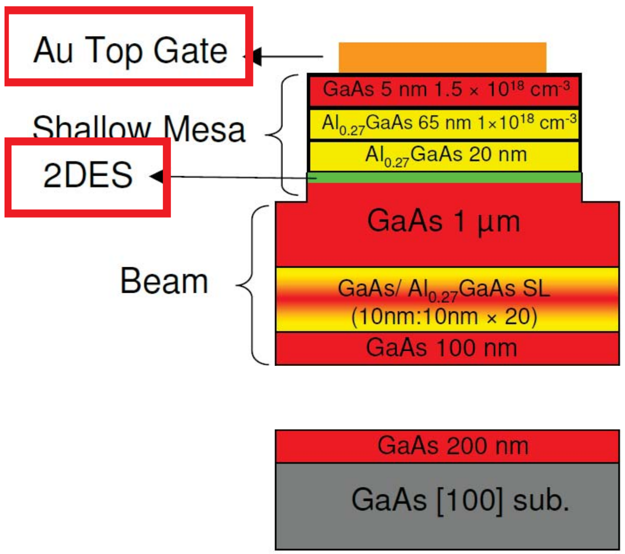
Fabricated device (top view)

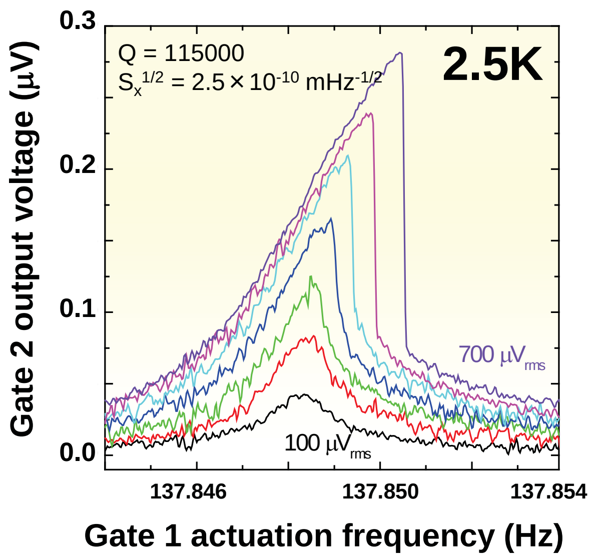


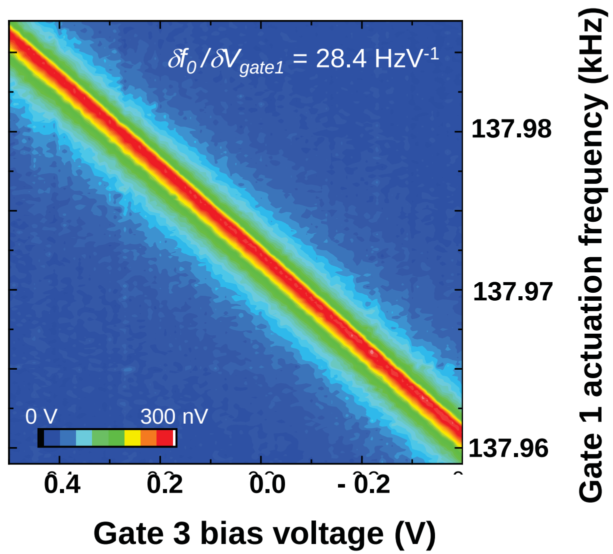
Gate 1: Application of AC voltage
→ Actuation through bending moment

Gate 2: Measurement of generated voltage
→ Beam-motion detection

Gate 3: Application of DC voltage
→ Resonance frequency modulation







I. Mahboob and H. Y., Nature Nanotechnology 3, 275 (2008)

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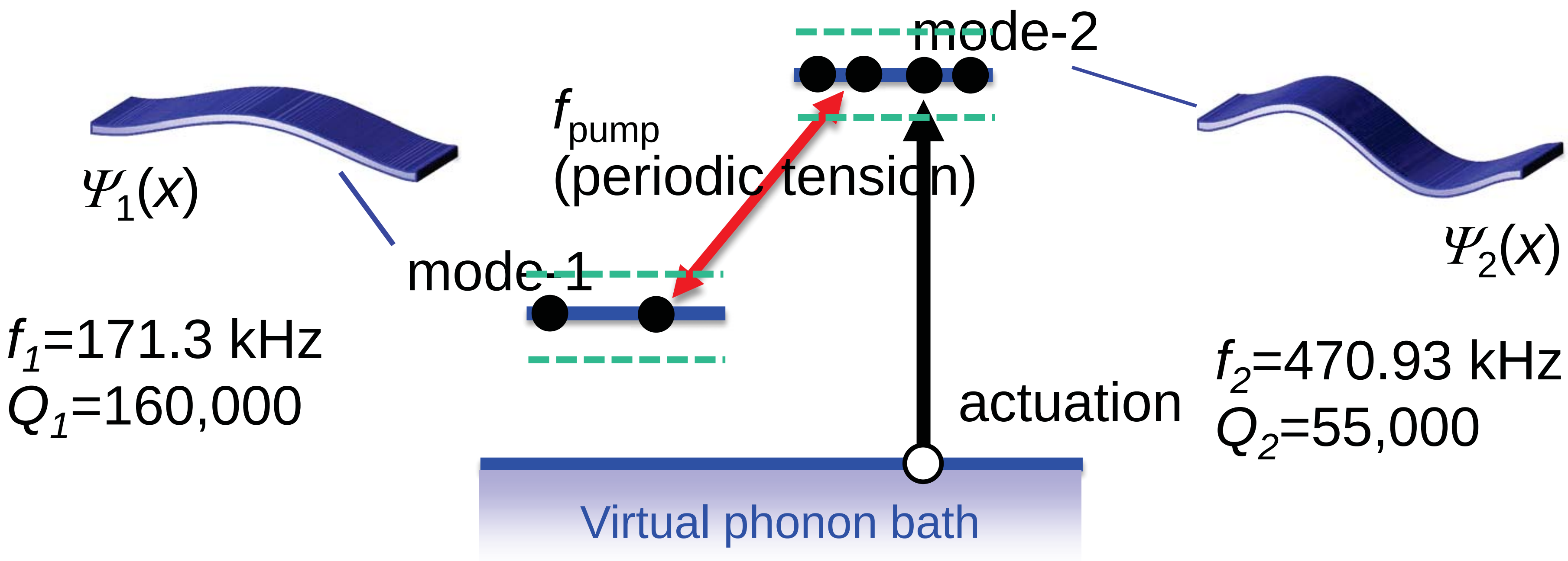
Mechanical two-mode system



piezoelectric tension

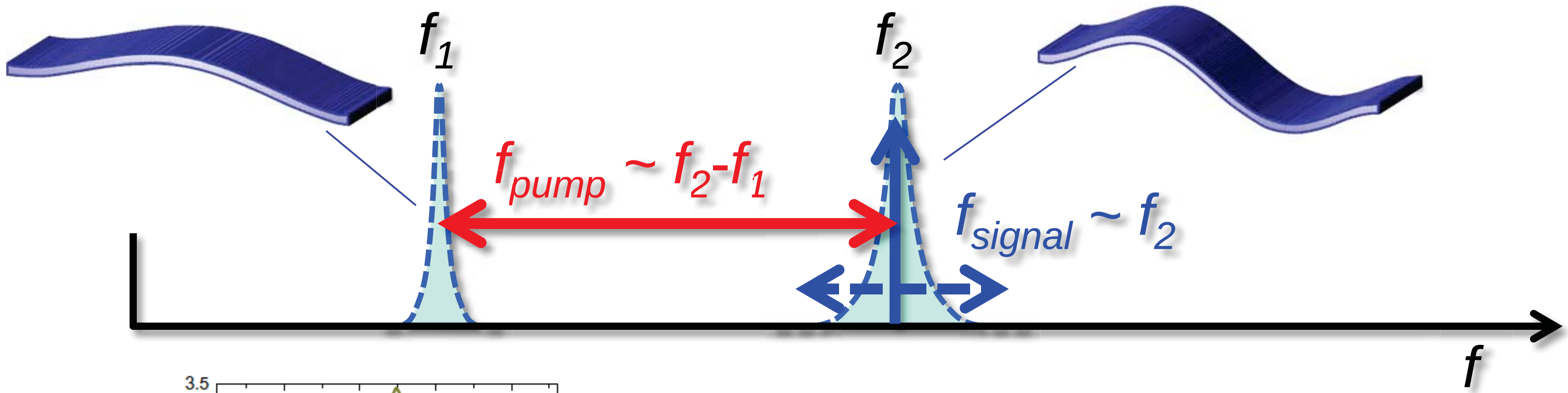
$$\Psi_1(x) \longrightarrow \Psi'_1(x) = a\Psi_1(x) + \boxed{b}\,\Psi_2(x) \quad (+\text{ higher modes})$$

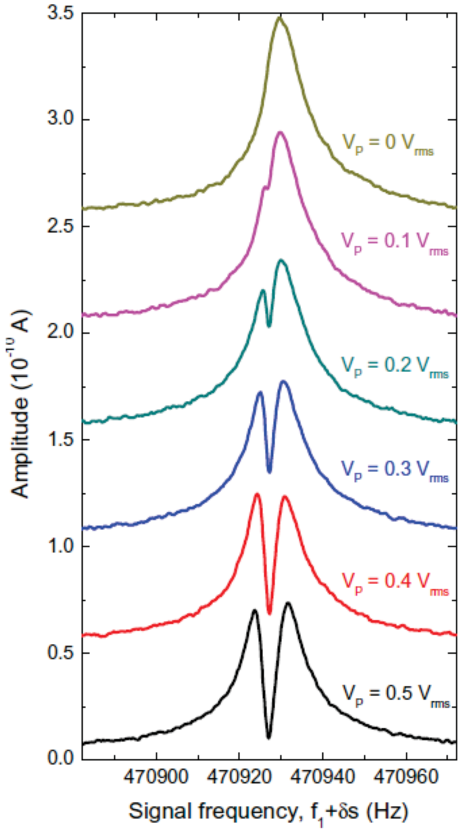
$$\Psi_2(x) \longrightarrow \Psi'_2(x) = \boxed{c}\,\Psi_1(x) + d\Psi_2(x) \quad (+\text{ higher modes})$$

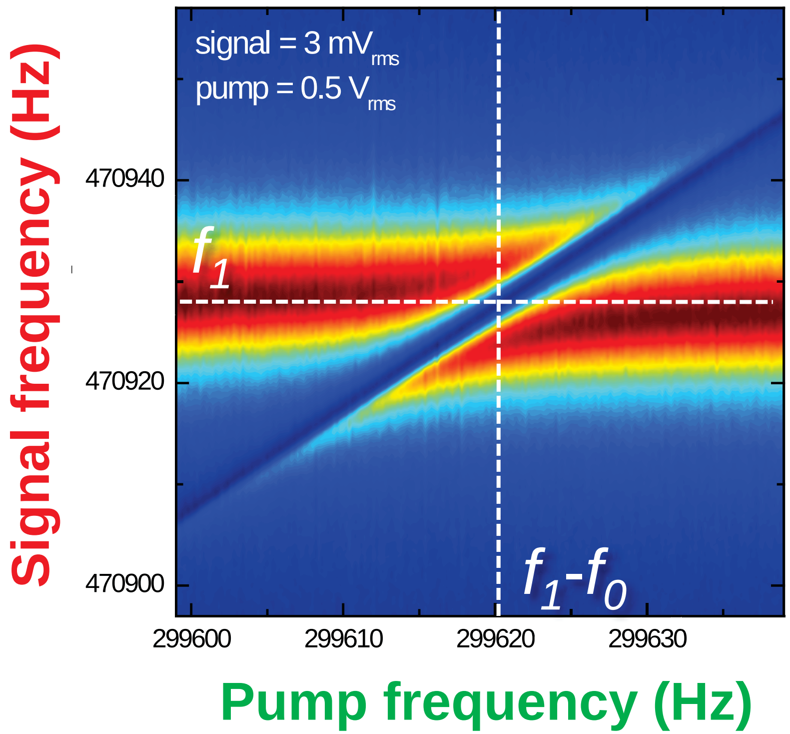
Tension-induced intermodal mixing

T. Dunn, J.-S. Wenzler, and P. Mohanty APL. 97, 123109 (2010)
H. J. R. Westra, M. Poot, H.S.J. van der Zant, W. J. Venstra, PRL 105, 117205 (2010)
H. Yamaguchi and I. Mahboob, New J. Phys. 15, 015023 (2013)

Dynamic inter-mode coupling







I. Mahboob, et al. Nature Phys. 8 387 (2012).

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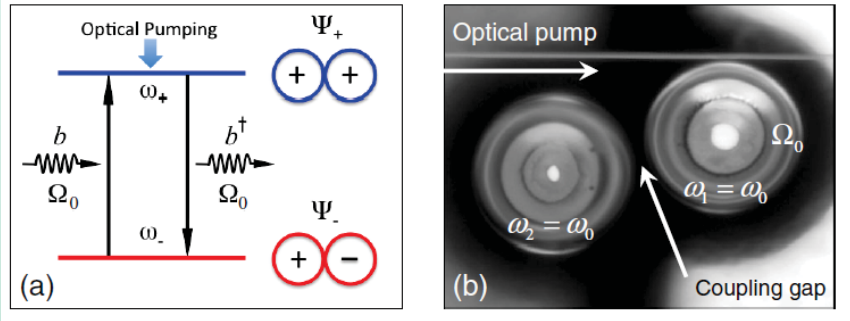
Phonon lasing operation

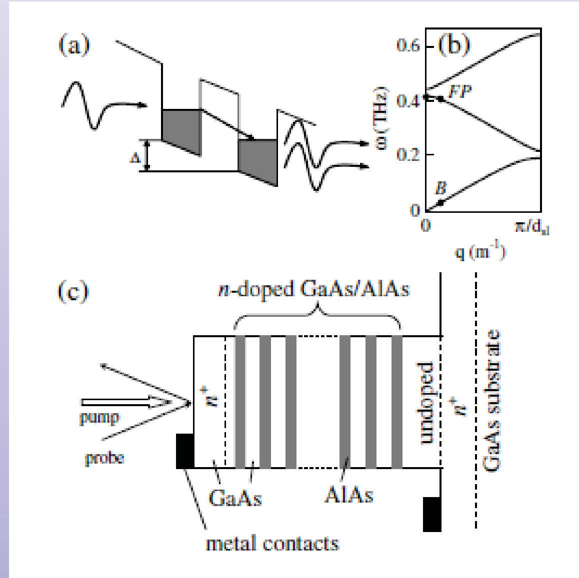
Optomechanical systems



Ivan S. Grudinin et al. PRL2010

- ***Photon transition between two optical modes induce the stimulated emission of phonon.***
- ***Mechanical oscillation at 23.4 MHz was induced by optical pump.***
- ***Threshold characteristics was confirmed.***

Stark Ladder Superlattices



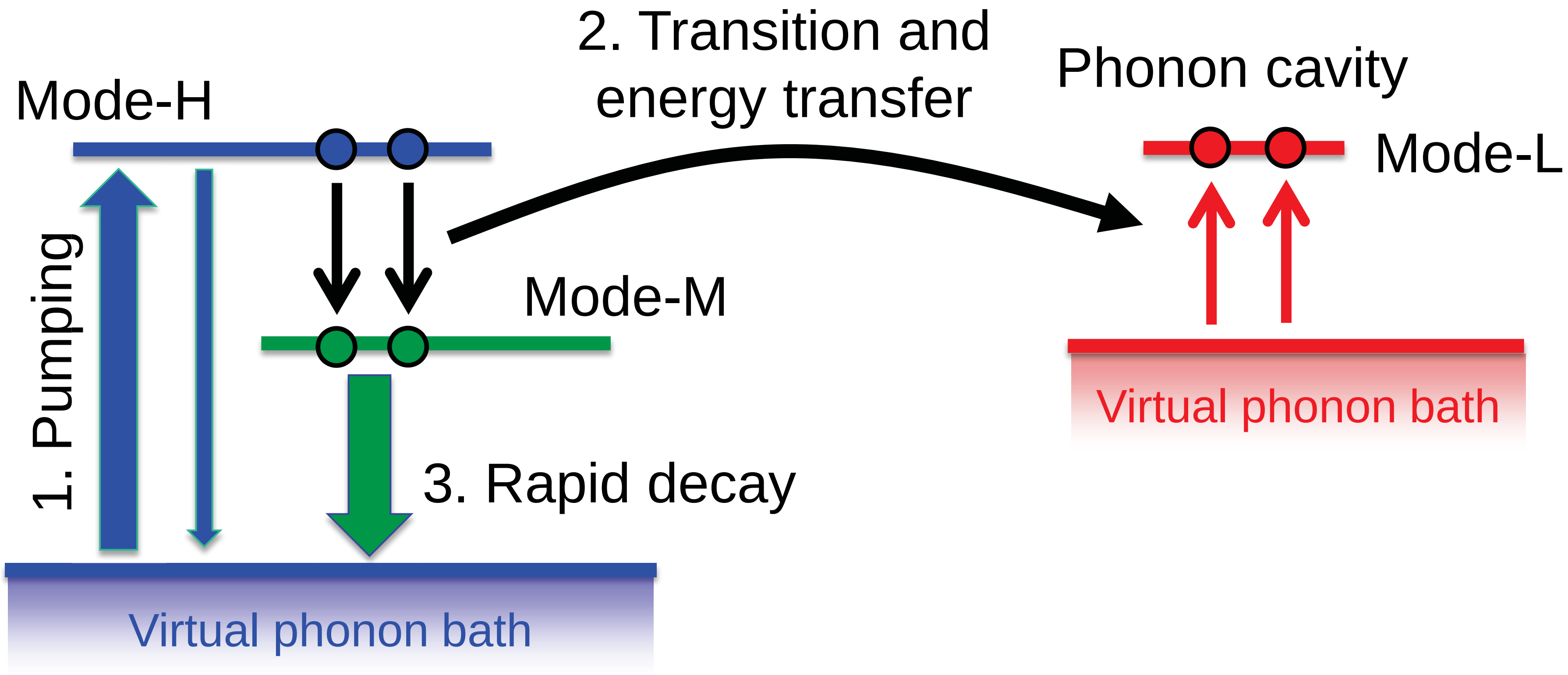
R. P. Beardsley et al. PRL2010

- ***Phonon generation by the transition of electron states.***
- ***Amplification for 0.4 THz lattice vibration was confirmed.***

Two-level systems coupled with the frequency-matched phonon cavity are required for the phonon lasing operation.

Phonon/Sound amplification by stimulated emission of raddiation (PHSER/SASER)

Basic processes for "phonon lasing" operation



Requirements:

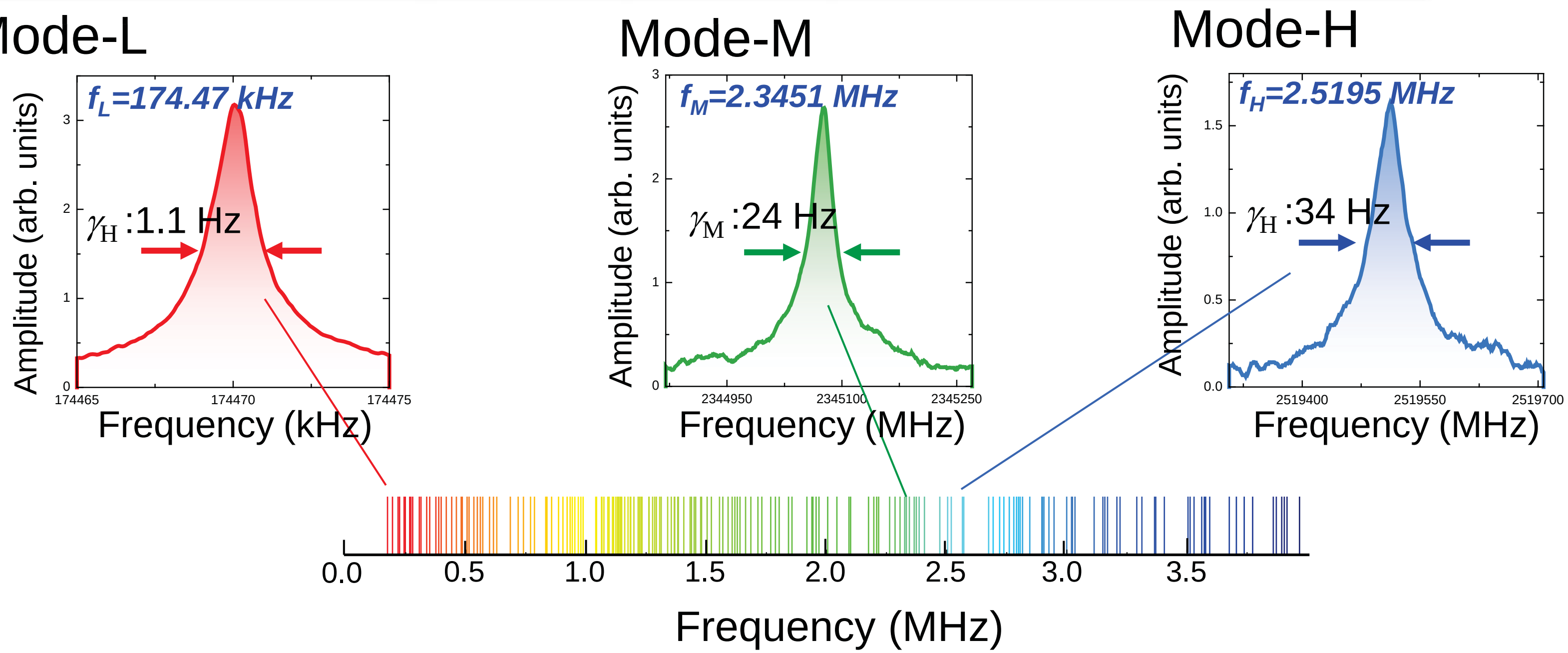
1. Two level systems (can be both fermionic and bosonic)
   → **Two mechanical modes (mode-H and –M)**
2. Energy-matched phonon cavity
   → **One mechanical mode (Mode-L)**
3. Coupling between two level systems and cavity phonon
   → **Parametric coupling**

Three mechanical modes (items 1–2)

Frequency-matched mechanical 3-modes systems

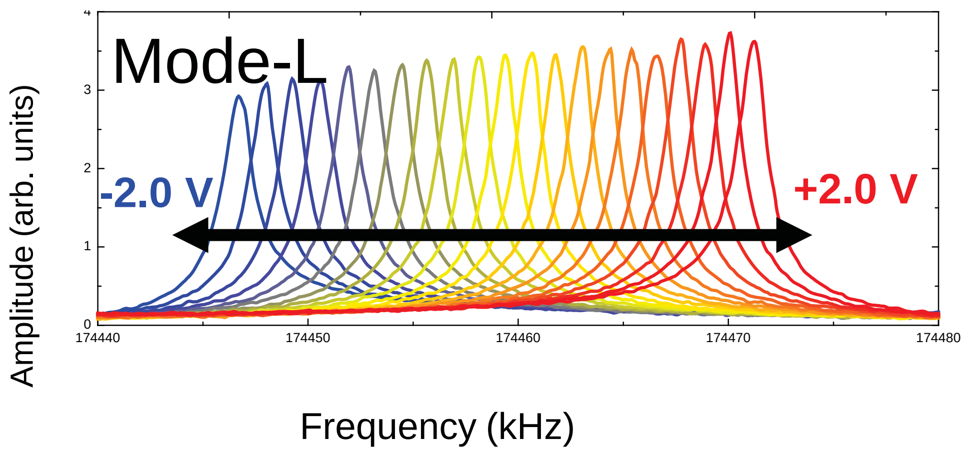
NTT

1. Choosing three modes with frequency matching: $f_H - f_M \sim f_L$

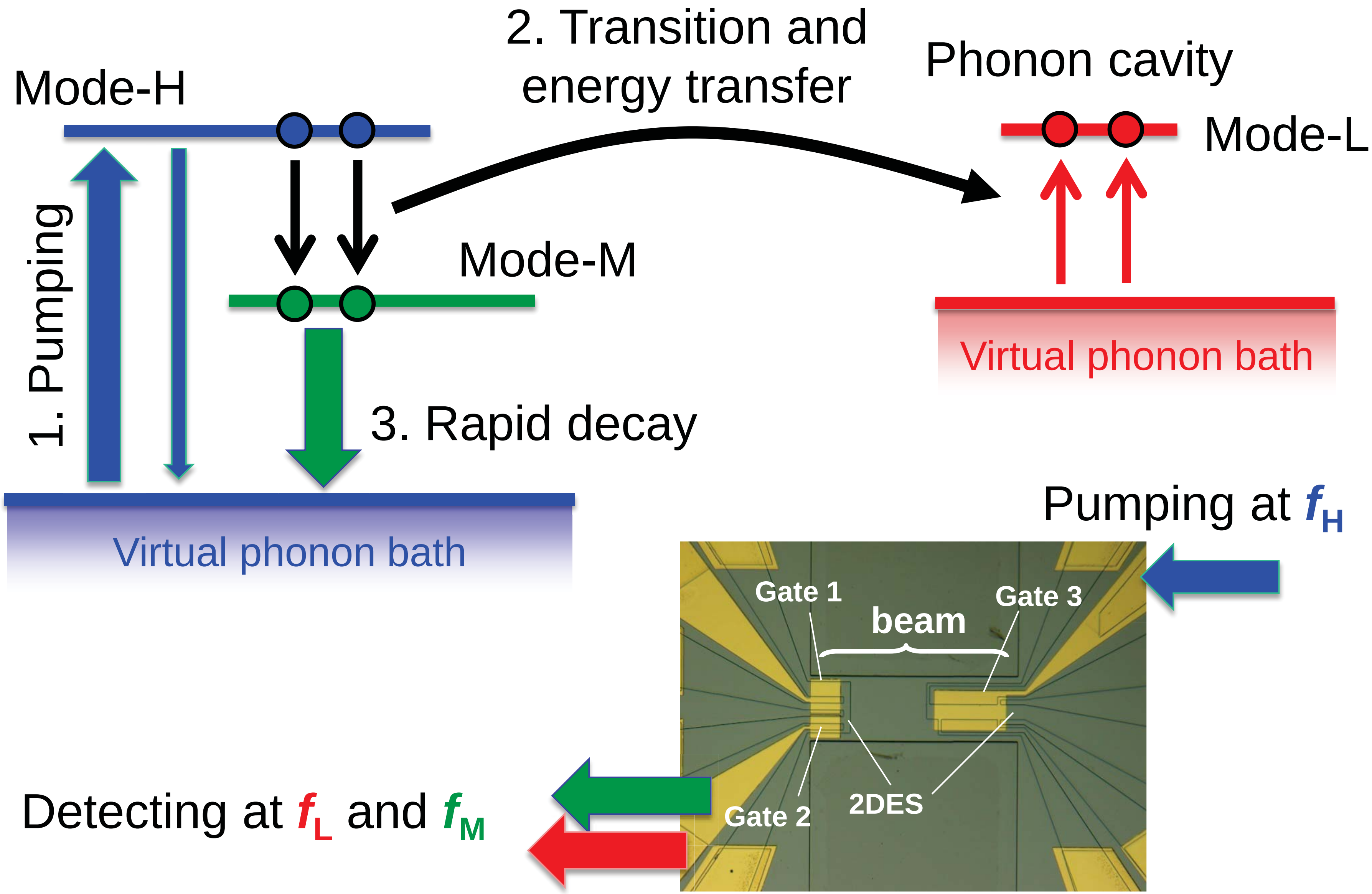


2. Fine tuning by applying a gate voltage

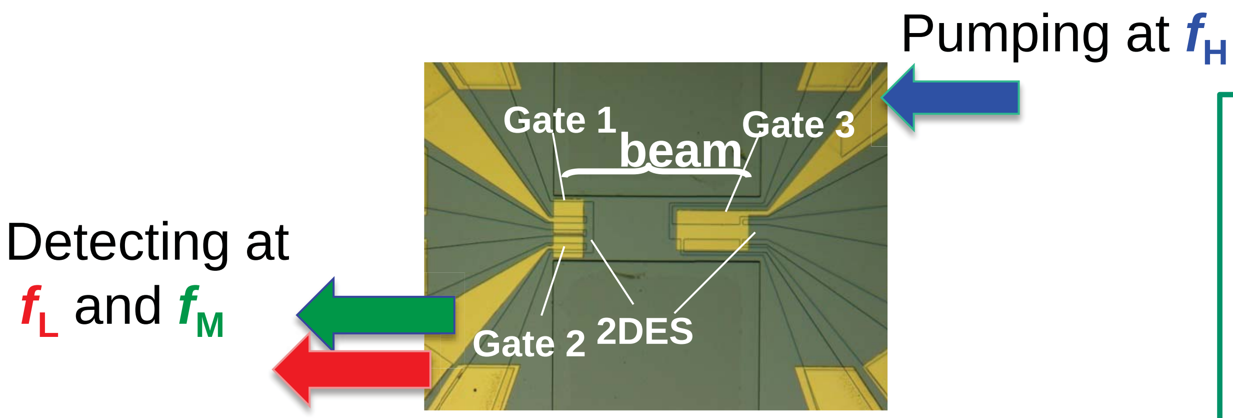
→ $f_H - f_M - f_L < \gamma_H, \gamma_M$

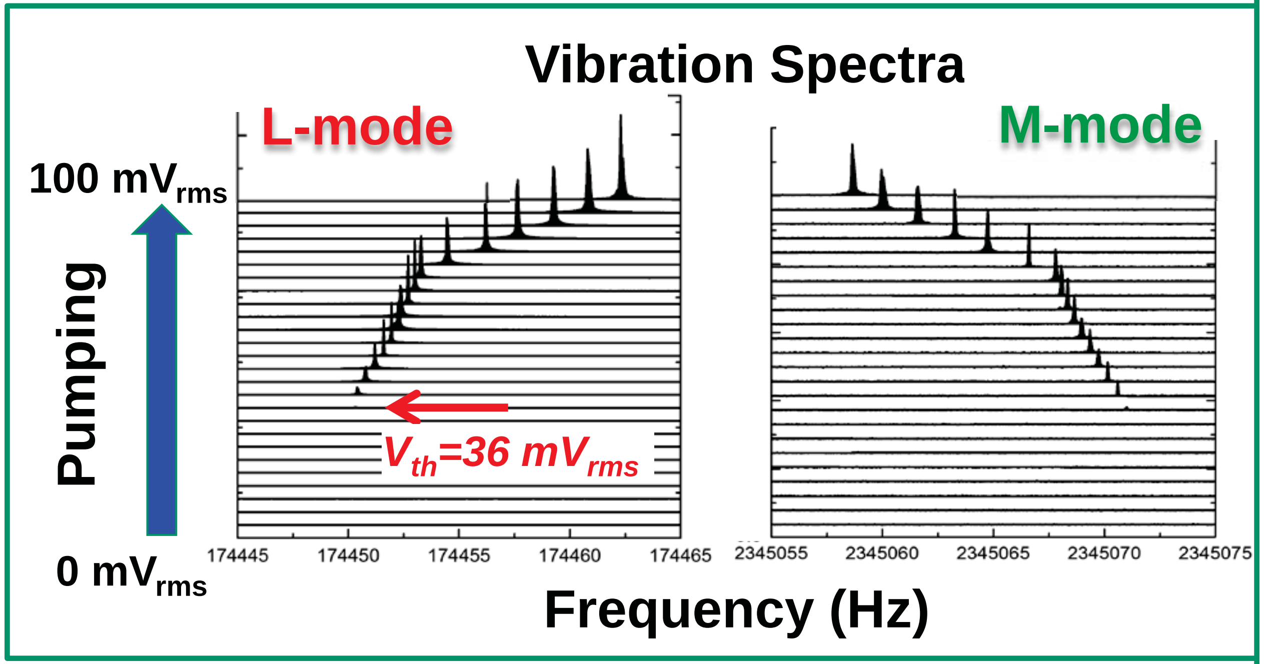


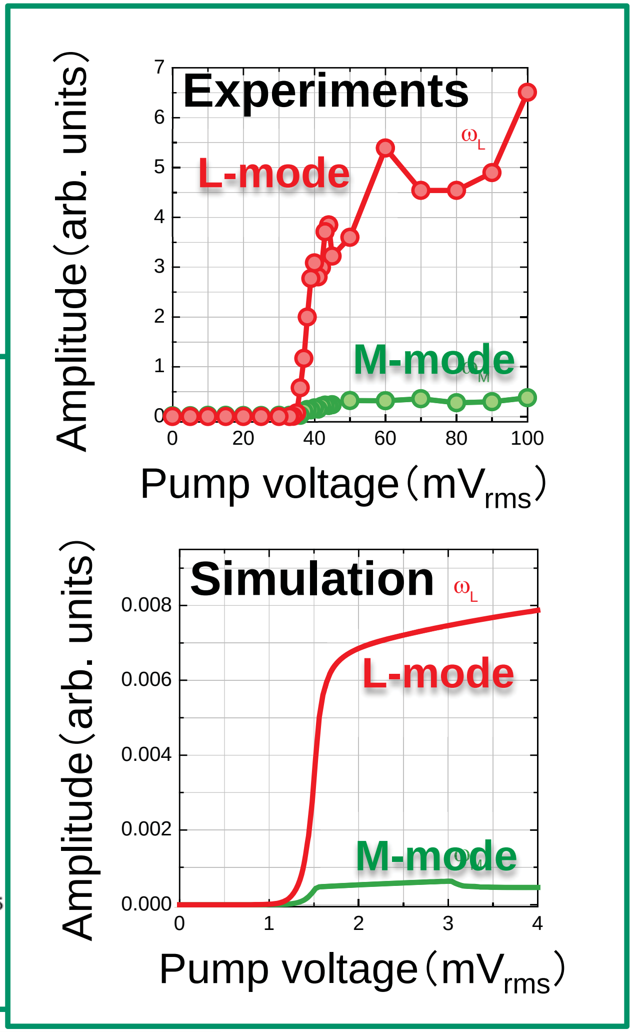
Basic processes for “phonon lasing” operation



Threshold characteristics

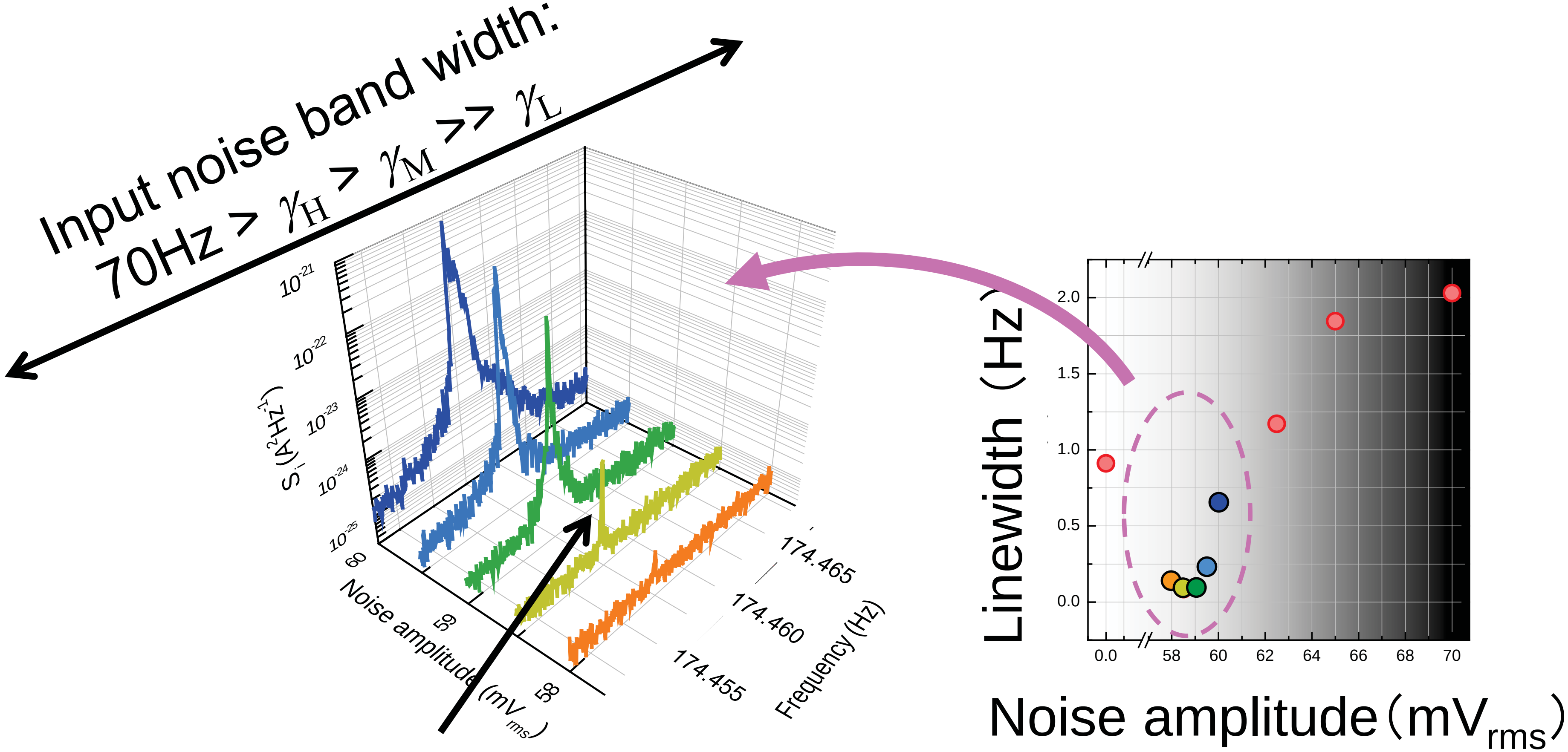






- ✓ *Clear threshold characteristics was observed*
- ✓ *Good agreement with theoretical simulation*

Response to noise pumping



Minimum spectral linewidth :80 mHz

- ✓ *Noise pumping can also induce oscillation.*
- ✓ *Line narrowing was observed.*
- ✓ *Incoherent pumping results in coherent oscillation.*

I. Mahboob et al. PRL 110, 127202 (2013)

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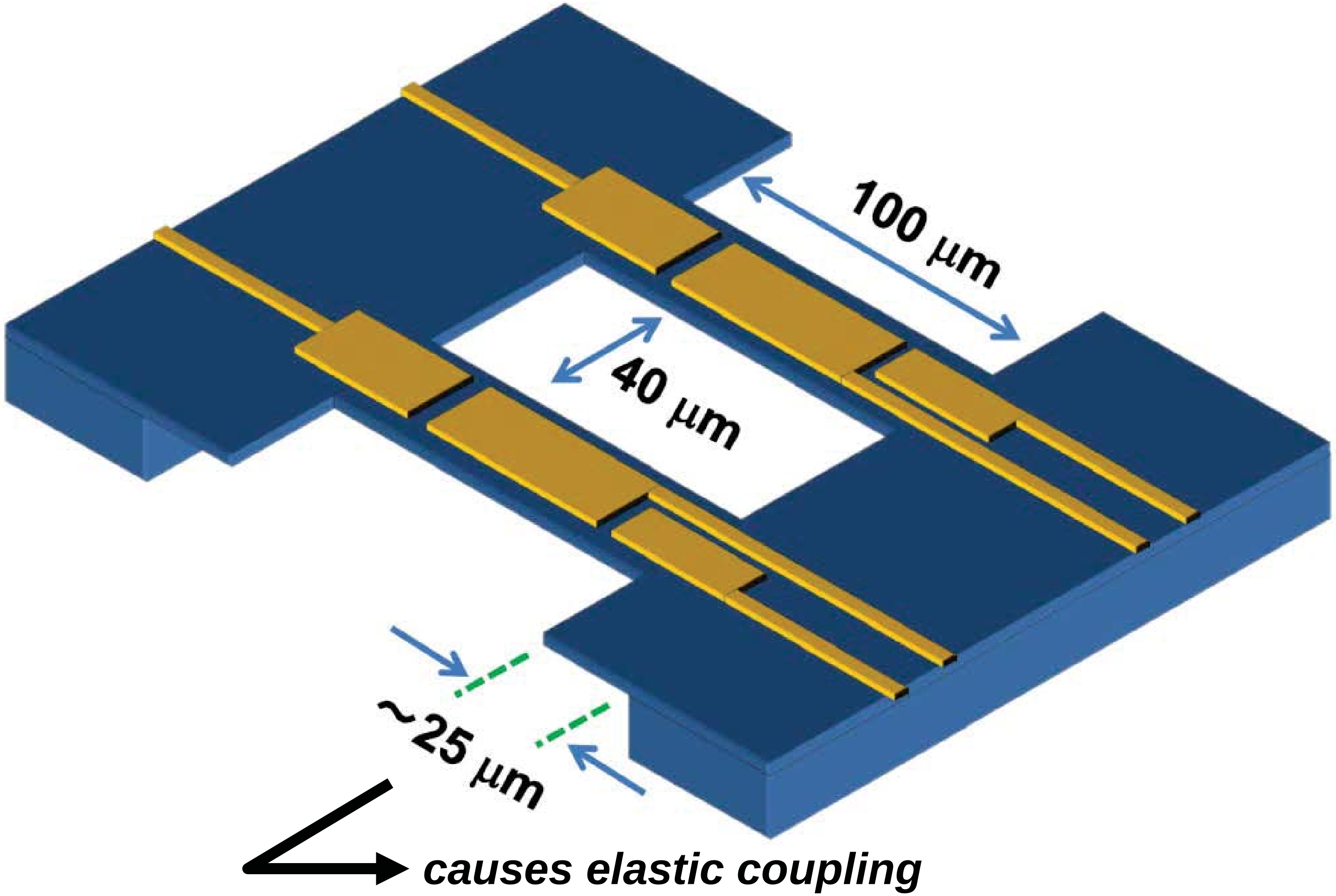
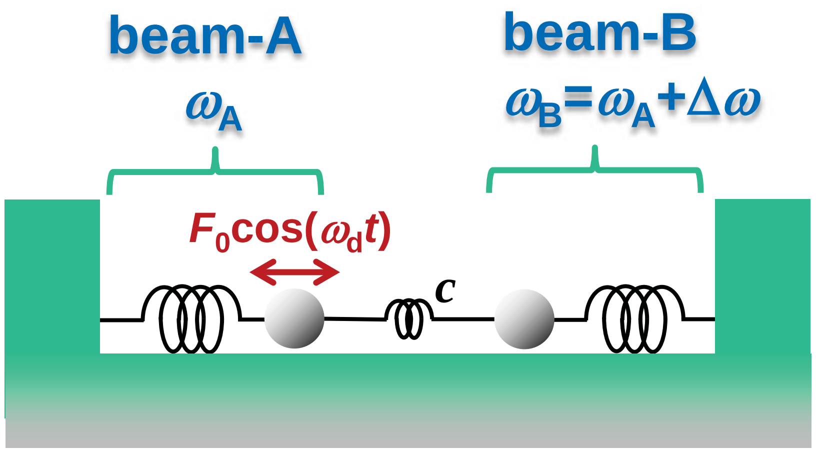
Coupled parametric electromechanical resonators



H. Okamoto et al. Appl. Phys. Express 2, 062202 (2009)
Appl. Phys. Lett. 98, 014103 (2011)

Coupled mechanical oscillation

Mass and spring model

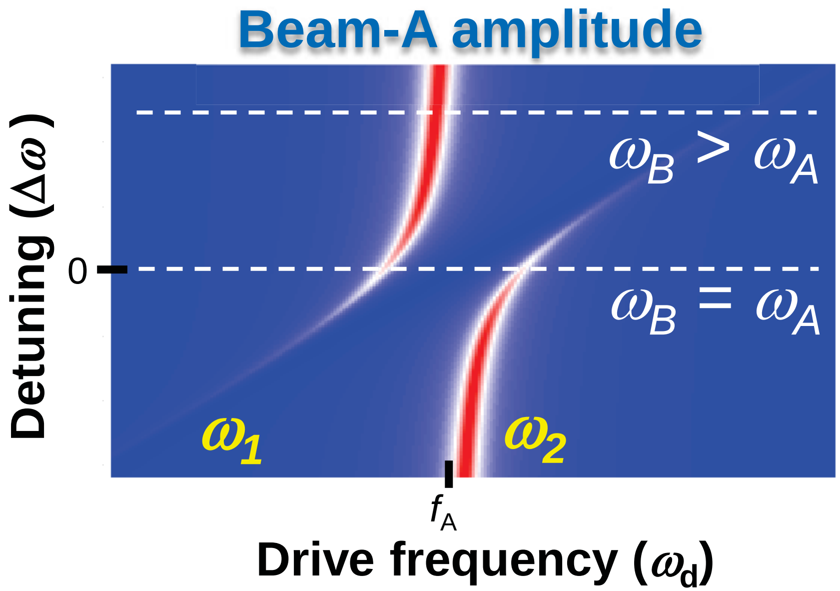


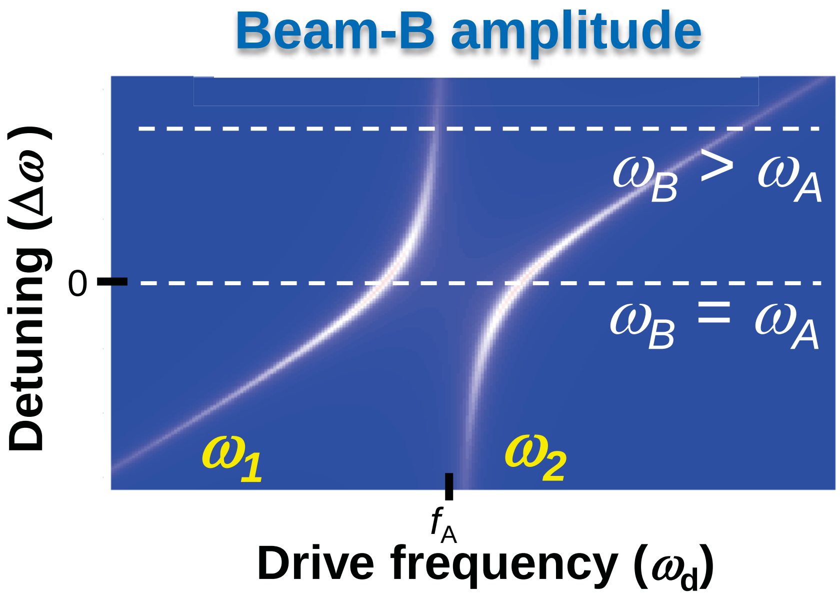
$$\ddot{x}_A + \gamma\dot{x}_A + \omega_A{}^2 x_A = c(x_B - x_A) + F_0\cos(\omega_d t)$$

$$\ddot{x}_B + \gamma\dot{x}_B + \omega_B{}^2 x_B = c(x_A - x_B)$$

labels A and B → beams
labels 1 and 2 → modes

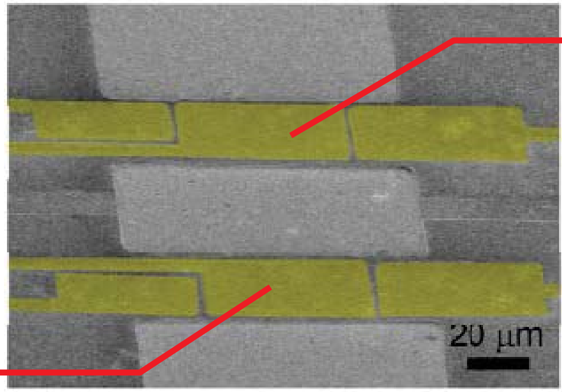
When ω_B is detuned from ω_A, the coupling is inefficient.





Dynamical coupling by parametric pumping

$\omega_* = 2\pi f_*$



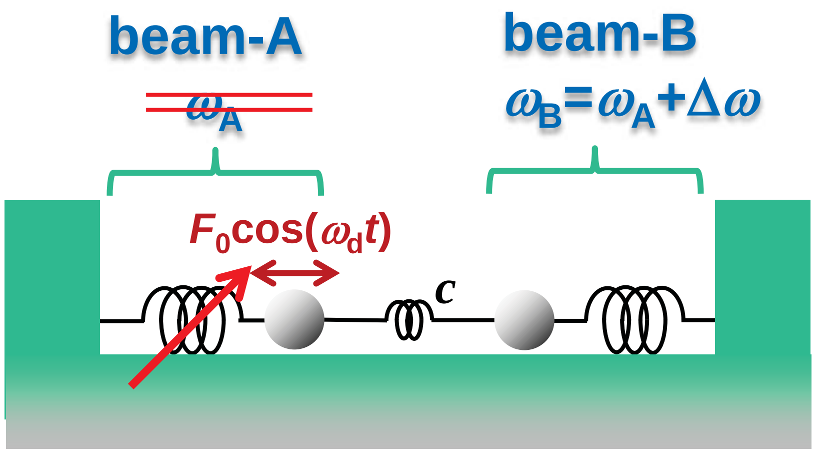
beam-B
f_2=294.37 kHz
(intrinsic f_B=294.34 kHz)

beam-A
f_1=293.93 kHz
(intrinsic f_A=293.96 kHz)

f_2 - f_1 = 440 Hz
(Δf = f_B - f_A = 380 Hz)

➡ ***Coupling is inefficient.***

Parametric pumping



For the modulation at $f_p=\omega_p/2\pi=(f_2 - f_1)$

The frequency mismatch is compensated by the spring constant modulation.

Two resonators are coupled despite the frequency mismatch.

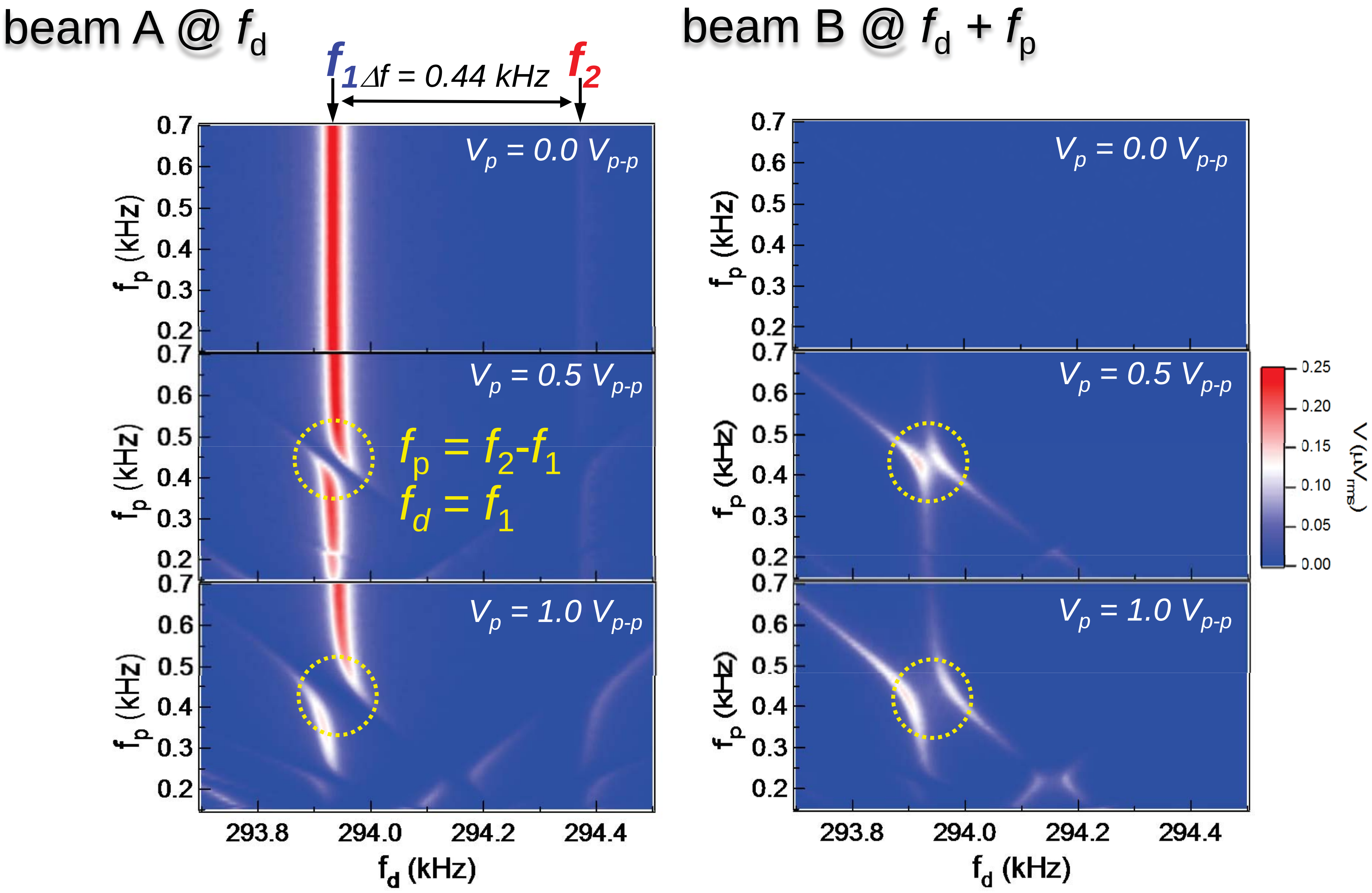
The coupling efficiency can be externally controlled by changing the pump amplitude and frequency.

$$\omega_A(t) = \omega_0+(\Gamma/2\omega_0)\cos(\omega_p t)$$

$$\ddot{x}_A + \gamma\dot{x}_A + [\omega_0^{\,2} + \Gamma\cos(\omega_p t)]x_1 = F_0\cos(\omega_d t + \delta) + c(x_B - x_A)$$

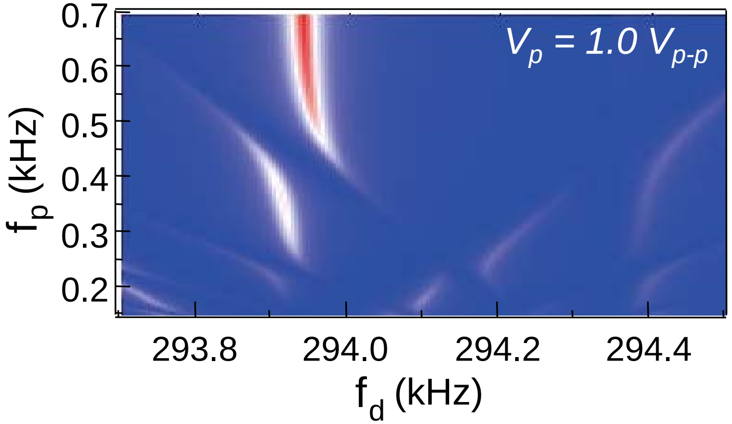
$$\ddot{x}_B + \gamma\dot{x}_B + (\omega_0 + \Delta\omega)^2 x_B = c(x_A - x_B)$$

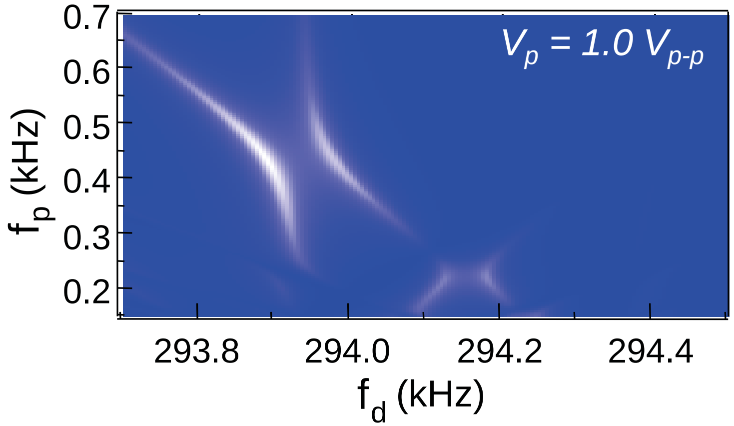
Results of experiments



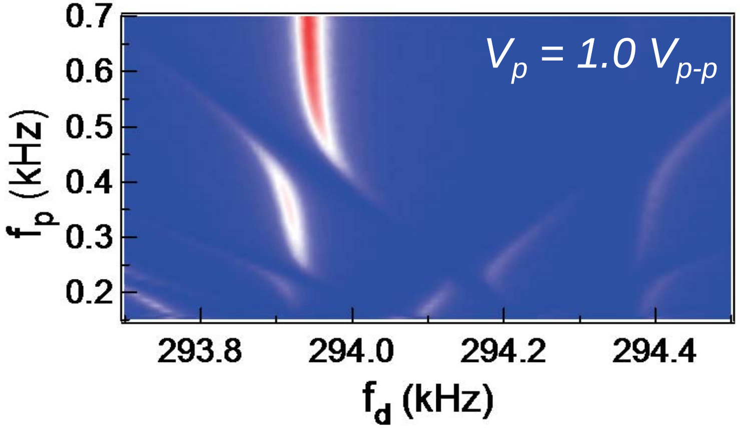
Results of experiments

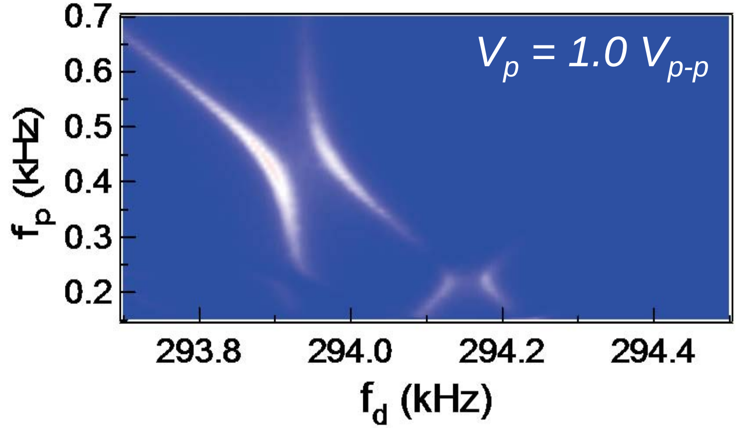
Simulation



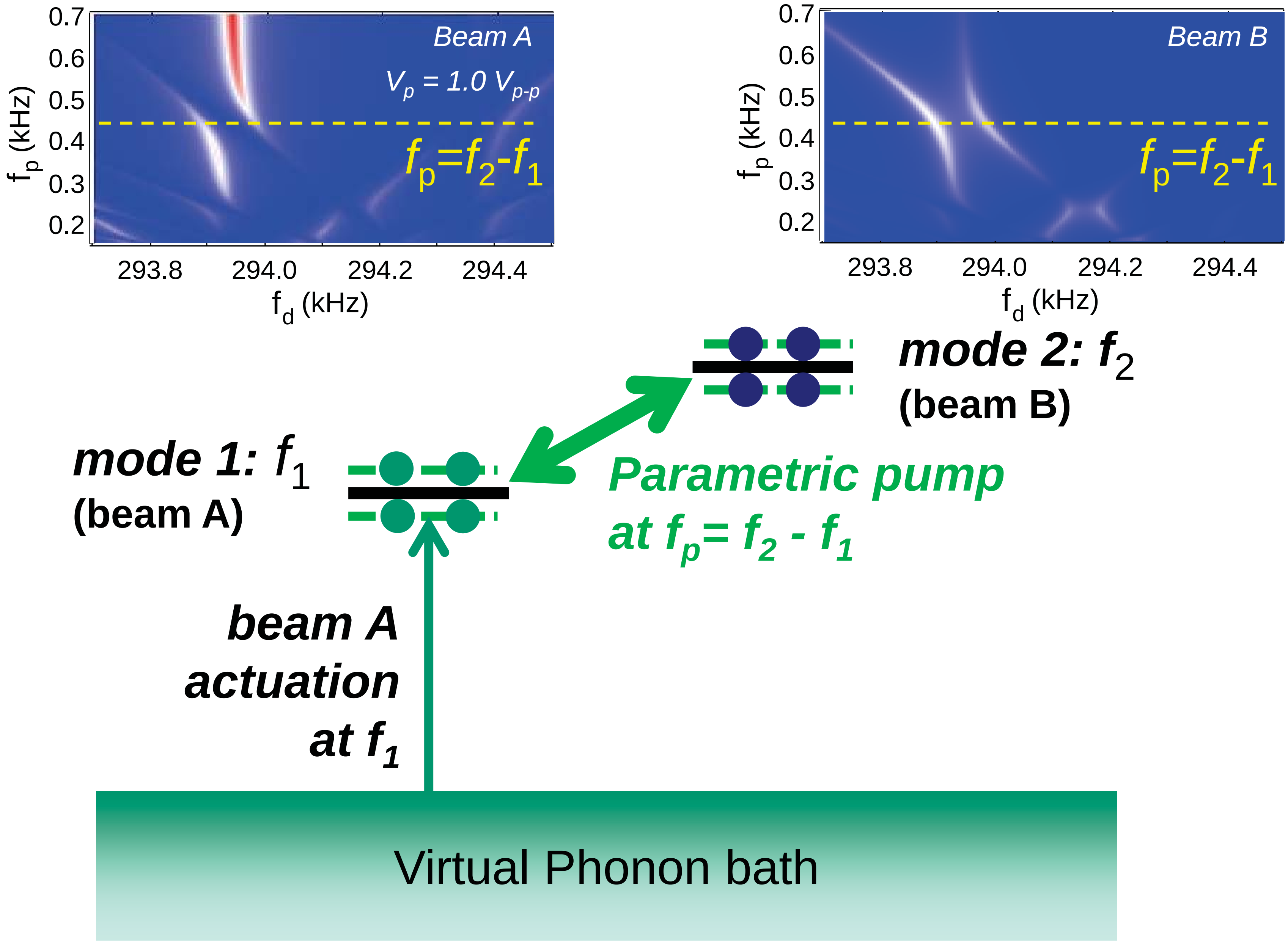


Experiment





Quantum mechanical picture using phonon mode

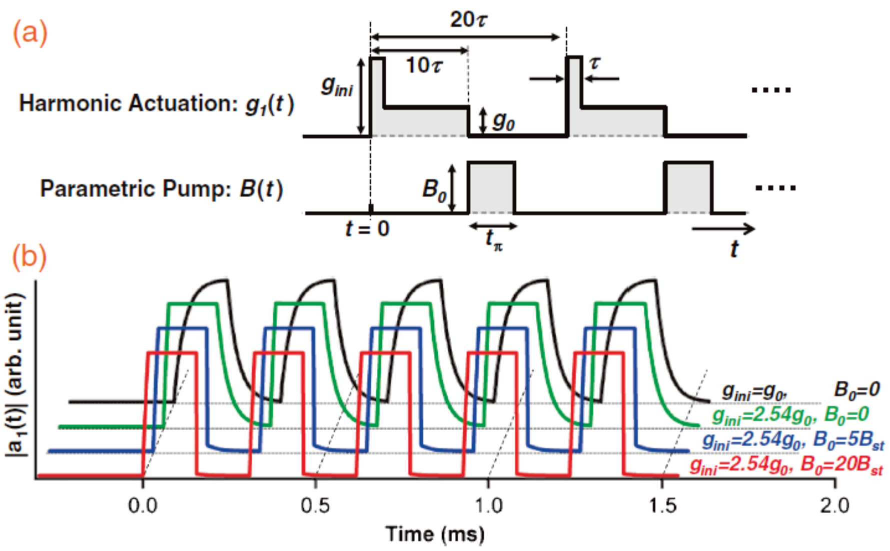


How we utilize the dynamic strong coupling ?

-The coupling between two modes can be controlled electrically.

-Strong coupling (Rabi splitting) allows the mode transition more rapidly than the energy relaxation rate.

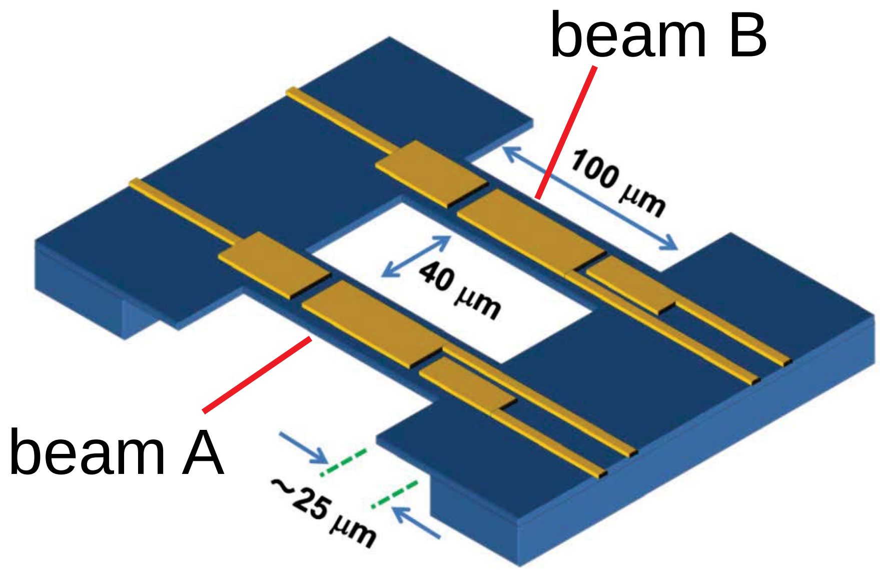
-Therefore, the pulse pumping enables the quick transfer of oscillation energy than the damping rate using Rabi oscillation.



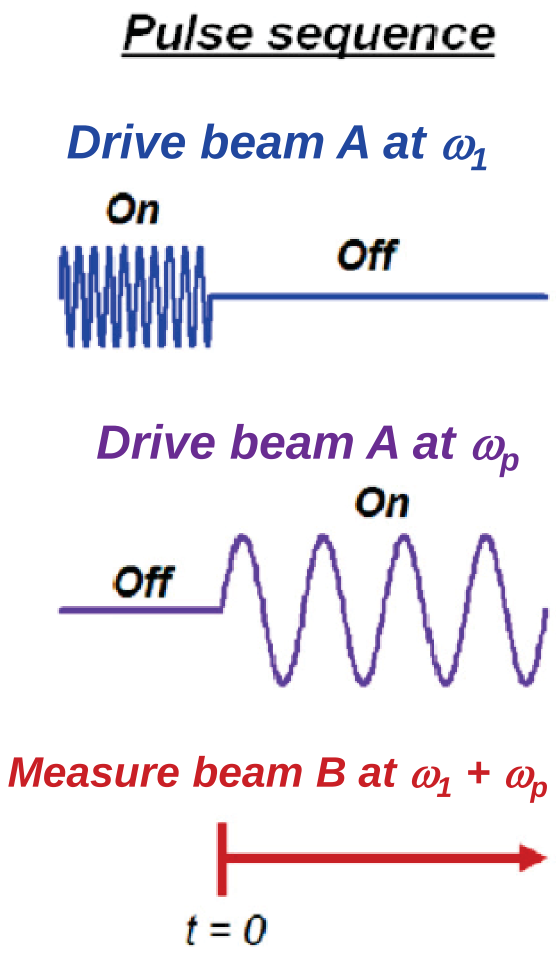
H. Yamaguchi et al., Appl. Phys. Express 5, 014001 (2012)

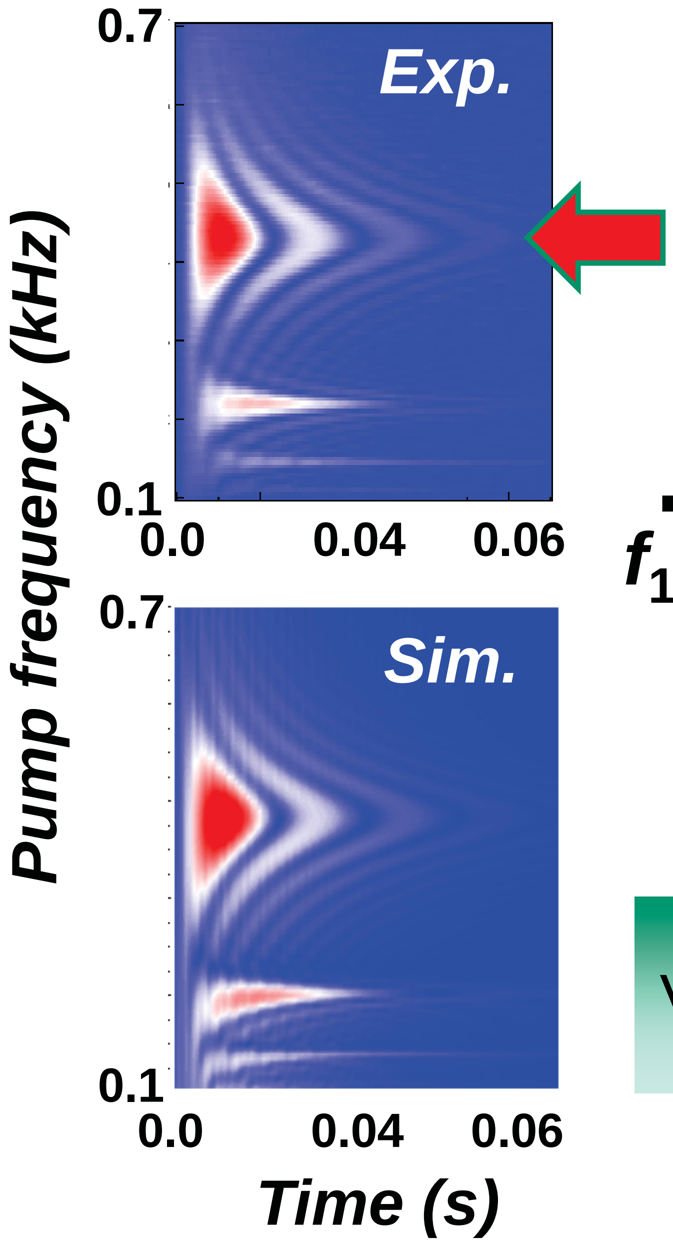
Actuation → beam A

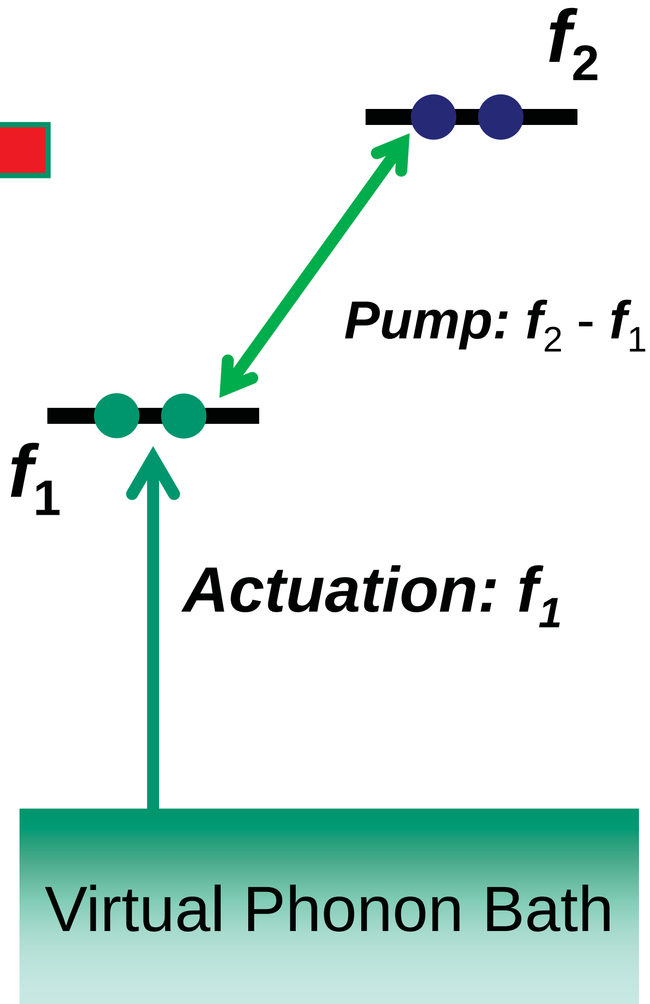
Detection → beam B



Pump Frequency Dependence of Rabi Oscillation







$V_p = 0.5\ V_{p\text{-}p}$

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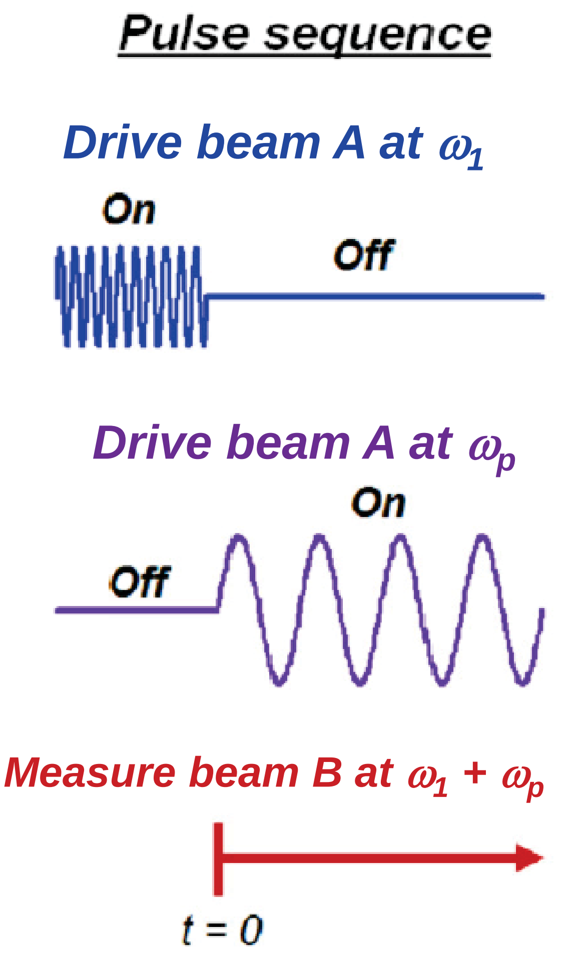
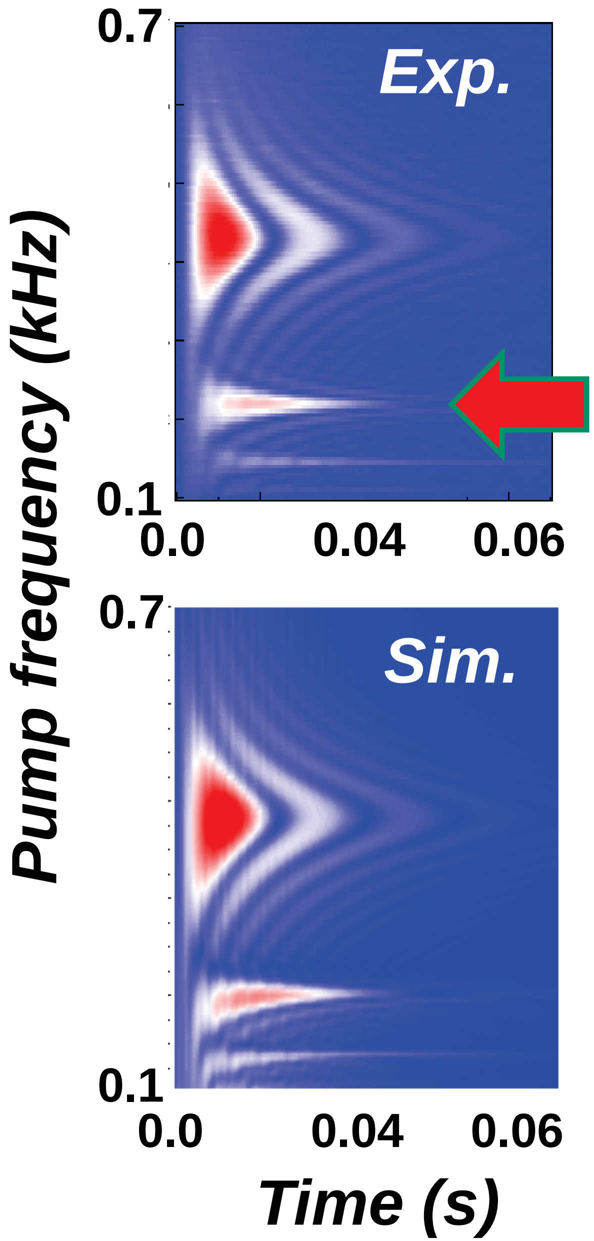
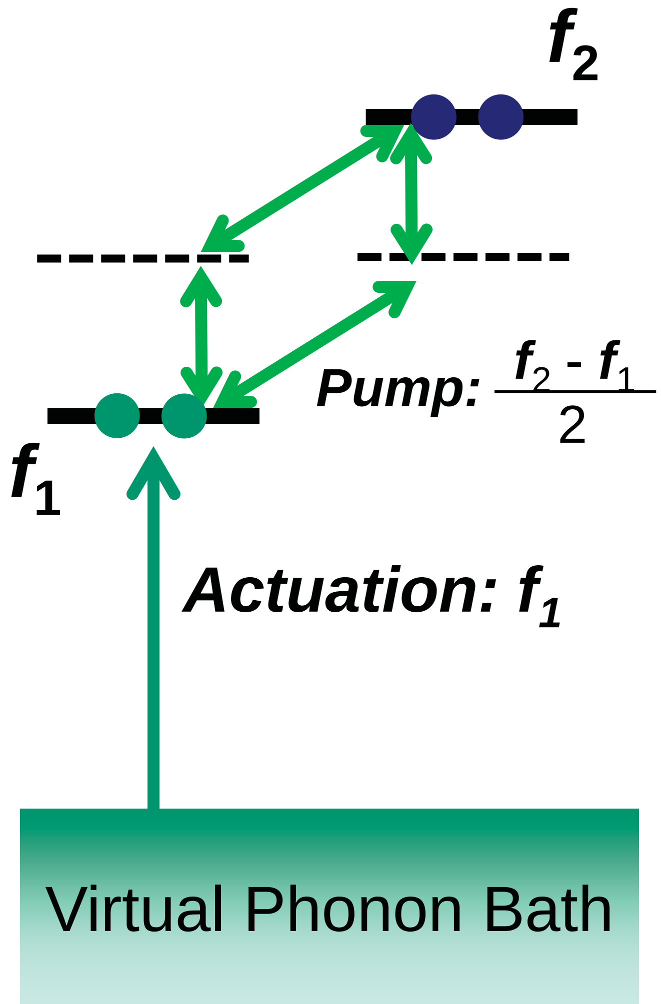
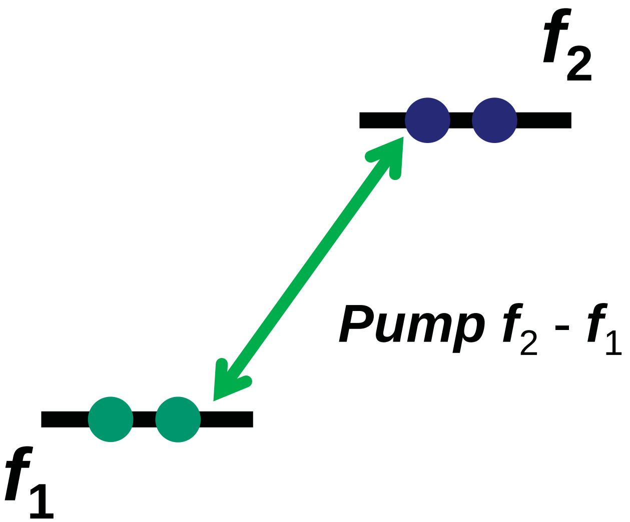


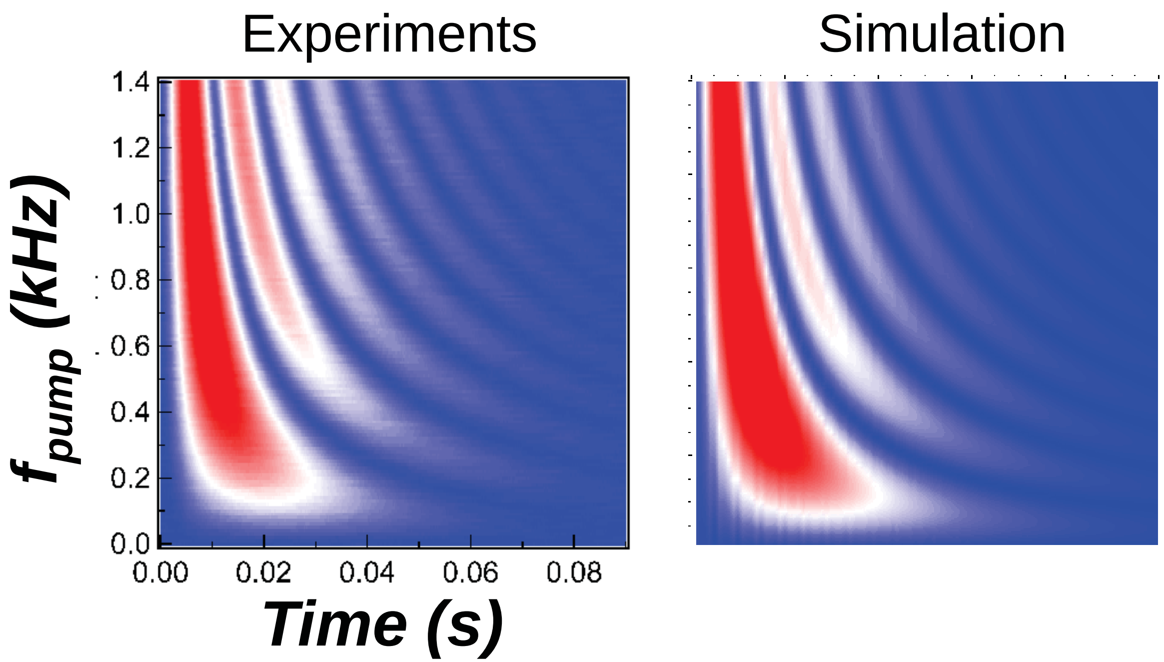
$V_p = 0.5\ V_{p\text{-}p}$

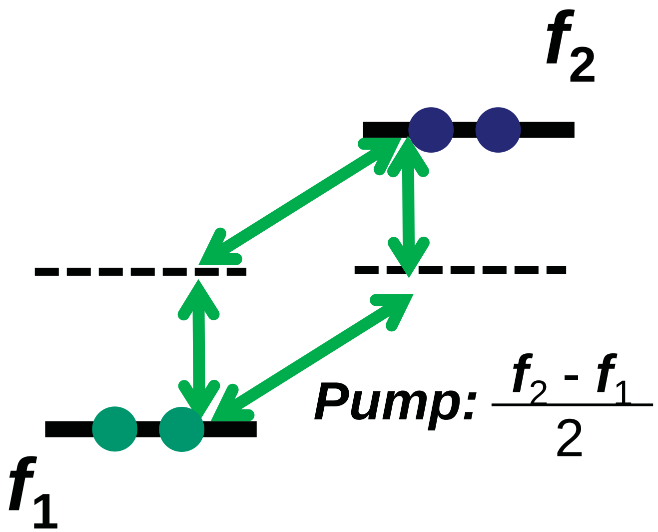


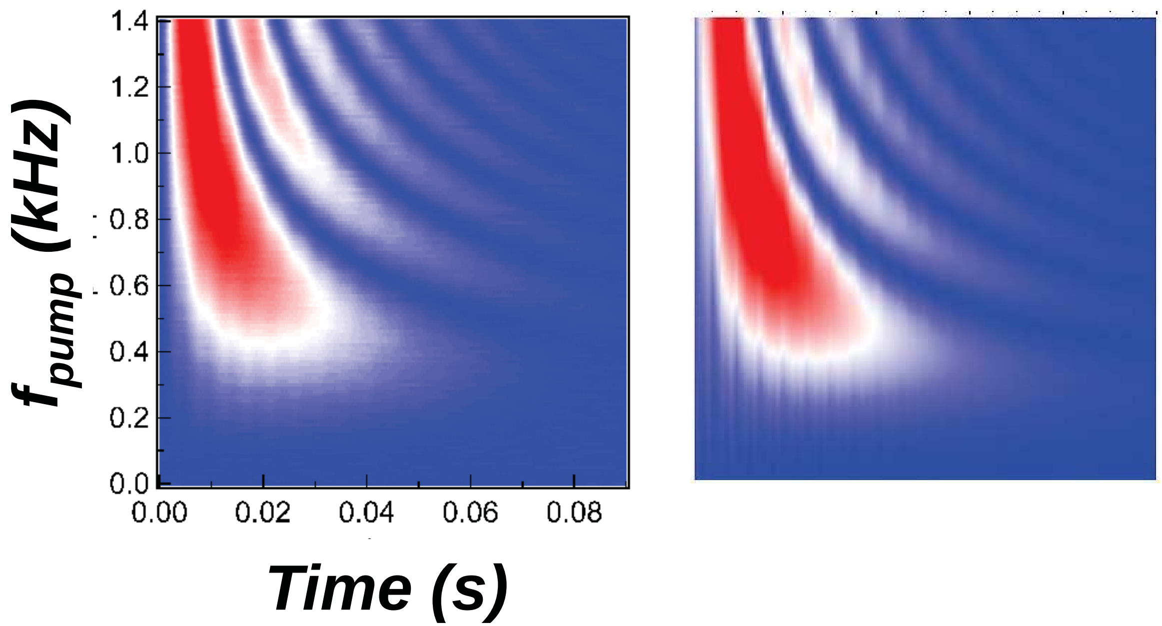


Pump Power Dependence of Rabi Oscillation

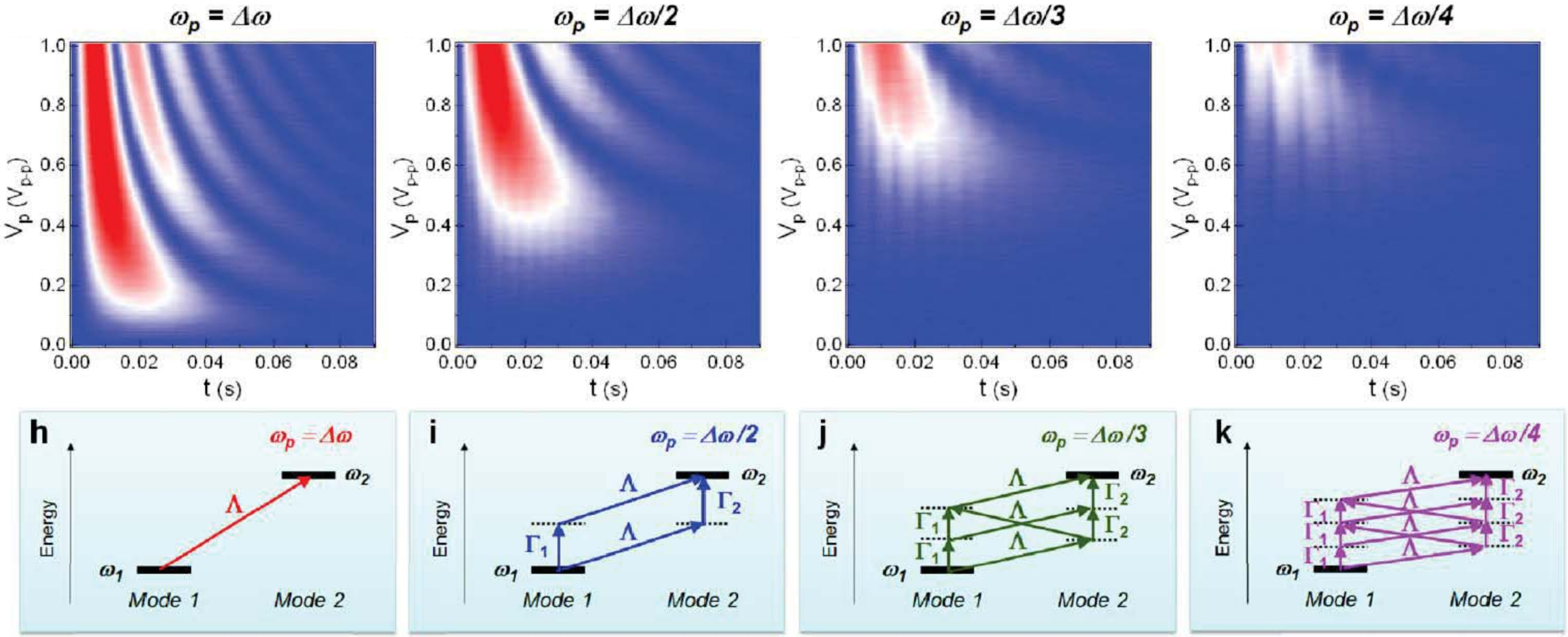








Higher order coherent oscillation



Equations of motion for ***mode*** *variables*

$$\ddot{X}_1 + \gamma\dot{X}_1 + \omega_1^{\,2} X_1 + \Gamma_1 \cos(\omega_p t) X_1 + \Lambda \cos(\omega_p t) X_2 = U_{11} f_0 \cos(\omega t + \delta)$$

$$\ddot{X}_2 + \gamma\dot{X}_2 + \omega_2^{\,2} X_2 + \Gamma_2 \cos(\omega_p t) X_2 + \Lambda \cos(\omega_p t) X_1 = U_{21} f_0 \cos(\omega t + \delta)$$

$$\Gamma_1 = \Gamma(1 + \omega_0 \Delta\omega / \sqrt{c^2 + \omega_0^{\,2} \Delta\omega^2}) / 2$$

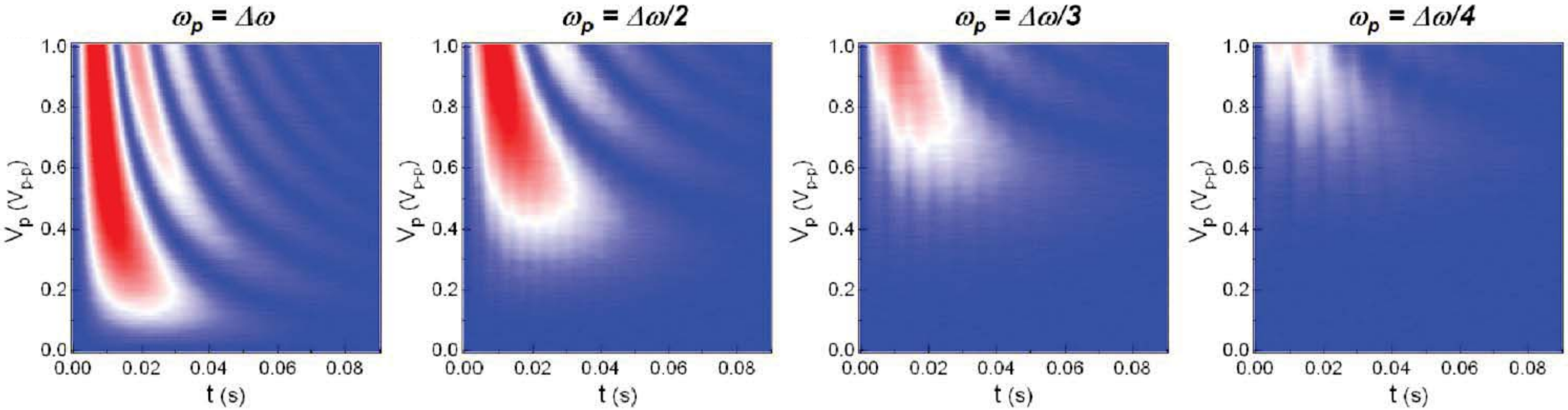
$$\Gamma_2 = \Gamma(1 - \omega_0 \Delta\omega / \sqrt{c^2 + \omega_0^{\,2} \Delta\omega^2}) / 2$$

intra-modal coupling

$$\Lambda = \Gamma c / \sqrt{c^2 + \omega_0^{\,2} \Delta\omega^2} / 2$$

inter-modal coupling

Higher order coherent oscillation



$$g^{(1)} \sim \frac{\Lambda}{2\sqrt{\omega_1\omega_2}}$$

$$g^{(2)} \sim \frac{\Lambda\left|-\Gamma_1/\omega_1+\Gamma_2/\omega_2\right|}{4(\omega_2-\omega_1)\sqrt{\omega_1\omega_2}}$$

$$g^{(3)} \sim \frac{9\Lambda[2\Gamma_1{}^2/\omega_1{}^2+2\Gamma_2{}^2/\omega_2{}^2-(4\Gamma_1\Gamma_2+\Lambda^2)/\omega_1\omega_2]}{128(\omega_2-\omega_1)^2\sqrt{\omega_1\omega_2}}$$

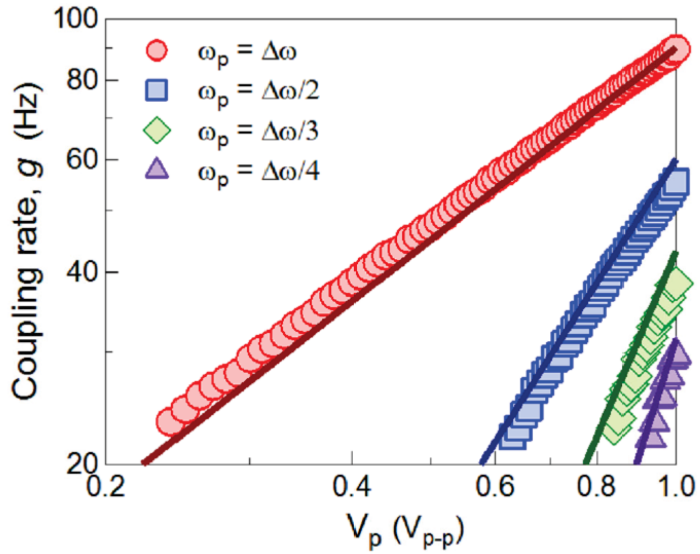
$$g^{(4)} \sim \frac{\Lambda\,|-3\Gamma_1{}^3/\omega_1{}^3+9\Gamma_1{}^2\Gamma_2/\omega_1{}^2\omega_2+4\Lambda^2\Gamma_1/\omega_1{}^2\omega_2-9\Gamma_1\Gamma_2{}^2/\omega_1\omega_2{}^2-4\Lambda^2\Gamma_2/\omega_1\omega_2{}^2+3\Gamma_2{}^3/\omega_2{}^3\,|}{36(\omega_2-\omega_1)^3\sqrt{\omega_1\omega_2}}$$

$$\Gamma_1 = \Gamma(1+\omega_0\Delta\omega/\sqrt{c^2+\omega_0{}^2\Delta\omega^2})/2$$

$$\Gamma_2 = \Gamma(1-\omega_0\Delta\omega/\sqrt{c^2+\omega_0{}^2\Delta\omega^2})/2$$

$$\Lambda = \Gamma c/\sqrt{c^2+\omega_0{}^2\Delta\omega^2}\,/2$$

good agreement with no fitting parameter !

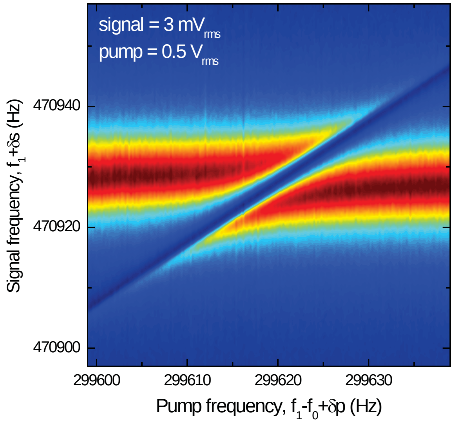


H. Okamoto et al. Nature Phys. 9, 480 (2013)

Summary

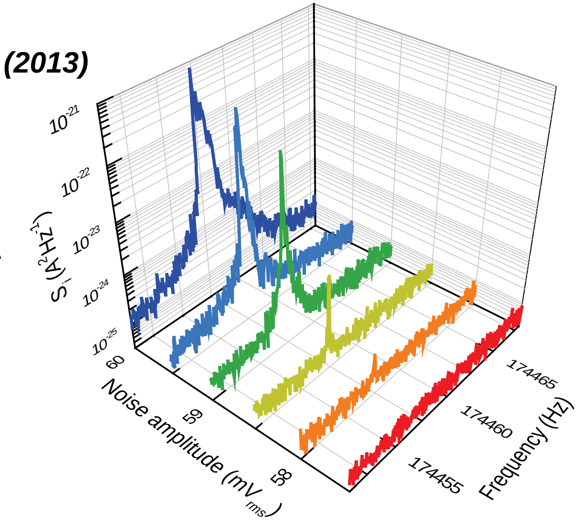
✓ ***Piezoelectric transduction in a GaAs/AlGaAs parametric resonator***

I. Mahboob and H. Yamaguchi, Nature Nanotechnology 3, 275 (2008)



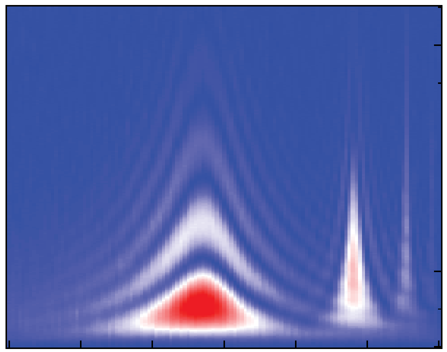
✓ ***Mechanical two-mode systems and tension-induced strong coupling***

I. Mahboob, et al. Nature Phys. 8, 387 (2012)
H. Yamaguchi and I. Mahboob, New J. Phys. 15, 015023 (2013)



✓ ***Mechanical three-mode systems and phonon lasing operation***

I. Mahboob et al. PRL 110, 127202 (2013)
J. T. Mendonça, Physics 6, 32 (2013)



✓ ***Coherent oscillation and higher-order parametric mixing in paired mechanical resonators***

H. Okamoto et al. Nature Phys. 9, 480 (2013)
H. Yamaguchi et al., Appl. Phys. Express 5, 014001 (2012)