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Relaxation, Pre-thermalization and Diffusion in a Noisy Quantum Ising Chain

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Relaxation, pre-thermalization and diffusion in a noisy Quantum Ising Chain

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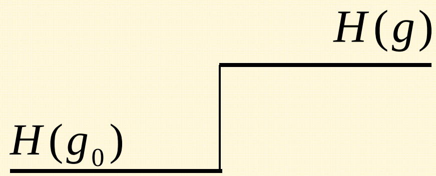
Relaxation Dynamics

$t = 0$ $\qquad$ $t > 0$

$$H(g_0) \longrightarrow H(g)$$

$$|\psi(g_0)\rangle \longrightarrow e^{-iH(g)t}|\psi(g_0)\rangle$$

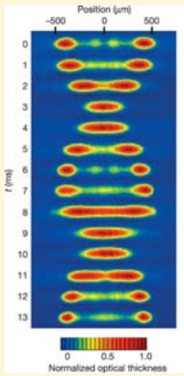


Quantum Quench

Thermal state? *Srednicki (1994), Kollath et al. (2007), Rigol et al. (2008), ...*

but

Integrable systems



Kinoshita et al. (2006)

Generalized Gibbs Ensemble

$$\rho \sim e^{-\sum_\alpha \lambda_\alpha I_\alpha}$$

Prethermalization

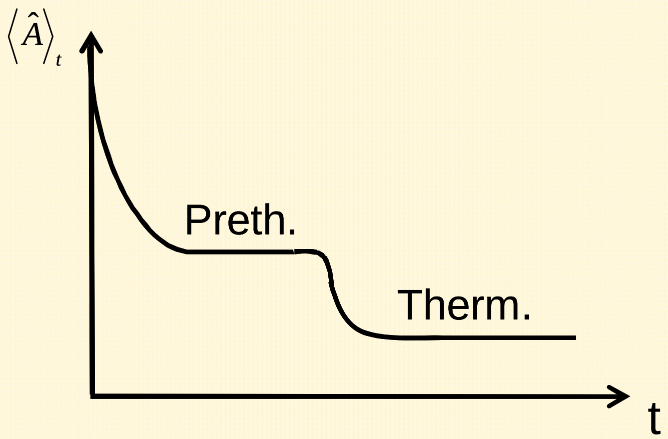
H_0 Integrable Hamiltonian

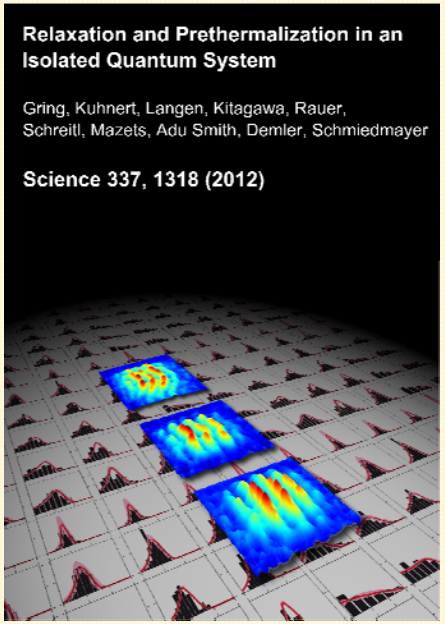
Kehrein et al. (2008), Eckstein et al. (2009, 2011)

Quench ⟶ $H = H_0 + gH_1$ ⟶ breaks integrability $|g| << 1$

Prethermalization:

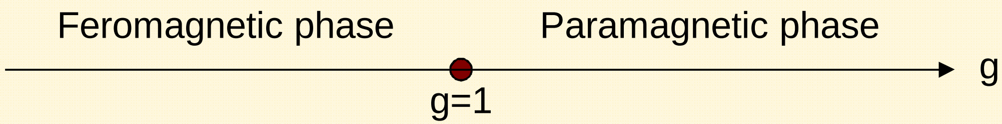
$$\langle \hat{A} \rangle_t \approx \langle \hat{A} \rangle_{GGE\, of\, \tilde{H}} \qquad \frac{1}{g} << t << \frac{1}{g^2}$$





The Quantum Ising Chain

$$H_0 = -J\sum_i \sigma_i^x \sigma_{i+1}^x + g\sigma_i^z$$



$$\Delta = |g - 1|$$

Jordan Wigner Transformation:

$$c_j^+ = \left(\prod_{m=1}^{j-1} \sigma_m^z\right)\sigma_j^+ \quad c_j^- = \left(\prod_{m=1}^{j-1} \sigma_m^z\right)\sigma_j^-$$

Fourier transformation and Bogolyubov Rotation:

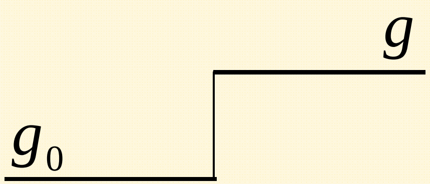
$$H = \sum_k E_k \gamma_k^+ \gamma_k$$

Free fermions

$$E_k = \sqrt{(g - \cos k)^2 + \sin^2 k}$$

Quench of the Quantum Ising Chain

Quantum Quench of the Transverse Field



Rossini et al. (2010)
Calabrese et al. (2011)
Foini et al. (2011)
Fagotti et al. (2013)
...

$$\rho_{GGE} = \frac{1}{Z}\exp\left(\int_{-\pi}^{\pi}\frac{dk}{2\pi}\lambda_k \gamma_k^+ \gamma_k\right)$$

GGE in terms of mode occupation numbers

$$n_k = \gamma_k^+ \gamma_k$$

Fermion occupation number

The model and the out of equilibrium protocol

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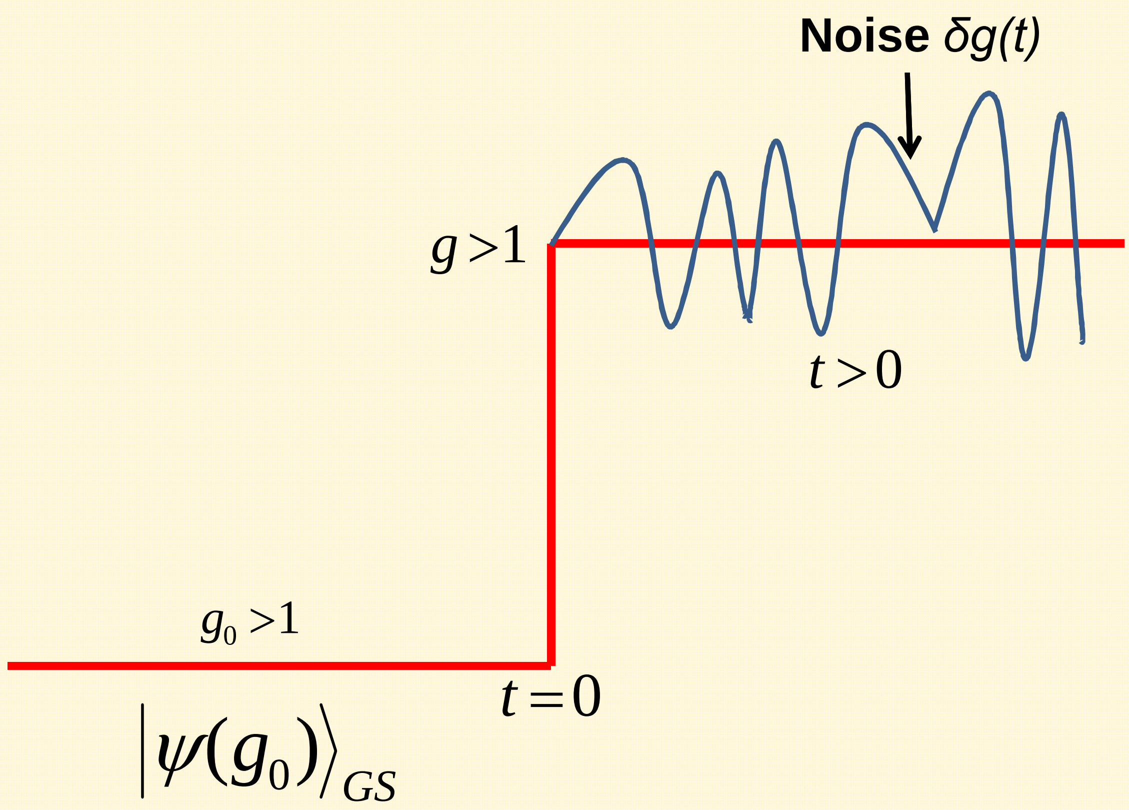
$$H = H_0 + V$$

$$H_0 = -J\sum_i \sigma_i^x \sigma_{i+1}^x + g\sigma_i^z \qquad \Delta = |g-1|$$

$$V = \sum_i \delta g(t)\sigma_i^z \qquad \langle \delta g(t)\delta g(t')\rangle = \frac{\Gamma}{2}\delta(t-t')$$

We perform a quantum quench from the paramagnetic phase at g_0 to the paramagnetic phase at g and we compute observables of physical interest during the dynamics

Out of Equilibrium Protocol



Dynamics of Thermalization

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$$\frac{\Gamma}{\Delta} << 1$$

- ***Prethermalization***

  drfits towards GGE (*Kollar et al.* 2010)

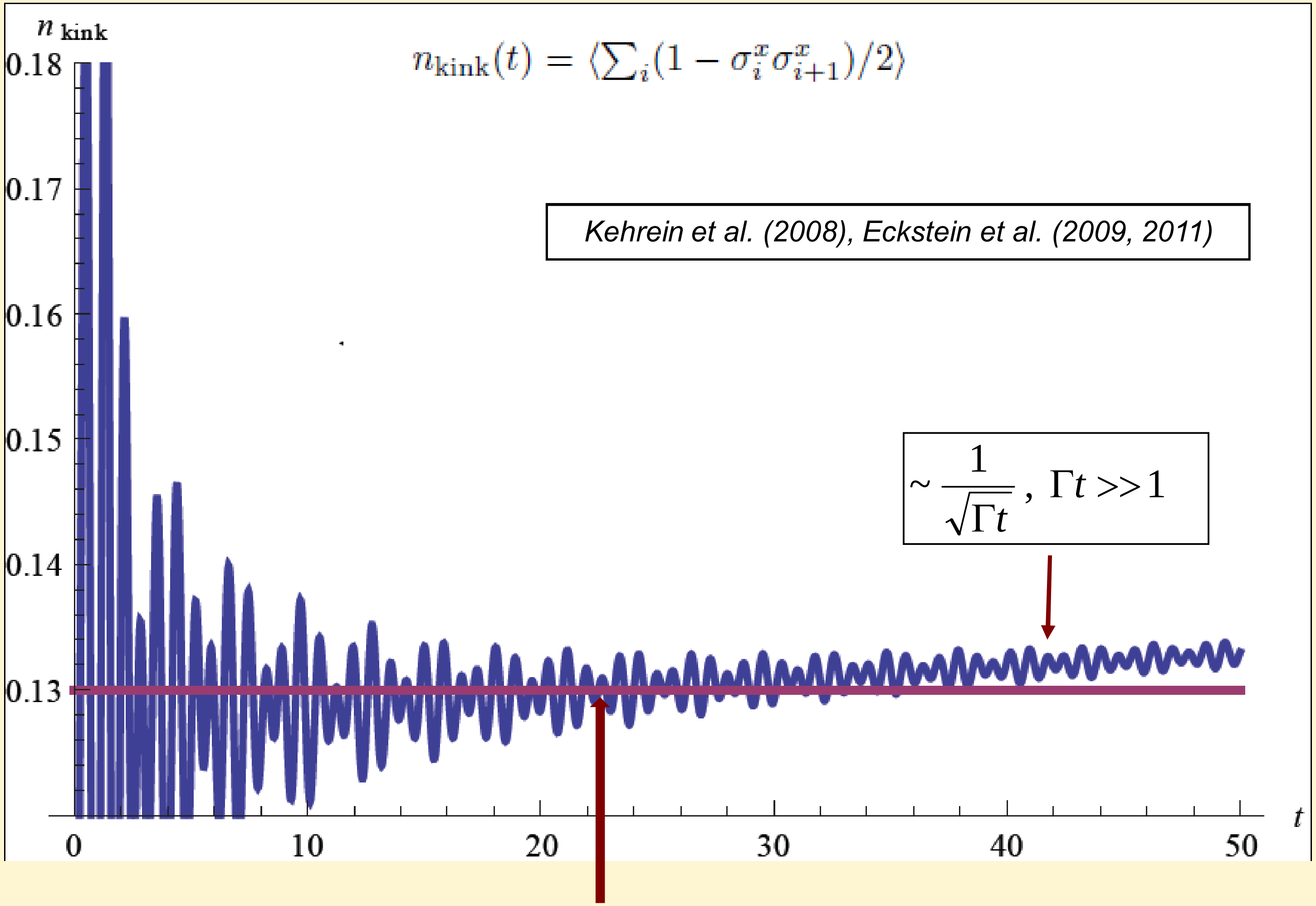
- ***Dephasing*** $t \approx \frac{1}{\Gamma}$: coherences killed

- ***Thermalization*** population heating towards $T \rightarrow \infty$

Fig.1

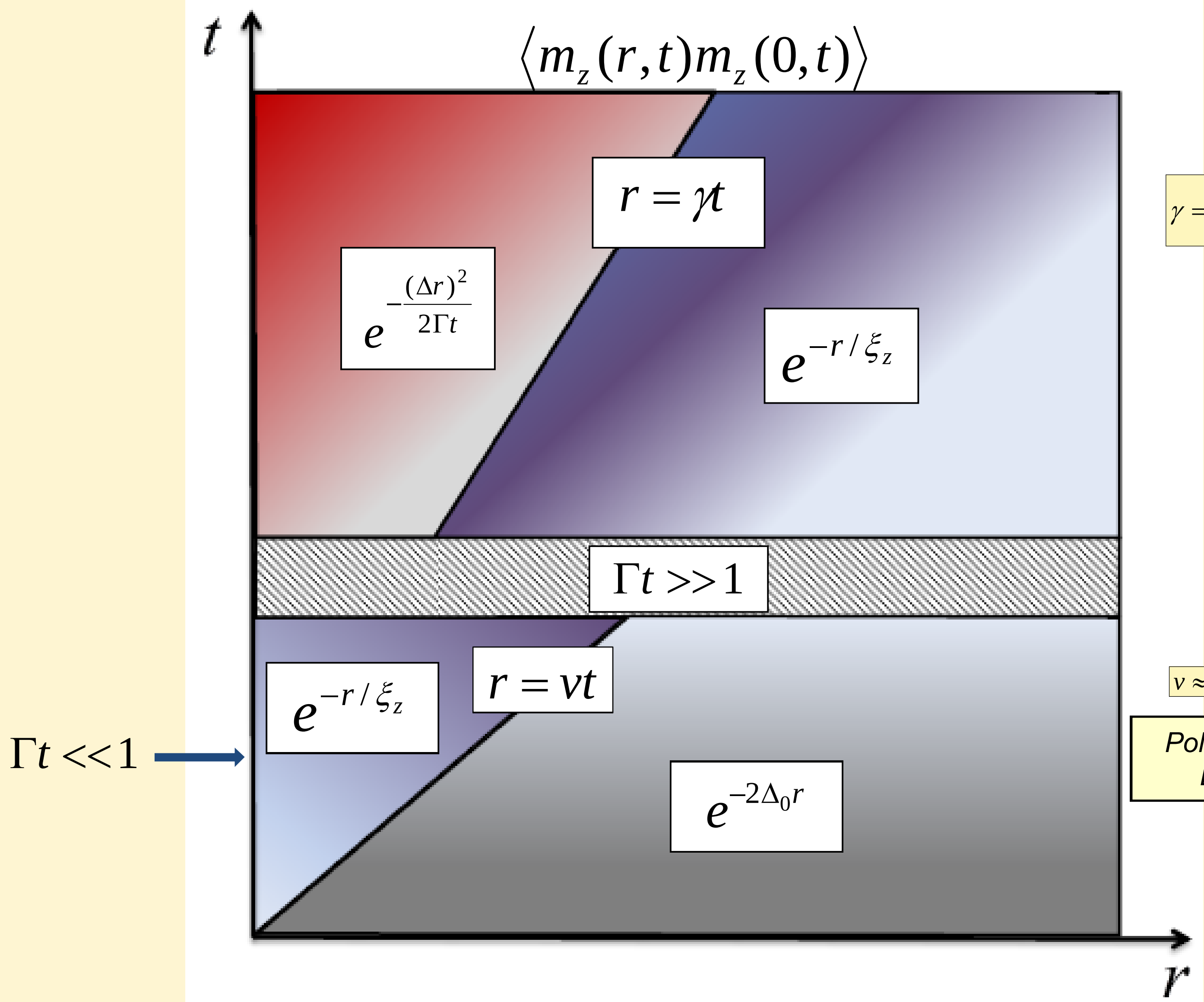
Fig.2

The density of kinks vs. time



Ising GGE , $t \sim \frac{1}{J}$ $\Gamma t << 1$

The spreading of quantum and thermal correlations



$\gamma = \frac{\Gamma}{\Delta}$

$v \approx J$

Polkovnikov et al. (2010)
Essler et al. (2012)

Order parameter correlations

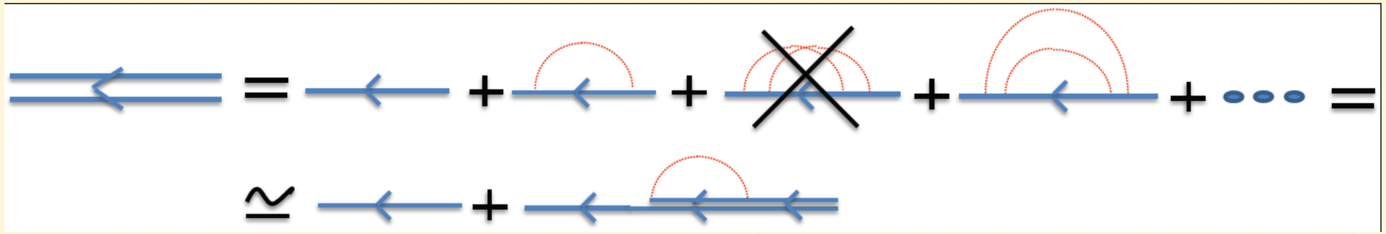
$$\left\langle m_x(r,t)m_x(0,t)\right\rangle = r^{-1/2}e^{\frac{r}{\xi(t)}}, \qquad \frac{1}{\xi(t)} = \frac{1}{\xi} + \frac{\Gamma t}{2(1+\Delta)^2}$$

No Quench

Different signatures in different observables.

A bit of details …

$$\gamma = \frac{\Gamma}{\Delta} << 1$$



Keldysh

$$\rho_k = \begin{pmatrix} \langle \gamma_k^\dagger \gamma_k \rangle & \langle \gamma_k^\dagger \gamma_{-k}^\dagger \rangle \\ \langle \gamma_{-k} \gamma_k \rangle & \langle \gamma_{-k} \gamma_{-k}^\dagger \rangle \end{pmatrix}$$

$$\partial_t \rho_k = -i[H_k^0, \rho_k] + \frac{\Gamma}{2}(\sigma \rho_k \sigma - \rho_k)$$

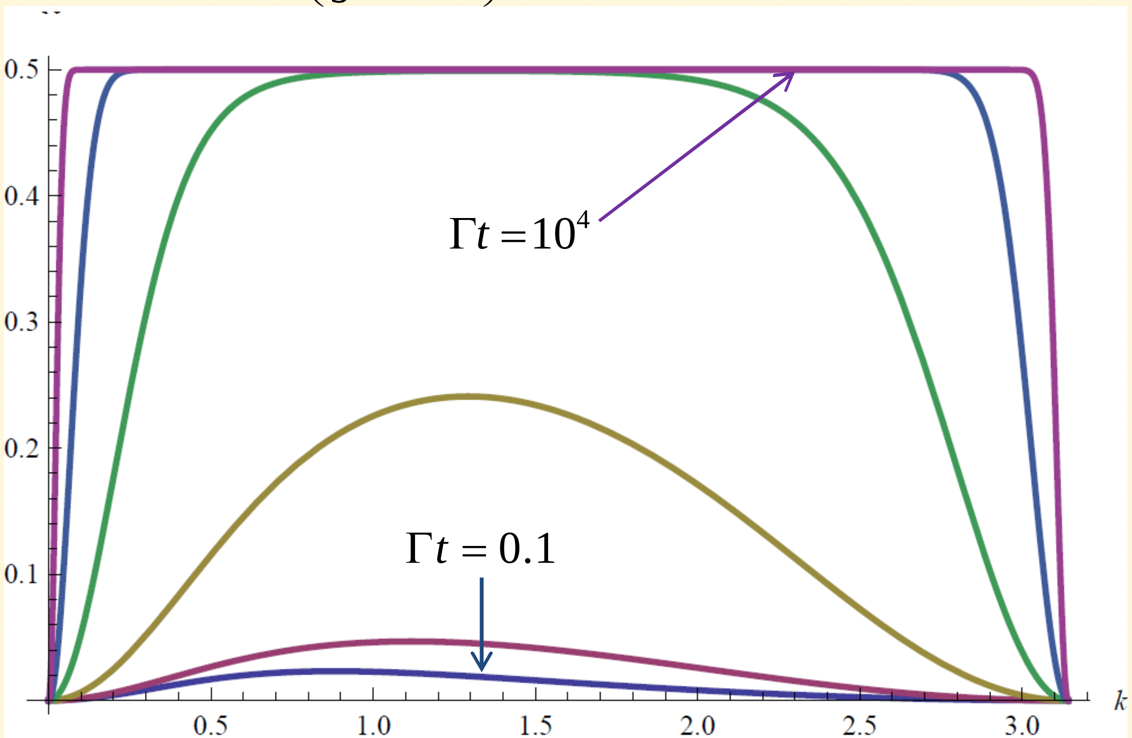
$$\sigma \equiv \cos 2\theta_k \sigma_z + \sin 2\theta_k \sigma_y$$

$$\theta_k(g) = \frac{1}{2} \arctan\left(\frac{\sin k}{g - \cos k} \right)$$

Populations

$$\left\langle \gamma_k^+(t)\gamma_k(t)\right\rangle = \frac{1}{2} + (\sin^2(\Delta\alpha_k) - \frac{1}{2})e^{-\Gamma \sin^2\theta_1 t}$$ *Populations*

$$\theta(g) = \frac{1}{2}\arctan\left(\frac{\sin k}{g-\cos k}\right) \quad \Delta\alpha_k = \theta(g_1) - \theta(g_0)$$



The relaxation rates tend to vanish close to the band edge for: $k = 0, \pm\pi$ $\quad \Gamma t >> 1$

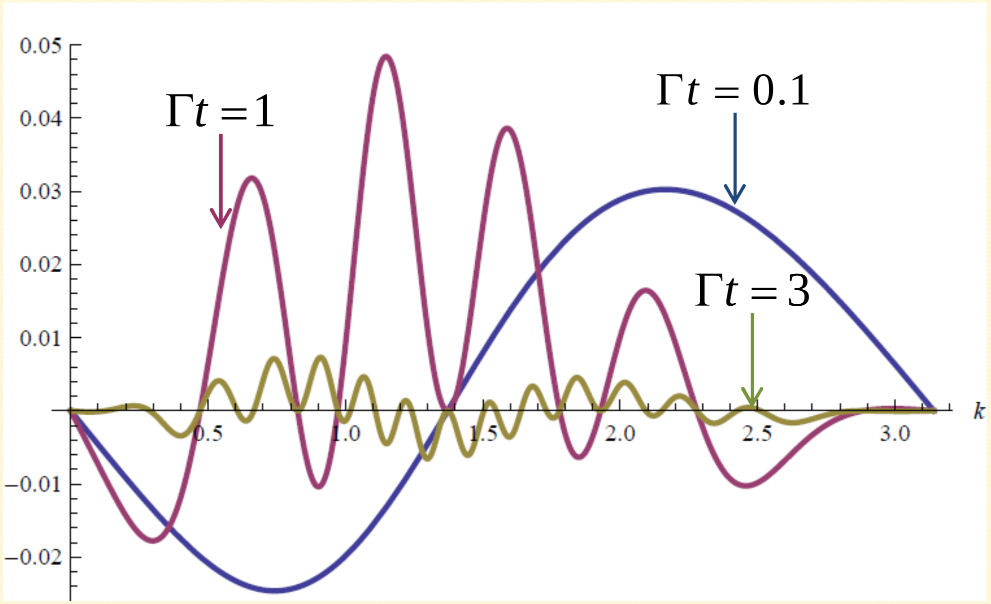
$$\Gamma_k \approx_{(k\approx 0)} \Gamma \frac{k^2}{\Delta^2}$$

Most of the modes relax on time scales of the order of

$1/\Gamma$

Coherences

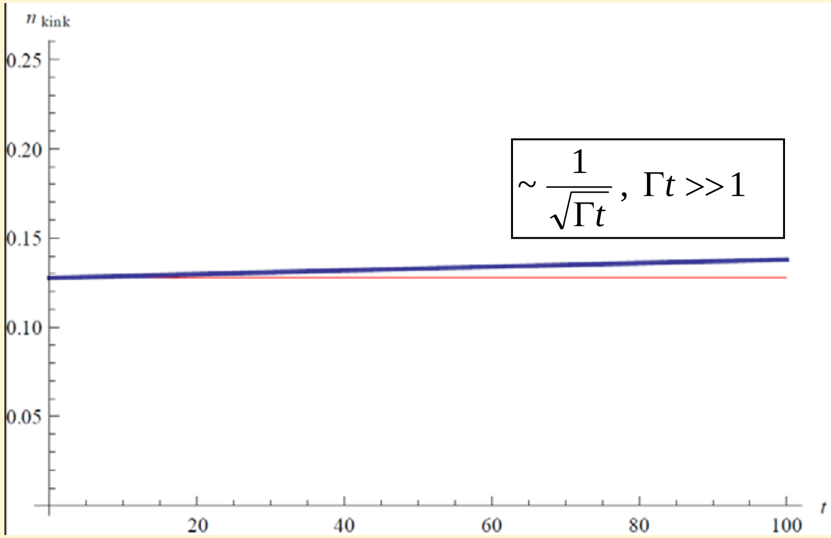
$$\left\langle \gamma_k^+(t)\gamma_{-k}^+(t) \right\rangle$$

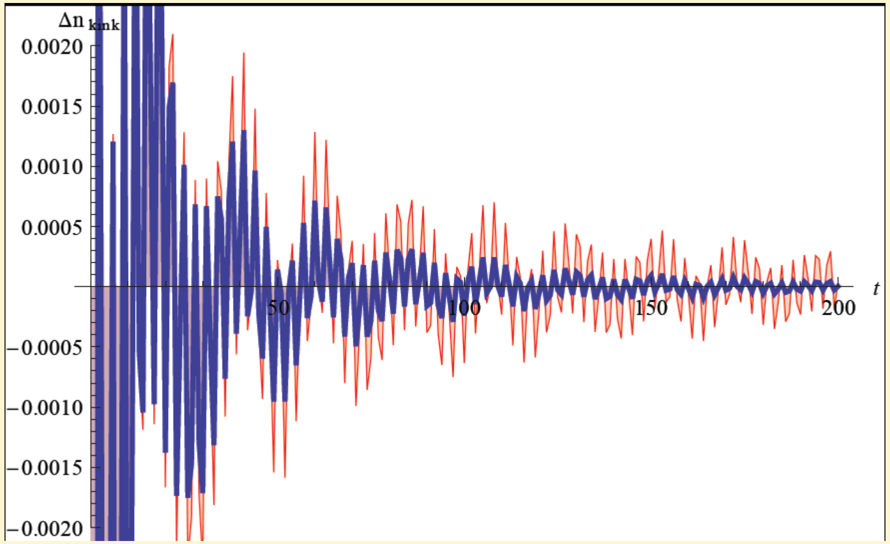


Coherences decay exponentially fast as $e^{-\Gamma t}$, and they are subleading compared to populations for $\Gamma t >> 1$.

Number of kinks

$$n_{kink}(t) = n_{drift}(t) + \Delta n(t)$$





$$E(t) \sim \int dk E_k \left\langle \gamma_k^+ \gamma_k \right\rangle \sim \frac{1}{\sqrt{\Gamma t}}$$

Conclusions

- We study the dynamics of thermalization resulting from a time-dependent noise in a Quantum Ising Chain subject to a sudden quench of *g*.

- For weak noises the dynamics shows a pre-thermalized state at intermediate time scales, eventually drifting towards an asymptotic inifinite temperature state, charatezied by a diffusive behaviour in the transverse magnetization correlations.

Appendix

Appendix B

Appendix C

Transverse Magnetization Correlator

$$\langle m(r,t)m(0,t)\rangle = \langle m(r,t)m(0,t)\rangle_{pop.} +$$
$$+ \langle m(r,t)m(0,t)\rangle_{coh.} + \langle m(r,t)m(0,t)\rangle_{mix.}$$

$e^{-\Gamma t}$ $\Gamma t >> 1$ $e^{-\Gamma t}$

$$\int_{-\infty}^{+\infty} dk\, e^{ikr} e^{-\frac{\Gamma k^2 t}{\Delta^2}}$$

$$\langle m(r,t)m(0,t)\rangle_c = 4\int_{-\infty}^{\infty}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}e^{i(k-k')r}$$
$$\left(\frac{1}{4} + \frac{k}{E_k}\delta f_k(t)\frac{k'}{E'_k}\delta f_{k'}(t) - \frac{m^2}{E_k E_{k'}}\delta f_k(t)\delta f_{k'}(t)\right)$$

$$\delta f_k(t) = \frac{1}{2}\left(\frac{k^2}{2\Delta_1^2}\rho_-^2 - 1\right)e^{\frac{-\Gamma k^2 t}{\Delta_1^2}}$$

Order Parameter Correlator

$$\rho^x_{lm} \quad = \quad \langle 0|B_l A_{l+1} B_{l+1} \dots A_{m-1} B_{m-1} A_m|0\rangle$$

$$A_i = c_i^\dagger + c_i \quad B_i = c_i^\dagger - c_i$$

$$D_n[f] = det(T_n) = det|s(j-k)|^n_{j,k=0}$$

$$T_n = \begin{pmatrix} s(0) & s(-1) & s(-2) & \dots & s(-n) \\ s(1) & s(0) & s(-1) & \dots & s(1-n) \\ s(2) & s(1) & s(0) & \dots & s(2-n) \\ \dots & \dots & \dots & \dots & \dots \\ s(n) & s(n-1) & s(n-2) & \dots & s(0) \end{pmatrix}$$

$$\langle B_i A_j \rangle = s(n) = \frac{dk}{2\pi} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} e^{-ik(i-j)} e^{i2\theta_k} \delta\ f_k$$

$$\delta f_k = -\frac{1}{2} e^{-\Gamma t \sin^2 2\vartheta(g)}$$

$$i - j = n$$

For large *n* we apply Fisher-Hartwig conjecture.