The Abdus Salam
International Centre
for Theoretical Physics

2449-6

38th Conference of the Middle European Cooperation in Statistical Physics - MECO38

25 - 27 March 2013

Quantum Phases and Non-equilibrium Dynamics with Rydberg Gases

Tommaso MACRI'
Max Planck Institute for the Physics of Complex Systems
Dresden
Germany

Quantum phases and non-equilibrium dynamics with Rydberg gases

Tommaso Macrì

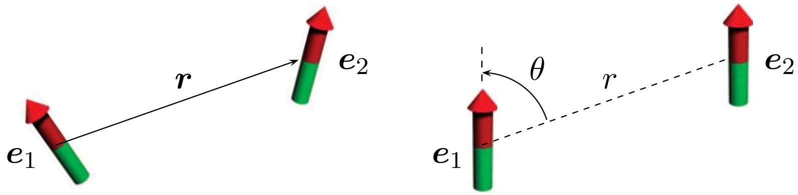
Max Planck Institute
for the Physics of Complex Systems, Dresden (Germany)

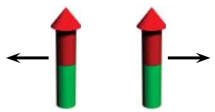
Trieste, March 25 2013

Motivation

Manipulation of ultracold atomic gases with long range interactions

Magnetic atoms & Polar molecules





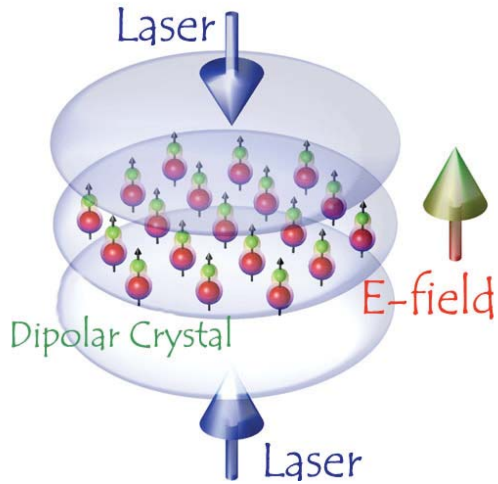
Chromium BEC (Stuttgart - Pfau)

Dysprosium BEC (Illinois - Lev)

Erbium BEC (Innsbruck - Ferlaino)

K-Rb Molecules (Jila - Jin, Ye)

Na-K Molecules (MIT, Zwierlein & Trento, Ferrari and Lamporesi)



H. P. Büchler, E. Demler, M.Lukin, A. Micheli, N. Prokof’ev, G. Pupillo and P. Zoller, Phys. Rev. Lett. **98**, 060404 (2007)

T. Lahaye, C. Menotti, L. Santos, M. Lewenstein and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009)

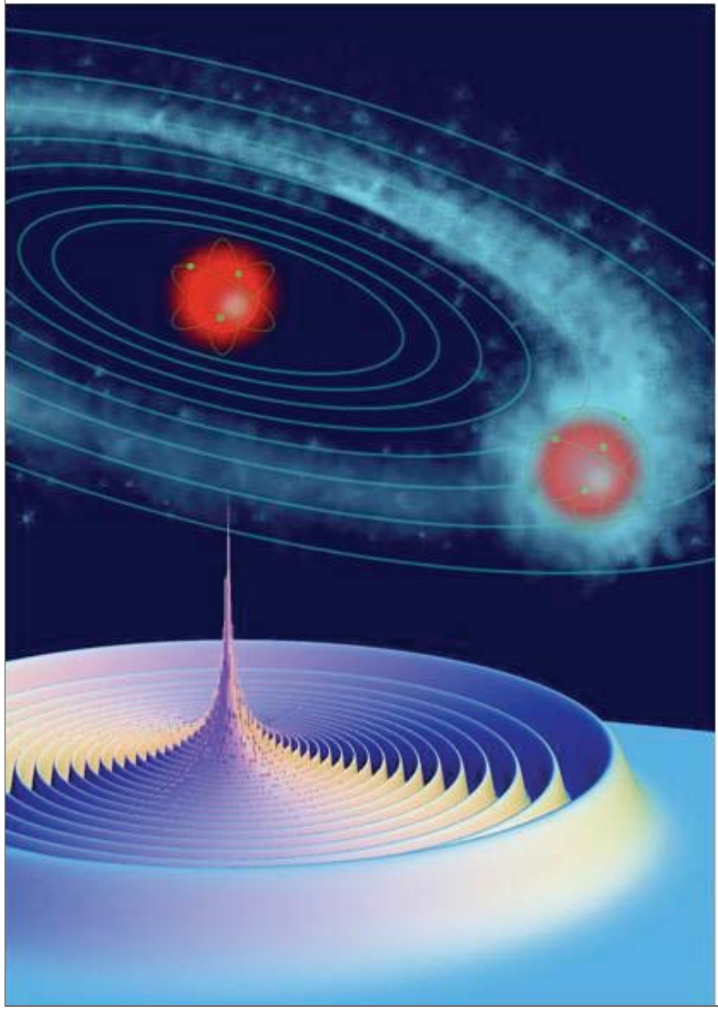
C. Trefzger, C. Menotti, B. Capogrosso-Sansone and M. Lewenstein, J. Phys. B **44**, 193001 (2011)

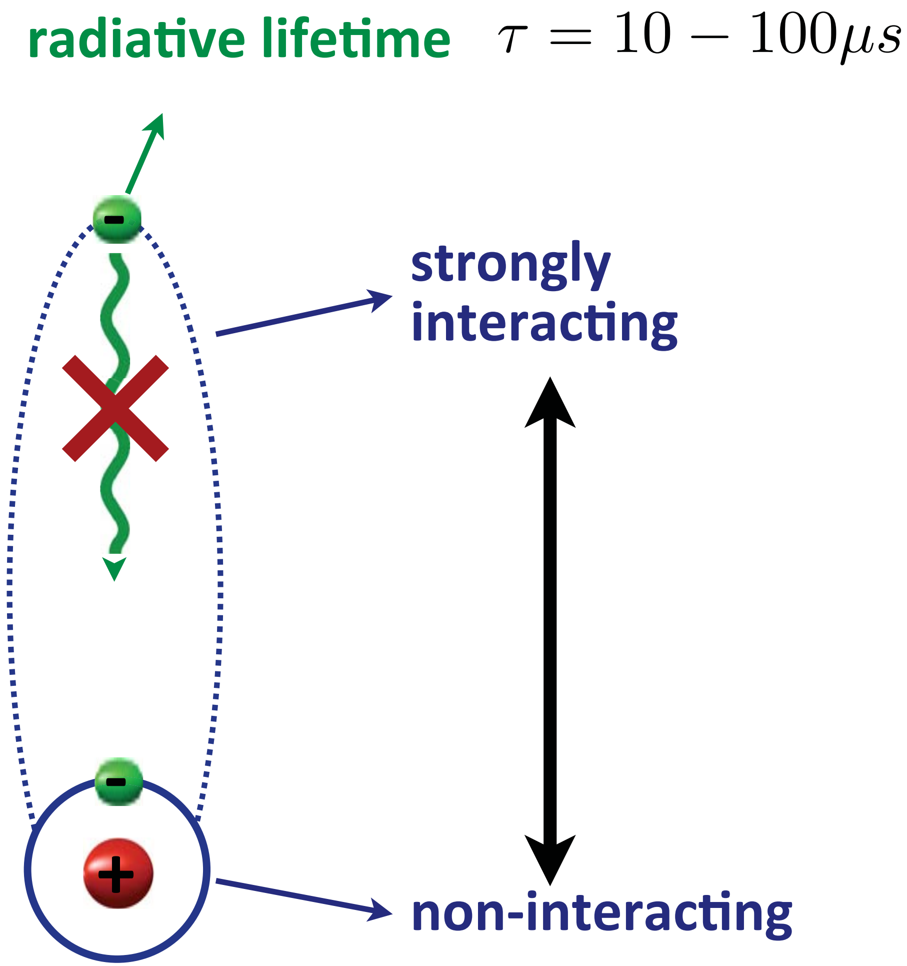
M. Baranov, M. Dalmonte, G. Pupillo and P. Zoller, Chemical Reviews **112**, 5012 (2012)

Motivation

Manipulation of ultracold atomic gases with long range interactions

Rydberg gases

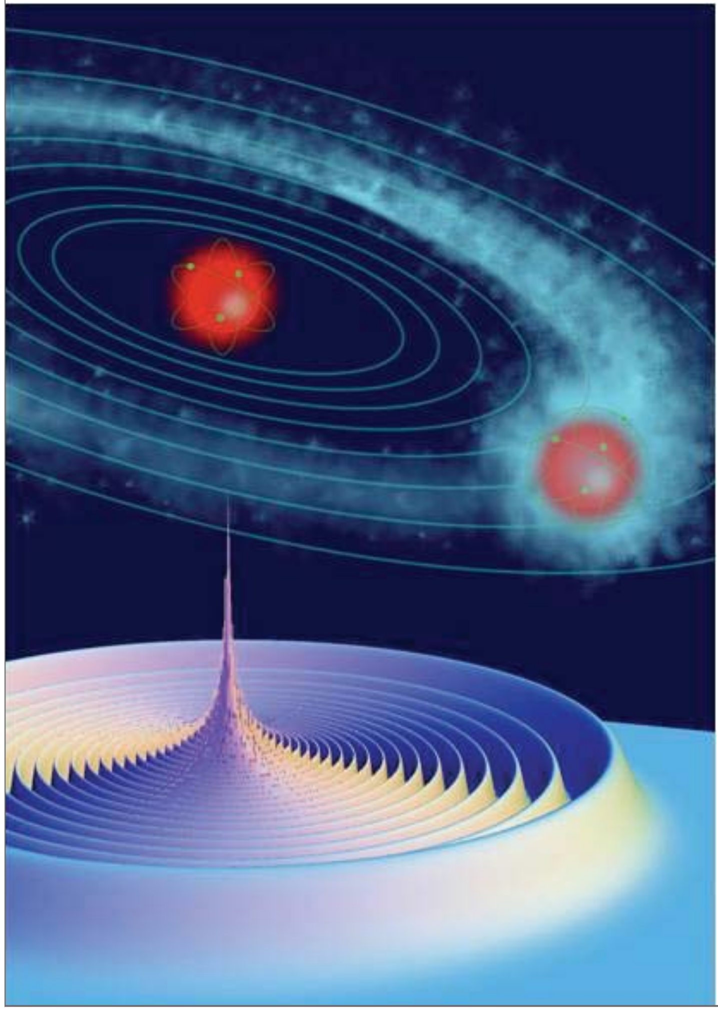




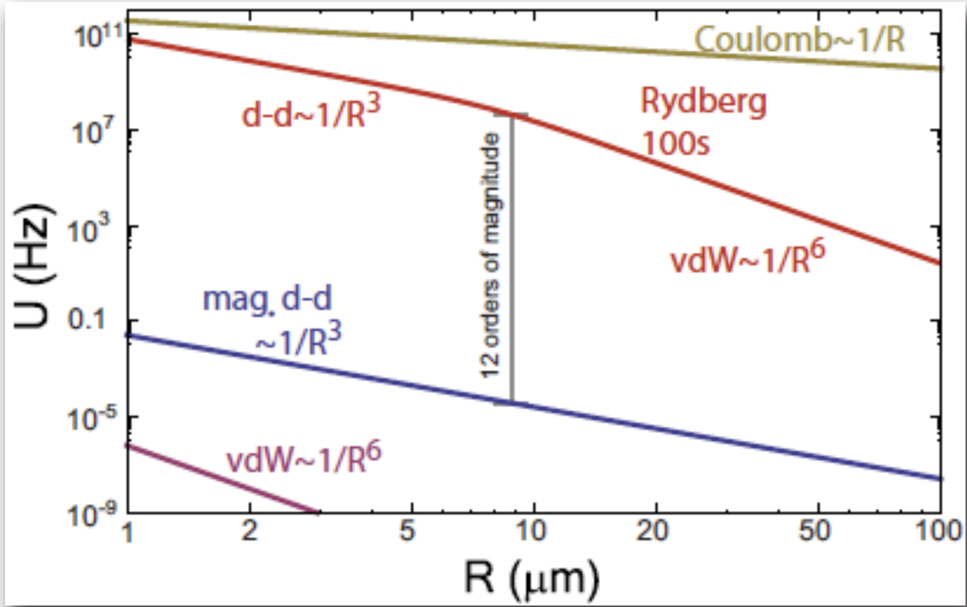
Motivation

Manipulation of ultracold atomic gases with long range interactions

Rydberg gases



Strong interactions compared to other systems



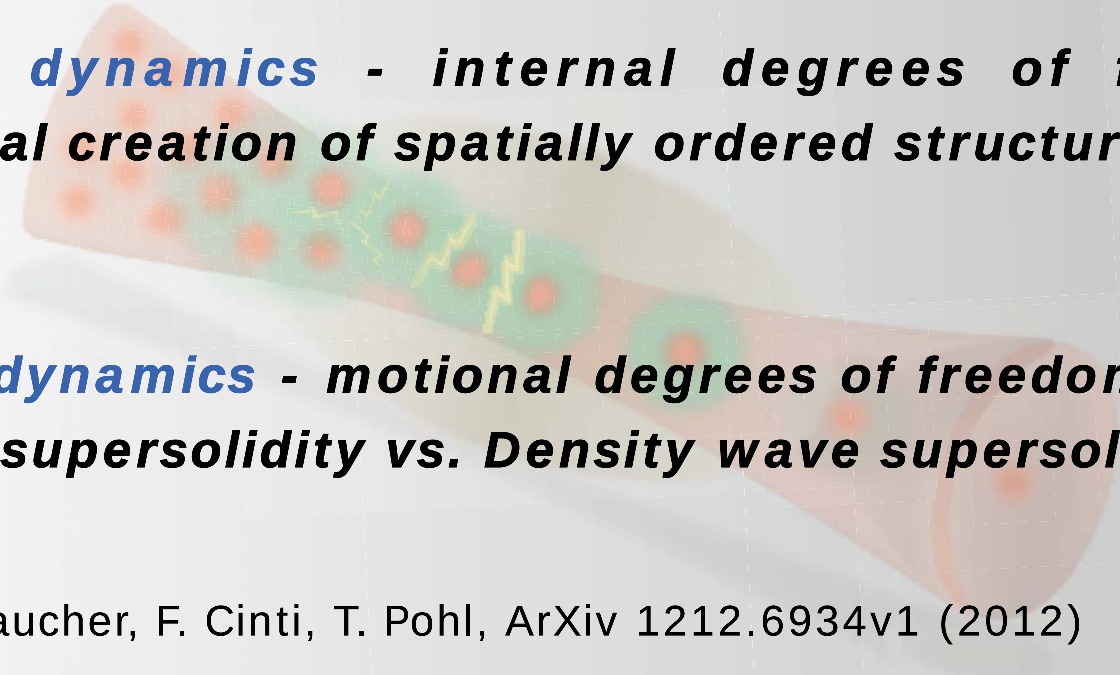
M. Saffman et al., Rev. Mod. Phys. **82**, 2313 (2010)

Motivation

Manipulation of ultracold atomic gases with long range interactions

Rydberg gases

COHERENT MANY BODY PHYSICS



- ***Fast dynamics - internal degrees of freedom: dynamical creation of spatially ordered structures in spin models***

- ***Slow dynamics - motional degrees of freedom: defect-induced supersolidity vs. Density wave supersolidity***

T. M., F. Maucher, F. Cinti, T. Pohl, ArXiv 1212.6934v1 (2012)

F. Cinti, T. M., W. Lechner G. Pupillo, and T. Pohl, ArXiv 1302.4576v1 (2013)

Motivation

Rydberg gases

COHERENT MANY BODY PHYSICS

- ***Fast dynamics*** *- internal degrees of freedom: dynamical creation of spatially ordered structures in spin models*


Günes Soyler


Thomas Pohl


Immanuel Bloch's group

- ***Slow dynamics*** *- motional degrees of freedom: defect-induced supersolidity vs. Density wave supersolidity*


Fabio Cinti


Fabian Maucher

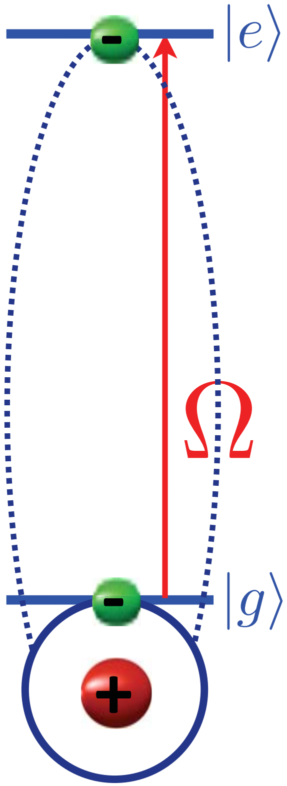

Wolfgang Lechner


Guido Pupillo


Thomas Pohl

Quantum dynamics

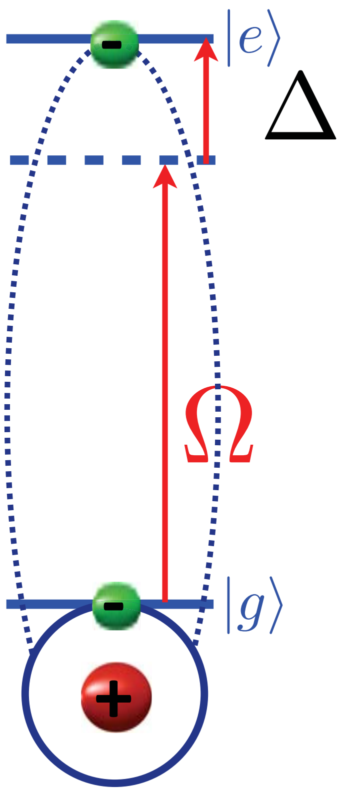
$$\hat{H} = \frac{\Omega}{2} \sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)}$$



Ω Rabi frequency

Quantum dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i}$$



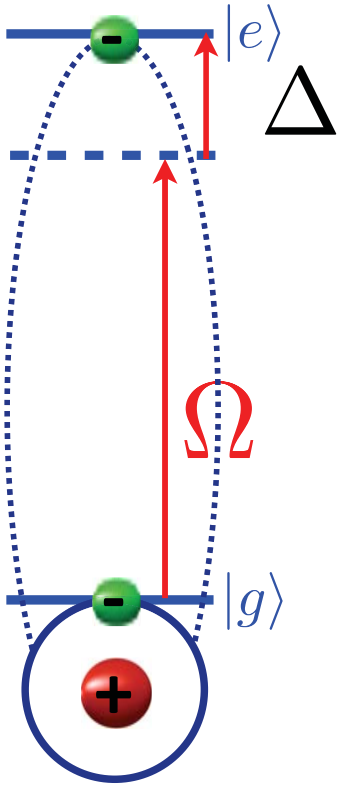
Ω Rabi frequency

Δ Detuning

Quantum dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

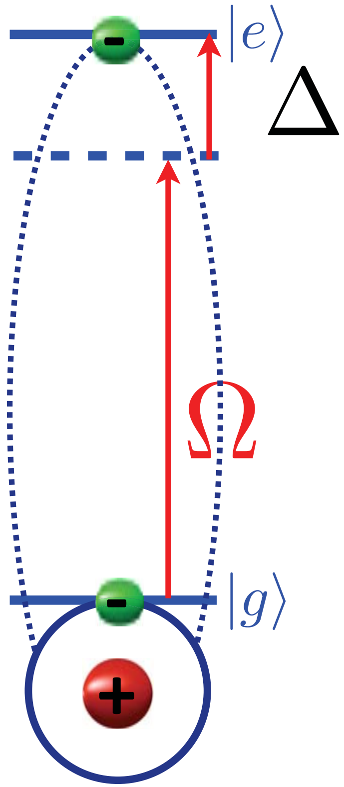
Quantum simulator for long range spin models with longitudinal and transverse field



Ω Rabi frequency

Δ Detuning

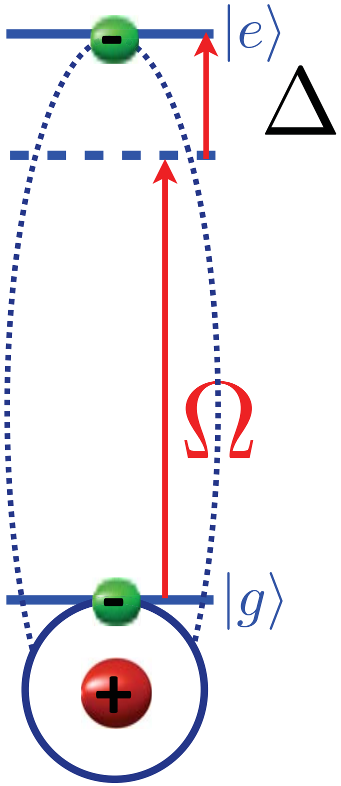
$\dfrac{\tilde{C}_6}{a^6}$ van der Walls coefficient



Quantum dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

Quantum simulator for long range spin models with longitudinal and transverse field

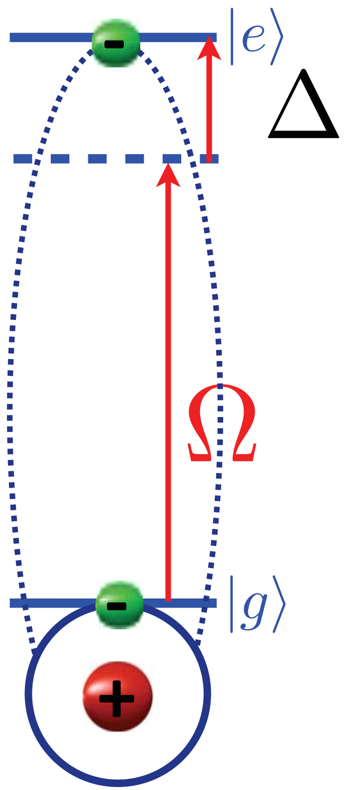


$\Omega \sim 0.3$ h MHz

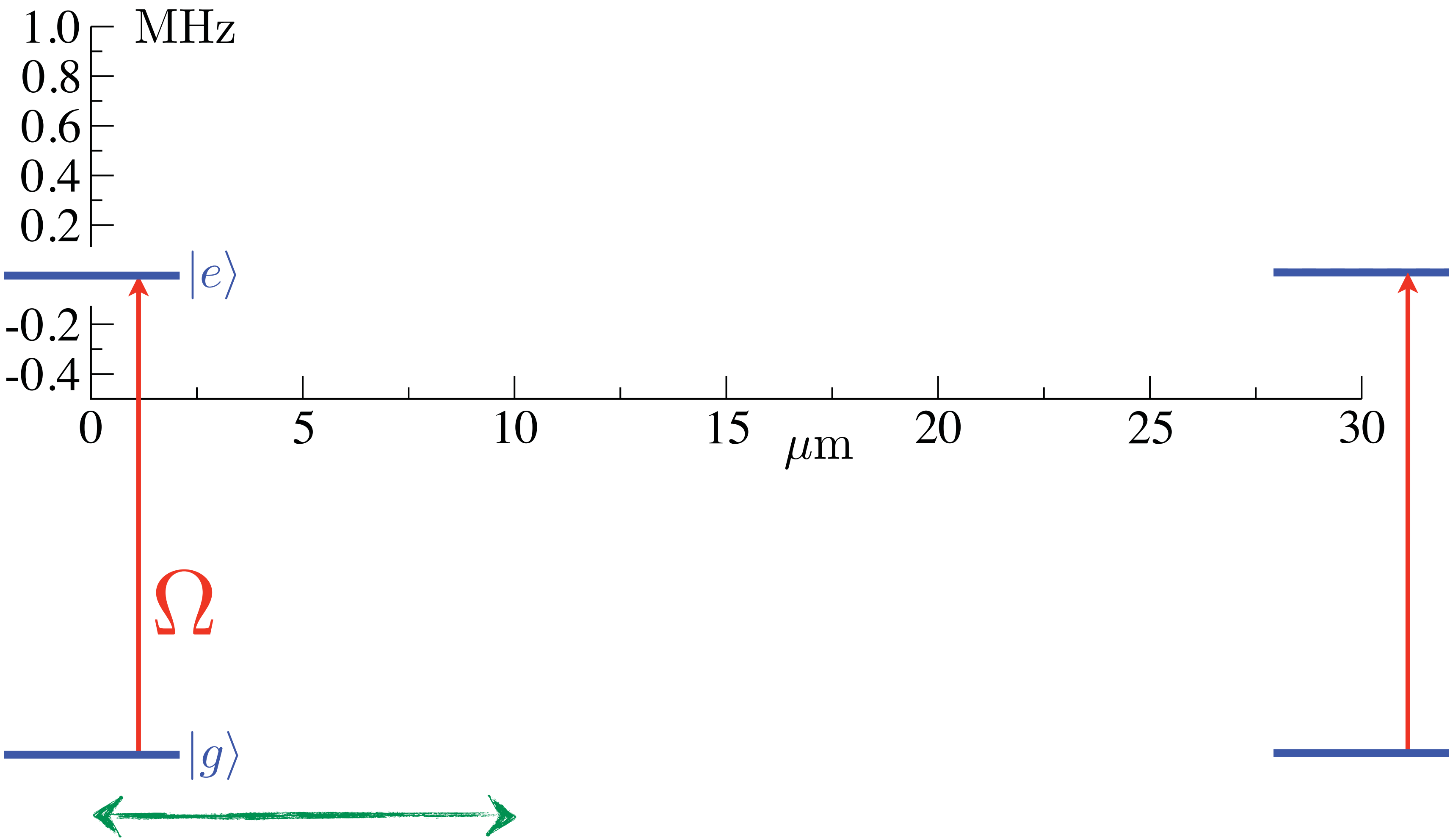
$\Delta \sim 1 - 5$ h MHz

$\frac{\tilde{C}_6}{a^6} \sim 100$ h GHz @ n = 43

Energy scales: interaction is by far the largest scale



Excitation blockade

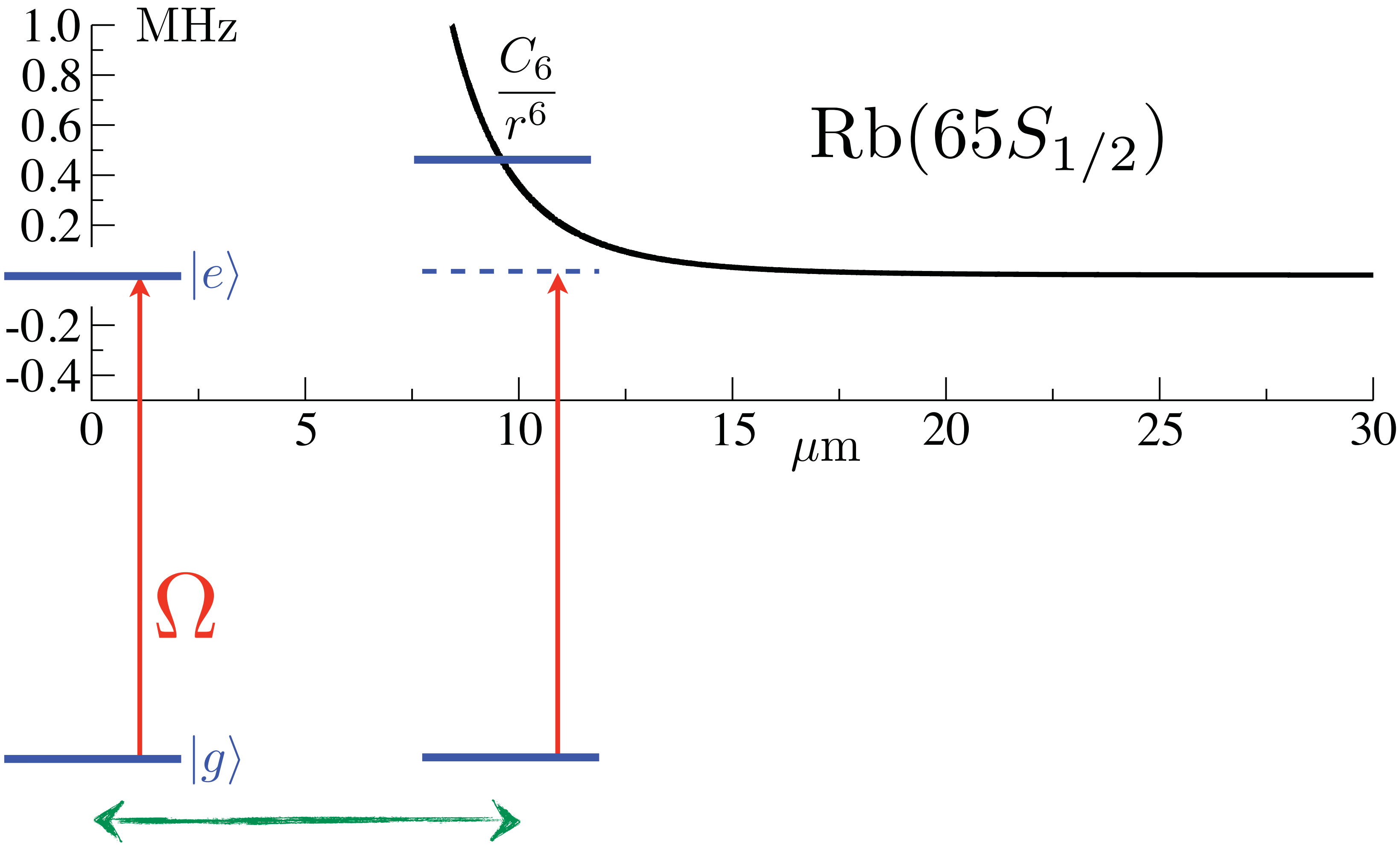


$R_0 \approx 10\mu m$ $a \approx 0.1\mu m$

Blockade radius $\gg$ **Interparticle distance**

Simultaneous excitations blocked within blockade sphere

Excitation blockade



$\mathrm{R}_0 \approx 10\mu\mathrm{m}$ $\qquad$ $a \approx 0.1\mu\mathrm{m}$

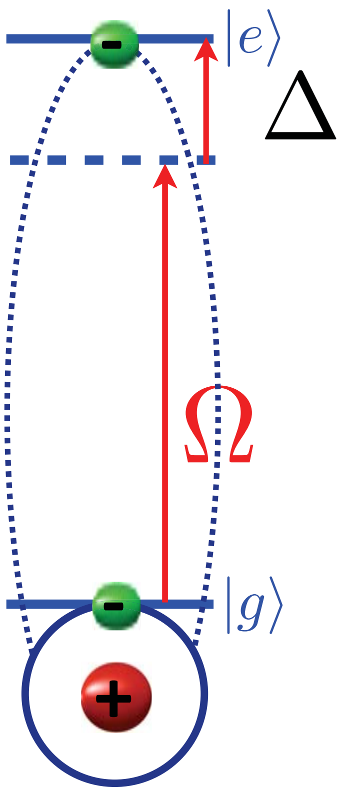
Blockade radius $\gg$ **Interparticle distance**

Simultaneous excitations blocked within blockade sphere

Quantum dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

Quantum simulator for long range spin models with longitudinal and transverse field

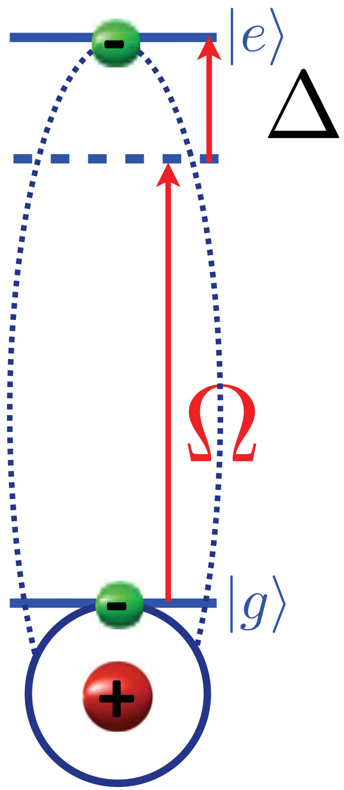


$\Omega \quad \sim 0.3 \text{ h MHz}$

$\Delta \quad \sim 1 - 5 \text{ h MHz}$

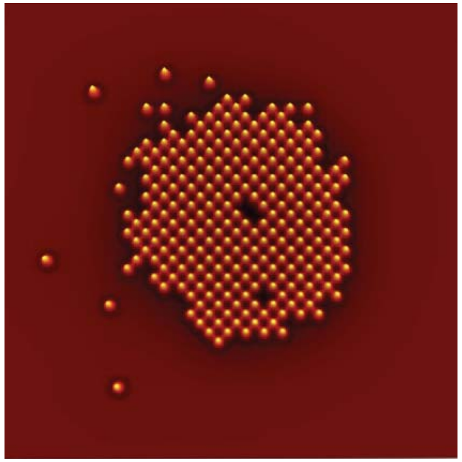
$\frac{\tilde{C}_6}{a^6} \quad \sim 100 \text{ h GHz @ n} = 43$

- *Energy scales: interaction is by far the largest scale*
- *Time scales: atoms are frozen, only spin degrees of freedom*



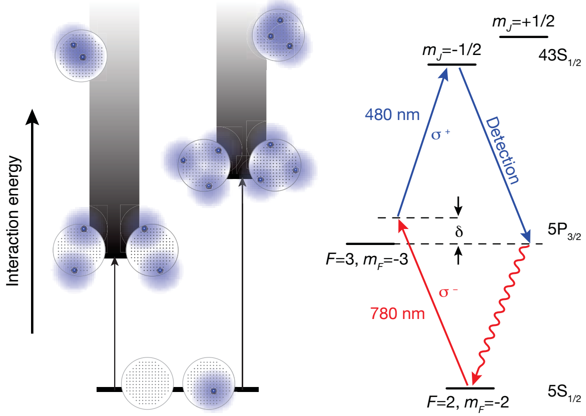
Experiment

Step 1: Deep Mott insulator

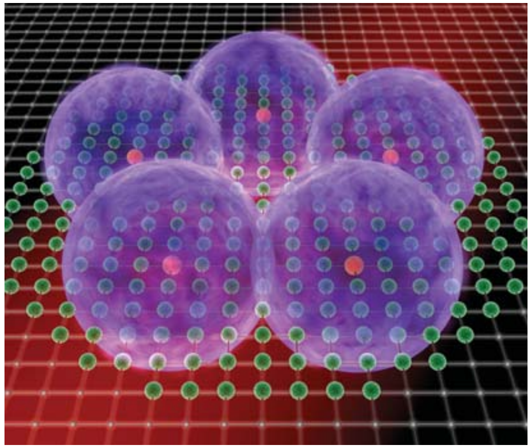


Step 2: Fast resonant excitation

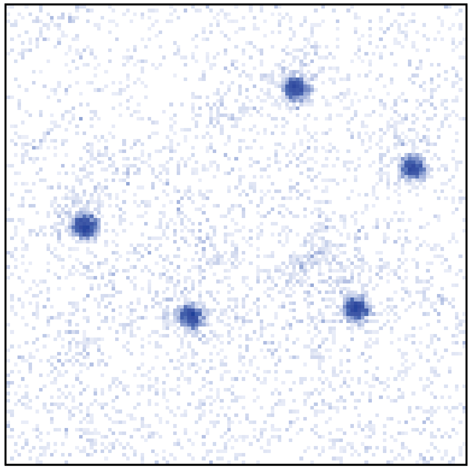
coherent dynamics



Step 3: Remove optically the ground state atoms

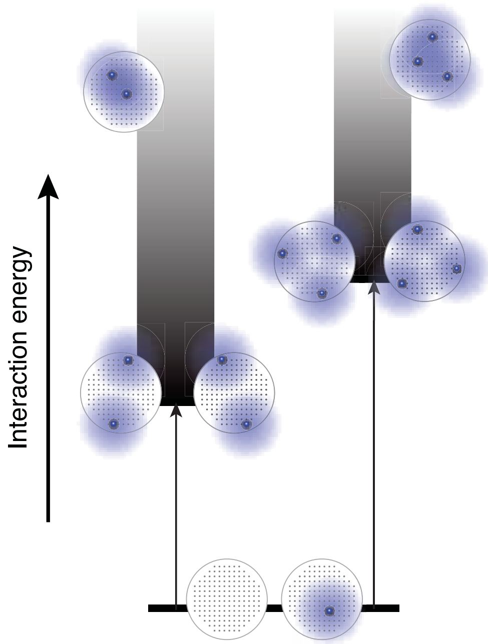


Step 4: De-excite and detect Rydberg atoms

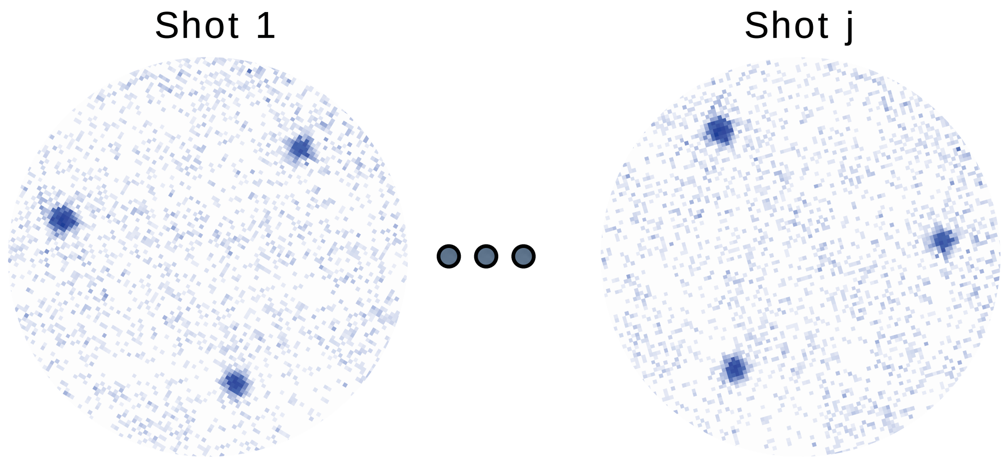


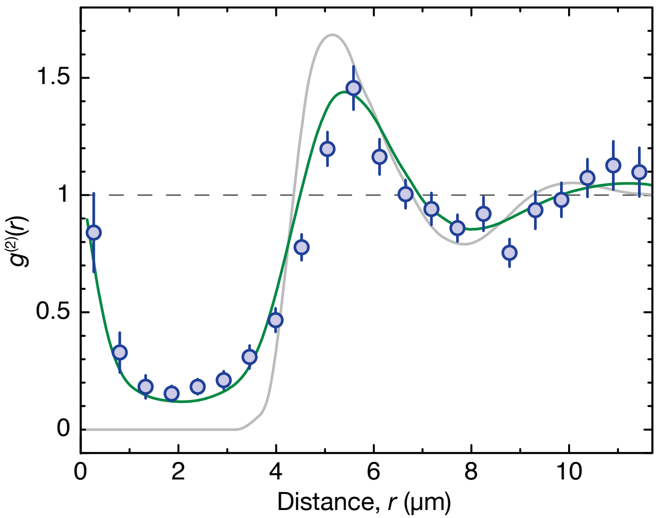
P. Schauß, M. Chenau, M. Endres, T. Fukuhara, S. Hild, A. Omran, T. Pohl, C. Gross, S. Kuhr, I. Bloch, Nature 491, 87 (2012)

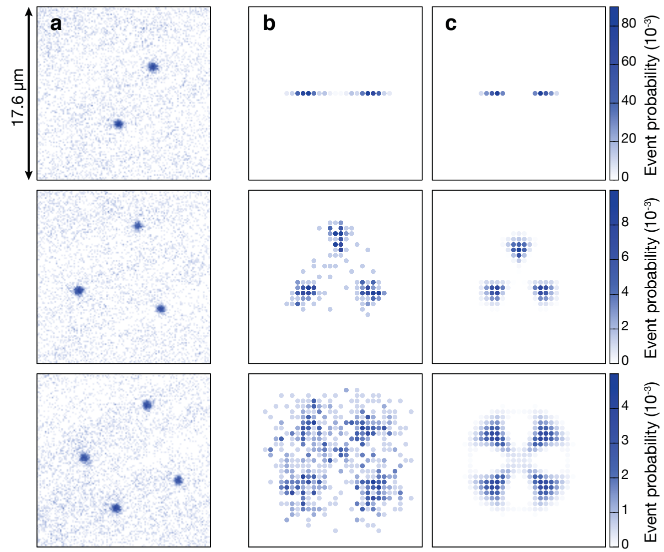
Quench dynamics



Step 5: alignement

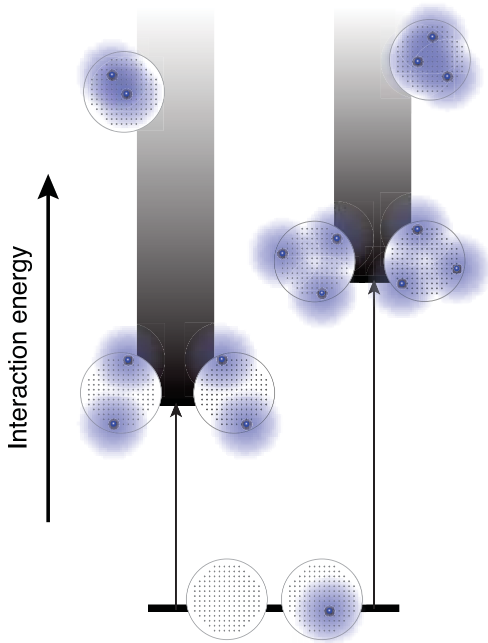




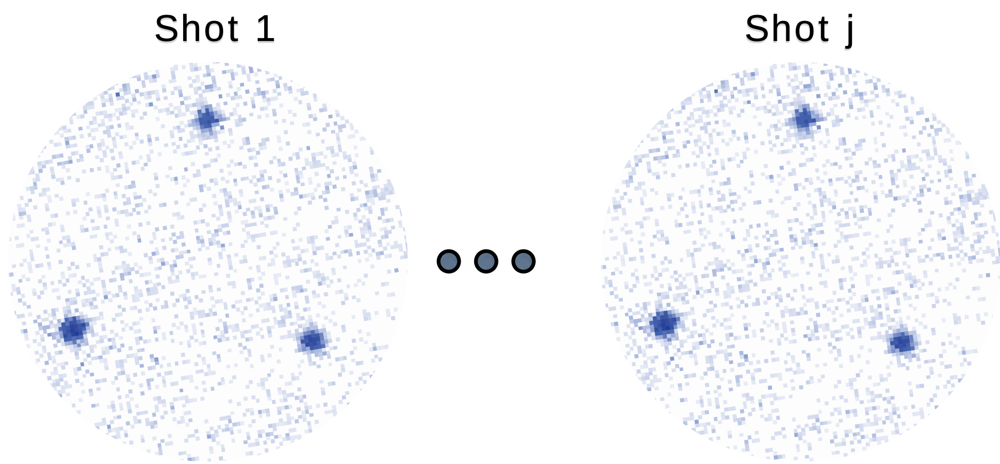


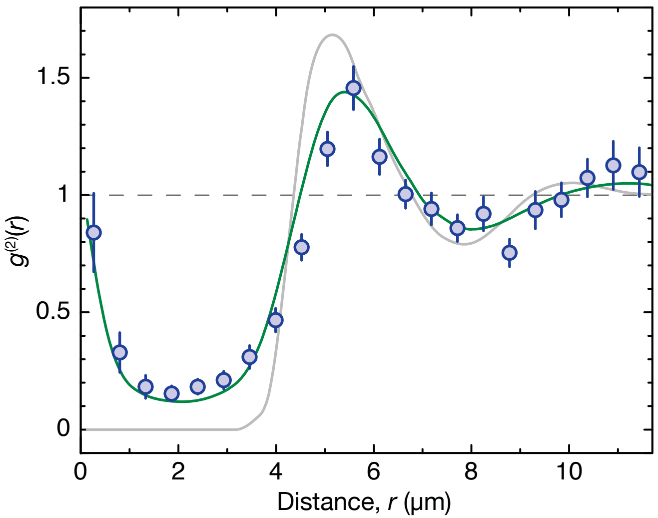
P. Schauß, M. Chenau, M. Endres, T. Fukuhara, S. Hild, A. Omran, T. Pohl, C. Gross, S. Kuhr, I. Bloch, Nature 491, 87 (2012)

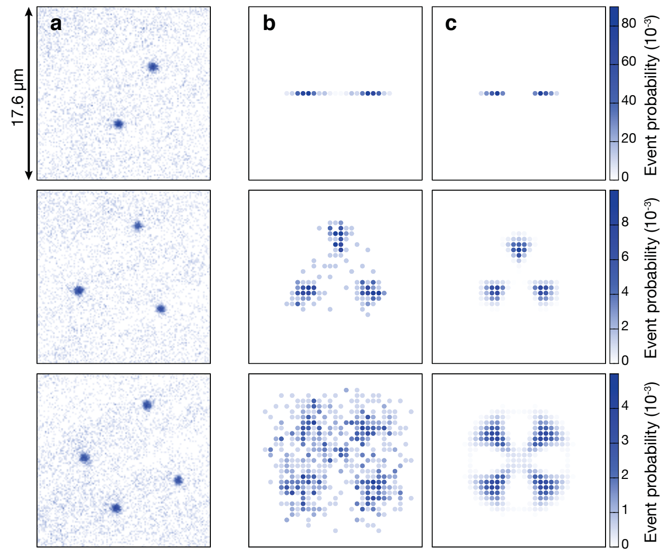
Quench dynamics



Step 5: alignement



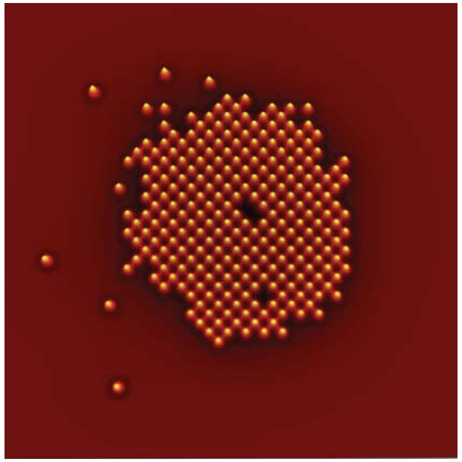




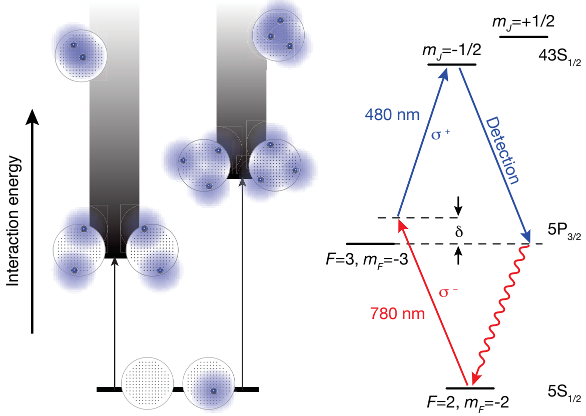
P. Schauß, M. Chenau, M. Endres, T. Fukuhara, S. Hild, A. Omran, T. Pohl, C. Gross, S. Kuhr, I. Bloch, Nature 491, 87 (2012)

Experiment (ongoing)

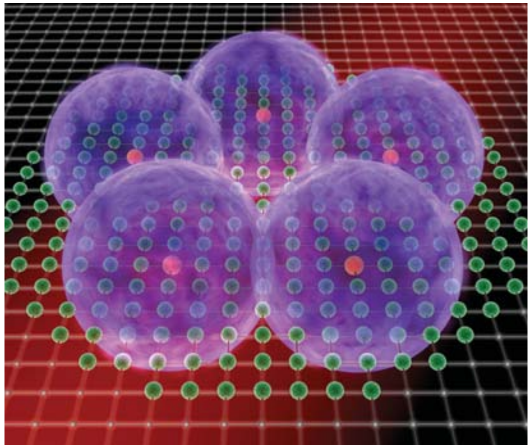
Step 1: Deep Mott insulator



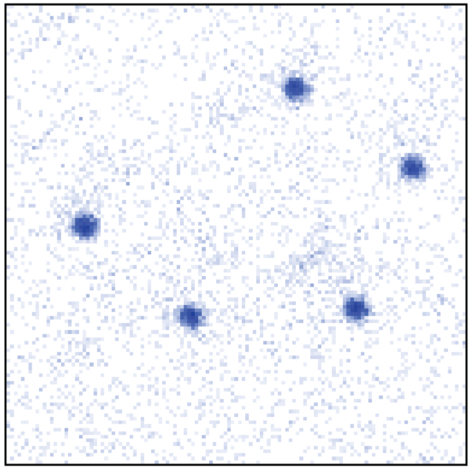
Step 2: Off-resonant non constant excitation coherent dynamics



Step 3: Remove optically the ground state atoms



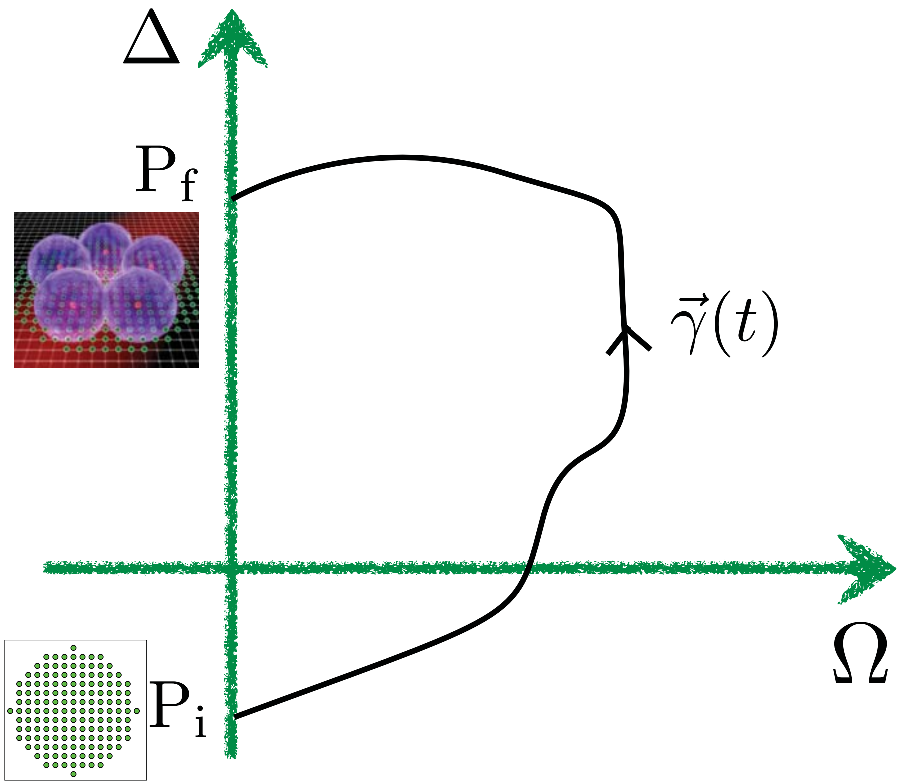
Step 4: De-excite and detect Rydberg atoms

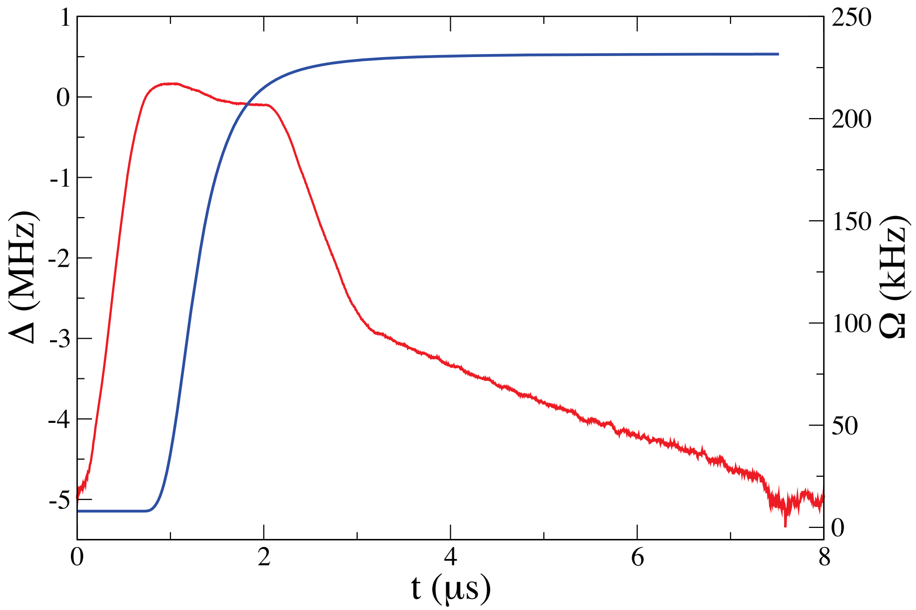


Finite time dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

- Create ordered structures
- Optimal (chirped) pulse to fit into few μs

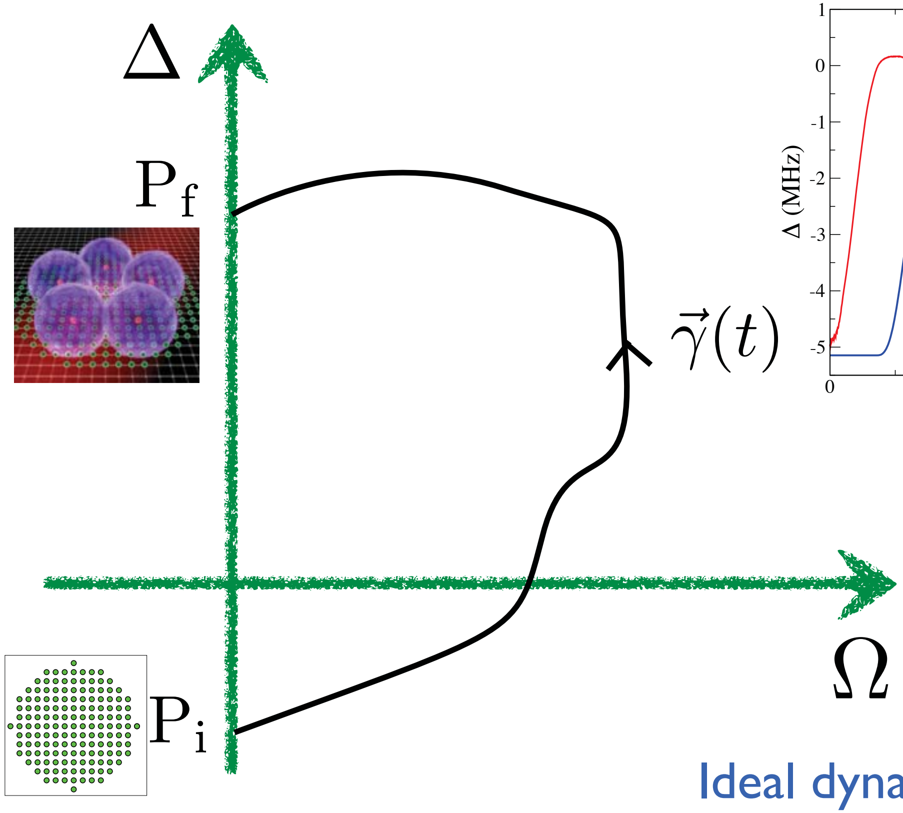


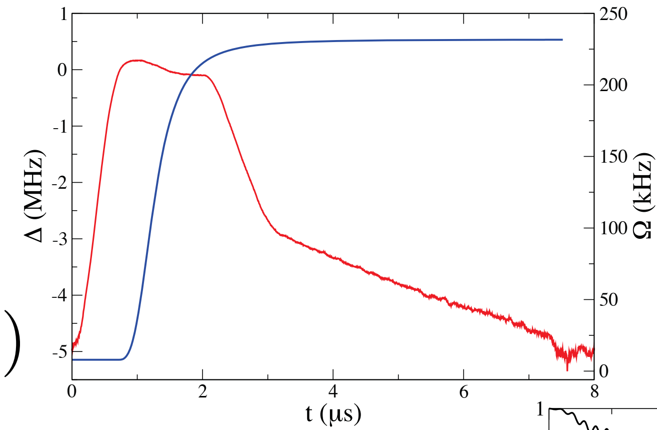


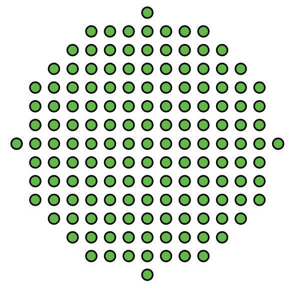
Finite time dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

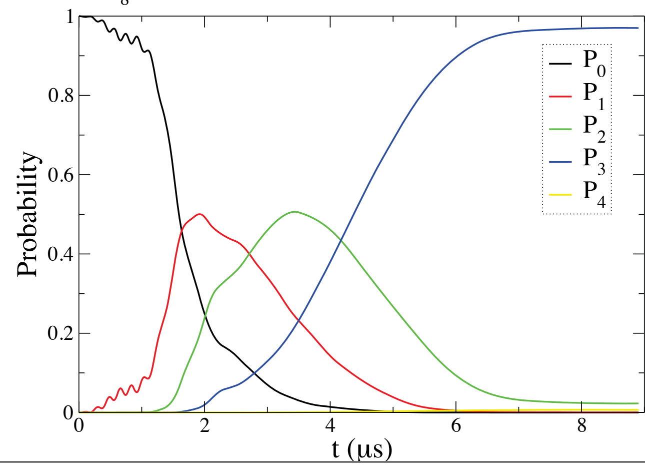
- Create ordered structures
- Optimal (chirped) pulse to fit into few μs







97% probability to excite a state with 3 excitations in a ideal Mott insulator with 149 particles

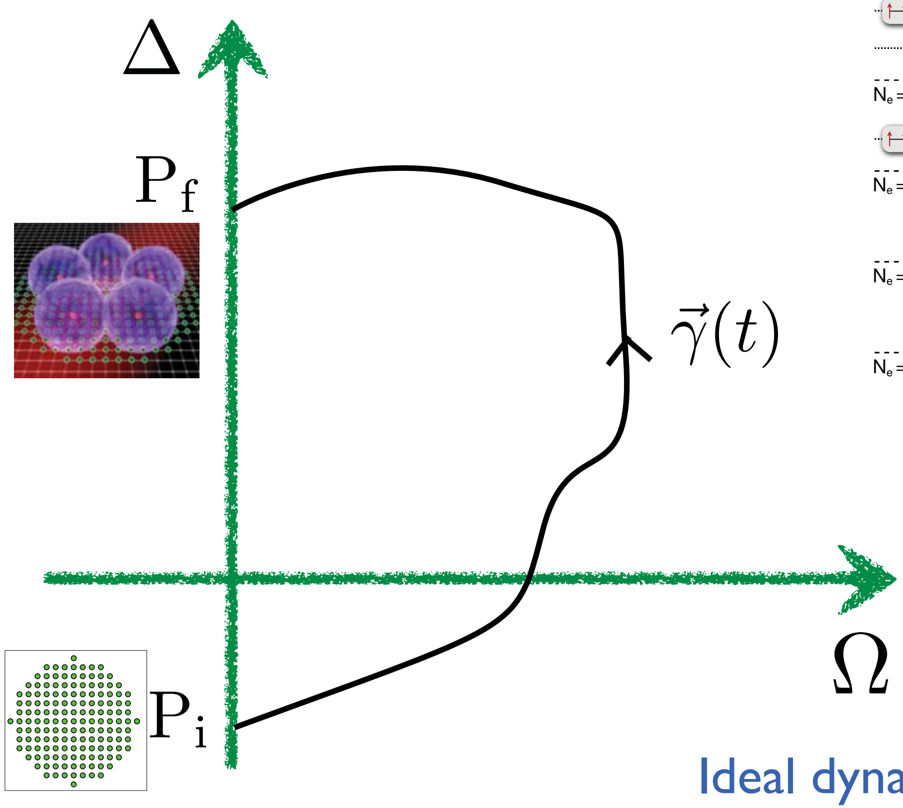


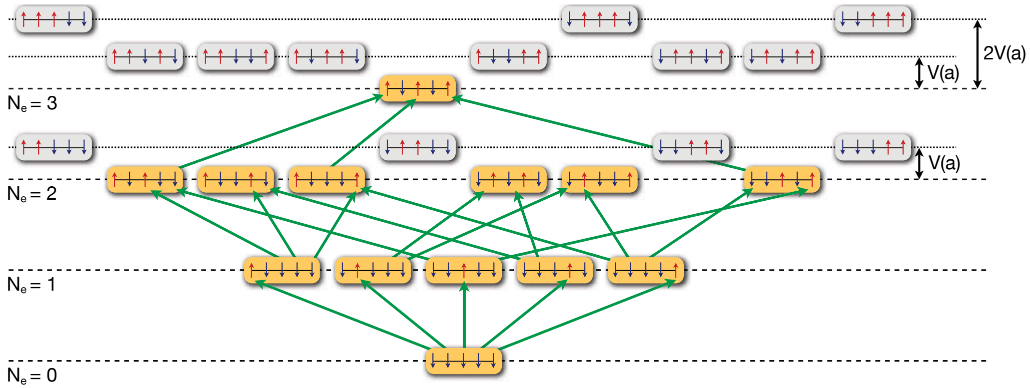
Ideal dynamical evolution

Finite time dynamics

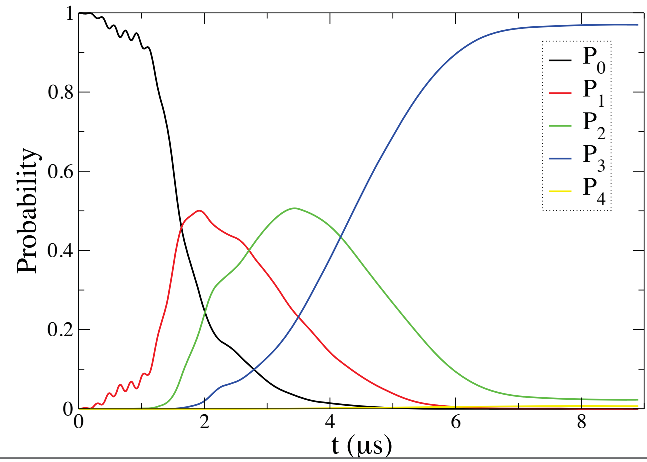
$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6}\hat{\sigma}_{ee}^{(i)}\hat{\sigma}_{ee}^{(j)}$$

- Create ordered structures
- Optimal (chirped) pulse to fit into few μs





Truncated Hilbert space by removing excitation blockaded configurations

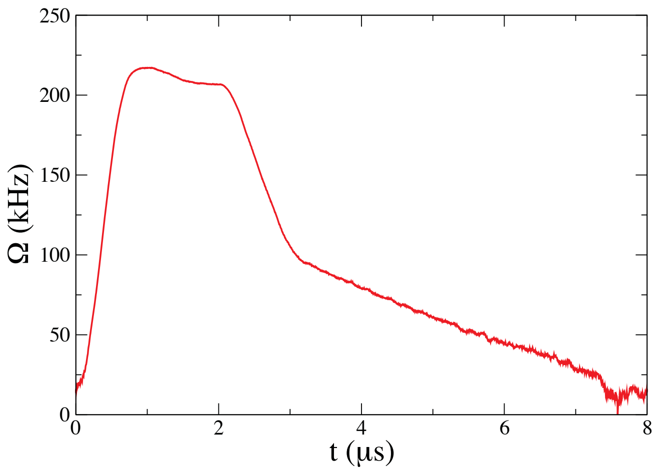


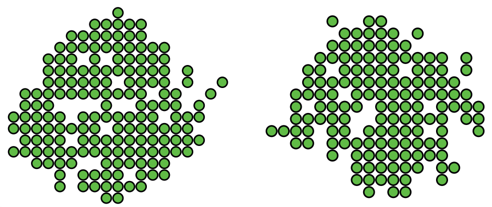
Ideal dynamical evolution

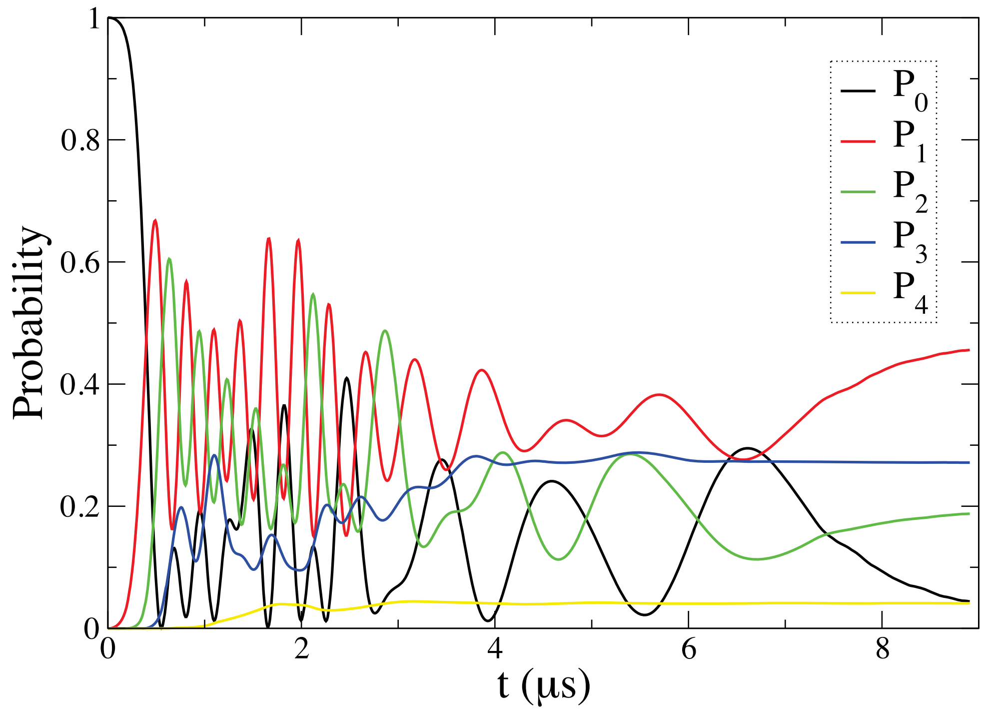
Finite time dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

Dynamics without longitudinal field Δ=0 (*preliminary experimental results*)





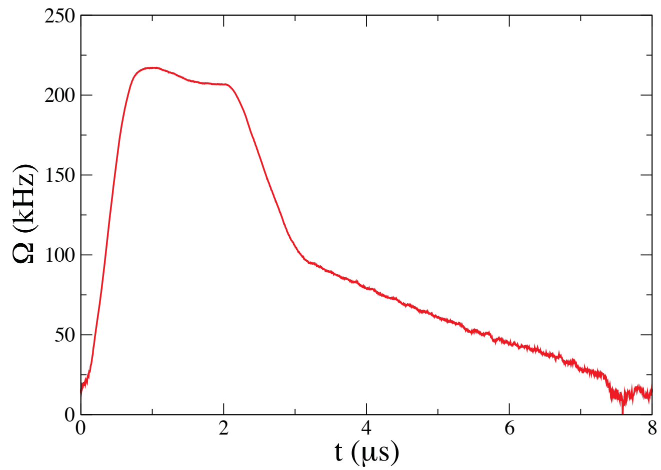


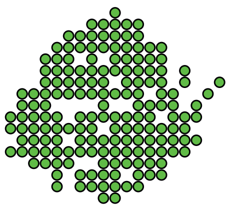
- Average over non-ideal 84 lattice configurations from experiment
- Detection efficiency to image Rydberg atoms ~ 70%

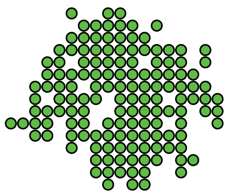
Finite time dynamics

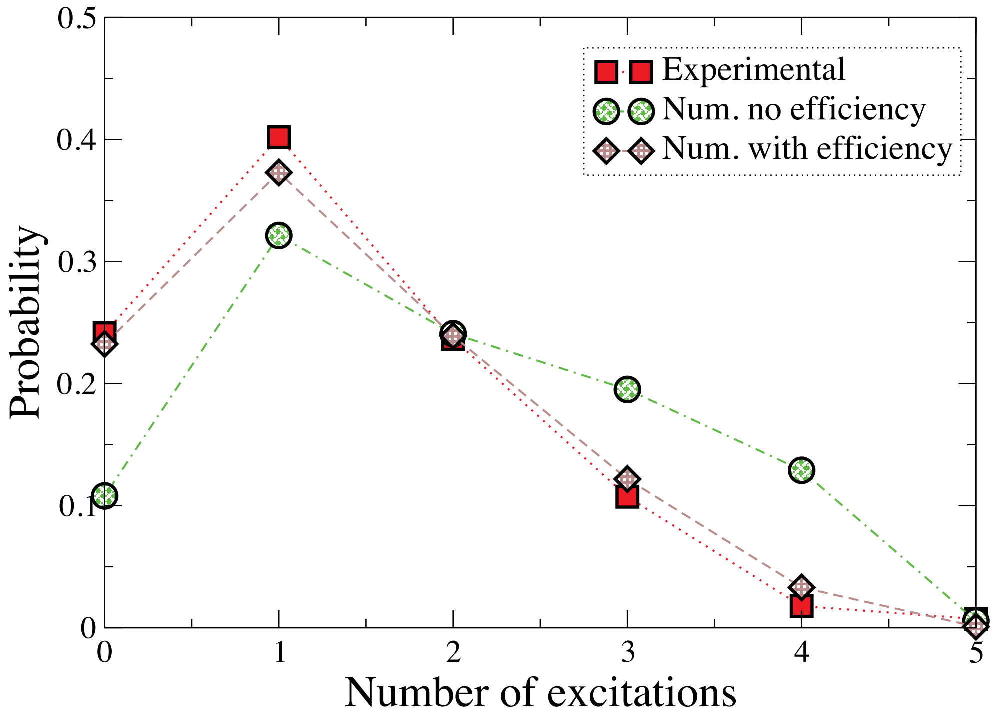
$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6}\hat{\sigma}_{ee}^{(i)}\hat{\sigma}_{ee}^{(j)}$$

Dynamics without longitudinal field Δ=0 (*preliminary experimental results*)







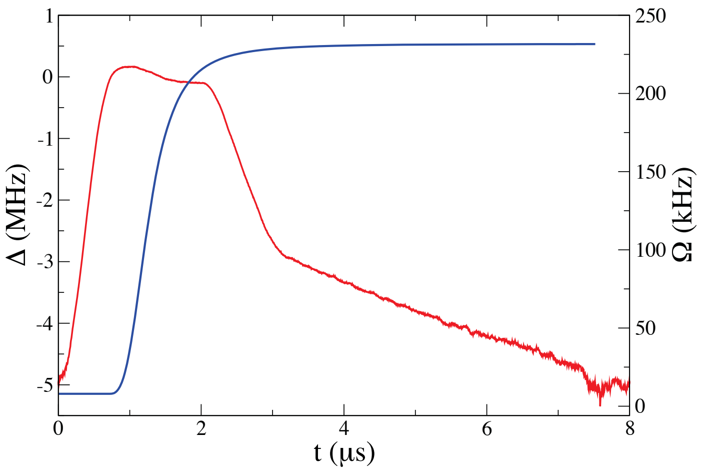


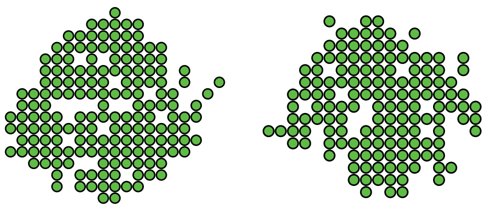
- Average over non-ideal 84 lattice configurations from experiment
- Detection efficiency to image Rydberg atoms ~ 70%

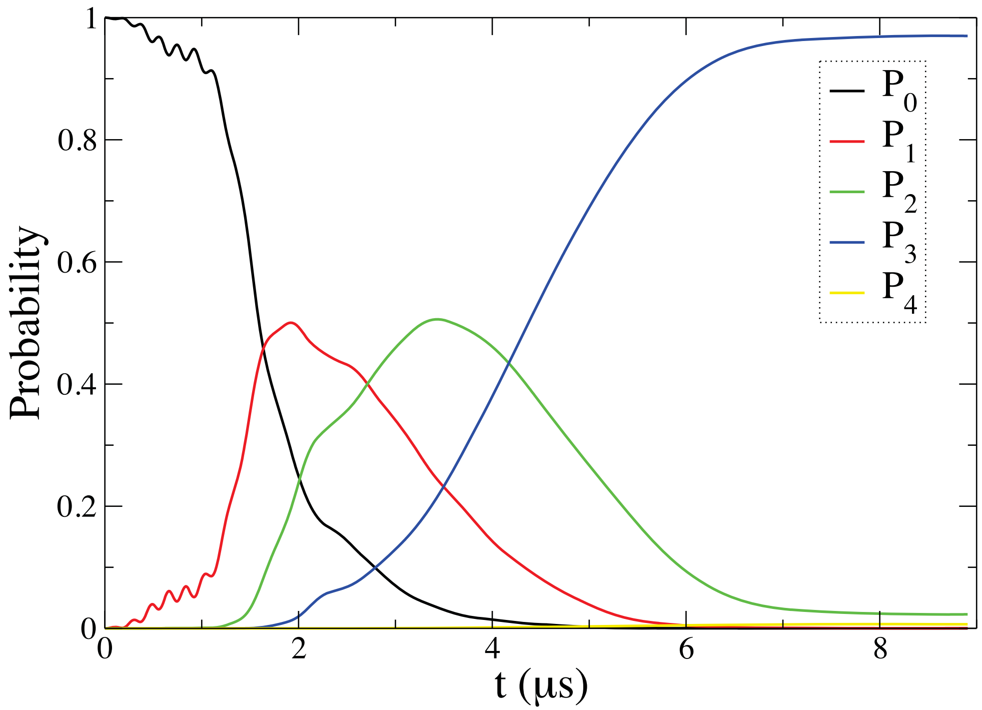
Finite time dynamics

$$\hat{H} = \frac{\Omega}{2}\sum_i \hat{\sigma}_{eg}^{(i)} + \hat{\sigma}_{ge}^{(i)} - \Delta \sum_i \hat{\sigma}_{ee}^{i} + \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6} \hat{\sigma}_{ee}^{(i)} \hat{\sigma}_{ee}^{(j)}$$

Dynamics with longitudinal and transverse fields (*numerical results only*)





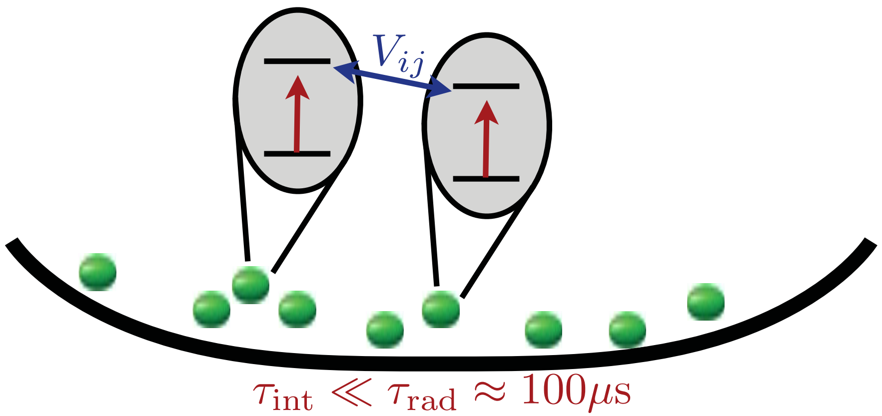


- Average over non-ideal 84 lattice configurations from experiment
- Detection efficiency to image Rydberg atoms ~ 70%

Slow dynamics

ATOMIC MOTION

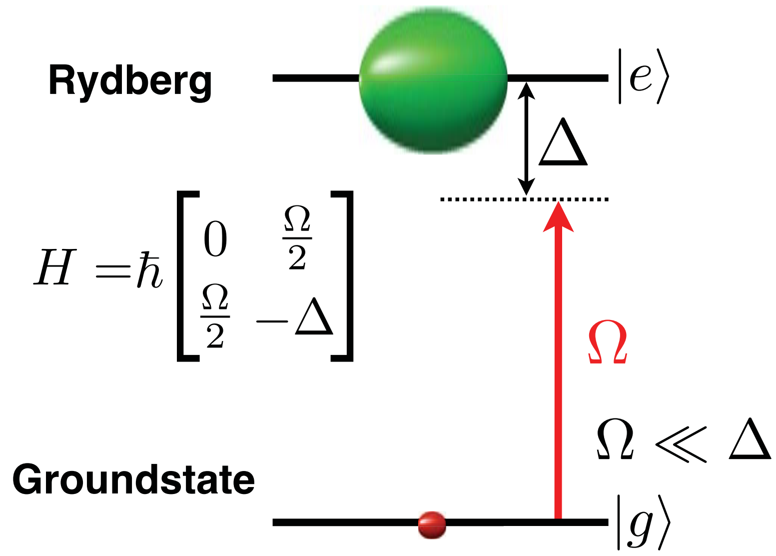
Correlated internal dynamics (spin dynamics)



motional dynamics $\tau_{\rm mot} \gg \tau_{\rm rad}$

... too slow compared to Rydberg state lifetime

RYDBERG DRESSING

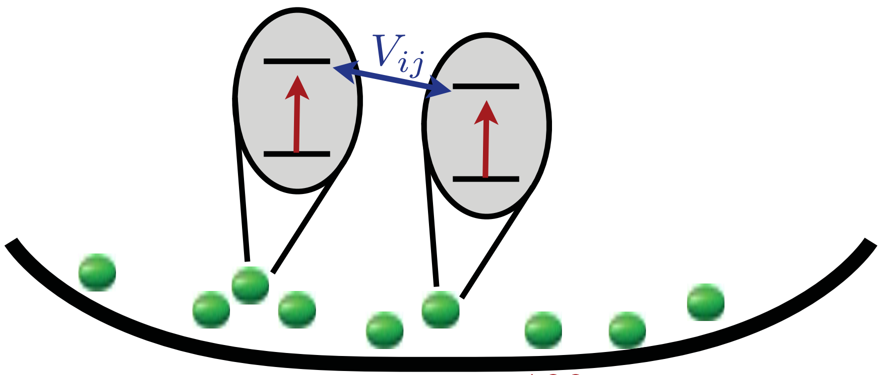


Effective description with one level

Slow dynamics

ATOMIC MOTION

Correlated internal dynamics (spin dynamics)



$\tau_{\text{int}} \ll \tau_{\text{rad}} \approx 100\mu\text{s}$

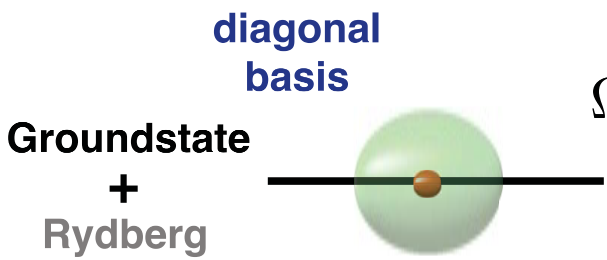
motional dynamics $\tau_{\text{mot}} \gg \tau_{\text{rad}}$

... too slow compared to Rydberg state lifetime

RYDBERG DRESSING

Excitation probability $\text{p}_{\text{ex}} = \left(\frac{\Omega}{2\Delta}\right)^2$

Enhanced lifetime $\tau_{\text{eff}} = \text{p}_{\text{e}}^{-1}\tau_{\text{rad}} \approx 1\text{s}$



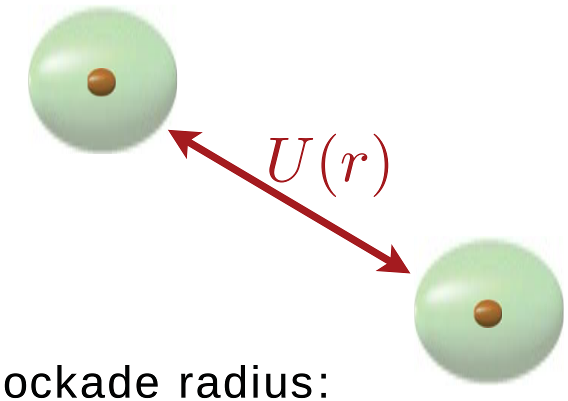
$\Omega \ll \Delta$

$|\tilde{g}\rangle = |g\rangle + \frac{\Omega}{2\Delta}|e\rangle$

Effective description with one level

Soft-core interactions

Rydberg blockade



Blockade radius:

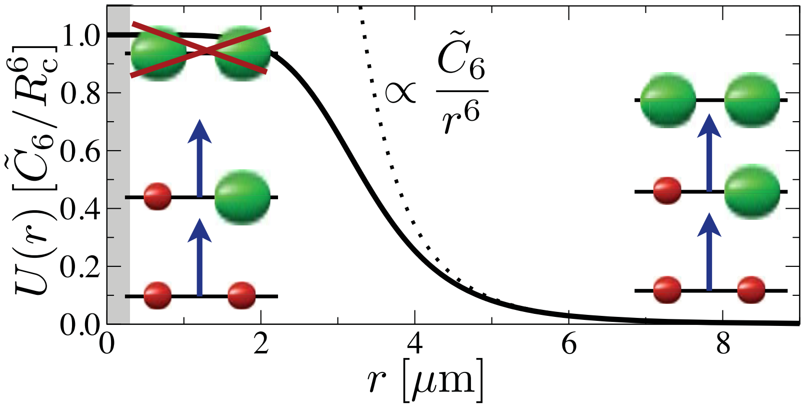
$$R_c = \left(\frac{C_6}{2\Delta}\right)^{1/6}$$

Effective interaction:

$$U(r) = \frac{\tilde{C}_6}{r^6 + R_c^6} \qquad \tilde{C}_6 = \left(\frac{\Omega}{2\Delta}\right)^4 C_6$$

Simultaneous excitations blocked

Rydberg dressing



Many body description
perturbation expansion in $\Omega/\Delta << 1$:

$$E_N = \sum_{i<j} \frac{\tilde{C}_6}{r_{ij}^6 + R_c^6}$$

Effective description with one level

N. Henkel, R. Nath, and T. Pohl, PRL **104**, 195302 (2010)

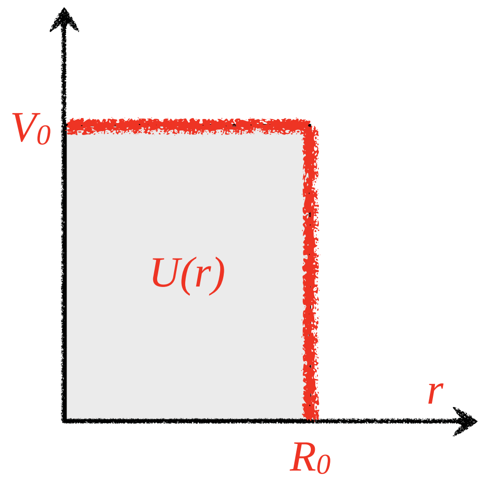
F. Maucher, N. Henkel, M. Saffman, W. Królikowski, S. Skupin, and T. Pohl, PRL **106**, 170401 (2011)

N. Henkel, F. Cinti, P. Jain, G. Pupillo and T. Pohl, PRL **108**, 265301 (2012)

Many body framework

Model potential:

$$U(\mathbf{r}) = \begin{cases} V_0, & |\mathbf{r}| < R_0 \\ 0, & \text{otherwise} \end{cases}$$



Approaches to the problem:

- Monte Carlo

$$\hat{H} = -\sum_{i=1}^{N} \frac{1}{2}\nabla^2 + \alpha' \sum_{i<j} \theta\left(1 - |\mathbf{r_i} - \mathbf{r_j}|\right)$$

- Mean field theory
  Gross-Pitaevskii equation

$$i\frac{\partial\psi(\mathbf{r},t)}{\partial t} = -\frac{1}{2}\nabla^2\psi(\mathbf{r},t) + \alpha\int d\mathbf{r}'\theta(1 - |\mathbf{r} - \mathbf{r}'|)|\psi(\mathbf{r}',t)|^2\psi(\mathbf{r},t)$$

Dimensionless parameters

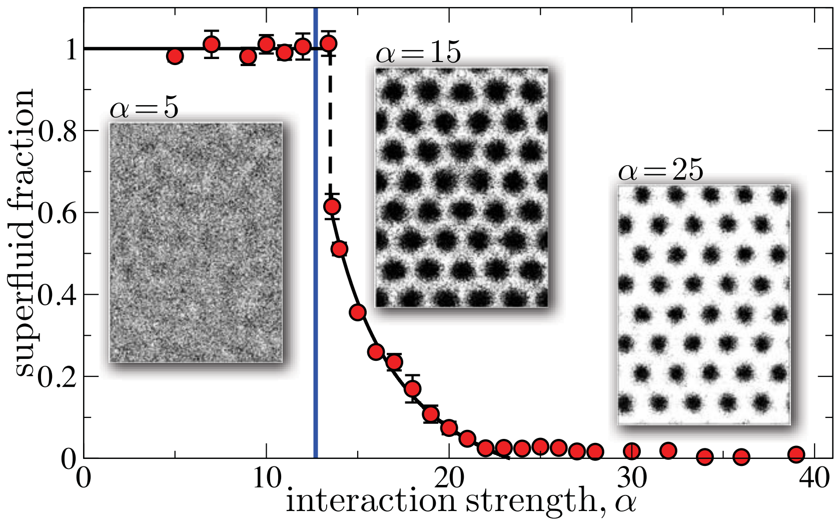
1) $\alpha' = \frac{V_0 m R_0^2}{\hbar^2}$

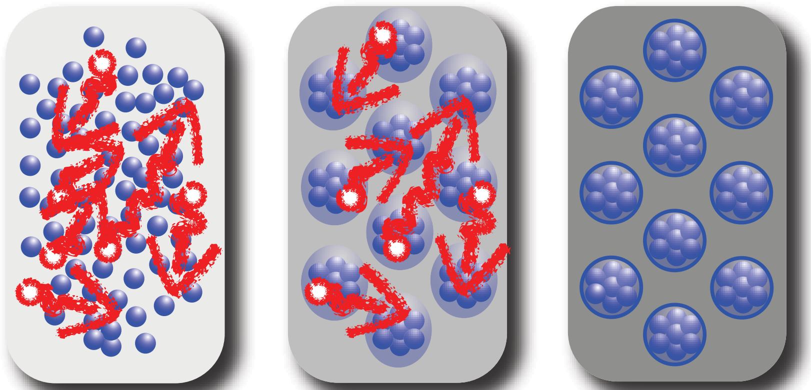
2) ρR_0^2

$$\alpha = \alpha' \rho R_0^2$$

Phase diagram

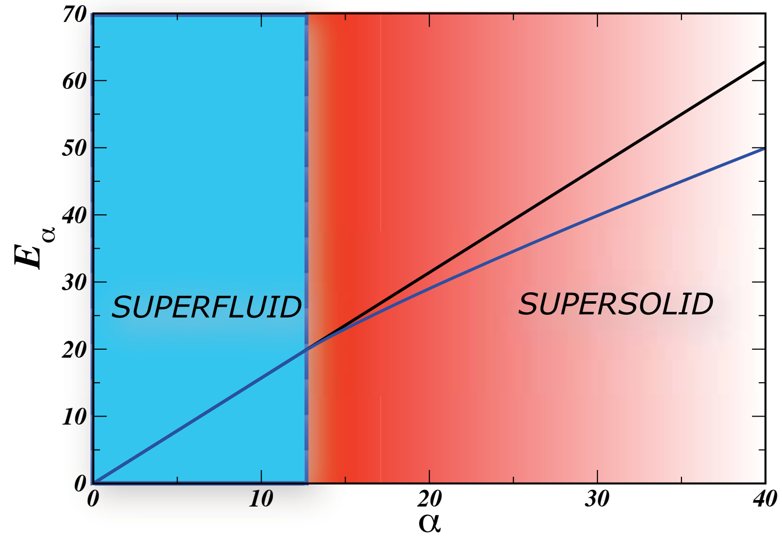
PATH INTEGRAL MC IN 2D





Superfluid *Supersolid* *Crystal*

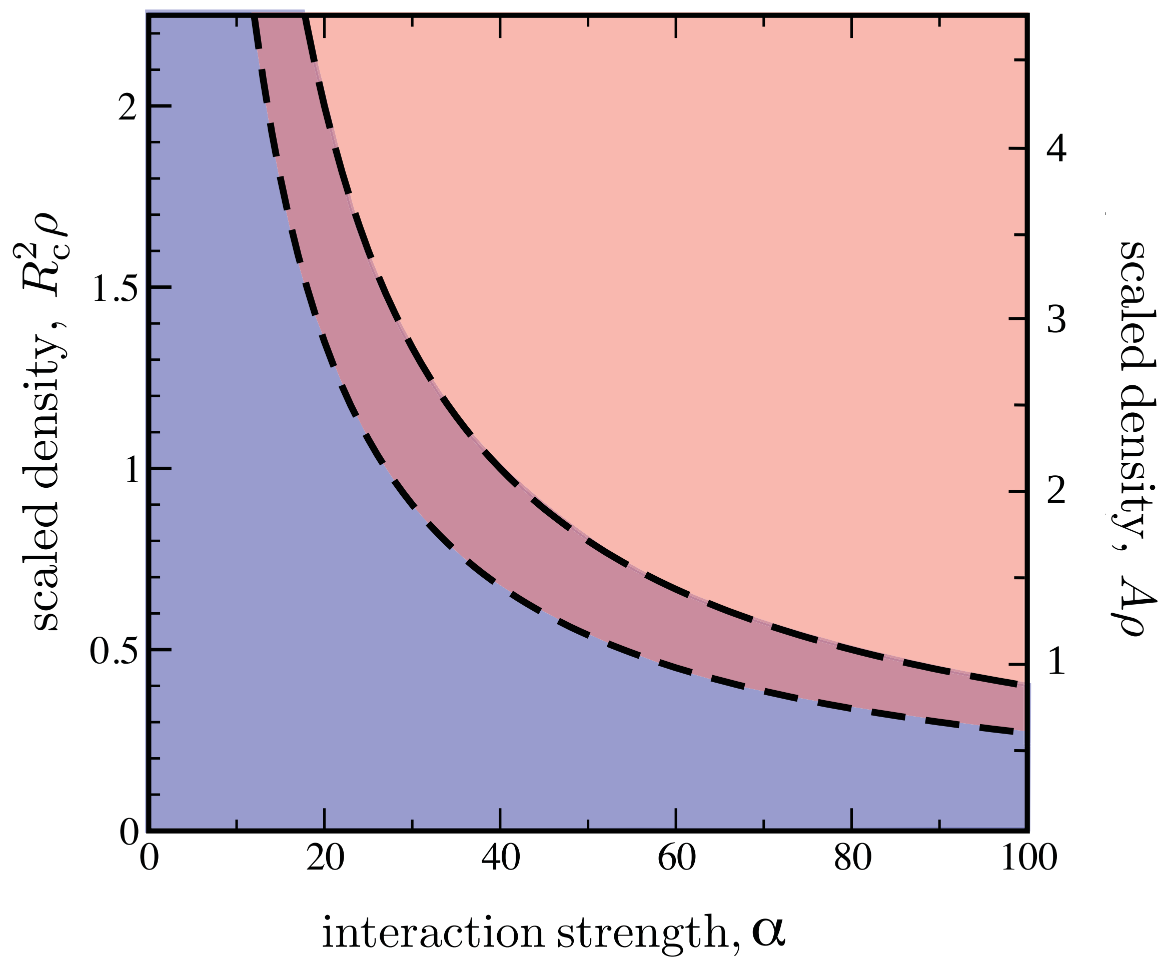
MEAN FIELD APPROACH



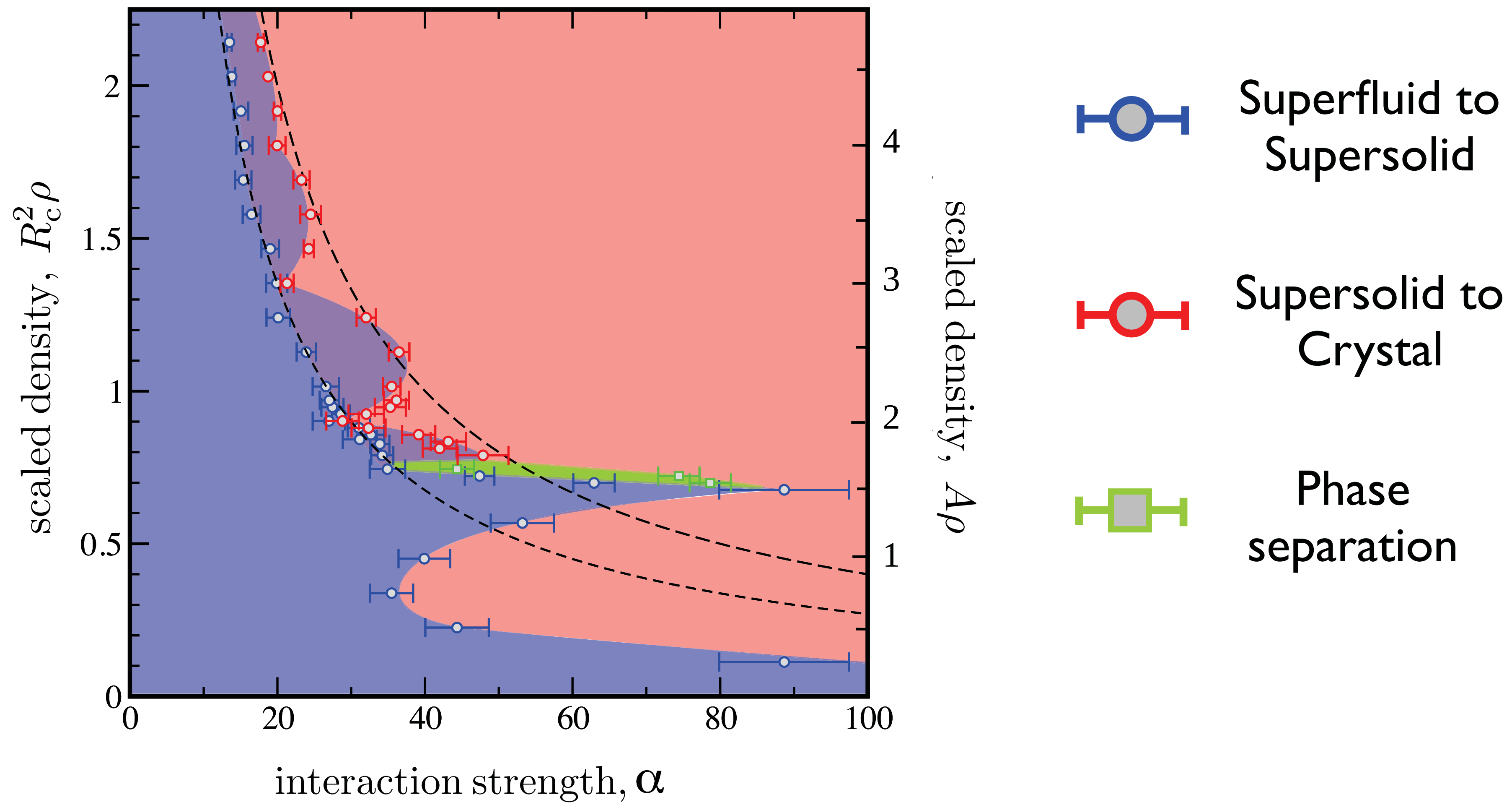
Variational wavefunction:
Gaussians in a triangular lattice

Both approaches predict cluster supersolid in the high density/low interaction limit, what else?

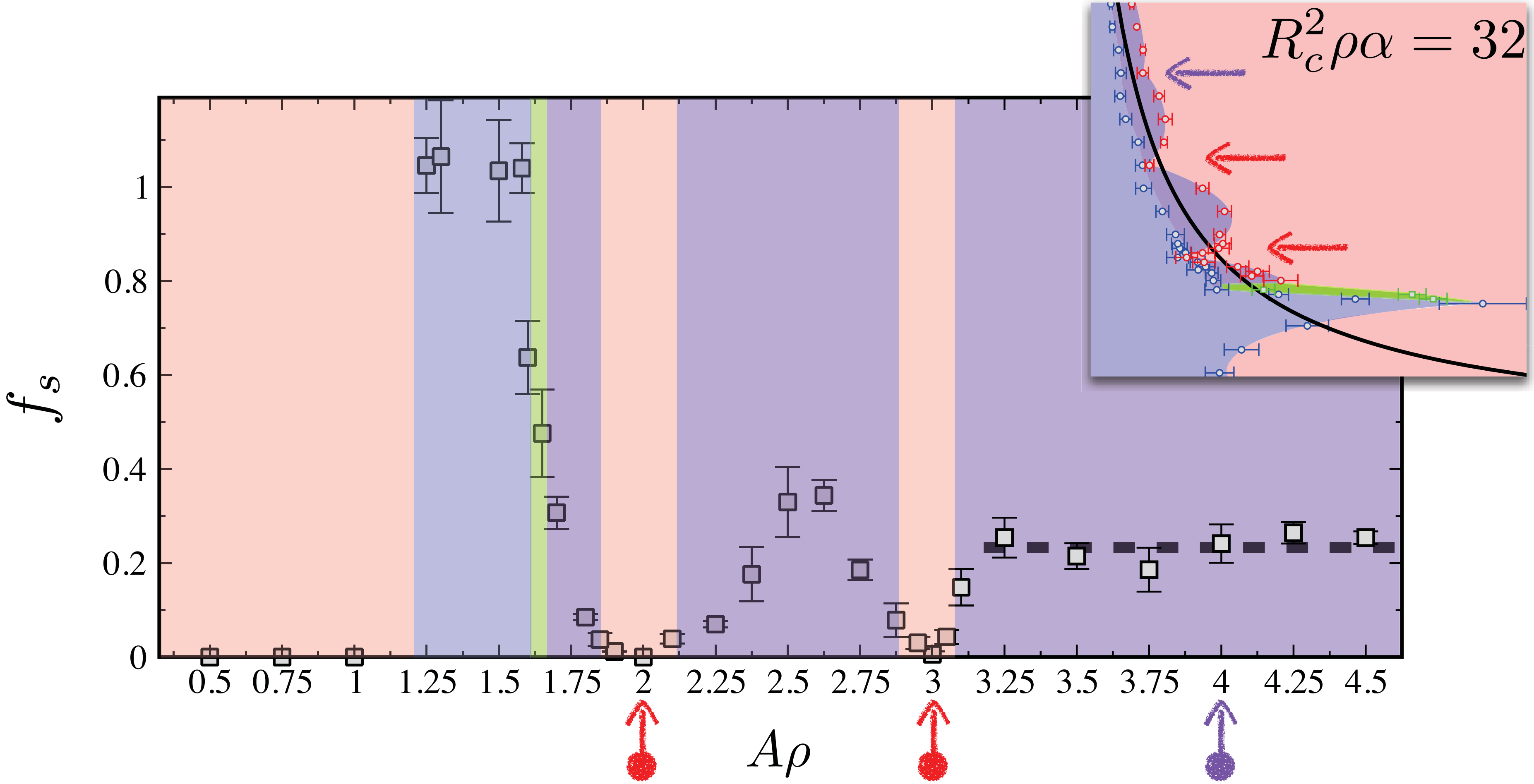
Low density phase diagram



Low density phase diagram



Low density phase diagram

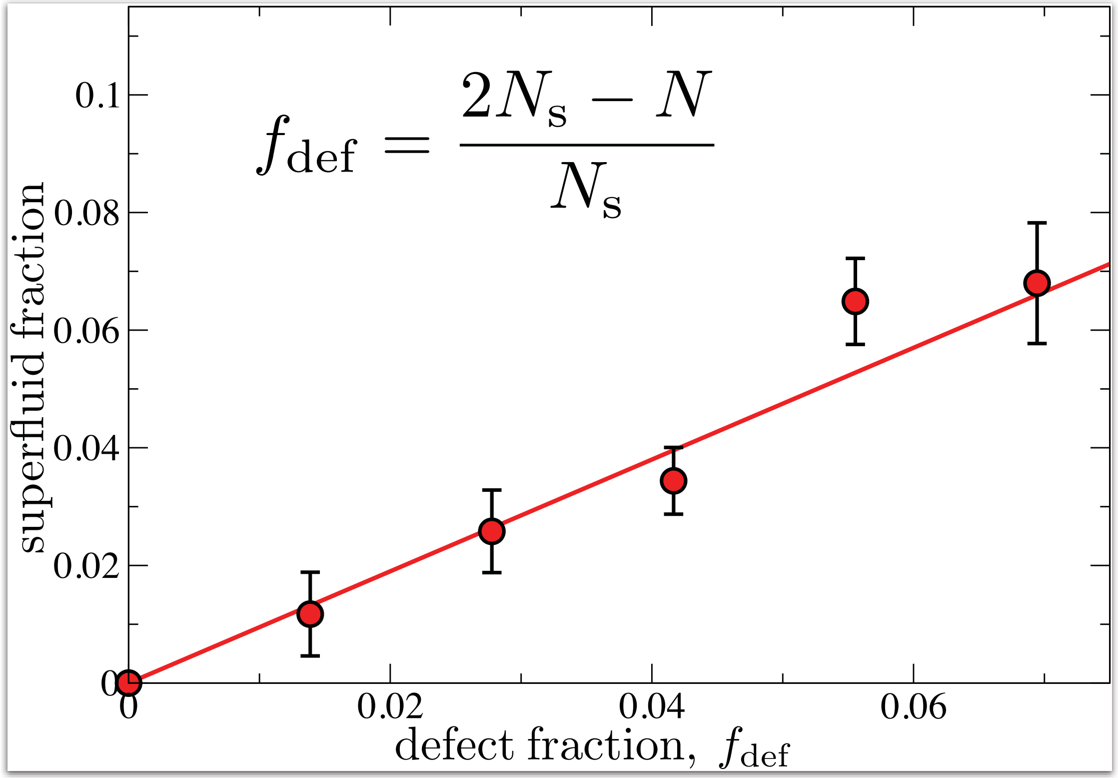


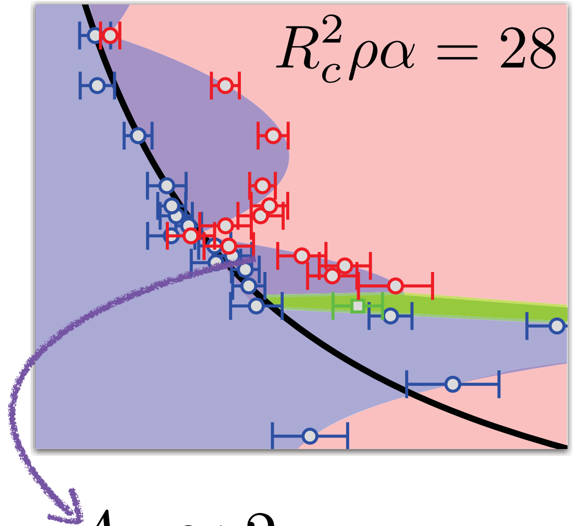
superfluid fraction from density wave supersolid

Leggett bound A. Leggett, Phys. Rev. Lett. 25 154 (1970)

Defect-induced supersolidity

f_s linearly depends on f_{def}





$A\rho \approx 2$

From crystal to SS phase removing particles randomly

Excitation spectrum

- Detect the phases experimentally
- Mean field: analytical results in the homogeneous, good for wide energy range
- QMC: dyn. struct. factor, long. excitations at low energies, high computational effort

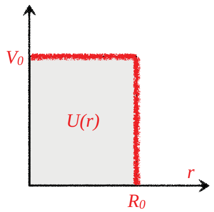
Mean field approach

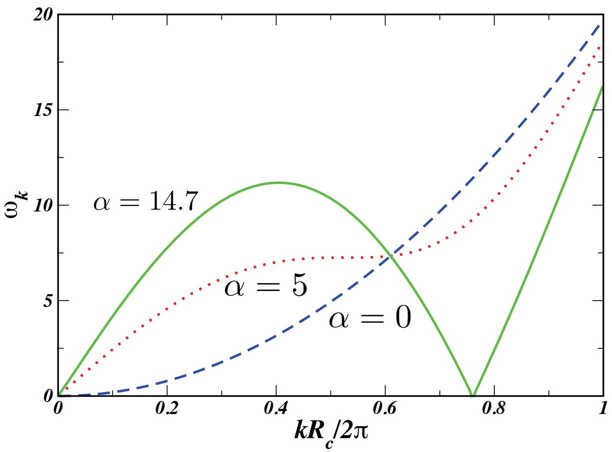
Bogoliubov equations for homogeneous ground state $\delta\psi(\mathbf{r},t) = e^{-i\mu t}\left(u(\mathbf{r})e^{-i\omega t} - v^*(\mathbf{r})e^{i\omega t}\right)$

$$\begin{cases} \left(-\frac{1}{2}\nabla^2 - \omega\right) u(\mathbf{r}) + \alpha R_0^2 n \int d\mathbf{r}'\; U(\mathbf{r}-\mathbf{r}')\left(u(\mathbf{r}) - v(\mathbf{r}')\right) & = & 0 \\ \left(-\frac{1}{2}\nabla^2 - \omega\right) v(\mathbf{r}) + \alpha R_0^2 n \int d\mathbf{r}'\; U(\mathbf{r}-\mathbf{r}')\left(v(\mathbf{r}) - u(\mathbf{r}')\right) & = & 0 \end{cases}$$

Excitation energy $\omega_k = \pm\sqrt{\frac{k^2}{2}\left(\frac{k^2}{2} + 2\alpha U_k\right)}$

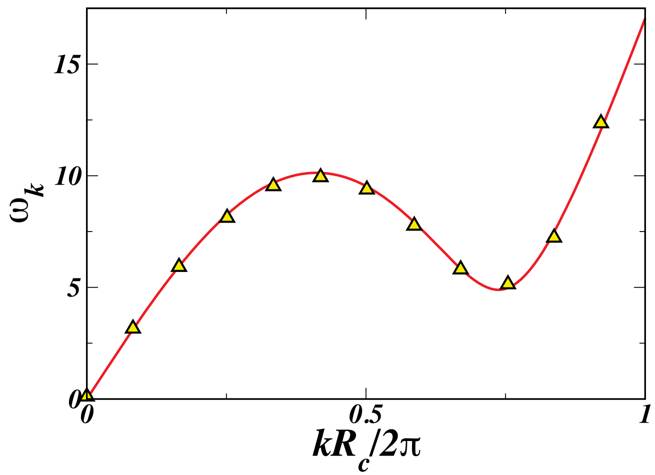
$$U_k \propto \frac{J_1(k)}{k}$$



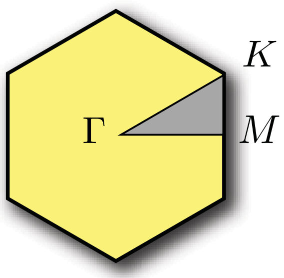


Spectrum of supersolid phase?

Excitation spectrum

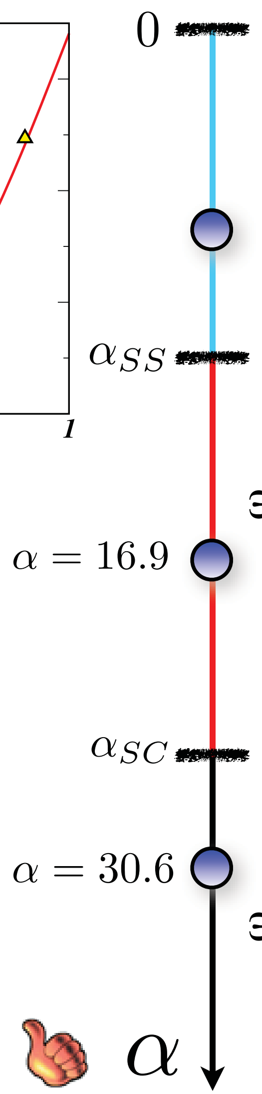


Brillouin zone

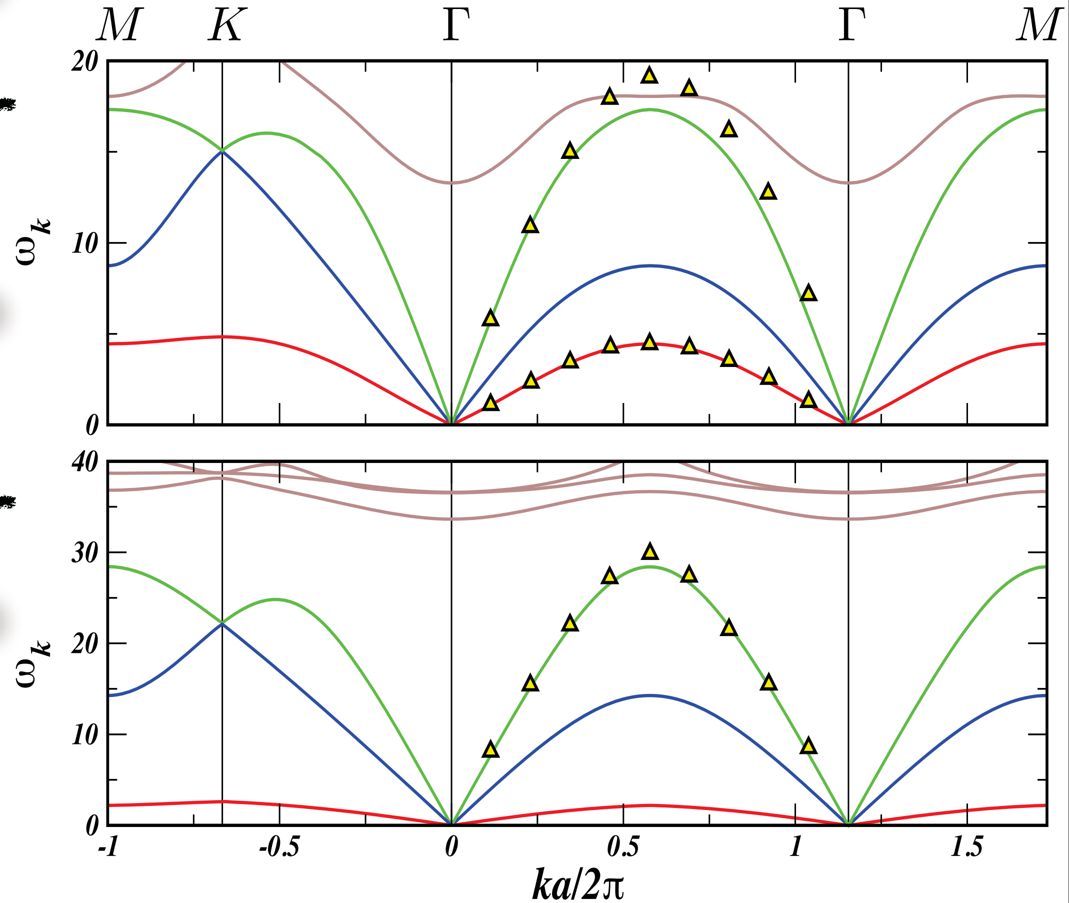


Extemely good agreement!

Also in the crystal phase



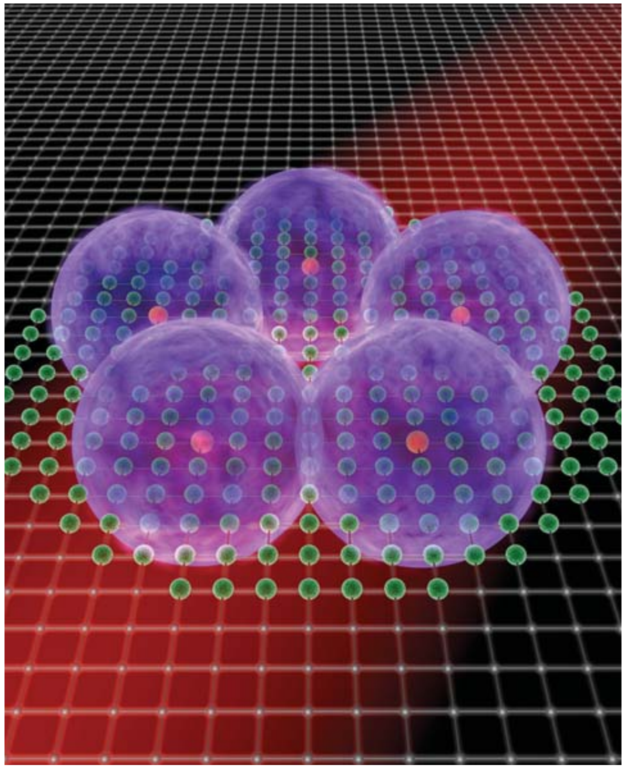
Comparison with MC data of the dynamical structrure factor from
Saccani, Moroni, Boninsegni, PRL (2012)



Conclusions

- *Rydberg gases as quantum simulators of long range spin models and exotic quantum phases.*
- *Fast dynamics: limited coherence time ~10 μs*

Dynamical creation of mesoscopic crystalline phases.



Conclusions

- *Rydberg gases as quantum simulators of long range spin models and exotic quantum phases.*

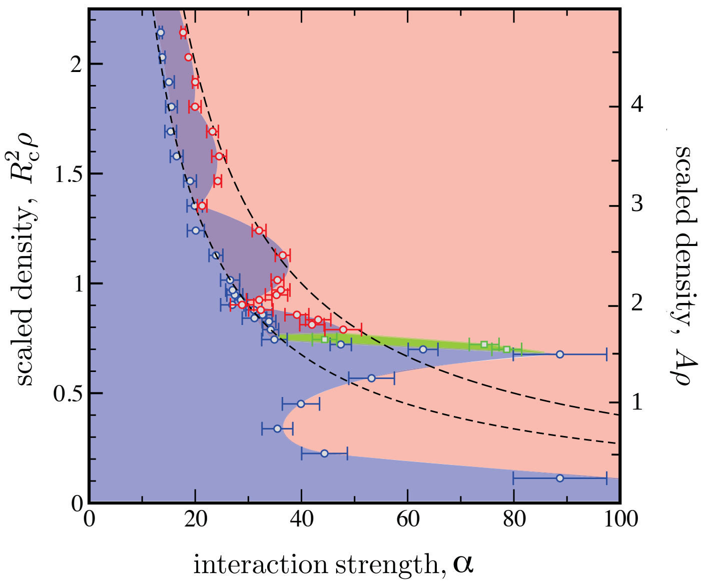
- *Fast dynamics: limited coherence time ~10 μs*

Dynamical creation of mesoscopic crystalline phases.

- *Slow dynamics: coherence time enhanced by dressing ~ 1 s*

Defect-induced vs. Density wave supersolidity.

Excitation spectra to probe the many body phases.



Conclusions

- *Rydberg gases as quantum simulators of long range spin models and exotic quantum phases.*
- *Fast dynamics: limited coherence time ~10 μs*

  Dynamical creation of mesoscopic crystalline phases.
- *Slow dynamics: coherence time enhanced by dressing ~ 1 s*

  Defect-induced vs. Density wave supersolidity.

  Excitation spectra to probe the many body phases.