The Abdus Salam International Centre for Theoretical Physics

**2451-4**

**Workshop on Interferometry and Interactions in Non-equilibrium Meso- and Nano-systems**

***8 - 12 April 2013***

# Fractional statistics and interferometry in saddle potentials and mesoscopic rings

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*UNITED STATES OF AMERICA*

# Fractional statistics and interferometry in saddle potentials and mesoscopic rings


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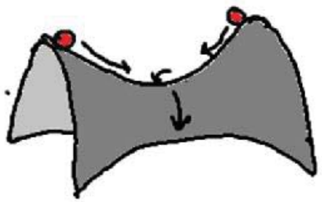
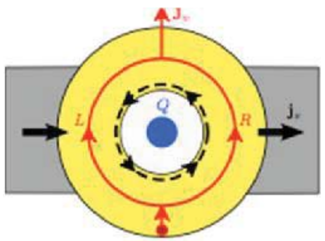
Babak Seradjeh, Michael Stone, **Smitha Vishveshwara**

**Department of Physics, UIUC**
**On Sabbatical at IISC**

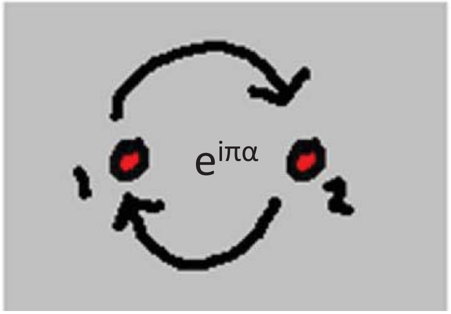





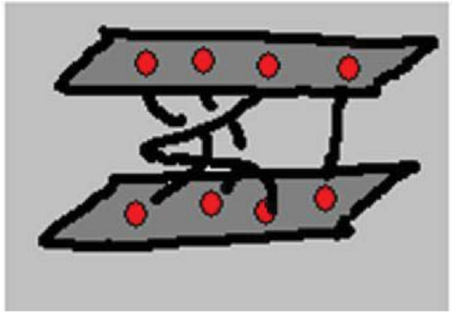


*Workshop on Interferometry and Interactions in Non-equilibrium Meso- and Nano-systems*
*April 2013; ICTP, Trieste, Italy*

# Low dimensions and fractional statistics

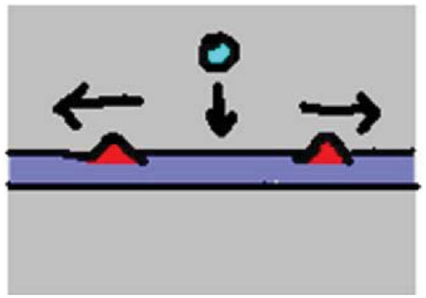



***Abelian***

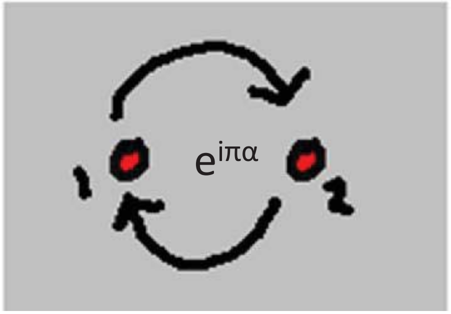


***Non-Abelian***

***Also***
***Charge***
***Fractionalization***

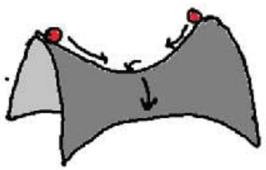


# In this talk

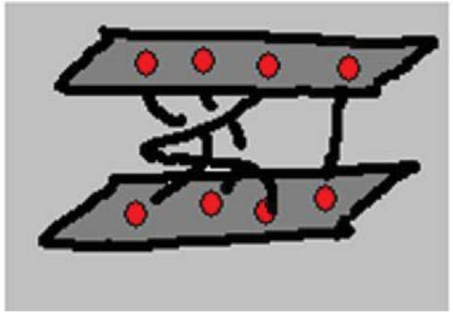



*Abelian*

**Quantum Hall System**
**HBT bulk correlations**



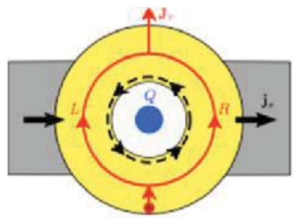
**Saddle potential**
**Beam splitter**



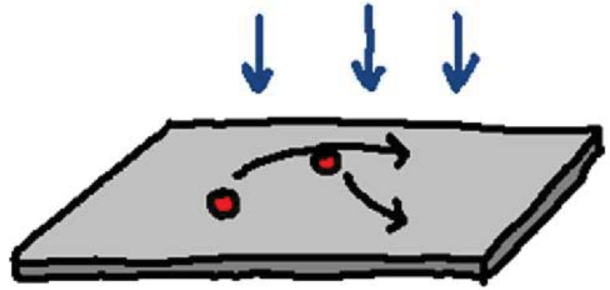
*Non-Abelian*

**Interferometry ideas**
**In Qtm Hall Systems**

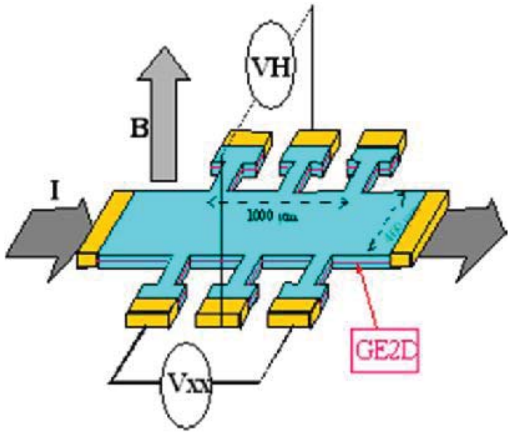
**Ring geometry**
**Chiral p-wave sc**

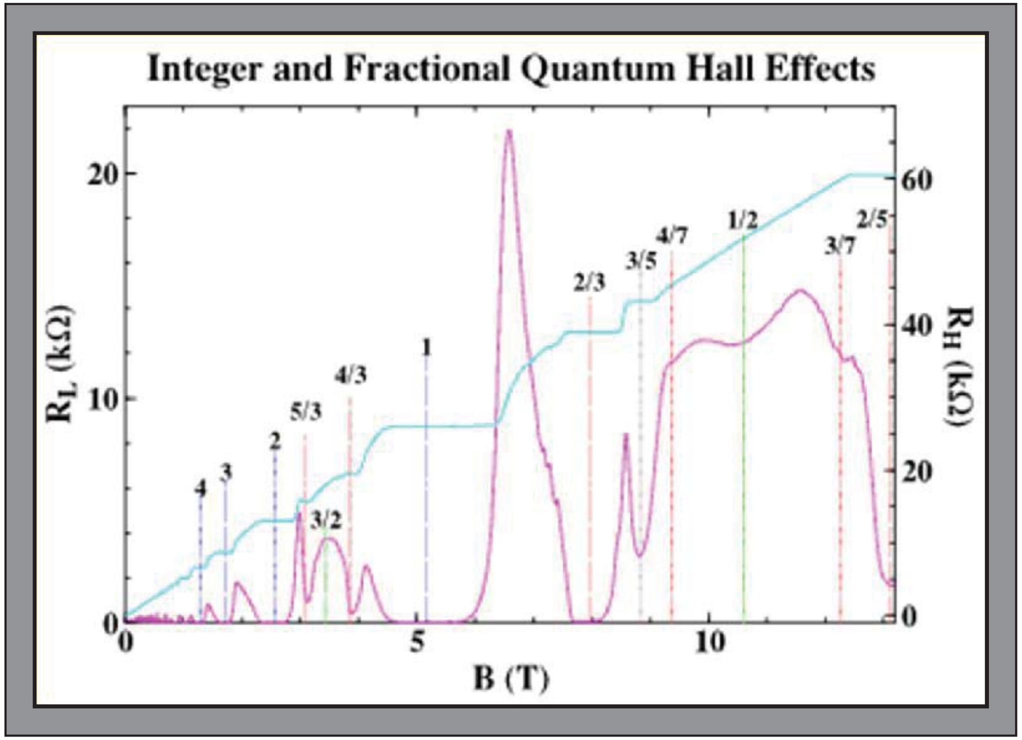


# Quantum Hall bulk Abelian anyons and saddle potential beam splitters



# Fractional quantum Hall system



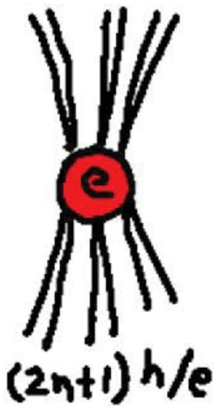




*For e.g.,*
*D. C. Tsui, H. L. Stormer, A. C. .Gossard, 1982*

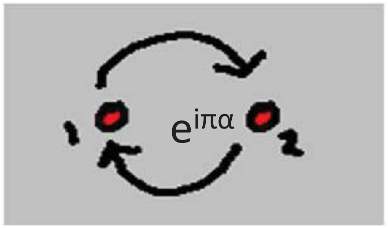
# Fractional quantum Hall system

2-dimensional electron gas in
strong magnetic field;
Interactions + disorder

Incompressible
dissipationless
bulk fluid


Laughlin state: Filling ν = 1/(2n+1)
Quantized Conductance – $\nu e^2/h$




Quasiparticle/hole

$e^* = \nu e$ Fractional Charge

$e^{\pm i\pi\nu}$ Fractional Statistics

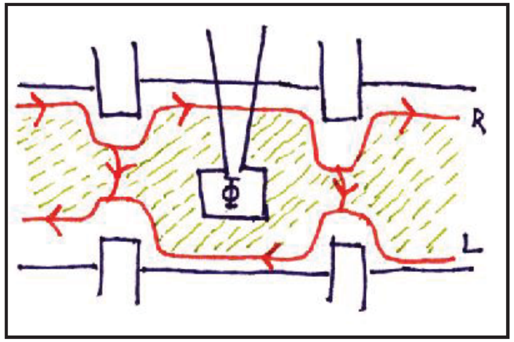
Breakdown of Fermi-liquid theory; Absence of Landau quasiparticles

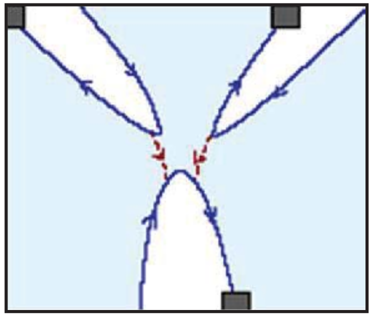
# Fractional statistics in QH systems

## Probes employing edge state properties

*X. G. Wen,1995*

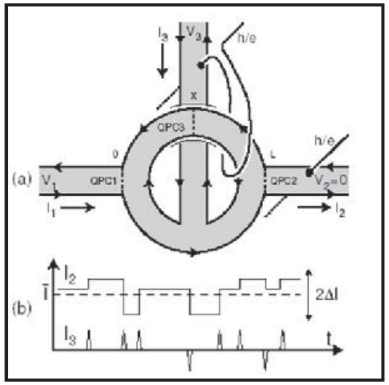
### Anomalous Aharonov-Bohm period



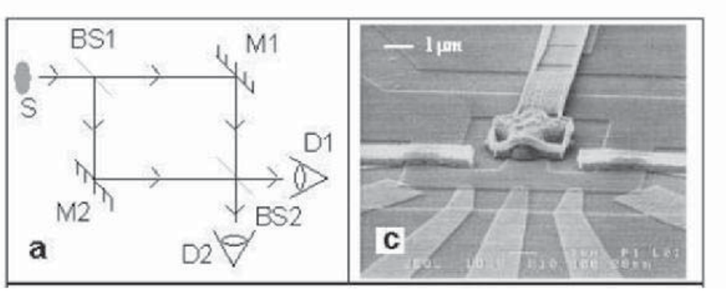
### Partitioning Noise

### Telegraph Noise




*C. Kane,2003*

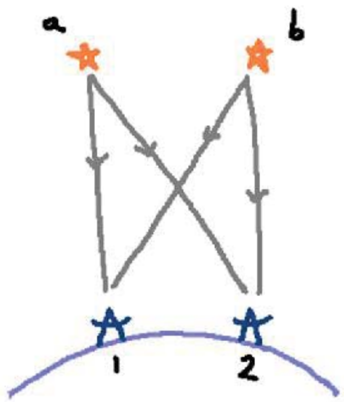
### Mach-Zehnder Interferometry



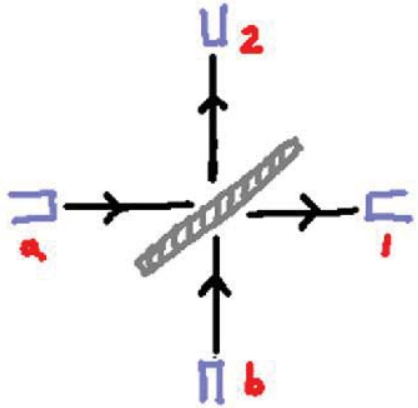

*Ji et al.,2003*

# Quantum statistics: two-particle properties

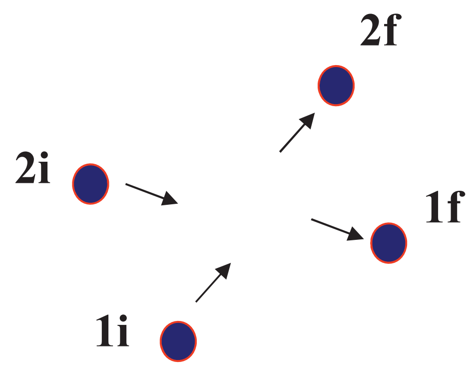
**Hanbury-Brown Twiss**




**Beam-splitters**

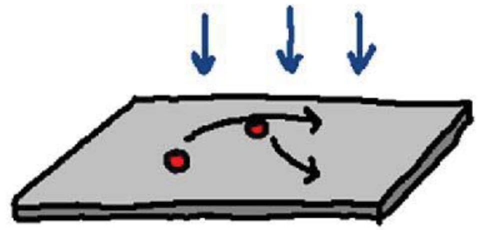


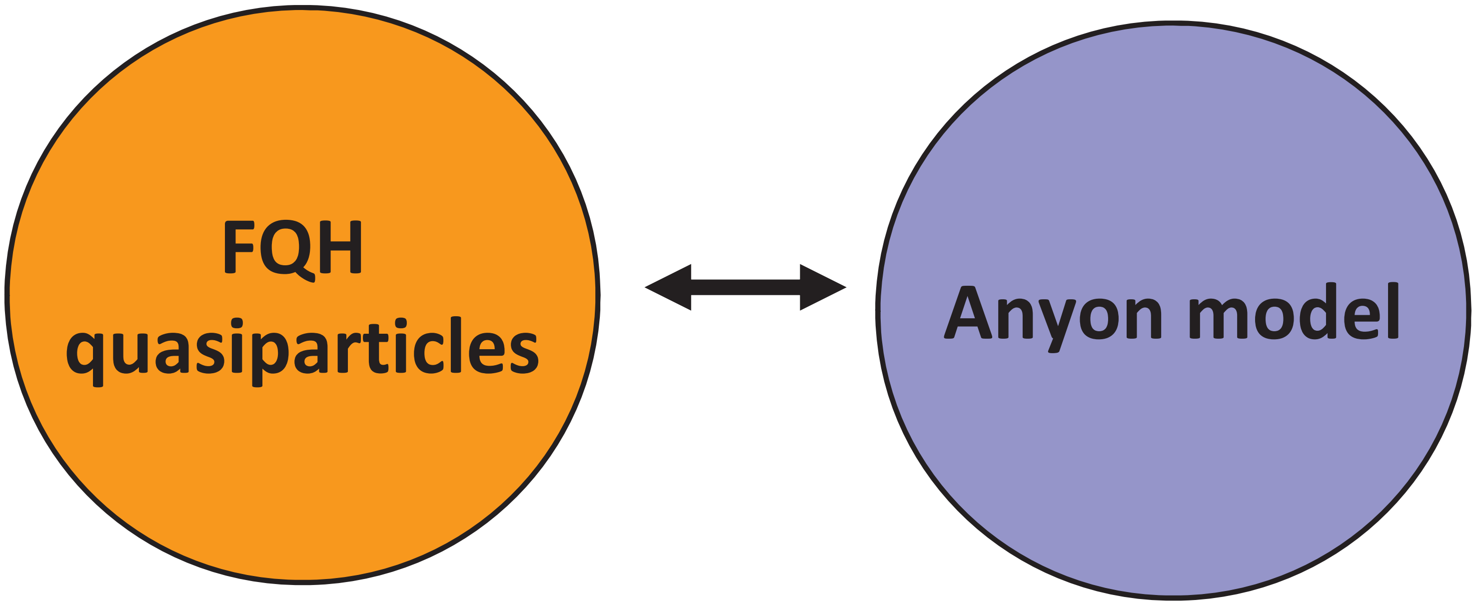

**Scattering processes**




Signatures of statistics in bunching behavior
and angular dependence

**Manifestations in QH bulk?**






*B. I. Halperin, '87; R. B. Laughlin '87*
*T. H. Hansson, J. M. Leinaas, J. Myrheim, '92*
*H. Kjonsberg and J. M. Leinaas, '97*
*J.  Myerheim  in Les Houches series '99;*
*J. Jain, Composite Fermions '07*

# Two-anyon model

$$H = \frac{1}{4\mu}\left(P_x + \frac{qB}{c}Y\right)^2 + \frac{1}{4\mu}\left(P_y - \frac{qB}{c}X\right)^2 + \frac{1}{\mu}\left(p_x + \frac{qB}{4c}y\right)^2 + \frac{1}{\mu}\left(p_y - \frac{qB}{4c}x\right)^2$$

$l = \sqrt{\hbar c/qB}$

Center of mass: $(\vec{R}, \vec{P})$ Relative co-ordinates: $(\vec{r}, \vec{p})$


<u>Anyonic feature</u>

$$\psi(-\vec{r}) = e^{\pm i\pi\alpha}\psi(\vec{r})$$

(Symmetric gauge; lowest Landau level (LLL); Laughlin: $q = \nu e$, $\alpha = \nu = 1/m$)

# Two-anyon LLL Hilbert space

## Center of mass:

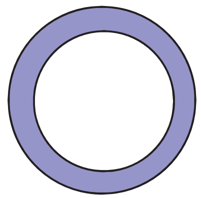
Guiding center coordinates

$$\hat{X} = l(\hat{A} + \hat{A}^\dagger)/2, \quad \hat{Y} = il(\hat{A} - \hat{A}^\dagger)/2$$

$$[A, A^\dagger] = 1$$

Angular momentum eigenstates

$$\hbar A^\dagger A|n\rangle_c = n\hbar|n\rangle_c \qquad \psi_n(\vec{R}) \propto \frac{Z^n}{\sqrt{n!}}$$



Localized (coherent) states

$$|Z\rangle_c = e^{-|Z|^2/2} \sum_{n=0}^{\infty} \frac{(Z^*)^n}{\sqrt{n!}} |n\rangle_c$$

$Z=(z_1+z_2)/2$


*T. H. Hansson, J. M. Leinaas, J. Myrheim ,1992 (HLM92); H. Kjonsberg and J. M. Leinaas, 1997 (KL97)*

# Two-anyon LLL Hilbert space

**Relative coordinates:**

Guiding center coordinates

$$\hat{a} \equiv (\hat{x}^2 + \hat{y}^2)/8l^2, \hat{b} \equiv (\hat{x}^2 - \hat{y}^2)/8l^2$$
$$\hat{c} \equiv (\hat{x}\hat{y} + \hat{y}\hat{x})/8l^2, \hat{L} = \hbar(2\hat{a} - 1/2)$$

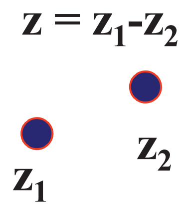
Respect sp(1,R) algebra

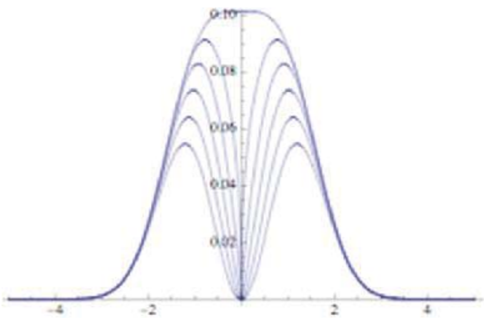
Angular momentum eigenstates

$$\hat{L}|k,\alpha\rangle_r = (2k+\alpha)\hbar|k,\alpha\rangle_r$$
$$\psi_k(\vec{r}) \propto \frac{z^{2k+\alpha}}{\sqrt{\Gamma(2k+\alpha+1)}}$$

Localized states
(not coherent states)

$$|z\rangle_\alpha = N_{\alpha,z} \sum_{k=0}^{\infty} \frac{(z^*/2)^{2k+\alpha}}{\sqrt{\Gamma(2k+\alpha+1)}} |k,\alpha\rangle_r$$




Fermions/
Bosons

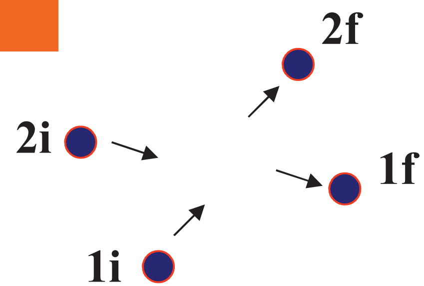
$$|z\rangle_{1/0} = e^{|z|^2/8} N_{1/0,z}[|z\rangle_d \mp |-\ z\rangle_d],$$

*HLM92; KL97*

# Propagators

Single particle

$$K_1(\vec{r}_f;\vec{r}_i) = \sum_n \psi_n(\vec{r}_f)\psi_n^*(\vec{r}_i)e^{-iE_nt/\hbar}$$

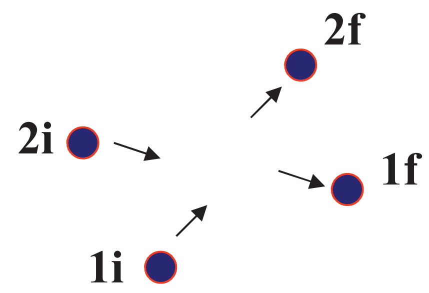



Two particles

$$K_2(\vec{r}_{1f},\vec{r}_{2f};\vec{r}_{1i},\vec{r}_{2i}) = \sum_n \psi_n(\vec{R}_f)\psi_n^*(\vec{R}_i)e^{-iE_nt/\hbar} \times \sum_p \psi_p(\vec{r}_f)\psi_p^*(\vec{r}_i)e^{-iE_pt/\hbar}$$

# Propagators

## Lowest Landau level




### Single particle

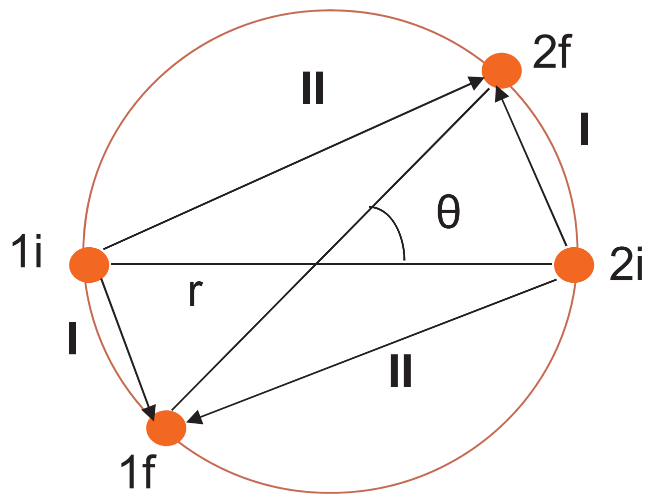
$$K_1(z_f; z_i) = \frac{1}{2\pi} \; \exp[-\tfrac{1}{4} \; (|z_f|^2 + |z_i|^2) \; + \; \tfrac{1}{2} z_f z_i^*]$$

### Two particles

$$\begin{aligned} & K_2(z_{1f}, z_{2f}; z_{1i}, z_{2i}; \tau) \\ = & \frac{1}{(2\pi m)^2} \; \exp \; [-\frac{1}{4m}(|z_{1f}|^2 + |z_{2f}|^2 + |z_{1i}|^2 + |z_{2i}|^2) \\ & \qquad + \frac{1}{4m}(z_{1f} + z_{2f})(z_{1i}^* + z_{2i}^*)] \\ \times & \; \sum_{p=0}^{\infty} \; \frac{[(z_{1f} - z_{2f})(z_{1i}^* - z_{2i}^*)/4m]^{2p+1/m}}{\Gamma(2p+1/m+1)} \end{aligned}$$

*R. B. Laughlin '87*
*S.V., M. Stone and D. Sen, '07; '08*

# Two-particle propagator in LLL - fermions and bosons




Destructive Interference

**Magnitudes of I and II equal:**

$\theta = \pi/2$

**Relative phase 0/π:**

$2Br^2/\Phi_0 = (2n+0/1)/2$

**Aharonov-Bohm**
**+ Quantum statistics**

$$K_2(z_{1f}, z_{2f}; z_{1i}, z_{2i}) = K_1(z_{1f}; z_{1i})K_1(z_{2f}, z_{2i}) \mp K_1(z_{2f}; z_{1i})K_1(z_{1f}, z_{2i})$$

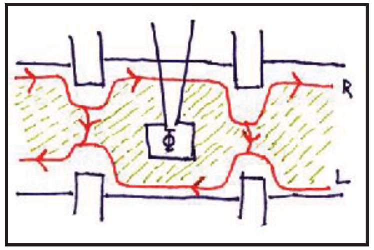
# Two-particle propagator - anyons

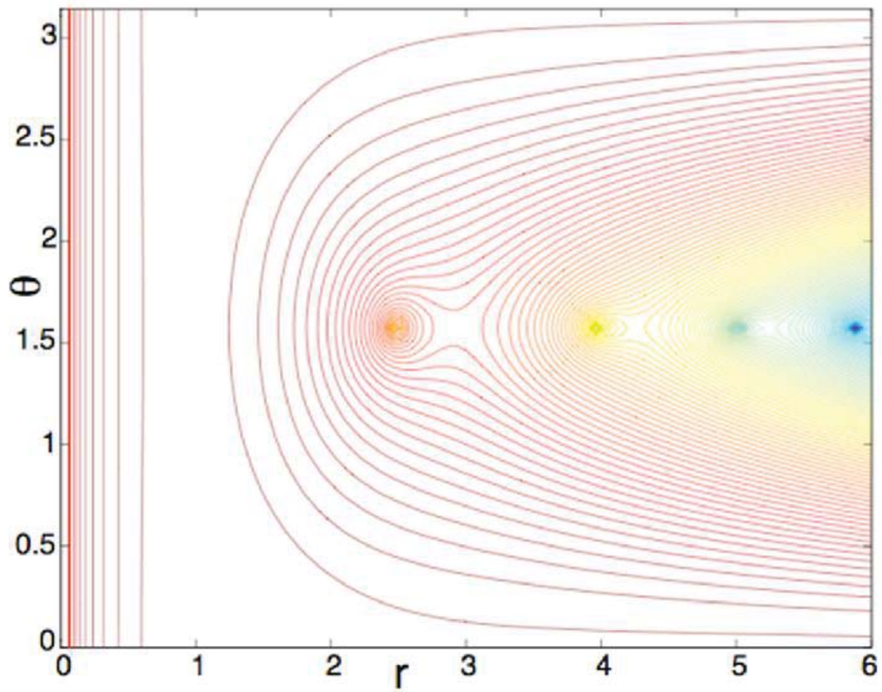
## Destructive Interference

**Similar geometric picture:**

**Also at $\theta = \pi/2$**

**$2Br^2/\Phi_0 \approx (2n+1+\alpha)/2$**



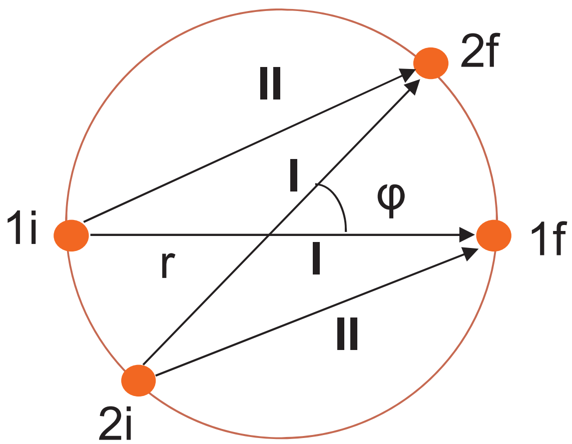

*S.V., M. Stone and D. Sen, '07; '08*

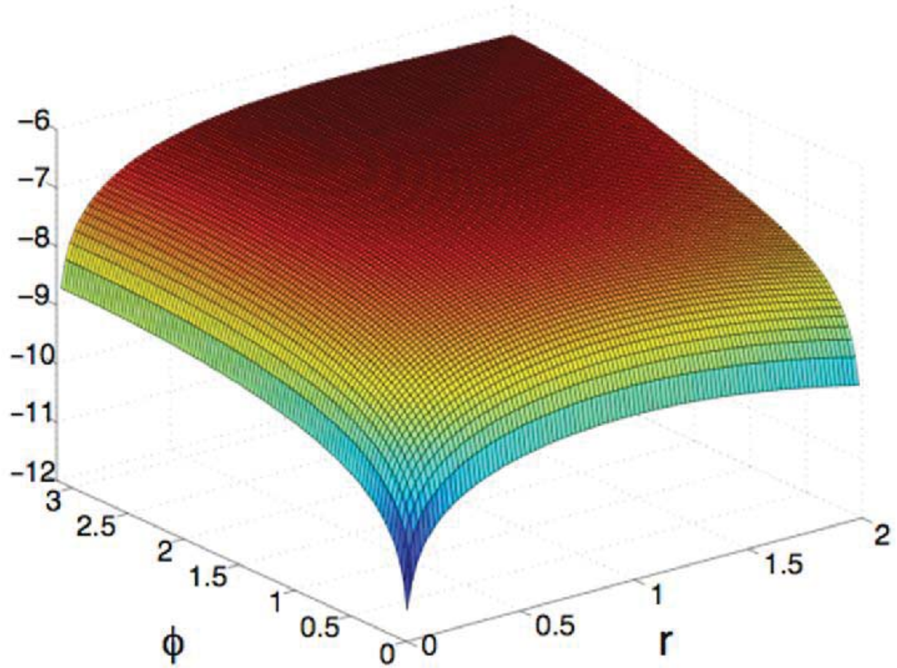
# Two-particle propagator - anyons

## Exclusion behavior

*Haldane, 1991*

$$K_2 \sim |\varphi|^{2\alpha}\ ,\ \varphi \to 0$$




**For 1i=2i=0, 1f=2f=r,** $K_2 \sim |r|^{2\alpha}\ ,\ r \to 0$

# Bunching properties

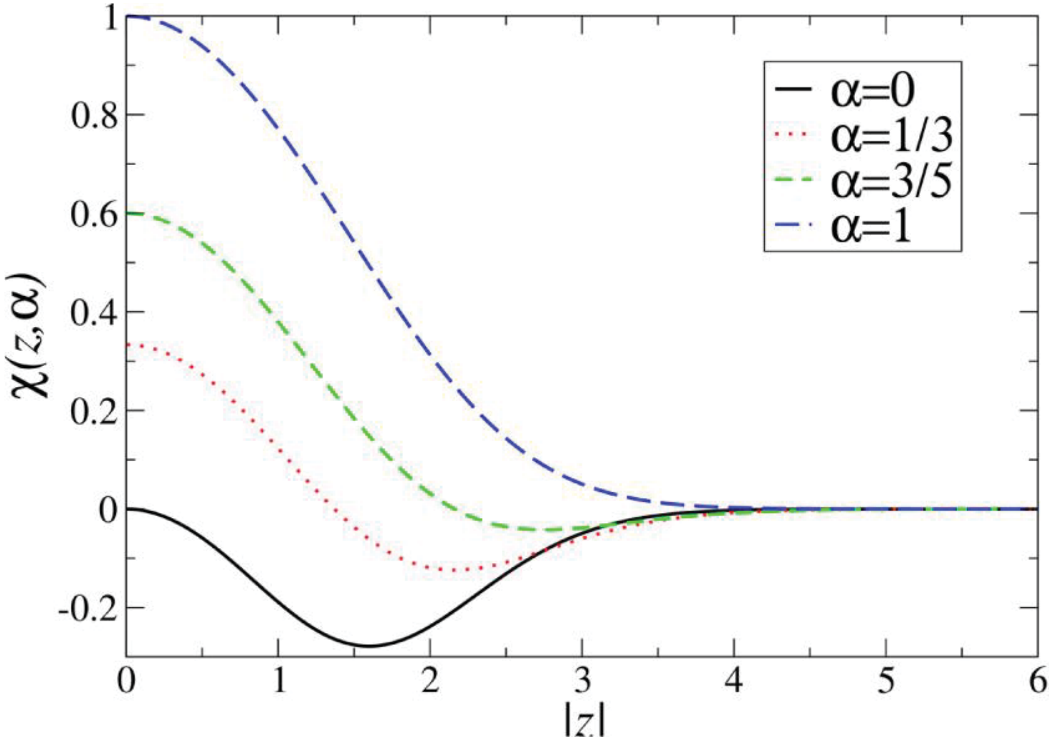
Bosons Fermions

Bunching parameter

$$\chi(|z|,\alpha) \equiv \frac{1}{4l^2}\left[\langle z|\hat{r}^2|z\rangle_\alpha - \langle z|\hat{r}^2|z\rangle_d\right]$$

$$\langle \hat{r}^2\rangle \equiv \langle \hat{x}^2 + \hat{y}^2\rangle$$

# Bunching parameter

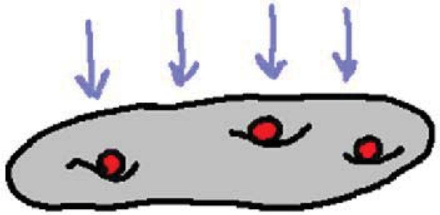



$$\chi = (|z|^2/4)[\coth(|z|^2/4) - 1] \quad \text{Fermions}$$

$$= (|z|^2/4)[\tanh(|z|^2/4) - 1] \quad \text{Bosons}$$

*S.V., N. R. Cooper, 2010*

# Probing Correlations

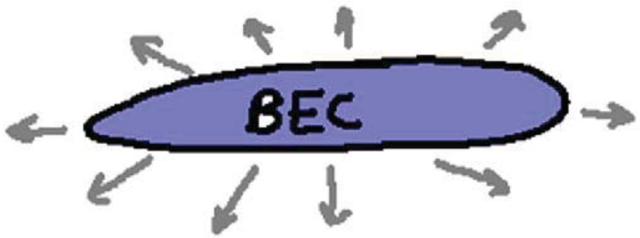


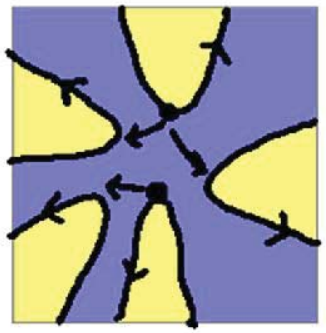
**Localized quantum Hall bulk quasiparticles**

*J. Martin et al, 2004*

**Correlations in ultracold gases**

*M. Schellehens et al, 2005*

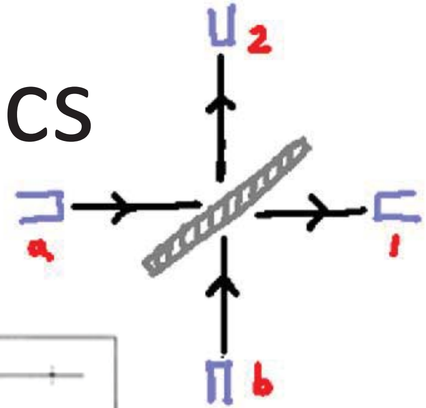


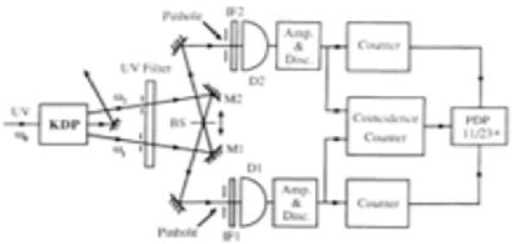
**Tunneling into multiple edge states**

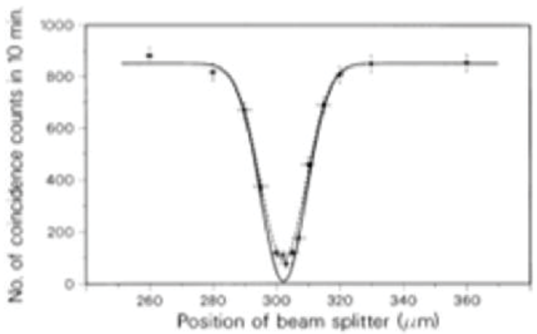
*E. Comforti et al, 2002*

# Beam Splitters and Statistics



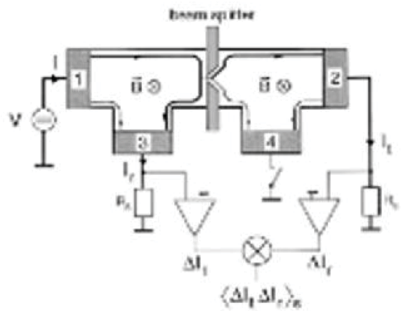

**Coincidence measurements for photons**

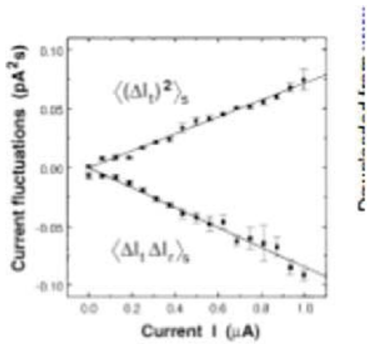


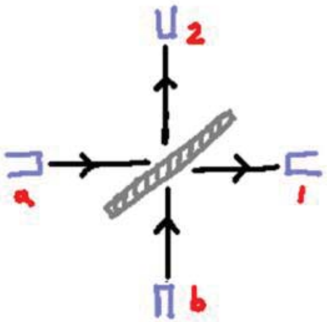

*C. K. Hong at al., 1987*

**Current correlation measurements for electrons**

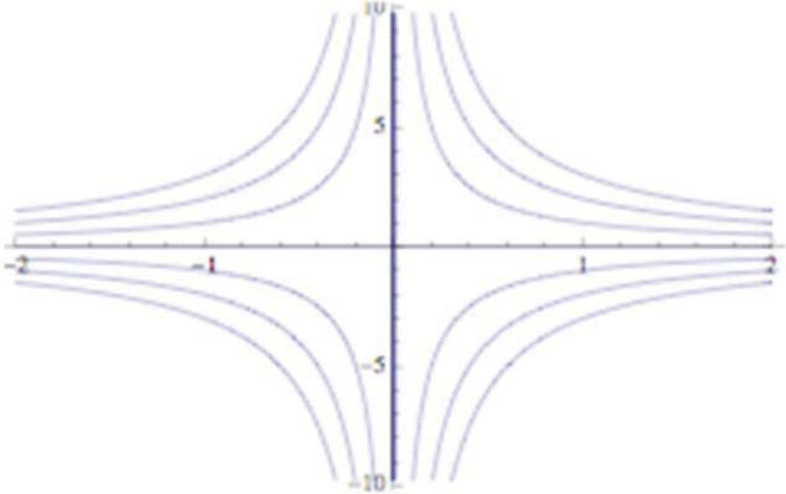



*M. Henny et al; W. D. Oliver at al, 1999*




# QH beam splitter – Saddle potential



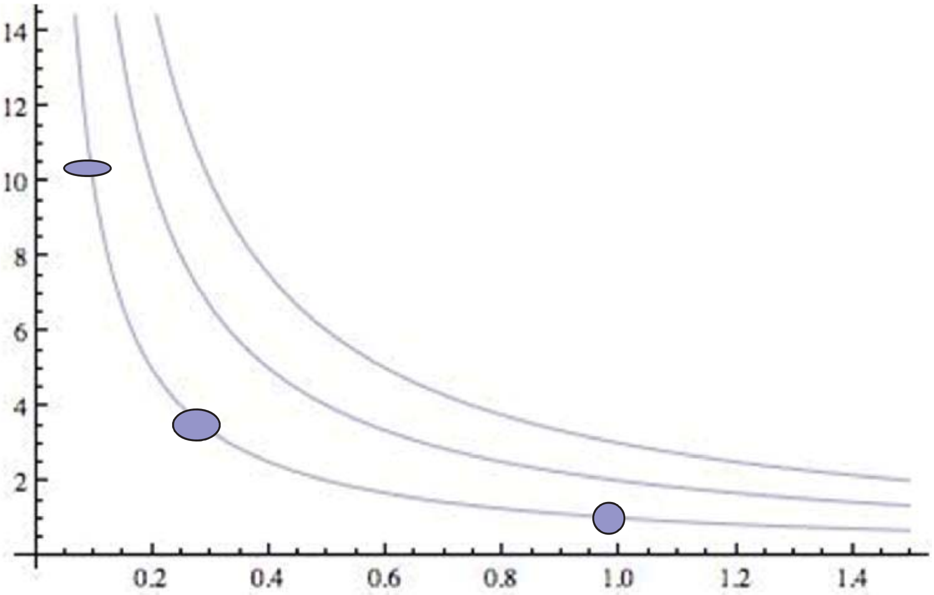

$$\hat{H}_s = \sum_{\gamma=1,2} U \hat{x}_\gamma \hat{y}_\gamma$$

Separable,
Preserves anyon b.c.

<u>LLL projected Hamiltonian</u>

$$\hat{H}_s^P = \frac{1}{2} i U l^2 [\hat{A}^2 - (\hat{A}^\dagger)^2] + 2 U l^2 \hat{c}$$

# Saddle dynamics




## C.o.m motion

$$|Z\rangle_c = \exp(Z\hat{A}^\dagger - Z^*\hat{A})|0\rangle_c$$
$$|Z(t)\rangle_c = e^{-i\hat{H}_s^P t/\hbar}|Z\rangle_c$$

**Analogy from qtm optics**

Squeeze along $re^{i\phi} \equiv Utl^2/\hbar$
Follow

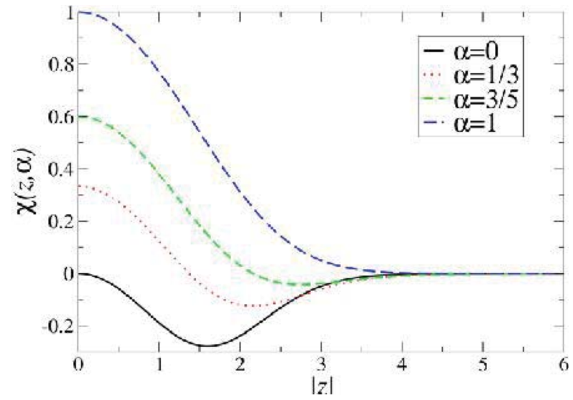
$$(Xe^{-Utl^2/\hbar}, Ye^{Utl^2/\hbar})$$

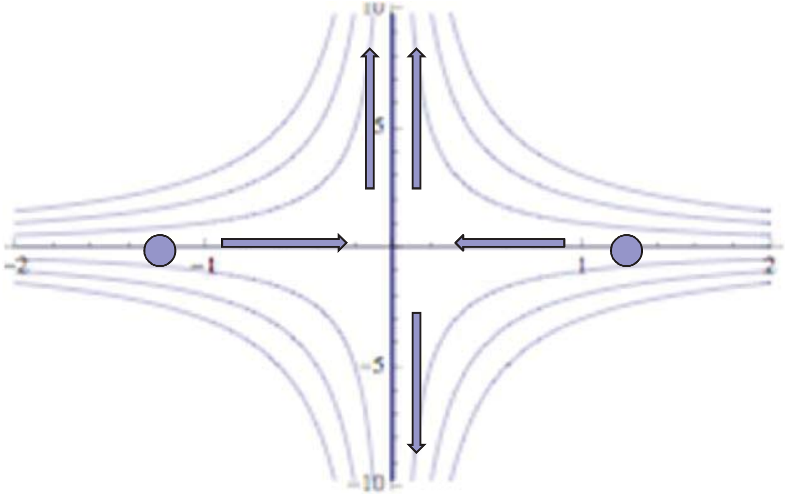
## Relative motion

(Initial amplitude peaked at +z and -z)

$$\hat{x}^2(t) = e^{-2Utl^2/\hbar}\hat{x}^2(0), \hat{y}^2(t) = e^{2Utl^2/\hbar}\hat{y}^2(0)$$

# Beam splitter properties



**Did the particles go in the same direction or different ones?**

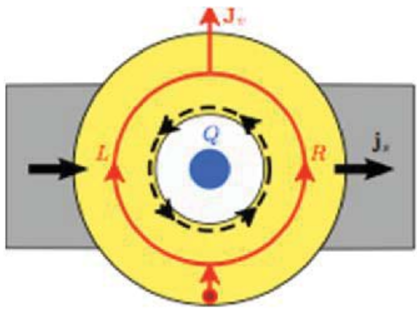
Depends on statistics
and bunching parameter

Behavior of <$y_1 y_2$>

$$\langle \hat{y}_1 \hat{y}_2 \rangle = l^2 e^{2Utl^2/\hbar} \left[ \mathrm{Im}[Z]^2 - \frac{1}{4}\mathrm{Im}[z]^2 - \frac{1}{2}\chi + \delta \right]$$

*S.V., N. R. Cooper, 2010*

# Non-Abelian statistics in chiral p-wave superconductor rings

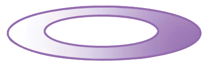


# Non-Abelian quasiparticles

**Quantum Hall (ν=5/2)**

e/4 fractional quasiparticles


**Chiral p-wave superconductors**



Zero energy state in vortex core

h/(2e) or h/(4e) (One or two component sc)

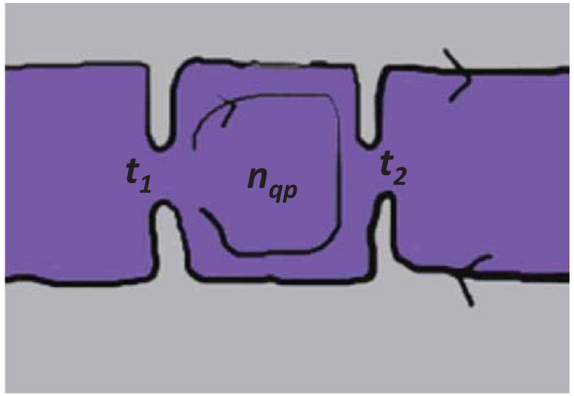
**Majorana mode**

Upon exchange

$$\nu_1 \rightarrow \nu_2$$
$$\nu_2 \rightarrow -\nu_1$$

# Non-Abelian quasiparticles: detection

## Quantum Hall (ν=5/2)




Interferometry

$$\sigma_{xx}^{int} \propto Re[t_1^* t_2 e^{i\phi} \langle \psi_0 | U_1^{-1} U_2 | \psi_0 \rangle]$$

*For e.g., E. Fradkin et al., 1998; S. Das Sarma et al, 2005; Stern & Halperin, 2006; P. Bonderson et al., 2006*

## Chiral p-wave superconductors (CpSC)

**DUALITY!!?**

# Two component CpSc

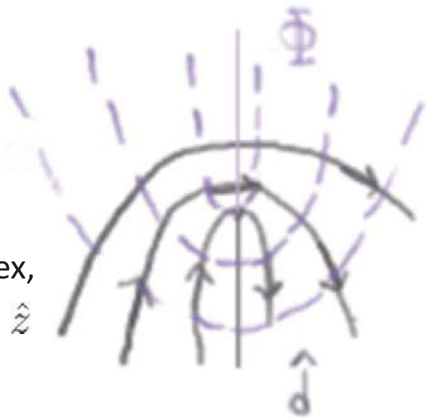
2d CpSc ($^3$He-A): S=1, L=1; $M_L$=1 ($\hat{l}$), $M_S$=0 ($\hat{d}$)

$$H = \begin{pmatrix} h & \Delta \\ \Delta^\dagger & -h^T \end{pmatrix}, \quad h = (\frac{p^2}{2m} - \mu)I$$

$$\Delta = \frac{i}{2} v_\Delta e^{i\Phi/2}\{\partial_x - i\partial_y, (\boldsymbol{\sigma}\cdot\mathbf{d})\sigma_2\}e^{i\Phi/2}$$

*For e.g. Leggett, 1975; Volovik 1992; Ivanov, 2001; Chung and Stone, 2006 Gurarie and Radzihovsky, 2007*

h/(4e) vortex, $\hat{l}$, $\hat{d}$ along $\hat{z}$




## Half Quantum Vortices(h/(4e)):

Sc phase and d-vector: Each wind by ±π

Zero-energy bound state at core

$\Gamma_0 = \Gamma_0^\dagger \equiv \gamma$ Majorana mode

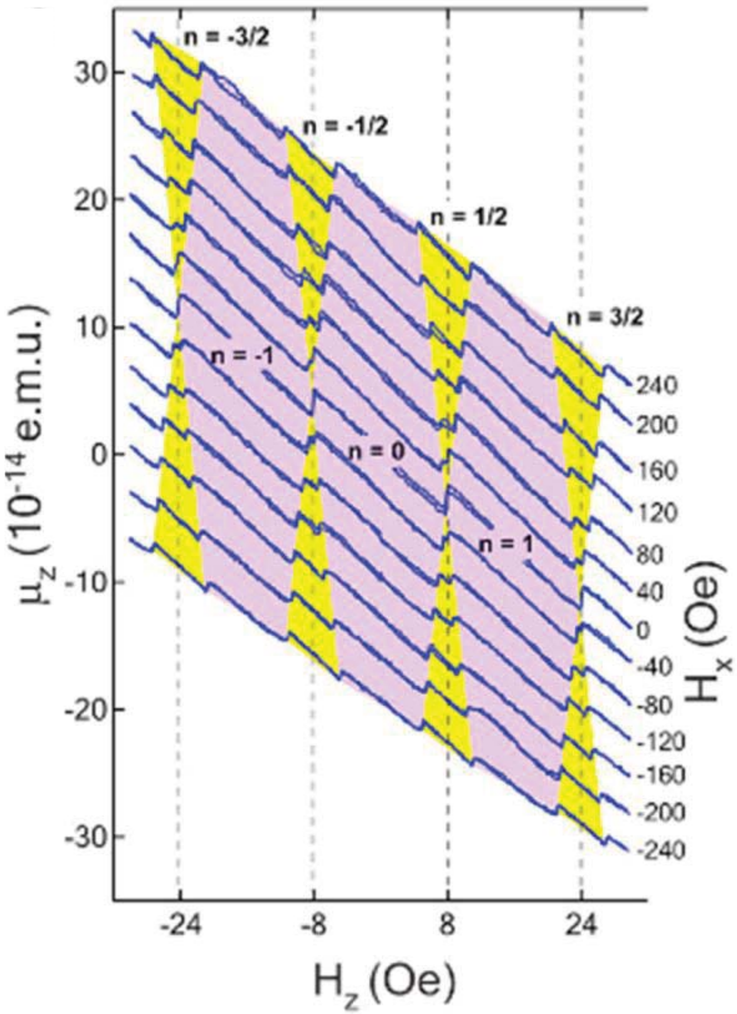
*S. B. Chung et al, 2007*

*Stabilized by ring geometry*

# One component CpSc:

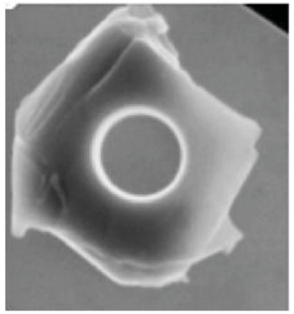
Candidate – Topological Insulator+Proximity Induced SC

# Measurements from the lab of Raffi Budakian




d-vector pinning in SRO? – *S. Raghu et al, 2010*

Strontium Ruthenate (SRO):
Candidate CpSc material



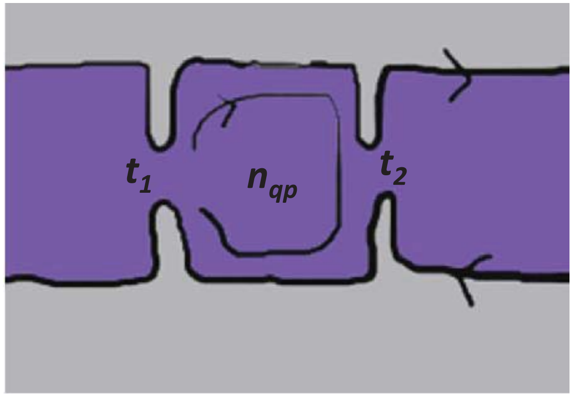
SRO sample –
Mesoscopic ring
Order 1500nm

Magnetic response: Splitting as a function of $H_x$ indicates **Half Quantum Vortices (HQV)**

*Courtesy R. Budakian*
*J. Jang et al, 2011*

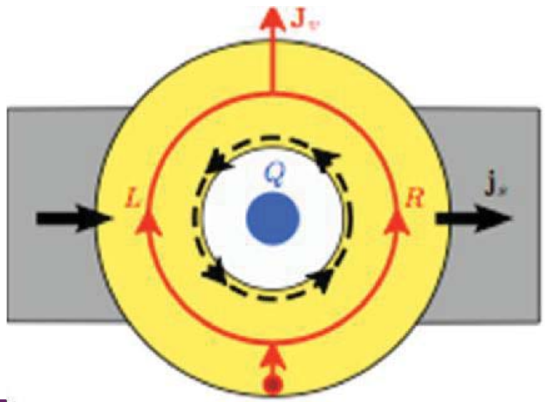
# Non-Abelian quasiparticles: detection

**Quantum Hall (ν=5/2)**




$$\sigma_{xx}^{int} \propto Re[t_1^* t_2 e^{i\phi} \langle\psi_0|U_1^{-1}U_2|\psi_0\rangle]$$

**Chiral p-wave superconductors**




$$J_v^{int} \propto |t_L||t_R| \left[e^{i\varphi} \langle\psi_L|\psi_R\rangle\right]$$

Interferometry

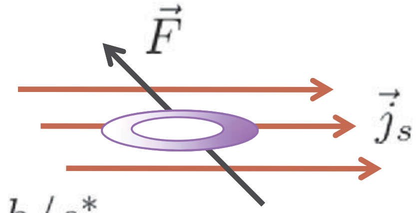
*E. Grosfeld et al., 2011*

# Vortex current and interferometry

Magnus force on vortex:

*For e.g.,Tinkham '96; Thouless '99; Sonin '01*

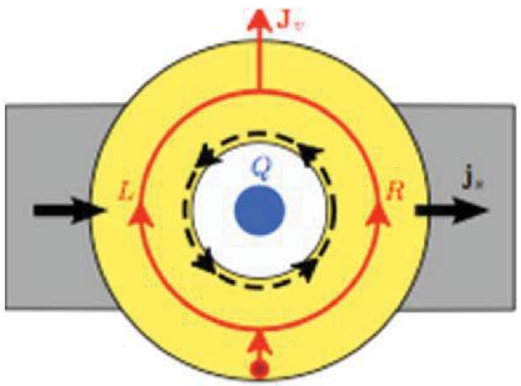
$$\vec{j}_s \times \Phi_V \hat{z}$$




$\Phi_V \equiv h/e^*$

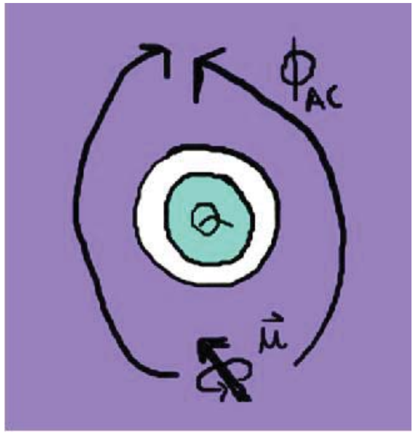
Vortex current:

$$J_v \propto |t_L|^2 + |t_R|^2 + 2|t_L||t_R| \left[e^{i\varphi}\langle\psi_L|\psi_R\rangle\right]$$




Josephson relation: $V_s \propto J_v$
Measure V or dV/dI

# Aharonov-Casher effect



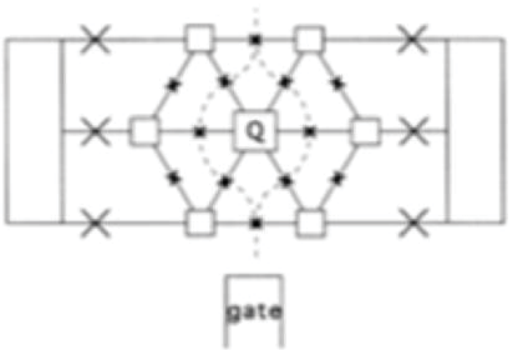
Magnetic moment moving in static electric field – geometric phase:

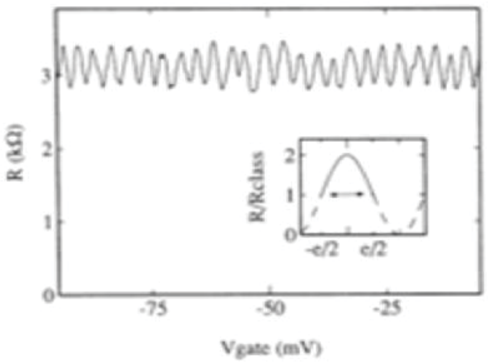
$$\phi_{AC} = \frac{1}{\hbar c^2} \oint \vec{\mu} \times \vec{E} \cdot d\vec{l}$$

2d vortex encircling charge Q: $\phi_{AC} = \Phi_V Q/\hbar = 2\pi Q/e^*$

*Aharonov and Casher, '84; Reznik and Aharonov, '89*

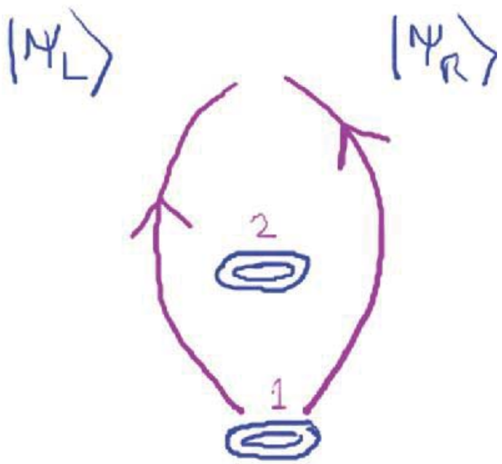
*Observation of the AC-interference in Josephson junction arrays*




*W. J. Elion et al., '93*

# Encircling Majorana vortices



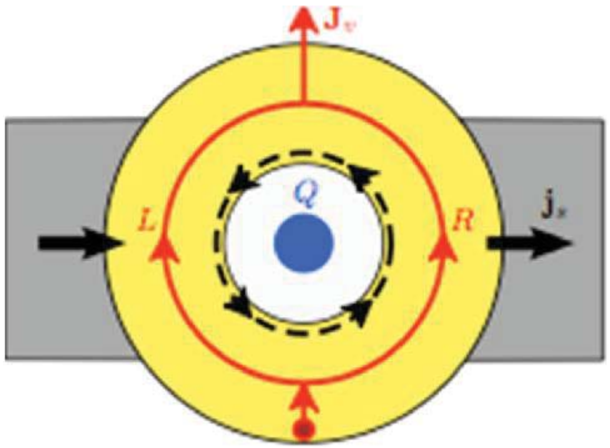
Majorana structure:

$$\begin{array}{l} \nu_1 \to \nu_2 \\ \nu_2 \to -\nu_1 \end{array} \to \langle \psi_L | \psi_R \rangle = 0$$

Two-component CpSc:

Holds for HQV around HQV as well as around FQV (FQV: can consider one Majorana in each spin component)

# Proposed Experiment




$$J_v^{int} \propto |t_L||t_R| \left[ e^{i\varphi} \langle \psi_L | \psi_R \rangle \right]$$

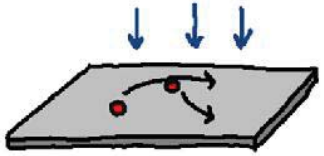
- In zero field, drive $j_s$
- Vary charge Q, see dI/dV oscillation of period 2e or 4e
- Turn on field to nucleate HQV
- Repeat AC procedure
- Oscillations should disappear

*Many thanks to R. Budakian and D. VanHarlingen*
*E. Grosfeld et al, 2011*

Alternative: Josephson vortices

*Also: E. Grosfeld and Stern, 2011*
*Beenakker 2011; Alicea 2012*

# Thanks to….



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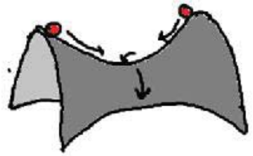
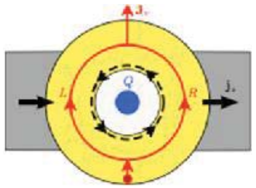
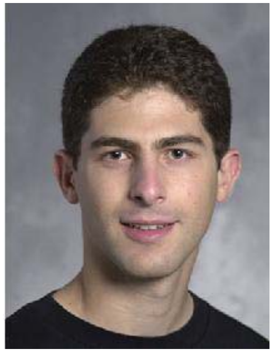


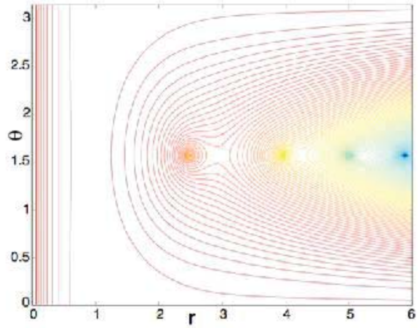




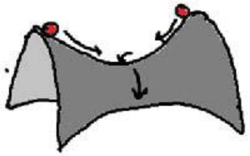
Eytan Grosfeld
Ben Gurion Univ.

Babak Seradjeh
Indiana Univ.

# In Conclusion

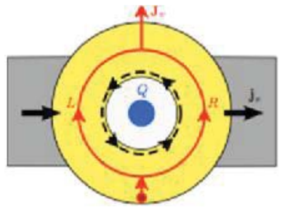



- **Abelian statistics in the quantum Hall system:**



  - **- Two-particle correlators show several signatures**
  - **- Saddle potential acts as beam for bulk anyons**

- **Non-Abelian statistics in CpSc rings:**
  - **- Experimental evidence for fractional vortices**
  - **- Aharonov-Casher interference as a proposed means of detecting Majorana fermions**



***Measures of bulk correlation? Connecting to edge states? Non-Abelian qp's in saddle potential? Integrating braiding schemes into ring geometry?***

# Physical considerations

## Quantum Interference of vortex:

### A. Mass of vortex – Estimate from Volovik's treatment

$$M_V = E_v/s^2 + M_{bd.st.},\ s \sim p_F/m$$

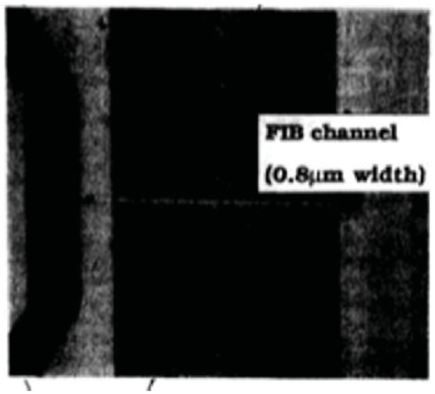
*G. E. Volovik, JETP Lett.(1997)*

Obtain estimate $10^3$-$10^4 m_e$; Compare fullerene ~$10^6 m_e$

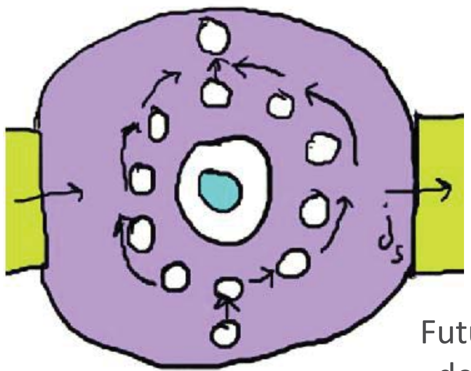
*Arndt et al, Nature (1999)*

### B. Patterning pathways

Eg. Focus ion beam channel for flux flow in high $T_c$ SC




*Miyara et al., Appl. Supercond., IEEE* (1995)



Futuristic device

## Temperature:

Suppression of vortex core states $kT < \hbar^2/(2mR\xi) \approx k10^{-3}K$

# Physical considerations (contd.)

## Rate of vortex motion:

Prevent decoherence    Adiabaticity

$$\lambda 2R/\hbar \ll v_V \ll \hbar/(m\xi)$$

Bound on Majorana coupling

Voltage Sensitivity

$$V_s \approx N_v h v_V/(4eR)$$

## Perturbations:

$$H_P = i\lambda\gamma_1\gamma_2$$

Splits zero mode – no good!

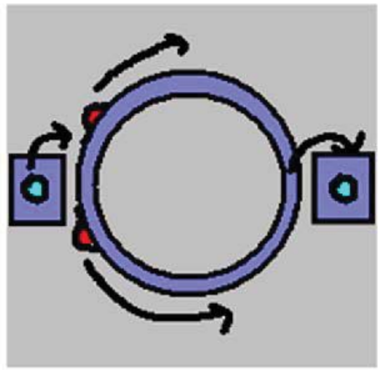
E.g. Zeeman splitting – $\lambda = -g\mu_B \mathbf{B}\cdot\mathbf{d}/2$

(leading order; mode preserved for $\mathbf{B}\perp\mathbf{d}$

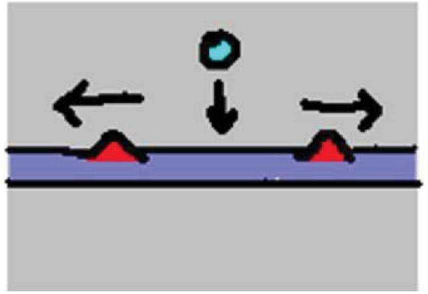
*Evidence for such a condition: Murakawa et al., '04 Yoshioka and Miyake, '09*)

**Coupling between layers? Other sources of decoherence?**

# Charge fractionalization in etched ring geometries



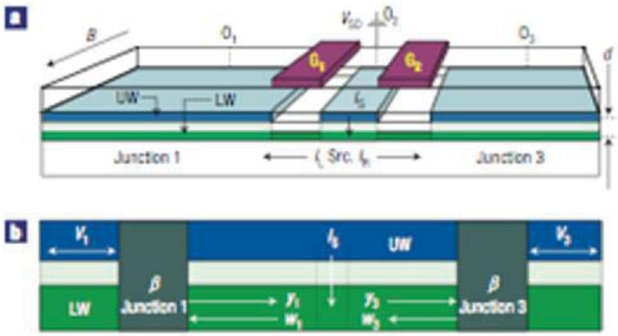
# Fractionalization in quantum wires



Tunneling of R-moving electron:
$(1+g_c)e/2$ to the right,
$(1-g_c)e/2$ to the left

$$\left(g_c \approx \left[1 + U/(2E_F)\right]^{-1/2}\right)$$

*Safi, 1997; Pham et al., 2000*
*Leinaas et al., 2009*




*Steinberg et al., 2008;*
*Le Hur et al., 2008*

Experimental realization,
Evidence for fractionalization

# Quantum Wire – Collective modes

**Low-energy plasmon-like excitations**

Tomonaga-Luttinger-liquid Hamiltonian:

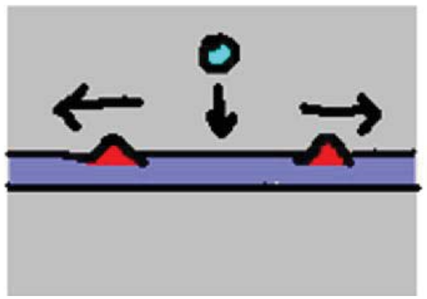
$$H_{LL} = \sum_{a=c,s} \frac{\hbar u_a}{2} \int_0^L dx \left[ g_a^{-1} \left(\partial_x \theta_a\right)^2 + g_a \left(\partial_x \phi_a\right)^2 \right]$$

Densities: $\rho_a = (\partial_x \theta_a)/\pi$
Velocities: $u_a = v_F/g_a$
$g<1$ – repulsive, $g>1$ - attractive

Electron operator:

$$\Psi_\sigma(x) = \sum_r e^{irk_F x} \psi_{r\sigma}(x)$$
$$\psi_{r\sigma}(x) \sim e^{i\sqrt{\pi}(\phi_\sigma + r\theta_\sigma)}$$

Electron propagator: Shows R-L fractionalization,
Spin-charge separation



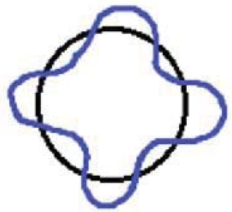
# Ring geometry – basic features

## Periodic boundary conditions

Low-energy excitations:

$$\theta_\sigma = \theta_\sigma^0 + \bar{\theta}_\sigma$$

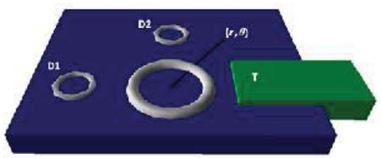
Topological modes
+ Quantized plasmons



Topological modes:
Quantum numbers for particle number and circulation

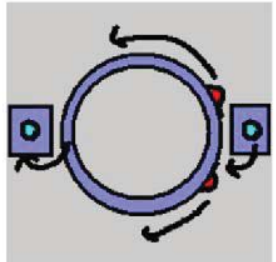
$$\theta_\sigma^0 = \theta_{J\sigma} + \sqrt{\pi}\frac{x}{L}M_\sigma$$

$$\phi_\sigma^0 = \phi_{M\sigma} + \sqrt{\pi}\left(\frac{x + L/2}{L}\right)(J_\sigma + 2\Phi/\Phi_0)$$



Localized electron: Built up of all modes

# Adding an extra electron



With momentum boost: $$|\Omega\rangle = \psi^{\dagger}_{+\sigma}(x=0)|G\rangle$$

Current: $$I(x) = \sqrt{\frac{2}{\pi}}\partial_x \phi_c(x)$$

Quantum averaged current carried by electron:

$$\langle I(x,t)\rangle_\Omega = \frac{ev_F}{g}\left[\frac{e}{2}(1+g)\delta(x-vt) - \frac{e}{2}(1-g)\delta(x+vt)\right]$$

Applicable within coherence time

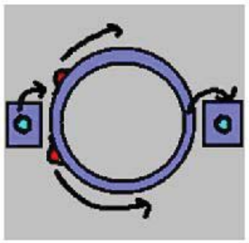
# Magnetic field and related non-invasive measures

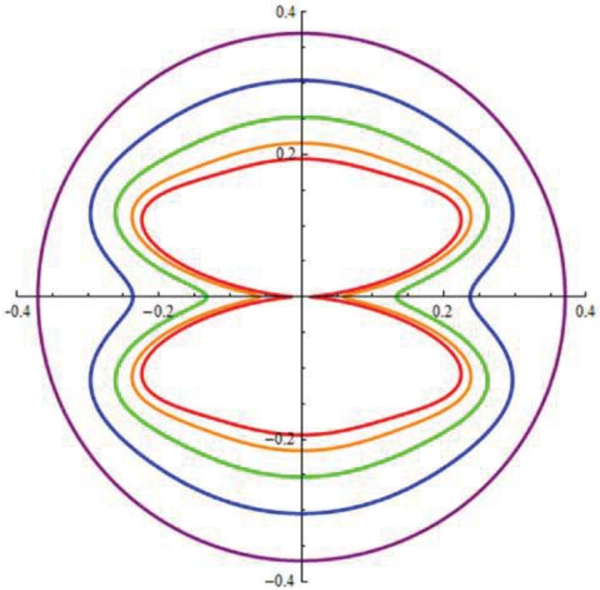
Biot-Savart: $d\hat{\vec{B}} = \frac{\mu_0}{4\pi}\frac{\hat{I}d\vec{l}\times\vec{r}}{|r|^3}$

*DeGottardi et al., 2013*

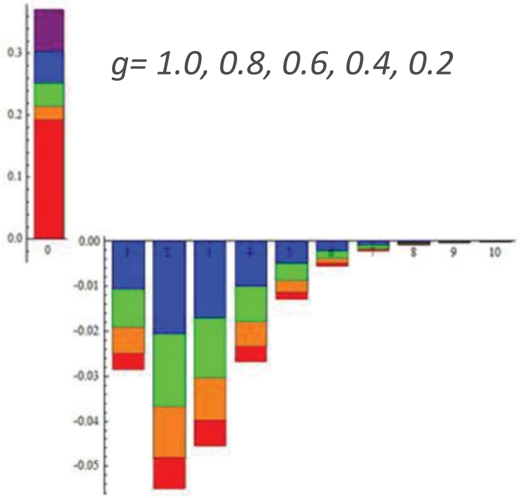
***Measures of fractionalization:***

Power ( $\langle\langle\partial_t B\rangle_\Omega^2\rangle_T$ ) ; $\langle\langle B^2\rangle_\Omega\rangle_T$

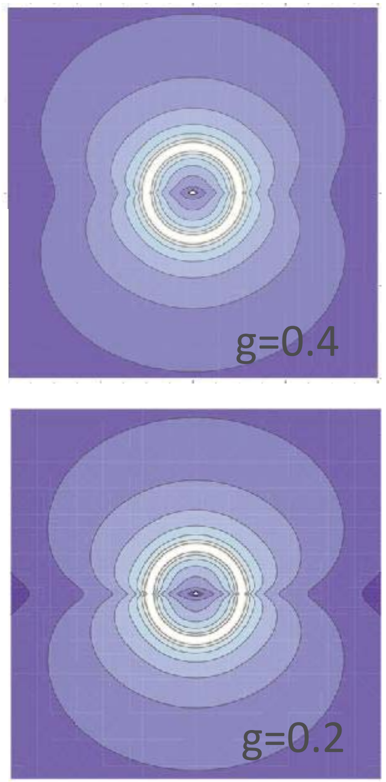





Maps of $\langle\langle\partial_t B\rangle_\Omega^2\rangle_T$ at r =2R




Fourier modes



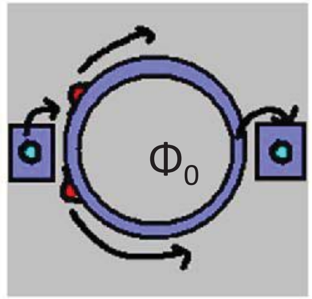

Spatial map of $\langle\langle\partial_t B\rangle_\Omega^2\rangle_T$

# Measuring ‘g’- Coulomb blockade

Determining quantum numbers:

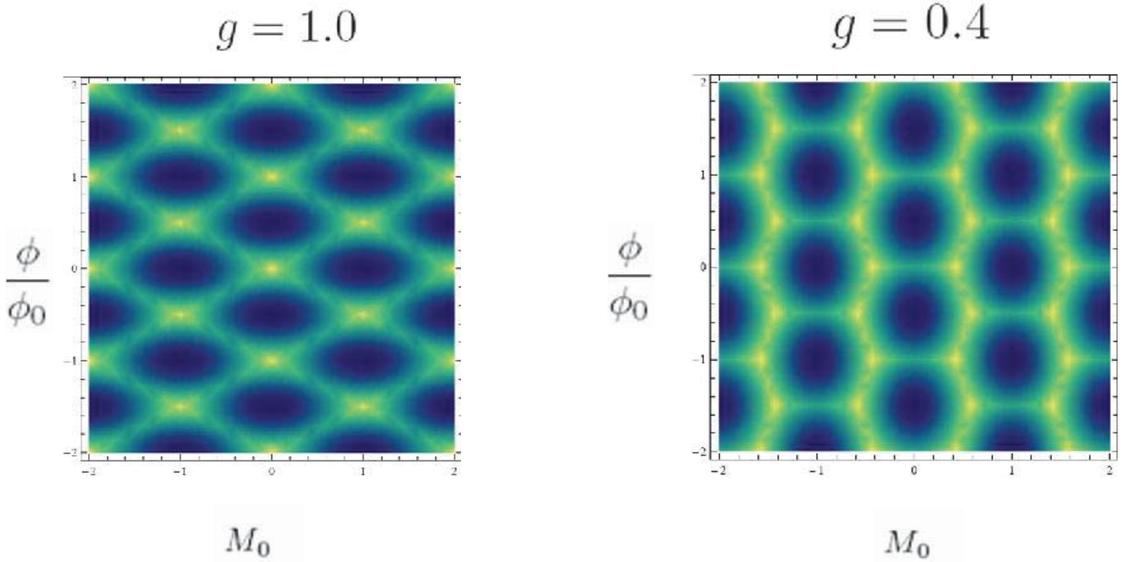
$$E^0 = \frac{\pi v_F}{2g^2 L}(M - M_0)^2 + \frac{\pi v_F}{2L}\left(J - \frac{2\phi}{\phi_0}\right)^2$$

*Kinaret et al, 1998*




Adiabatic limit

Stability regions dependent on g




Horizontal regions are of size $1\text{-}g^2$

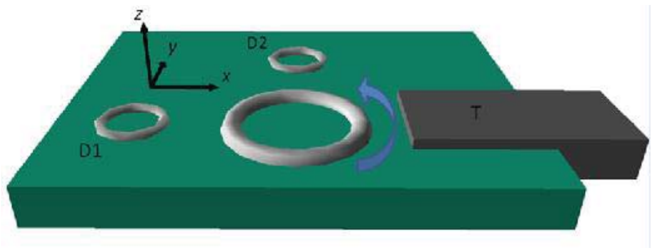
Measuring J: Test of momentum resolution

# Making measurements

## Proposed geometry

Etched ring and two pick loops
Estimates: Ring radius 1μm
g=0.4 – Asymmetry 1:10
$\langle(\Delta B)^2\rangle$ 1μm away O(mG$^2$)

Loop radius 0.1μm, R=1Ω
Induced current O(100nA)
Induced power O(10attowatts)




*E.g. Keyser et al, 2003*

*DeGottardi et al., 2013*

Asymmetric g-dependent distribution

## *Quantum mechanical scenarios*:

**A**.Probabilistic (g=1) **B**.Superposition (g=1) **C**.Fractionalization

$\langle\langle \partial_t B\rangle^2_\Omega\rangle_T$ - Asymmetry for B and C

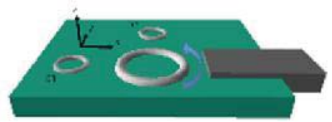
$\langle\langle B^2\rangle_\Omega\rangle_T$ - Asymmetry for C; *J* - Fluctuates for A and B

*Fluctuations also discussed by Leinaas et al., 2009*

**Coherence times? Temperature effects?**

# In Summary

- Ring geometries can serve to probe fractionalization
- Charge "fractionalization" in thin mesoscopic rings:



- Signatures in asymmetric <$(\Delta B)^2$>and power profile
- Measurements can distinguish distinctly different quantum scenarios

# Some References

*Safi, Ann. Phys. Fr. 1997*
*Pham et al., Phys. Rev. B 2000*
*Leinaas et al., Phys. Rev B 2009*

*Steinberg et al. Nature 2008*
*Le Hur et al. Ann. Phys. 2008*

*Kinaret et al, Phys. Rev. B 1998*

*E.g. Keyser et al, PRL 2003*

*Fluctuations also discussed by*
*Leinaas et al., Phys. Rev B 2009*

*For e.g.,*
*D. C. Tsui, H. L. Stormer, A. C. .Gossard*
*PRL 48, 1559, 1982*

*For e.g., E. Fradkin et al, Nuc. Phys. B '98;*
*S. Das Sarma et al, PRL '05;*
*Stern & Halperin, PRL '06; P. Bonderson et al., PRL '06*

*For e.g. Leggett, Rev. Mod. Phys '75*
*Volovik '92; Ivanov, Phys. Rev. Lett '01*
*Chung and Stone, Phys. Rev. B. '06*
*Gurarie and Radzihovsky, Phys. Rev. B '07*

*Courtesy R. Budakian*
*J. Jang et al, Science 2011*

*Aharonov and Casher, Phys. Rev. Lett. '84; Reznik and Aharonov, Phys. Rev. D '89*

E.g. Zeeman splitting –
(leading order; mode preserved for
*Evidence for such a condition: Murakawa et al., Phys. Rev. Lett. '04*
*Yoshioka and Miyake, J. Phys. Soc. Jpn, '09*)

Anyon Mapping

*B. I. Halperin, PRL (1984)*
*R. B. Laughlin in The Quantum Hall Effect (1987)*
*T. H. Hansson, J. M. Leinaas, J. Myrheim, Nucl Phys B (1992)*
*H. Kjonsberg and J. M. Leinaas, Int. J. Mod. Phys. A (1997)*
*J.  Myerheim  in Les Houches series (1999);*
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Edge States

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