The Abdus Salam International Centre for Theoretical Physics

2451-8

Workshop on Interferometry and Interactions in Non-equilibrium Meso- and Nano-systems

8 - 12 April 2013

Magnetic field effects on the finite-frequency noise of a kondo quantum dot out of equilibrium

Sabine ANDERGASSEN

University of Vienna, Boltzmanngasse 5, 1090 Vienna Austria

Magnetic field effects on the finite-frequency noise of a Kondo quantum dot out of equilibrium

S. Andergassen

S. Müller, M. Pletyukhov, D. Schuricht

Trieste, 11.4.2013

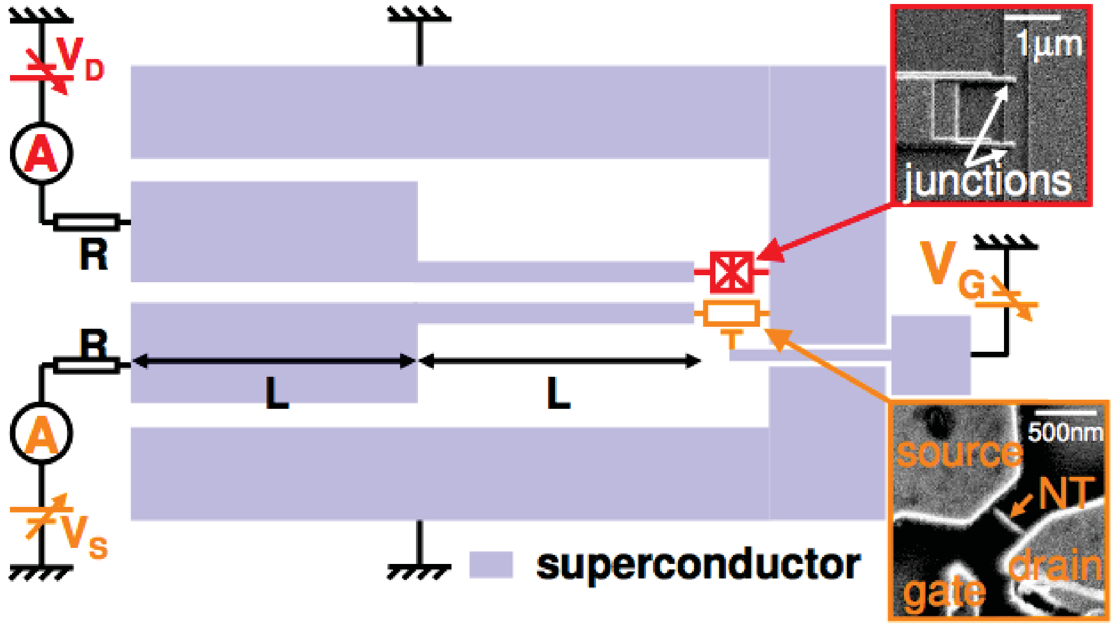
universität wien

Motivation

finite-frequency current noise

$$S^{\pm}(\Omega) = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}t \, e^{i\Omega t} \langle [I(t) - \langle I \rangle, I(0) - \langle I \rangle]_{\pm} \rangle$$

now accessible experimentally in Kondo regime



Basset, Kasumov, Moca, Zarand, Simon, Bouchiat, Deblock, PRL (2012)

Motivation

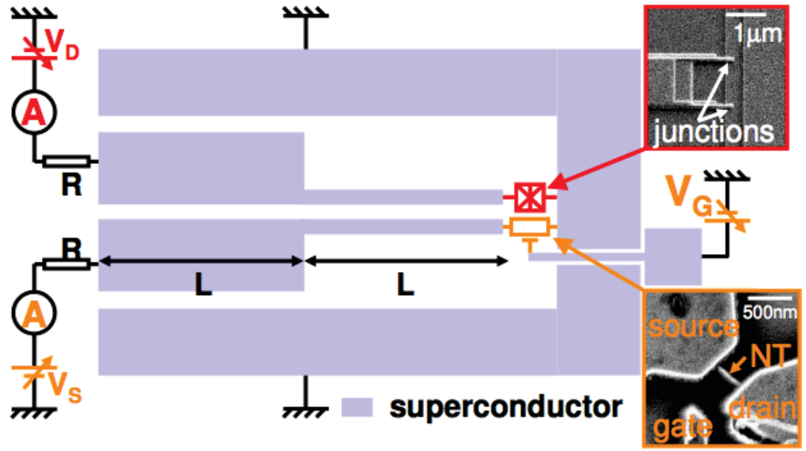
finite-frequency current noise

$$S^{\pm}(\Omega) = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}t \, e^{i\Omega t} \langle [I(t) - \langle I \rangle, I(0) - \langle I \rangle]_{\pm} \rangle$$

now accessible experimentally in Kondo regime

→ signatures of quantum many-body effects in non-equilibrium dynamics

- reveals characteristic time scales
- excitation dynamics



→ understanding of *basic mechanisms*

Basset, Kasumov, Moca, Zarand, Simon, Bouchiat, Deblock, PRL (2012)

Outline

→ Motivation

→ Method: Real-time RG

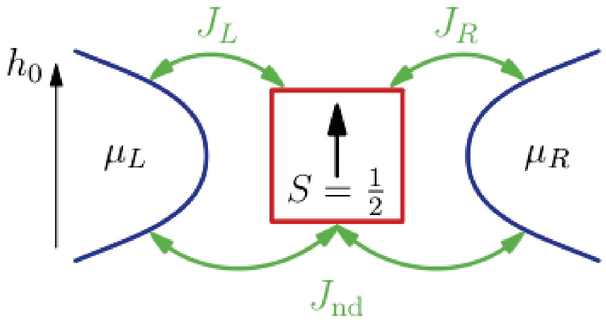
→ Results for *non-equilibrium Kondo quantum dot*

- magnetic field effects on finite-frequency noise
- experimental observation
- ac conductance

→ Conclusion

Kondo quantum dot

minimal model for quantum dot dominated by spin fluctuations



$$H = H_{\mathrm{res}} + h_0 S^z + \frac{1}{2} \sum_{\alpha\alpha' k k' \sigma\sigma'} J_{\alpha\alpha'} a^\dagger_{\alpha k \sigma} \boldsymbol{S} \cdot \boldsymbol{\sigma}_{\sigma\sigma'} a_{\alpha' k' \sigma'}$$

isotropic exchange interaction $J_{\mathrm{nd}}^2 = J_L J_R$

$\mu_{L/R} = \pm V/2$

with asymmetry $r = J_L / J_R$

calculation of noise

introduce auxiliary function $C^{\pm}_{II}(\Omega) = \int_{-\infty}^{0} \mathrm{d}t\, e^{-i\Omega t} \langle [I(0), I(t)]_{\pm} \rangle$

$\rightarrow$ $$\boxed{S^{\pm}(\Omega) = \mathrm{Re}\, C^{\pm}_{II}(\Omega)}$$

Schuricht, Schoeller, PRB (2009)

Real-time RG

Schoeller, EJP (2010)

von Neumann equation $\dot{\rho}(t) = -\mathrm{i}\,[H, \rho(t)] = -\mathrm{i}\, L\, \rho(t)$

$$\rho(0) = \rho_D(0)\, \rho_L^{eq}\, \rho_R^{eq}$$

Liouvillian $L = L_{res} + L_D + L_{ex}$

→ quantum kinetic equation for $\rho_D(t) = \mathrm{Tr}_{res}\, \rho(t)$

$$\boxed{\dot{\rho}_D(t) = -iL_D\rho_D(t) - i\int_0^t dt'\Sigma(t-t')\rho_D(t')}$$

Laplace transform

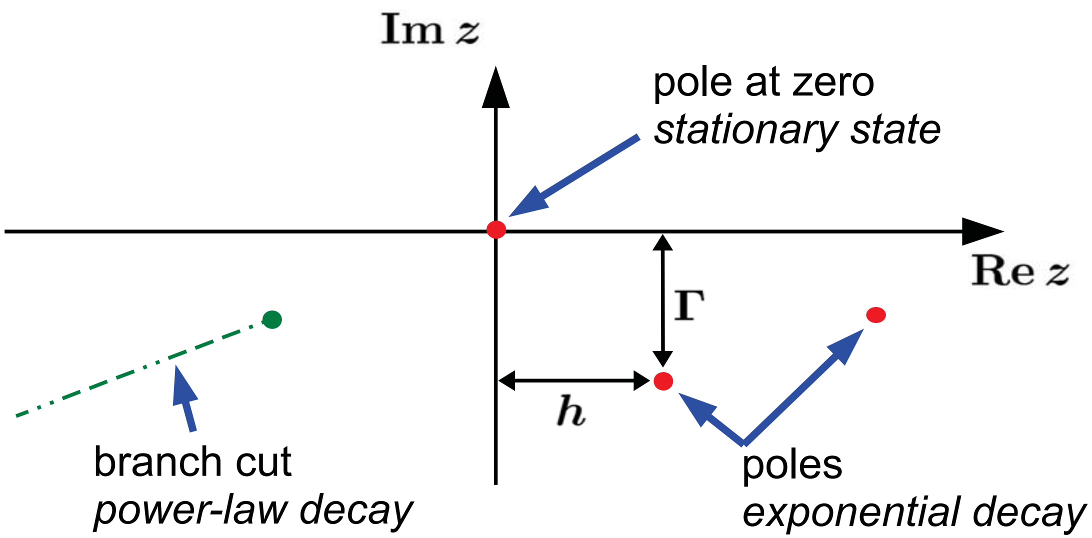
$$\rho_D(z) = \int_0^\infty dt\, e^{izt}\, \rho_D(t) = \frac{i}{E - L_D - \Sigma(z)}\rho_D(t=0)$$

→ formulate RG for $L^{eff}(z) = L_D + \Sigma(z)$

Stationary state and time evolution

$$\rho_D(z) = \frac{i}{z - L_D^{eff}(z)}\,\rho_D(0)$$ determines *relaxation dynamics*

analytic structure of $L_D^{eff}(z)$



→ *relaxation rates* identified by microscopic cutoff scales

→ *closed analytic form* of $L_D^{eff}(z)$ beyond Markovian theories

→ time evolution determined by inverse Laplace transform

Pletyukhov, Schuricht, Schoeller, PRL (2010)

Symmetric and antisymmetric noise

diagrammatic expansion in renormalized J_{nd}

$\rightarrow$ *analytic solution* of flow equations at weak coupling $\Lambda_c = \max\{|\Omega|, |V|, |h|\} \gg T_K$

$$S^+(\Omega) = \frac{\pi}{8} J_{\mathrm{nd}}^2 M h + \frac{\pi}{8} J_{\mathrm{nd}}^2 \sum_{\sigma,\eta=\pm} |\Omega + \sigma V + \eta h|_2 + \frac{\pi}{2} J_{\mathrm{nd}}^2 \sum_{\sigma=\pm} \left[M^2 |\Omega + \sigma V| - \left(M^2 - \frac{1}{4} \right) |\Omega + \sigma V|_1 \right]$$

$$S^-(\Omega) = \frac{3\pi}{4} J_{\mathrm{nd}}^2 \Omega + \frac{\pi}{4} J_{\mathrm{nd}}^2 M \sum_{\sigma,\eta=\pm} \eta |\Omega + \sigma V + \eta h|_2$$

with $|x|_i = \frac{2x}{\pi} \arctan\left(\frac{x}{\Gamma_i} \right)$

- renormalized magnetic field $h = h_0(1 - J)$
- magnetization $M = -\frac{(1+r^2)h}{2(1+r^2)|h| + 2r(|V-h| + |V+h|)}$

Symmetric and antisymmetric noise

diagrammatic expansion in renormalized J_{nd}

$\longrightarrow$ *analytic solution* of flow equations at weak coupling $\Lambda_c = \max\{|\Omega|, |V|, |h|\} \gg T_K$

$$S^+(\Omega) = \frac{\pi}{8} J_{\mathrm{nd}}^2 M h + \frac{\pi}{8} J_{\mathrm{nd}}^2 \sum_{\sigma,\eta=\pm} |\Omega + \sigma V + \eta h|_2 + \frac{\pi}{2} J_{\mathrm{nd}}^2 \sum_{\sigma=\pm} \left[M^2 |\Omega + \sigma V| - \left(M^2 - \frac{1}{4} \right) |\Omega + \sigma V|_1 \right]$$

$$S^-(\Omega) = \frac{3\pi}{4} J_{\mathrm{nd}}^2 \Omega + \frac{\pi}{4} J_{\mathrm{nd}}^2 M \sum_{\sigma,\eta=\pm} \eta |\Omega + \sigma V + \eta h|_2$$

with $|x|_i = \frac{2x}{\pi} \arctan\left(\frac{x}{\Gamma_i}\right)$

- rates: *longitudinal* $\Gamma_1 = \pi (J_L^2 + J_R^2) h/2 + \pi J_{\mathrm{nd}}^2 \max\{|V|, |h|\}$

  transverse $\Gamma_2 = \pi J_{\mathrm{nd}}^2 V/2 + \Gamma_1/2$

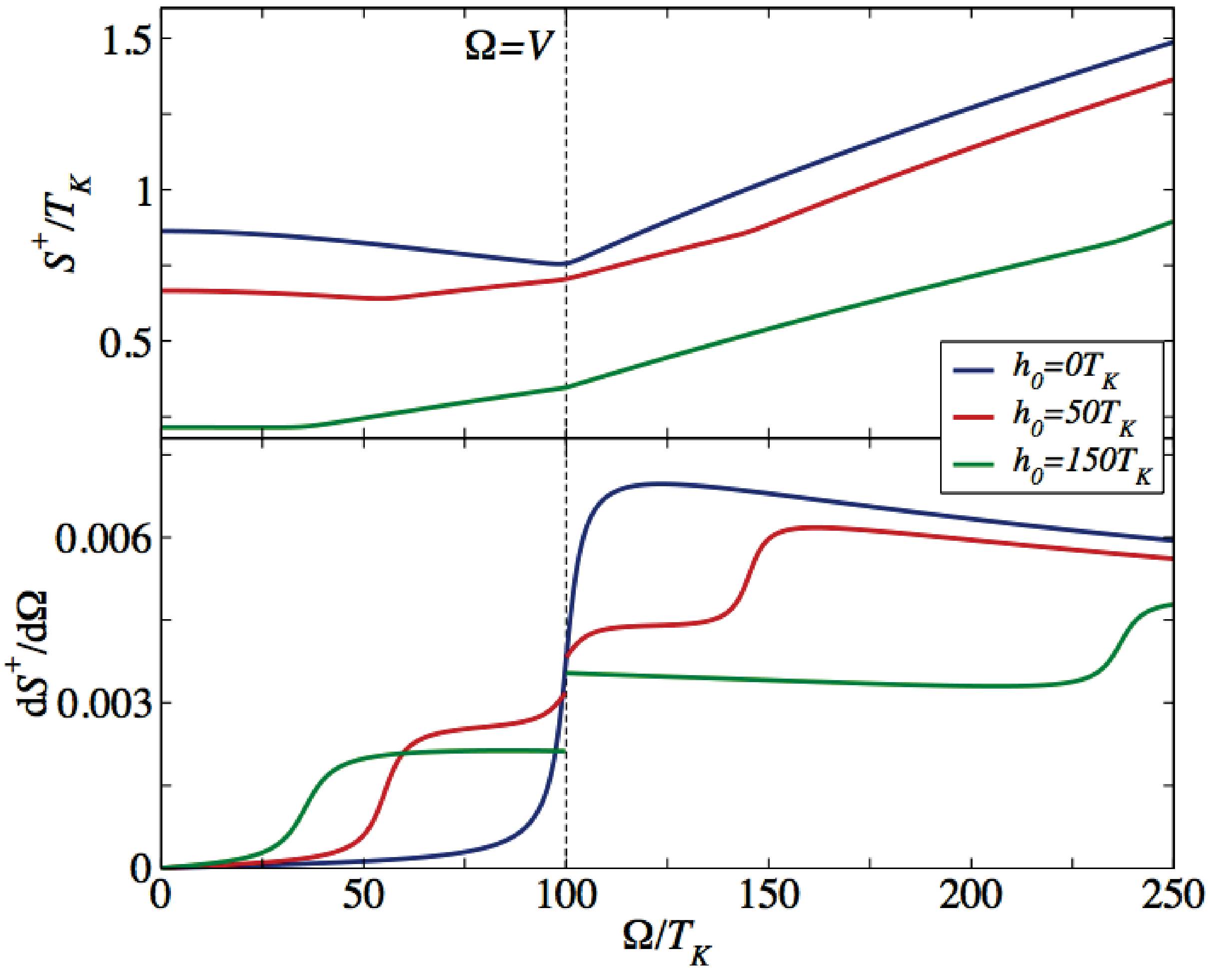
  $\longrightarrow$ define lineshape at resonances

- new contribution to S^+ not broadened by any decay rate

  $\longrightarrow$ *discontinuity in derivative* $V = \pm\,\Omega$

Symmetric noise

$h = 0$ $$S^+(\Omega) = \frac{3}{4} J_{\rm nd}^2 \sum_{\sigma=\pm} (\Omega+\sigma V) \arctan\left(\frac{\Omega+\sigma V}{\Gamma}\right)$$ $(\Gamma_1 = \Gamma_2)$



→ characteristic *resonance* in derivative at $V = \Omega$

$$S^{+}(\Omega) = \frac{\pi}{8} J_{\mathrm{nd}}^2 M h + \frac{\pi}{8} J_{\mathrm{nd}}^2 \sum_{\sigma,\eta=\pm} |\Omega+\sigma V+\eta h|_2 + \frac{\pi}{2} J_{\mathrm{nd}}^2 \sum_{\sigma=\pm} \left[M^2|\Omega+\sigma V| - \left(M^2 - \frac{1}{4}\right) |\Omega+\sigma V|_1 \right]$$

$h > 0$

$\rightarrow$ additional features at

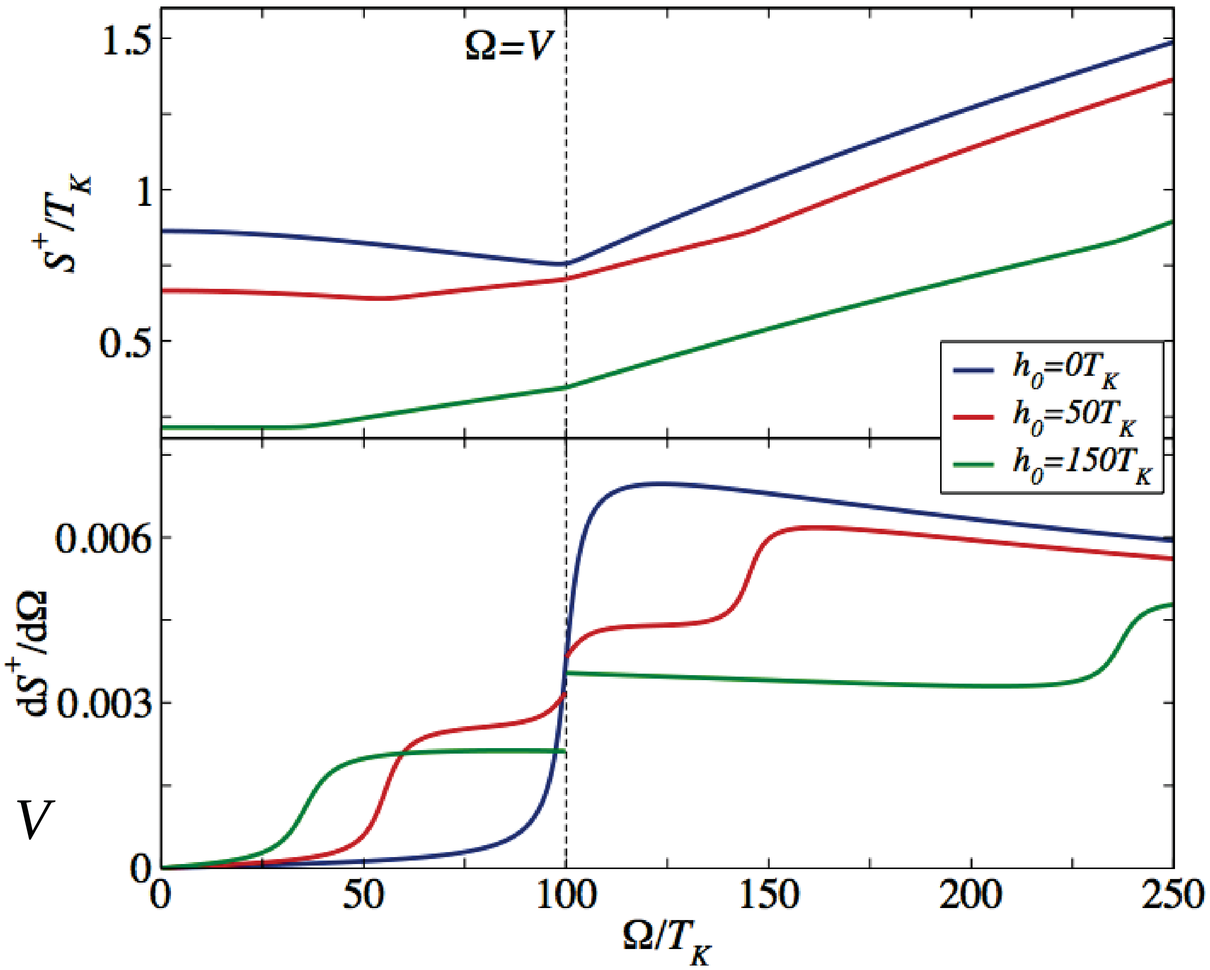
$\Omega = |V \pm h|$ in derivative

broadened by Γ_2

$\rightarrow$ *discontinuity* at $\Omega = V$

$$\Delta = \pi J_{\mathrm{nd}}^2 M^2$$

- superposition with continuous enhancement broadened by Γ_1 for $h < V$
- no broadening for $h > V$



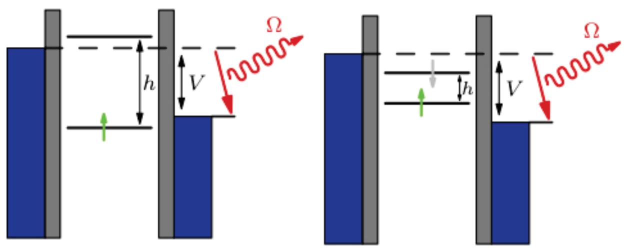
reported also in strong-coupling Toulouse limit

Schiller, Hershfield, PRB (1998)

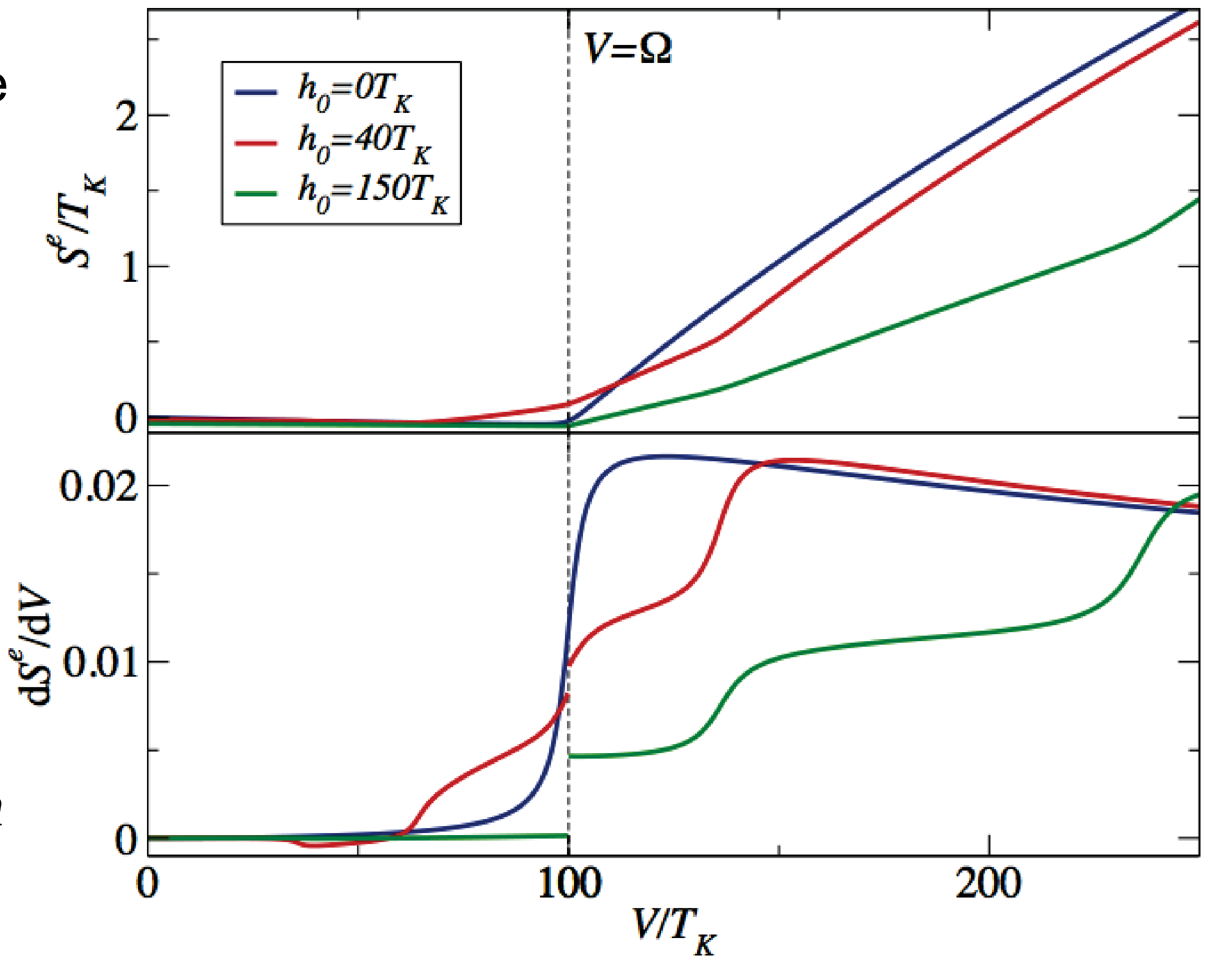
Emission noise

noise induced by photon emission $S^e(\Omega) = S^+(\Omega) - S^-(\Omega)$

→ same characteristic feature as S^+ with *discontinuity* at $\Omega = V$ for finite h

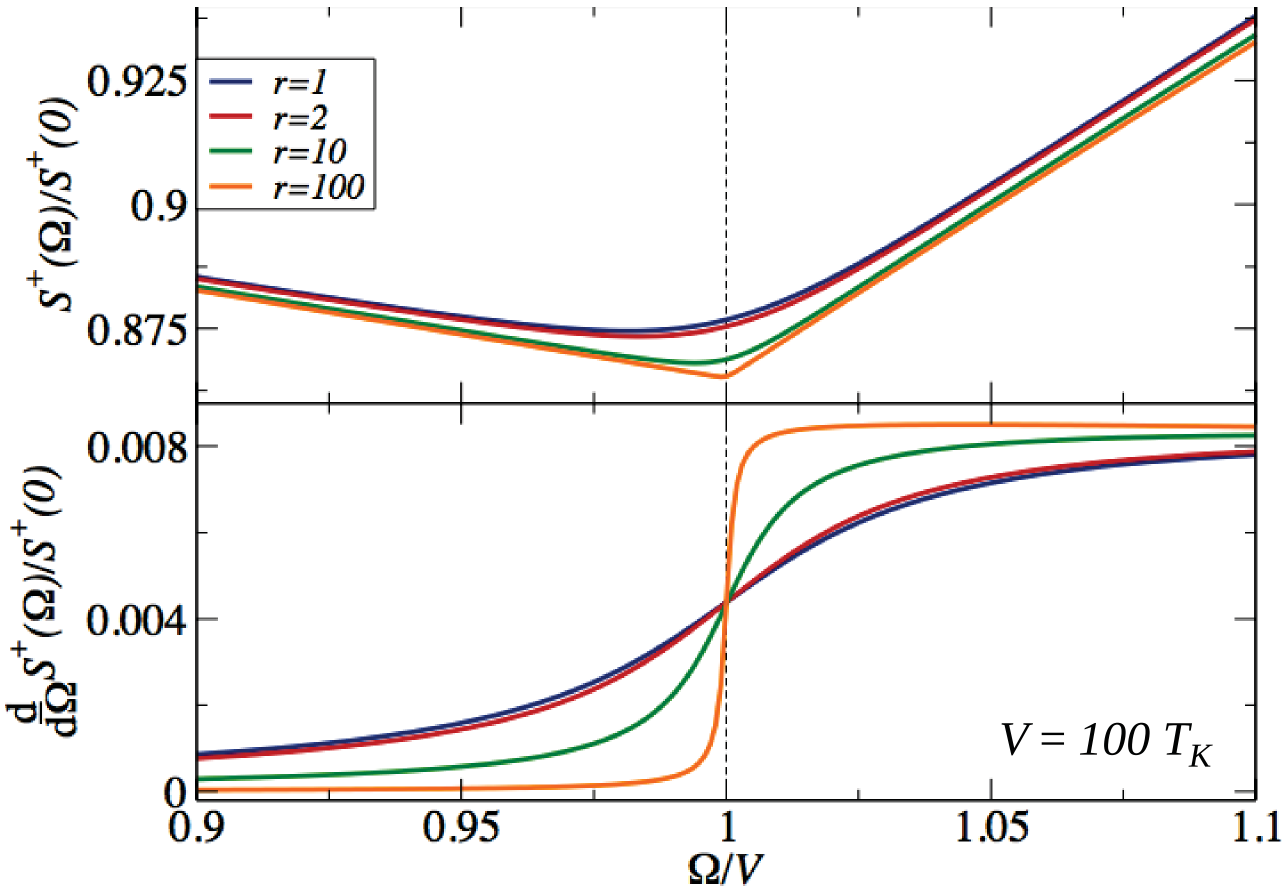


→ magnetic field introduces *additional feature* at $V = h$ due to magnetization



→ V - *dependence* qualitatively similar to Ω - dependence

Asymmetry effects



rescaling of exchange couplings $\rightarrow$ sharpening of the features

Comparison to experiment

noise measurement on carbon nanotube quantum dot

Basset, Kasumov, Moca, Zarand, Simon, Bouchiat, Deblock, PRL (2012)

$h = 0$ $$\frac{\mathrm{d}S^e(\Omega)}{\mathrm{d}V} = \frac{3}{4} J_{\mathrm{nd}}^2 \left(\arctan \frac{\Omega + V}{\Gamma} - \arctan \frac{\Omega - V}{\Gamma} \right)$$

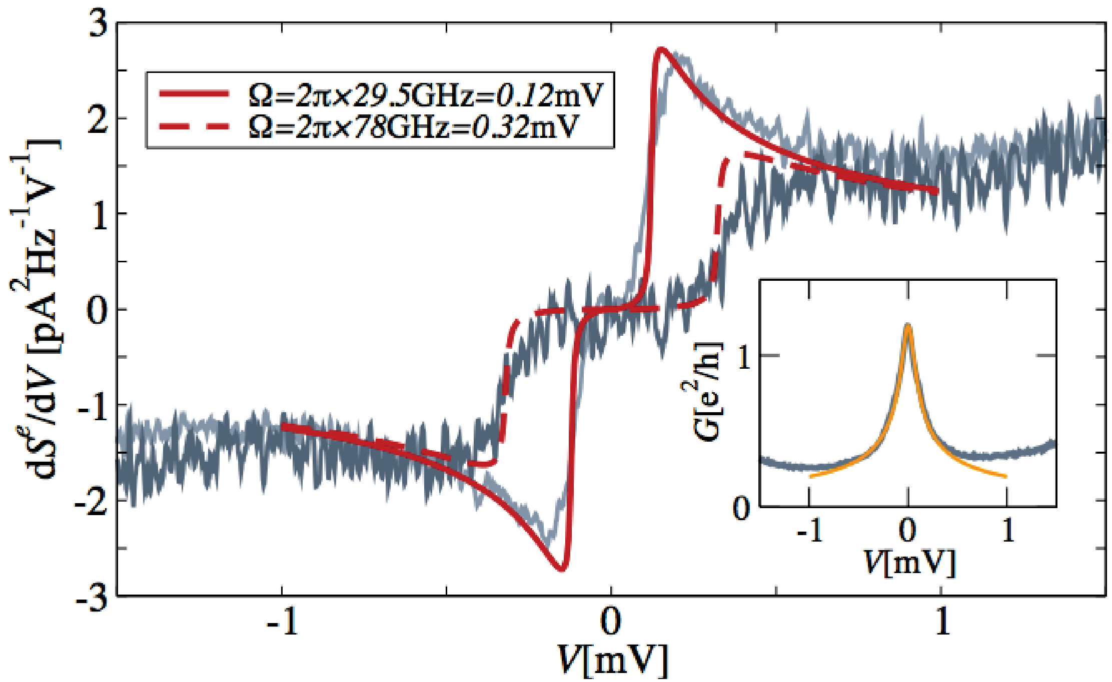
parameters determined by $G(V)$

in strong-coupling regime

asymmetry $G(V=0) = 1.194 \frac{e^2}{h}$

T_K $G(V = T_K^*) = \frac{2}{3} G(V=0)$

with $T_K^* = 10.57\, T_K$



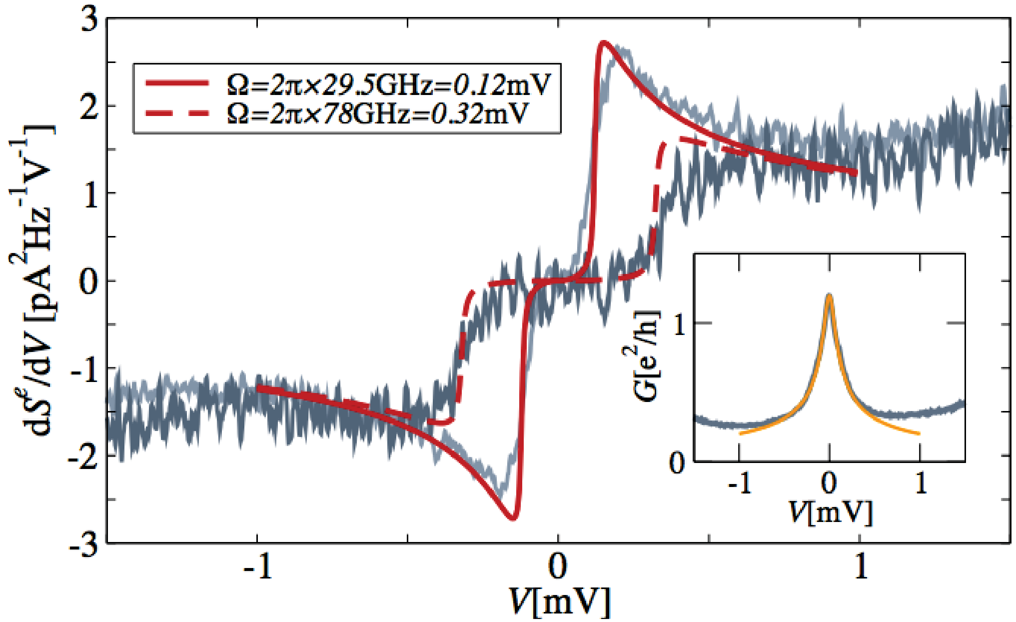
Pletyukhov, Schoeller, PRL (2012)

→ *resonances* at $V = \pm\,\Omega$ broadened by Γ

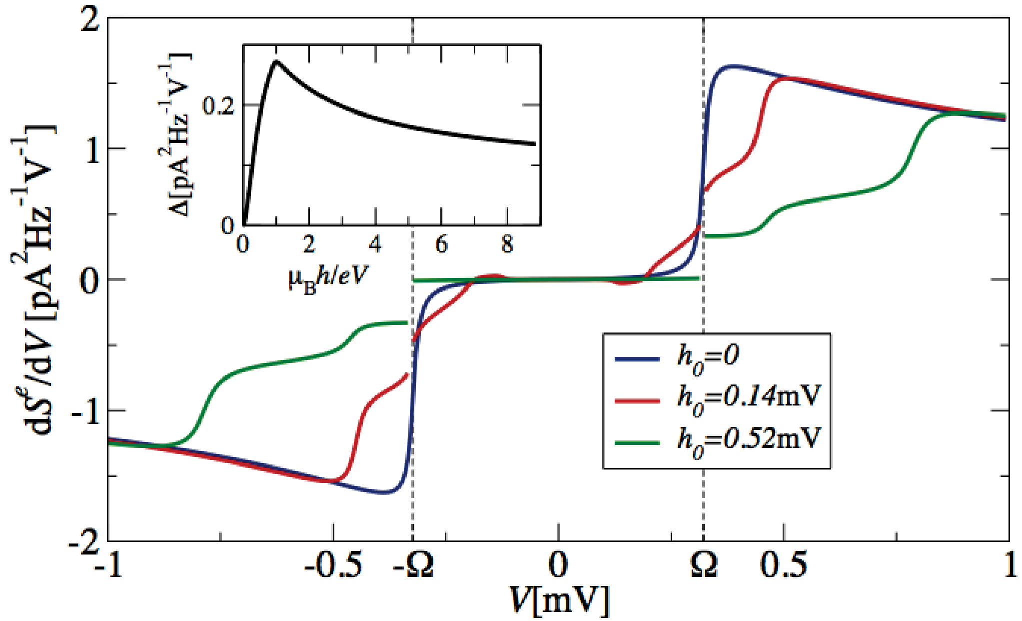
→ consistent description for both frequencies *without adjustable parameters*

Prediction for measurement in magnetic field

$h = 0$



$h > 0$



→ additional features at $V = |\Omega \pm h|$

→ resonance at $\Omega = \pm V$ only broadened by temperature and experimental resolution

→ discontinuity most pronounced for $h = V$

AC conductance

dc bias modulated by small ac voltage $V(t) = V + \delta V \; e^{-i\Omega t}$

generalization of Kubo to non-equilibrium

Safi, Bena, Crepieux, PRB (2008)
Kubala, Marquardt, PRB (2010)

$$\boxed{G(\Omega) = \frac{1}{\Omega}\left[C^{-}_{II}(\Omega) - C^{-}_{II}(0)\right]}$$

AC conductance

dc bias modulated by small ac voltage $V(t) = V + \delta V\ e^{-i\Omega t}$

analytic results

$$\mathrm{Re}\, G(\Omega) = \frac{3\pi}{4} J_{\mathrm{nd}}^2 + \frac{\pi M}{4\Omega} J_{\mathrm{nd}}^2 \sum_{\sigma,\eta=\pm} \eta\, |\Omega + \sigma V + \eta h|_2$$

$$\mathrm{Im}\, G(\Omega) = -\frac{M}{2\Omega} J_{\mathrm{nd}}^2 \sum_{\sigma,\eta=\pm} \eta \left[\mathcal{L}_2(\Omega + \sigma V + \eta h) - \mathcal{L}_2(\sigma V + \eta h)\right]$$

with $\mathcal{L}_i(x) = x\ \ln(\Lambda_c / \sqrt{x^2 + \Gamma_i^2})$

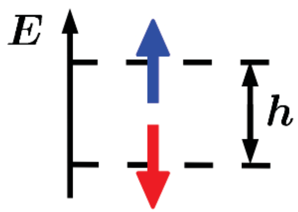
- *logarithmic behavior* in $\mathrm{Im}\, G$ characteristic Kondo feature
- *no feature* at $\Omega = V$
- in contrast to noise *all* resonances broadened by Γ_2

→ conductance contains less information on relaxation processes

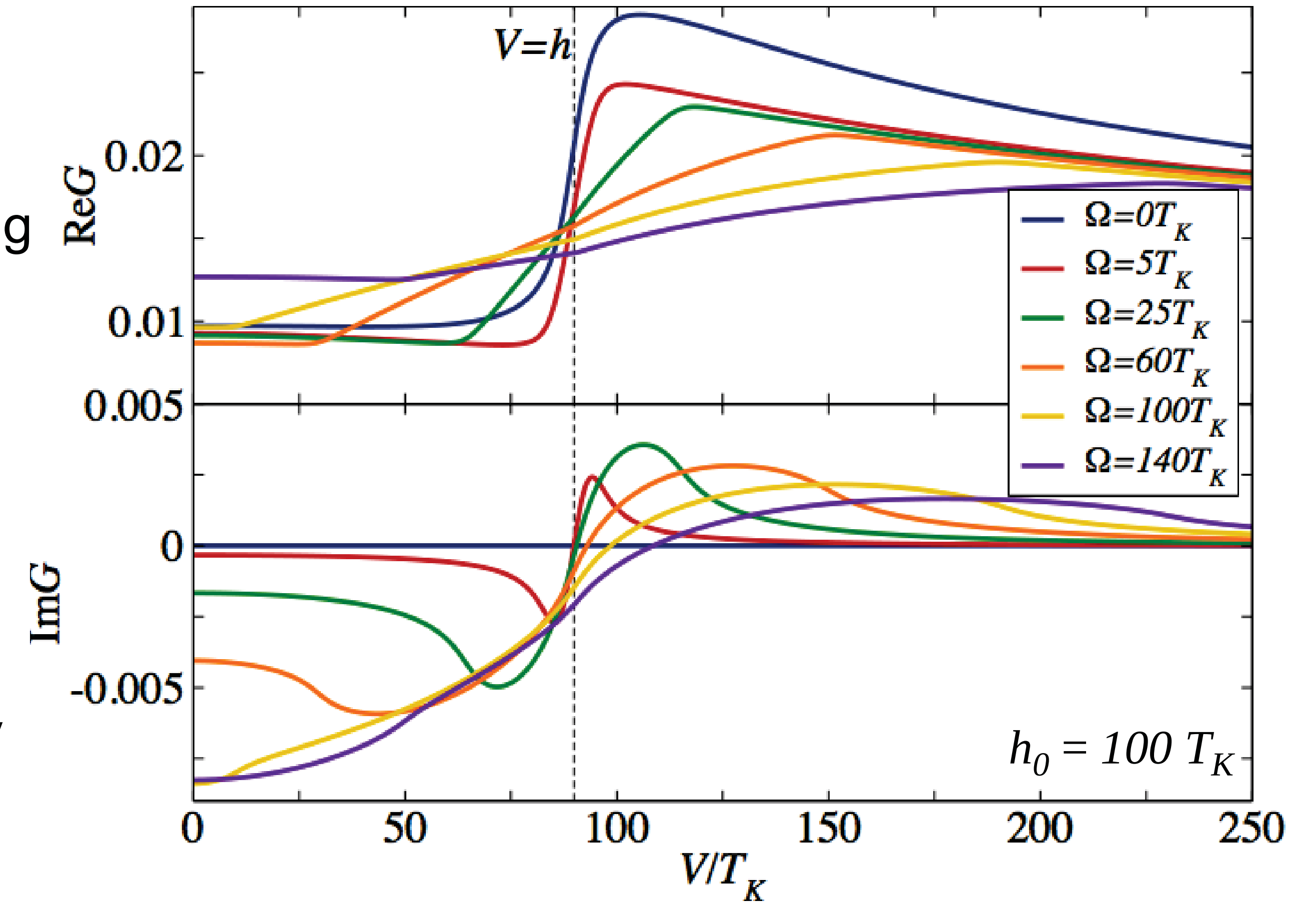
AC conductance

voltage dependence

- for $\Omega = 0$ *characteristic resonance* at $V = h$ due to inelastic cotunneling



- for $\Omega > 0$ step-like enhancement replaced by *continuous increase* in range $V = |h \pm \Omega|$



Conclusion

finite-frequency current noise of non-equilibrium Kondo dot

$\longrightarrow$ real-time RG in Liouville space provides *analytic results*

$\longrightarrow$ novel *magnetic field effects*

- characteristic resonances broadened by longitudinal and transverse spin relaxation rates
- sharp feature at $\Omega = \pm V$ characterized by absence of any decay rate

$\longrightarrow$ prediction for *experimental observation* at finite field

Ref.: Müller, Schuricht, Pletyukhov, Andergassen, arXiv:1211.7072

Thanks to ...

Richard Deblock

Herbert Schoeller

Pascal Simon

Gergely Zarand

DFG-FG 912

FOR723

Functional Renormalization Group
for Correlated Fermion Systems