Digital Gamma-Ray Spectroscopy: Trapezoidal Filtering

This activity serves an introduction to Pulse Shape Analysis (PSA) with HPGe detectors. Participants will analyse digitised preamplifier signals from a HPGe detector using trapezoidal filtering.

Each participant will have the opportunity to test a trapezoidal filtering algorithm, which will be applied to real gamma-ray data in order to produce a gamma-ray spectrum. Analysis of this spectrum can be used to optimise the algorithm. Trapezoidal filtering will be demonstrated initially with simple arithmetic using a spreadsheet.

Task 1: FilterEnergy.m

The program FilterEnergy.m, reads in signal data, performs signal-by-signal filtering using ge_trace_to_energy.m, and generates the resulting gamma-ray spectrum. You can run the program with the 'debug_mode' parameter set to one to see the signal shapes at various stages of filtering. Running with this parameter set to zero will analyse a user specified number of signals and generate a gamma-ray spectrum. Set the values which define the pole zero correction, peaking time, and gap time, and investigate their effect on the filtering process and spectral shape.

The traces were recorded using 100 MHz sampling.

Question: How do the parameters influence the filtering?

Question: What are the features of the gamma-ray spectrum? What was the source?

Question: What happens if the gap-time is too short?

Question: What could happen if the gap time was too long?

Task 2: Trapezoid.xls

The spreadsheet, Trapezoid.xls, contains a single signal from a HPGe detector which is to be shaped, graphed, and analysed. The result of this analysis will be the relative energy of the incident gamma ray which produced the signal. The digitized signal was sampled at a rate of 100 MHz and comprises 1000 samples.

For the present task, the raw trace [Figure 1(a), attached] should first be corrected for the offset in the baseline [Figure 1(b), attached] and the exponential decay (pole zero correction) [Figure 1(c), attached]. Following this, a trapezoidal filter should be applied [Figure 1(d), attached]. It should be noted that all orders of operation commute with each other (i.e., **B**x**P**x**T** = **B**x**T**x**P** = **T**x**P**x**B** = **T**x**B**x**P** = **P**x**T**x**B** = **P**x**B**x**T**).

Step 1. Baseline Correction

The baseline offset can be corrected by calculating the average baseline at the start of the trace (e.g. over the first 100 samples) and subtracting that average from the raw signal, according to

$$Tr(n) = Traw(n) - \frac{1}{100}\sum_{i=0}^{99} Traw(i)$$

Question: What is the effect of not performing this step?

Step 2: Pole-Zero Correction

The exponential decay of the signal can be corrected by applying the relation below. This process is known as a pole-zero correction and is an essential step in ensuring good energy resolution.

$$Tr'(n) = Tr'(n-1) + \left[Tr(n) - Tr(n-1)e^{-1/\tau}\right] \quad (1)$$

$$= Tr'(n-1) + Tr(n) - Tr(n-1)[1 - \frac{1}{\tau}]$$

$$= Tr'(n-1) + Tr(n) - Tr(n-1) + \frac{Tr(n-1)}{\tau}$$

$$= Tr'(n-1) + Gz(n)$$

$$= \sum_{i=0}^{n} Gz(i), \quad (2)$$

where

$$Gz(n) = Tr(n) - Tr(n-1) + \frac{Tr(n-1)}{\tau}$$

or equivalently by

$$Tr'(n) = Tr(n) + \frac{Pz(n)}{\tau} \tag{3}$$

where

$$Pz(n) = Pz(n-1) + Tr(n-1) = \sum_{i=1}^{n} Tr(i-1)$$

Question: Where does the exponential decay arise from?

Question: What is the effect of an incorrectly applied pole-zero correction?

Step 3: Application of Trapezoidal Filter

The corrected trace, *Tr*, can be shaped by the trapezoidal filter for energy determination by the following relation,

$$S(n) = \sum_{i=n-k}^{n} Tr'(i) - \sum_{i=n-l-k}^{n-l} Tr'(i)\,. \tag{4}$$

The filter can be viewed as a sliding window which subtracts the components of the trace within the "left" window of the filter from those components in the "right". This is illustrated by Figure 2, attached.

While Eq. 4 and Figure 2 are pictorially useful and easy to implement in a spreadsheet, a recursive form is required for efficiency and speed in a program. This can be achieved by noting that

$$S(n-1) = \sum_{i=n-k-l}^{n-1} Tr'(i) - \sum_{i=n-l-k-1}^{n-l-1} Tr'(i)$$

and therefore,

$$S(n) = S(n-1) + Tr'(n) - Tr'(n-k-1) - Tr(n-l) + Tr'(n-l-k-1)$$

$$= S(n-1) + Tr'(n) - Tr'(n-k') - Tr'(n-l) + Tr'(n-l-k')$$

$$= S(n-1) + d_{k',l}(n)$$

$$= \sum_{0}^{n} d_{k',l}(i), \qquad (5)$$

where

$$k' = k + 1$$

and

$$d_{k',l}(n) = Tr'(n) - Tr'(n-l) - Tr'(n-k') + Tr'(n-l-k').$$

Question: What do the values of k and l correspond to?

References:

[1] V.T. Jordanov and G.F. Knoll, Nucl. Instrum. Methods Phys. Res. A **345**, 337 (1994).
[2] G.F. Knoll, *Radiation Detection and Measurements*, 2nd ed. (Wiley, New York, 1989).