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List of Abstracts

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Differentiability of Lipschitz functions with respect to singular measures

Giovanni Alberti
Università degli Studi di Pisa, Pisa, Italy

Abstract: According to Rademacher theorem every Lipschitz function on the Euclidean space is differentiable almost everywhere, where "almost everywhere" refers to the Lebesgue measure. In this talk I will explain how the statement of Rademacher theorem should be modified when the Lebesgue measure is replaced by an arbitrary measure. In particular I will show that the differentiability properties of Lipschitz functions with respect to such a measure are related to the decompositions of this measure in terms of one-dimensional rectifiable measures.
This result is connected to recent work by many authors, including David Bate, Marianna Csornyei, Peter Jones, Andrea Marchese, and David Preiss.

Calibrations and blow up analysis

Costante Bellettini
IAS, Princeton NJ, USA

Abstract: Calibrated currents naturally appear when dealing with several geometric questions: Plateau's problem was probably the first connection, but recent years have witnessed the birth of new sources of interest, for example Taubes's work on Seiberg-Witten and Gromov invariants or some special setting for high dimensional gauge theory (such as anti self-dual instantons).
Some aspects of these geometric problems require a deep understanding of regularity properties of calibrated currents. The issues are generally very hard. We will focus mostly on the two-dimensional case where we will show a surprising connection with pseudo-holomorphic currents and perform a blow up analysis for 2-dimensional calibrated currents, where we will be interested in the uniqueness of tangent cones and in the rate of decay for the mass ratio.

On the area of the graph of a piecewise smooth map from the plane to the plane with a curve discontinuity

Giovanni Bellettini
Roma Tor Vergata and INFN, Roma, Italy

Abstract: We discuss an estimate from above for the value of the area of the graph of a map from the plane to the plane, discontinuous along a smooth simple curve. We show that the area of the graph does not exceed the area of the regular part, with the addition of a singular term measuring the area of a disk-type solution of the Plateau problem spanning the two traces of the map on its discontinuity set. The construction leading to this result is valid also when the involved minimal surface has self-intersections.

Analytic tools for the study of spaces with Ricci curvature bounded from below

Nicola Gigli
Université de Nice, Nice, France

Abstract: I will show that every metric measure space admits a weak first order differentiable structure which allows to integrate by parts. I will present some use of this structure in relation with the Lott-Sturm-Villani synthetic notion of lower bound on the Ricci curvature. In particular, I will discuss the Cheeger-Colding-Gromoll splitting theorem in such context.

Rigidity of Sobolev isometric immersions

Robert L. Jerrard
University of Toronto, Toronto, Canada

Abstract: It has been known for more than 50 years that a C^2 isometric immersion of the n-dimensional Euclidean unit ball into $(n+k)$-dimensional Euclidean space enjoys certain rigidity properties if $1\leq k<n$. For example, its image cannot be contained in a ball of radius much less than 1. This is easily seen to be false if $k \ge n$, and there are dramatic counterexamples, starting with work of Nash and Kuiper, showing that it is also false if one considers isometric immersions that are merely $C^{1,\alpha}$ for small enough α. We show that if an isometric immersion of the Euclidean n-ball into R^{n+k}, $k<n$, belongs to the Sobolev space $W^{2,p}$ for $p\ge \min\{ 2k,n\}$, then it is C^1 and enjoys rigidity properties similar to those of C^2 isometric immersions.
This is joint work with Reza Pakzad.

Most contractions are isometries

Bernd Kirchheim
University of Leipiz, Leipzig, Germany

Abstract: We investigate whether isometric mappings form a very big or rather pathological subclass of the space of all 1-lipschitz mappings (between Euclidean spaces or Riemannian manifolds).
This question is particularly interesting if source and target space are of the same dimension. The answer depends obviously on the notion of isometries considered, it turns out that there is a very geometric one (in particular implying that lengths of all curves in preserved) which presents the typical behaviour of (non-strict) contractions. A very important tool is the existence of locally strictly short extensions of given 1-lipschitz maps to be discussed in some detail.
This is joint work with Emanuele N. Spadaro and Laszlo Szekelyhidi Jr.

Geometric properties of perimeter minimizing soap bubble clusters

Francesco Maggi
University of Texas at Austin, Austin TX, USA

Abstract: We study the stability properties of perimeter minimizing clusters. In particular, we address the diffeomorphic representation of clusters that minimize (under a volume constraint) a free energy consisting of the perimeter plus a small potential term. We represent this clusters with respect to the absolute perimeter minimizing clusters having the same enclosed volumes. We also address a quantitative stability inequality with sharp decay rates for planar double bubbles.
This is a joint work with Marco Cicalese (U. Bonn) and Gian Paolo Leonardi (U. Modena e Reggio Emilia).

Weakly differentiable functions on varifolds

Ulrich Menne
AEI - University of Potsdam, Postdam, Germany

Abstract: The purpose of this talk is to exhibit several distinct natural classes of weakly differentiable functions on rectifiable varifolds whose first vari ations is suitably bounded. Amongst the topics studied are Sobolev Poincaré type emdeddings into Lebesgue spaces, approximate differen- tiability and a coarea formula. Finally, connections to the notion of inde- composability will be discussed.

On timelike extremal surfaces in Minkowski spacetime

Giandomenico Orlandi
Università di Verona, Verona, Italy

Abstract: We discuss a few properties of timelike extremal surfaces in Minkowski spacetime focusing in particular on the case of relativistic strings. The corresponding geometric evolution law for time-slices, a relativistic hyperbolic mean curvature flow, allows the formation of singularities in finite time. We propose suitable notions of weak or generalized solutions of this flow (in both a parametric and non-parametric, measure theoretic sense) globally defined in time, and discuss also possible approximations by singular limits of non linear wave equations. Some of the presented results are based on **joint works with G. Bellettini (Roma 2), M. Novaga (Pisa), J. Hoppe (Stockholm) and R.L. Jerrard (Toronto).**

On a recent variational model for the growth of islands

Aldo Pratelli
Universitaet Erlangen Nuernberg, Erlangen , Germany

Abstract: We will present a one-dimensional variational model which has been recently introduced in order to study the epitaxially strained growth of quantum dots, or "islands", that is, agglomerates which are formed when letting a thin film of some material on some substrate. This model has been done for the case of Silicium over Germanium, which is a particularly important case (quantum dot lasers and many optical or optoelectronic devices). The energy to be minimised is the sum of two parts, a surface energe and an elastic one. There are three peculiarities of this model, with respect to the similar ones already proposed in the past: first of all, the elastic energy is a concave functional, which gives of course troubles to the existence of minimizers. The second one is that, due to the constraint coming from the crystallin structure of the materials, the "admissible shapes" of these islands are only those for which the slope belongs a.e. to a given finite set of admissible angles, and this of course prevents the compactness of the set of the admissible shapes. Finally, among these admissible angles there is not the horizontal one, thus the islands cannot start nor end tangentially to the substrate: this phenomenon, usually called "mismatch", is very important for the applications (e.g., for all the observed cases where the islands are not symmetric), but it prevents the well known result of the "zero contact angle" which usually holds in this kind of models. In our talk, after having described the general model and its main characteristics, we will present the relaxation result and we will show some first important properties of the minimizers, which were already been experimentally and numerically observed, but of which a mathematical proof was still missing. We will also describe the other main properties which have been observed, and have yet not been formally proved.
(Joint works with I. Fonseca and B. Zwicknagel, and with E. Radici).

Sharpness of Rademacher's theorem

David Preiss
University of Warwick, Coventry, UK

Abstract: An old theorem of Rademacher states that a Lipschitz mapping from one Euclidean space to another is differentiable almost everywhere. Only results announced very recently show in what way this result may or may not be sharpened:

Theorem. The statement that, given a set E of Lebesgue measure zero, there is a Lipschitz mapping from R^n to R^k that is nowhere differentiable on E holds if and only if k<n.

I will concentrate on the case k<n proved very recently by Speight and Preiss. (The opposite case follows by combining announced results of Alberti, Csornyei and Preiss with those announced by Csornyei and Jones, and may be partly treated in some other talks.) If time permits, I will also discuss intriguing infinite dimensional connections.

Non integrable Plateau Problems

Tristan Rivière
ETH-Zentrum, Zurich, Switzerland

Plateau's problem in infinite-dimensional spaces

Thomas Schmidt
University of Zurich, Zurich, Switzerland

Abstract: Plateau's problem consists in finding a surface of minimal area among competitors with a prescribed boundary. The talk will be concerned with a generalized formulation of this problem in terms of currents, and the focus will be on some recent existence and regularity results for the case of infinite-dimensional ambient spaces.

A sharp strong maximum principle and a sharp unique continuation theorem for singular minimal hypersurfaces

Neshan Wickramasekera
University of Cambridge, Cambridge, UK

Abstract: If two smooth, connected, embedded minimal hypersurfaces with no singularities satisfy the property that locally near every common point p, one hypersurface lies on one side of the other, then it is a direct consequence of the Hopf maximum principle that either the hypersurfaces are disjoint or they coincide. It is a natural question to ask if the same result must

extend to pairs of singular minimal hypersurfaces (stationary codimesion 1 integral varifolds) with connected supports; in this case the above "one hypersurface lies locally on one side of the other" hypothesis can naturally be imposed for each common point p which is a regular point of at least one hypersurface.

The answer to this question in general is no in view of simple examples such as two pairs of transversely interecting hyperplanes with a common axis. The answer however is yes if the singular set of one of the hypersurfaces has $(n-1)$-dimesional Hausdorff measure zero, where n is the dimension of the hypersurfaces. The talk will present a proof of this result, which generalizes and unifies previous maximum principles of Ilmanen and Solomon-White (and thereby unifies all previously known strong maximum principles for singular minimal hypersurfaces). A sharp unique continuation result for singular minimal hypersurfaces will also be discussed.