



The Abdus Salam
**International Centre
for Theoretical Physics**



SMR 2460

Advanced School and Workshop in Real and Complex Dynamics

20 - 31 May 2013

List of Abstracts

Infinitesimal dynamics: holomorphic maps, connections and geodesics

Marco Abate (abate@dm.unipi.it)

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Abstract: At first glance, the dynamics on a fixed point set seems to be trivial: by definition, nothing moves there. But a more careful inspection shows that, at least in the holomorphic setting, the dynamics in a neighborhood of the fixed point set actually induces an infinitesimal dynamics on the fixed point set itself, expressed in terms of geodesics for a meromorphic connection along an induced foliation. Conversely, the study of this infinitesimal dynamics might provide important insights on the behavior of the original dynamical system. In this talk I shall in particular describe how this program can be carried out completely for homogeneous vector fields, obtaining for instance the description of the dynamics in a full neighborhood of the fixed point for a substantial class of 2-dimensional holomorphic germs tangent to the identity; and I shall also point out some directions for future research.

(Joint work with F. Tovena and with F. Bianchi).

Lyapunov exponents and bifurcations for endomorphisms of $\mathbb{P}^k$

François Berteloot

Université Paul Sabatier, Toulouse, France

Abstract: For any holomorphic family of endomorphisms of $\mathbb{P}^k$, the sum L of Lyapunov exponents, with respect to the maximum entropy measure, is known to be a $\mathcal{H}^s$ function of the parameter.

We characterize the support of the current L by different properties and show that it is a reasonable bifurcation current. When $k = 1$, our results provide another proof of the classical Mane-Sad-Sullivan theorem.

This is a joint work with C. Dupont.

Antipode Preserving Cubic Rational Maps and Herman Rings

Araceli Bonifant (bonifant@math.uri.edu)

Abstract: In this talk, I will discuss a family of cubic rational maps which carry antipodal points of the Riemann sphere to antipodal points, with emphasis on the abundance of Herman rings.

About the Inou Shishikura renormalization

Arnaud Chéritat (arnaud.cheritat@math.univ-toulouse.fr)

Abstract: I will report on an ongoing project to extend the near parabolic renormalization of Inou and Shishikura to unisingular maps.

Rigidity of Smooth Critical Circle Maps

Wellington De Melo (demelo@impa.br)

Abstract: I will discuss a joint work with Pablo Guarino where we prove that any two C^3 critical circle maps with the same criticality and the same rotation number of bounded type are $C^{1+\alpha}$ conjugate where $\alpha > 0$ is universal.

Stability and bifurcations for polynomial automorphisms of C^2

Romain Dujardin (romain.dujardin@univ-mlv.fr)

Abstract: We study stability and bifurcations in holomorphic families of polynomial automorphisms of C^2 . We introduce a notion of "weak stability" which parallels in many ways the classical notion of J-stability in one-dimensional dynamics.

Then we prove that under an assumption of moderate dissipativity, the parameters displaying homoclinic tangencies are dense in the bifurcation locus. This confirms one of Palis' Conjectures in the complex setting. The proof relies on the formalism of semi-parabolic bifurcation and the construction of "critical points" in semi-parabolic basins. **This is joint work with Misha Lyubich.**

Renormalization and rigidity for area-preserving maps

Denis Gaidashev (gaidash@math.uu.se)

University of Uppsala, Sweden

Abstract: Area-preserving maps have been observed to undergo a universal period-doubling cascade, analogous to the famous Feigenbaum-Coullet-Tresser period doubling cascade in one-dimensional dynamics.

We describe a construction of "stable" invariant Cantor sets for infinitely renormalizable area-preserving maps. Similarly to the one-dimensional situation, but unlike for the dissipative Henon-like maps, the dynamics on these sets turns out to be rigid.

This is a joint work with Marco Martens and Tomas Johnson.

Sums of Cantor sets, convolutions of singular measures, and spectral properties of quasicrystals.

Anton Gorodetski, Associate Professor

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Abstract: Questions on structure of a sum of Cantor sets and on a type of a convolution of singular measures appear in a natural way in different settings. In particular, convolutions of measures of maximal entropy on dynamically defined Cantor sets are related to spectral properties of the so called Square Fibonacci Hamiltonian, a model for a two-dimensional quasicrystal (**this is a joint result with D.Damanik**). **In the joint work with D.Damanik and B.Solomyak** we show that under some natural assumptions these convolutions must be absolutely continuous for most values of the parameters. This implies that for most small values of the coupling constants the density of states measure of the Square Fibonacci Hamiltonian is absolutely continuous.

Trinitas for the Henon maps

Yutaka Ishii (Kyushu University)

Abstract: In this talk I relate several combinatorial descriptions of the Julia sets for hyperbolic polynomial diffeomorphisms of $\mathbb{C}^2$; quotients of solenoids by Bedford and Smillie, automata by Oliva, and Hubbard trees by myself. The notion of iterated monodromy groups are defined for such polynomial diffeomorphisms and are used to construct automata from Hubbard trees.

Quasianalytic spaces and small divisors

Stefano Marmi, Scuola Normale Superiore, Pisa

Singular measures of circle homeomorphisms with several break points

Habib Marzougui

Faculté des Sciences de Bizerte, Bizerte, Tunisia

Abstract: Let f be a circle-homeomorphism with break point singularities, that is maps f that are derivable except at some singular points where the derivative has a jump. Let f have irrational rotation number and Df be absolutely continuous on every continuity interval of Df . We study whether the f -invariant measure μ_f is singular with respect to the Haar measure or not, using the cross ratio distortion tool, and the combinatorial related to the product of jumps at break points.

Thurston's work on entropy in dimension one.

Jack Milnor (jack@math.sunysb.edu)

Abstract: In his last (and not yet published) paper, Thurston studied the topological entropy h of a postcritically finite one dimensional map, and in particular the relations between dynamics and the number theory of e^h . I will try to describe some of his amazing constructions.

The Julia sets of a class of post-critically finite endomorphisms of $\mathbb{CP}^n$.

Volodymyr Nekrashevych (nekrash@math.tamu.edu)

Texas A&M University, College Station, TX, USA

Abstract: I will describe topological structure of the Julia sets of post-critically finite endomorphism of $\mathbb{CP}^n$ coming from maps induced by hyperbolic polynomials on the moduli space of a punctured sphere.

Fatou components in two complex variables

Han Peters (hanpeters77@gmail.com)

Abstract: Fatou components for rational functions on the Riemann sphere have been precisely described. We study invariant Fatou components in two complex variables, in particular for polynomial automorphisms and for holomorphic endomorphisms of projective space. I will present joint work with Misha Lyubich and with Luka Boc-Thaler and John Erik Fornaess.

On Poincare series of unicritical polynomials at the critical point

Weixiao Shen (matsw@nus.edu.sg)

Abstract: We show that for a unicritical polynomial having a priori bounds, the unique conformal measure of minimal exponent has no atom at the critical point.

For a conformal measure of higher exponent, we give a necessary and sufficient condition for the critical point to be an atom, in terms of the growth rate of the derivatives at the critical value. **This is a joint work with Rivera-Letelier.**

Satellite renormalization for complex quadratic polynomials

Mitsuhiro Shishikura

Abstract: The renormalization has been one of the main focus of the theory of one dimensional complex dynamics. It is connected to the conjectures of on the density of hyperbolicity and the local connectivity of the Mandelbrot set. For quadratic polynomials, there are two different types of renormalizations -- primitive and satellite. The primitive renormalizations has been successfully studied by Kahn and Lyubich and now there are powerful a priori bounds. The satellite type has a very different nature and our knowledge is limited. In this talk, we discuss the difference between two types of renormalizations and explain recent results on the satellite renormalizations.

Holomorphic Dynamics of automorphisms: Rigidity and equidistribution.

Nessim Sibony (nessim.sibony@math.u-psud.fr)

Abstract: Polynomial automorphisms in $\mathbb{C}^k$ or in a compact Kähler manifold admit a very rich dynamics.

One should expect that properties of Hénon maps in dimension 2, should extend to higher dimension.

However computing with positive closed currents of bidegree (p,p) , $p>1$ requires new tools: super-potentials and Densities.

This gives results on the "rigidity" for Green currents, and equidistribution properties for orbits of varieties and for periodic saddles.

The talk is based on joint work with T.C. Dinh.

Circle diffeomorphisms with several break points: symmetry and hyperbolicity of the renormalization operator

Alexey Teplinsky

Institute of Mathematics NASU, Kiev, Ukraine

Abstract: In our previous project, we showed that any two $C^{2+\alpha}$ -smooth circle diffeomorphisms, $\alpha>0$, each having a single break (i.e. a jump in the first derivative), with the same size of breaks and with the same irrational rotation number from certain class are C^1 -smoothly conjugate (Commun. Math. Phys., 320:2, 2013). In order to achieve this, in particular, we have constructed a renormalization operator in appropriate functional space and proved its uniform hyperbolicity. Our present aim is to expand this theory to the case of circle diffeomorphisms with N breaks. We introduce the renormalization procedure based on the S.Ferenczi's induction for interval

exchanges and show that the consecutive renormalizations, which are arrays of $2N$ smooth functions acting on definite intervals, approach a $2N$ -parameter family of (normalized) arrays of linear-fractional functions intertwined in N commutation relations. We explicitly describe the time-reverse symmetry of the renormalization operator acting on that family and explain the essence of its contracting and expanding directions. We believe this will eventually lead to the following rigidity statement: any two $C^{2+\alpha}$ -smooth, $\alpha > 0$, circle diffeomorphisms with breaks with the same irrational rotation number from certain class (containing all numbers of bounded type), the same sizes of corresponding breaks and the same invariant measures of intervals between corresponding break points are C^1 -smoothly conjugate. **This is work in progress. (Joint with K.Khanin.)**

Decidability of Thurston equivalence and geometrization of branched covering maps

Michael Yampolsky (yampol@math.toronto.edu)