ULTRACOLD COLOR SUPERCONDUCTORS

Massimo Mannarelli

INFN-LNGS

massimo@lngs.infn.it

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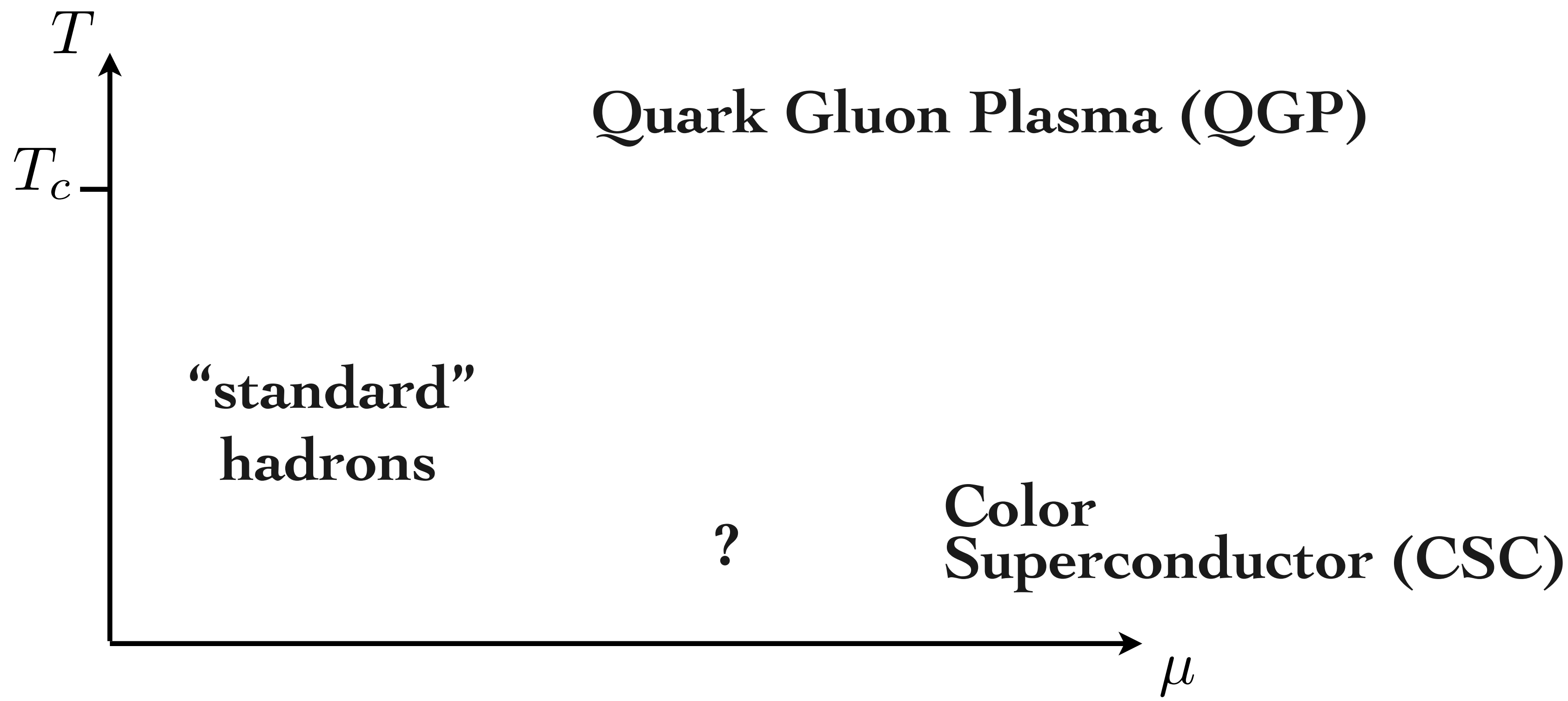
Outline

- Motivations
- Color Superconductors
- Color flavor locked phase (CFL)
- Crystalline color superconductors (CCSC)
- Low energy effective field theories

Reviews: hep-ph/0011333, hep-ph/0202037, 0709.4635, 1302.4624

MOTIVATIONS

QCD phase diagram



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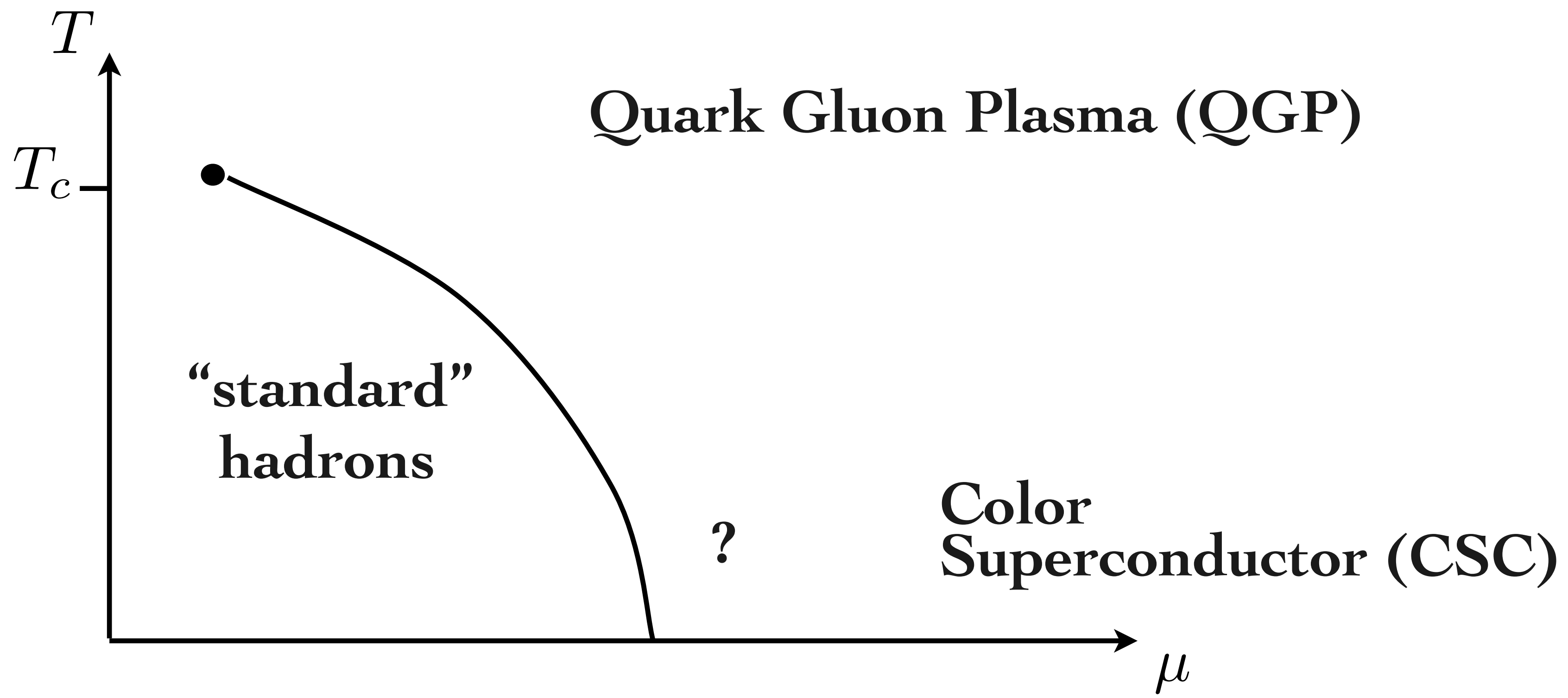

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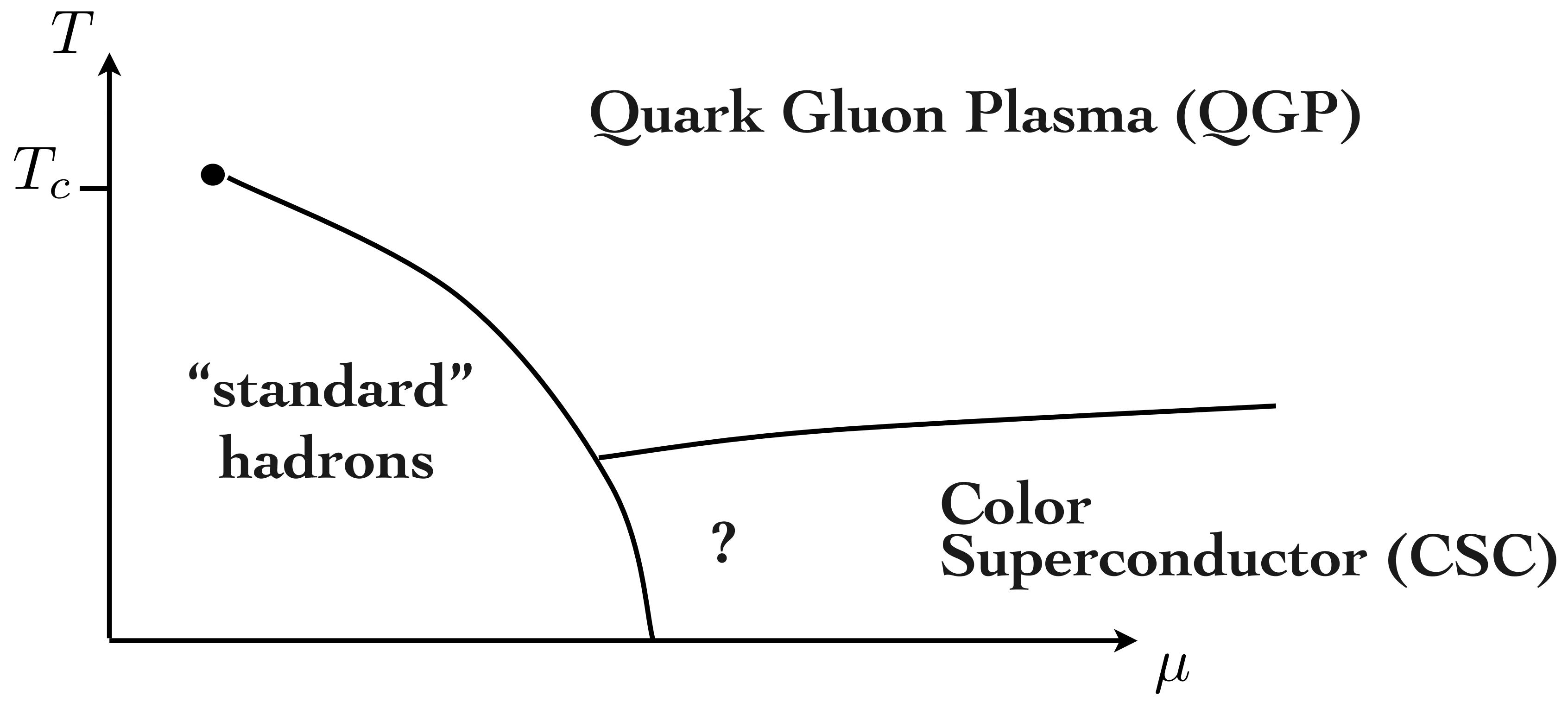

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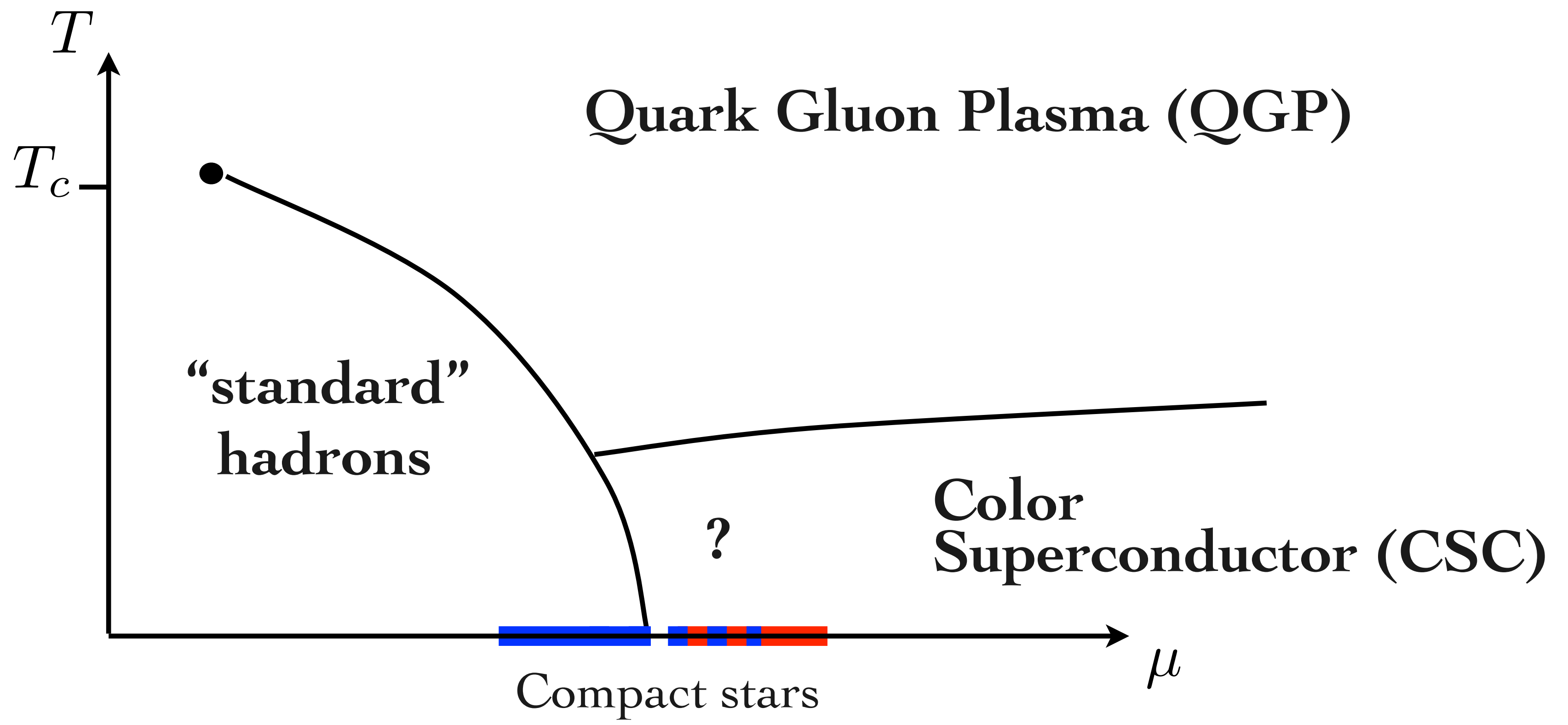

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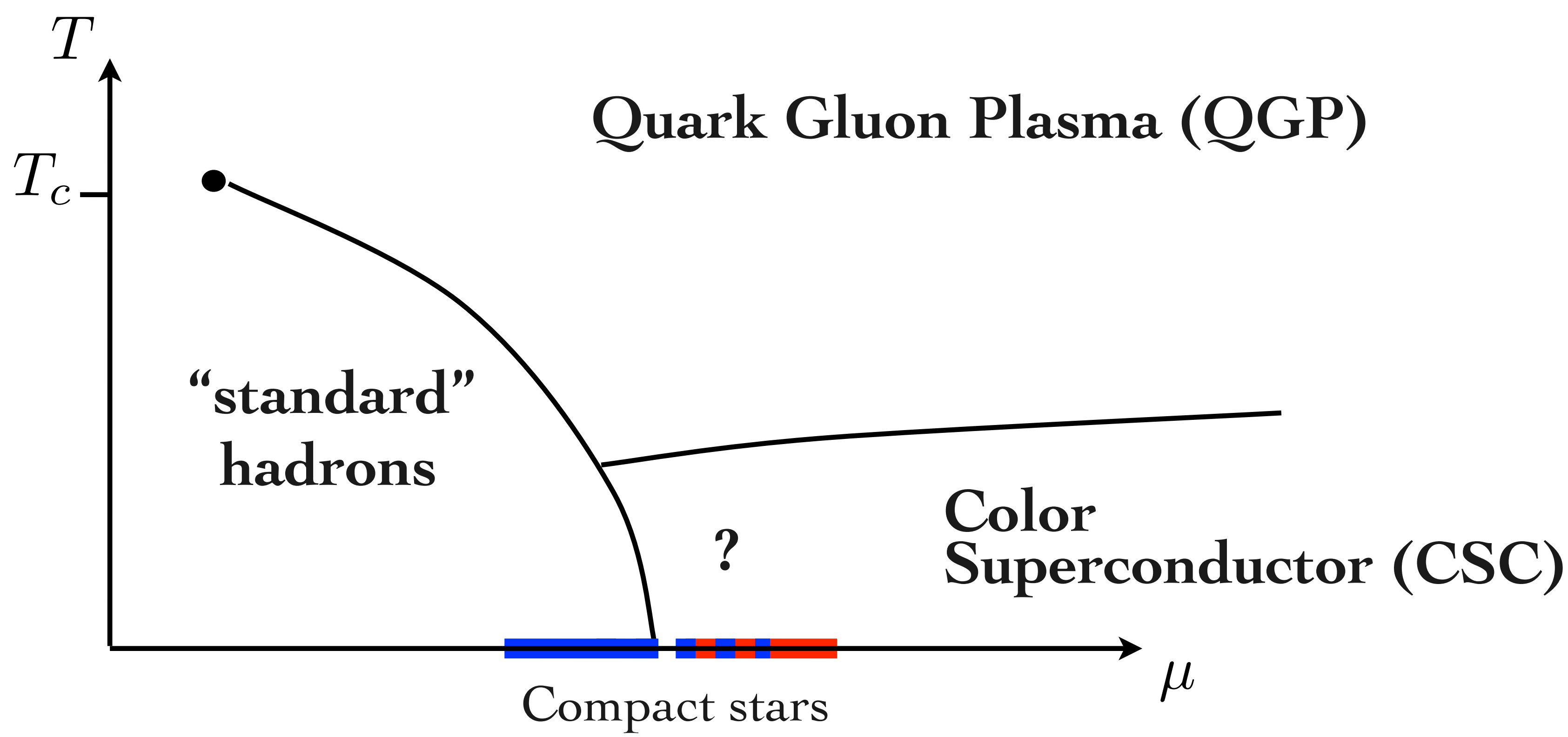

Warning: QCD is perturbative only at asymptotic energy scales

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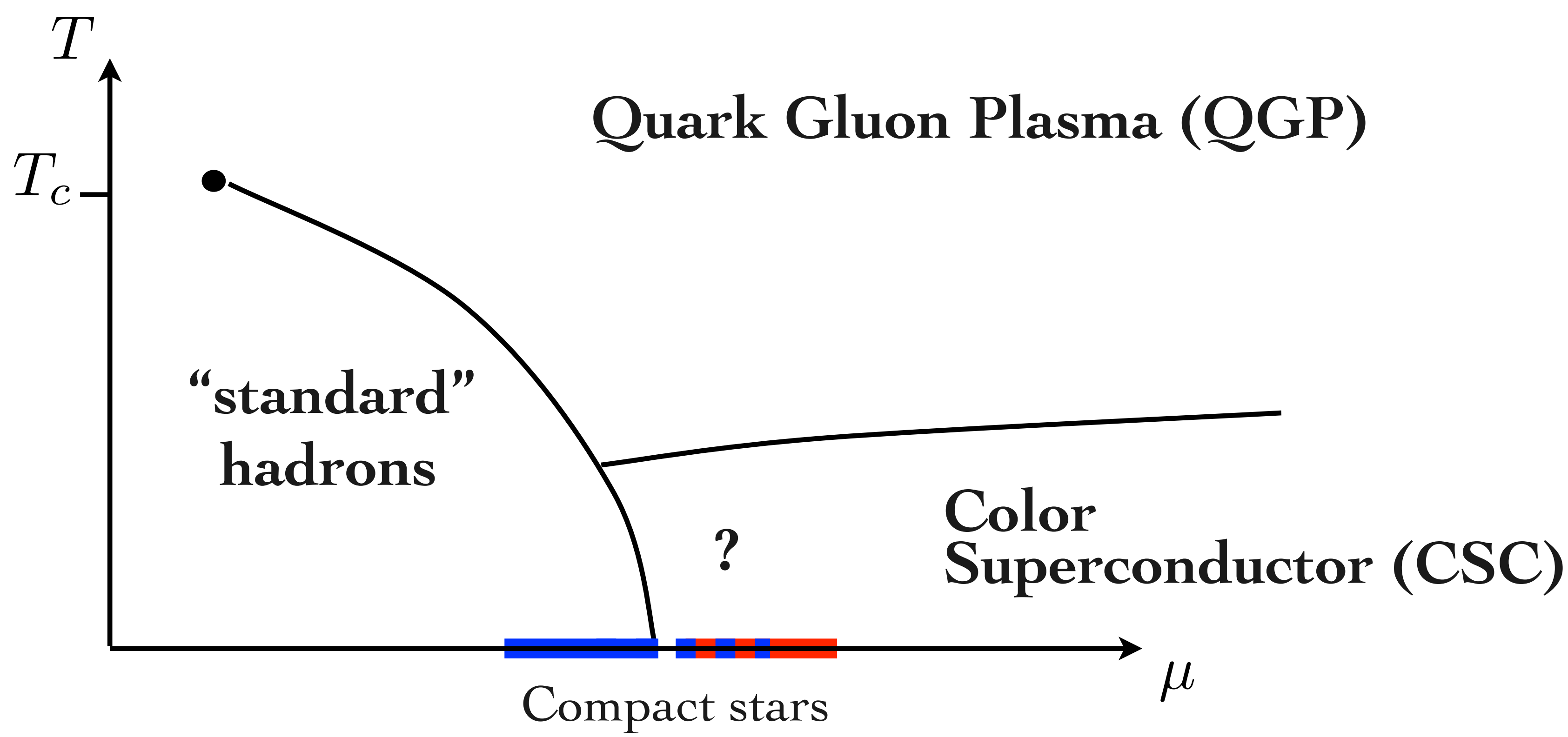

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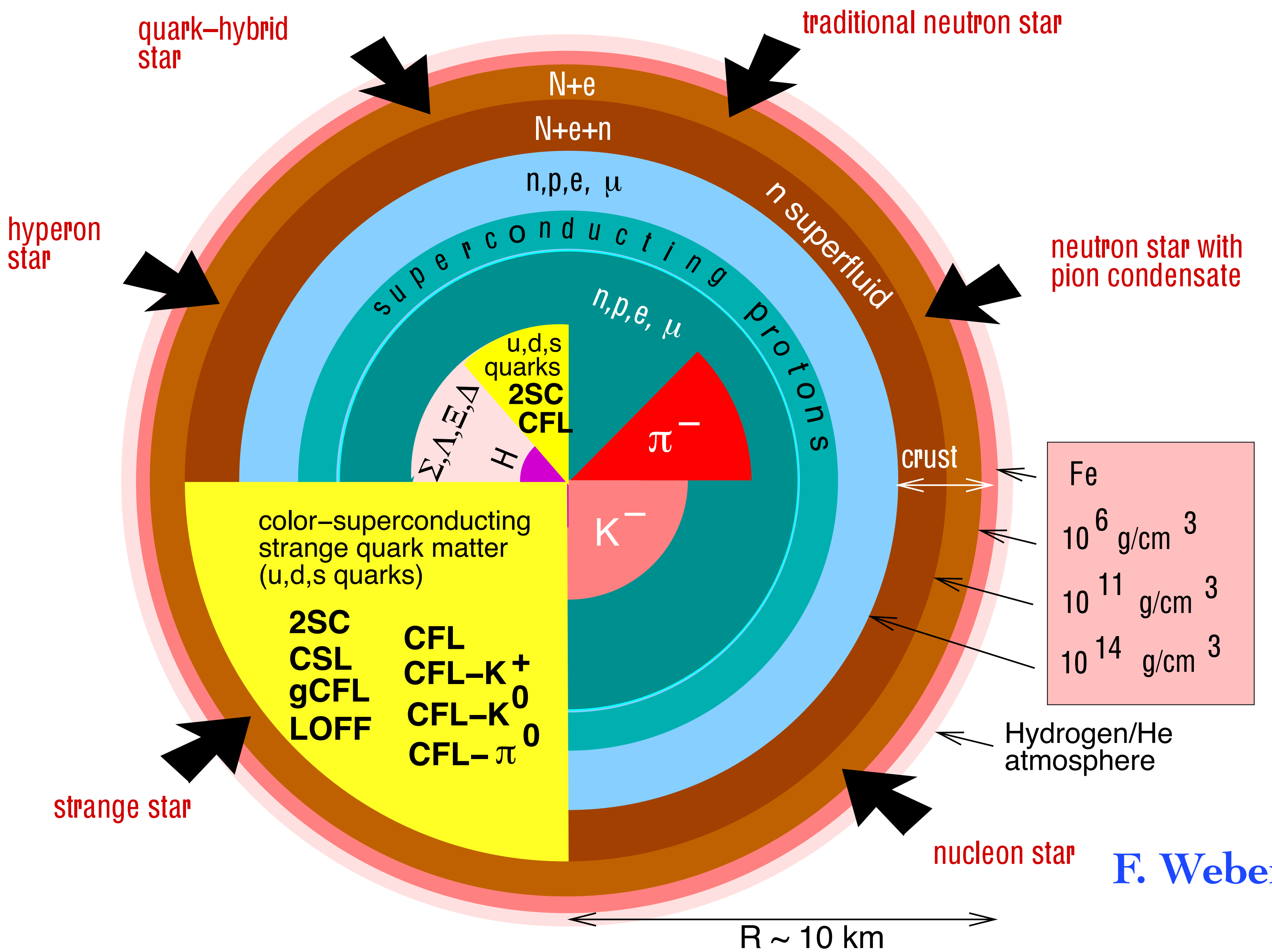
	HOT MATTER	ENERGY-SCAN	EMULATION
EXPERIMENTS	RHIC LHC	RHIC NA61/SHINE@CERN-SPS CBM@FAIR/GSI MPD@NICA/JINR	Ultracold fermionic atoms

Compact stellar objects (CSO)



F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193

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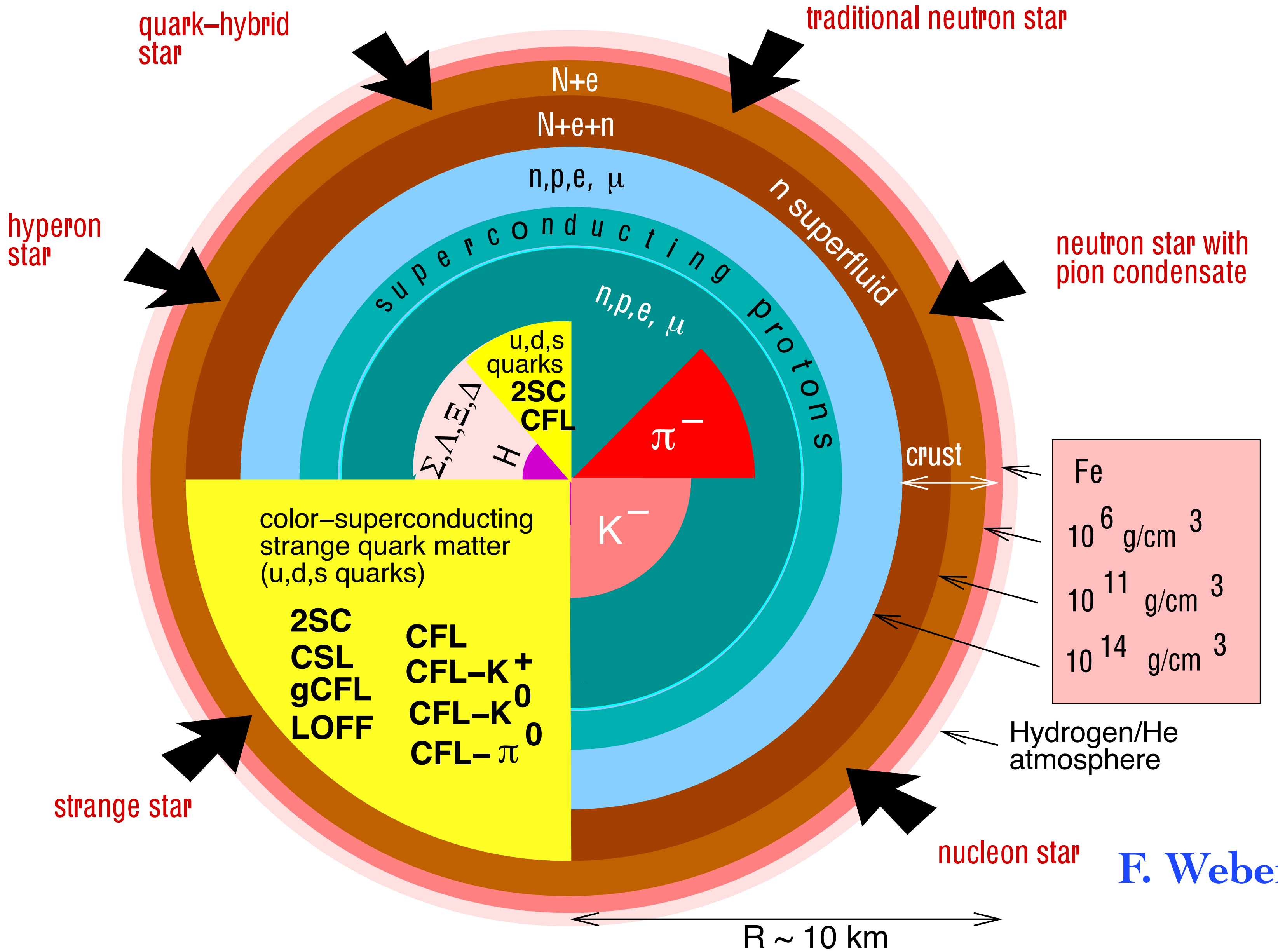

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cooling
glitches
instabilities
mass-radius
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……

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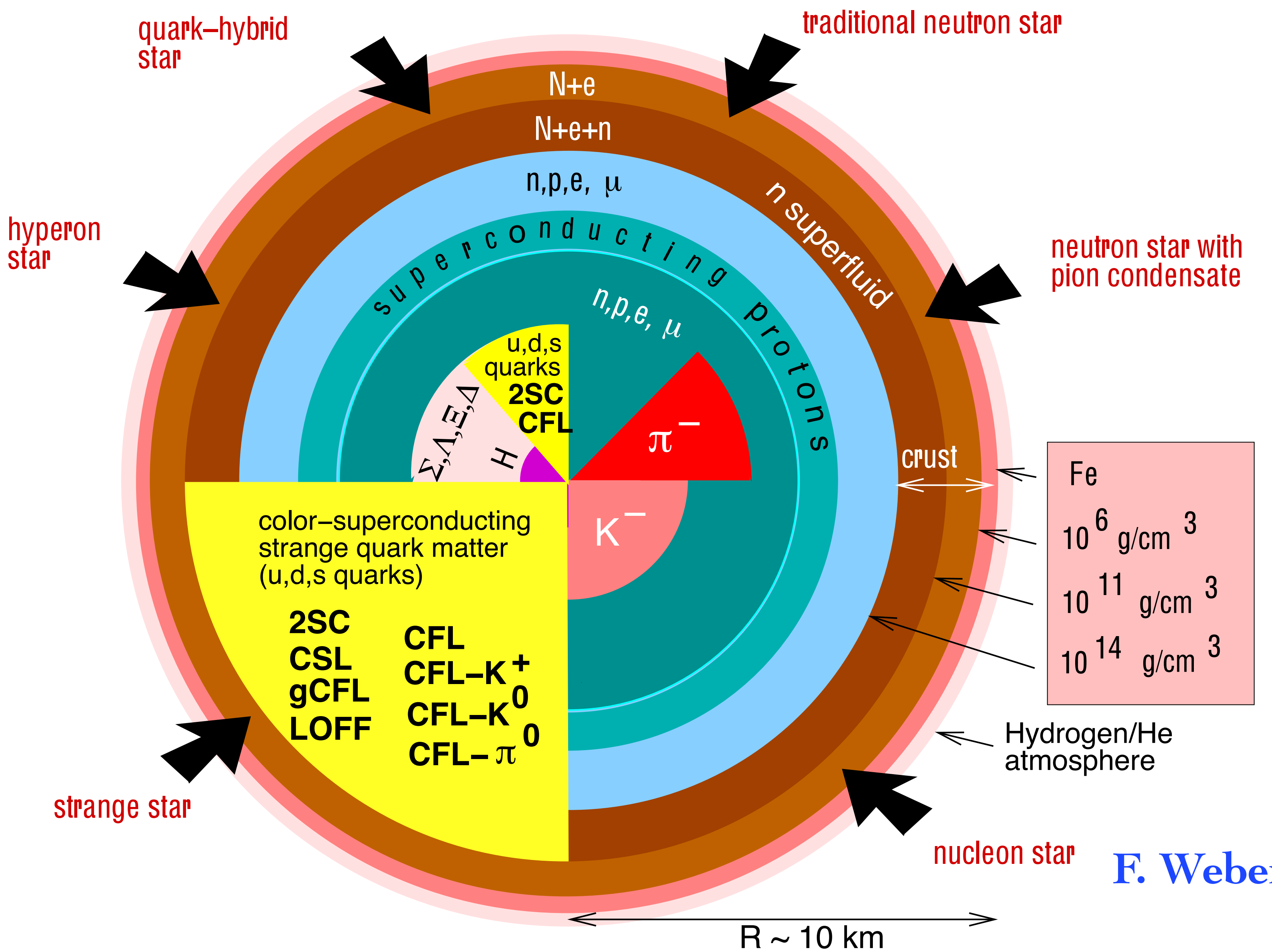

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Surface temperature in sufficiently old compact stars $< 10^6\mathrm{K}$

Mass $1.2M_\odot < M < 2M_\odot$

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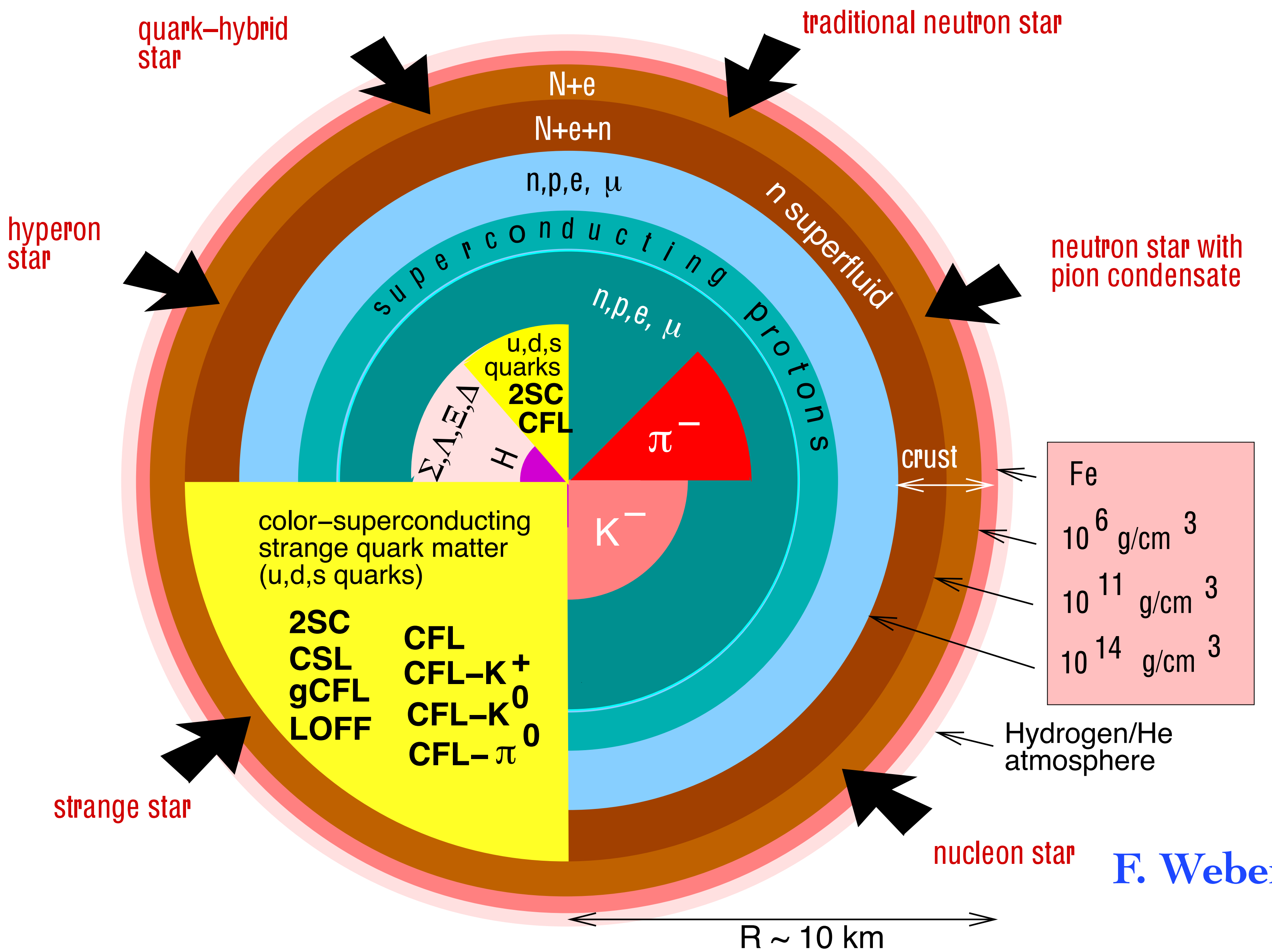

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PSR J1614-2230 mass $M \sim 2\, M_\odot$ Demorest et al Nature 467, (2010) 1081
hard to explain with quark matter models Bombaci et al. Phys. Rev. C 85, (2012) 55807

COLOR SUPERCONDUCTORS

A bit of history

- **Quark matter inside compact stars, Ivanenko and Kurdgelaidze (1965), Paccini (1966) ...**

- **Quark Cooper pairing was proposed by Ivanenko and Kurdgelaidze (1969)**

- **With asymptotic freedom (1973) more robust results by Collins and Perry (1975), Baym and Chin (1976)**

- **Classification of some color superconducting phases: Bailin and Love (1984)**

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- A large gap with instanton models by Alford et al. (1998) and by Rapp et al. (1998)
- The color flavor locked (CFL) phase was proposed by Alford et al. (1999)

The idea with a cartoon

PARTICLE

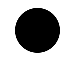
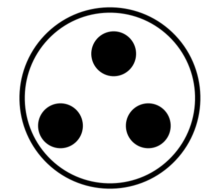
CARTOON

SIZE

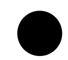
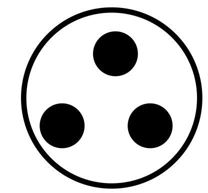
The idea with a cartoon

PARTICLE	quark
CARTOON	●
SIZE	point-like

The idea with a cartoon

PARTICLE	quark	baryon
CARTOON		
SIZE	point-like	~1 fm

The idea with a cartoon

PARTICLE	quark	baryon	diquark
CARTOON			
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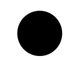
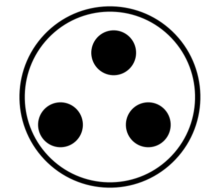
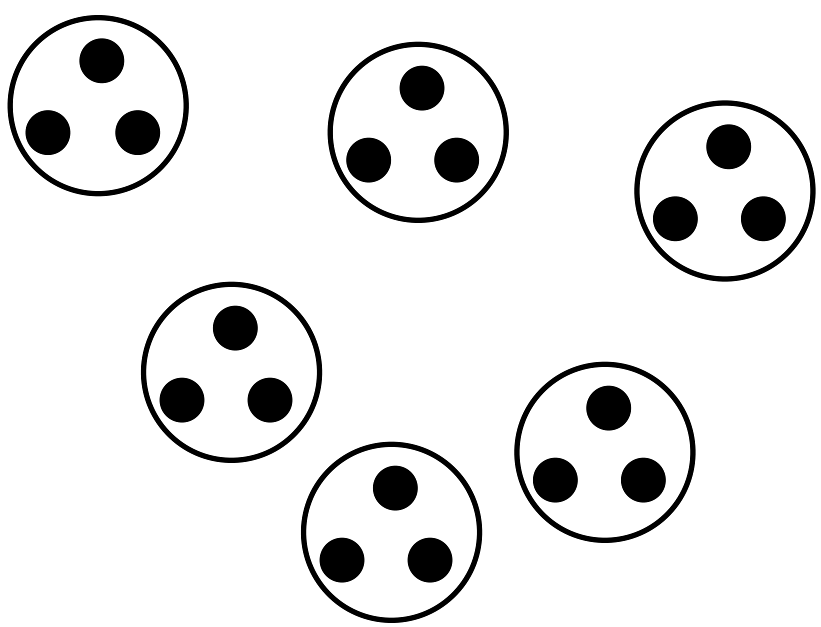
CARTOON			
SIZE	point-like	~1 fm	~10 fm

High density (CSO's outer core)



Liquid of neutrons

The idea with a cartoon

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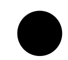
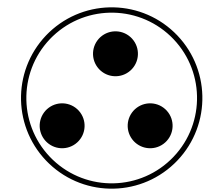
CARTOON			
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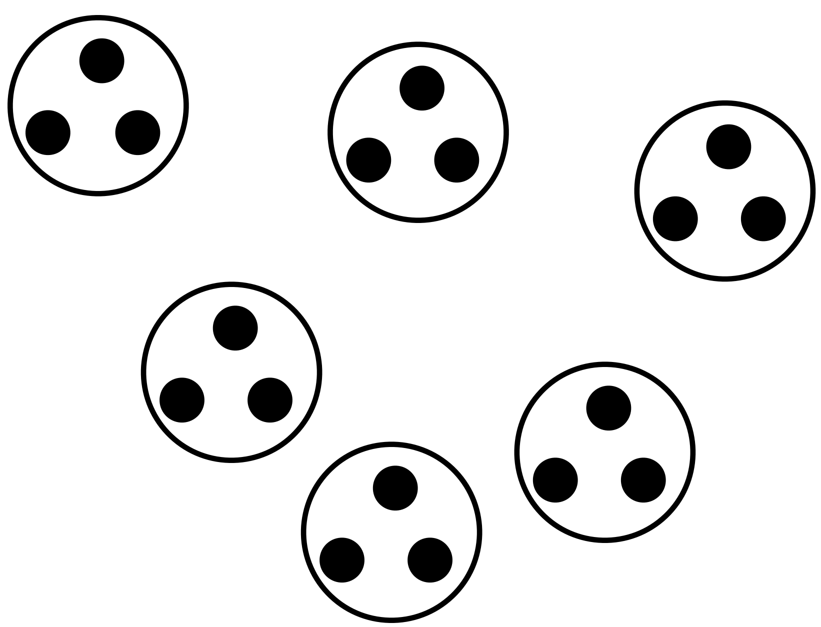

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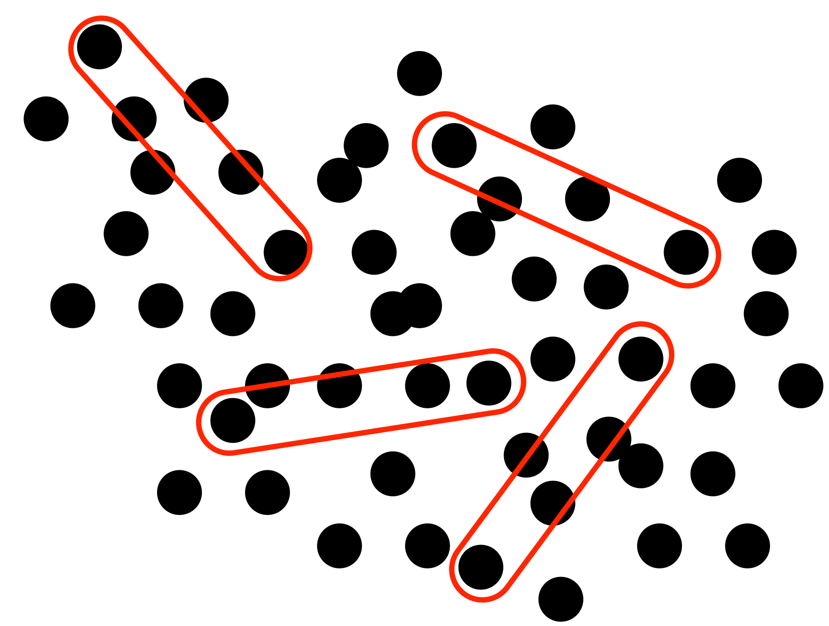

Liquid of quarks with correlated diquarks

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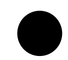
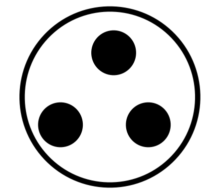
CARTOON			
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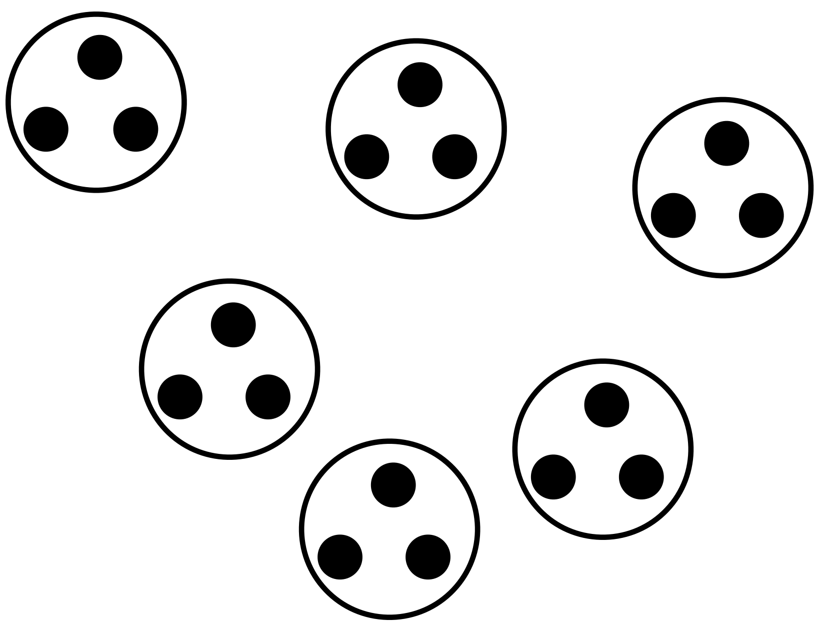

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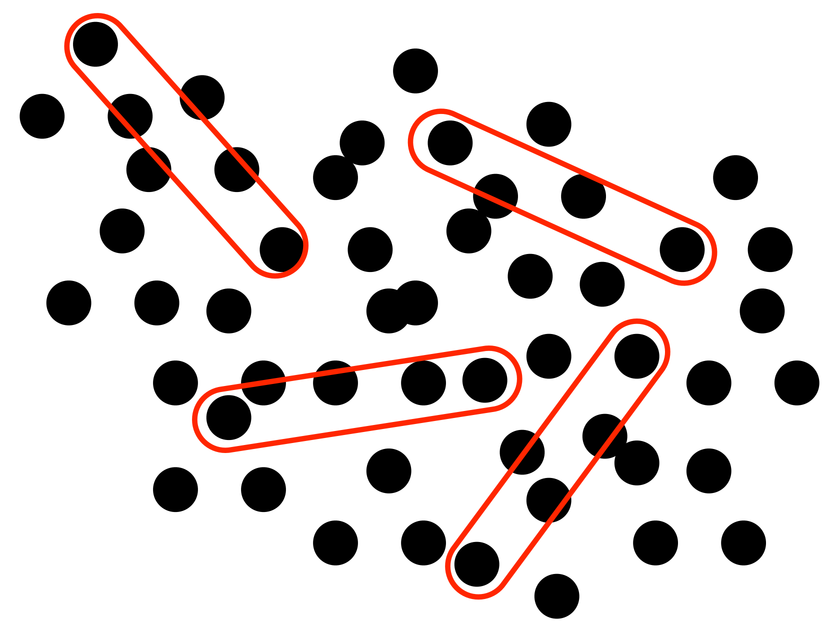
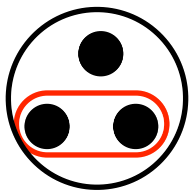


Liquid of quarks with correlated diquarks

Models for the lowest-lying baryon excited states with **diquarks**
Anselmino et al. Rev Mod Phys 65, 1199 (1993)



Do we have the ingredients?

Recipe for superconductivity

- Degenerate system of fermions
- Attractive interaction (in some channel)
- $T < T_c$

Color superconductivity

- At large μ, degenerate system of quarks
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- We expect $T_c \sim (10 - 100)$ MeV >> $T_{star\text{-}core} \sim 10 \div 100$ keV
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Oss. Quarks have color, flavor as well as spin degrees of freedom: complicated dishes.
A long menu of colored dishes.

The main course: Color Flavor Locked phase

Condensate

(Alford, Rajagopal, Wilczek hep-ph/9804403)

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta_{\mathrm{CFL}}\, \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

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Symmetry breaking

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\rm Q}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\rm Q}}} \times Z_2$$

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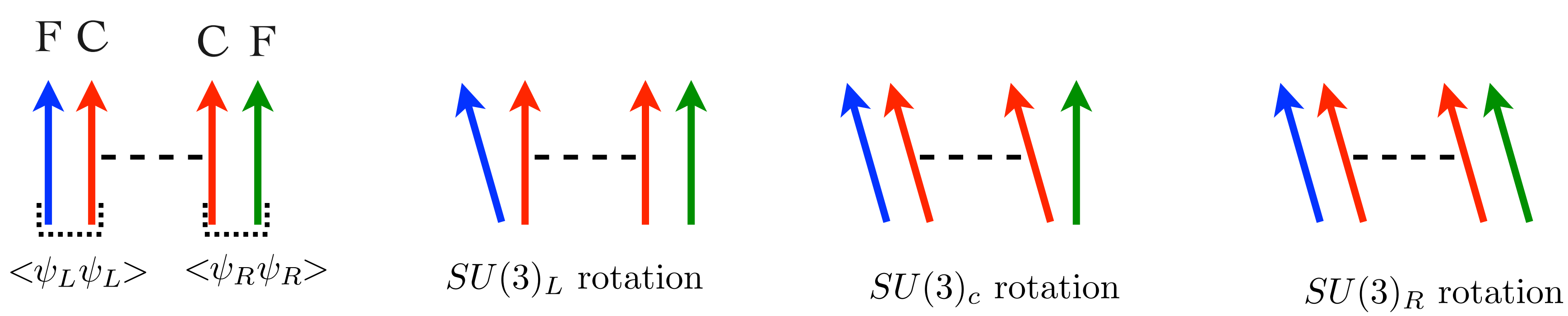

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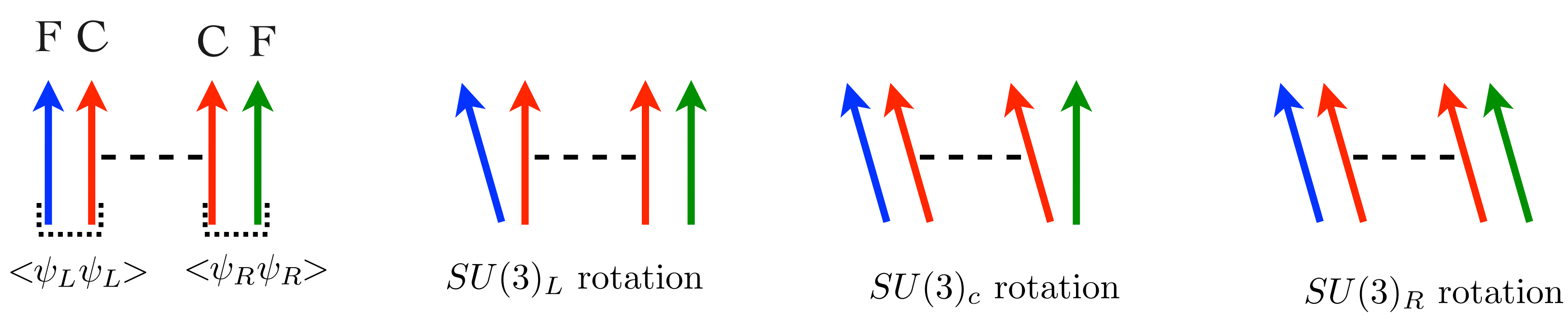

- Higgs mechanism: all gluons acquire “magnetic mass”; quarks become “massive”
- χSB: 8 (pseudo) Nambu-Goldstone bosons (NGBs)
- $U(1)_B$ breaking: 1 NGB
- “Rotated” electromagnetism, mixing angle $\cos\theta = \dfrac{g}{\sqrt{g^2 + 4e^2/3}}$ (analog of the Weinberg angle)

NGBs of the CFL phase

Since the color superconductors are realized in compact stars (ultracold systems) we can integrate out the fermionic degrees of freedom, focusing on the low energy NGBs

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Octet

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_0\Sigma\partial_0\Sigma^\dagger - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger]$$

$$f_\pi^2 = \frac{21-8\log 2}{18}\frac{\mu^2}{2\pi^2}$$

$$v_\pi^2 = \frac{1}{3}$$

$\Sigma = e^{i\phi^a\lambda_a/f_\pi}$ $\quad \phi^a$ describes the octet $\quad (\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$

Casalbuoni and Gatto, Phys. Lett. B 464, (1999) 111

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$$\Delta\mathcal{L}_{\rm eff} = -c\,{\rm Det}M\,{\rm Tr}(M^{-1}\Sigma) - c'\,{\rm Det}\Sigma\,[{\rm Tr}(M\Sigma^\dagger)^2 + ({\rm Tr}M\Sigma^\dagger)^2] + {\rm h.c.}$$

$$\boxed{\begin{aligned} m^2_{\pi^\pm} &= A\,(m_u+m_d)m_s \\ m^2_{K^\pm} &= A\,(m_u+m_s)m_d \\ m^2_{K^0,\bar{K}^0} &= A\,(m_d+m_s)m_u \end{aligned}}$$

kaons are lighter than mesons!

$$A = \frac{3\Delta^2}{\pi^2 f_\pi^2}$$

$$\pi^+ \sim (\bar{d}\bar{s})(us)$$

$$K^+ \sim (\bar{d}\bar{s})(ud)$$

Son and Sthephanov, Phys. Rev. D 61, (2000) 74012

“Phonons”

There is an additional massless NGB, φ, associated to $U(1)_B \longrightarrow Z_2$

Quantum numbers φ ~ <Λ Λ > like the H-dibaryon of **Jaffe, Phys. Rev. Lett. 38, 195 (1977)**

Effective Lagrangian up to quartic terms

$$\mathcal{L}_{\rm eff}(\varphi) = \frac{3}{4\pi^2}\left[(\mu - \partial_0\varphi)^2 - (\partial_i\varphi)^2\right]^2$$

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$$\varphi(x) = \overset{\text{bulk}}{\underset{\text{classical field}}{\bar{\varphi}(x)}} + \overset{\text{“sound” or phonon}}{\underset{\text{long-wavelength fluctuations}}{\phi(x)}}$$

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bulk **"sound" or phonon**

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

classical field **long-wavelength fluctuations**

Phenomenology

Dissipative processes due to vortex-phonon interaction **damp r-mode oscillations** of CFL stars rotating at frequencies < 1 Hz

MM et al., Phys. Rev. Lett. 101, 241101 (2008)

CRYSTALLINE COLOR SUPERCONDUCTORS

More realistic conditions

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

$\longrightarrow$

mismatch of Fermi momenta

More realistic conditions

sizable strange quark mass

+

weak equilibrium

+

electric neutrality

→ mismatch of Fermi momenta

No pairing case

Fermi momenta $p_u^F = \mu_u \quad p_d^F = \mu_d \quad p_s^F = \sqrt{\mu_s^2 - m_s^2}$

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weak decays

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

$\longrightarrow$

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality $\longrightarrow$ $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

More realistic conditions

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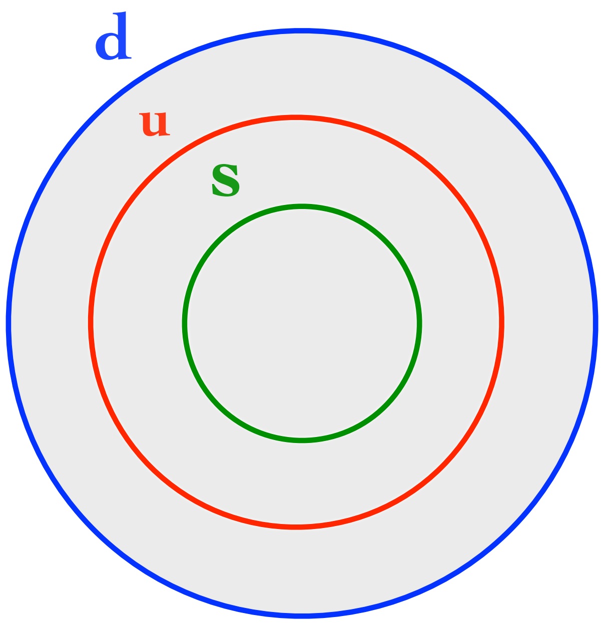

Fermi spheres of u,d, s quarks

weak decays

$u \to d + \bar{e} + \nu_e$
$u \to s + \bar{e} + \nu_e$
$u + d \leftrightarrow u + s$

→ $\mu_u = \mu_d - \mu_e$

$\mu_d = \mu_s$

electric neutrality → $\frac{2}{3} N_u - \frac{1}{3} N_d - \frac{1}{3} N_s - N_e = 0$

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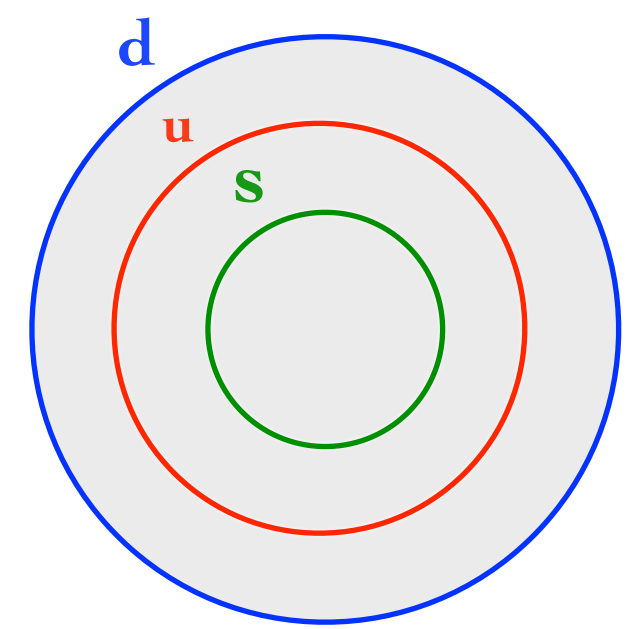


Fermi spheres of u,d, s quarks

weak decays

$u \to d + \bar{e} + \nu_e$
$u \to s + \bar{e} + \nu_e$
$u + d \leftrightarrow u + s$

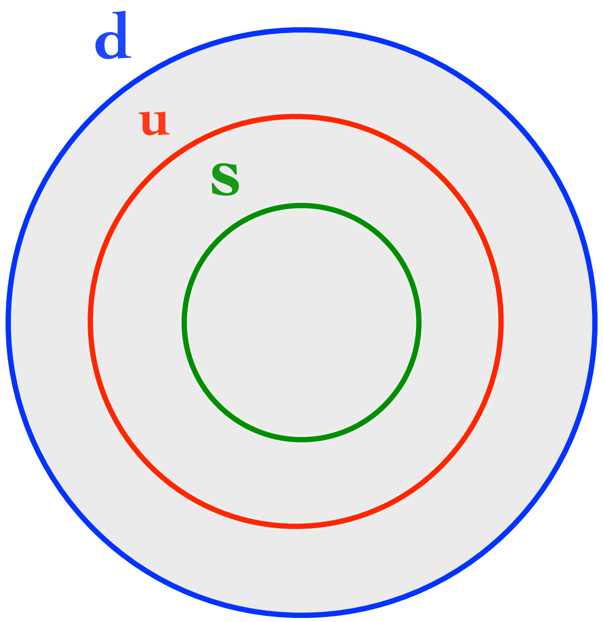
→ $\mu_u = \mu_d - \mu_e$, $\mu_d = \mu_s$

electric neutrality → $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

$$\mu = \frac{\mu_u + \mu_d + \mu_s}{3} \qquad \mu_e \simeq \frac{m_s^2}{4\mu} \qquad p_d^F = \mu + \frac{1}{3}\mu_e \qquad p_u^F = \mu - \frac{2}{3}\mu_e \qquad p_s^F \simeq \mu - \frac{5}{3}\mu_e$$

Alford and Rajagopal, JHEP 0206 (2002) 031

Mismatch vs Pairing



- **Energy gained in pairing** $\sim 2\Delta_{CFL}$
- **Energy cost of pairing** $\sim \delta\mu \sim \frac{m_s^2}{\mu}$

The CFL phase is favored for $\frac{m_s^2}{\mu} \lesssim 2\Delta_{CFL}$

Forcing the superconductor to a homogenous gapless phase $E(p) = -\delta\mu + \sqrt{(p-\mu)^2 + \Delta^2}$

leads to the "chromomagnetic instability" $M^2_{\text{gluon}} < 0$

Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

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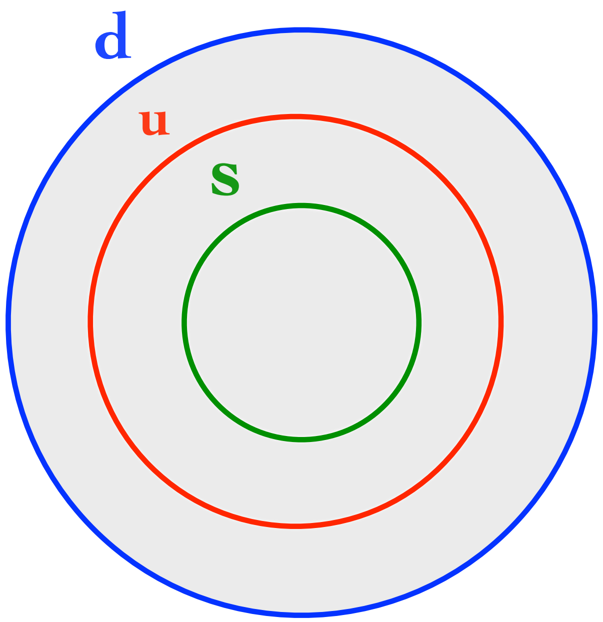

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Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

For $\dfrac{m_s^2}{\mu} \gtrsim 2\Delta_{CFL}$ **some less symmetric CSC phase should be realized**

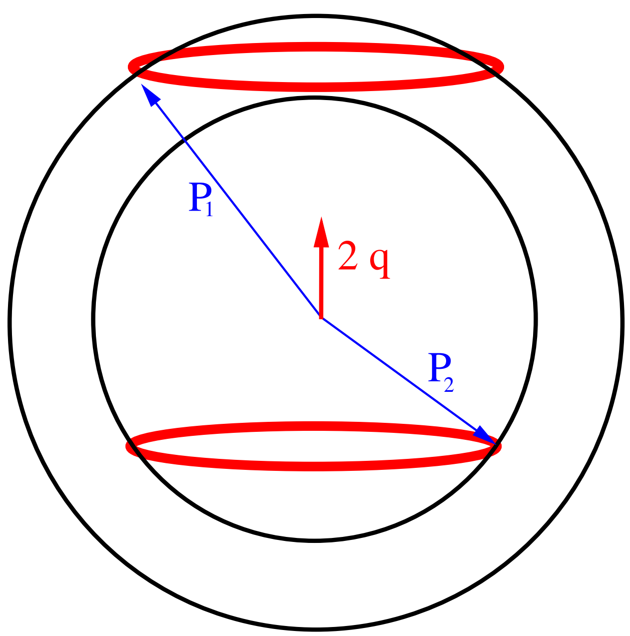
LOFF (or FFLO)-phase

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ the superconducting phase named LOFF is favored with Cooper pairs of non-zero total momentum

For two flavors

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$

LOFF: Larkin-Ovchinnikov and Fulde-Ferrel



- In momentum space

$$< \psi(\mathbf{p_1})\psi(\mathbf{p_2}) > \sim \Delta\,\delta(\mathbf{p_1}+\mathbf{p_2}-\mathbf{2q})$$

- In coordinate space

$$< \psi(\mathbf{x})\psi(\mathbf{x}) > \sim \Delta\, e^{i\mathbf{2q}\cdot\mathbf{x}}$$

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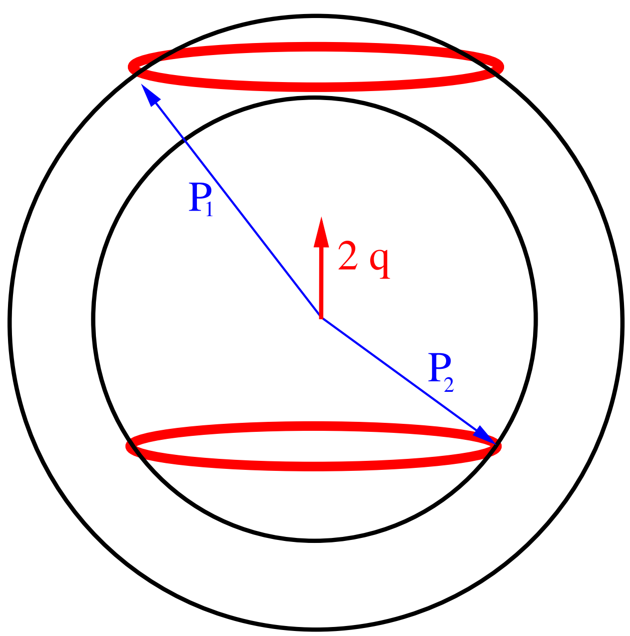

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$$< \psi(\mathbf{p_1})\psi(\mathbf{p_2}) > \sim \Delta\,\delta(\mathbf{p_1} + \mathbf{p_2} - \mathbf{2q})$$

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The LOFF phase: non-homogeneous superconductor, with a spatially modulated condensate in the spin 0 channel

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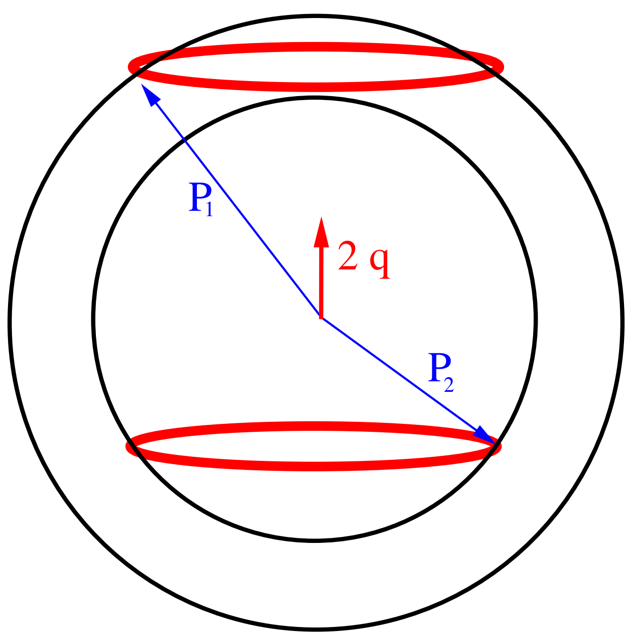

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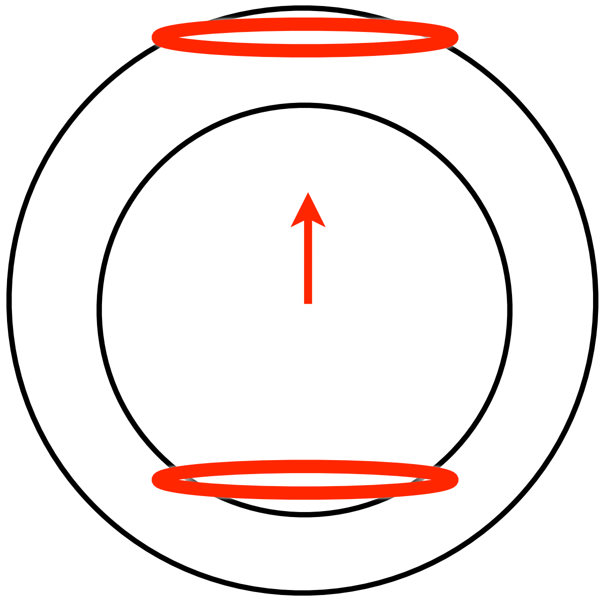
The LOFF phase: non-homogeneous superconductor, with a spatially modulated condensate in the spin 0 channel

The dispersion law of quasiparticles is gapless in some specific directions.
No chromomagnetic instability.

Crystalline structures: CCSC phase



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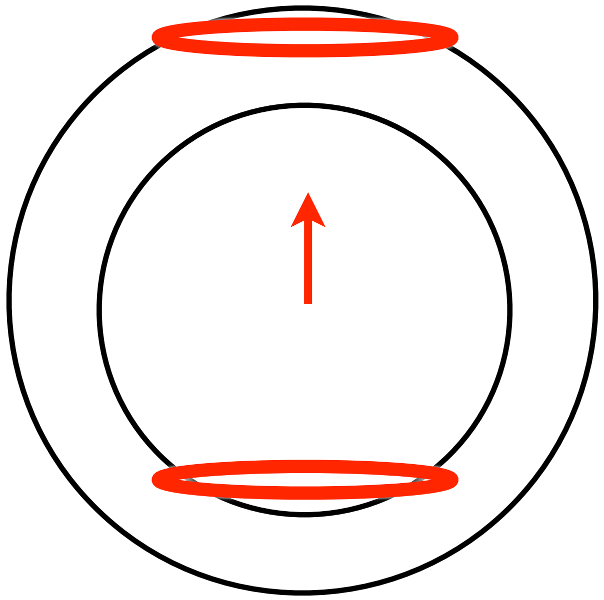

- Compliacted structures can be obtained combining more plane waves

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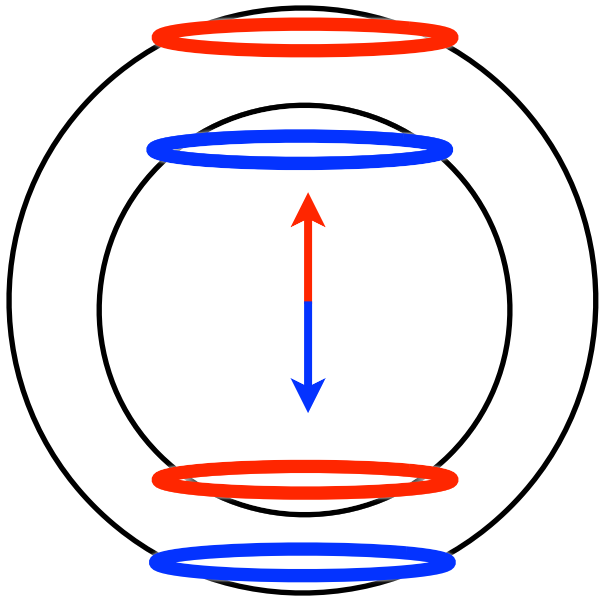

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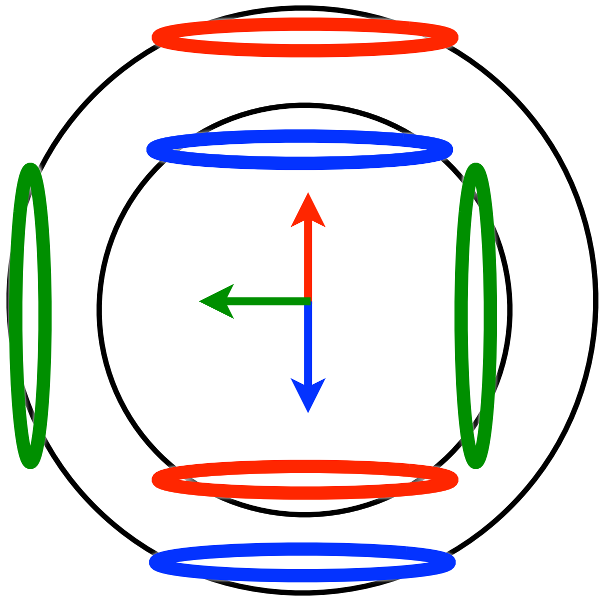

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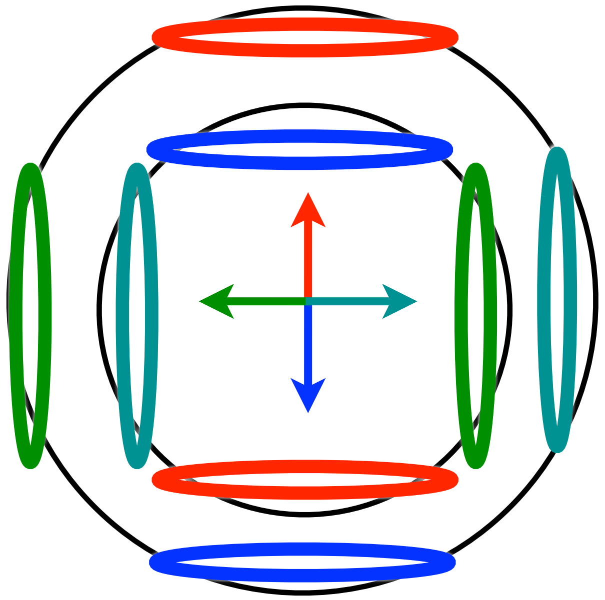

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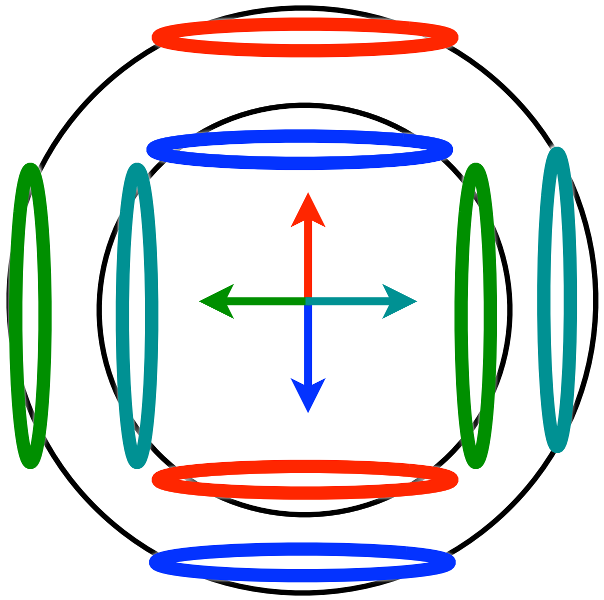

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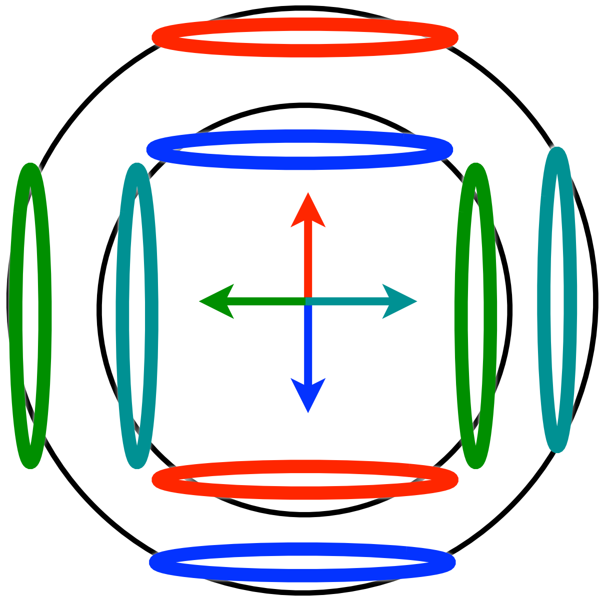

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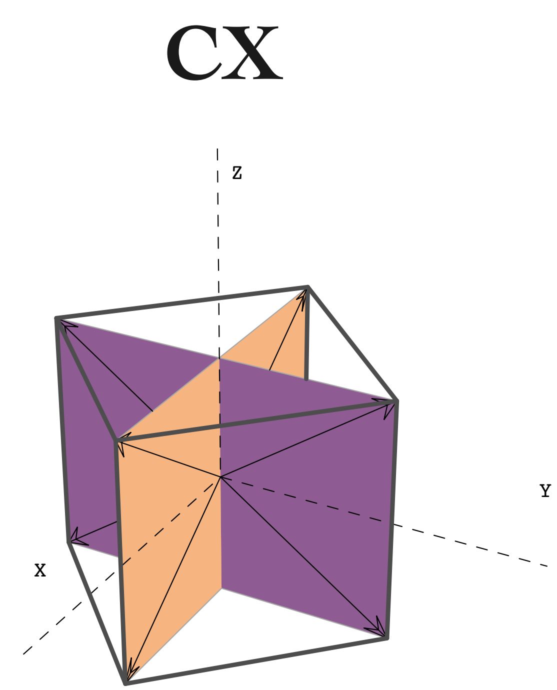

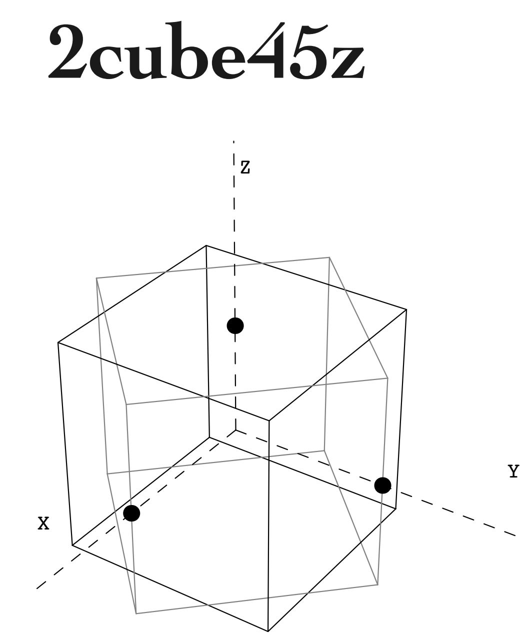

- Three flavors

$$< \psi_{\alpha i} C\gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I^m\}} e^{2i\mathbf{q}_I^m\cdot\mathbf{r}} \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

simplifications

$$\mathbf{q}_I^m = q\,\mathbf{n}_I^m$$

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Phonons in the CCSC phase

Phonon fields, $\mathbf{u}_I$, describe the fluctuations of the condensate $\Delta_I(\mathbf{r}) \to \Delta_I(\mathbf{r} - \mathbf{u}_I)$

Low-energy Lagrangian (from GL and momentum expansions)

$$\mathcal{L}^{\Delta^2} = \frac{1}{2}\sum_I \kappa_I \sum_{\mathbf{n}_I^m} \left[\partial_0(\mathbf{n}_I^m \cdot \mathbf{u}_I)\partial_0(\mathbf{n}_I^m \cdot \mathbf{u}_I) - (\mathbf{n}_I^m \cdot \partial)(\mathbf{n}_I^m \cdot \mathbf{u}_I)(\mathbf{n}_I^m \cdot \partial)(\mathbf{n}_I^m \cdot \mathbf{u}_I)\right]$$

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interaction channel

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interaction channel **crystalline structure**

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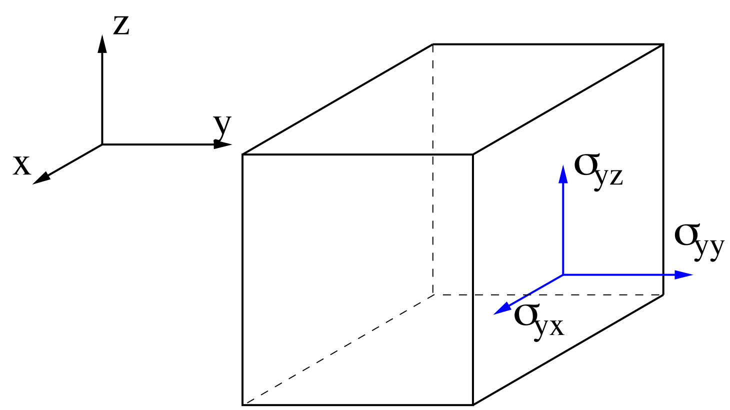
alternative description $\varphi_I^m = 2\mathbf{n}_I^m \cdot \mathbf{u}_I$

$$\kappa_I \equiv \frac{2\mu^2|\Delta_I|^2}{\pi^2(1 - z_q^2)}$$

This quantity multiplies also the "potential term": and is thus the energy price needed to produce these fluctuations

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Shear modulus



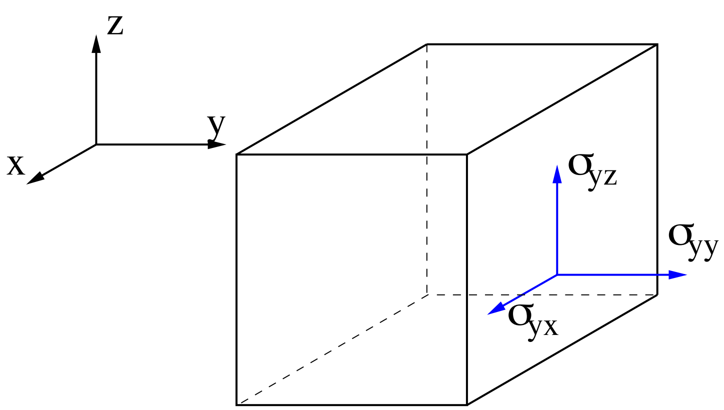
The shear modulus describes the response of a crystal to a shear stress

$$\nu^{ij} = \frac{\sigma^{ij}}{2s^{ij}} \qquad \text{for} \quad i \neq j$$

σ^{ij} **stress tensor acting on the crystal**

s^{ij} **strain (deformation) matrix of the crystal**

Shear modulus



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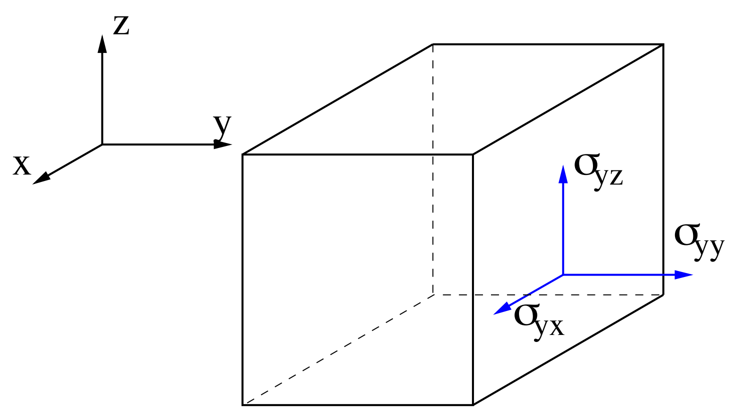

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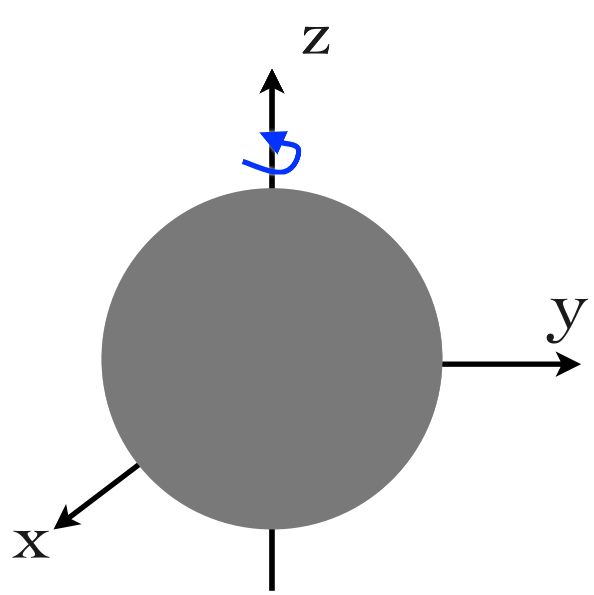
$$\nu = 2.47\,\frac{\mathrm{MeV}}{\mathrm{fm}^3}\left(\frac{\Delta}{10\mathrm{MeV}}\right)^2\left(\frac{\mu}{400\mathrm{MeV}}\right)^2$$

More rigid than diamond!!
20 to 1000 times more rigid than the crust of neutron stars

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Gravitational waves from "mountains"



If the star has a non-axial symmetric deformation (mountain) it can emit gravitational waves

ellipticity

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

GW amplitude

$$h = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{r}$$

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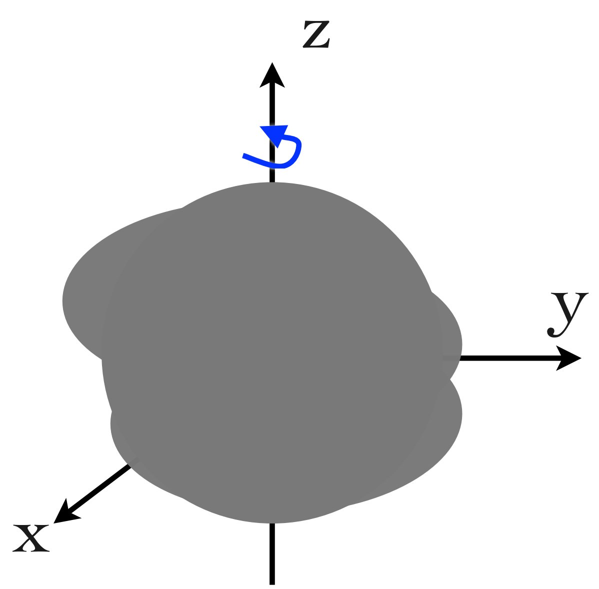

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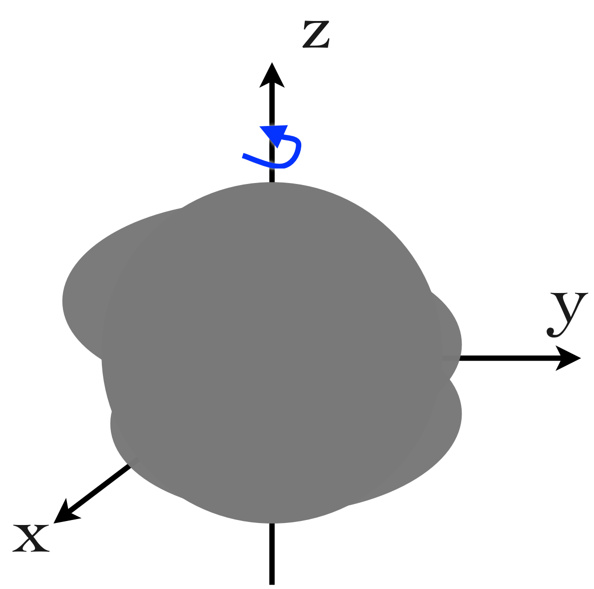

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- **Deformation depends on the breaking strain and the shear**

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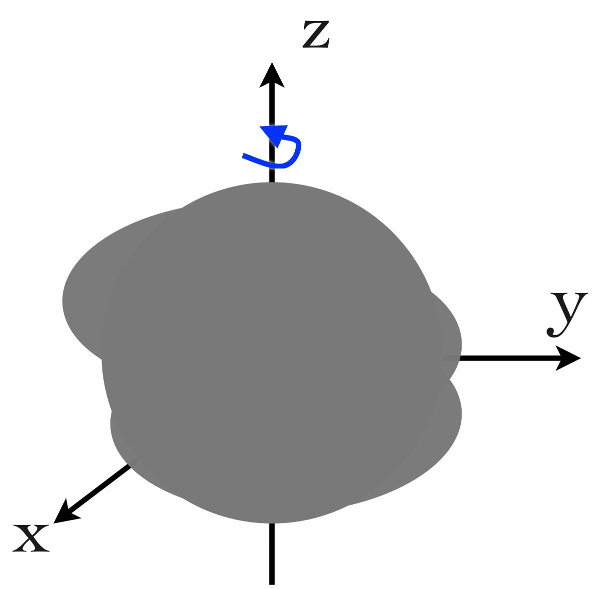

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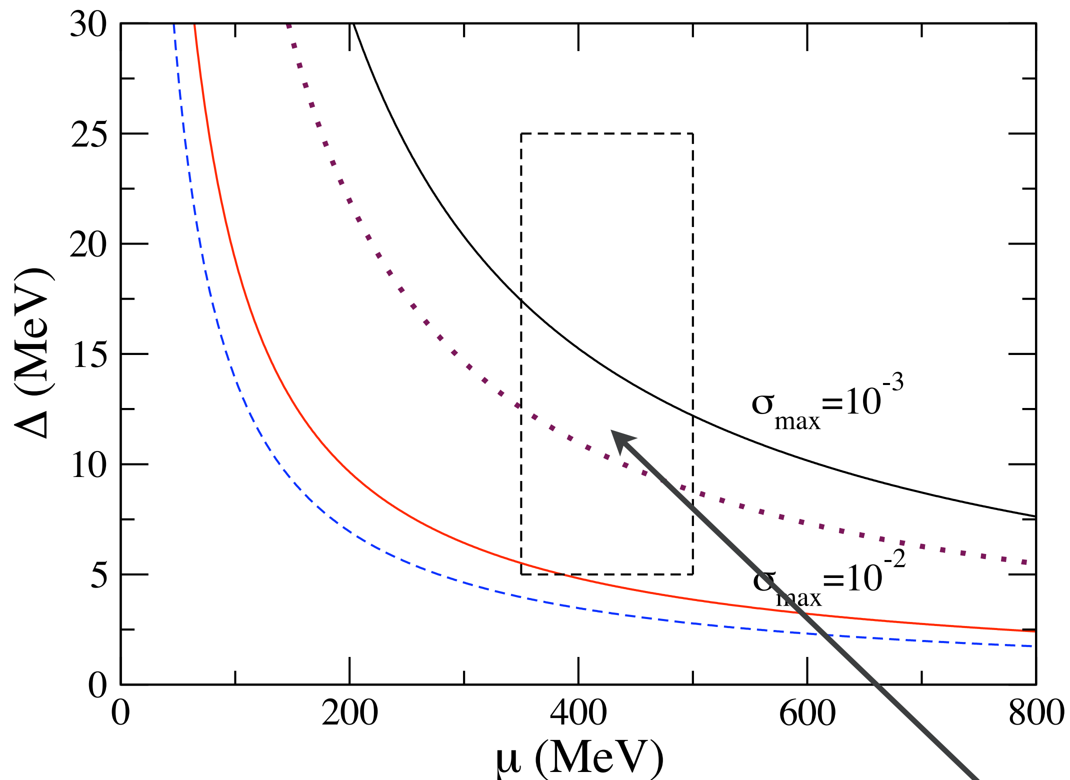
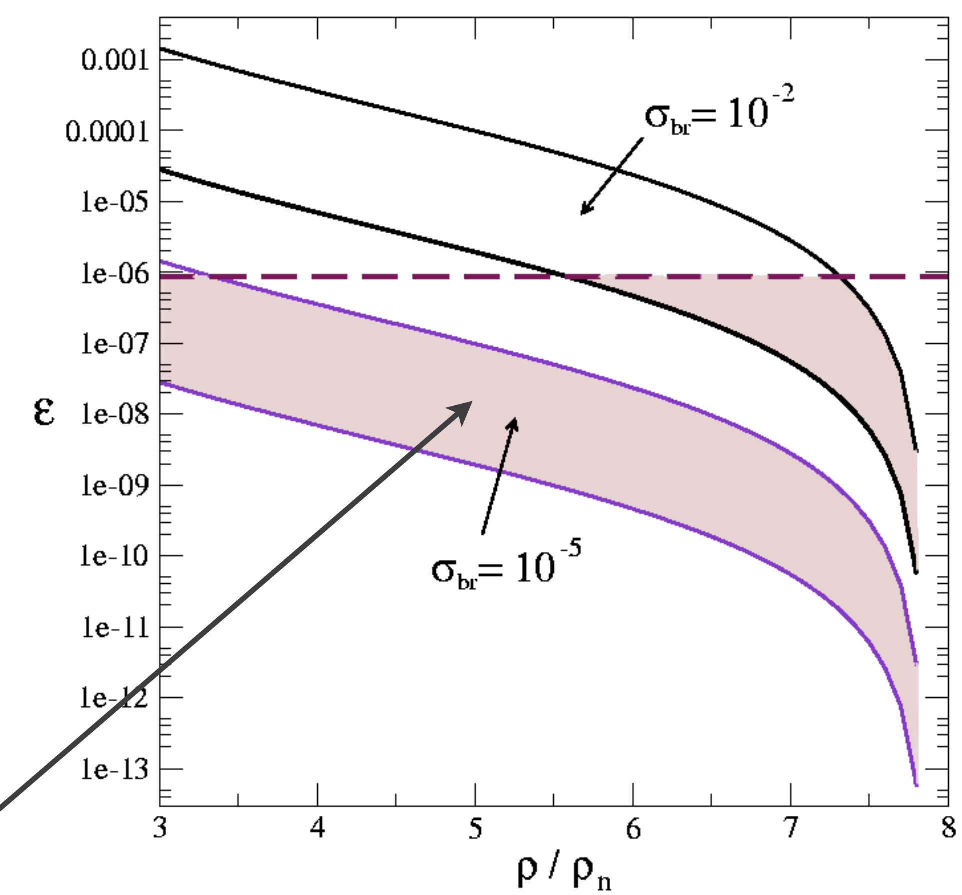
To have a “large” GW amplitude

- Large shear modulus
- Large breaking strain

Gravitational waves

Using the non-observation of GW from the Crab by the LIGO experiment





Lin, Phys.Rev. D76 (2007) 081502

allowed regions

Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

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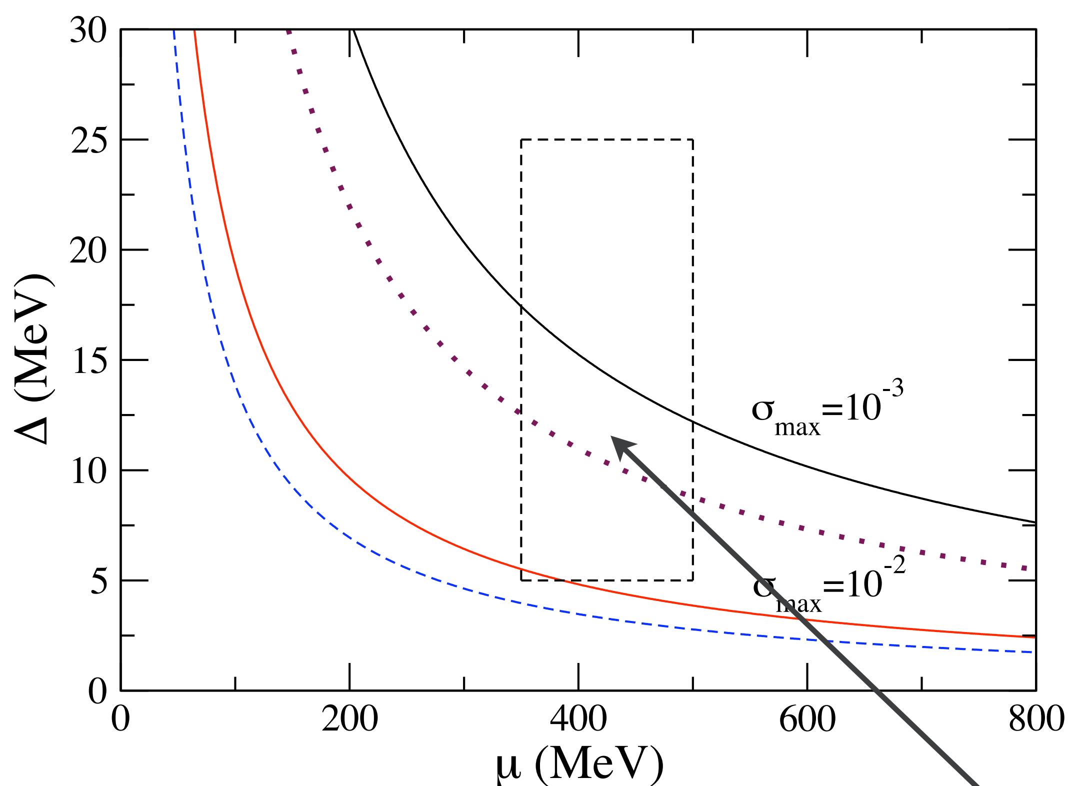
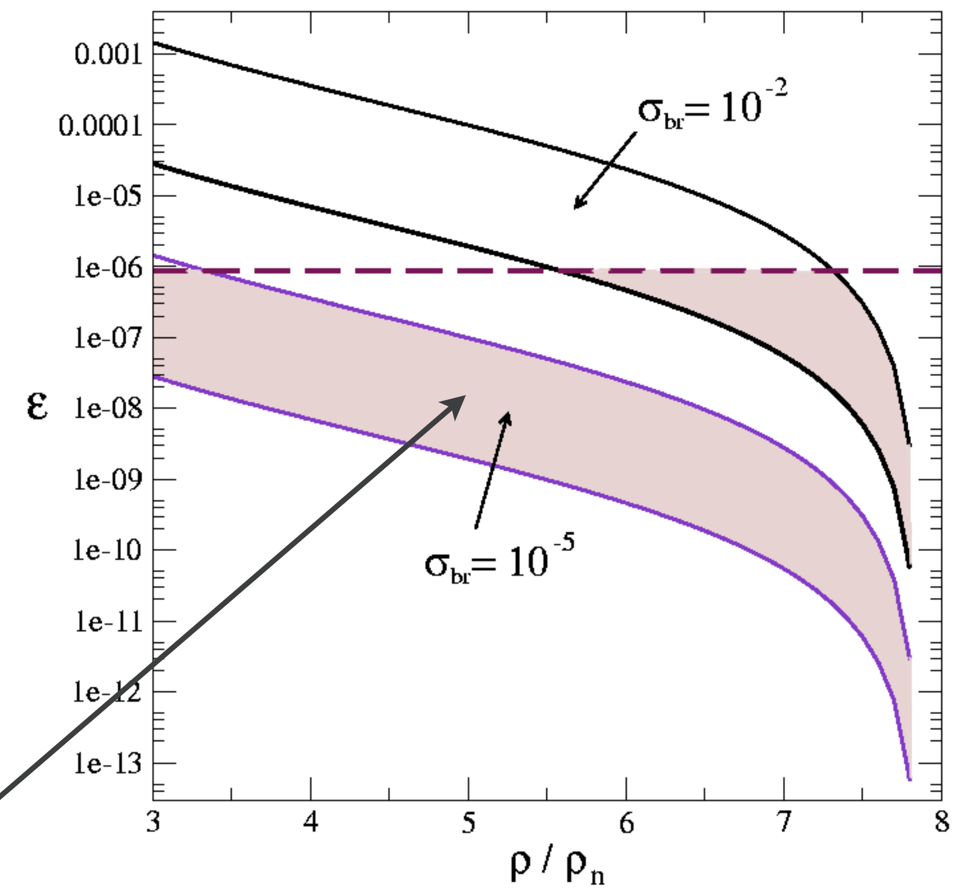




Lin, Phys.Rev. D76 (2007) 081502

allowed regions

Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

...we can restrict the parameter space!

Summary

- Motivated by compact stellar observations, the study of matter in extreme conditions allows to shed light on some properties of QCD

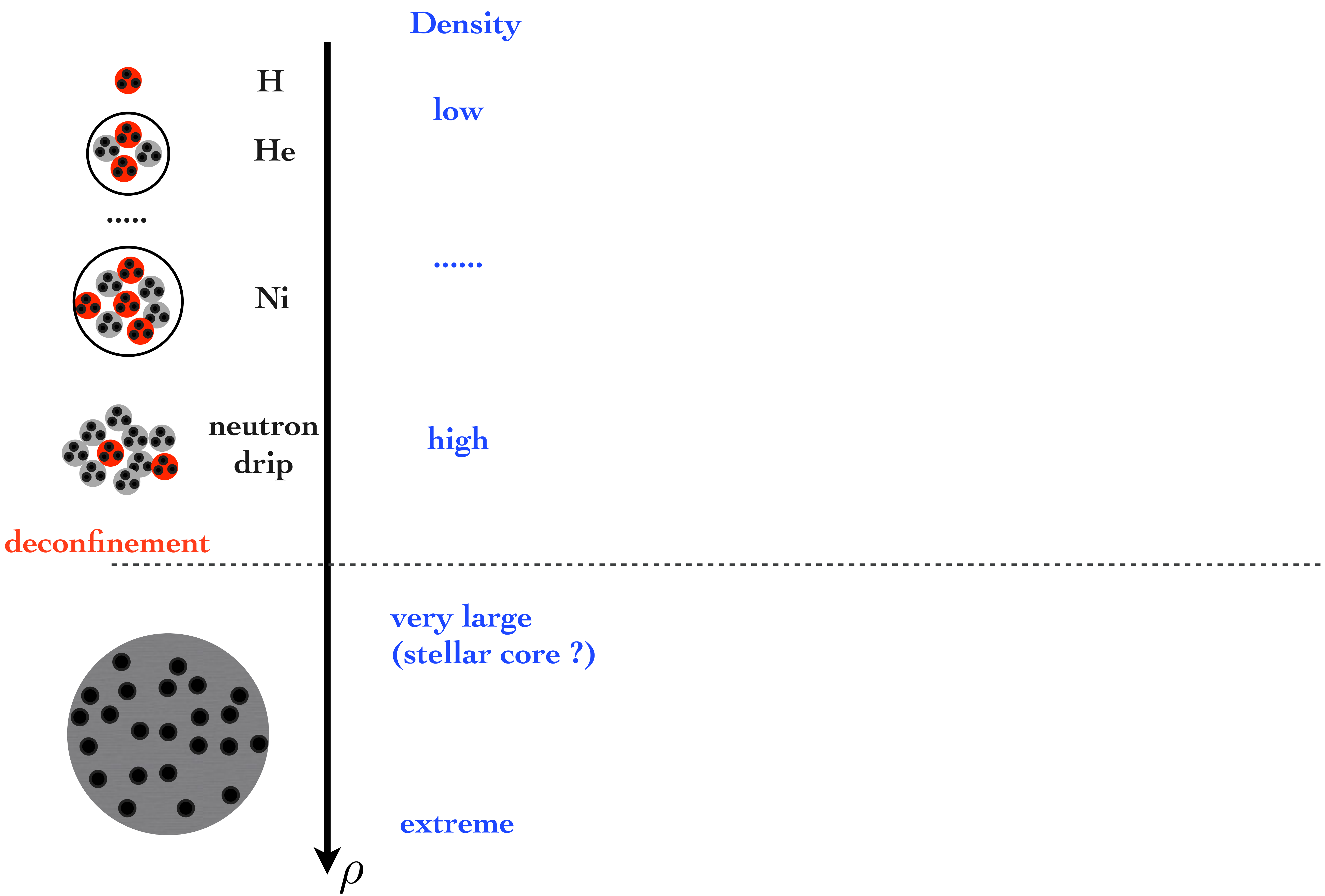
- Color superconductivity is a phase of matter predicted by QCD

- The temperature is so low that only NGBs are relevant

- The study of NGBs is linked to some observables of compact stars

Back-up slides

Increasing the baryonic density



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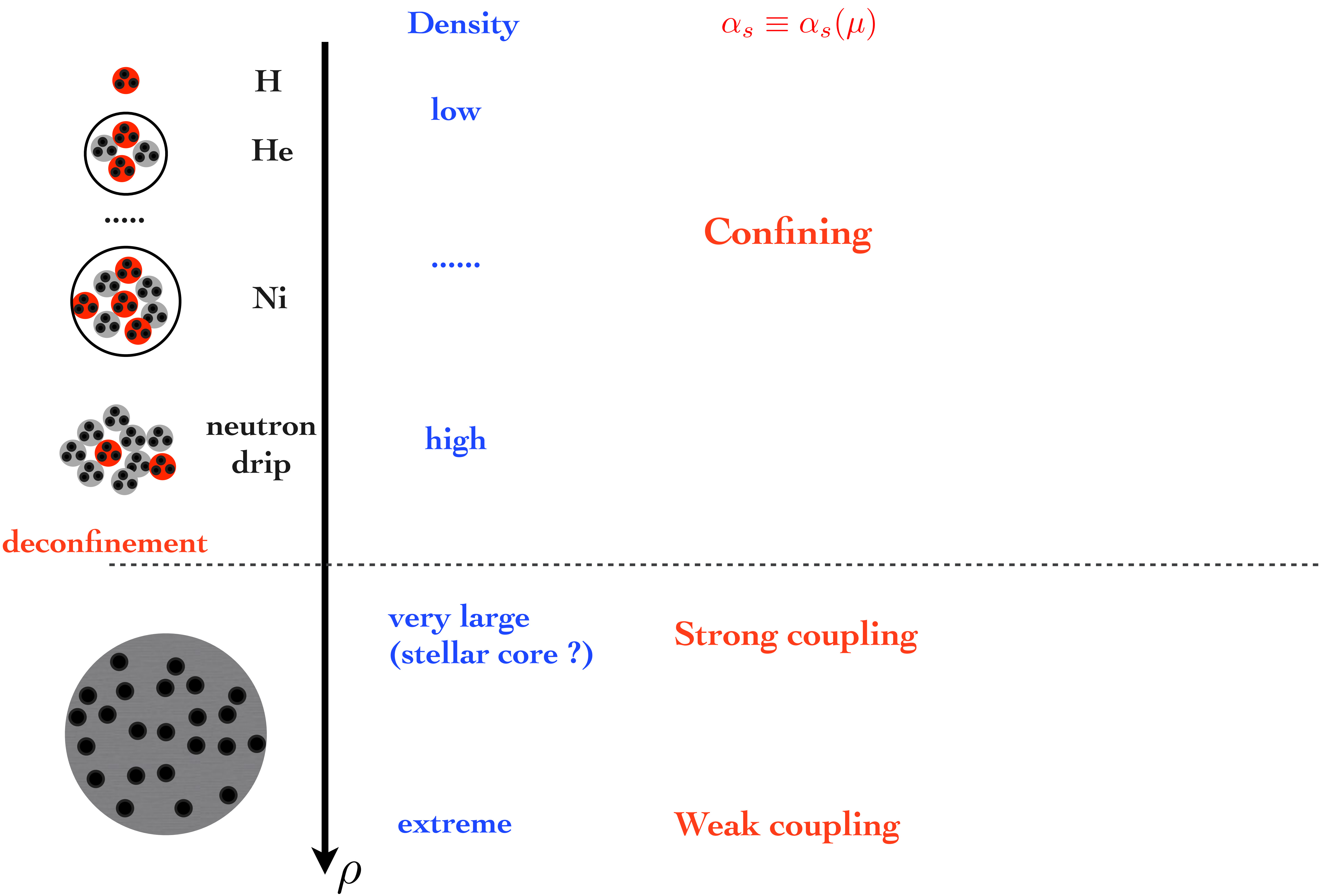

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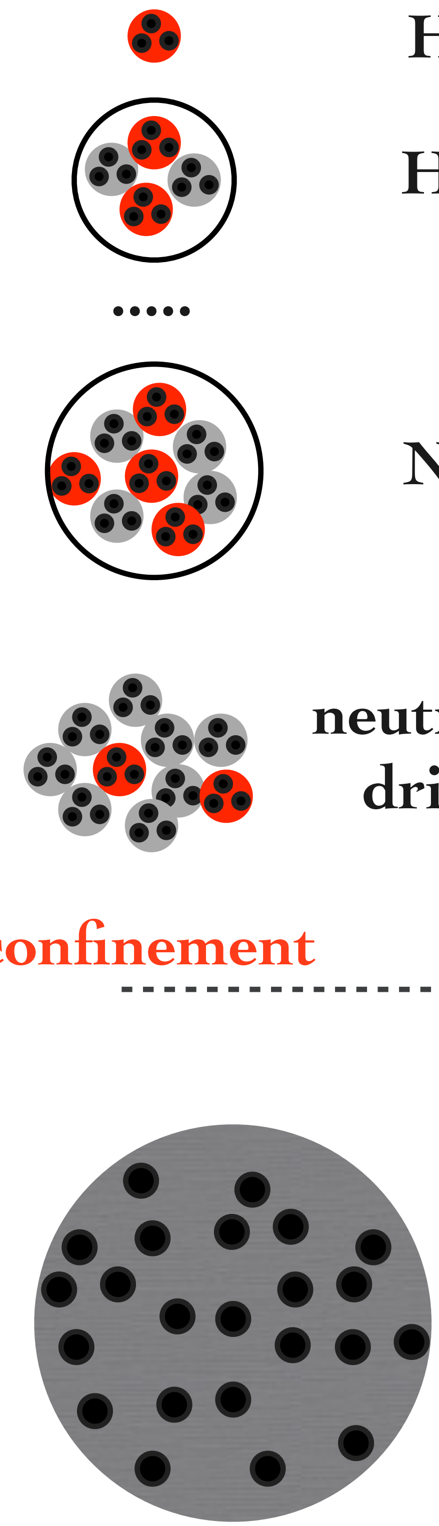


	Density	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H	low		light nuclei
He			
.....	......	Confining	.....
Ni			heavy nuclei
neutron drip	high		neutrons and protons
deconfinement			
	very large (stellar core ?)	Strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
	extreme	Weak coupling	Cooper pairs of quarks NGBs

ρ

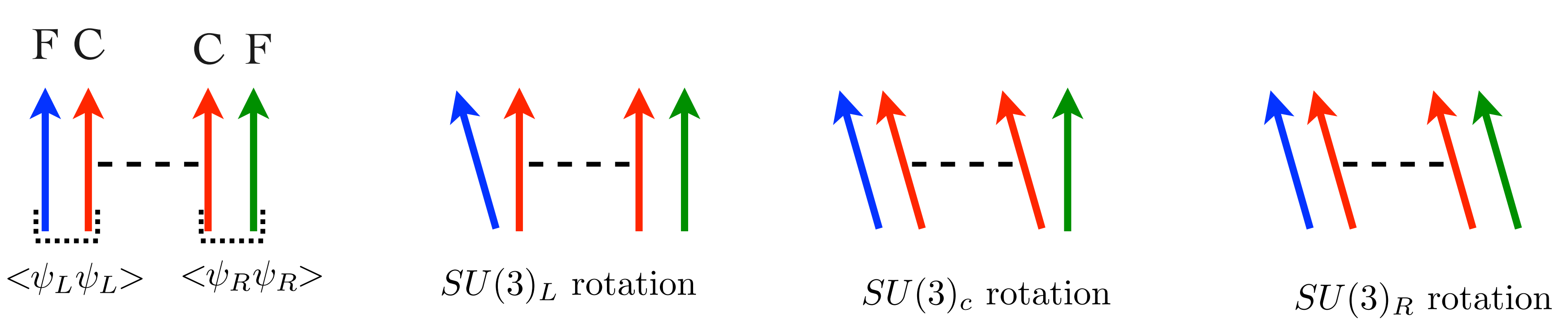
Chiral symmetry breaking

At low density the χSB is due to the condensate that locks left-handed and right-handed fields $\langle \bar{\psi}\,\psi \rangle$

In the CFL phase we can write the condensate as

$$\langle \psi^L_{\alpha i} \psi^L_{\beta j} \rangle = -\langle \psi^R_{\alpha i} \psi^R_{\beta j} \rangle = \kappa_1 \delta_{\alpha i} \delta_{\beta j} - \kappa_2 \delta_{\alpha j} \delta_{\beta i}$$

Color is locked to both left-handed and right-handed rotations.



Two good dishes ...

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \epsilon_{I\alpha\beta} \epsilon_{Iij} \Delta_{\mathrm{I}}$$

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CFL

Color superconductor
Baryonic superfluid
"e.m." insulator

$$\Delta_3 = \Delta_2 = \Delta_1 > 0$$

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

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$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

2SC

Color superconductor
"e.m." conductor

$$\Delta_3 > 0\,, \; \Delta_2 = \Delta_1 = 0$$

$$SU(3)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_S}_{\supset U(1)_{\mathrm{Q}}} \to SU(2)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}} \times U(1)_S}_{\supset U(1)_{\tilde{\mathrm{Q}}}}$$