Dark Energy as a cosmic scalar field

ϕ: a pioneer for new physics!

zeldovich: most important field for cosmology

"QUINTESSENCE" $\qquad \mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$ [PRL 4424; PRL 80 1582 (1998)]

quick list of examples

$V = \frac{1}{2}m^2\phi^2, \quad M^4(1+\cos\phi/f), \quad M^{4+n}\phi^{-n}, \ldots$

examples nearly as numerous as models of inflation

[MANY REFS.]

Properties / Problems

Since we require $w \approx -1$, then $\dot\phi^2 \ll V$

non-clustering, then $m = \sqrt{V''} \lesssim H$

dominant $\quad V \simeq M_p^2 H^2$

so $\quad \frac{V''}{V} \frac{4}{R} M_p^2 \simeq 1$

which for $V = \frac{1}{2}m^2\phi^2$ means $\phi \simeq M_p$

EXTREME!

It is a challenge to build a particle physics model of such a light field, which has Planckian ϕ, and remain DARK

[Carroll, PRL 81, 3067 (1998); Pecci hep-ph/0009030; Kolda + Lyth PLB 458, 197 (1999)]

EQUATION OF STATE

$$w = \frac{P}{\rho}$$ (NOT A TRUE E.O.S. SINCE ρ, P ARE HOMOG.)

reverse engineer: given $w(a)$

$$\rho(a) = \rho(a_0) \exp\left[3\int_a^{a_0} \frac{da'}{a'}(1+w(a'))\right]$$

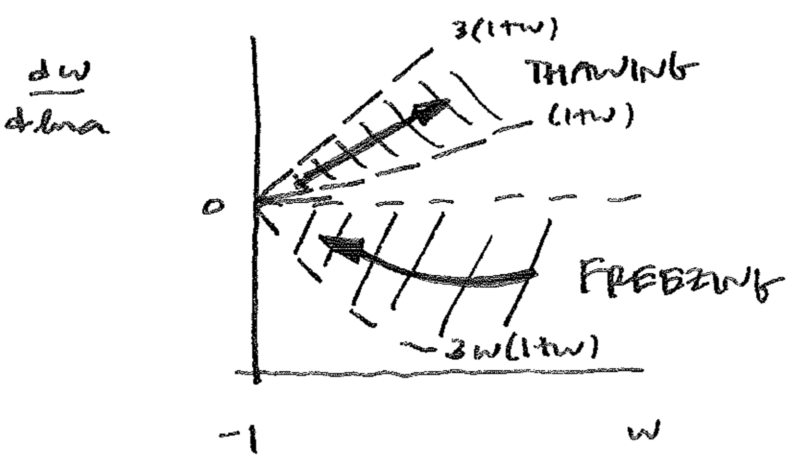
<u>SIMPLISTIC</u>: $w =$ constant, $\rho = \rho_0 (a_0/a)^{3(1+w)}$

<u>POPULAR MODEL</u>: $w(a) = w_0 + w_a(1 - a/a_0)$

use to diagnose for $w' \neq 0$

$$\text{so} \quad \rho = \rho_0 (a_0/a)^{3(1+w_0+w_a)} \exp\left[3w_a\left(\frac{a_0}{a} - 1\right)\right]$$

<u>BROAD CLASSIFICATION</u>: THAWING VS. FREEZING



BOUNDARIES ARE PHENO. BASED ON VARIOUS MODELS.

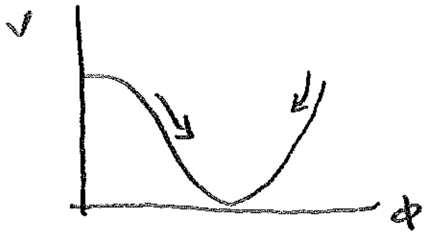
Scherrer PRL 95, 141301 (2005)

THAWING: SUGGESTS $\frac{dw}{d\ln a} \simeq C(1+w)$ $\quad 1 \lesssim C \lesssim 3$

or $w = -1 + (1+w_0)\left(\frac{a}{a_0}\right)^C$

POTENTIALS

$V = \frac{1}{2}m^2\phi^2,\ \lambda(\phi^2-\sigma^2),\ M^4(\cos\frac{\phi}{f}+1),\ \ldots$



rolling slowly today

how did they start?

must choose $\phi, \dot{\phi}$ + parameters $\to$ DELICATE

However, H-friction damps ϕ motion at early times

$|\dot{\phi}/\phi| \ll H$

so it seems fair to set $\dot{\phi} \to 0$ at early times

call such models "frozen" $\to$ $\boxed{\text{THAWING}}$

$V = M^{4+n}\phi^{-n}$ "TRACKER"

This model has $w < w_B$ while $\Omega_\phi \ll 1$

so it eventually catches up to dominate.

Into the future $w \to -1$ $\boxed{\text{FREEZING}}$

let's see: $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

use $H = 2/3(1+w_B)t$ & $\phi = At^B$

show $\dot{\phi}^2, \phi^{-n} \propto t^{-2n/(2+n)}$

$w = -1 - \frac{1}{3}\frac{1}{H}\frac{d\ln\rho}{dt} = -1 + \frac{n}{2+n}(1+w_B)$

or more generally $w = \dfrac{w_B - 2(\Gamma - 1)}{1 + 2(\Gamma - 1)}$, $\Gamma \equiv \dfrac{V'' V}{(V')^2}$

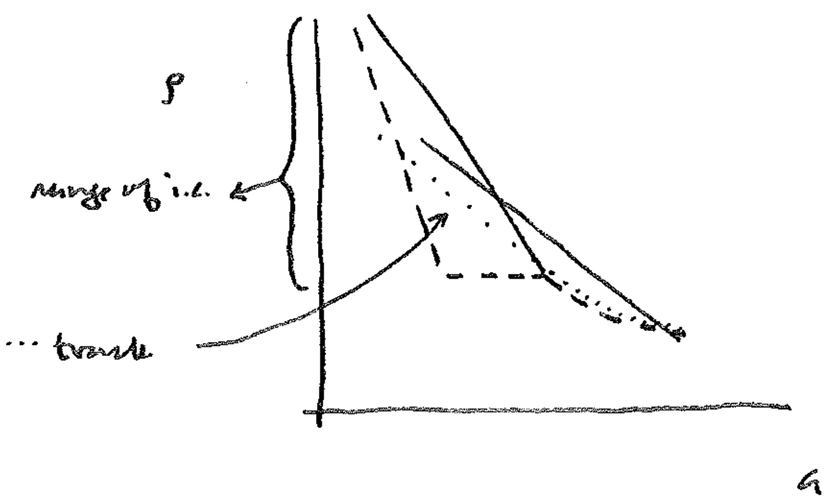
tracking conditions : $\Gamma \sim$ constant)

$\Gamma > 1$ for $w < w_B$

features: broad insensitivity to initial conditions

tracker is an attractor.

challenge: Need $0 < n \ll 1$ or $\Gamma \gg 1$ to get a sufficiently negative w by today.



see Steinhardt et al, PRD59, 123504 (1999)

Further examples

$$V = \sum_{i=1}^{N} V_i \, e^{-\alpha_i \phi / M_p}$$ one field, many exponential potentials

suppose $N=2$ with $\alpha_1 > \sqrt{24\pi}$ & $\alpha_2 < \sqrt{24\pi}$

so at early times ϕ scales

" late " dominates

good example of early dark energy

$$V = \sum_i V_i \, e^{-\alpha_i \phi_i / M_p}$$ many fields

For all the fields that are in play, ϕ_i's behave as a single fluid with

$$\frac{1}{\alpha_{eff}^2} = \sum_i \frac{1}{\alpha_i^2}$$

[NOT IN PLAY? $V_i \ll V_{others}$]

Asymptotic EOS: $w \to -1 + \frac{\alpha_{eff}^2}{24\pi}$

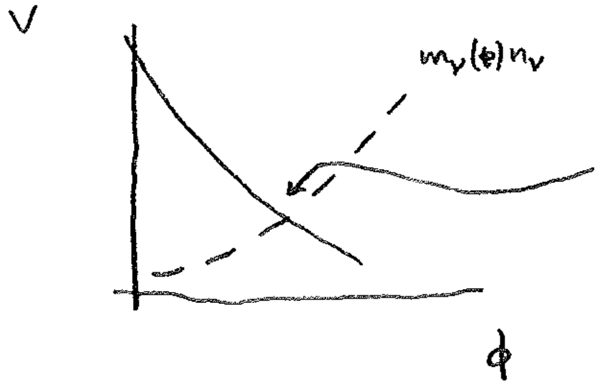
"assisted inflation" Liddle et al, PRD58 061301 (1998)

MASS VARYING NEUTRINOS [eg Fardon et al JCAP 0410 005 (2004)]

suppose $m_\nu(\phi)$ depends on a cosmic field

MODEL $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - m_\nu(\phi)\bar{\Psi}\Psi$

ignore fermion kinetic term
for non-rel ν's



effective potential has a minimum ; can this ΔV provide dark energy?

$$\rho_T = \rho_\phi + \rho_\nu$$

$$\dot\rho_T = \left(\tfrac{1}{2}\dot\phi^2 + V\right)^{\cdot} + \dot\rho_\nu = -3H(\rho_T + p_T)$$

$$\dot\phi\ddot\phi + V'\dot\phi + \dot\rho_\nu = -3H(\dot\phi^2 + \rho_\nu)$$

$$\rho_\nu = m_\nu(\phi)\, n_\nu$$

$$\dot\rho_\nu = \frac{\partial m_\nu}{\partial\phi}\dot\phi\, n_\nu + m_\nu \dot n_\nu$$

$\dot n_\nu = -3H n_\nu$ (# ν's are conserved)

$$\dot\phi\left(\ddot\phi + V' + 3H\dot\phi + \frac{\partial m_\nu}{\partial\phi} n_\nu\right) = 0$$

so $\ddot\phi + 3H\dot\phi + V' = -Q(\phi)\rho_\nu$, $Q = \dfrac{\partial \ln m_\nu}{\partial\phi}$

Now let's suppose ϕ moves along $V_{T,\min}$

$$V_T = V + m_\nu n_\nu$$

$$\frac{\partial V_T}{\partial \phi} = V' + \frac{\partial \ln m_\nu}{\partial \phi} \rho_\nu = 0$$

$$V' = -\frac{\partial \ln m_\nu}{\partial \phi} \rho_\nu \quad \longrightarrow \quad \dot{\phi}^2 \ll V_T$$

in which case $\rho_T \simeq V + m_\nu n_\nu$

$$= V - \frac{\partial V}{\partial \ln m_\nu}$$

and $P_T = -V$

The eq'n of state is $w = \frac{P_T}{\rho_T} = \frac{-V}{V + m_\nu n_\nu} = -1 + \frac{\Omega_\nu}{\Omega_\phi + \Omega_\nu}$

ACCELERATES! SUPE.

real mass? $V'' > 0$ easily satisfied ✓

SOUND SPEED? Here is the problem

$$c_a^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{(-V)^{\cdot}}{(V + m_\nu n_\nu)^{\cdot}} = \frac{(-V'\dot{\phi})}{(\underbrace{V'\dot{\phi} + \frac{\partial m_\nu}{\partial \phi}\dot{\phi} n_\nu}_{\text{cancel}} + m \underbrace{\dot{n}_\nu}_{-3Hn_\nu})}$$

$$= \frac{\frac{\partial m}{\partial \phi} n_\nu \dot{\phi}}{m_\nu(-3Hn_\nu)} = -\frac{1}{3}\frac{\dot{\phi}}{H}\frac{\partial \ln m_\nu}{\partial \phi}$$

GENERALLY NEGATIVE

"adiabatic instability" ruins this scenario*

(see Bean et al, PRD 78 023009 (2008))

Living with adiabatic instability?

If c_a^2 is not too negative

Then growth of perturbations is not too fast.

~ Compare $c_a^2 k^2$ vs. $\mathcal{H}^2$

See Corasaniti, PRD 78 083538 (2008)

This also leads to "Growing Neutrino" Cosmology

eg Wetterich + Pettorino arxiv: 0905.0715

But here, $\frac{\partial \ln m_\nu}{\partial \phi}$ is large

Large lumps of neutrinos result, so that

non-linear effects ($\delta_\nu > 1$) become important.

HOW NEGATIVE Eq'N OF State? "PHANTOM"

$w < -1$? Consistent w/ SNe data [RC, PLB 545, 23 (2002)

violates energy condition

$\rho > 0$, $p < -\rho$ in cosmic frame means

there is a boosted frame in which $\rho' < 0$!

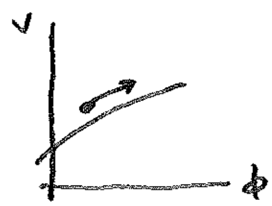
nevertheless... what would it imply?

A simple realization $\mathcal{L}_\phi = +\frac{1}{2}(\partial\phi)^2 - V$

↑ wrong sign leads to instabilities.

EOM $\ddot{\phi} + 3H\dot{\phi} - V' = 0$ $\cdot = \frac{d}{dt}$

so field runs up hill!



look at perturbs

STD: $\ddot{\delta\phi} + 3H\dot{\delta\phi} + (k^2 + V'')\delta\phi = -\frac{1}{2}\dot{h}\dot{\phi}$ $\cdot = \frac{\partial}{\partial\tau}$

Here: $\ddot{\delta\phi} + 3H\dot{\delta\phi} + (k^2 - V'')\delta\phi = -\frac{1}{2}\dot{h}\dot{\phi}$

need $V'' < 0$

but not so bad..

$|V''| \lesssim H^2$

NO INSTABILITY DUE TO DRIVING TERM

[see V'' eq'n in terms of $w, \dot{w}$]

CLASSICALLY, SUCH A FIELD THEORY IS 'OK'

BIG RIP

Let's say $w =$ constant, < -1

$$a(t) = \begin{cases} a_m \left(t/t_m \right)^{2/3} & t < t_m \\ a_m \left[-w + (1+w) \frac{t}{t_m} \right]^{\frac{2}{3(1+w)}} & t > t_m \end{cases}$$

patch phantom DE onto matter era.

$\downarrow$

USING $a(t)$

$$H = \frac{2}{3} \left[-w t_m + (t)(1+w) \right]^{-1}$$

$$R = 6 \left(\frac{\ddot{a}}{a} + H^2 \right)$$

$$= \frac{4}{3} \frac{1 - 3w}{(-w t_m + (1+w) t)^2}$$

OBSERVE $a(t)$, $H(t)$, $R(t)$ all diverge in finite time

$$t_{\text{BIG RIP}} = \frac{w}{1+w} t_M$$

EVENTS PRIOR TO $t_{\text{BIG RIP}}$: EXPANSION "FORCE" DIVERGES, PULLS APART OBJECTS

Further interest: "SUDDEN FUTURE SINGULARITIES" and other exotic behavior resulting from dark energy.

NON-STANDARD SCALAR FIELD THEORIES

k-essence: $\mathcal{L}_\phi = P(\phi, X)$, $X = -\frac{1}{2}(\partial\phi)^2$

(so $-+++$ $\rightarrow$ $X = +\frac{1}{2}\dot{\phi}^2$)

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G} + P(\phi, X) + \mathcal{L}_m\right]$$

$$T_{\mu\nu}^{(\phi)} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,P)}{\delta g^{\mu\nu}} = P g_{\mu\nu} + P_{,X}\,\partial_\mu\phi\,\partial_\nu\phi$$

like a fluid with pressure P

energy density $\rho = 2XP_{,X} - P$

and 4-vel $u_\mu = \partial_\mu\phi/\sqrt{2X}$

EQ'N OF STATE: $w = \dfrac{P}{2XP_{,X} - P} = \left(2\cdot\dfrac{\partial \ln P}{\partial \ln X} - 1\right)^{-1}$

EQ'N OF MOTION

$$\dot{\rho} = \frac{d}{dt}(2XP_{,X} - P) = 2\dot{X}P_{,X} + 2X\dot{X}P_{,XX} + 2X\dot{\phi}P_{,X\phi} - \dot{X}P_{,X} - \dot{\phi}P_{,\phi}$$

$$= -3H(2XP_{,X})$$

$$\rightarrow \dot{X}(P_{,X} + 2XP_{,XX}) + \dot{\phi}(-P_{,\phi} + 2XP_{,X\phi}) = -6HXP_{,X}$$

k-essence perturbations

v. similar to std quintessence

$$\dot{\delta} = -(1+w)\left(\theta + \tfrac{1}{2}\dot{h}\right) - 3\mathcal{H}\left(\frac{\delta p}{\delta \rho} - w\right)\delta$$

$$\dot{\theta} = (3c_s^2 - 1)\mathcal{H}\theta + c_s^2 k^2 \delta / (1+w)$$

$$\delta p = c_s^2 \delta\rho + \frac{\theta \rho}{k^2}\left[3\mathcal{H}(1+w)(c_s^2 - w) + \dot{w}\right]$$

$$\dot{} = \frac{d}{d\tau}$$

$c_s^2 = 1$ for quintessence w/ canonical kinetic term

$$c_s^2 = \frac{P_{,X}}{2XP_{,XX} + P_{,X}} = \frac{1}{1 + 2X\frac{P_{,XX}}{P_{,X}}}$$

warning – $c_s^2 > 1$ may occur!

EXAMINE THE PERTURBED HAMILTONIAN

$$H = (P_{,X} + 2X P_{,XX}) \frac{(\dot{\delta\phi})^2}{2} + P_{,X} \frac{(\nabla\delta\phi)^2}{2} - P_{,\phi\phi} \frac{(\delta\phi)^2}{2}$$

$H \geq 0$ IF

$$\xi_1 \equiv P_{,X} + 2X P_{,XX} \geq 0$$

$$c_s^2 = \xi_2 / \xi_1$$

$$\xi_2 \equiv P_{,X} \geq 0$$

$$\xi_3 \equiv -P_{,\phi\phi} \geq 0$$

#3 $\xi_3 \geq 0$ for stability, but not essential.
$\xi_3 < 0$ leads to long-wavelength, power-law growth of perturbations which are typical in inflation, etc.

#1-2 QUANTUM STABILITY demands $\xi_{1,2} \geq 0$
or else quintessence will populate these negative energy, ghost states via quantum gravitational processes.

Refs: Carroll et al PRD 68 023509 (2003)
Cline et al PRD 70 043543 (2004)

QM STABILITY

Phantom : $P = -X - V$

$\xi_1 = -1 \qquad \xi_2 = -1$ looks bad!

so vacuum $\to 2\phi + 2\gamma$ will run away!

Only recourse is to cut-off properties of gravitons that mediate this decay [see Carroll et al.]

Dilatonic Ghost Condensate : $P = -X + \dfrac{X^2}{M^4} e^{\lambda \kappa \phi}$

call $Q = \dfrac{X}{M^4} e^{\lambda \kappa \phi}$

$\xi_1 = -1 + 6Q$

$\xi_2 = -1 + 2Q \geqslant 0$ MEANS $Q \geqslant \frac{1}{2}$

THIS ALSO ENSURES $c_s^2 = \dfrac{2Q-1}{6Q-1}$ in $[0,1]$.

Further $w = \dfrac{Q-1}{3Q-1}$ so $w \geqslant -1$

Despite "$-X$" it is not much of a ghost.

COUPLING ϕ / QUINTESSENCE TO STANDARD MODEL?

In most cases, this is ruled out!

$\phi F_{\mu\nu} F^{\mu\nu}$ leads to variation of α_{em}

$\phi \bar{\Psi} \Psi$ long range force between lumps of fermions

Symmetry to prevent most couplings: $\phi \to \phi + c$
that still allows derivative interactions

$$\int d^4x \sqrt{-g}\ \phi F_{\mu\nu} \tilde{F}^{\mu\nu} = \int d^4x \sqrt{-g}\ 2\phi \nabla_\mu A_\nu \tilde{F}^{\mu\nu}$$

$$= \int d^4x \sqrt{-g}\ 2\left[\underbrace{\nabla_\mu(\phi A_\nu \tilde{F}^{\mu\nu})}_{\to 0} - \nabla_\mu(\phi \tilde{F}^{\mu\nu}) A_\nu\right] \qquad \nabla_\mu \tilde{F}^{\mu\nu} = 0$$

$$= -2 \int d^4x \sqrt{-g}\ A_\nu \nabla_\mu \phi \tilde{F}^{\mu\nu}$$ respects shift ✓

So let's pursue this... COSMIC AXION ELECTRODYNAMICS

$$\mathcal{L}_I = -\frac{1}{4} \frac{\phi}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

recall $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\tau} F_{\sigma\tau}$

MAXWELL + COSMIC FIELD:

$$\nabla_\mu F^{\mu\nu} + J^\nu + \frac{1}{M} \nabla_\mu \phi \tilde{F}^{\mu\nu} = 0 \quad , \quad \nabla_\mu \tilde{F}^{\mu\nu} = 0$$

and $\Box \phi = V' + \frac{1}{4M} F_{\mu\nu} \tilde{F}^{\mu\nu}$

Maxwell's new equations ... in SI units!

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} - \frac{1}{Mc}\vec{\nabla}\phi\cdot\vec{B} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} + \frac{1}{Mc^3}(\dot{\phi}\vec{B} + \vec{\nabla}\phi\times\vec{E})$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad \frac{\Box\phi}{hc} = V' - \frac{\epsilon_0}{Mc}\vec{E}\cdot\vec{B}$$

Note: $\vec{D} = \epsilon_0(\vec{E} + \frac{\phi}{Mc}\vec{B})$

$$\vec{H} = \frac{1}{\mu_0}(\vec{B} - \frac{\phi}{Mc^3}\vec{E})$$

restores std form: $\vec{\nabla}\cdot\vec{D} = \rho$ & $\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t} = \vec{J}$

Interesting behavior!

Let's study EM waves, so set $\epsilon_0 = \mu_0 = c = 1$ for now.

$$\frac{\partial^2}{\partial t^2}\vec{E} - \vec{\nabla}^2\vec{E} = \frac{1}{M}\left[\vec{\nabla}(\vec{\nabla}\phi\cdot\vec{B}) - \frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\vec{B} + \vec{\nabla}\phi\times\vec{E}\right)\right]$$

$$\frac{\partial^2}{\partial t^2}\vec{B} - \nabla^2\vec{B} = \frac{1}{M}\left[\vec{\nabla}\times\left(\frac{\partial\phi}{\partial t}\vec{B} + \vec{\nabla}\phi\times\vec{E}\right)\right]$$

In an expanding universe

$$\vec{B}\to a^2\vec{B}, \quad \vec{E}\to a^2\vec{E}, \quad \frac{\partial}{\partial t}\to\frac{\partial}{\partial \tau}$$

If ϕ is a cosmic field, then it evolves slowly.

For EM waves with freq, wavenumber ω, k

$$\omega \gg \frac{1}{\phi}\frac{\partial \phi}{\partial t} \qquad k \gg \frac{1}{\phi}\nabla\phi$$

In which case

$$\frac{\partial^2}{\partial t^2}\vec{E} - \nabla^2\vec{E} = \frac{\dot{\phi}}{M}\left(-\frac{\partial \vec{B}}{\partial t}\right) + \frac{1}{M}(\vec{\nabla}\phi\cdot\vec{\nabla})\vec{B}$$

$$\frac{\partial^2}{\partial t^2}\vec{B} - \nabla^2\vec{B} = \frac{\dot{\phi}}{M}\frac{\partial \vec{E}}{\partial t} - \frac{1}{M}(\vec{\nabla}\phi\cdot\vec{\nabla})\vec{E}$$

Consider plane waves: $E, B \propto e^{i\omega t - i\vec{k}\cdot\vec{x}}$

$$\vec{P} \equiv \vec{E} + i\vec{B}, \quad \vec{Q} = \vec{E} - i\vec{B}$$

$$P: \quad \omega^2 = k^2 + \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi}{M}\cdot\vec{k}\right)$$

$$Q: \quad \omega^2 = k^2 - \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi}{M}\cdot\vec{k}\right)$$

Let's write $\vec{k} = k\hat{n}$ where $\hat{n}$ is the direction of propagation

$$\omega^2 = k^2 \pm \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi\cdot\hat{n}}{M}k\right)$$

$$k_\pm = \mp\frac{1}{2}\left(\frac{\dot{\phi}}{M} + \frac{\vec{\nabla}\phi\cdot\hat{n}}{M}\right) + \omega + O\left(\left(\frac{\phi}{M}\right)^2\right)$$

The phase velocity of L & R - circularly polarized waves is different. For a linearly polarized wave

$$\vec{E} = \frac{1}{2}(\vec{P}+\vec{R}) = \frac{1}{2}E_0 e^{i\omega t}\left(e^{-ik_+ z}[\hat{x}+i\hat{y}] + e^{-ik_- z}[\hat{x}-i\hat{y}]\right)$$

let's write $k_\pm = \omega \mp Q$, $Q = \frac{1}{2}\left(\frac{\dot{\phi}}{M} + \frac{\partial_z \phi}{M}\right)$

$$= \frac{1}{2}E_0 e^{i\omega(t-z/c)}\left(\left[e^{iQz} + e^{-iQz}\right]\hat{x} + i\left[e^{iQz} - e^{-iQz}\right]\hat{y}\right)$$

$$= E_0 e^{i\omega(t-z/c)}\left(\cos\theta\,\hat{x} + -\sin\theta\,\hat{y}\right)$$

$\theta = Qz$

But over long distances, $\dot{\phi}, \nabla\phi$ vary so that

$$\theta = \int Q\,dz = \frac{1}{2M}\int(\dot{\phi} + \partial_z\phi)\,dz$$

$$= \frac{1}{2M}\int\left[\frac{d}{dt}\phi(t, z(t))\right]dt$$

$$= \frac{\Delta\phi}{2M} \checkmark$$

In terms of Stokes parameters

$$\Pi = \begin{pmatrix} Q \\ U \end{pmatrix}, \quad \Pi' = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\Pi$$

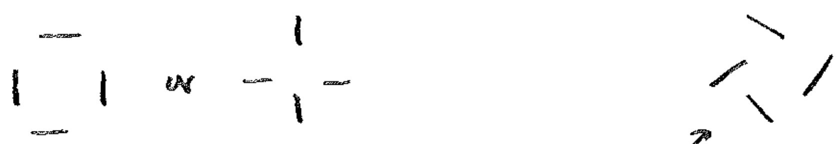
where $Q = |\hat{\epsilon}_1\cdot\vec{E}|^2 - |\hat{\epsilon}_2\cdot\vec{E}|^2$, $U = 2\,\mathrm{Re}[(\hat{\epsilon}_1\cdot\vec{E})^*(\hat{\epsilon}_2\cdot\vec{E})]$

so the rotation angle is $\theta = \frac{\Delta\phi}{2M}$.

This phenomena, known as COSMIC BIREFRINGENCE, is constrained by observations of CMB polarization.

CMB distinguishes two patterns of polarization

"E-type" "B-type"



headless vectors, the grad of a scalar on the sky.

a swirl: curl of a vector on the sky

The rotation of polarization will turn $E \to B$, $B \to E$.

The non-detection of B-type implies

$$-1.41^\circ < \Delta\theta < 0.91^\circ \quad \text{at } 95\% \text{ CL}$$

$$\text{or} \quad \left|\frac{\Delta\phi}{M}\right| < 0.035$$

See WMAP 7, Komatsu et al, ApJ Supp. 192 18 (2011)

Fluctuations of the scalar field at last scattering can further contribute to a position (on the sky)-dependent rotation of polarization.

CONSIDER QUINTESSENCE - ELECTRO/MAGNETO-STATICS

For a cosmic field $\frac{\dot{\phi}}{M}, \frac{\nabla\phi}{M} \sim H$ TINY!

But consider weak gravity in vicinity of Earth

$$\frac{d^2}{dt^2}\phi_0 + 3H\frac{d}{dt}\phi_0 + V' = 0$$

$$\frac{d^2}{dt^2}\delta\phi + 3H\frac{d}{dt}\delta\phi + V''\delta\phi - \frac{\nabla^2}{a^2}\delta\phi$$

$$= 4\dot{\phi}_0\dot{\Phi} - 2V'\Phi$$

SINCE FORMATION OF EARTH $\delta\phi \propto \Phi$

CONSTANT OF PROP. $\rightarrow \quad \delta\phi = b\,\Phi$

$$\ddot{b} + 3H\dot{b} + V''b = -2V'$$

$$b \sim 2\Delta\phi_0$$

S.T. $\quad \delta\phi \simeq 2\Delta\phi_0\Phi$

$$\vec{\nabla}\delta\phi \simeq -2\Delta\phi_0\vec{g}$$

$$\hbar\frac{\nabla\delta\phi}{Mc} = \hbar\epsilon_Q\frac{g}{c} \sim 10^{-32}\text{ GeV} \quad \checkmark$$

$(\text{IF } \epsilon_Q \sim 1)$

Return to Maxwell's Eq'ns

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} - \frac{\vec{\nabla}\phi}{Mc}\cdot\vec{B} \quad \rightarrow \quad \vec{\nabla}\cdot\vec{E} - \rho/\epsilon_0 = -\frac{\epsilon_Q}{c}\,\vec{g}\cdot\vec{B}$$

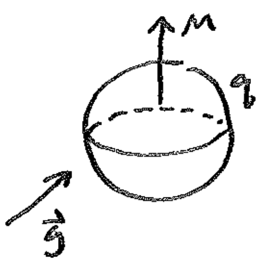
anomalous charge

$$\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} + \frac{1}{Mc^3}\vec{\nabla}\phi\times\vec{E}$$

$$\rightarrow \vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} - \mu_0\vec{J} = \frac{\epsilon_Q}{c^2}\,\vec{g}\times\vec{E}$$

anomalous current

look at a few cases: charged and magnetized sphere



inside, due to $\vec{E}$ (Coulomb) there is an anomalous $\vec{B}$

$$\vec{B} = \frac{\mu_0 q\,\epsilon_Q}{20\pi Rc}\left(5\vec{g} + \frac{r^2}{R^2}\left((\vec{g}\cdot\hat{r})\,\hat{r} - 2\vec{g}\right)\right)$$

assuming uniform q in radius R

It points in $\vec{g}$ direction, so if $\vec{g}$ is not aligned with the magnetization $\vec{\mu}$, there will be a torque.

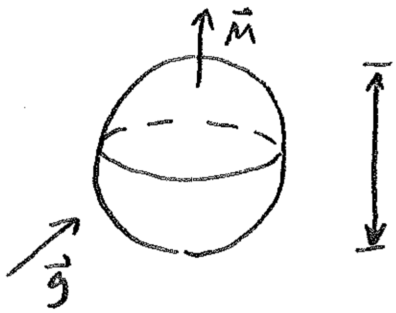
$$\vec{\tau} = \frac{2\mu_0 q\,\epsilon_Q}{5\pi Rc}\,\vec{\mu}\times\vec{g}$$

Spin-flip energy

$$\Delta\mathcal{E} = \frac{2\mu_0 q\,\epsilon_Q}{5\pi Rc}\,\mu g$$

For the proton as a classical object: $\Delta\mathcal{E} \simeq 10^{-34}\,\epsilon_Q$ GeV

Consider a uniformly magnetized sphere

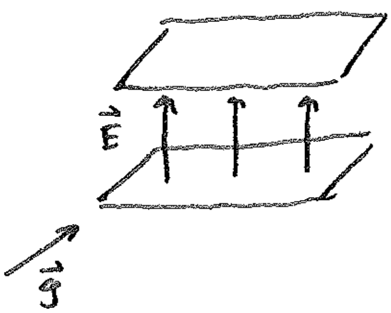


POTENTIAL DIFF.

$$\Delta V = \frac{\epsilon_Q R^2}{5c}\, \vec{g}\cdot\vec{B}$$

$$\simeq 0.3\, \epsilon_Q \left(\frac{B}{T}\right)\left(\frac{R}{0.1\,\text{m}}\right)^2 \text{ nV}$$

Consider a uniform electric field



Anomalous magnetic field

$$\vec{B} = -\frac{1}{2}\frac{\epsilon_Q}{c^3}\, \vec{r}\times(\vec{g}\times\vec{E})$$

$$\simeq 10^{-25}\, \epsilon_Q \left(\frac{\Delta V}{\text{volts}}\right) \text{ T}$$

also see: Flambaum et al PRD 80, 105021 (2009)

Bailey + Kostelecky PRD 70, 076006 (2004)

(14) HISTORY ?

Θ, R are derived quantities in SCM

In theories with novel matter or gravitational couplings,

eg scalar-tensor, massive gravity,

chameleon scalar

there may be interesting effects

Ex. $$S = \int d^4x \sqrt{-g}\, \mathcal{L}$$

$$\mathcal{L} = \frac{R}{2K^2} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \xi\phi^2 R + \mathcal{L}_{SM}$$

ACCEPT NO CONFORMAL TRANSFORMATIONS WITHOUT THE FULL LAGRANGIAN!

R vs. Θ MEDIATED BY ϕ

$$R = K^2\left[\frac{\Theta - (\partial\phi)^2(12\xi - 1) + 4V - 12\xi\phi V'}{1 - 2K^2\xi\phi^2(1-12\xi)}\right]$$

SPECIAL CASES: $\xi = \frac{1}{12}$, $V = \lambda\phi^4$

or $|\xi| \gg 1$, $|\xi K\phi| \ll 1$

$$R = K^2\,\Theta$$

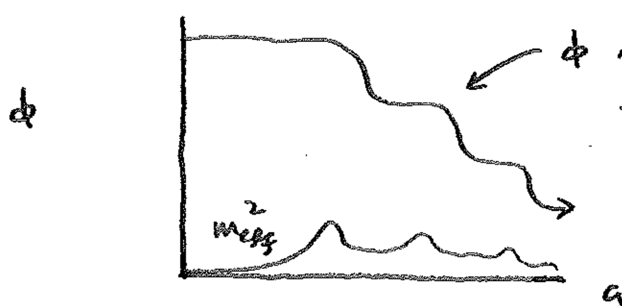
EOM: $\Box\phi = V' + 2\xi R \phi$

$$m_{eff}^2 = V'' + 2\xi R$$

$\downarrow$

R, T plays special role in guiding evolution of ϕ.

Ex: $\xi = \frac{1}{12}$, $\lambda = 0$



ϕ receives a "kick" from T

HOW TO KEEP QUINTESSENCE "DARK"

HOW TO KEEP IT FROM COUPLING TO SM?

"CHAMELEON" SOLUTION: COUPLE STRONGLY!

$$S = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} M_p^2 R - \tfrac{1}{2}(\partial\phi)^2 - V(\phi) \right]$$
$$+ \int d^4x \sqrt{-\hat{g}}\, \mathcal{L}_{SM}(\psi, \hat{g})$$

WHERE $\hat{g}_{\mu\nu} = \exp(2\beta\phi/M_p)\, g_{\mu\nu}$

SO QUINTESSENCE COUPLES TO EVERYTHING, VIA $\hat{g}$.

EQ'N FOR ϕ: $\quad \Box\phi = V' + \frac{\beta}{M_p} e^{4\beta\phi/M_p}\, \hat{\Theta}$

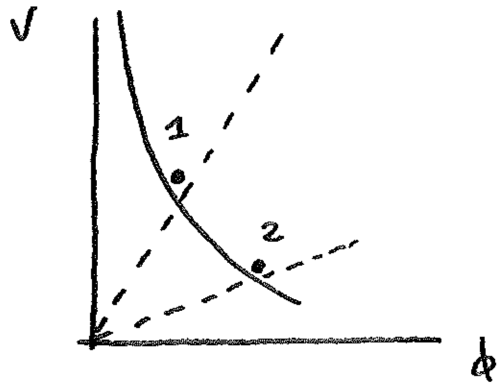
$\hat{\Theta}$ IS JORDAN (COUPLED) FRAME STRESS ENERGY TRACE (−)

$$\Theta = e^{4\beta\phi/M_p}\, \hat{\Theta}$$

EFFECTIVE POTENTIAL: $\quad V_{eff} = V + \frac{\beta}{M_p} \phi\, \Theta$

INTERESTING ROLE FOR Θ.

SUPPOSE $V \propto \phi^{-N}$



1. HIGH DENSITY REGION:

$$m_{eff}^2 = V'' \quad \text{LARGE}$$

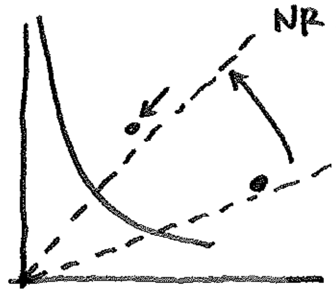
$$U \propto \frac{1}{r} e^{-m_{eff} r} \quad \text{SHORT RANGE.}$$

2. LOW DENSITY REGION:

SLOW ROLL $\rightarrow$ COSMIC ACCELERATION

COSMIC HISTORY OF

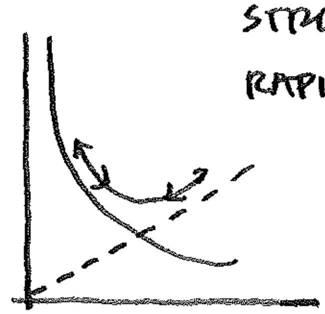
NR SPECIES KICK CHAMELEON



RADIATION ERA

BRAX et al PRD 70 123518 (2004)

STRONG KICKS,
RAPID OSCILLATIONS
= QM FLUCTUATIONS



ERICKCEK et al, 1304.0009