

Higgs Physics

Abdelhak DJOUADI (LPT Paris-Sud)

I: EWSB in the SM

- The Standard Model in brief
- The Higgs mechanism
- Constraints on M_H

II: Higgs decays

III: Higgs production a hadron colliders

IV: Higgs discovery and after

?? EWSB in SUSY theories ??

1. The Standard Model in brief

The SM of the electromagnetic, weak and strong interactions is:

- a relativistic quantum field theory,
- based on local gauge symmetry: invariance under symmetry group,
- more or less a carbon-copy of QED, the theory of electromagnetism.

QED: invariance under local transformations of the abelian group $U(1)_Q$

– transformation of electron field: $\Psi(\mathbf{x}) \rightarrow \Psi'(\mathbf{x}) = e^{ie\alpha(\mathbf{x})} \Psi(\mathbf{x})$

– transformation of photon field: $A_\mu(\mathbf{x}) \rightarrow A'_\mu(\mathbf{x}) = A_\mu(\mathbf{x}) - \frac{1}{e} \partial_\mu \alpha(\mathbf{x})$

The Lagrangian density is invariant under above field transformations

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} + i\bar{\Psi} \mathbf{D}_\mu \gamma^\mu \Psi - m_e \bar{\Psi} \Psi$$

field strength $\mathbf{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and cov. derivative $\mathbf{D}_\mu = \partial_\mu - ieA_\mu$

Very simple and extremely successful theory!

- minimal coupling: the interactions/couplings uniquely determined,
- renormalisable, perturbative, unitary (predictive), very well tested...

1. The Standard Model: brief introduction

The SM is based on the local gauge symmetry group

$$G_{\text{SM}} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$

• The group $SU(3)_C$ describes the strong force:

– interaction between quarks which are $SU(3)$ triplets: $\mathbf{q}, \mathbf{q}, \mathbf{q}$

– mediated by 8 **gluons**, G_μ^a corresponding to 8 generators of $SU(3)_C$

Gell-Man 3×3 matrices: $[T^a, T^b] = if^{abc}T_c$ with $\text{Tr}[T^a T^b] = \frac{1}{2}\delta_{ab}$

– asymptotic freedom: interaction “weak” at high energy, $\alpha_s = \frac{g_s^2}{4\pi} \ll 1$

The Lagrangian of the theory is given by:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + i \sum_i \bar{q}_i (\partial_\mu - ig_s T_a G_\mu^a) \gamma^\mu q_i \quad \left(- \sum_i m_i \bar{q}_i q_i \right)$$

$$\text{with } G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c$$

The interactions/couplings are then uniquely determined:

– fermion gauge boson couplings : $-g_i \bar{\psi} \mathbf{V}_\mu \gamma^\mu \psi$

– \mathbf{V} self-couplings : $ig_i \text{Tr}(\partial_\nu \mathbf{V}_\mu - \partial_\mu \mathbf{V}_\nu) [\mathbf{V}_\mu, \mathbf{V}_\nu] + \frac{1}{2} g_i^2 \text{Tr}[\mathbf{V}_\mu, \mathbf{V}_\nu]^2$

1. The Standard Model: brief introduction

• $SU(2)_L \times U(1)_Y$ describes the electroweak interaction:

– between the three families of quarks and leptons: $\mathbf{f}_{L/R} = \frac{1}{2}(1 \mp \gamma_5)\mathbf{f}$

$$\mathbf{I}_f^{3L,3R} = \pm\frac{1}{2}, 0 \Rightarrow \mathbf{L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \mathbf{R} = e^-_R, \mathbf{Q} = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

$$Y_f = 2Q_f - 2I_f^3 \Rightarrow Y_L = -1, Y_R = -2, Y_Q = \frac{1}{3}, Y_{u_R} = \frac{4}{3}, Y_{d_R} = -\frac{2}{3}$$

Same holds for the two other generations: $\mu, \nu_\mu, c, s; \tau, \nu_\tau, t, b.$

There is no ν_R (and neutrinos are and stay exactly massless)

– mediated by the W_μ^i (isospin) and B_μ (hypercharge) gauge bosons

the gauge bosons, corresp. to generators, are exactly massless

$$\mathbf{T}^a = \frac{1}{2}\boldsymbol{\tau}^a; \quad [\mathbf{T}^a, \mathbf{T}^b] = i\epsilon^{abc}\mathbf{T}_c \quad \text{and} \quad [Y, Y] = 0$$

Lagrangian simple: with fields strengths and covariant derivatives

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi = \left(\partial_\mu - ig\mathbf{T}_a W_\mu^a - ig' \frac{Y}{2} B_\mu \right) \psi, \quad \mathbf{T}^a = \frac{1}{2}\boldsymbol{\tau}^a$$

$$\mathcal{L}_{SM} = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\mathbf{F}}_{Li} iD_\mu \gamma^\mu \mathbf{F}_{Li} + \bar{\mathbf{f}}_{Ri} iD_\mu \gamma^\mu \mathbf{f}_{Ri}$$

1. The Standard Model: brief introduction

But if gauge boson and fermion masses are put by hand in \mathcal{L}_{SM}

$\frac{1}{2}M_V^2 V^\mu V_\mu$ and/or $m_f \bar{f}f$ terms: breaking of gauge symmetry.

This statement can be visualized by taking the example of QED where the photon is massless because of the local $U(1)_Q$ local symmetry:

$$\Psi(\mathbf{x}) \rightarrow \Psi'(\mathbf{x}) = e^{ie\alpha(\mathbf{x})} \Psi(\mathbf{x}), \quad A_\mu(\mathbf{x}) \rightarrow A'_\mu(\mathbf{x}) = A_\mu(\mathbf{x}) - \frac{1}{e} \partial_\mu \alpha(\mathbf{x})$$

• For the photon (or B field for instance) mass we would have:

$$\frac{1}{2}M_A^2 A_\mu A^\mu \rightarrow \frac{1}{2}M_A^2 (A_\mu - \frac{1}{e} \partial_\mu \alpha)(A^\mu - \frac{1}{e} \partial^\mu \alpha) \neq \frac{1}{2}M_A^2 A_\mu A^\mu$$

and thus, gauge invariance is violated with a photon mass.

• For the fermion masses, we would have (e.g. for the electron):

$$m_e \bar{e}e = m_e \bar{e} \left(\frac{1}{2}(1 - \gamma_5) + \frac{1}{2}(1 + \gamma_5) \right) e = m_e (\bar{e}_R e_L + \bar{e}_L e_R)$$

manifestly non-invariant under $SU(2)$ isospin symmetry transformations.

We need a less “brutal” way to generate particle masses in the SM:

\Rightarrow The Brout-Englert-Higgs mechanism \Rightarrow the Higgs particle H.

1. The Standard Model: brief introduction

⇒ High precision tests of the SM performed at quantum level: 1%–0.1%

The SM describes precisely (almost) all available experimental data!

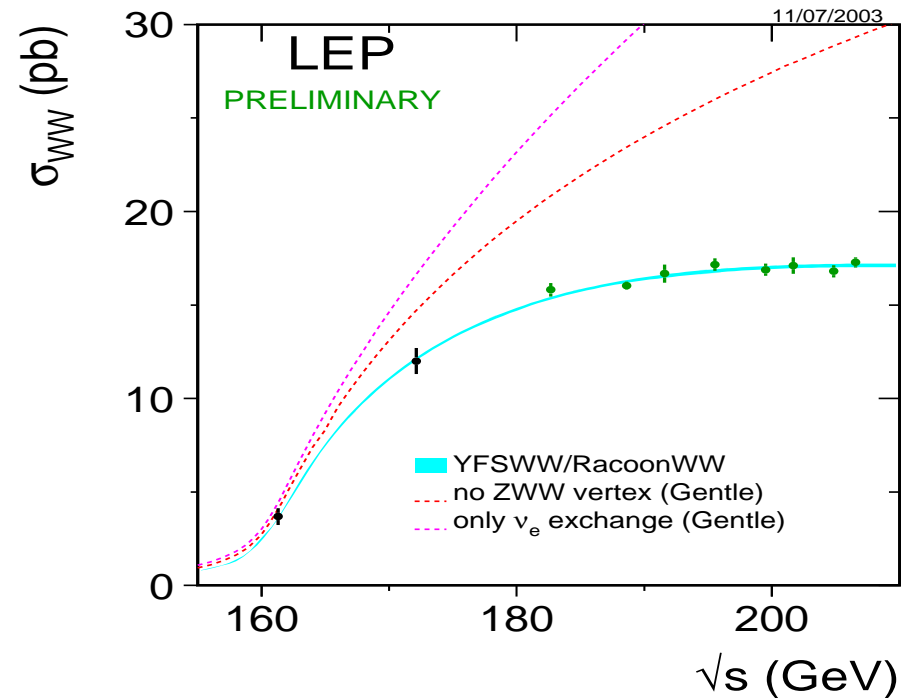
- Couplings of fermions to γ, Z
- Z, W boson properties
- measure/running of α_S

- SM gauge structure
- Properties of W bosons

	Measurement	Fit	$ O^{\text{meas}} - O^{\text{fit}} /\sigma^{\text{meas}}$
$\Delta\alpha_{\text{had}}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02766	0.1
m_Z [GeV]	91.1875 ± 0.0021	91.1874	0.05
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957	0.2
σ_{had}^0 [nb]	41.540 ± 0.037	41.477	1.7
R_f	20.767 ± 0.025	20.744	0.9
$A_{\text{fb}}^{0,l}$	0.01714 ± 0.00095	0.01640	1.5
$A_f(P_f)$	0.1465 ± 0.0032	0.1479	0.4
R_b	0.21629 ± 0.00066	0.21585	0.8
R_c	0.1721 ± 0.0030	0.1722	0.02
$A_{\text{fb}}^{0,b}$	0.0992 ± 0.0016	0.1037	2.8
$A_{\text{fb}}^{0,c}$	0.0707 ± 0.0035	0.0741	1.1
A_b	0.923 ± 0.020	0.935	0.6
A_c	0.670 ± 0.027	0.668	0.05
$A_f(\text{SLD})$	0.1513 ± 0.0021	0.1479	1.6
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{fb}})$	0.2324 ± 0.0012	0.2314	0.8
m_W [GeV]	80.392 ± 0.029	80.371	0.7
Γ_W [GeV]	2.147 ± 0.060	2.091	1.0
m_t [GeV]	171.4 ± 2.1	171.7	0.1

LEP1, SLC, LEP2, Tevatron

LEP2, Tevatron



- Physics of top&bottom quarks, QCD
- Tevatron, HERA and B factories

2. EWSB in the SM

In the SM, if gauge boson and fermion masses are put by hand in \mathcal{L}_{SM}

breaking of gauge symmetry \Rightarrow spontaneous EW symmetry breaking

\Rightarrow introduce a doublet of complex scalar fields: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = +1$

with a Lagrangian that is invariant under $SU(2)_L \times U(1)_Y$

$$\mathcal{L}_S = (\mathbf{D}^\mu \Phi)^\dagger (\mathbf{D}_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$\mu^2 > 0$: 4 scalar particles.

$\mu^2 < 0$: Φ develops a vev:

$$\langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

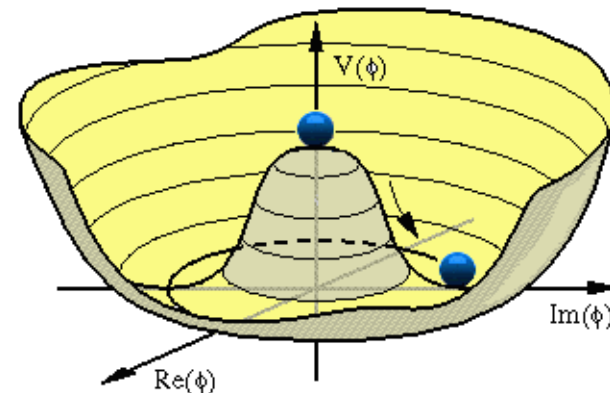
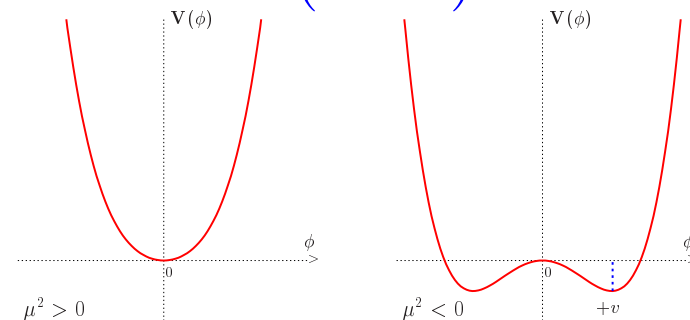
with $\text{vev} \equiv v = (-\mu^2/\lambda)^{\frac{1}{2}}$

– symmetric minimum: instable

– true vacuum: degenerate

\Rightarrow to obtain the physical states,

write \mathcal{L}_S with the true vacuum:



2. EWSB in SM: mass generation

- Write Φ in terms of four fields $\theta_{1,2,3}(\mathbf{x})$ and $H(\mathbf{x})$ at 1st order:

$$\Phi(\mathbf{x}) = e^{i\theta_a(\mathbf{x})\tau^a(\mathbf{x})/v} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H(\mathbf{x}) \end{pmatrix} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2+i\theta_1 \\ v+H-i\theta_3 \end{pmatrix}$$

- Make a gauge transformation on Φ to go to the unitary gauge:

$$\Phi(\mathbf{x}) \rightarrow e^{-i\theta_a(\mathbf{x})\tau^a(\mathbf{x})} \Phi(\mathbf{x}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H(\mathbf{x}) \end{pmatrix}$$

- Then fully develop the term $|\mathbf{D}_\mu \Phi|^2$ of the Lagrangian \mathcal{L}_S :

$$\begin{aligned} |\mathbf{D}_\mu \Phi|^2 &= \left| \left(\partial_\mu - i\mathbf{g}_1 \frac{\tau_a}{2} \mathbf{W}_\mu^a - i\frac{\mathbf{g}_2}{2} \mathbf{B}_\mu \right) \Phi \right|^2 \\ &= \frac{1}{2} \left| \begin{pmatrix} \partial_\mu - \frac{i}{2}(\mathbf{g}_2 \mathbf{W}_\mu^3 + \mathbf{g}_1 \mathbf{B}_\mu) & -\frac{i\mathbf{g}_2}{2}(\mathbf{W}_\mu^1 - i\mathbf{W}_\mu^2) \\ -\frac{i\mathbf{g}_2}{2}(\mathbf{W}_\mu^1 + i\mathbf{W}_\mu^2) & \partial_\mu + \frac{i}{2}(\mathbf{g}_2 \mathbf{W}_\mu^3 - \mathbf{g}_1 \mathbf{B}_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \\ &= \frac{1}{2} (\partial_\mu H)^2 + \frac{1}{8} \mathbf{g}_2^2 (v+H)^2 |\mathbf{W}_\mu^1 + i\mathbf{W}_\mu^2|^2 + \frac{1}{8} (v+H)^2 |\mathbf{g}_2 \mathbf{W}_\mu^3 - \mathbf{g}_1 \mathbf{B}_\mu|^2 \end{aligned}$$

- Define the new fields \mathbf{W}_μ^\pm and \mathbf{Z}_μ [\mathbf{A}_μ is the orthogonal of \mathbf{Z}_μ]:

$$\mathbf{W}^\pm = \frac{1}{\sqrt{2}} (\mathbf{W}_\mu^1 \mp \mathbf{W}_\mu^2), \quad \mathbf{Z}_\mu = \frac{\mathbf{g}_2 \mathbf{W}_\mu^3 - \mathbf{g}_1 \mathbf{B}_\mu}{\sqrt{\mathbf{g}_2^2 + \mathbf{g}_1^2}}, \quad \mathbf{A}_\mu = \frac{\mathbf{g}_2 \mathbf{W}_\mu^3 + \mathbf{g}_1 \mathbf{B}_\mu}{\sqrt{\mathbf{g}_2^2 + \mathbf{g}_1^2}}$$

$$\text{with } \sin^2 \theta_W \equiv \mathbf{g}_2 / \sqrt{\mathbf{g}_2^2 + \mathbf{g}_1^2} = e / \mathbf{g}_2$$

2. EWSB in SM: mass generation

- And pick up the terms which are bilinear in the fields W^\pm, Z, A :

$$M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_A^2 A_\mu A^\mu$$

\Rightarrow 3 degrees of freedom for W_L^\pm, Z_L and thus M_{W^\pm}, M_Z :

$$M_W = \frac{1}{2} v g_2, \quad M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2}, \quad M_A = 0,$$

with the value of the vev given by: $v = 1/(\sqrt{2}G_F)^{1/2} \sim 246$ GeV.

\Rightarrow The photon stays massless, $U(1)_{\text{QED}}$ is preserved.

- For fermion masses, use same doublet field Φ and its conjugate field $\tilde{\Phi} = i\tau_2 \Phi^*$ and introduce \mathcal{L}_{Yuk} which is invariant under $SU(2) \times U(1)$:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} &= -f_e (\bar{e}, \bar{\nu})_L \Phi e_R - f_d (\bar{u}, \bar{d})_L \Phi d_R - f_u (\bar{u}, \bar{d})_L \tilde{\Phi} u_R + \dots \\ &= -\frac{1}{\sqrt{2}} f_e (\bar{\nu}_e, \bar{e}_L) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R \dots = -\frac{1}{\sqrt{2}} (v + H) \bar{e}_L e_R \dots \\ &\Rightarrow m_e = \frac{f_e v}{\sqrt{2}}, \quad m_u = \frac{f_u v}{\sqrt{2}}, \quad m_d = \frac{f_d v}{\sqrt{2}} \end{aligned}$$

With same Φ , we have generated gauge boson and fermion masses, while preserving $SU(2) \times U(1)$ gauge symmetry (which is now hidden)!

What about the residual degree of freedom?

2. EWSB in SM: the Higgs boson

It will correspond to the physical spin-zero scalar Higgs particle, H.

The kinetic part of H field, $\frac{1}{2}(\partial_\mu \mathbf{H})^2$, comes from $|\mathbf{D}_\mu \Phi|^2$ term.

Mass and self-interaction part from $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$:

$$V = \frac{\mu^2}{2}(\mathbf{0}, \mathbf{v} + \mathbf{H}) \begin{pmatrix} 0 \\ \mathbf{v} + \mathbf{H} \end{pmatrix} + \frac{\lambda}{2} |(\mathbf{0}, \mathbf{v} + \mathbf{H}) \begin{pmatrix} 0 \\ \mathbf{v} + \mathbf{H} \end{pmatrix}|^2$$

Doing the exercise you find that the Lagrangian containing H is,

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu \mathbf{H})(\partial^\mu \mathbf{H}) - V = \frac{1}{2}(\partial^\mu \mathbf{H})^2 - \lambda \mathbf{v}^2 \mathbf{H}^2 - \lambda \mathbf{v} \mathbf{H}^3 - \frac{\lambda}{4} \mathbf{H}^4$$

The Higgs boson mass is given by: $M_H^2 = 2\lambda \mathbf{v}^2 = -2\mu^2$.

The Higgs triple and quartic self-interaction vertices are:

$$g_{H^3} = 3i M_H^2 / \mathbf{v}, \quad g_{H^4} = 3i M_H^2 / \mathbf{v}^2$$

What about the Higgs boson couplings to gauge bosons and fermions?

They were almost derived previously, when we calculated the masses:

$$\mathcal{L}_{M_V} \sim M_V^2 (1 + \mathbf{H}/\mathbf{v})^2, \quad \mathcal{L}_{m_f} \sim -m_f (1 + \mathbf{H}/\mathbf{v})$$

$$\Rightarrow g_{Hff} = im_f / \mathbf{v}, \quad g_{HVV} = -2i M_V^2 / \mathbf{v}, \quad g_{HHVV} = -2i M_V^2 / \mathbf{v}^2$$

Since \mathbf{v} is known, the only free parameter in the SM is M_H or λ .

2. EWSB in SM: W/Z/H at high energies

Propagators of gauge and Goldstone bosons in a general ζ gauge:

$$\begin{array}{l}
 \begin{array}{c} \text{wavy line} \\ \longrightarrow q \end{array} \quad \frac{-i}{q^2 - M_V^2 + i\epsilon} \left[g_{\mu\nu} + (\zeta - 1) \frac{q_\mu q_\nu}{q^2 - \zeta M_V^2} \right] \quad \begin{array}{l} \zeta = \infty: \text{Landau gauge} \\ \zeta = 1: \text{'t Hooft-Feynman} \end{array} \\
 \omega^\pm, \omega^0 : \quad \begin{array}{c} \text{dashed line} \\ \longrightarrow q \end{array} \quad \frac{-i}{q^2 - \zeta M_V^2 + i\epsilon}
 \end{array}$$

- In unitary gauge, Goldstones do not propagate and gauge bosons have usual propagators of massive spin-1 particles (old IVB theory).
- At very high energies, $s \gg M_V^2$, an approximation is $M_V \sim 0$. The V_L components of V can be replaced by the Goldstones, $V_L \rightarrow w$.
- In fact, **the electroweak equivalence theorem** tells that at high energies, massive vector bosons are equivalent to Goldstones. In VV scattering e.g. $A(V_L^1 \dots V_L^n \rightarrow V_L^1 \dots V_L^{n'}) = (i)^n (-i)^{n'} A(w^1 \dots w^n \rightarrow w^1 \dots w^{n'})$

Thus, we simply replace V by w in the scalar potential and use w :

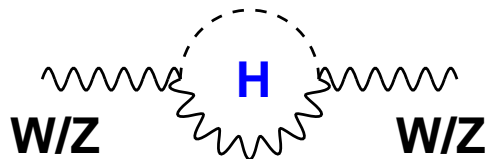
$$V = \frac{M_H^2}{2v} (\mathbf{H}^2 + w_0^2 + 2w^+ w^-) \mathbf{H} + \frac{M_H^2}{8v^2} (\mathbf{H}^2 + w_0^2 + 2w^+ w^-)^2$$

3. Constraints on M_H

First, there were constraints from pre-LHC experiments....

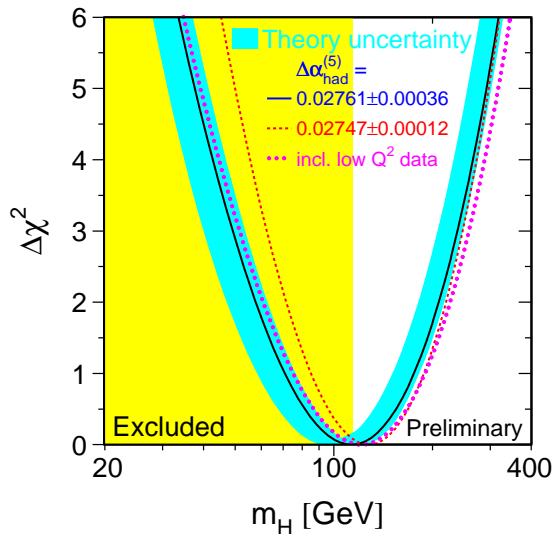
Indirect Higgs searches:

H contributes to RC to W/Z masses:



Fit the EW precision measurements:

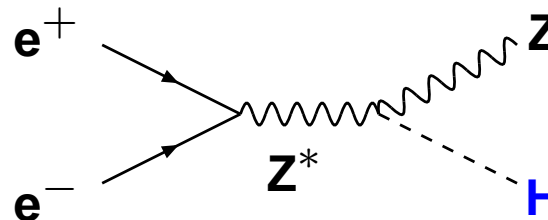
we obtain $M_H = 92^{+34}_{-26}$ GeV, or



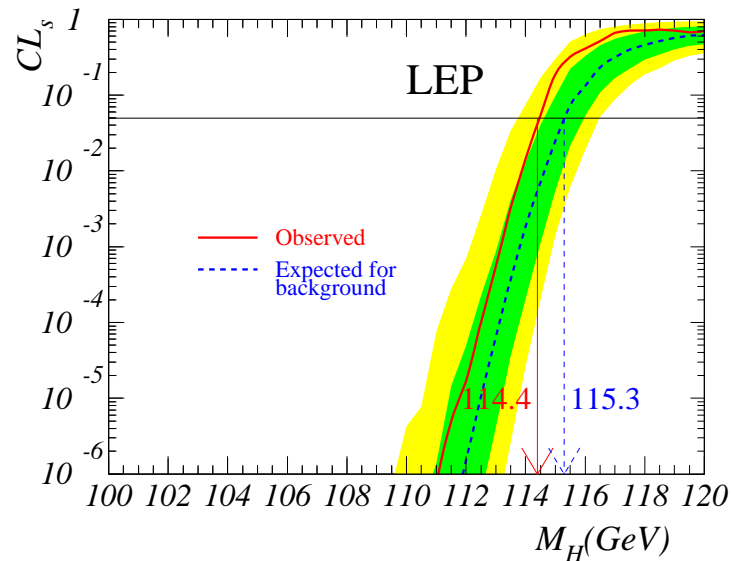
$M_H \lesssim 160$ GeV at 95% CL

Direct searches at colliders:

H looked for in $e^+e^- \rightarrow ZH$



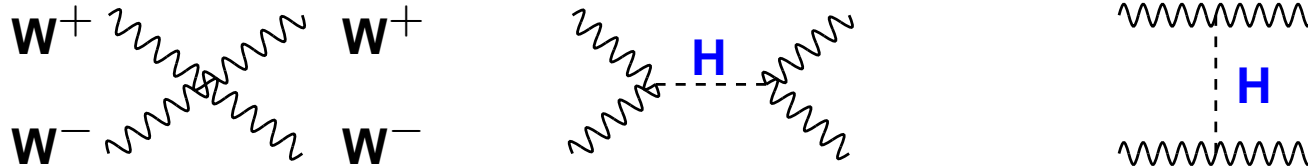
$M_H > 114.4$ GeV @95% CL



Tevatron $M_H \neq 160 - 175$ GeV

3. Constraints on M_H : perturbative unitarity

Scattering of massive gauge bosons $V_L V_L \rightarrow V_L V_L$ at high-energy



Because w interactions increase with energy (q^μ terms in V propagator),
 $s \gg M_W^2 \Rightarrow \sigma(w^+ w^- \rightarrow w^+ w^-) \propto s \Rightarrow$ **unitarity violation possible!**

Decomposition into partial waves and choose $J=0$ for $s \gg M_W^2$:

$$a_0 = -\frac{M_H^2}{8\pi v^2} \left[1 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{s} \log \left(1 + \frac{s}{M_H^2} \right) \right]$$

For unitarity to be fulfilled, we need the condition $|\text{Re}(a_0)| < 1/2$.

- At high energies, $s \gg M_H^2, M_W^2$, we have: $a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$

$$\text{unitarity} \Rightarrow M_H \lesssim 870 \text{ GeV} \quad (M_H \lesssim 710 \text{ GeV})$$

- For a very heavy or no Higgs boson, we have: $a_0 \xrightarrow{s \ll M_H^2} -\frac{s}{32\pi v^2}$

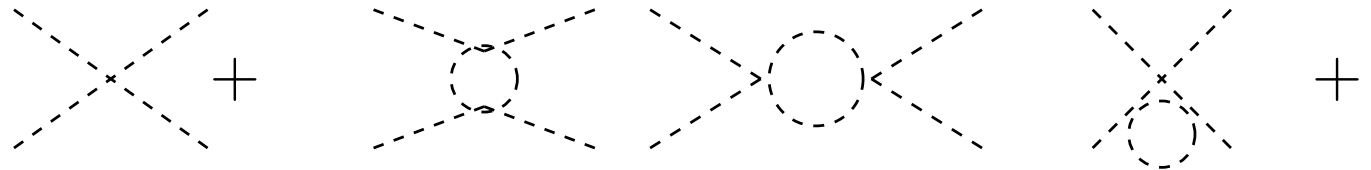
$$\text{unitarity} \Rightarrow \sqrt{s} \lesssim 1.7 \text{ TeV} \quad (\sqrt{s} \lesssim 1.2 \text{ TeV})$$

Otherwise (strong?) New Physics should appear to restore unitarity.

3. Constraints on M_H : triviality

The quartic coupling of the Higgs boson $\lambda (\propto M_H^2)$ increases with energy.

If the Higgs is heavy: the H contributions to λ is by far dominant



The RGE evolution of λ with Q^2 and its solution are given by:

$$\frac{d\lambda(Q^2)}{dQ^2} = \frac{3}{4\pi^2} \lambda^2(Q^2) \Rightarrow \lambda(Q^2) = \lambda(v^2) \left[1 - \frac{3}{4\pi^2} \lambda(v^2) \log \frac{Q^2}{v^2} \right]^{-1}$$

- If $Q^2 \ll v^2$, $\lambda(Q^2) \rightarrow 0_+$: the theory is trivial (no interaction).
- If $Q^2 \gg v^2$, $\lambda(Q^2) \rightarrow \infty$: Landau pole at $Q = v \exp\left(\frac{4\pi^2 v^2}{M_H^2}\right)$.

The SM is valid only at scales before λ becomes infinite:

$$\text{If } \Lambda_C = M_H, \lambda \lesssim 4\pi \Rightarrow M_H \lesssim 650 \text{ GeV}$$

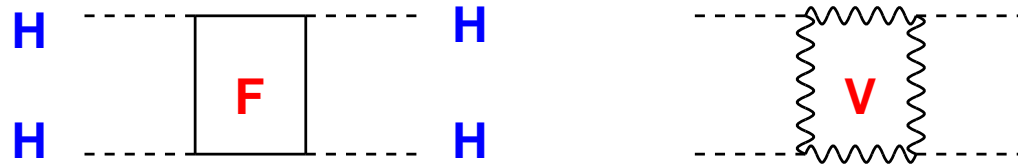
(comparable to results obtained with simulations on the lattice!)

$$\text{If } \Lambda_C = M_P, \lambda \lesssim 4\pi \Rightarrow M_H \lesssim 180 \text{ GeV}$$

(comparable to exp. limit if SM extrapolated to GUT/Planck scales)

3. Constraints on M_H : vacuum stability

The top quark and gauge bosons also contribute to the evolution of λ .
(contributions dominant (over that of H itself) at low M_H values)



The RGE evolution of the coupling at one-loop is given by

$$\lambda(Q^2) = \lambda(v^2) + \frac{1}{16\pi^2} \left[-12 \frac{m_t^4}{v^4} + \frac{3}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log \frac{Q^2}{v^2}$$

If λ is small (H is light), top loops might lead to $\lambda(0) < \lambda(v)$:

v is not the minimum of the potential and EW vacuum is unstable.

\Rightarrow Impose that the coupling λ stays always positive:

$$\lambda(Q^2) > 0 \Rightarrow M_H^2 > \frac{v^2}{8\pi^2} \left[-12 \frac{m_t^4}{v^4} + \frac{3}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log \frac{Q^2}{v^2}$$

Very strong constraint: $Q = \Lambda_C \sim 1 \text{ TeV} \Rightarrow M_H \gtrsim 70 \text{ GeV}$

(we understand why we have not observed the Higgs before LEP2...)

If SM up to high scales: $Q = M_P \sim 10^{18} \text{ GeV} \Rightarrow M_H \gtrsim 130 \text{ GeV}$

3. Constraints on M_H : triviality+stability

Combine the two constraints and include all possible effects:

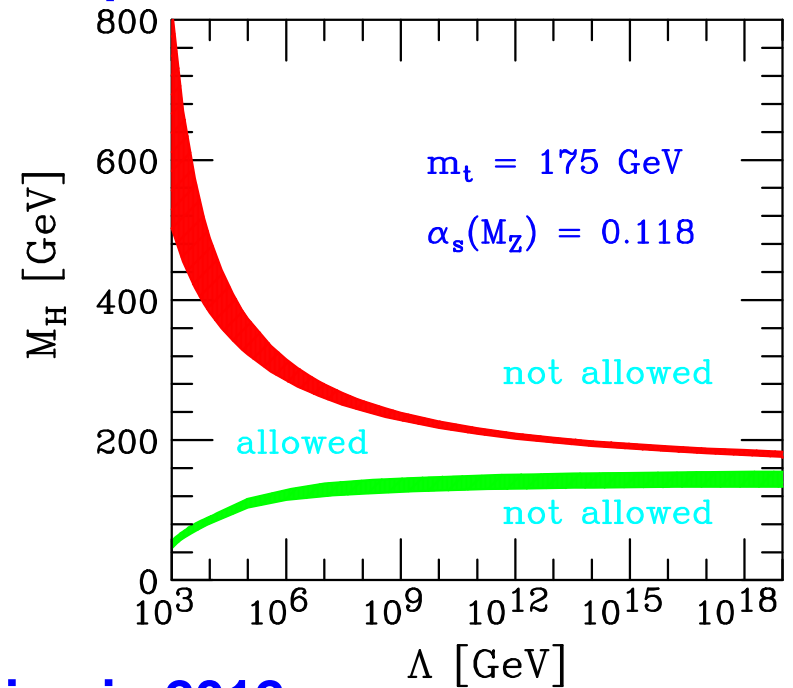
- corrections at two loops
- theoretical+exp. errors
- other refinements ...

$$\Lambda_C \approx 1 \text{ TeV} \Rightarrow 70 \lesssim M_H \lesssim 700 \text{ GeV}$$

$$\Lambda_C \approx M_{\text{Pl}} \Rightarrow 130 \lesssim M_H \lesssim 180 \text{ GeV}$$

Cabibbo, Maiani, Parisi, Petronzio

Hambye, Riesselmann



A more up-to date (full two loop) calculation in 2012:

Degrassi et al., Berzukov et al.

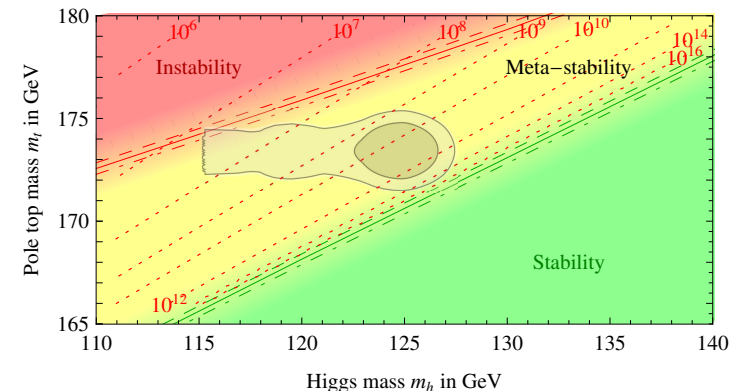
At 2-loop for $m_t^{\text{pole}} = 173.1 \text{ GeV}$:

fully stable vacuum $M_H \gtrsim 129 \text{ GeV}$...

but vacuum metastable below!

metastability OK: unstable vacuum

but very long lived $\tau_{\text{tunnel}} \gtrsim \tau_{\text{univ}}$...

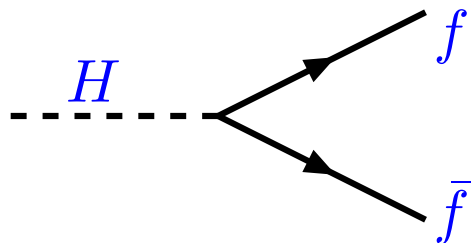


4. Higgs decays

Higgs couplings proportional to particle masses: once M_H is fixed,

- the profile of the Higgs boson is determined and its decays fixed,
- the Higgs has tendency to decay into heaviest available particle.

Higgs decays into fermions:



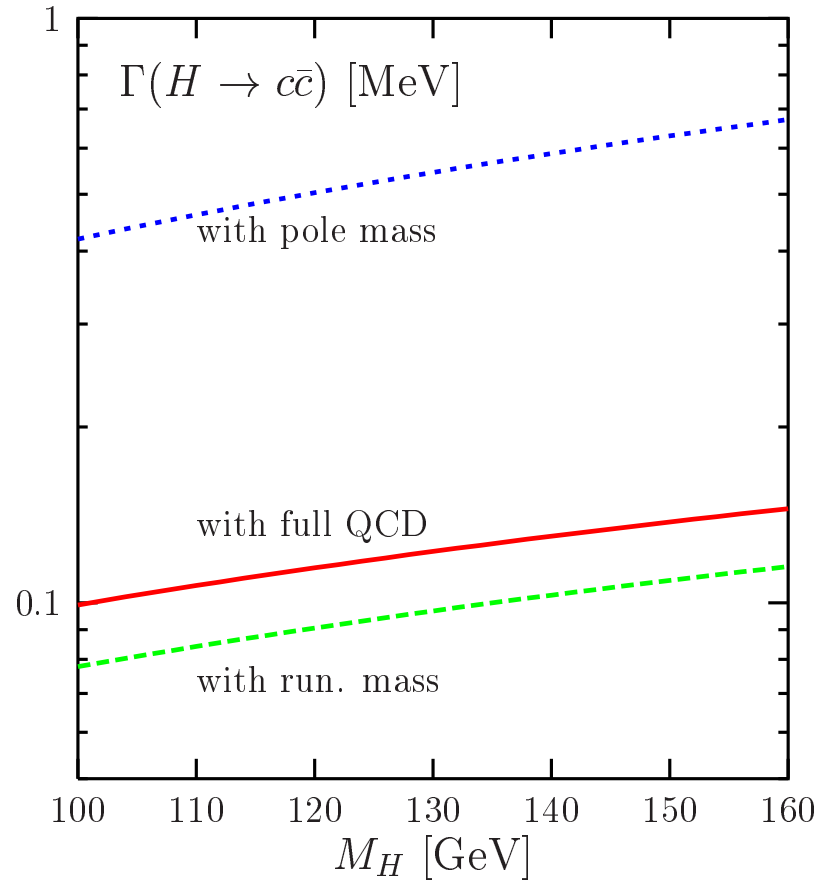
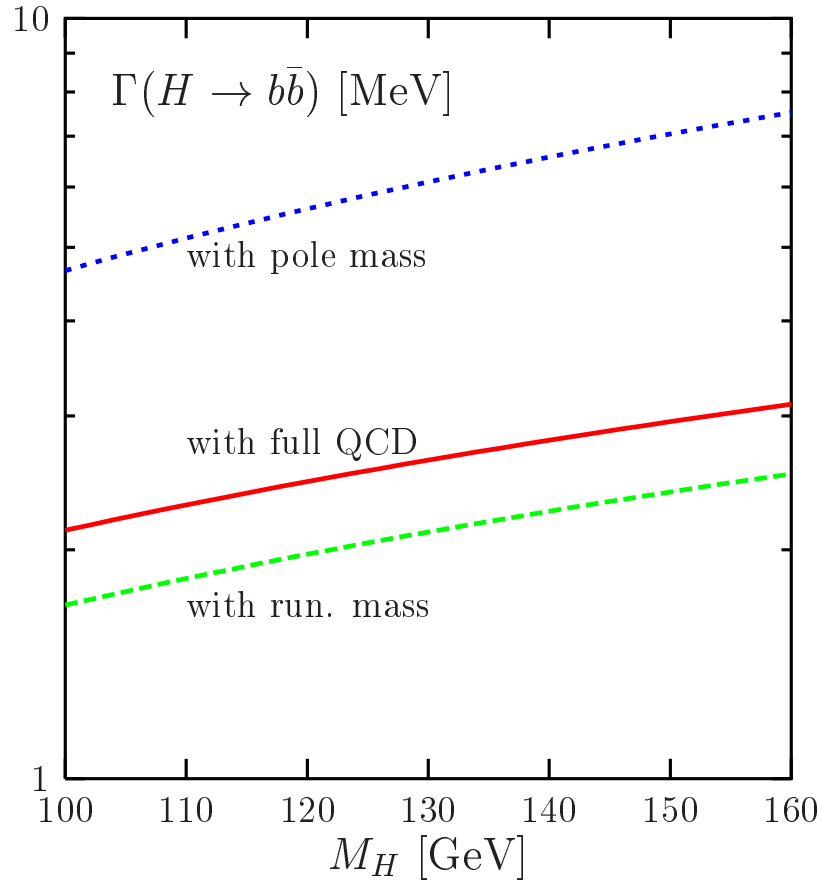
$$\Gamma_{\text{Born}}(\text{H} \rightarrow f\bar{f}) = \frac{G_\mu N_c}{4\sqrt{2}\pi} M_H m_f^2 \beta_f^3$$

$$\beta_f = \sqrt{1 - 4m_f^2/M_H^2} : f \text{ velocity}$$

$$N_c = \text{color number}$$

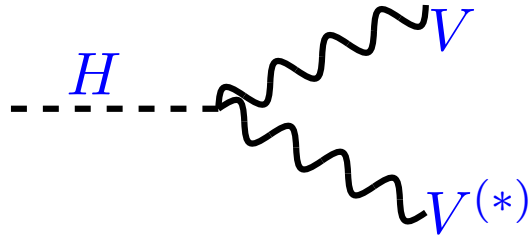
- Only $b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$, $\mu^+\mu^-$ for $M_H < 350$ GeV, also $t\bar{t}$ beyond.
- $\Gamma \propto \beta^3$: H is CP-even scalar particle ($\propto \beta$ for pseudoscalar H).
- Decay width grows as M_H : moderate growth....
- QCD RC: $\Gamma \propto \Gamma_0 [1 - \frac{\alpha_s}{\pi} \log \frac{M_H^2}{m_q^2}] \Rightarrow$ very large: absorbed/summed using running masses at scale M_H : $m_b(M_H^2) \sim \frac{2}{3} m_b^{\text{pole}} \sim 3$ GeV.
- Include also direct QCD corrections (3 loops) and EW (one-loop).

4. Higgs decays: QCD corrections



Partial widths for the decays $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$ as a function of M_H .

4. Higgs decays: decays into gauge bosons



$$\Gamma(H \rightarrow VV) = \frac{G_\mu M_H^3}{16\sqrt{2}\pi} \delta_V \beta_V (1 - 4x + 12x^2)$$

$$x = M_V^2/M_H^2, \beta_V = \sqrt{1 - 4x}$$

$$\delta_W = 2, \delta_Z = 1$$

- For a very heavy Higgs boson:

$$\Gamma(H \rightarrow WW) = 2 \times \Gamma(H \rightarrow ZZ); \Rightarrow \text{BR}(WW) \sim \frac{2}{3}, \text{BR}(ZZ) \sim \frac{1}{3}$$

$$\Gamma(H \rightarrow WW + ZZ) \propto \frac{1}{2} \frac{M_H^3}{(1 \text{ TeV})^3} \text{ because of contributions of } V_L:$$

heavy Higgs is obese: width very large, comparable to M_H at 1 TeV.

EW radiative corrections from scalars large because $\propto \lambda = \frac{M_H^2}{2v^2}$.

- For a light Higgs boson:

$M_H < 2M_V$: possibility of off-shell V decays, $H \rightarrow VV^* \rightarrow Vff$.

Virtuality and addition EW cplg compensated by large g_{HVV} vs g_{Hbb} .

In fact: for $M_H \gtrsim 130 \text{ GeV}$, $H \rightarrow WW^*$ dominates over $H \rightarrow b\bar{b}$

4. Higgs decays: decays into gauge bosons

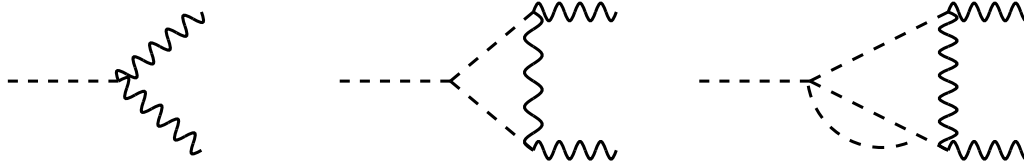
Electroweak radiative corrections to $H \rightarrow VV$:

Using the low-energy/equivalence theorem for $M_H \gg M_V$, Born easy..

$$\Gamma(H \rightarrow ZZ) \sim \Gamma(H \rightarrow W_0 W_0) = \left(\frac{1}{2M_H} \right) \left(\frac{2!M_H^2}{2v} \right)^2 \frac{1}{2} \left(\frac{1}{8\pi} \right) \rightarrow \frac{M_H^3}{32\pi v^2}$$

$H \rightarrow WW$: remove statistical factor: $\Gamma(H \rightarrow W^+ W^-) \simeq 2\Gamma(H \rightarrow ZZ)$.

Include now the one- and two-loop EW corrections from H/W/Z only:



$$\Gamma_{H \rightarrow VV} \simeq \Gamma_{\text{Born}} \left[1 + 3\hat{\lambda} + 62\hat{\lambda}^2 + \mathcal{O}(\hat{\lambda}^3) \right] ; \quad \hat{\lambda} = \lambda/(16\pi^2)$$

$M_H \sim \mathcal{O}(10 \text{ TeV}) \Rightarrow$ one-loop term = Born term.

$M_H \sim \mathcal{O}(1 \text{ TeV}) \Rightarrow$ one-loop term = two-loop term

\Rightarrow for perturbation theory to hold, one should have $M_H \lesssim 1 \text{ TeV}$.

Approx. same result from the calculation of the fermionic Higgs decays:

$$\Gamma_{H \rightarrow ff} \simeq \Gamma_{\text{Born}} \left[1 + 2\hat{\lambda} - 32\hat{\lambda}^2 + \mathcal{O}(\hat{\lambda}^3) \right]$$

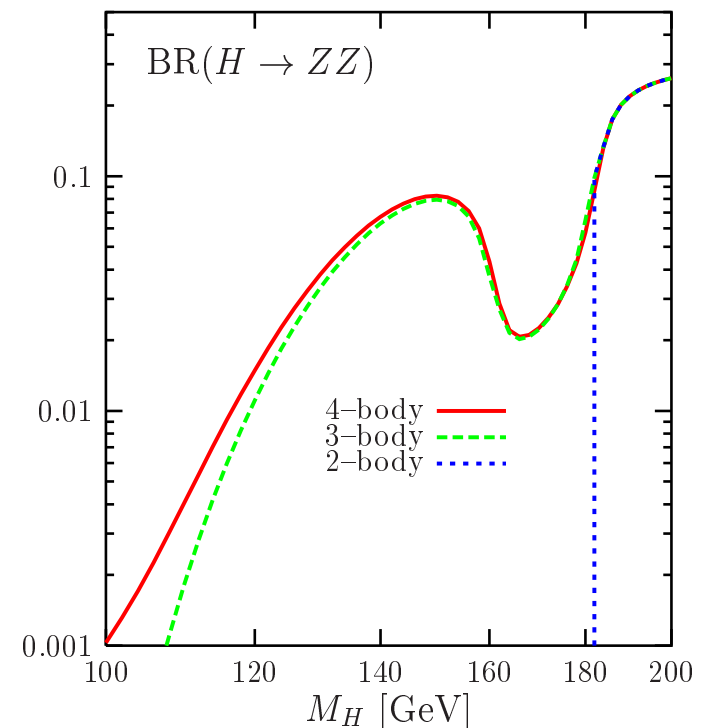
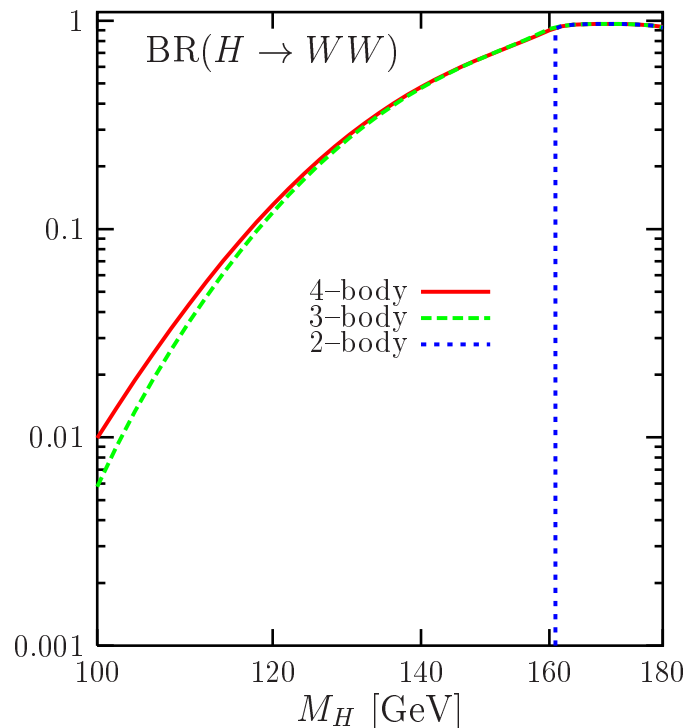
4. Higgs decays: decays into gauge bosons

gd 2+3+4 body decay calculation of $H \rightarrow V^*V^*$:

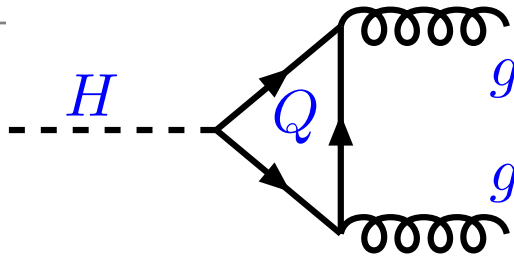
$$\Gamma(H \rightarrow V^*V^*) = \frac{1}{\pi^2} \int_0^{M_H^2} \frac{dq_1^2 M_V \Gamma_V}{(q_1^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \int_0^{(M_H - q_1)^2} \frac{dq_2^2 M_V \Gamma_V}{(q_2^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \Gamma_0$$

$$\lambda(\mathbf{x}, \mathbf{y}; \mathbf{z}) = (1 - \mathbf{x}/\mathbf{z} - \mathbf{y}/\mathbf{z})^2 - 4\mathbf{x}\mathbf{y}/\mathbf{z}^2 \text{ with } \delta_{W/Z} = 2/1 \text{ and}$$

$$\Gamma_0 = \frac{G_\mu M_H^3}{16\sqrt{2}\pi} \delta_V \sqrt{\lambda(q_1^2, q_2^2; M_H^2)} \left[\lambda(q_1^2, q_2^2; M_H^2) + \frac{12q_1^2 q_2^2}{M_H^4} \right]$$



4. Higgs decays: decays into gluons



$$\Gamma(H \rightarrow gg) = \frac{G_\mu \alpha_s^2 M_H^3}{36 \sqrt{2} \pi^3} \left| \frac{3}{4} \sum_Q A_{1/2}^H(\tau_Q) \right|^2$$

$$A_{1/2}^H(\tau) = 2[\tau + (\tau - 1)f(\tau)] \tau^{-2}$$

$$f(\tau) = \arcsin^2 \sqrt{\tau} \text{ for } \tau = M_H^2/4m_Q^2 \leq 1$$

- Gluons massless and Higgs has no color: must be a loop decay.
- For $m_Q \rightarrow \infty$, $\tau_Q \sim 0 \Rightarrow A_{1/2} = \frac{4}{3} = \text{constant}$ and Γ is finite!

Width counts the number of strong inter. particles coupling to Higgs!

- In SM: only top quark loop relevant, b-loop contribution $\lesssim 5\%$.
- Loop decay but QCD and top couplings: comparable to cc, $\tau\tau$.
- Approximation $m_Q \rightarrow \infty/\tau_Q = 1$ valid for $M_H \lesssim 2m_t = 350 \text{ GeV}$.

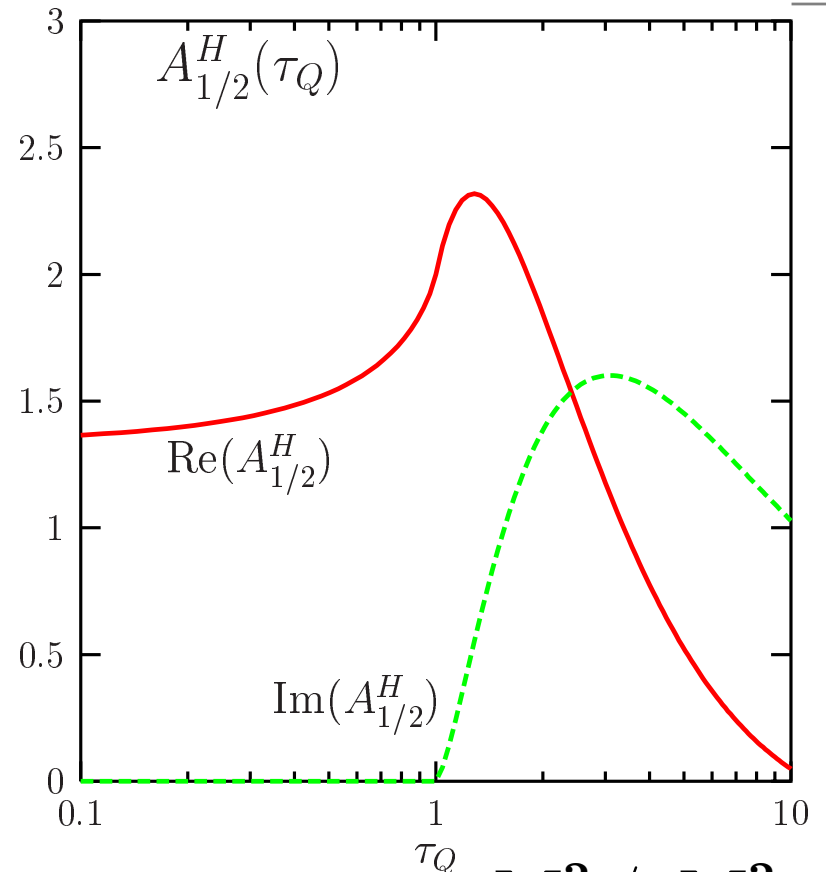
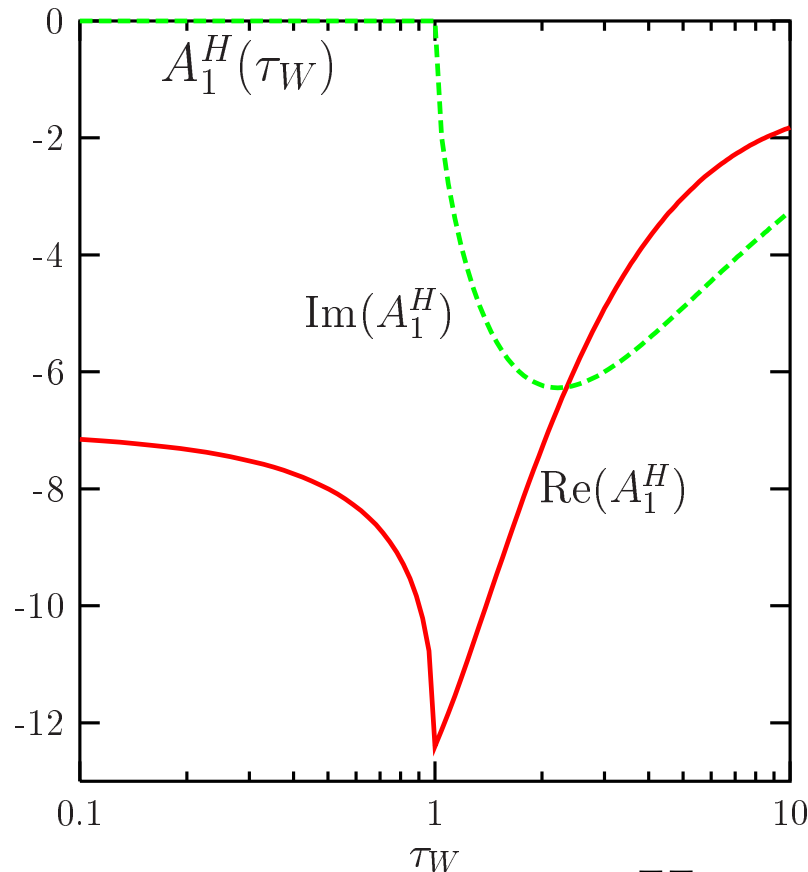
Good approximation in decay: include only t-loop with $m_Q \rightarrow \infty$. But:

- Very large QCD RC: the two- and three-loops have to be included:

$$\Gamma = \Gamma_0 \left[1 + 18 \frac{\alpha_s}{\pi} + 156 \frac{\alpha_s^2}{\pi^2} \right] \sim \Gamma_0 [1 + 0.7 + 0.3] \sim 2\Gamma_0$$

- Reverse process $gg \rightarrow H$ very important for Higgs production in pp!

4. Higgs decays: loop form factors

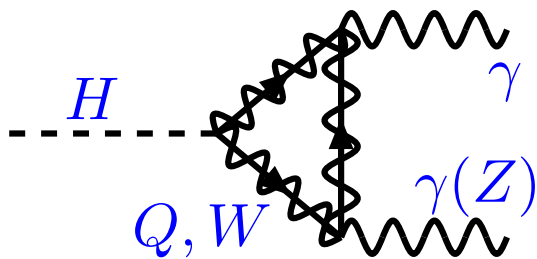


W and fermion amplitudes in $\mathbf{H} \rightarrow \gamma\gamma$ as function of $\tau_i = \mathbf{M}_H^2/4\mathbf{M}_i^2$.

We could repeat the calculation of $\mathbf{H} \rightarrow \gamma\gamma$ and check that Barroso+Pulido+Romao (1986) and Higgs Hunters Guide were correct??...

Trick for an easy calculation: low energy theorem for $\mathbf{M}_H \ll \mathbf{M}_i$

4. Higgs decays: decays into photons



$$\Gamma = \frac{G_\mu \alpha^2 M_H^3}{128 \sqrt{2} \pi^3} \left| \sum_f N_c e_f^2 A_{\frac{1}{2}}^H(\tau_f) + A_1^H(\tau_W) \right|^2$$

$$A_{\frac{1}{2}}^H(\tau) = 2[\tau + (\tau - 1)f(\tau)] \tau^{-2}$$

$$A_1^H(\tau) = -[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)] \tau^{-2}$$

- Photon massless and Higgs has no charge: must be a loop decay.
- In SM: only W-loop and top-loop are relevant (b-loop too small).
- For $m_i \rightarrow \infty \Rightarrow A_{1/2} = \frac{4}{3}$ and $A_1 = -7$: W loop dominating!
(approximation $\tau_W \rightarrow 0$ valid only for $M_H \lesssim 2M_W$: relevant here!).

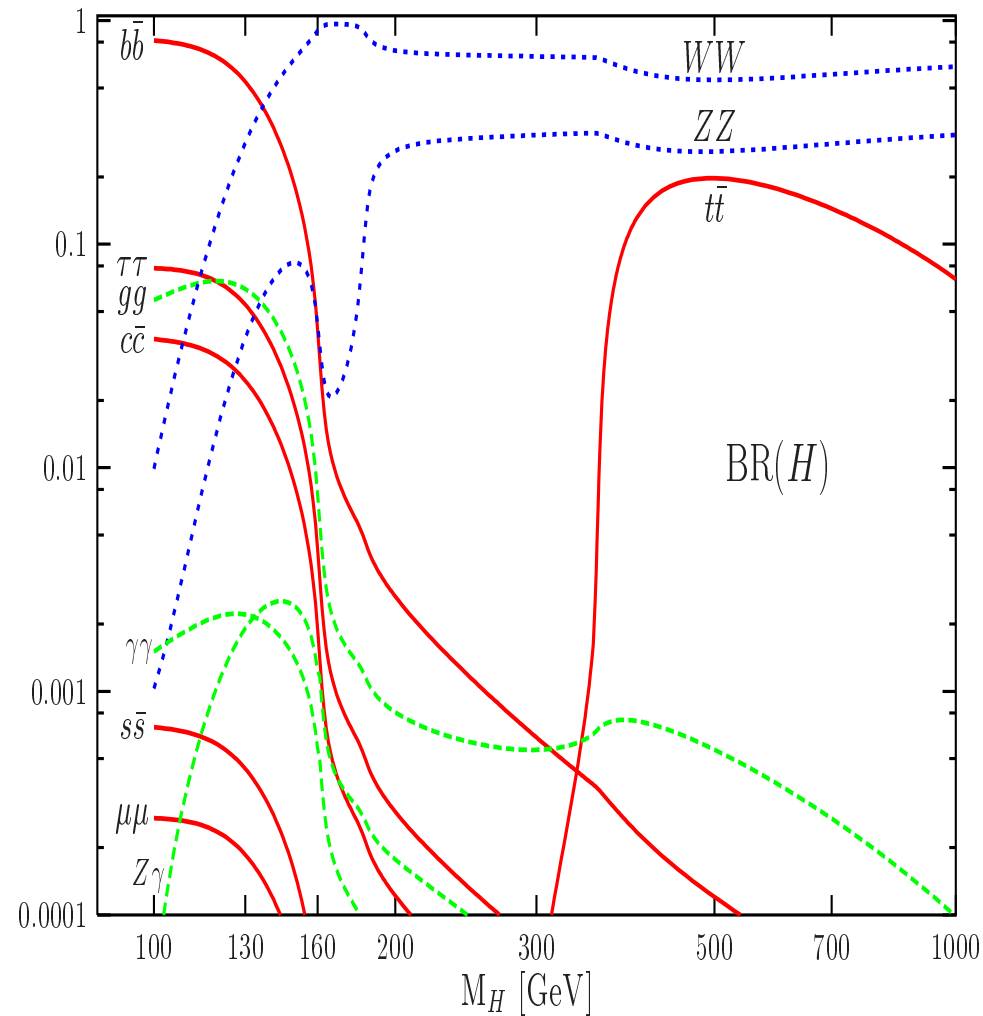
$\gamma\gamma$ width counts the number of charged particles coupling to Higgs!

- Loop decay but EW couplings: very small compared to $H \rightarrow gg$.
- Rather small QCD (and EW) corrections: only of order $\frac{\alpha_s}{\pi} \sim 5\%$.
- Reverse process $\gamma\gamma \rightarrow H$ important for H production in $\gamma\gamma$.
- Same discussions hold qualitatively for loop decay $H \rightarrow Z\gamma$.

4. Higgs decays: branching ratios

Branching ratios: $BR(H \rightarrow X) \equiv \frac{\Gamma(H \rightarrow X)}{\Gamma(H \rightarrow \text{all})}$

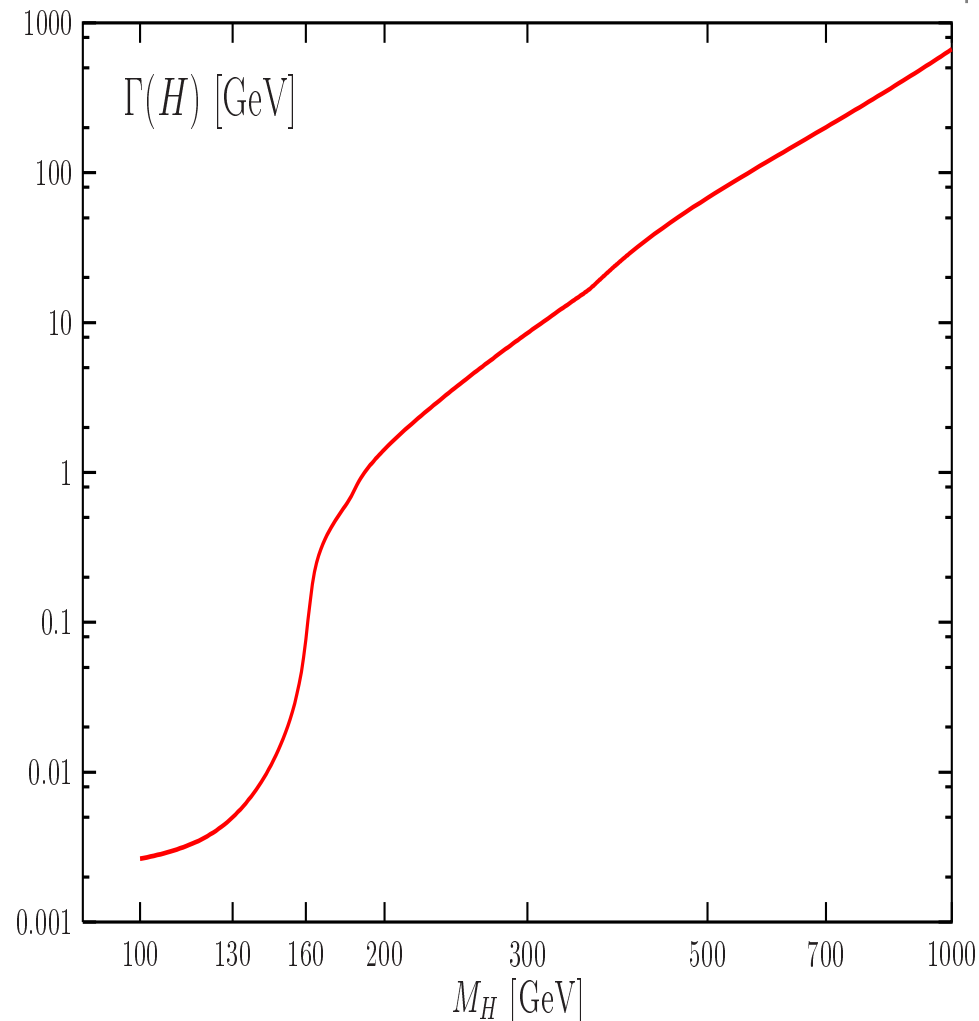
- 'Low mass range', $M_H \lesssim 130 \text{ GeV}$:
 - $H \rightarrow b\bar{b}$ dominant, $BR = 60\text{--}90\%$
 - $H \rightarrow \tau^+\tau^-$, $c\bar{c}$, gg $BR = \text{a few } \%$
 - $H \rightarrow \gamma\gamma, \gamma Z$, $BR = \text{a few permille.}$
- 'High mass range', $M_H \gtrsim 130 \text{ GeV}$:
 - $H \rightarrow WW^*, ZZ^*$ up to $\gtrsim 2M_W$
 - $H \rightarrow WW, ZZ$ above ($BR \rightarrow \frac{2}{3}, \frac{1}{3}$)
 - $H \rightarrow t\bar{t}$ for high M_H ; $BR \lesssim 20\%$.
- Total Higgs decay width:
 - $\mathcal{O}(\text{MeV})$ for $M_H \sim 100 \text{ GeV}$ (small)
 - $\mathcal{O}(\text{TeV})$ for $M_H \sim 1 \text{ TeV}$ (obese).



4. Higgs decays: total width

$$\text{Total decay width: } \Gamma_H \equiv \sum_X \Gamma(H \rightarrow X)$$

- 'Low mass range', $M_H \lesssim 130$ GeV:
 - $H \rightarrow b\bar{b}$ dominant, BR = 60–90%
 - $H \rightarrow \tau^+\tau^-$, $c\bar{c}$, gg BR = a few %
 - $H \rightarrow \gamma\gamma, \gamma Z$, BR = a few permille.
- 'High mass range', $M_H \gtrsim 130$ GeV:
 - $H \rightarrow WW^*, ZZ^*$ up to $\gtrsim 2M_W$
 - $H \rightarrow WW, ZZ$ above (BR $\rightarrow \frac{2}{3}, \frac{1}{3}$)
 - $H \rightarrow t\bar{t}$ for high M_H ; BR $\lesssim 20\%$.
- Total Higgs decay width:
 - $\mathcal{O}(\text{MeV})$ for $M_H \sim 100$ GeV (small)
 - $\mathcal{O}(\text{TeV})$ for $M_H \sim 1$ TeV (obese).



4. Higgs decays: theory uncertainties

However: there are theoretical uncertainties....

- Input quark masses in $H \rightarrow b\bar{b}, c\bar{c}$

$$M_Q^{\text{pole}} \rightarrow \bar{m}_Q(\mu = M_H)$$

$$- \bar{m}_b(M_b) = 4.19^{+0.018}_{-0.006} \text{ GeV}$$

$$- \bar{m}_c(M_c) = 1.27^{+0.007}_{-0.009} \text{ GeV}$$

- Theory+experimental error on α_s :

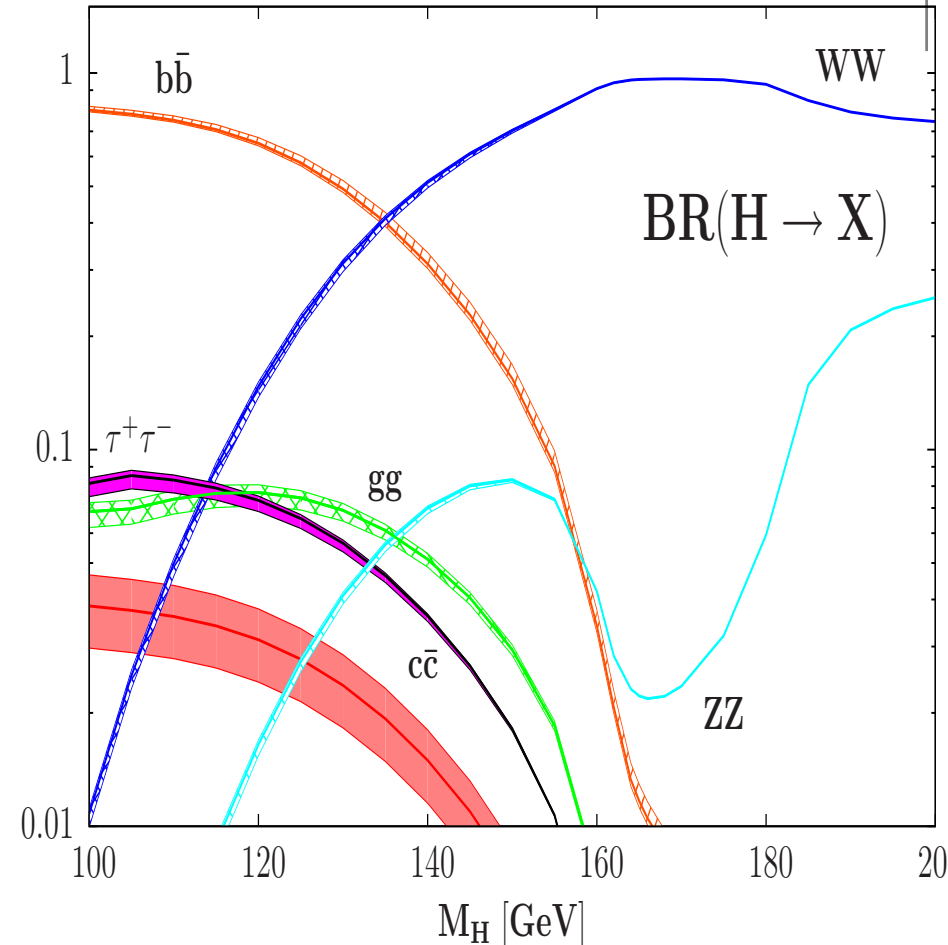
$$\alpha_s(M_Z^2) = 0.1171 \pm 0.0014 \text{ @NNLO}$$

- Scale error: measure of higher orders

$$\frac{1}{2}M_H \leq \mu \leq 2M_H$$

- Scale and α_s errors in $H \rightarrow gg$

$$\Gamma(H \rightarrow gg) \propto \alpha_s^2 + \text{large } \mathcal{O}(\alpha_s^3)$$

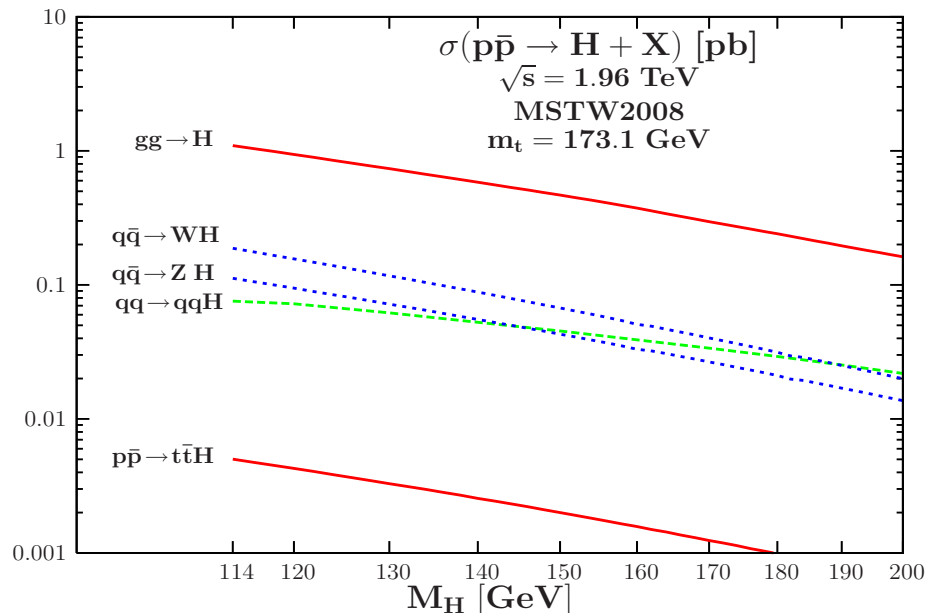
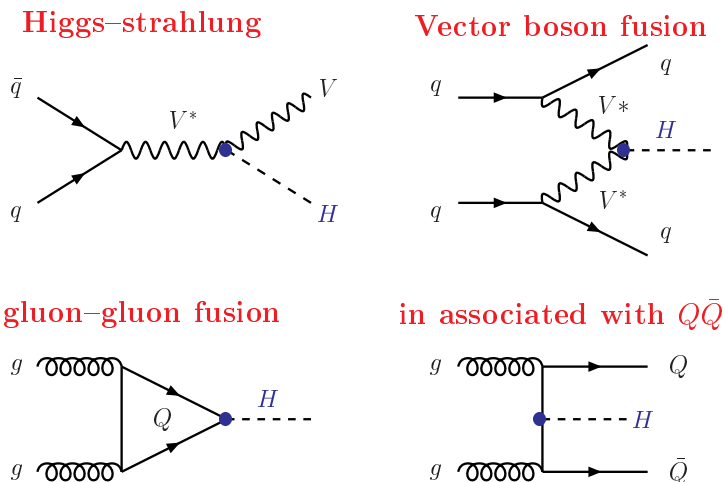


Include all items \Rightarrow non-negligible uncertainties.

esp. for $M_H \approx 120\text{--}150$ GeV: 5–10% for $H \rightarrow b\bar{b}$ and $H \rightarrow WW^*$

5. SM Higgs at hadron colliders

Main Higgs production channels



Large production cross sections

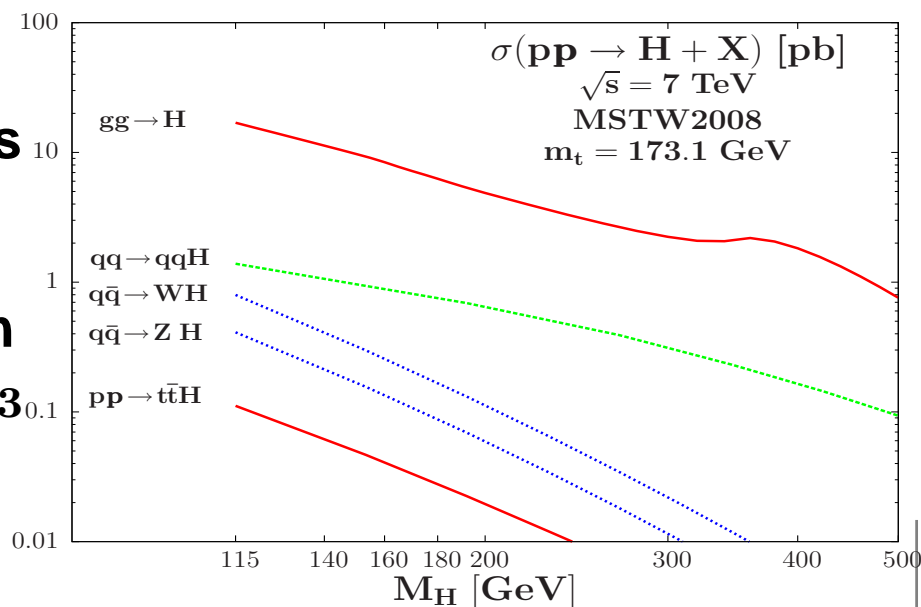
with $gg \rightarrow \text{H}$ by far dominant process

$1 \text{ fb}^{-1} \Rightarrow \mathcal{O}(10^4)$ events @ IHC

$\Rightarrow \mathcal{O}(10^3)$ events @ Tevatron

but eg $\text{BR}(\text{H} \rightarrow \gamma\gamma, \text{ZZ} \rightarrow 4\ell) \approx 10^{-3}$

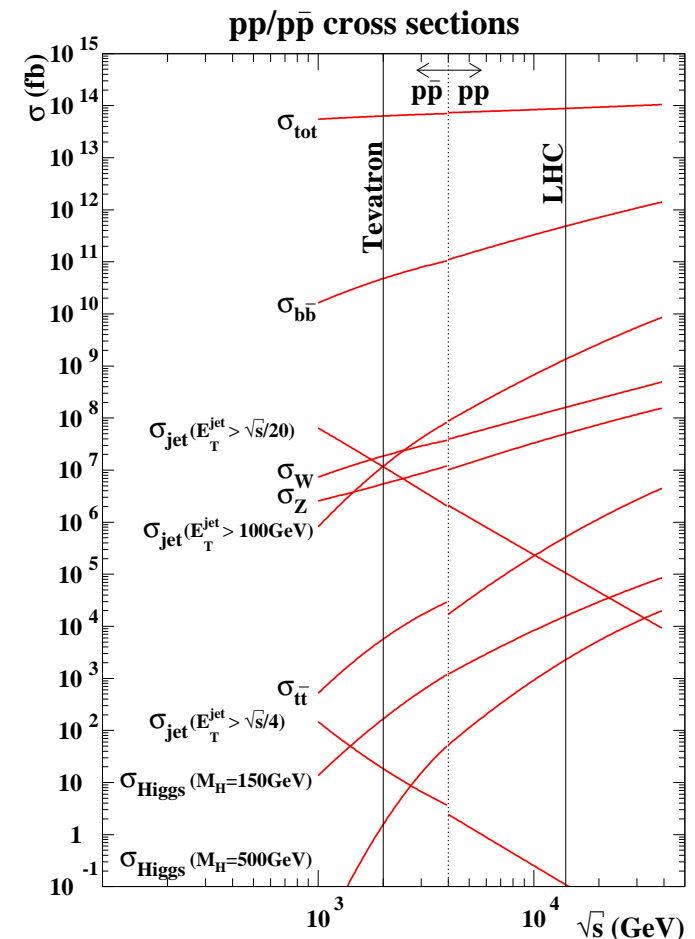
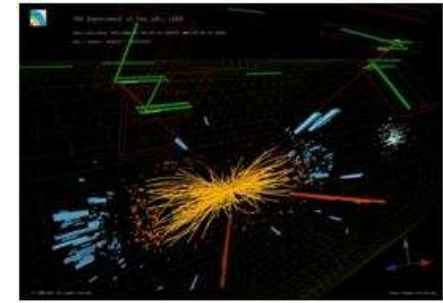
... a small # of events at the end...



5. SM Higgs at hadron colliders: generalities

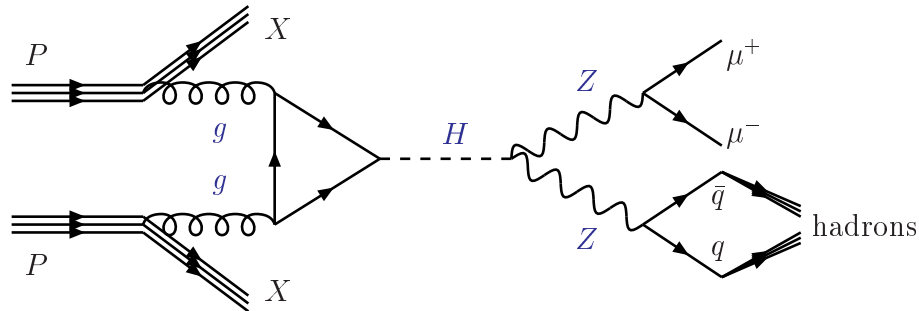
⇒ an extremely challenging task!

- Huge cross sections for QCD processes
 - Small cross sections for EW Higgs signal
 $S/B \gtrsim 10^{10} \Rightarrow$ a needle in a haystack!
 - Need some strong selection criteria:
 - trigger: get rid of uninteresting events...
 - select clean channels: $H \rightarrow \gamma\gamma, VV \rightarrow \ell\ell$
 - use specific kinematic features of Higgs
 - Combine # decay/production channels (and eventually several experiments...)
 - Have a precise knowledge of S and B rates (higher orders can be factor of 2! see later)
 - Gigantic experimental + theoretical efforts (more than 30 years of very hard work!)
- For a flavor of how it is complicated from the theory side: a look at the $gg \rightarrow H$ case



5. SM Higgs at hadron colliders: generalities

Example of process at LHC to see how things work: $gg \rightarrow H$



$$N_{\text{ev}} = \mathcal{L} \times P(g/p) \times \hat{\sigma}(gg \rightarrow H) \times B(H \rightarrow ZZ) \times B(Z \rightarrow \mu\mu) \times BR(Z \rightarrow qq)$$

For a large number of events, all these numbers should be large!

Two ingredients: hard process (σ , B) and soft process (PDF, hadr).

Factorization theorem! Here discuss production/decay process.

The partonic cross section of the subprocess, $gg \rightarrow H$, is:

$$\hat{\sigma}(gg \rightarrow H) = \int \frac{1}{2\hat{s}} \times \frac{1}{2.8} \times \frac{1}{2.8} |\mathcal{M}_{Hgg}|^2 \frac{d^3\mathbf{p}_H}{(2\pi)^3 2E_H} (2\pi^4) \delta^4(\mathbf{q} - \mathbf{p}_H)$$

Flux factor, color/spin average, matrix element squared, phase space.

Convolute with gluon densities to obtain total hadronic cross section

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \frac{\pi^2 M_H}{8\hat{s}} \Gamma(H \rightarrow gg) g(x_1) g(x_2) \delta(\hat{s} - M_H^2)$$

5. SM Higgs at hadron colliders: generalities

The calculation of σ_{born} is not enough in general at pp colliders: need to include higher order radiative corrections which introduce terms of order $\alpha_s^n \log^m(Q/M_H)$ where Q is either large or small...

- Since α_s is large, these corrections are in general very important.
- Choose a (natural scale) which absorbs/resums the large logs.

Since we truncate pert. series: only NLO/NNLO corrections available.

- The (hope small) not known HO corrections induce a theoretical error.
- The scale variation is a (naive) measure of the HO: must be small.

Also, precise knowledge of σ is not enough: need to calculate some kinematical distributions (e.g. $p_T, \eta, \frac{d\sigma}{dM}$) to distinguish S from B.

In fact, one has to do this for both the signal and background (unless directly measurable from data): the important quantity is $\sigma = \frac{N_S}{\sqrt{N_{\text{bjg}}}}$
 \Rightarrow a lot of theoretical work is needed!

But most complicated thing is to actually see the signal for $S/B \ll 1!$