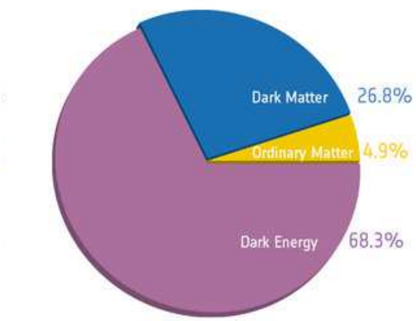
NEW PHYSICS WITHOUT NEW ENERGY SCALE

Oleg RUCHAYSKIY



After Planck

Summer School on Particle Physics
ICTP. Trieste

June 14–19, 2013

Plan of the lectures

Lecture I: BSM problem 1 – **neutrino oscillations**. Type I see-saw Lagrangian and properties of sterile neutrinos. Experimental searches for sterile neutrinos

Lecture II: BSM problem 2 — **baryon asymmetry of the Universe**. Early Universe. Sakharov conditions. Sterile neutrinos in the early Universe. Leptogenesis

Lectire III: BSM problem 3 — **dark matter of the Universe**. What do we know about dark matter. Cold vs. warm dark matter. Decay vs. annihilation.

Lectire IV: bringing the pieces together. New physics without new energy scale. Theory of everything? Theoretical motivations. Fermi and Planck scales. Bigger picture?

The whole course on one slide

- In these lectures I will argue that on the one hand
  - **particle physics suggest the existence of new massive neutral super-weakly interacting particles**

    ... while on the other hand,
  - **cosmology asks for massive, neutral, weaker-than-neutrino-interacting particles**.

- The two can be happily brought together which results in the Standard-Model-like theory free of all the "beyond-the-Standard-Model" problems.

- Detailed experimental predictions follow

List of abbreviations/notations

If there is some professional jargon that I use but do not explain – please, tell me, I'll put it here

SM: Standard Model of elementary particles; particles in the Standard Model Lagrangian.

Active neutrinos: in the context of these lectures the SM neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ will be called sometimes "active" (or "ordinary" neutrinos). We will also mean *charge* (rather than *mass*) eigenstates

BSM: (= "beyond the Standard Model") – phenomena (puzzles) that cannot be explained by the conventional particle physics (Standard Model) coupled to the Eisteinian gravity

BAU: baryon asymmetry of the Universe: absence of primordial anti-matter in the visible part of the Universe

BBN: (= "Big Bang Nucleosynthesis") primordial synthesis of light elements (Deuterium, Helium, Lithium). Abundances of these elements, predicted by the hot Big Bang theory have been confirmed experimentally which serves as the **most distant** clue about the history of the Universe

CMB: cosmic microwave background – relic radiation from recombination of protons and electrons into hydrogen when the Universe was about $380,000$ years old

DM: dark matter – at galactic scales and above the motion of tracers of gravitational potential are not described by Newtonian gravity sourced by the observed matter

Ordinary matter: (also sometimes "**baryons**") in the cosmological context by this name one calls all matter that exists in the form of gas, stars, etc. and is made of the ordinary particles (baryons + electrons).

Sterile neutrinos: (denoted by N or N_I, where $I = 1, 2, \ldots$) the right-handed counterparts of the active neutrinos inert with respect to the SM interactions

PMNS matrix: Pontecorvo-Maki-Nakagawa-Sakata matrix

Planck mass/scale: for particle of such mass Compton wave length equals to its Schwarzschild radius, $M_{\mathrm{Pl}} = 1.2 \times 10^{19}$ GeV.

Indexes: $\alpha, \beta = \{e, \mu, \tau\}$ – flavour indexes; $I = 1, 2, \ldots \mathcal{N}$ – index numbering right-handed fermions.

$\mathcal{M}$**:** Mass matrix of active neutrinos (size 3×3)

m_D**:** (sometimes, m_{Dirac}) – Dirac matrix, mixing active and sterile neutrinos generated by the Yukawa interaction with the Higgs boson (size $3 \times \mathcal{N}$)

M_N**:** (sometimes M_I, where $I = 1, 2, \ldots$) – Majorana mass of sterile neutrino, with good precision coinciding with its propagation mass ($p^2 = M_N^2$)

List of abbreviations/notations

Higgs vev: $v = 174$ GeV. This means that Dirac mass $m_D = Fv$ (rather than $m_D = \frac{1}{\sqrt{2}}Fv$ used e.g. in Peskin & Schroeder)

M_*: Reduced Planck mass used in cosmology ($M_* \equiv \sqrt{\frac{3}{8\pi g_{\text{EFF}}}} M_{\text{Planck}}$)

Unless otherwise stated $\hbar = c = k_{\text{Boltzmann}} = 1$.

Before we start

Ask questions! You can interrupt me, ask after the lecture or at the beginning of the next one.

"*A minute of shame – years of health*" (V.L. Ginzburg)

You can e-mail me at `oleg.ruchayskiy@epfl.ch`

Beyond neutrino masses

Front page of International Herald Tribune

From β-decay to LHC

- Discovered in 1891, radioactivity of the nuclei was the actual birth of the Standard Model of elementary particles
- Exploration of α decay led to the discovery of baryons and mesons, their classification according to SU(2)/SU(3); prediction and discovery of quarks $\Rightarrow$ **QCD**
- Attempts to explain the β decay led to Fermi four-interaction, prediction of neutrino, discovery of parity violation, prediction of intermediate vector boson $\Rightarrow$ **electroweak unification and eventually the Higgs boson**

For example, see the timeline here **Novaes, *"Standard model: An Introduction"* [hep-ph/0001283]**

Why did it take so long?

During this 100+ years the particle physics was never in the position when

– All the predicted particles are discovered

– All the discovered particles/phenomena are accounted for by the model

– The model is mathematically consistent

> The discovery of the Higgs boson was a necessary step in this programme.
>
> Can we finally say that we "understood the radioactivity"?[1]

[1] If you find this question too much of an exaggeration – think of the "ultraviolet catastrophe" of the late XIX century

New challenge ahead?

☺ All the predicted particles are discovered (Higgs was the last of such particles)

☺ The model is mathematically consistent (Within experimental uncertainties on the top mass, the SM can be valid quantum field theory up to the Planck scale)[2]

☹ All the discovered particles/phenomena are accounted for by the model? — **Not at all!**

[2]See lectures by A. Djouadi. See also M. Shaposhnikov's lecture on Saturday.

Why "new physics"?

Why do we think that there should be any "new physics" not described by the Standard Model of particle physics? There are different lines of motivation for that:

Particle physics: neutrino oscillations

Cosmology: particle physics (coupled to Einstein gravity) applied to the Universe as a whole faces the challenges of

- dark matter
- matter-antimatter asymmetry of the Universe
- inflation

Fine-tuning problems: some parameters of the Standard Model Lagrangian are "unnaturally" small

- Gauge hierarchy problem (lecture by M. Shaposhnikov tomorrow)
- Strong CP-problem
- Cosmological constant problem

What kind of new physics do we expect?

- Pre-LHC particle physics community strongly focused on the idea that **new physics should show up at the TeV scale**
  - supersymmetry
  - extra dimensions
  - strong dynamics ("technicolor")

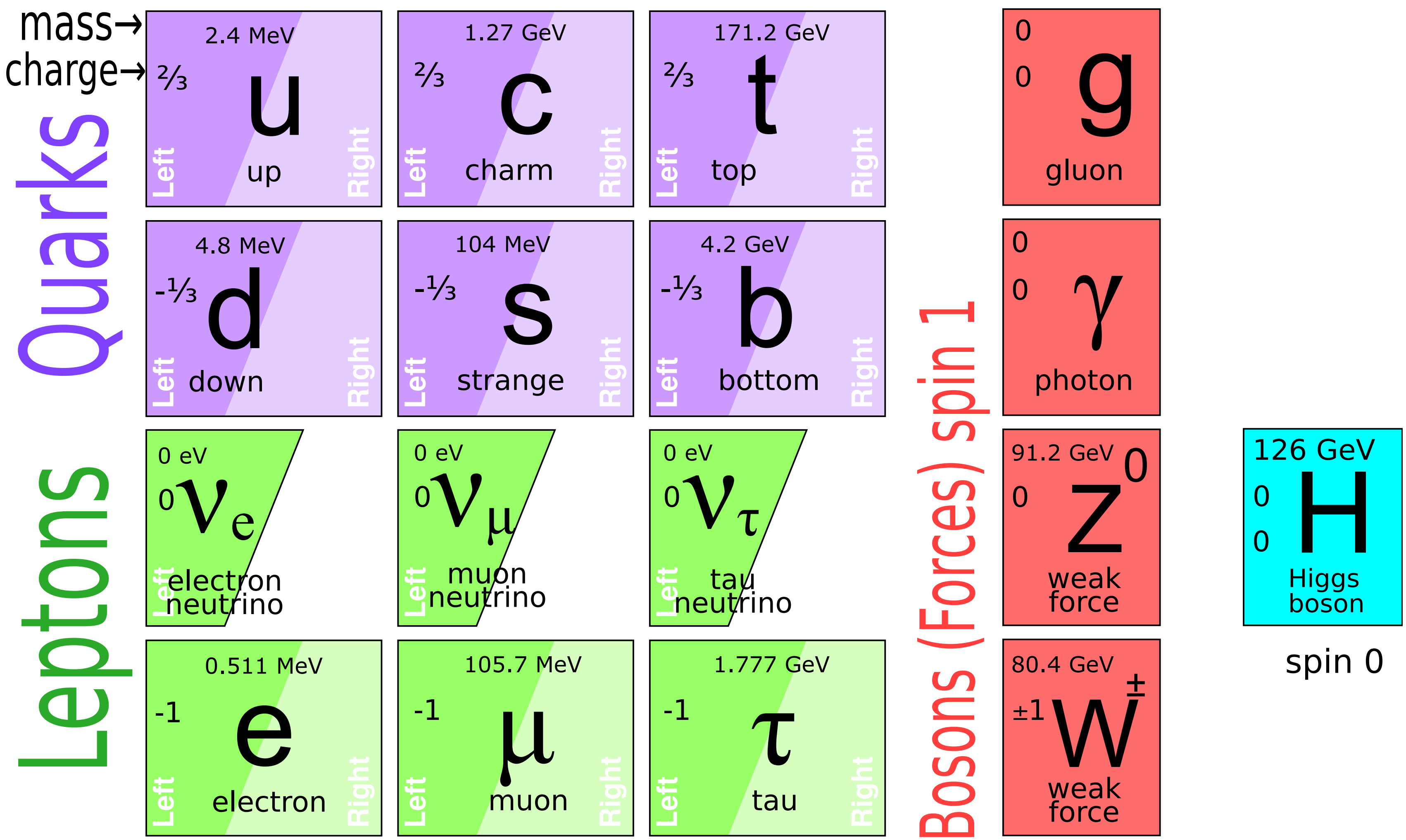
  Many lectures on this school focus on these subjects

- Post-LHC @ 8 TeV community:

  What should we do with **beyond-the-Standard-Model problems** if no new physics will be found at LHC?

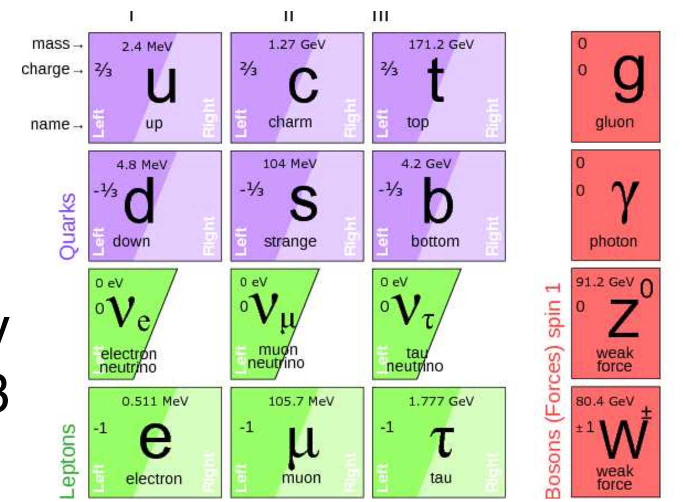
In these lectures we will not follow the road of "new physics at TeV scale", but rather analyze one-by-one the observational BSM problems and attempt to identify a minimal parameter space that encompasses all of them.

Standard Model of particle physics



Standard Model of particle physics

- $SU(3) \times SU(2) \times U(1)$ gauge group



- Three generations of fermions

- $SU(2) \times U(1)$ is spontaneously broken by the Higgs field to massless photon γ and 3 intermediate vector bosons $W^{\pm}$, Z

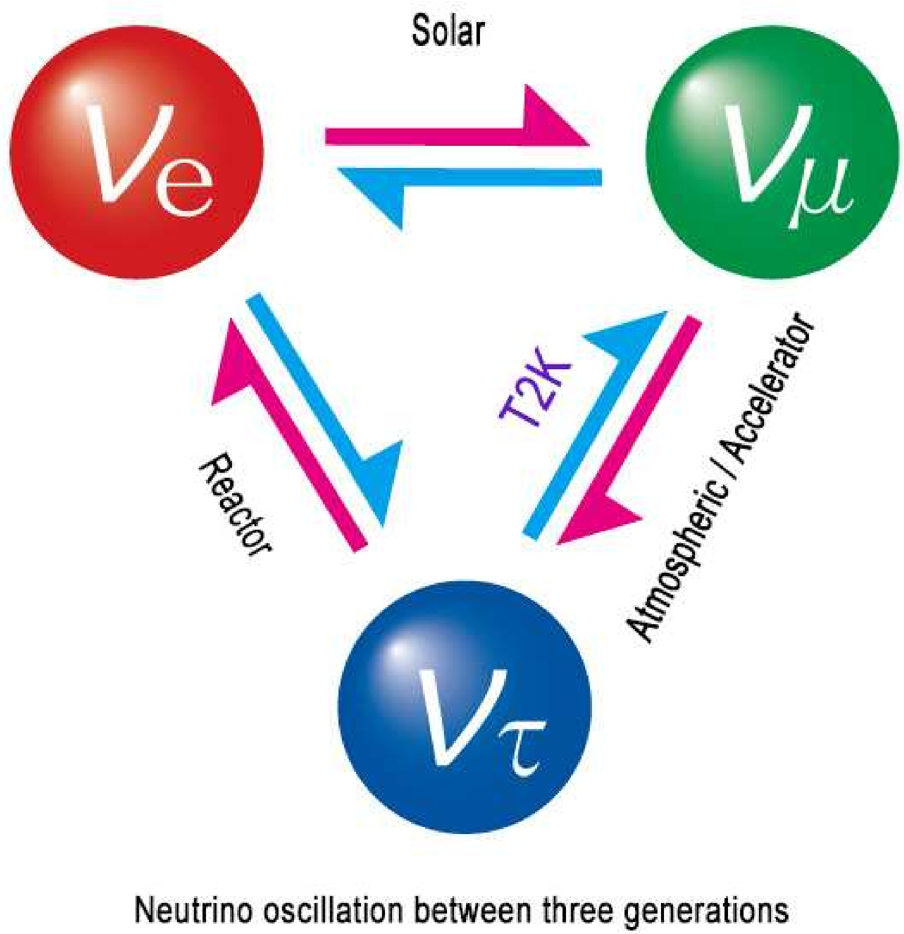
- Chiral structure of the electroweak sector means that
  - Left $SU(2)$ doublets (darker upper-left corners) couple to $W^{\pm}$, Z
  - Right $SU(2)$ singlets (lighter lower-right corners) couple to Z only

- Conservation laws (at zero temperature)[3]
  - Baryon number B
  - Three flavour lepton numbers L_α

---

[3] Of these 4 charges only $Q_\alpha = B/3 - L_\alpha$ are *non-anomalous* i.e. conserved always. Non-conservation of $B + L$ will be important in the early Universe (see Lecture 3).

Neutrino oscillations



From `http://j-parc.jp`

See lectures by Pilar Hernández

Charge-mass state oscillations

- Neutrino oscillations mean that flavour lepton numbers are **not** conserved (only their sum, total lepton number is conserved)

- Is this something we never saw before in the SM?

- Recall: **charge eigenstates** are the states that are created in the acts of interaction:

$$\mathcal{L} = \begin{pmatrix}\bar\psi_1\\ \bar\psi_2\end{pmatrix}\left[i\partial\!\!\!/\begin{pmatrix}1&0\\0&1\end{pmatrix} + A\!\!\!/\begin{pmatrix}Q_1&0\\0&Q_2\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} + \begin{pmatrix}\bar\psi_1\\ \bar\psi_2\end{pmatrix}\begin{pmatrix}m_{11}&m_{12}\\m_{21}&m_{22}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

- **Mass eigenstates** – eigenvectors of the free (propagation) Hamiltonian

$$\mathcal{L} = \begin{pmatrix}\bar\psi_1'\\ \bar\psi_2'\end{pmatrix}\left[i\partial\!\!\!/\begin{pmatrix}1&0\\0&1\end{pmatrix} + \begin{pmatrix}m_1&0\\0&m_2\end{pmatrix}\right]\begin{pmatrix}\psi_1'\\ \psi_2'\end{pmatrix} + \begin{pmatrix}\bar\psi_1'\\ \bar\psi_2'\end{pmatrix}A\!\!\!/\begin{pmatrix}q_{11}&q_{12}\\q_{21}&q_{22}\end{pmatrix}\begin{pmatrix}\psi_1'\\ \psi_2'\end{pmatrix}$$

Charge-mass state oscillations

- Unitary transformation rotates $(\psi_1, \psi_2) \leftrightarrow (\psi'_1, \psi'_2)$:

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}}_{\text{matrix } U}\begin{pmatrix}\psi'_1\\ \psi'_2\end{pmatrix}$$

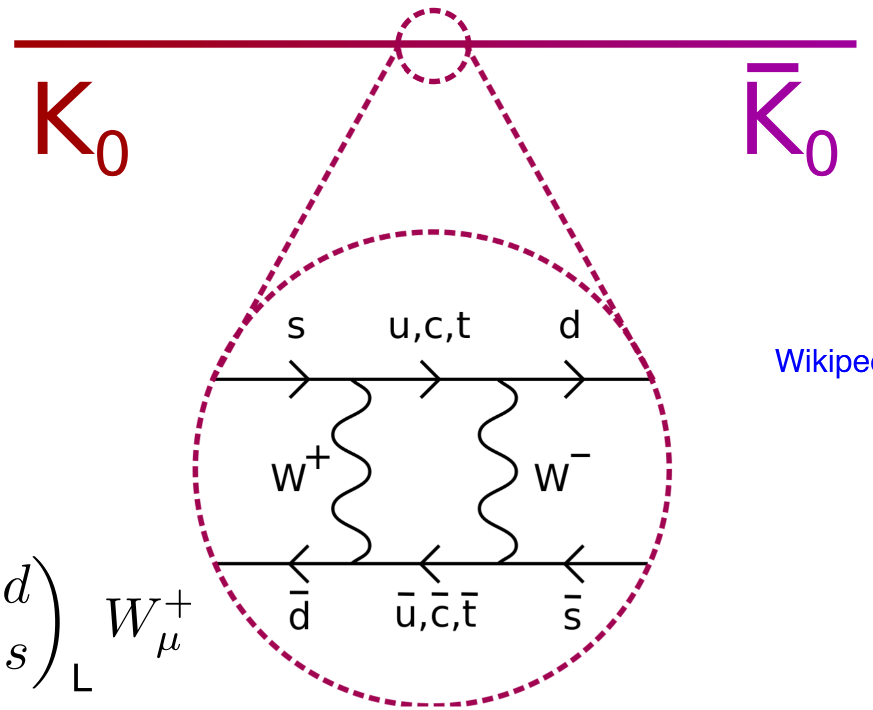
- Example? Neutral kaon oscillations:

– s-quark carries a quantum number (**strangeness**) conserved in strong interactions

– There are two neutral kaons: $|K_0\rangle \equiv |d\bar{s}\rangle$ and $|\bar{K}_0\rangle \equiv |\bar{d}s\rangle$

– In the QCD (propagation) basis the interaction with $W^{\pm}$ is not diagonal:

$$\mathcal{L} = \frac{g}{\sqrt{2}}(\bar{u}, \bar{c})_L \gamma^\mu \begin{pmatrix}\cos\theta_c & \sin\theta_c\\ -\sin\theta_c & \cos\theta_c\end{pmatrix}\begin{pmatrix}d\\ s\end{pmatrix}_L W^+_\mu$$



Wikipedia

- Neutrinos are always created or detected with a well defined flavour ν_e, ν_μ, ν_τ ($W^+ \to e^+ + \nu_e$) – **charge** (or **gauge**, or **flavour**) eigenstates

- Experiments on neutrino oscillations determined two mass differences between neutrino **mass eigenstates**

- This means that there is at least **three** mass states ν_1, ν_2, ν_3

- And there exists a 3×3 unitary transformation U that relates mass eigenstates (ν_1, ν_2, ν_3) to flavour eigenstates (ν_e, ν_μ, ν_τ)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

See lectures by Pilar Hernández at this school

Neutrino mixing matrix (reminder)

- **Recall** that neutrinos $\nu_{e,\mu,\tau}$ couple to $W^{\pm}$ bosons and to charged leptons (neutrinos are part of SU(2) doublet)

$$\mathcal{L}_{\rm CC} = \bar\nu_e \not{W}^+ e^- + \bar\nu_\mu \not{W}^+ \mu^- + \dots$$

  Invariant under $\nu_e \to \nu_e e^{i\alpha}$ simultaneously with $e^- \to e^- e^{i\alpha}$, etc.

- All other terms in the Lagrangian have the form $\bar\psi \not{D} \psi$ or $m\bar\psi\psi$ — i.e. are invariant if $\psi \to \psi e^{i\alpha}$ (here ψ is any of $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau$)

- Additionally, we can rotate each of the $\nu_{1,2,3}$ by an independent phase

- 5 of 9 parameters of the mixing matrix U can be absorbed in the redefinitions of $\nu_{1,2,3}$ and $\nu_{e,\mu,\tau}$ (6th phase does is overall redefinition of all fields – does not change U).

Neutrino mixing matrix (reminder)

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

- The rest $9 - 5 = 4$ parameters are usually chosen as follows: **3 mixing angles** $\theta_{12}, \theta_{23}, \theta_{13}$ and **1 phase** ϕ (since 3×3 real orthogonal matrix has 3 parameters only)

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix} \quad (1)$$

where one denotes $\cos\theta_{12} = c_{12}$, $\sin\theta_{23} = s_{23}$, etc.

Three rotations plus **one** phase ϕ:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\phi}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\phi}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Neutrino oscillations

- So transitions between flavours can be either:

$$\delta\mathcal{L}_1 = \underbrace{M(\bar\nu_e\nu_\mu + \bar\nu_\mu\nu_e)}_{\nu_\mu \text{ and } \nu_e \text{ charge eigenstates}} \qquad \text{or} \qquad \delta\mathcal{L}_2 = \underbrace{(\bar\nu_1 \not{A}\nu_2 + \bar\nu_2\not{A}\nu_1)}_{\nu_1 \text{ and } \nu_2 \text{ are mass eigenstates}}$$

- Similar to quarks where QCD Lagrangian reads:

$$\mathcal{L}_{\text{QCD}} = \bar u \not{D} u + \bar d \not{D} d + \bar s \not{D} s + m_u \bar u u + m_d \bar d d + m_s \bar s s + \dots$$

while weak Lagrangian is non-diagonal

$$\mathcal{L}_{\text{Weak}} = \frac{g}{\sqrt{2}}(\bar u, \bar c)_{\text{L}}\gamma^\mu \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_{\text{L}} W^+_\mu$$

- Why did we call this **beyond** the Standard Model problem?

Dirac and Majorana mass terms

- Mass eigenstates $\nu_{1,2,3}$ are **freely propagating massive fermions**
- Only two types of such fermions are possible which differ by their mass terms:
  - **Dirac mass term** requires **adding new particles** $N_1, N_2, \ldots$:

$$\mathcal{L}_{Dirac} = \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} + h.c. \qquad (2)$$

  - **Majorana mass term:**

$$\mathcal{L}_{\text{Majorana}} = \begin{pmatrix} \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (3)$$

m_1, m_2, m_3 can be complex

Majorana mass term

- Majorana mass term couples ν and its charge conjugate. Requires no new particles

- However, neutrino is a part of the SU(2) doublet $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ and therefore a Majorana mass term reads in the

$$\bar{\nu}^c_\alpha \nu_\beta \to \frac{c_{\alpha\beta}(\bar{L}_\alpha \cdot \tilde{H}^\dagger)(L_\beta \cdot \tilde{H})}{\Lambda}$$

  where Λ is some constant with the dimension of **mass**

- This is an “operator of dimension 5” or “non-renormalizable” interaction

- For many people this was a satisfactory viewpoint: in the logic of **effective field theory** one expects the operator of dimensions 5, 6, etc. whose contributions are **small** at energies $E \ll \Lambda$.

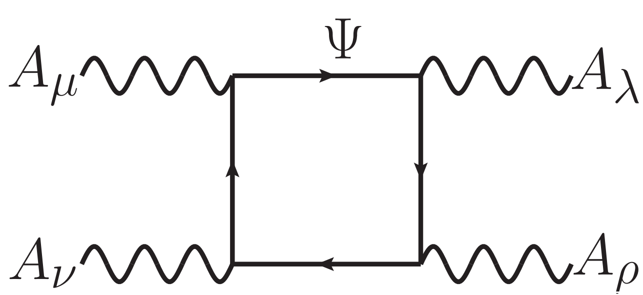
Neutrino masses and effective field theory

$$\text{Neutrino mass term} = \frac{c_{\alpha\beta}(\bar{L}_\alpha \cdot H^\dagger)(L_\beta \cdot H)}{\Lambda}$$

- Assuming $c_{\alpha\beta} \sim \mathcal{O}(1)$ one gets

$$\Lambda \sim \frac{v^2}{m_{\text{atm}}} \sim 10^{15}\ \text{GeV}$$

- In the logic of EFT one expects that some “heavy” particles had mediated this type of interaction and that at energies $E \lesssim \Lambda$ new particles should appear



$\Rightarrow$?

Neutrino masses and effective field theory

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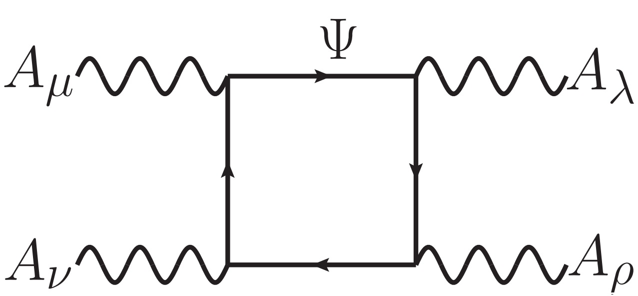
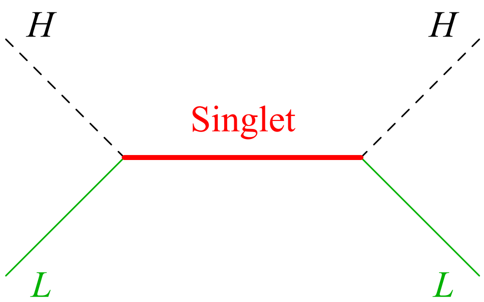
- **Recall:** In the logic of EFT one expects that some "heavy" particles had mediated this type of interaction and that at energies $E \lesssim \Lambda$ new particles should appear

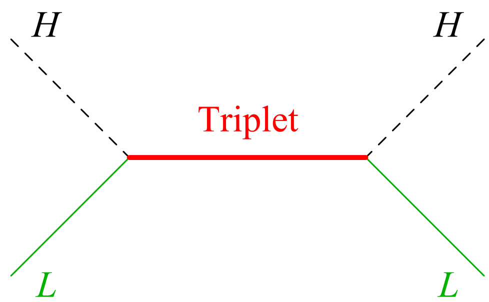


$$\frac{\alpha^2}{45m^4}\Big((E^2 - B^2)^2 + 7(E \cdot B)^2\Big)$$

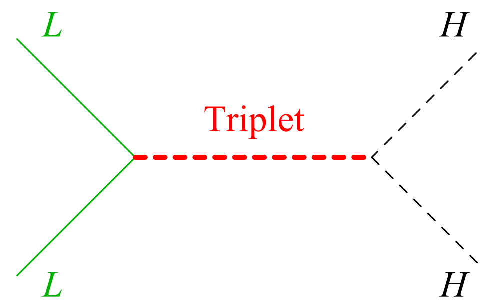
$\Rightarrow$ Integrating out massive electrons leads to **Heisenber-Euler effective action** and light-to-light scattering



Type I see-saw
extra singlet fermion



Type III see-saw
extra SU(2) triplet fermion
with zero hypercharge

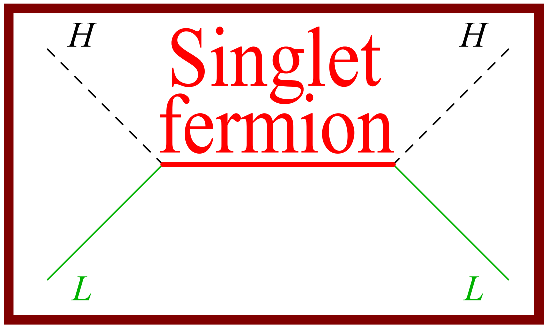


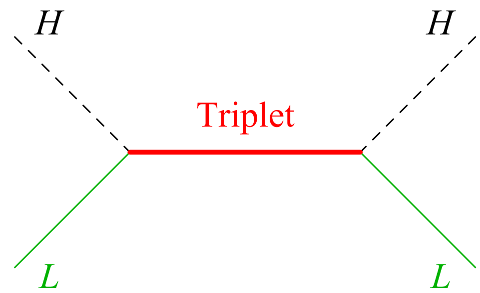
Type II see-saw
extra SU(2) triplet scalar
with hypercharge 1

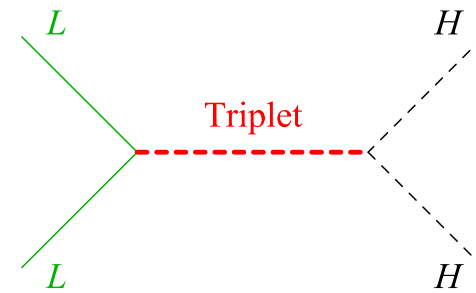
There are models with “loop mediated neutrino masses”, etc.

Strumia & Vissani ***“Neutrino masses and mixings and. . . ”*** **[hep-ph/0606054v3]**

"Resolving" Majorana mass term







Type I see-saw
extra singlet fermion

Type III see-saw
extra SU(2) triplet fermion
with zero hypercharge

Type II see-saw
extra SU(2) triplet scalar
with hypercharge 1

- If neutrino masses are due to **type-I see-saw mechanism**, this implies existence of new particles — sterile neutrinos
- Can they affect any other observables beyond neutrino masses?
- Can they be probed (with "effective energy scale" being 10^{15} GeV)?

Boyarsky, O.R., Shaposhnikov Ann. Rev. Nucl. Part. Sci. (2009), [0901.0011]

Dirac mass term

- **Recall:** Massless fermions can be left and right-chiral (left and right moving). The Dirac equation[4] reads

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancel{-m}^{\,0} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancel{-m}^{\,0} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

  where $\gamma_5\psi_{R,L} = \pm\psi_{R,L}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

- Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- (Dirac) mass term mixes left and right chiral parts of the 4-component spinor

[4] In the γ-matrix basis of Peskin & Schroeder

Type I see-saw mechanism

Recall gauge charges of neutrinos and Higgs:

– $\nu_{e,\mu,\tau}$: upper component of the SU(2) doublet, U(1)$_Y$ charge = -1
– Higgs boson: SU(2) doublet, U(1)$_Y$ charge = 1

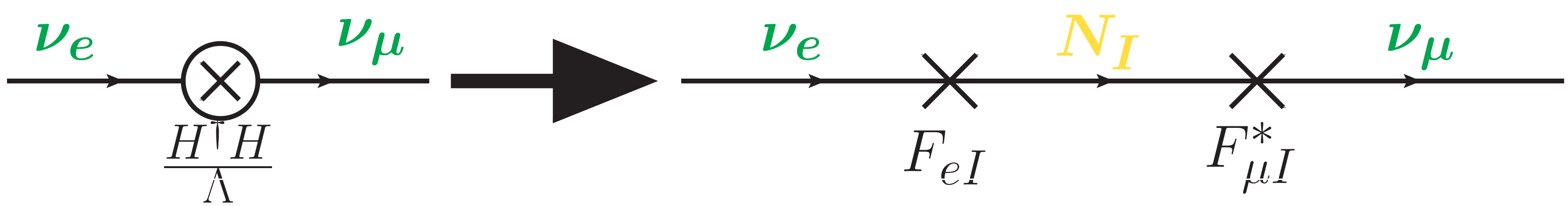
- **Dirac mass term:** $$(m_{\text{Dirac}})_{\alpha I} = \langle H \rangle F_{\alpha I}\, \bar{\nu}_\alpha\, N_I$$
  – N_I – new particles, *right-handed* fermions, $I = 1, 2, \ldots, \mathcal{N}$
  – $F_{\alpha I}$ – Yukawa matrix, size $3 \times \mathcal{N}$ (Neutrino Yukawa matrix needs not be square. One can have any number of sterile neutrinos $\mathcal{N} = 1, 2, 3, 4, 5, \ldots$)

- Right-chiral neutrinos N_I **carry no charge** under the SM interactions (**"sterile neutrinos"**)

- Neutral leptons can have **Majorana mass term**:

$$(M_M) = M_{IJ} \bar{N}_I^c N_J \quad I, J = 1, 2 \ldots \mathcal{N}$$

Type I see-saw mechanism



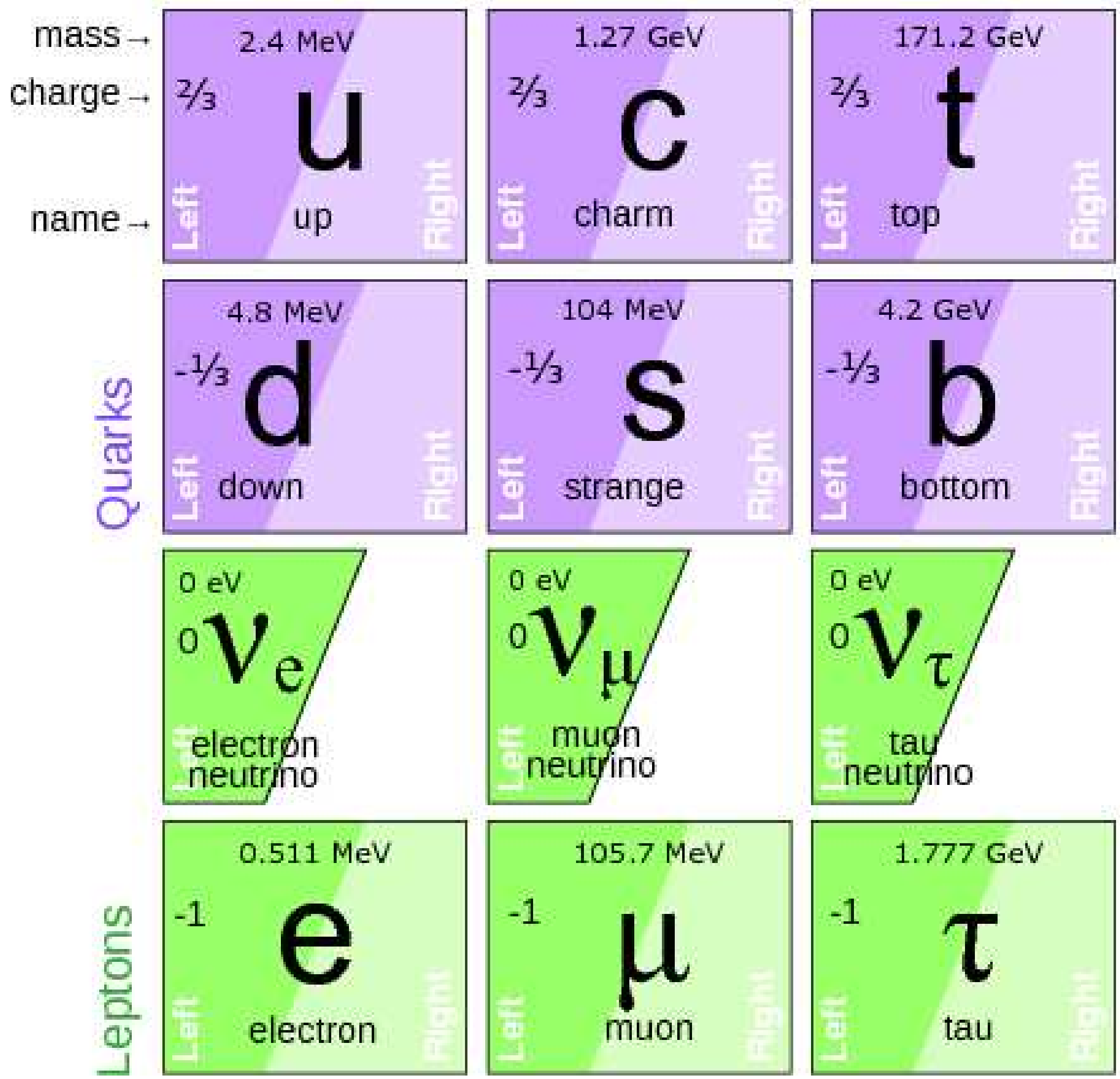
- The full neutrino mass matrix (3 active and $\mathcal{N} = 3$ sterile)

$$\text{Neutrino mass term} = \begin{pmatrix} \bar\nu_e & \bar\nu_\mu & \bar\nu_\tau & \bar N_1 & \bar N_2 & \bar N_3 \end{pmatrix} \begin{pmatrix} \boxed{0} & \boxed{\text{Dirac mass}} \\ \boxed{\text{Dirac mass}} & \boxed{\text{Majorana mass}} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

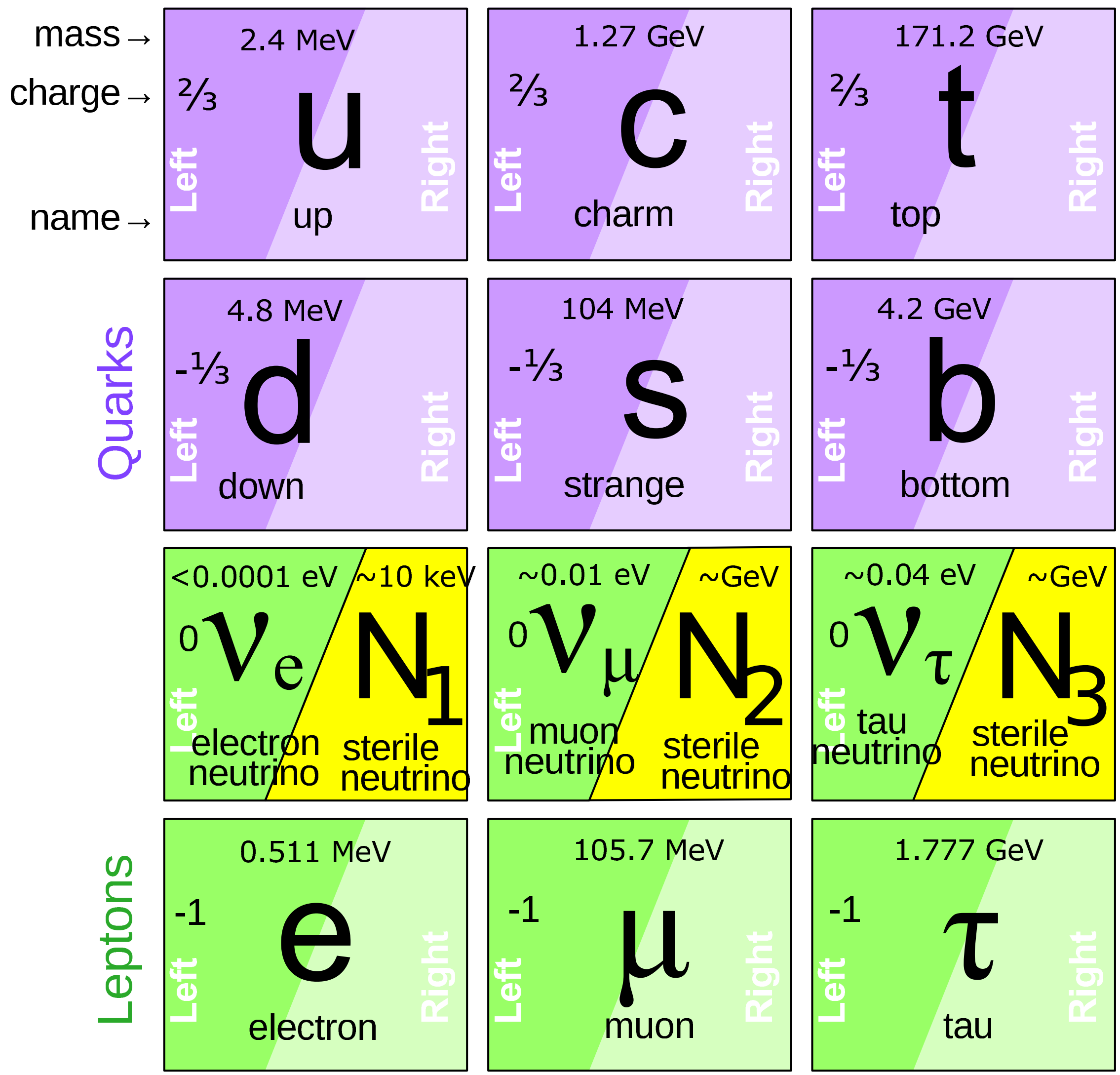
- Neutrino masses are given by the **see-saw formula**:

Neutrino masses $\mathcal{M}_{\text{active}} = -m_{\text{Dirac}} \dfrac{1}{M_{\text{Majorana}}} m^T_{\text{Dirac}}$

Neutrino oscillations and sterile neutrinos



Neutrino oscillations and sterile neutrinos



Scale of sterile neutrino masses?

- See-saw formula

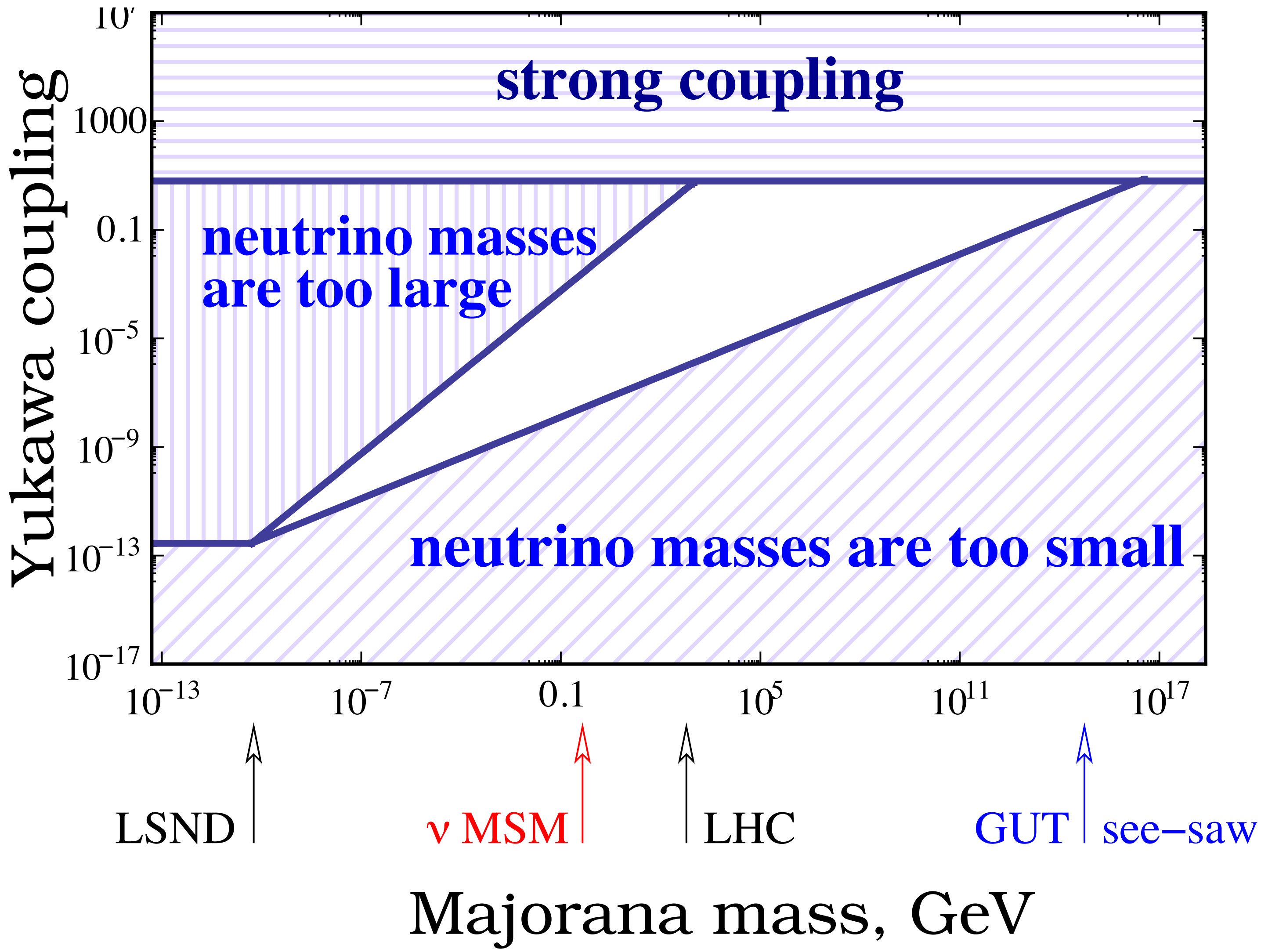
$$\boxed{\mathcal{M}_{\text{active}} \sim \frac{v^2|F|^2}{M_{\text{sterile}}}}$$

- At least some of the Yukawa couplings in the matrix $F_{\alpha I}$ are

$$|F| \sim \left(\frac{M_{\text{sterile}}\mathcal{M}_{\text{active}}}{v^2}\right)^{1/2} \approx 4 \times 10^{-8} \left(\frac{M_{\text{sterile}}}{1\ \text{GeV}}\right)^{1/2}$$

- The scale of sterile neutrino masses is **not determined by neutrino oscillations**

Scale of sterile neutrino masses?



Sterile neutrino white paper [**1204.5379**]

Number of free parameters?

Active neutrino masses $\mathcal{M}_{\text{active}} = -m_{\text{Dirac}} \frac{1}{M_{\text{Majorana}}} m_{\text{Dirac}}^T$

- Rank of the active neutrino mass matrix $\leq \mathcal{N}$ – the number of sterile neutrinos.
- At least **two** sterile neutrinos are required to explain two mass splittings (in which case $\sum \mathcal{M}_i \approx (1$ or $2) m_{\text{atm}}$
- Number of new parameters for $\mathcal{N}$ sterile neutrinos:

$\mathcal{N}$ real Majorana masses $+ 3 \times \mathcal{N}$ complex Yukawas (Dirac masses) $-$ 3 phases absorbed in redefinitions of ν_e, ν_μ, ν_τ.

- In total this brings us $7 \times \mathcal{N} - 3$ new parameters with $\mathcal{N}$ sterile neutrinos.

Number of free parameters?

- Number of new parameters

$$7 \times \mathcal{N} - 3 = \begin{cases} 11, & \mathcal{N} = 2 \\ 18, & \mathcal{N} = 3 \end{cases}$$

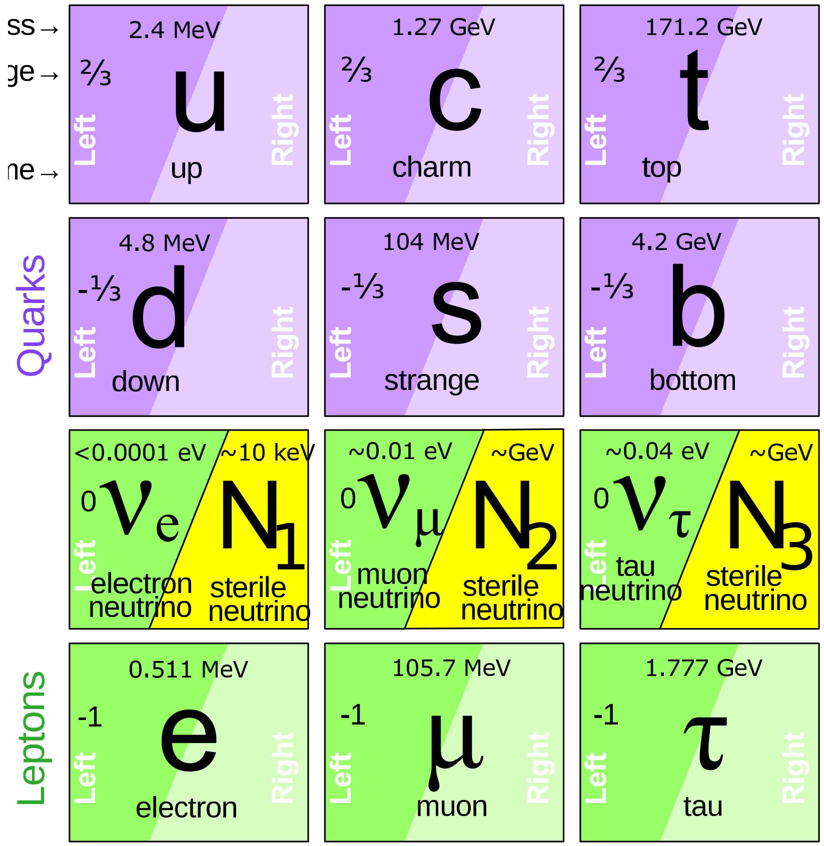
- Neutrino oscillation experiment may determine **9** parameters (**3** masses, **3** mixing angles, **2** Majorana phases and **1** Dirac CP phase)

- Undetermined parameters are: $\mathcal{N}$ Majorana masses + some ratios of Yukawas (for example, one replace $F_{\alpha I} \leftrightarrow F_{\alpha J}(M_I/M_J)^{1/2}$ for some pairs $I \neq J$.)

- With the **full knowledge** of PMNS and active neutrino masses/phases we will be able to determine

7 out of 11 parameters $\mathcal{N} = 2$
9 out of 18 parameters $\mathcal{N} = 3$
$\ldots$

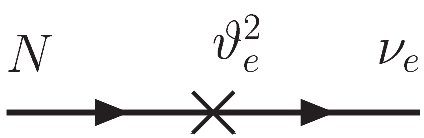
Properties of sterile neutrinos



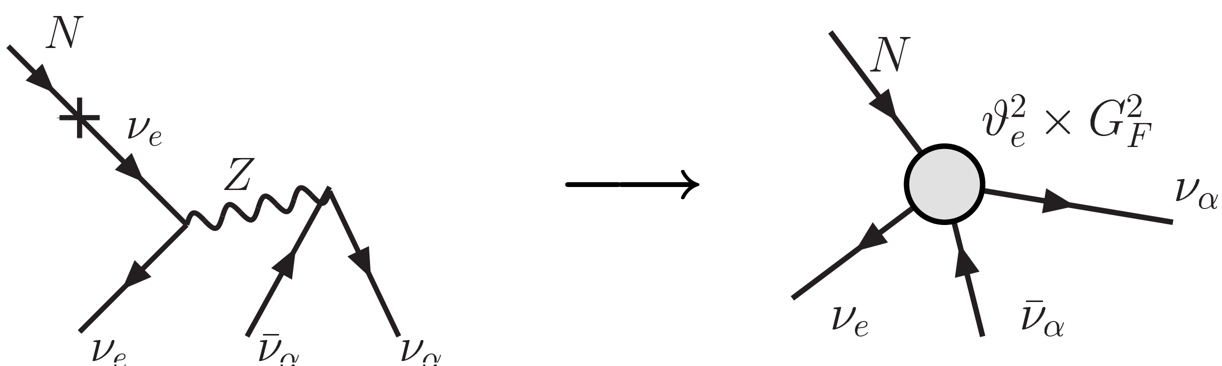
What are the properties of the new particles?

Sterile neutrinos behave as ***superweakly interacting* heavy neutrinos with a smaller Fermi constant**

$$G_F \longrightarrow \vartheta \times G_F$$



Quadratic mixing $N_s \leftrightarrow \nu$ of sterile neutrinos N_s to $\nu_{e,\mu\,\tau}$



Mixing angles $\vartheta^2_{e,\mu,\tau} \equiv \dfrac{|M_{\text{Dirac}}|^2}{M^2_{\text{Majorana}}} = \dfrac{\mathcal{M}_{\text{active}}}{M_{\text{sterile}}} \approx 5\times 10^{-11}\left(\dfrac{1\text{ GeV}}{M_{\text{sterile}}}\right)$

Interaction strength of sterile neutrino

$$\text{Yukawa coupling} \sim \left(\frac{M_{\text{sterile}}\mathcal{M}_{\text{active}}}{v^2}\right)^{1/2} \approx 4\times 10^{-8}\left(\frac{M_{\text{sterile}}}{1\text{ GeV}}\right)^{1/2}$$

$$\text{Mixing angles } \vartheta^2_{e,\mu,\tau} = \frac{\mathcal{M}_{\text{active}}}{M_{\text{sterile}}} \approx 5\times 10^{-11}\left(\frac{1\text{ GeV}}{M_{\text{sterile}}}\right)$$

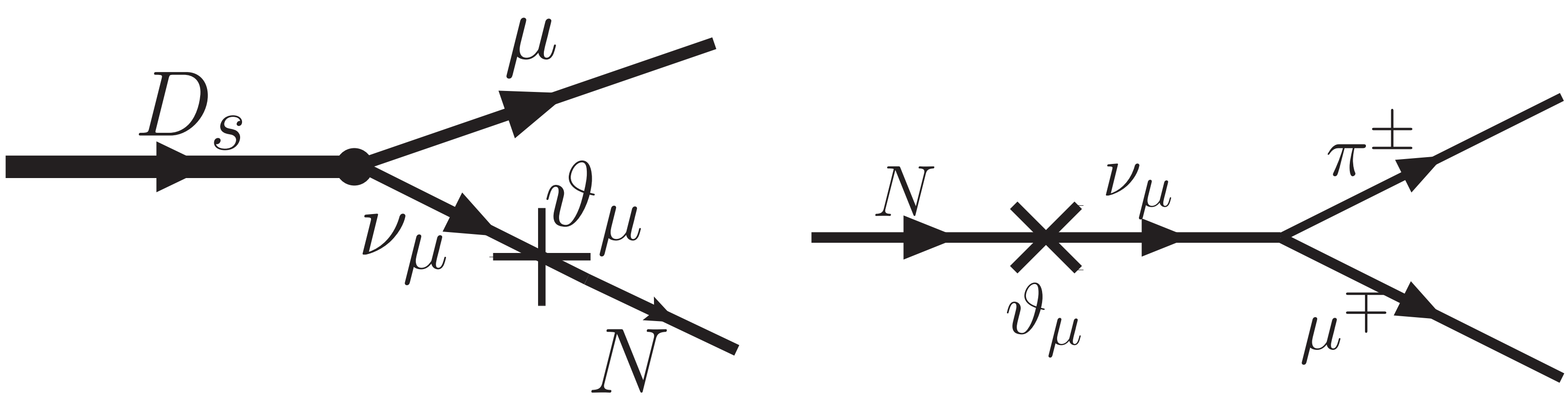
Mass	Yukawa coupling	ϑ^2
1 eV	1.3×10^{-12}	5×10^{-2}
1 keV	4.1×10^{-11}	5×10^{-5}
m_e	9.2×10^{-10}	1×10^{-7}
1 MeV	1.3×10^{-9}	5×10^{-8}
m_π	1.5×10^{-8}	4×10^{-10}
m_K	3×10^{-8}	1×10^{-10}
1 GeV	4.1×10^{-8}	5×10^{-11}
m_t	5.3×10^{-7}	3×10^{-13}
1 TeV	1.3×10^{-6}	5×10^{-14}
10^{15} GeV	1.3	5×10^{-26}

Interaction properties of sterile neutrinos

- Sterile neutrinos behave as **superweakly interacting** **heavy neutrinos** whose "Fermi constant" is flavour dependent $\vartheta_\alpha G_F$.

- If light enough they are produced in the decays of mesons/baryons and decay to SM particles

See e.g. Pal & Wolfenshtein (1982); Gorbunov & Shaposhnikov (2007)



$M_I < 1$ MeV	$M_I \gtrsim 1$ MeV	$M_I \gtrsim 140$ MeV	...
$N_I \to \nu\nu\bar{\nu}$	$N_I \to \nu e^+ e^-$	$N_I \to \pi^\pm e^\mp$	
$N_I \to \nu\gamma$		$N_I \to \pi^0 \nu$	

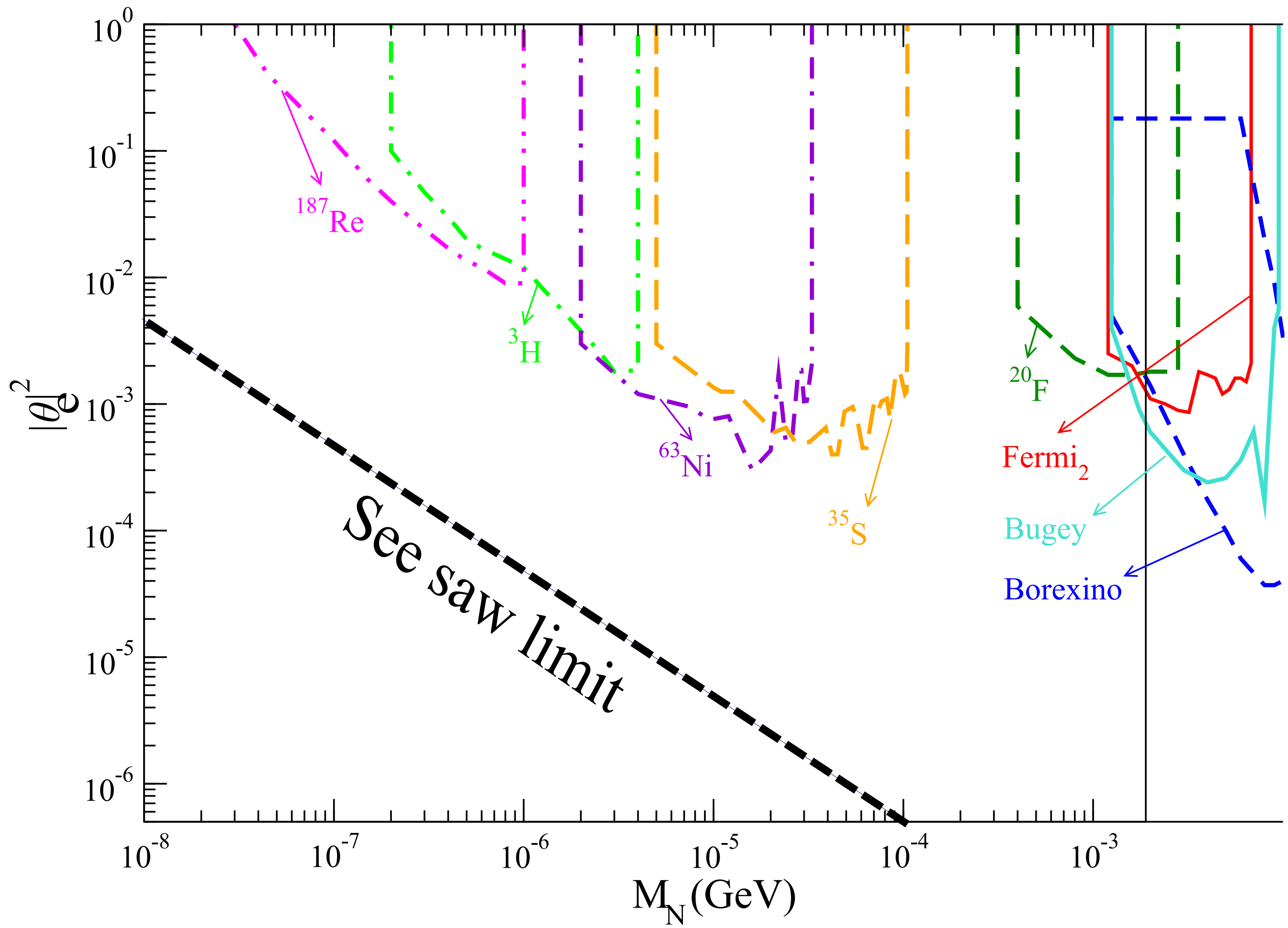
Phenomenology of sterile neutrinos

- Sterile neutrino is produced "as neutrino" if the mass of the decaying particle is "big enough"
- For example:
  $\pi \to e + \nu \;\Rightarrow \pi \to e + N$ if $M_N < m_\pi - m_e$;
  $K \to e + \nu \;\Rightarrow K \to e + N$ if $M_N < m_K - m_e$;
  $K \to \mu + \nu \;\Rightarrow K \to \mu + N$ if $M_N < m_K - m_\mu$, etc.
- Even if the kinematics is right, for each X neutrinos that you produce you also get $\vartheta^2 X$ sterile neutrinos (recall $\vartheta^2 \sim 10^{-11}$ for $M_N \sim 1$ GeV!)
- Interaction cross-section of sterile neutrinos with matter is ϑ^2 times **smaller** than that of ordinary neutrino

How can we search for them?

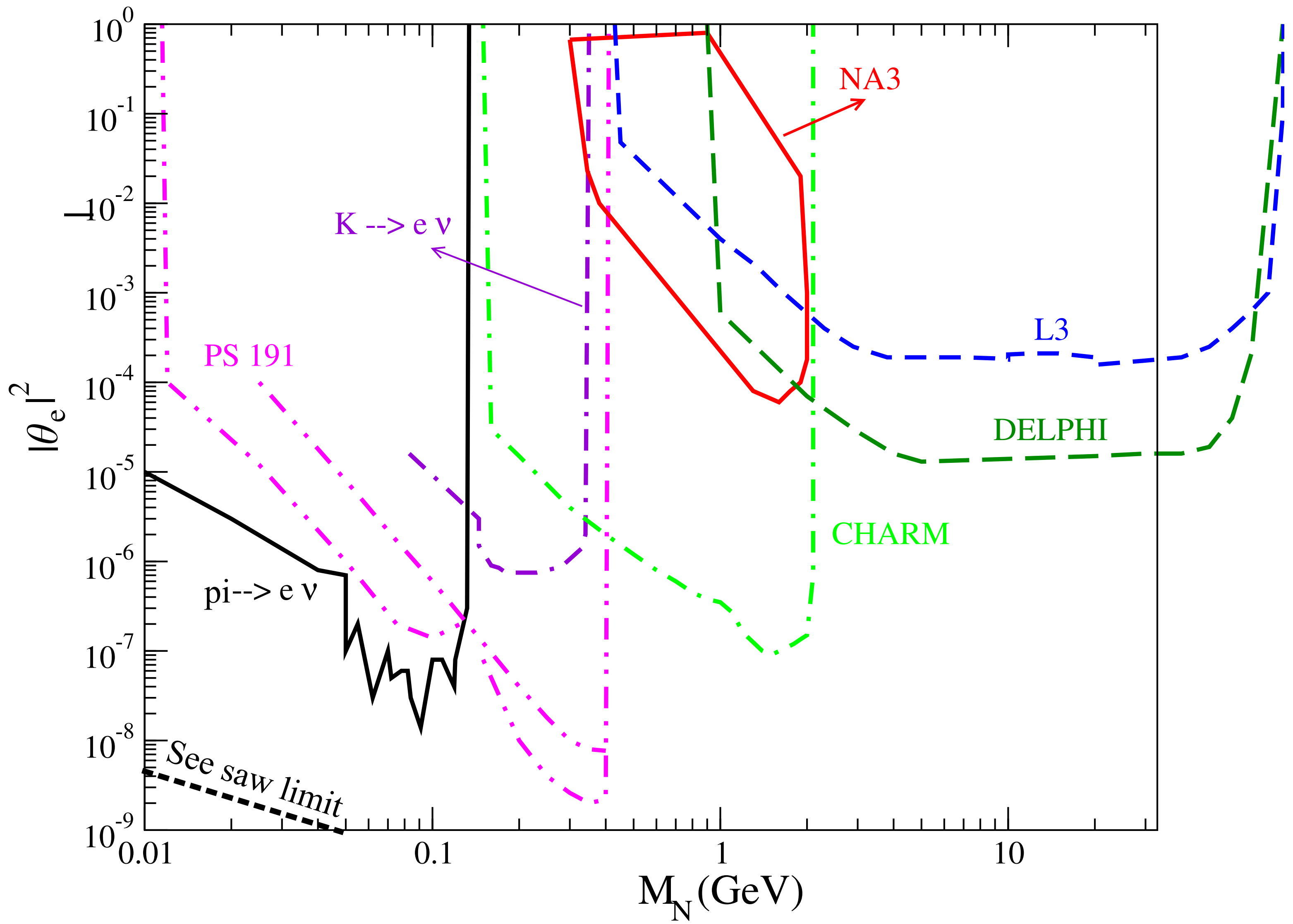
If "super-Kamiokande" (5×10^4 tons of water) is needed to detect ordinary neutrinos, how can we detect something orders of magnitude weaker?!

Kink searches in the spectrum of β-decay



Adopted from **Atre et al.** ***"The Search for Heavy Majorana Neutrinos"*****[0901.3589]**

Peak searches for larger masses



Peak searches?

- Accumulate a lot of pions. They decay through $\pi \to e + \nu$ and $\pi \to \mu + \nu$

- Pion decay width

$$\Gamma_{\pi\to e\nu} = \frac{G_F^2 f_\pi^2 \cos^2\theta_c m_\pi^3}{8\pi}\left(\frac{m_e}{m_\pi}\right)^2\left(1-\frac{m_e^2}{m_\pi^2}\right)$$

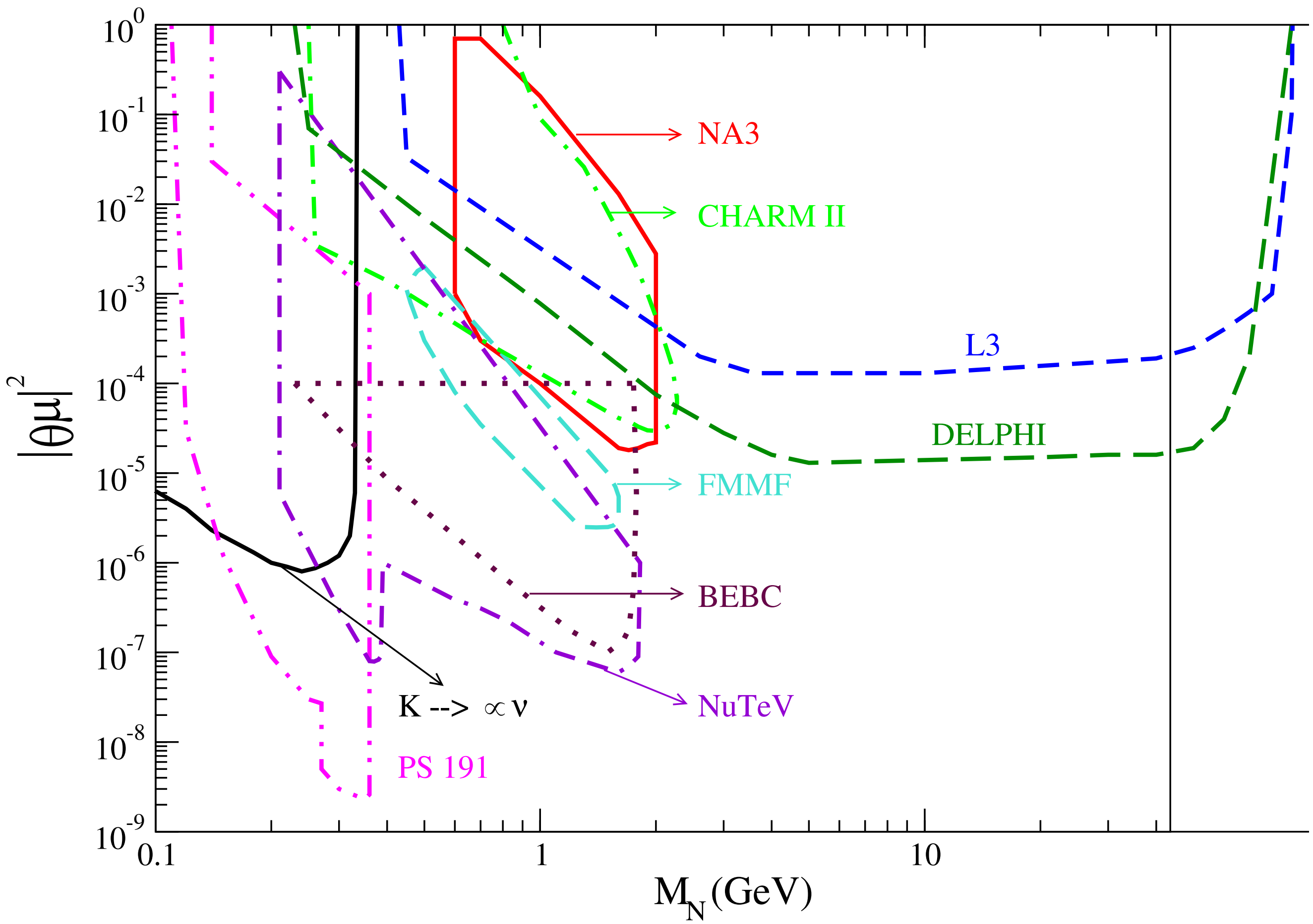
See Okun "Leptons & quarks"

- Factor $\left(\frac{m_e}{m_\pi}\right)^2$ is due to the **helicity suppression** (because charge currents couple to left particles only and pion is a scalar)

- Sterile neutrino has both left and right components. Therefore

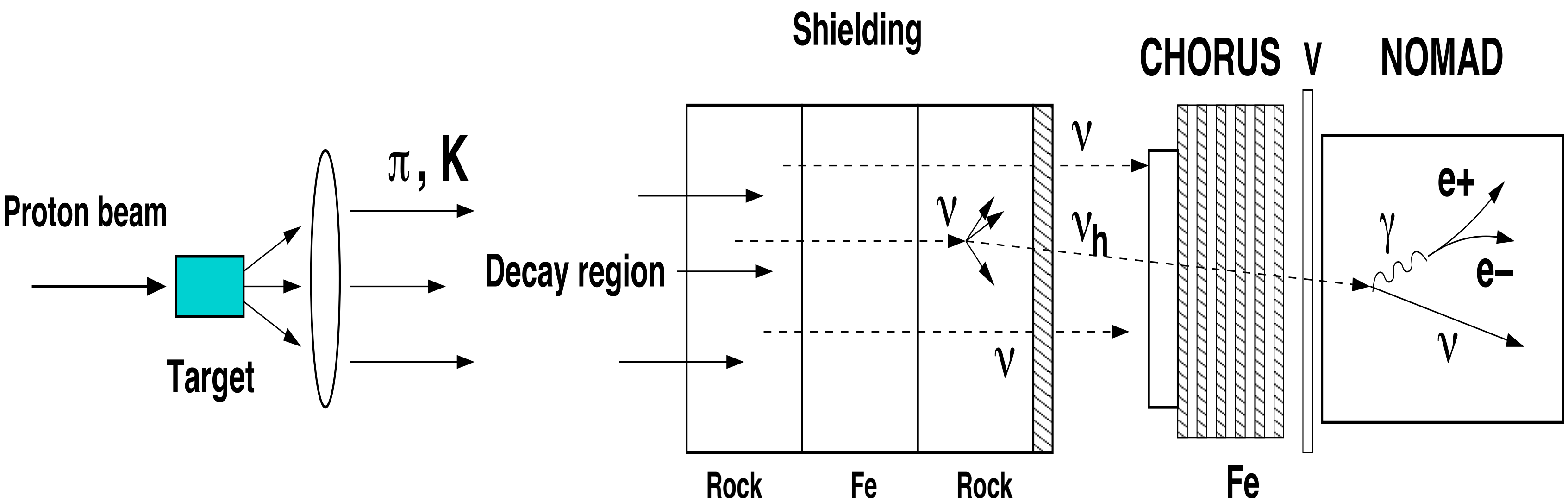
$$\Gamma_{\pi\to eN} = \Gamma_{\pi\to e\nu}\times\vartheta_e^2\times\left(\frac{M_N}{m_e}\right)^2$$

Other searches?



– **Gorbunov & Shaposhnikov** ***"How to nd neutral leptons of the νMSM?"*** **[0705.1729]**
– **Atre et al.** ***"The Search for Heavy Majorana Neutrinos"*****[0901.3589]**
– . . . with some important corrections in **O.R & Ivashko** ***"Experimental bounds on sterile neutrino mixing angles"*** **[1112.3319]**

Beam dump (fixed target) experiment



Experiment	E, GeV	N_{POT}	M_{pp}	χ_s	χ_c	χ_b
CERN, SPS	400	4.5×10^{19}	13	$1/7$	$5 \cdot 10^{-4}$	$3 \cdot 10^{-8}$
NuMi, Fermilab	120	5×10^{19}	11	$1/7$	10^{-4}	10^{-10}
T2K	50	10^{21}	7	$1/7$	10^{-5}	10^{-12}
NuTeV, Fermilab	800	5×10^{19}	15	$1/7$	10^{-3}	$2 \cdot 10^{-7}$

- N_{POT} – number of protons per target
- M_{pp} – multiplicity (average number of secondary particles)
- $\chi_{s,c,b}$ – fraction of strange, charmed, beauty quarks produced

Gorbunov & Shaposnikov (2007)

Sterile neutrino production...

- The total number of sterile neutrino produced is given by the product of all these numbers

$$N_N(E) = \sum_{Q=s,c,b} \xi_Q \cdot \chi_Q(E) \cdot N_{POT}(E) \cdot M_{pp}(E) \; .$$

- ...where ξ_Q is the product of branching ratios $\xi_Q \equiv \text{Br}(Q \to \mathcal{X}) \cdot \text{Br}(\mathcal{X} \to N)$ (where $\mathcal{X}$ is each of the strange, charmed, beauty mesons and baryons produced in collisions)

- The branching ratio $\text{Br}(\mathcal{X} \to N)$ is proportional to ϑ^2 ($\vartheta^2 \sim 5 \times 10^{-11}$ for $M_N \sim 1$ GeV)

- ...and sterile neutrino decay, each decay suppressed as ϑ^2

For all the gory details see **Gorbunov & Shaposhnikov** ***"How to nd neutral leptons of the νMSM?"*** **[0705.1729]**

Lifetime of sterile neutrinos

- Mode that always exists $N \to \nu\bar{\nu}\nu$

Pal & Wolfenshtein'8

$$\text{Lifetime}_N = \left(\frac{\vartheta^2 G_F^2 M_N^5}{192\pi^3}\right)^{-1} \sim 0.6\,\text{sec}\left(\frac{1\text{ GeV}}{M_N}\right)^4$$

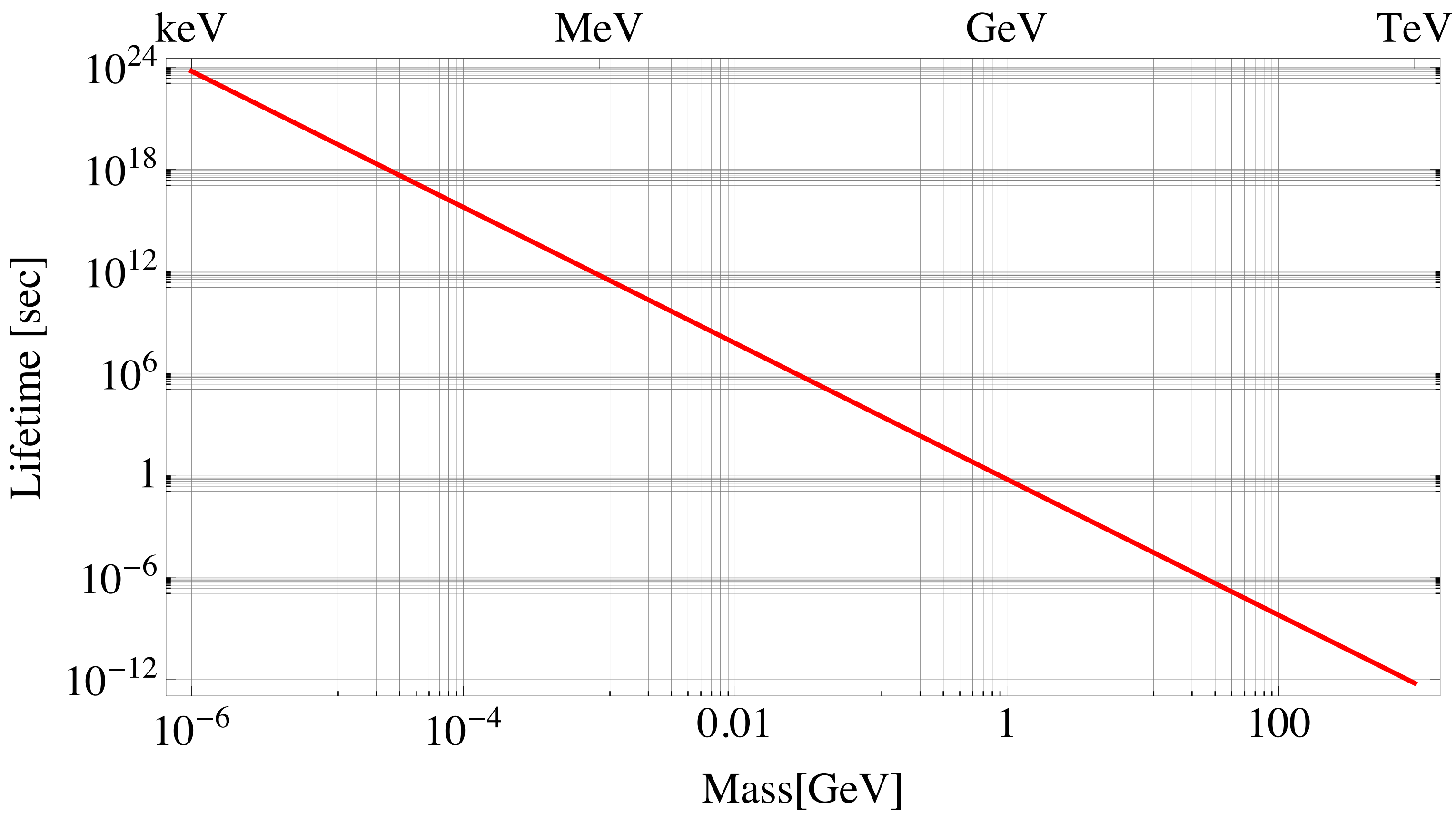
- this formula remains (almost) the same for $N \to e^+e^-\nu$ if $M_N \gg m_e, m_\mu$, etc.

- This is an **upper bound** for sterile neutrino lifetime

- For decays into mesons $N \to \pi^+e^-$, etc. the formula becomes

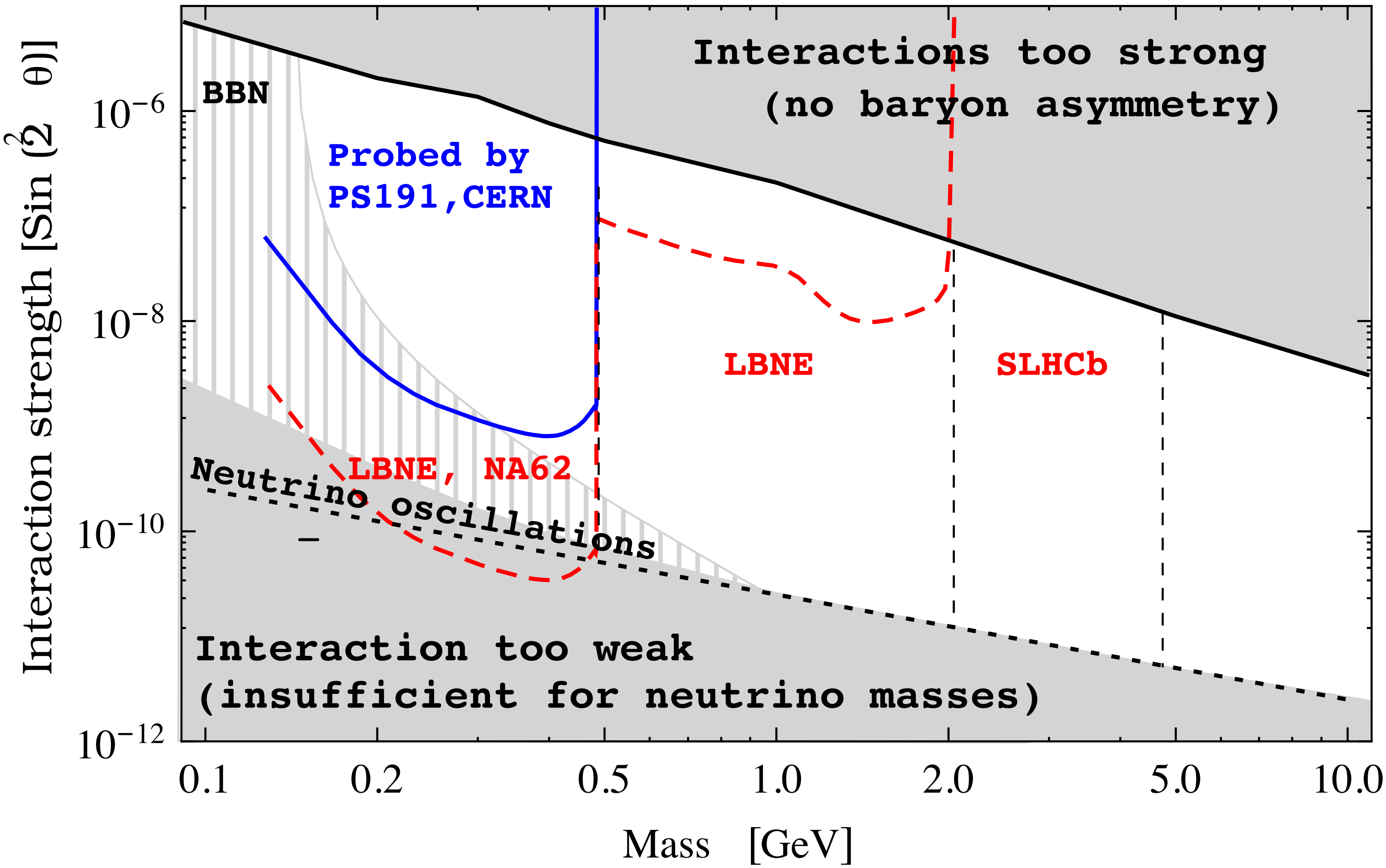
$$\text{Lifetime}_N = \frac{\vartheta^2 G_F^2 M_N^3 f_\pi^2}{32\pi} \sim 0.6\,\text{sec}\left(\frac{1\text{ GeV}}{M_N}\right)^2$$

Upper bound on τ_N



- Upper bound ($N \to 3\nu$ channel) for sterile neutrinos responsible for neutrino oscillations
- The fraction of decaying particles is additionally suppressed by the factor $\frac{L_{\text{detector}}}{c\tau_N} \sim 6 \times 10^{-7} \left(\frac{L_{\text{detector}}}{100\,\text{m}}\right)\left(\frac{M_N}{1\text{ GeV}}\right)^4$

Future experiments



Take home messages

1) If singlet sterile neutrinos are responsible for neutrino oscillations – there should be **at least two of them** ($\mathcal{N}$ need not be 3)

2) Full determination of PMNS matrix + masses + Majorana phases of active neutrinos **will not fully determine** the properties of sterile neutrinos

3) Most importantly, **their masses are not fixed** and can be anything from m_{atm} to 10^{15} GeV

4) Sterile neutrinos behave as massive neutral particles, interacting **weaker than neutrinos** (suppression of interaction strength $\vartheta^2 \sim \frac{matm}{M_{\text{sterile}}}$)

5) Phenomenologically one can describe their interaction as 4-fermion interaction with $G_F \to G_F\vartheta$

6) These particle has been searched in the range up to $\mathcal{O}(1)$ eV $\lesssim M_N \lesssim \mathcal{O}(10^2)$ GeV, but **searches never approached the interaction strength as dictated by neutrino oscillation**

7) For masses in MeV–GeV range it is possible to explore this interesting region of interaction strength (LBNE, NA62, LHCb, . . .)

They could be right here and we would not know about them!

What's next?


WANTED
Massive neutral particle
Interacting weaker than neutrino

Particle physics meets early Universe

Particle physics of the early Universe

- The laws of gravity are the same all over the Universe
  Has this worked in the past? **Yes!** Prediction by Friedmann and others that the Universe expands

- The laws of particle physics are the same all over the Universe
  Has this worked in the past? **Yes!** Prediction by Gamow and others of CMB and primordial abundance of light elements

- All together this led to the **hot Big Bang model**

- This theory, although written in every textbook today (as an “obvious thing”) has not been immediately accepted by the community at the time of creation (50s–60s).
  the very name “big bang” was a mocking name, intended as a joke/insult to the Gamow’s idea

See e.g. **Ya. Zel’dovich, *“Theory of the Expanding Universe as Originated by A.A. Friedmann”***
http://adsabs.harvard.edu/abs/1964SvPhU...6..475Z

Cosmology in a couple of slides

- FRW metric: (**Scale factor** → $a^2(t)$) $$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$
- Friedmann equation $$\left[\frac{\dot{a}(t)}{a(t)}\right]^2 \equiv H^2(t) = \frac{8\pi}{3M_{\text{Pl}}^2}\rho_{\text{tot}}(t)$$
- Redshift: $$1 + z = \frac{E_{\text{then}}}{E_{\text{now}}} = \frac{a_{\text{now}}}{a_{\text{then}}} \gtrsim 1$$

Main substances:

- **Radiation** is the gas of relativistic particles: $\rho_{\text{rad}} = \frac{1}{3}$pressure
- **Matter** is the gas of **non**-relativistic particles:
  $\rho_{\text{mat}} =$ mass $\times$ number density, pressure $= 0$
- **Cosmological constant** is the substance with negative pressure
  $\rho_{\Lambda} = -$pressure

Cosmology in a couple of slides

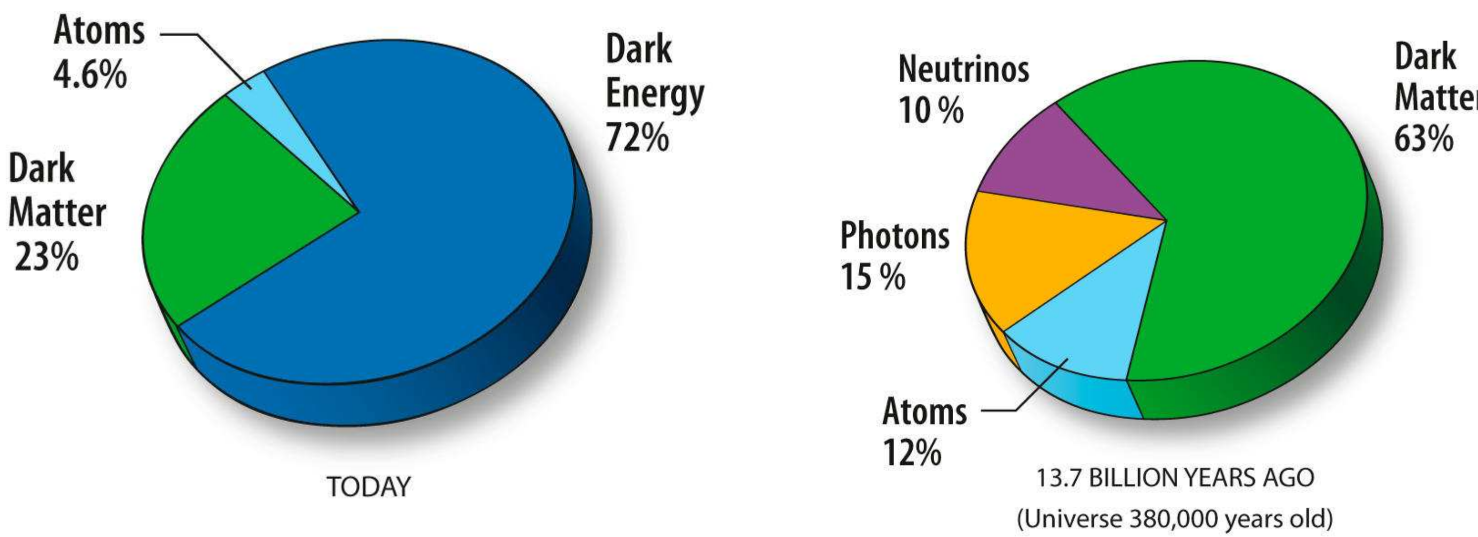
- Friedmann equation (for spatially flat Universe!)

$$H^2(z) = H_0^2\left(\Omega_\Lambda + \Omega_{\text{mat}}(1+z)^3 + \Omega_{\text{rad}}(1+z)^4\right)$$

($z_{\text{today}} = 0$, z increases into the past)

- Ω is a fraction of a given substance in the total energy of the Universe **today**

- Relative importance of components changes with time (redshift) WMAP website



Primordial plasma

- $H^2(z) = H_0^2\Big(\Omega_\Lambda + \Omega_{\text{mat}}(1+z)^3 + \Omega_{\text{rad}}(1+z)^4\Big)$

- As one goes into the past, the matter contribution grows. The density of matter increases.

- (It was Gamow who understood) that at some moment the density will bring matter into equilibrium and for that there are two scales to compare :

$$\Gamma \sim \sigma \times n \times \langle v \rangle$$

and $H(t)$ – Hubble expansion rate **or** inverse "age of the Universe" at redshift z.[5]

- Whenever you have $\Gamma \gg H$ – you can think about plasma of particles being in equilibrium and having some **temperature**.

[5]Relation $H(t) \approx t_{\text{age}}^{-1}$ holds for matter and radiation dominated Universe. In the Λ-dominated Universe the two become unrelated (as $H(t) \approx \text{const}$).

Primordial plasma

- In the opposite limit dynamics is out-of-equilibrium and it is "physical kinetics" rather than "thermodynamics" that describes your physics.

- If $\Gamma \gg H$ – the Universe is expanding **adiabatically** – processes do not feel the temperature change and thermodynamic equilibrium gets established at each plasma temperature.

- When this is the case you can treat temperature $T(t) \sim \dfrac{1}{a(t)}$

- Although most of its history the primordial plasma is in equilibrium, the most important processes (CMB, BBN, BAU) are happening when some particles **go out of equilibrium** (=="freeze out"):
  - Electromagnetic interactions freeze-out — CMB is formed
  - Weak interactions freeze-out — onset of BBN
  - Sterile neutrinos freeze-out — . . . (this will be the subject of our next lecture)

TOC

TOC