Supersymmetry and the LHC

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Topics:

- Why: Motivation for supersymmetry (SUSY)
- What: SUSY Lagrangians, SUSY breaking and the Minimal Supersymmetric Standard Model
- How: to look for supersymmetry at the LHC
- Where: SUSY might be hiding
- Who: Sorry, not covered.

For some more details and a slightly better attempt at proper referencing:

- *A supersymmetry primer*, `hep-ph/9709356`, version 6, September 2011
- *TASI 2011 lectures notes: two-component fermion notation and supersymmetry*, `arXiv:1205.4076`.

If you find corrections, please do let me know!

Lecture 1: Motivation and Introduction to Supersymmetry

- Motivation: The Hierarchy Problem
- Supermultiplets
- Particle content of the Minimal Supersymmetric Standard Model (MSSM)
- Need for “soft” breaking of supersymmetry
- The Wess-Zumino Model

People have cited many reasons why the next discoveries beyond the Standard Model might involve **supersymmetry (SUSY).**

Some of them are:

- A possible cold dark matter particle
- A light Higgs boson, $M_h = 125.5$ GeV
- Unification of gauge couplings
- Mathematical beauty

However, they are all insignificant compared to the one really good reason:

- **The Hierarchy Problem**

An analogy: Coulomb self-energy correction to the electron's mass

A point-like electron would have an infinite classical electrostatic energy.

Instead, suppose the electron is a solid sphere of uniform charge density and radius R:

$$\Delta E_{\text{Coulomb}} = \frac{3e^2}{20\pi\epsilon_0 R}$$

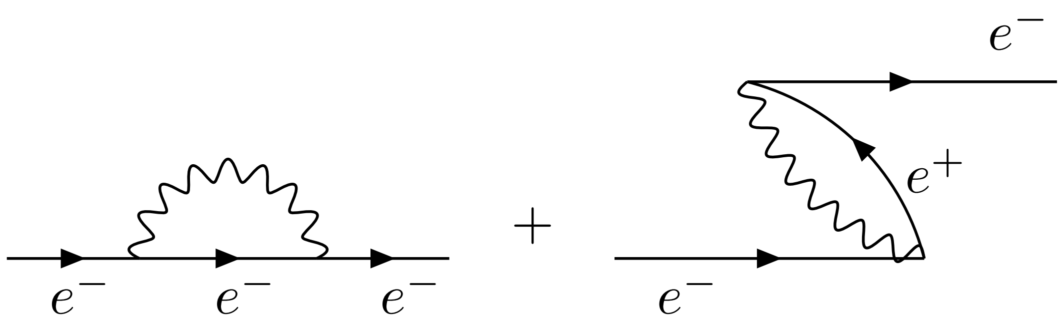
Interpreting this as a correction $\Delta m_e = \Delta E_{\text{Coulomb}}/c^2$ to the electron mass:

$$m_{e,\text{physical}} = m_{e,\text{bare}} + (1\,\text{MeV}/c^2)\left(\frac{0.86 \times 10^{-15}\,\text{meters}}{R}\right).$$

A divergence arises if we try to take $R \to 0$. Naively, we might expect $R \gtrsim 10^{-17}$ meters, to avoid having to tune the bare electron mass to better than 1%, for example:

$$0.511\,\text{MeV}/c^2 = -100.000\,\text{MeV}/c^2 + 100.511\,\text{MeV}/c^2.$$

However, there is another important quantum mechanical contribution:



The virtual positron effect cancels most of the Coulomb contribution, leaving:

$$m_{e,\mathrm{physical}} = m_{e,\mathrm{bare}} \left[1 + \frac{3\alpha}{4\pi} \ln \left(\frac{\hbar/m_e c}{R} \right) + \ldots \right]$$

with $\hbar/m_e c = 3.9 \times 10^{-13}$ meters. Even if R is as small as the Planck length 1.6×10^{-35} meters, where quantum gravity effects become dominant, this is only a 9% correction.

The existence of a "partner" particle for the electron, the positron, is responsible for eliminating the dangerously huge contribution to its mass.

This “reason” for the positron’s existence can be understood from a **symmetry**, namely the Poincaré invariance of Einstein’s relativity imposed on the quantum theory of electrons and photons (QED).

If we did not yet know about relativity or the positron, we would have had three options:

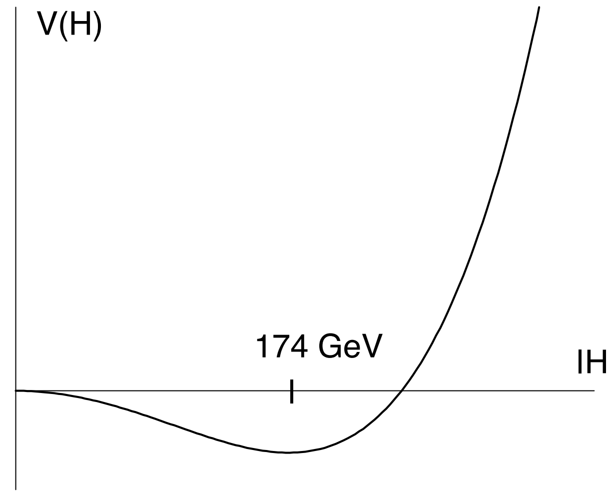
- **Assume that the electron has structure at a measurable size $R \gtrsim 10^{-17}$ meters.**
- **Assume that the electron is pointlike or very small, $R \ll 10^{-17}$ meters, and there is a mysterious fine-tuning between the bare mass and the Coulomb correction.**
- **Predict that the electron’s symmetry “partner”, the positron, must exist.**

Today we know that the last option is the correct one.

The Hierarchy Problem

Potential for H, the complex scalar field that is the electrically neutral part of the Standard Model Higgs field:

$$V(H) = m_H^2 |H|^2 + \frac{\lambda}{2} |H|^4$$



For $M_Z = \sqrt{(g^2 + g'^2)/2}\langle H \rangle$ to be 91.19 GeV, we need:

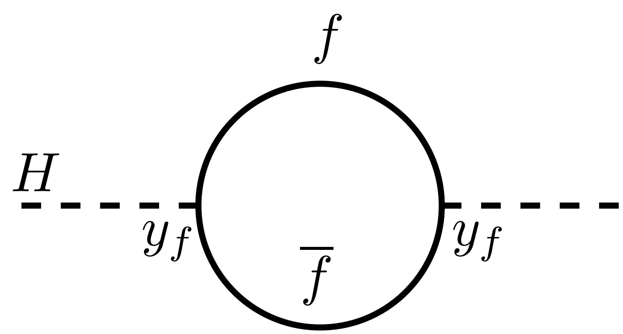
$$\langle H \rangle = \sqrt{-m_H^2/\lambda} \approx 174 \text{ GeV}$$

For the physical Higgs mass $M_H = \sqrt{2\lambda}\langle H \rangle$ to be 125.5 GeV, we need:

$$\lambda = 0.26, \qquad m_H^2 = -(89 \text{ GeV})^2$$

However, this appears fine-tuned (in other words, incredibly and mysteriously lucky!) when we consider the likely size of quantum corrections to m_H^2.

Contributions to m_H^2 from a Dirac fermion loop:



The correction to the Higgs squared mass parameter from this loop diagram is:

$$\Delta m_H^2 = \frac{y_f^2}{16\pi^2}\left[-2M_{\rm UV}^2 + 6m_f^2 \ln\left(M_{\rm UV}/m_f\right) + \ldots\right]$$

where y_f is the coupling of the fermion to the Higgs field H.

$M_{\rm UV}$ should be interpreted as (at least!) the scale at which new physics enters to modify the loop integrations.

Therefore, m_H^2 is directly sensitive to the **largest** mass scales in the theory.

For example, some people believe that String Theory is responsible for modifying the high energy behavior of physics, making the theory finite. Compared to field theory, string theory modifies the Feynman integrations over Euclidean momenta:

$$\int d^4p \, [\ldots] \;\rightarrow\; \int d^4p \; e^{-p^2/M^2_{\rm string}} \, [\ldots]$$

Using this, one obtains from each Dirac fermion one-loop diagram:

$$\Delta m_H^2 \;\sim\; -\frac{y_f^2}{8\pi^2} M^2_{\rm string} + \ldots$$

A typical guess is that $M_{\rm string}$ is comparable to $M_{\rm Planck} \approx 2.4 \times 10^{18}$ GeV.

These huge corrections make it difficult to explain how $-m_H^2$ could be as small as $(89 \text{ GeV})^2$.

The Hierarchy Problem

We already know:

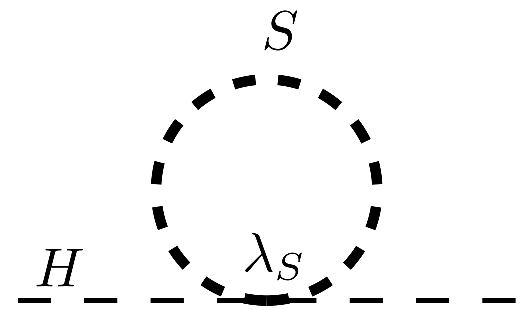
$$\frac{m_H^2}{M_{\rm Planck}^2} \approx -1.4 \times 10^{-33}$$

Why should this be so small, if individual radiative corrections Δm_H^2 can be of order $M_{\rm Planck}^2$ or $M_{\rm string}^2$, multiplied by loop factors?

This applies even if String Theory is wrong and some other unspecified effects modify physics at $M_{\rm Planck}$, or any other very large mass scale, to make the loop integrals converge.

An incredible coincidence seems to be required to make the corrections to the Higgs squared mass cancel to give a much smaller number.

The Higgs mass is also quadratically sensitive to other scalar masses.
Suppose S is some heavy complex scalar particle that couples to the Higgs.

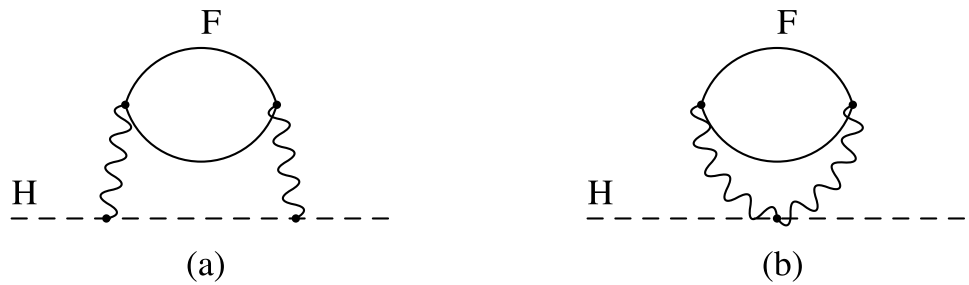


$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[M_{\mathrm{UV}}^2 - 2m_S^2 \ln\left(M_{\mathrm{UV}}/m_S\right) + \ldots\right]$$

(Note that the coefficients from a scalar loop have the **opposite sign** compared to a fermion loop.)

In dimensional regularization, the terms proportional to M_{UV}^2 do not occur.
However, this does *NOT* solve the problem, because the term proportional to m_S^2 is always there.

Indirect couplings of the Higgs to heavy particles still give a problem:



Here F is any heavy fermion that shares gauge quantum numbers with the Higgs boson. Its mass m_F does not come from the Higgs boson and can be arbitrarily large. From these diagrams one finds (C is a group-theory factor):

$$\Delta m_H^2 = C \left(\frac{g^2}{16\pi^2} \right)^2 \left[k M_{\mathrm{UV}}^2 + 48 m_F^2 \ln(M_{\mathrm{UV}}/m_F) + \ldots \right]$$

Here k depends on the choice of cutoff procedure (and is 0 in dimensional regularization). However, the contribution proportional to m_F^2 is always present.

More generally, *any* indirect communication between the Higgs boson and very heavy particles, or very high-mass phenomena in general, can give an unreasonably large contribution to m_H^2.

The systematic cancellation of loop corrections to the Higgs mass squared requires the type of conspiracy that is better known to physicists as a **symmetry**.

Fermion loops and boson loops gave contributions with opposite signs:

$$\Delta m_H^2 = -\frac{y_f^2}{16\pi^2}(2M_{\rm UV}^2) + \ldots \qquad \text{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\rm UV}^2 + \ldots \qquad \text{(complex scalar)}$$

SUPERSYMMETRY, a symmetry between fermions and bosons, makes the cancellation not only possible, but automatic.

There are two complex scalars for every Dirac fermion, and $\lambda_S = y_f^2$.

Supersymmetry

A SUSY transformation turns a boson state into a fermion state, and vice versa. So the operator Q that generates such transformations acts, schematically, like:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

This means that Q must be an anticommuting spinor. This is an intrinsically complex object, so $Q^\dagger$ is also a distinct symmetry generator:

$$Q^\dagger|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q^\dagger|\text{Fermion}\rangle = |\text{Boson}\rangle$$

The single-particle states of the theory fall into groups called **supermultiplets**, which are turned into each other by Q and $Q^\dagger$. Fermion and boson members of a given supermultiplet are **superpartners** of each other.

Each supermultiplet contains equal numbers of fermion and boson degrees of freedom.

Types of supermultiplets

Chiral (or “Scalar” or “Matter” or “Wess-Zumino”) supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. $(n_F = 2)$

2 real spin-0 scalars = 1 complex scalar. $(n_B = 2)$

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or “Vector”) supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. $(n_F = 2)$

1 real spin-1 massless gauge vector boson. $(n_B = 2)$

The Standard Model photon γ, gluon g, and weak vector bosons $Z, W^\pm$ must fit into these.

Gravitational supermultiplet:

1 two-component Weyl fermion gravitino, helicity $\pm\frac{3}{2}$. $(n_F = 2)$

1 real spin-2 massless graviton. $(n_B = 2)$

How do the Standard Model quarks and leptons fit in?

Each quark or charged lepton is 1 Dirac = 2 Weyl fermions

$$\text{Electron:}\quad \Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} \begin{array}{l} \leftarrow \text{two-component Weyl LH fermion} \\ \leftarrow \text{two-component Weyl RH fermion} \end{array}$$

Each of e_L and e_R is part of a chiral supermultiplet, so each has a complex, spin-0 superpartner, called $\widetilde{e}_L$ and $\widetilde{e}_R$ respectively. They are called the "left-handed selectron" and "right-handed selectron", although they carry no spin.

The conjugate of a right-handed Weyl spinor is a left-handed Weyl spinor. So, there are two left-handed chiral supermultiplets for the electron:

$$(e_L,\ \widetilde{e}_L) \quad \text{and} \quad (e_R^\dagger,\ \widetilde{e}_R^*).$$

The other charged leptons and quarks are similar. We do not need ν_R in the Standard Model, so there is only one neutrino chiral supermultiplet for each family:

$$(\nu_e,\ \widetilde{\nu}_e).$$

Chiral supermultiplets of the Minimal Supersymmetric Standard Model (MSSM):

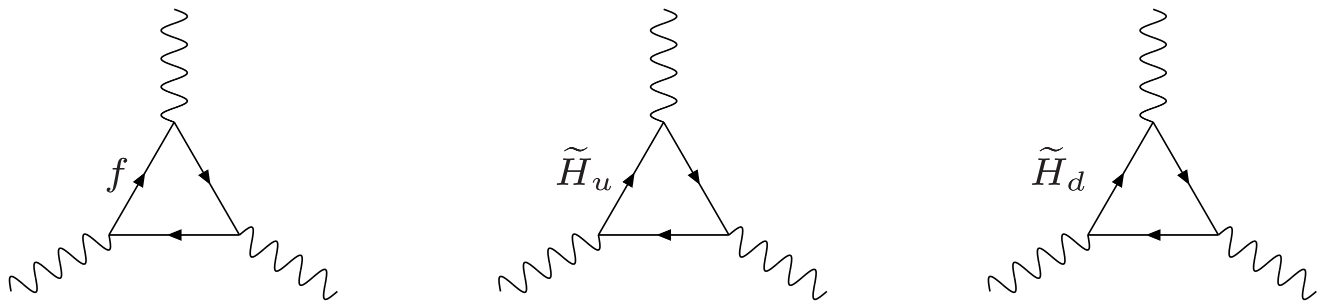
Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times 3$ families)	$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times 3$ families)	$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

The superpartners of the Standard Model particles are written with a $\widetilde{\ }$. The scalar names are obtained by putting an "s" in front, so they are generically called **squarks** and **sleptons**, short for "scalar quark" and "scalar lepton".

The Standard Model Higgs boson requires two different chiral supermultiplets, H_u and H_d. The fermionic partners of the Higgs scalar fields are called **higgsinos**. There are two charged and two neutral Weyl fermion higgsino degrees of freedom.

Why do we need two Higgs supermultiplets? Two reasons:

1) Anomaly Cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \quad = \quad 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present. Otherwise, the electroweak gauge symmetry would not be allowed!

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge +2/3 quarks (u, c, t).
Only the H_d Higgs scalar can give masses to charge -1/3 quarks (d, s, b) and the charged leptons (e, μ, τ). We will show this later.

The vector bosons of the Standard Model live in gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

The spin 1/2 **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\widetilde{\ }$.

The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

However, the winos and the bino are not mass eigenstate particles; they mix with each other and with the higgsinos of the same charge.

Recall that if supersymmetry were an exact symmetry, then superpartners would have to be exactly degenerate with each other. For example,

$$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511\ \mathrm{MeV}$$

$$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$$

$$m_{\tilde{g}} = m_{\mathrm{gluon}} = 0 + \mathrm{QCD\ \ effects}$$

etc.

New particles with these properties have been ruled out long ago, so:

Supersymmetry must be broken in the vacuum state chosen by Nature.

Supersymmetry is usually thought to be spontaneously broken and therefore hidden, the same way that the full electroweak symmetry $SU(2)_L \times U(1)_Y$ is hidden from very low-energy experiments.

A clue for SUSY breaking is given by our motivation in the Hierarchy Problem. The Higgs mass parameter gets corrections from each chiral supermultiplet:

$$\Delta m_H^2 = \frac{1}{16\pi^2}(\lambda_S - y_f^2)M_{\mathrm{UV}}^2 + \dots$$

If supersymmetry were exact and unbroken,

$$\lambda_S = y_f^2.$$

If we want SUSY to be a solution to the hierarchy problem, we must demand that this is still true even after SUSY is broken:

The breaking of supersymmetry must be “soft”. This means that the part of the Lagrangian with **dimensionless** couplings remains supersymmetric.

The effective Lagrangian has the form:

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$$

- $\mathcal{L}_{\rm SUSY}$ contains all of the gauge, Yukawa, and dimensionless scalar couplings, and preserves exact supersymmetry
- $\mathcal{L}_{\rm soft}$ violates supersymmetry, and contains only mass terms and couplings with *positive* mass dimension.

If $m_{\rm soft}$ is the largest mass scale in $\mathcal{L}_{\rm soft}$, then by dimensional analysis,

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(M_{\rm UV}/m_{\rm soft}) + \ldots \right],$$

where λ stands for dimensionless couplings. This is because Δm_H^2 must vanish in the limit $m_{\rm soft} \to 0$, in which SUSY is restored. Therefore, we expect that $m_{\rm soft}$ should not be much larger than **roughly** 1000 GeV.

This is the best reason to be optimistic that SUSY will be discovered at the CERN Large Hadron Collider, despite the searches so far.

Without further justification, “soft” SUSY breaking might seem like a rather arbitrary requirement.

Fortunately, it arises naturally from the spontaneous breaking of SUSY, as we will see later.

Is there any good reason why the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far?

Yes! The reason is electroweak gauge invariance.

- All of the particles discovered as of 1995 (quarks, leptons, gauge bosons) would be exactly massless if the electroweak symmetry were not broken. So their masses are expected to be at most of order $v = 174$ GeV, the electroweak breaking scale. **They were required to be light.**

- All of the particles in the MSSM that have **not** yet been discovered as of 2013 (squarks, sleptons, gauginos, Higgsinos, Higgs scalars) can get a mass even without electroweak symmetry breaking. **They are not required to be light.**

- The lightest Higgs scalar is an exception; its mass of ~ 125.5 GeV is within (and near the upper end of) the range predicted by supersymmetry.

Two-component spinor language is much more natural and convenient for SUSY, because the supermultiplets are in one-to-one correspondence with the LH Weyl fermions.

More generally, two-component spinor language is more natural for any theory of physics beyond the Standard Model, because it is an Essential Truth of Nature that parity is violated.

Nature does not treat left-handed and right-handed fermions the same, and the higher we go in energy, the more essential this becomes.

Notations for two-component (Weyl) fermions

Left-handed (LH) two-component Weyl spinor: $\psi_\alpha \quad \alpha = 1, 2$

Right-handed (RH) two-component Weyl spinor: $\psi^\dagger_{\dot\alpha} \quad \dot\alpha = 1, 2$

The Hermitian conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor, and vice versa:

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha} \equiv \psi^\dagger_{\dot\alpha}$$

All spin-$1/2$ fermionic degrees of freedom in **any** theory can be defined in terms of a list of left-handed Weyl spinors, $\psi_{i\alpha}$ where i is a flavor index. With this convention, right-handed Weyl spinors always carry a dagger: $\psi^{\dagger i}_{\dot\alpha}$.

I use metric signature ($-$,$+$,$+$,$+$).

Products of spinors are defined as:

$$\psi\xi \equiv \psi_\alpha \xi_\beta \epsilon^{\beta\alpha} \qquad \text{and} \qquad \psi^\dagger \xi^\dagger \equiv \psi^\dagger_{\dot\alpha} \xi^\dagger_{\dot\beta} \epsilon^{\dot\alpha\dot\beta}$$

Since ψ and ξ are anti-commuting fields, the antisymmetry of $\epsilon^{\alpha\beta}$ implies:

$$\psi\xi = \xi\psi = (\psi^\dagger \xi^\dagger)^* = (\xi^\dagger \psi^\dagger)^*.$$

To make Lorentz-covariant quantities, define matrices $(\overline{\sigma}^\mu)^{\dot\alpha\beta}$ and $(\sigma^\mu)_{\alpha\dot\beta}$ with:

$$\overline{\sigma}^0 = \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad \sigma^n = -\overline{\sigma}^n = (\vec{\sigma})_n \quad \text{(for } n = 1, 2, 3).$$

Then the Lagrangian for an arbitrary collection of LH Weyl fermions ψ_i is:

$$\mathcal{L} = i\psi^{\dagger i} \overline{\sigma}^\mu D_\mu \psi_i - \tfrac{1}{2} M^{ij} \psi_i \psi_j - \tfrac{1}{2} M_{ij} \psi^{\dagger i} \psi^{\dagger j}$$

where D_μ = covariant derivative, and the mass matrix M^{ij} is symmetric, with $M_{ij} \equiv (M^{ij})^*$.

Two LH Weyl spinors ξ, χ can form a 4-component Dirac or Majorana spinor:

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$$

In the 4-component formalism, the Dirac Lagrangian is:

$$\mathcal{L} \quad = \quad i\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m\overline{\Psi}\Psi, \qquad \text{where} \qquad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix},$$

In the two-component fermion language, with spinor indices suppressed:

$$\mathcal{L} = i\xi^\dagger \overline{\sigma}^\mu \partial_\mu \xi + i\chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi - m(\xi\chi + \xi^\dagger \chi^\dagger),$$

up to a total derivative.

A Majorana fermion can be described in 4-component language in the same way by identifying $\chi = \xi$, and multiplying the Lagrangian by a factor of $\frac{1}{2}$ to compensate for the redundancy.

The simplest SUSY model: a free chiral supermultiplet

The minimum particle content for a SUSY theory is a complex scalar ϕ and its superpartner fermion ψ. We must at least have kinetic terms for each, so:

$$S = \int d^4x \, (\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion})$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad\qquad \mathcal{L}_{\rm fermion} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

A SUSY transformation should turn ϕ into ψ, so try:

$$\delta\phi = \epsilon\psi; \qquad\qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

where $\epsilon =$ infinitesimal, anticommuting, constant spinor, with dimension [mass]$^{-1/2}$, that parameterizes the SUSY transformation. Then we find:

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi.$$

We would like for this to be canceled by an appropriate SUSY transformation of the fermion field. . .

To have any chance, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and must contain one spacetime derivative. There is only one possibility, up to a multiplicative constant:

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi; \qquad\qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*$$

With this guess, one obtains:

$$\delta\mathcal{L}_{\text{fermion}} = -\delta\mathcal{L}_{\text{scalar}} + \big(\text{total derivative}\big)$$

so the action S is indeed invariant under the SUSY transformation, justifying the guess of the multiplicative factor. This is called the free Wess-Zumino model.

Furthermore, if we take the commutator of two SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) = i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\phi$$

Since ∂_μ corresponds to the spacetime 4-momentum P_μ, This says that the commutator of two SUSY transformations is just a spacetime translation.

The fact that two SUSY transformations give back another symmetry (namely a spacetime translation) means that the SUSY algebra "closes".

If we do the same check for the fermion ψ:

$$\begin{aligned}\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) \quad &= \quad i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\psi_\alpha \\ &\quad\quad +i\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) - i\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)\end{aligned}$$

The first line is expected, but the second line only vanishes on-shell (when the classical equations of motion are satisfied). This seems like a problem, since we want SUSY to be a valid symmetry of the quantum theory (off-shell)!

To show that there is no problem, we introduce another bosonic spin-0 field, F, called an auxiliary field. Its Lagrangian density is:

$$\mathcal{L}_{\mathrm{aux}} = F^* F$$

Note that F has no kinetic term, and has dimensions [mass]^2, unlike an ordinary scalar field. It has the not-very-exciting equations of motion $F = F^* = 0$.

The auxiliary field F does not affect the dynamics, classically or in the quantum theory. But it does appear in modified SUSY transformation laws:

$$\begin{aligned}\delta\phi &= \epsilon\psi \\ \delta\psi_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F \\ \delta F &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi\end{aligned}$$

Now the total Lagrangian

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F$$

is still invariant, and also one can now check:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}X) - \delta_{\epsilon_1}(\delta_{\epsilon_2}X) = i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu X$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$, without using equations of motion.

So in the "modified" theory, SUSY does close off-shell as well as on-shell.

The auxiliary field F is really just a book-keeping device to make this simple. We can see why it is needed by considering the number of degrees of freedom on-shell (classically) and off-shell (quantum mechanically):

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

Going on-shell eliminates half of the propagating degrees of freedom of the fermion, because the Lagrangian density is linear in time derivatives, so that the fermionic canonical momenta are not independent phase-space variables. The momentum conjugate to ψ is $\psi^\dagger$, and the momentum conjugate to $\psi^\dagger$ is ψ.

The auxiliary field will also plays an important role when we add interactions to the theory, and in gaining a simple understanding of SUSY breaking.

Covered in Lecture 1:

- The Hierarchy Problem, $m_Z \ll m_{\rm Planck}$, is a strong motivation for supersymmetry (SUSY)
- In SUSY, all particles fall into:
  - Chiral supermultiplet $=$ complex scalar boson and fermion partner
  - Gauge supermultiplet $=$ vector boson and gaugino fermion partner
  - Gravitational supermultiplet $=$ graviton and gravitino fermion partner
- The Minimal Supersymmetric Standard Model (MSSM) introduces squarks, sleptons, Higgsinos, gauginos as the superpartners of Standard Model states
- Soft supersymmetry breaking
- Two-component fermion notation: ψ_α $=$ LH fermion, $\psi^\dagger_{\dot\alpha}$ $=$ RH fermion
- The Wess-Zumino Model Lagrangian describes a single chiral supermultiplet
- Auxiliary fields are a useful trick.

Lecture 2: Supersymmetric gauge theories and the Minimal SUSY Standard Model

- Supercurrents, supercharges, and the supersymmetry algebra
- Superpotentials and interactions
- Supersymmetric gauge interactions
- Soft SUSY breaking in general
- The MSSM superpotential
- R-parity and its consequences
- Soft SUSY breaking in the MSSM
- The MSSM particles

Recall: the Wess-Zumino model Lagrangian involves a scalar ϕ, a fermion ψ, and an auxiliary field F:

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F.$$

This describes a massless, non-interacting theory with supersymmetry.

Noether's Theorem: for every symmetry, there is a conserved current. In SUSY, the **supercurrent** J^μ_α is an anti-commuting 4-vector that also carries a spinor index.

By the usual Noether procedure, one finds for the supercurrent (and its conjugate $J^\dagger$), in terms of the variations of the fields δX for $X = (\phi, \phi^*, \psi, \psi^\dagger, F, F^*)$:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \quad \equiv \quad \sum_X \delta X \, \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu,$$

where K^μ satisfies $\delta\mathcal{L} = \partial_\mu K^\mu$. One finds:

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*; \qquad\qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi.$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0; \qquad\qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0,$$

as can be checked using the equations of motion.

From the conserved supercurrents one can construct the conserved charges:

$$Q_\alpha = \sqrt{2}\int d^3x\, J^0_\alpha; \qquad\qquad Q^\dagger_{\dot\alpha} = \sqrt{2}\int d^3x\, J^{\dagger 0}_{\dot\alpha},$$

As quantum mechanical operators, they satisfy:

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\,\delta X$$

for any field X. Let us also introduce the 4-momentum operator $P^\mu = (H, \vec{P})$, which satisfies:

$$[P_\mu, X] = i\partial_\mu X.$$

Now by using the canonical commutation relations of the fields, one finds:

$$\begin{aligned}\left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right] &= 2(\epsilon_1\sigma_\mu\epsilon_2^\dagger - \epsilon_2\sigma_\mu\epsilon_1^\dagger)\, P^\mu \\ \left[\epsilon Q + \epsilon^\dagger Q^\dagger,\, P\right] &= 0\end{aligned}$$

This implies…

The SUSY Algebra

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu,$$

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$$

$$[Q_\alpha, P^\mu] = [Q^\dagger_{\dot\alpha}, P^\mu] = 0$$

(The commutators turned into anti-commutators in the first two, when we extracted the anti-commutating spinors ϵ_1, ϵ_2.)

Masses and Interactions for Chiral Supermultiplets

The Lagrangian describing a collection of free, massless, chiral supermultiplets is

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i .$$

Question: How do we make mass terms and interactions for these fields, while still preserving supersymmetry invariance?

Answer: choose a **superpotential**,

$$W = \tfrac{1}{2} M^{ij} \phi_i \phi_j + \tfrac{1}{6} y^{ijk} \phi_i \phi_j \phi_k .$$

Must be holomorphic. In other words, only depends on the ϕ_i, not on the ϕ^{*i}.

The superpotential W contains masses M^{ij} and couplings y^{ijk}, which must be symmetric under interchange of i, j, k.

Supersymmetry is very restrictive; you cannot just do anything you want!

The resulting Lagrangian for interacting chiral supermultiplets is:

$$\begin{aligned}\mathcal{L} \quad = \quad & -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i \\ & -\tfrac{1}{2}\left(M^{ij}\psi_i\psi_j + y^{ijk}\phi_i\psi_j\psi_k\right) + \text{c.c.} \\ & -V(\phi_i, \phi^{*i})\end{aligned}$$

where the scalar potential is:

$$\begin{aligned}V(\phi_i, \phi^{*i}) \quad = \quad & M_{ik}M^{kj}\phi^{*i}\phi_j + \frac{1}{2}M^{in}y_{jkn}\phi_i\phi^{*j}\phi^{*k} \\ & +\frac{1}{2}M_{in}y^{jkn}\phi^{*i}\phi_j\phi_k + \frac{1}{4}y^{ijn}y_{kln}\phi_i\phi_j\phi^{*k}\phi^{*l}\end{aligned}$$

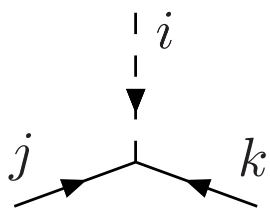
The superpotential W "encodes" all of the information about these masses and interactions.

The superpotential $W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$ **determines all non-gauge masses and interactions.**

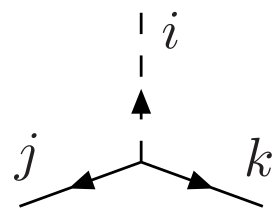
Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$.

The interaction Feynman rules for the chiral supermultiplets are:

Yukawa interactions:

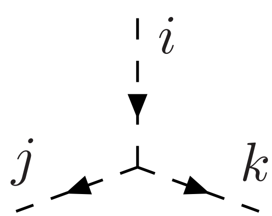


$-iy^{ijk}$

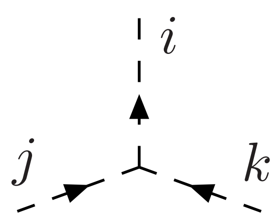


$-iy_{ijk}$

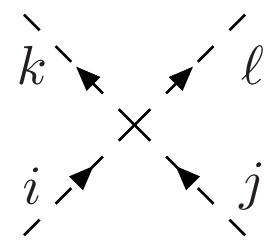
Scalar interactions:



$-iM^{in}y_{njk}$



$-iM_{in}y^{njk}$



$-iy^{ijn}y_{k\ell n}$

Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains physical fields:

- a gauge boson A_μ^a
- a gaugino λ_α^a.
- D^a, a real spin-0 auxiliary field with no kinetic term (non-propagating).

The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$, $1, 2, 3$ for $SU(2)_L$, and 1 for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i\lambda^{\dagger a}\overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a$$

where

$$\nabla_\mu \lambda^a \equiv \partial_\mu \lambda^a + g f^{abc} A_\mu^b \lambda^c.$$

The auxiliary field D^a is again needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell $(n_B = n_F = 2)$	2	2	0
off-shell $(n_B = n_F = 4)$	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, one must turn the ordinary derivatives into covariant ones:

$$\begin{aligned}\partial_\mu \phi_i \quad &\rightarrow \quad \nabla_\mu \phi_i = \partial_\mu \phi_i - igA^a_\mu (T^a \phi)_i \\ \partial_\mu \psi_i \quad &\rightarrow \quad \nabla_\mu \psi_i = \partial_\mu \psi_i - igA^a_\mu (T^a \psi)_i\end{aligned}$$

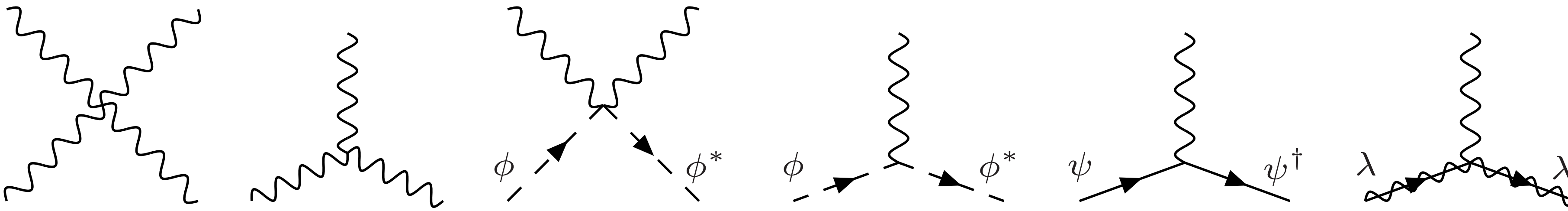
One must also add three new terms to the Lagrangian:

$$\begin{aligned}\mathcal{L} \quad = \quad & \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{chiral}} - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) \\ & + g(\phi^* T^a \phi) D^a .\end{aligned}$$

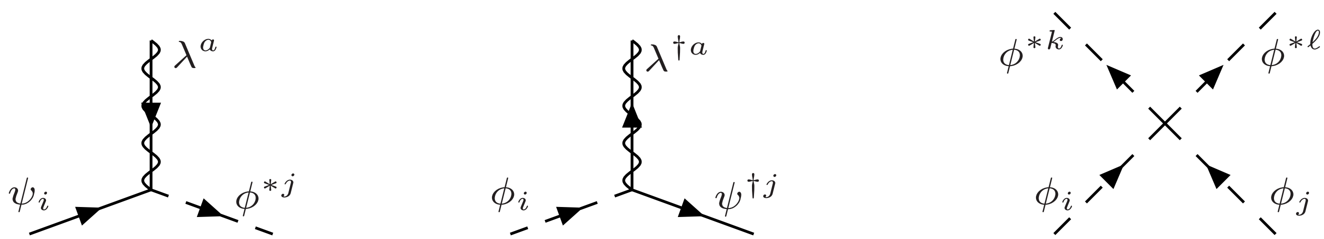
You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY transformations and gauge transformations.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



SUSY also predicts interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



$-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-ig_a^2(T_i^{ak}T_j^{a\ell}+T_i^{a\ell}T_j^{ak})$

These interactions are entirely determined by supersymmetry and the gauge group. Experimental measurements of the magnitudes of these couplings will provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown that the quadratic sensitivity to $M_{\rm UV}$ is still absent in SUSY theories with these SUSY-breaking terms added in:

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j\phi^{*j}\phi_i \\ &\quad -\Big(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\Big),\end{aligned}$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

How to make a SUSY Model:

- Choose a gauge symmetry group.

  In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.

- Choose a superpotential W; must be invariant under the gauge symmetry.

  In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses.

- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.

  Almost all of the unknowns and arbitrariness in the MSSM are here.

Let's do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$\boxed{W_{\text{MSSM}} = \tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{y_e}\tilde{L}H_d + \mu H_u H_d}$$

H_u, H_d, $\tilde{Q}$, $\tilde{L}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\bar{e}}$ are the scalar fields appearing in the left-handed chiral supermultiplets. Tricky notation:

$$Q = (u, d) \equiv (u_L, d_L), \qquad L = (e, \nu) \equiv (e_L, \nu_L),$$
$$\bar{e} \equiv e_R^\dagger, \qquad \bar{u} \equiv u_R^\dagger, \qquad \bar{d} \equiv d_R^\dagger$$

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, they are the same as in the Standard Model.

We need both H_u and H_d, because $\tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_d^*$ and $\tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_u^*$ are not analytic, and so not allowed in the superpotential.

In the approximation that only t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes (in $SU(2)_L$ components):

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+H_d^- - H_u^0H_d^0). \end{aligned}$$

The minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0\rangle = v_u$ and $\langle H_d^0\rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

$$m_t = y_t v_u, \qquad m_b = y_b v_d, \qquad m_\tau = y_\tau v_d.$$

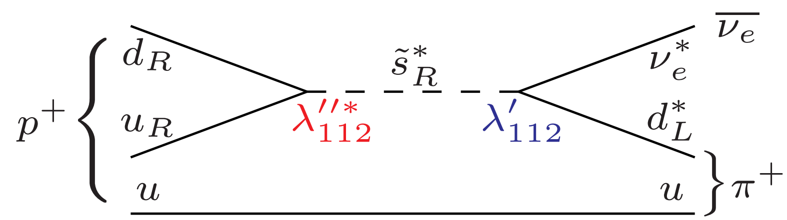
Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \tfrac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u$$

$$W_{\Delta B=1} = \tfrac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

These violate lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot require exact B and L conservation, since they are known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the renormalizable superpotential.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

All of the known Standard Model particles and the Higgs scalar bosons carry $P_R = +1$, while all of the squarks and sleptons and higgsinos and gauginos carry $P_R = -1$.

Matter parity and R-parity are **exactly equivalent**, because the product of $(-1)^{2S}$ for all of the fields in any interaction vertex that conserves angular momentum is always $+1$.

Consequences if R-parity is conserved

The particles with odd R-parity ($P_R = -1$) are the "supersymmetric particles" or "sparticles".

Every interaction vertex in the theory has an even number of $P_R = -1$ sparticles. Then:

- The lightest sparticle with $P_R = -1$, called the "Lightest Supersymmetric Particle" or LSP, is absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so could be the non-baryonic dark matter required by cosmology.
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSPs (usually just one). The LSP escapes the detector, with a missing momentum signature.

The Lightest SUSY Particle as Cold Dark Matter

Recent results in experimental cosmology require cold dark matter with density:

$$\Omega_{\text{CDM}} h^2 = 0.12 \qquad\qquad \text{(WMAP, Planck, ...)}$$

where $h \approx$ 0.7 is the Hubble constant in units of 100 km/(sec Mpc).

A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.12$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

As a crude estimate, a weakly interacting particle that annihilates in collisions with a characteristic mass scale M will have

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \left(\frac{150 \text{ GeV}}{M} \right)^2$$

So, a stable, weakly interacting particle with mass of order 100 GeV is a candidate. In particular, a neutralino LSP ($\tilde{N}_1$) may do it, if R-parity is conserved.

Is R-parity inevitable?

No! Maybe B violation is allowed but L violation isn't. OR maybe L violation is allowed, but B violation isn't.

Or maybe both types of couplings are allowed, but the ones relevant for proton decay are just very small.

Two specific alternatives:

- R-parity could be spontaneously broken, by the VEV of some scalar field with $P_R = -1$.

- **Baryon triality**, a Z_3 discrete symmetry:

$$Z_3^B = e^{2\pi i(B-2Y)/3}$$

  If Z_3^B is multiplicatively conserved, then the proton is absolutely stable, but the LSP is not.

The Soft SUSY-breaking Lagrangian for the MSSM

$$
\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\;& -\tfrac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + \text{c.c.} \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + \text{c.c.} \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_{\tilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\tilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\tilde{\overline{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\tilde{\overline{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\tilde{\overline{e}}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}
$$

The first line gives masses to the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).

The second line consists of (scalar)^3 interactions.

The third line is (mass)^2 terms for the squarks and sleptons.

The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$
\begin{gathered}
M_1,\ M_2,\ M_3,\ \mathbf{a_u},\ \mathbf{a_d},\ \mathbf{a_e}\ \sim\ m_{\rm soft}; \\
\mathbf{m^2_{\tilde{Q}}},\ \mathbf{m^2_{\tilde{L}}},\ \mathbf{m^2_{\tilde{\overline{u}}}},\ \mathbf{m^2_{\tilde{\overline{d}}}},\ \mathbf{m^2_{\tilde{\overline{e}}}},\ m^2_{H_u},\ m^2_{H_d},\ b\ \sim\ m^2_{\rm soft}
\end{gathered}
$$

where $m_{\rm soft}$ is not huge compared to 1 TeV.

The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals have been made.

The question can be answered experimentally by discovering the pattern of gaugino and squark and slepton masses, because they are the main terms in the SUSY-breaking Lagrangian.

Electroweak symmetry breaking and the Higgs bosons

In SUSY, there are two complex Higgs scalar doublets, (H_u^+, H_u^0) and (H_d^0, H_d^-), rather than one in the Standard Model.

The Higgs VEVs can be parameterized:

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle, \qquad \text{where}$$
$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\mathrm{GeV})^2$$
$$\tan\beta = v_u/v_d.$$

The quark and lepton masses are related to these VEVs and the superpotential Yukawa couplings by:

$$y_t = \frac{m_t}{v \sin\beta}, \qquad y_b = \frac{m_b}{v \cos\beta}, \qquad y_\tau = \frac{m_\tau}{v \cos\beta}, \qquad \text{etc.}$$

If we want the Yukawa couplings to avoid getting non-perturbatively large up to very high scales, we need:

$$1.5 \lesssim \tan\beta \lesssim 55$$

Define mass-eigenstate Higgs bosons: h^0, H^0, A^0, G^0, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

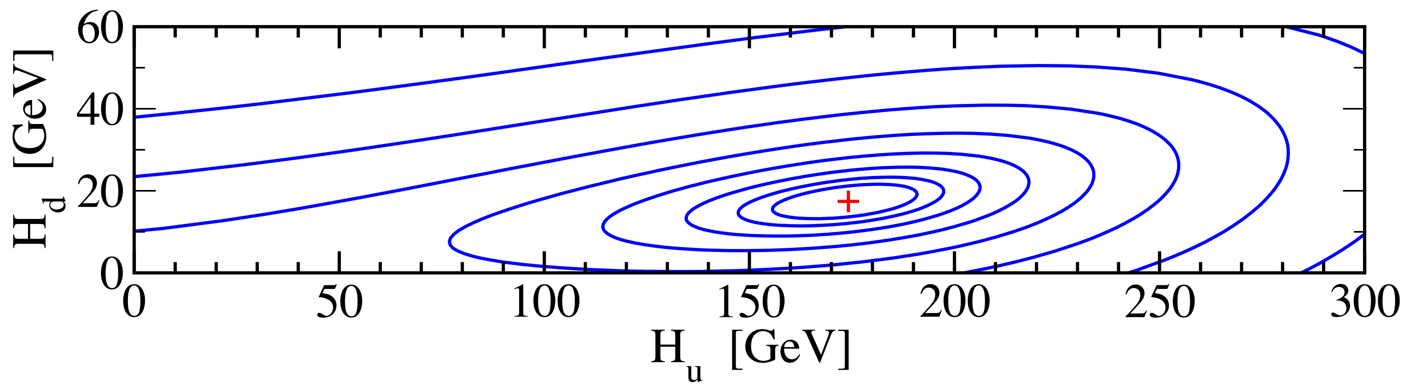
Now, expand the potential to second order in these fields to find:

$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{h^0,H^0}^2 &= \tfrac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\right), \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \\ \tan 2\alpha &= [(m_{A^0}^2 + m_Z^2)/(m_{A^0}^2 - m_Z^2)] \tan 2\beta \end{aligned}$$

Note only two independent parameters: $\tan\beta$, m_{A^0}.

The Goldstone bosons have $m_{G^0} = m_{G^\pm} = 0$; they are absorbed by the Z, $W^\pm$ bosons to give them masses, as in the Standard Model.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson h^0 corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level, it is easy to show from above that the lightest Higgs scalar would obey:

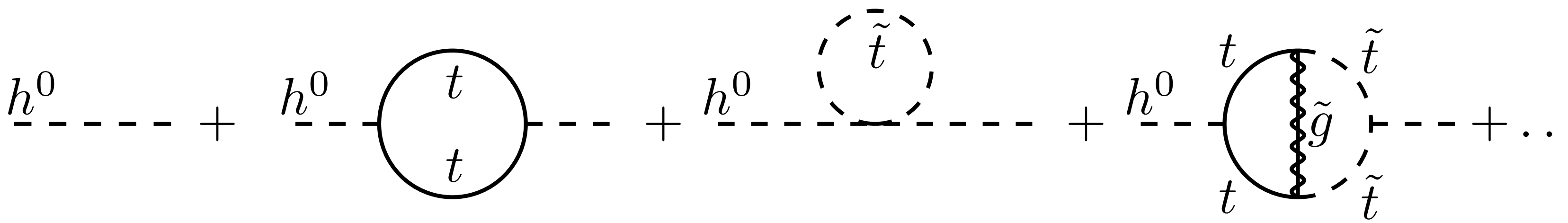
$$m_{h^0} < m_Z.$$

Naively, this disagrees with the recent discovery of $m_{h^0} = 125.5$ GeV.

However, taking into account loop effects, one can get the observed Higgs mass.

Radiative corrections to the Higgs mass in SUSY:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level: m_Z^2 pure electroweak

At one-loop: $y_t^2 m_t^2$ top Yukawa comes in

At two-loop: $\alpha_S y_t^2 m_t^2$ SUSYQCD comes in

At three-loop: $\alpha_S^2 y_t^2 m_t^2$

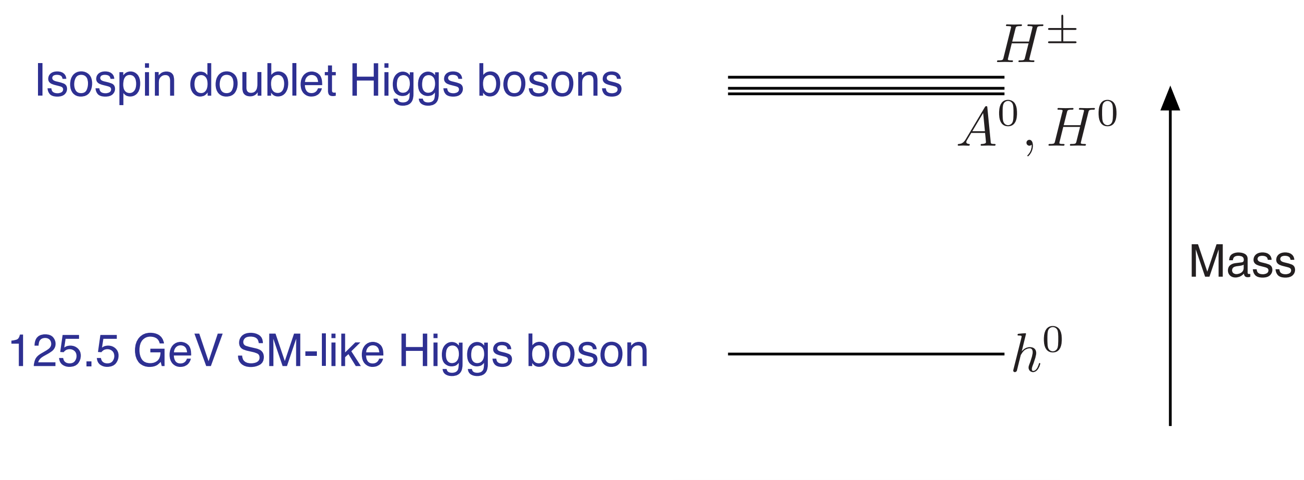
Even the three-loop corrections can add $\pm$1 GeV or so to m_{h^0}.

This is much larger than the eventual experimental uncertainty expected at the LHC, so we aren't done calculating yet!

The decoupling limit for the Higgs bosons

If $m_{A^0} \gg m_Z$, then:

- h^0 has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A^0, H^0, H^\pm$ form an isospin doublet, and are much heavier than h^0



Many (but not all) models of SUSY breaking approximate this decoupling limit.

Neutralinos

The neutral higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$, $\tilde{W}^0$) mix with each other because of electroweak symmetry breaking. In the gauge eigenstate basis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$,

$$\mathcal{L}_{\text{neutralino mass}} = -\tfrac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

The diagonal terms are just the gaugino masses in the soft SUSY-breaking Lagrangian. The $-\mu$ entries can be traced back to the superpotential. The off-diagonal terms come from the gaugino-Higgs-Higgsino interactions, and are always less than m_Z.

The physical neutralino mass eigenstates $\tilde{N}_i$ (another popular notation is $\tilde{\chi}_i^0$) are obtained by diagonalizing the mass matrix with a unitary matrix.

$$\tilde{N}_i = \mathbf{N}_{ij}\psi_j^0,$$

where

$$\begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix} = \mathbf{N}^*\mathbf{M}\mathbf{N}^{-1},$$

with $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$.

The lightest neutralino fermion, $\tilde{N}_1$, is a candidate for the cold dark matter that seems to be required by cosmology.

Charginos

Similarly, the charged higgsinos $\tilde{H}_u^+, \tilde{H}_d^-$ and the charged winos $\tilde{W}^+, \tilde{W}^-$ mix to form **chargino** fermion mass eigenstates.

$$\mathcal{L}_{\text{chargino mass}} = -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + \text{c.c.}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} \quad = \quad \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix} \qquad \text{with} \qquad \mathbf{X} = \begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix}$$

The mass eigenstates $\widetilde{C}^\pm_{1,2}$ (many other sources use $\tilde{\chi}^\pm_{1,2}$) are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}.$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions.

The chargino mixing matrices are chosen so that

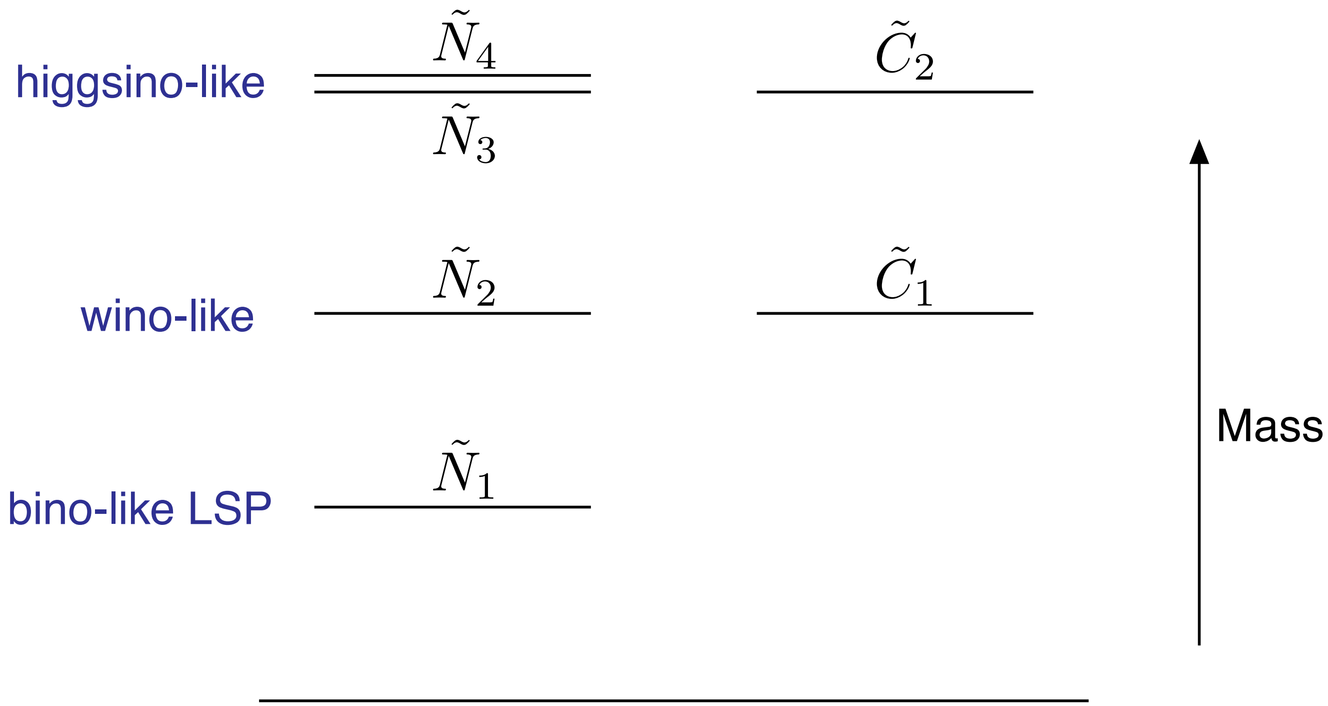
$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with positive real entries $m_{\widetilde{C}_i}$. In this case, one can solve for the tree-level (mass)^2 eigenvalues in simple closed form:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} \quad = \quad & \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \\ & \mp\sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \end{aligned}$$

In many models of SUSY breaking, one finds that $M_2 \ll |\mu|$, so the lighter chargino is mostly wino with mass close to M_2, and the heavier is mostly higgsino with mass close to $|\mu|$.

A typical mass hierarchy for the neutralinos and charginos, assuming $m_Z \ll |\mu|$ and $M_1 \approx 0.5 M_2 < |\mu|$.



Although this is a very popular scenario, it is NOT guaranteed. The lightest states could be the higgsinos, or the winos.

The Gluino

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. Therefore, at tree-level, its mass is the same as the corresponding parameter in the soft SUSY-breaking Lagrangian:

$$M_{\tilde{g}} = M_3$$

However, the quantum corrections to this are quite large (again, because this is a color octet!). If one calculates the one-loop pole mass of the gluino, one finds:

$$M_{\tilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_s}{4\pi}\big[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\big]\Big)$$

where Q is the renormalization scale, the sum is over all 12 squark multiplets, and

$$A_{\tilde{q}} = \int_0^1 dx\, x \ln\big[x m_{\tilde{q}}^2/M_3^2 + (1-x) m_q^2/M_3^2 - x(1-x) - i\epsilon\big].$$

This correction can be of order 5% to 25%, depending on the squark masses! It tends to **increase** the gluino mass, compared to the tree-level value.

Squarks and Sleptons

To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

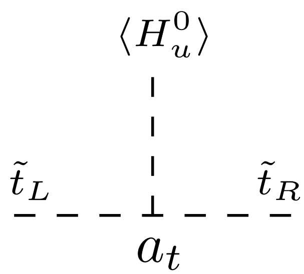
- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R$),
- a 3×3 matrix for sneutrinos ($\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau$)

In many popular models, the first- and second-family squarks and sleptons are in 7 very nearly degenerate, unmixed pairs:

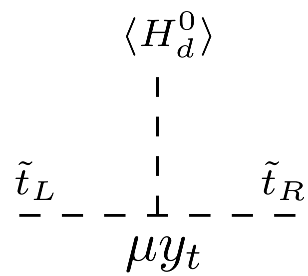
$(\widetilde{e}_R, \widetilde{\mu}_R),\ (\widetilde{\nu}_e, \widetilde{\nu}_\mu),\ (\widetilde{e}_L, \widetilde{\mu}_L),\ (\widetilde{u}_R, \widetilde{c}_R),\ (\widetilde{d}_R, \widetilde{s}_R),\ (\widetilde{u}_L, \widetilde{c}_L),\ (\widetilde{d}_L, \widetilde{s}_L),$

with mixing angles assumed small.

But, for the third-family squarks and sleptons, large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings are definitely important. For the top squark:



and



The first diagram comes directly from the soft SUSY-breaking Lagrangian, and the others from the F-term contribution to the scalar potential. So, in the $(\tilde{t}_L, \tilde{t}_R)$ basis, the top squark (mass)^2 matrix is:

$$\begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + \Delta_{\tilde{t}_L} & a_t^* v_u - \mu y_t v_d \\ a_t v_u - \mu^* y_t v_d & m^2_{\tilde{\bar{u}}_3} + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix}.$$

The off-diagonal terms imply significant $\tilde{t}_L, \tilde{t}_R$ mixing.

Diagonalizing the top squark mass^2 matrix, one finds mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s^*_{\tilde{t}} \\ s_{\tilde{t}} & c^*_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

where $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ by convention, and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$. If they are real, then $c_{\tilde{t}} = \cos\theta_{\tilde{t}}$ and $s_{\tilde{t}} = \sin\theta_{\tilde{t}}$.

Similarly, mixing for the bottom squark and tau slepton states:

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{b}} & -s^*_{\tilde{b}} \\ s_{\tilde{b}} & c^*_{\tilde{b}} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix};$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{\tau}} & -s^*_{\tilde{\tau}} \\ s_{\tilde{\tau}} & c^*_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

To avoid flavor constraints, often **assume** for the first- and second-family squarks and sleptons that the mixing is small, due to small Yukawa and a terms. However, the mixing could be large if they are heavy.

The undiscovered particles in the MSSM:

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	h^0 H^0 A^0 $H^\pm$	H_u^0 H_d^0 H_u^+ H_d^-
squarks	0	-1	$\widetilde{u}_L$ $\widetilde{u}_R$ $\widetilde{d}_L$ $\widetilde{d}_R$	" "
			$\widetilde{s}_L$ $\widetilde{s}_R$ $\widetilde{c}_L$ $\widetilde{c}_R$	" "
			$\widetilde{t}_1$ $\widetilde{t}_2$ $\widetilde{b}_1$ $\widetilde{b}_2$	$\widetilde{t}_L$ $\widetilde{t}_R$ $\widetilde{b}_L$ $\widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L$ $\widetilde{e}_R$ $\widetilde{\nu}_e$	" "
			$\widetilde{\mu}_L$ $\widetilde{\mu}_R$ $\widetilde{\nu}_\mu$	" "
			$\widetilde{\tau}_1$ $\widetilde{\tau}_2$ $\widetilde{\nu}_\tau$	$\widetilde{\tau}_L$ $\widetilde{\tau}_R$ $\widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\widetilde{N}_1$ $\widetilde{N}_2$ $\widetilde{N}_3$ $\widetilde{N}_4$	$\widetilde{B}^0$ $\widetilde{W}^0$ $\widetilde{H}_u^0$ $\widetilde{H}_d^0$
charginos	$1/2$	-1	$\widetilde{C}_1^\pm$ $\widetilde{C}_2^\pm$	$\widetilde{W}^\pm$ $\widetilde{H}_u^+$ $\widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "

Lecture 3: Experimental Signatures of Supersymmetry

- What flavor teaches us about SUSY breaking
- Planck-scale Mediated SUSY Breaking
- mSUGRA/CMSSM
- Patterns of SUSY breaking
- Superpartner Decays

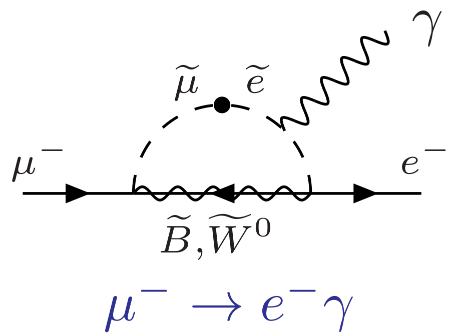
There are 105 new parameters associated with SUSY breaking in the MSSM.

How are we supposed to make any meaningful predictions in the face of this uncertainty?

Fortunately, we already have strong constraints on the MSSM soft terms, because of experimental limits on flavor violation.

Hints of an Organizing Principle

For example, if there is a smuon-selectron mixing (mass)^2 term $\mathcal{L} = -m^2_{\tilde{\mu}^*_L \tilde{e}_L} \tilde{\mu}^*_L \tilde{e}_L$, and $\tilde{M} =$ Max$[m_{\tilde{e}_L}, m_{\tilde{e}_R}, M_2]$, then by calculating this one-loop diagram, one finds the decay width:



$$\Gamma(\mu^- \to e^- \gamma) = 5 \times 10^{-19}\ \text{eV} \left(\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} \right)^2 \left(\frac{1000\ \text{GeV}}{\tilde{M}} \right)^4$$

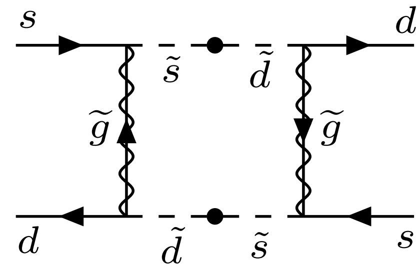
For comparison, the experimental limit is (from MEG at PSI):

$$\Gamma(\mu^- \to e^- \gamma) < 7 \times 10^{-22}\ \text{eV}.$$

So the amount of smuon-selectron mixing in the soft Lagrangian is limited by:

$$\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} < 0.038 \left(\frac{\tilde{M}}{1000\ \text{GeV}} \right)^2$$

Another example: $K^0 \leftrightarrow \overline{K^0}$ mixing:



$K^0 \leftrightarrow \overline{K^0}$

This constrains the flavor-violating SUSY breaking terms:

$$\mathcal{L} = -m^2_{\tilde{d}^*_L \tilde{s}_L} \tilde{d}^*_L \tilde{s}_L - m^2_{\tilde{d}_R \tilde{s}^*_R} \tilde{d}_R \tilde{s}^*_R.$$

Comparing this diagram with the observed Δm_{K_0} gives:

$$\frac{\mathrm{Re}[m^2_{\tilde{d}^*_L \tilde{s}_L} m^2_{\tilde{d}_R \tilde{s}^*_R}]^{1/2}}{\tilde{M}^2} \lesssim 0.002 \left(\frac{\tilde{M}}{\text{1000 GeV}} \right)$$

where $\tilde{M}$ is the dominant squark or gluino mass.

The experimental values of ϵ and ϵ'/ϵ in the effective Hamiltonian for the $K^0, \overline{K^0}$ system also give strong constraints on the amount of $\tilde{d}_L, \tilde{s}_L$ and $\tilde{d}_R, \tilde{s}_R$ mixing and CP violation in the soft terms.

Similarly:

The $D^0, \overline{D^0}$ system constrains $\tilde{u}_L, \tilde{c}_L$ and $\tilde{u}_R, \tilde{c}_R$ soft SUSY-breaking mixing.

The $B_d^0, \overline{B_d^0}$ system constrains $\tilde{d}_L, \tilde{b}_L$ and $\tilde{d}_R, \tilde{b}_R$ soft SUSY-breaking mixing.

To avoid experimental limits on flavor violation, the soft-SUSY breaking masses must be either

- **nearly flavor-bind**, or
- **aligned** (see Gilad Perez lecture), or
- **very heavy (well over 1000 GeV)**

The last option is becoming more popular recently.

The Flavor-Preserving Minimal Supersymmetric Standard Model

Idealized limit: the squark and slepton $(\text{mass})^2$ matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space.

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}} \mathbf{1}, \quad \mathbf{m^2_{\tilde{\bar{u}}}} = m^2_{\tilde{\bar{u}}} \mathbf{1}, \quad \mathbf{m^2_{\tilde{\bar{d}}}} = m^2_{\tilde{\bar{d}}} \mathbf{1}, \quad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}} \mathbf{1}, \quad \mathbf{m^2_{\tilde{\bar{e}}}} = m^2_{\tilde{\bar{e}}} \mathbf{1}.$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will.

Also assume:

$$\mathbf{a_u} = A_{u0}\, \mathbf{y_u}; \qquad \mathbf{a_d} = A_{d0}\, \mathbf{y_d}; \qquad \mathbf{a_e} = A_{e0}\, \mathbf{y_e},$$

and no new CP-violating phases:

$$M_1,\ M_2,\ M_3,\ A_{u0},\ A_{d0},\ A_{e0} = \text{real}.$$

The Flavor-Preserving Minimal Supersymmetric Standard Model (continued)

The new parameters, besides those already found in the Standard Model, are:

- M_1, M_2, M_3 (3 real gaugino masses)
- $m^2_{\tilde{Q}}, m^2_{\tilde{\bar{u}}}, m^2_{\tilde{\bar{d}}}, m^2_{\tilde{L}}, m^2_{\tilde{\bar{e}}}$ (5 squark and slepton mass^2 parameters)
- A_{u0}, A_{d0}, A_{e0} (3 real scalar^3 couplings)
- $m^2_{H_u}, m^2_{H_d}, b, \mu$ (4 real parameters)

So there are 15 real parameters in this model.

The parameters μ and $b \equiv B\mu$ are often traded for the known Higgs VEV $v = 174$ GeV, $\tan\beta$, and sign(μ).

Many SUSY breaking models are special cases of this.

However, these are Lagrangian parameters that run with the renormalization scale, Q. Therefore, one must also choose an "input scale" Q_0 where the flavor-independence holds.

What is the input scale Q_0 ?

Perhaps:

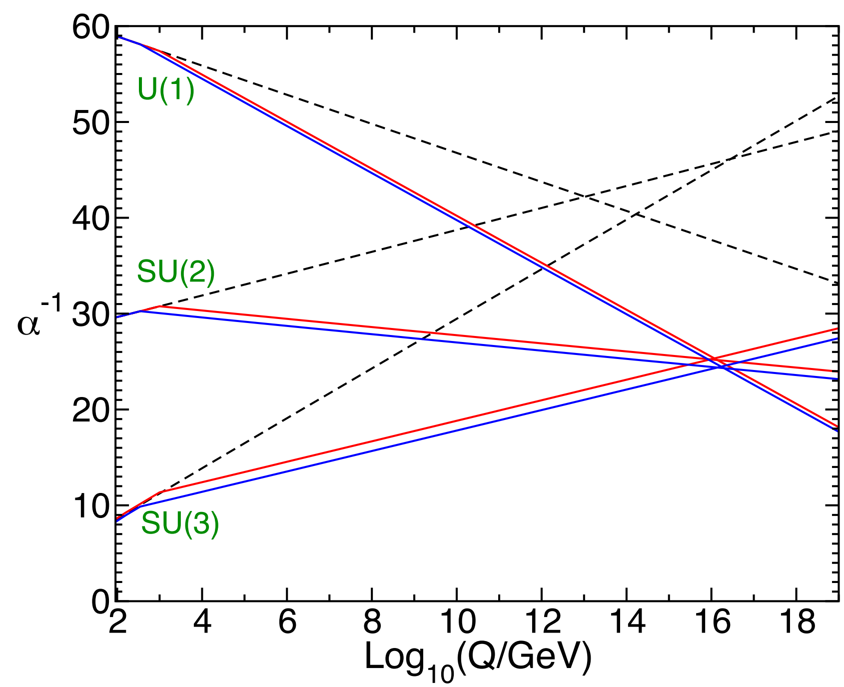
- $Q_0 = M_{\text{Planck}}$, or
- $Q_0 = M_{\text{string}}$, or
- $Q_0 = M_{\text{GUT}}$, or
- Q_0 is some other scale associated with the type of SUSY breaking.

In any case, the SUSY-breaking parameters are picked at Q_0 as boundary conditions, then run them down to the weak scale using their renormalization group (RG) equations.

Flavor violation will remain small, because the Yukawa couplings of the first two families are small.

At the weak scale, use the renormalized parameters to predict physical masses, decay rates, cross-sections, dark matter relic density, etc.

A reason to be optimistic that this program can succeed: the SUSY unification of gauge couplings. The measured α_1, α_2, α_3 are run up to high scales using the RG equations of the Standard Model (dashed lines) and the MSSM (solid lines).



At one-loop order, the RG equations are:

$$\frac{d}{d(\ln Q)}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad\qquad (a = 1, 2, 3)$$

with $b_a^{\rm SM}=(41/10,-19/6,-7)$ in the Standard Model, and $b_a^{\rm MSSM}=(33/5,1,-3)$ in the MSSM because of the extra particles in the loops. The results for the MSSM are in agreement with unification at $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV.

If this hint is real, we might hope that a similar extrapolation for the soft SUSY-breaking parameters can also work.

Origins of SUSY breaking

Up to now, we have simply put SUSY breaking into the MSSM explicitly.

For deeper understanding, how can SUSY spontaneously broken?

This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad\qquad Q^\dagger_{\dot\alpha}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\left(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\right) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

In SUSY, the potential energy can be written, using the equations of motion, as:

$$V = \sum_i |F_i|^2 + \tfrac{1}{2} \sum_a D^a D^a,$$

a sum of squares of auxiliary fields. So, for spontaneous SUSY breaking, one must arrange a stable (or quasi-stable) ground state with either $\langle F_i \rangle \neq 0$ or $\langle D^a \rangle \neq 0$, for at least one i or a.

Models of SUSY breaking where

- $\langle F_i \rangle \neq 0$ are called “O'Raifeartaigh models” or “F-term breaking models”
- $\langle D^a \rangle \neq 0$ are called “Fayet-Iliopoulis models” or “D-term breaking models”

F-term breaking is used in (almost) all known realistic models.
This can only happen if the chiral supermultiplet containing it is a singlet.

F-term breaking: the O'Raifeartaigh Model

The simplest example has $n = 3$ chiral supermultiplets, with ϕ_1 required to be a singlet, and:

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

Then the auxiliary fields are:

$$F_1 = \frac{\partial W}{\partial \phi_1} = k - \frac{y}{2}\phi_3^{*2}, \qquad F_2 = -m\phi_3^*, \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*.$$

The reason SUSY must be broken is that $F_1 = 0$ and $F_2 = 0$ are not compatible. The minimum of this potential is at $\phi_2 = \phi_3 = 0$, with ϕ_1 not determined (classically). Quantum corrections fix the true minimum to be at $\phi_1 = 0$. At the minimum:

$$F_1 = k, \qquad V = k^2 > 0.$$

F-term breaking (continued)

If you assume $m^2 > yk$ and expand the scalar fields around the minimum at $\phi_1 = \phi_2 = \phi_3 = 0$, you will find 6 real scalars with tree-level squared masses:

$$0,\ \ 0,\ \ m^2,\ \ m^2,\ \ m^2 - yk,\ \ m^2 + yk.$$

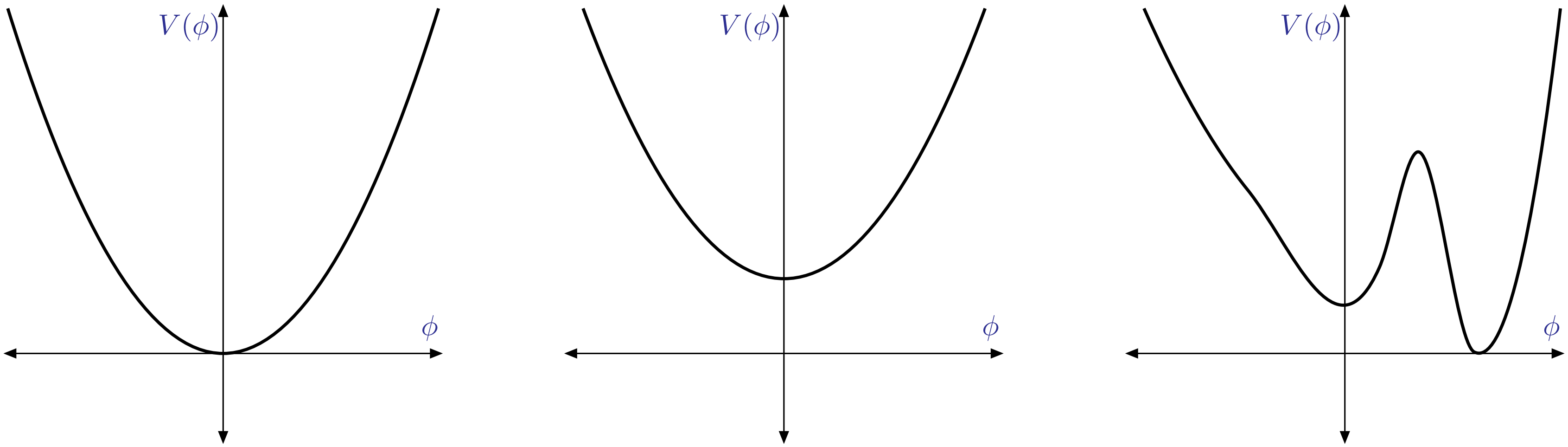
Meanwhile, there are 3 Weyl fermions with squared masses

$$0,\ \ m^2,\ \ m^2.$$

The fact that the fermions and scalars aren't degenerate is a clear sign that SUSY has indeed been spontaneously broken.

This theory always breaks SUSY at the true minimum of the potential, for any values of the superpotential parameters.

Behavior of the scalar potential as a function of some order parameter ϕ:



SUSY unbroken SUSY broken meta-stable SUSY breaking

Meta-stable SUSY breaking is acceptable if the tunneling lifetime to decay from our SUSY-breaking vacuum (with $\phi = 0$ here) to the global minimum SUSY-preserving vacuum is longer than the age of the universe.

Intriligator, Seiberg, Shih arXiv:hep-th/0602239 showed that this can work in simple, uncontrived SUSY Yang-Mills models.

An even simpler example: adding a small term $\epsilon\phi_2^2$ to the O'Raifeartaigh superpotential turns it into a meta-stable SUSY breaking model. (Try it!)

Spontaneous Breaking of SUSY requires us to extend the MSSM

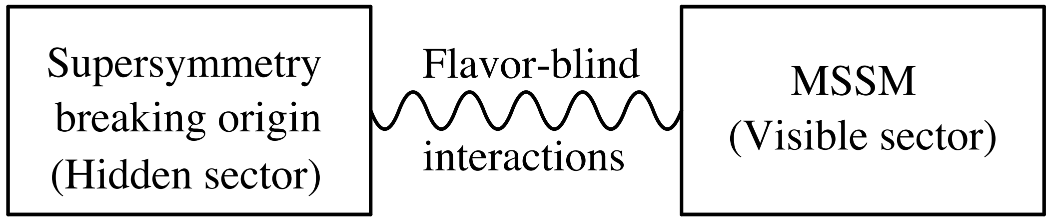
MSSM has no gauge-singlet chiral supermultiplet that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any (gaugino)-(gaugino)-(scalar) coupling that could turn into a gaugino mass term when a scalar gets a VEV.

We also have the clue that SUSY breaking must be essentially flavor-blind in order to not conflict with experiment.

This leads to the following general schematic picture of SUSY breaking. . .

The MSSM soft SUSY-breaking terms arise indirectly or radiatively, not from tree-level renormalizable couplings directly to the SUSY-breaking sector.



Spontaneous SUSY breaking occurs in a "hidden sector" of particles with no (or tiny) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly. As a bonus, if the mediating interactions are flavor-blind, then the soft SUSY-breaking terms of the MSSM will be also.

By dimensional analysis,

$$m_{\mathrm{soft}} \sim \frac{\langle F \rangle}{M}$$

where M is a mass scale associated with the physics that mediates between the two sectors.

The O'Raifeartaigh model has the mass scale of supersymmetry breaking put in by hand, as the parameter $k = \sqrt{\langle F \rangle}$.

More plausible: dynamical SUSY breaking, in which the scale of $\langle F \rangle$ arises from some strong dynamics, set by the scale at which a new gauge theory gets strong:

$$\Lambda = e^{-8\pi^2/bg^2} M_{\text{Planck}}$$

just as in QCD.

Then the field that breaks supersymmetry might be a composite made of strongly interacting fundamental fields.

Some great reviews on this subject:

Intriligator Seiberg hep-ph/0702069

Dine and Mason hep-th/1012.2836

Poppitz and Trivedi hep-th/9803107

Shadmi and Shirman hep-th/9907225

Planck-scale Mediated SUSY Breaking (also known as "gravity mediation")

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since m_{soft} must vanish in the limit that SUSY breaking is turned off $(\langle F \rangle \to 0)$ and in the limit that gravity becomes irrelevant $(M_P \to \infty)$.

Since we think $m_{\text{soft}} \sim 10^3$ GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\begin{aligned}\mathcal{L}_{\text{PMSB}} &= -\Big(\frac{f^a}{2M_P} F\lambda^a\lambda^a + \text{c.c.}\Big) - \frac{k_i^j}{M_P^2} FF^*\phi_i\phi^{*j} \\ &\quad -\Big(\frac{\alpha^{ijk}}{6M_P} F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P} F\phi_i\phi_j + \text{c.c.}\Big)\end{aligned}$$

This is (part of) a fully supersymmetric Lagrangian that arises in supergravity. When we replace F by its VEV $\langle F\rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian, with:

- Gaugino masses: $M_a = f^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious that these are flavor-blind!

A dramatically simplified parameter space is often called “Minimal Supergravity” (or “mSUGRA”) or the “Constrained MSSM”.

Assume only four parameters $m_{1/2}$, m_0^2, A_0, and B_0:

$$
\begin{aligned}
&M_3 = M_2 = M_1 = m_{1/2} \\
&\mathbf{m^2_{\tilde{Q}}} = \mathbf{m^2_{\bar{\tilde{u}}}} = \mathbf{m^2_{\bar{\tilde{d}}}} = \mathbf{m^2_{\tilde{L}}} = \mathbf{m^2_{\bar{\tilde{e}}}} = m_0^2\,\mathbf{1} \\
&m^2_{H_u} = m^2_{H_d} = m_0^2 \\
&\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e} \\
&b = B_0\mu.
\end{aligned}
$$

The most important thing to know about mSUGRA is that it is almost certainly wrong!

These soft relations should be true at the renormalization scale $Q_0 = M_P$, and then run down to the weak scale.

However, it is traditional to use $Q_0 = M_{\rm GUT}$ instead, **because nobody knows how to extrapolate above $M_{\rm GUT}$.** (Not a very good reason!)

Renormalization Group Running for an mSUGRA model with

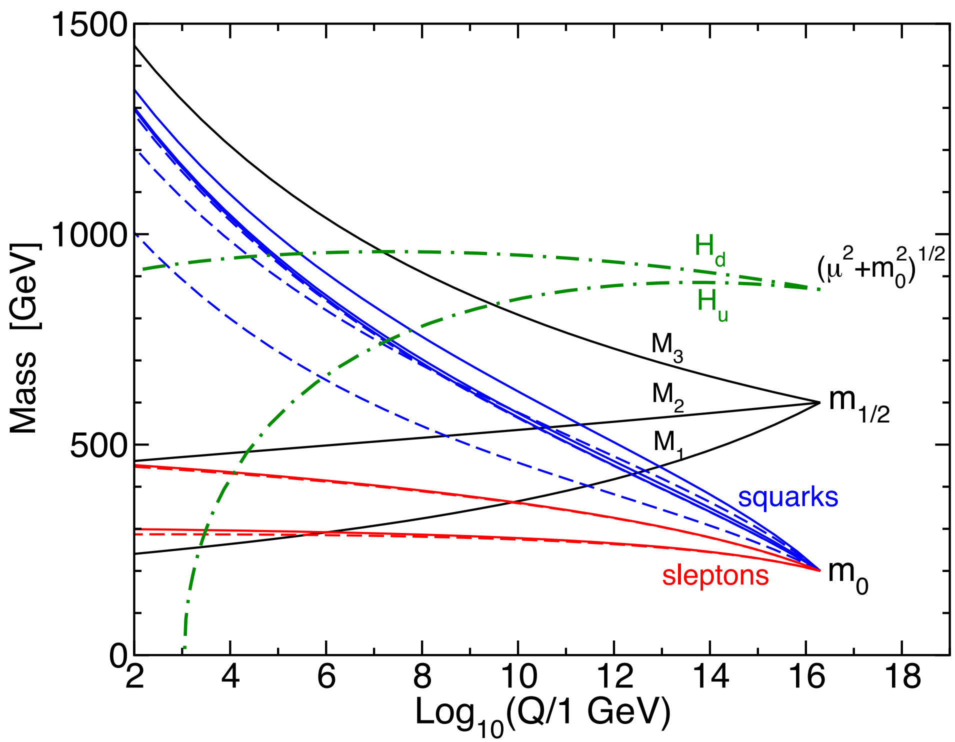
$m_{1/2} = 600$ GeV, $m_0 = 200$ GeV, $A_0 = -600$ GeV, $\tan\beta = 10$, $\mu > 0$

Gaugino masses M_1, M_2, M_3

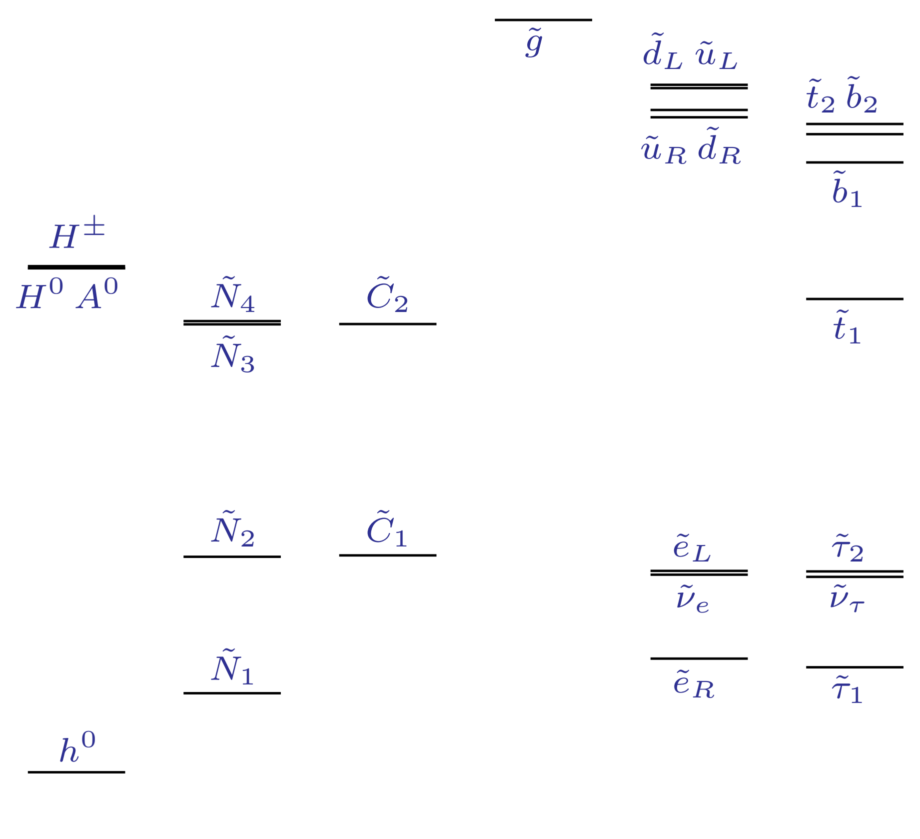
Slepton masses (dashed=stau)

Squark masses (dashed=stop)

Higgs: $(m^2_{H_d} + \mu^2)^{1/2}$,
$(m^2_{H_u} + \mu^2)^{1/2}$



Here is the resulting sparticle mass spectrum:



$M_{\tilde{g}} =$ 1500 GeV

$M_{\rm squarks} =$ 1300 GeV

$M_h =$ 118 GeV

This model was OK only last year, but is completely ruled out now!

Squarks and gluino would have been found at LHC in 20 fb^{-1} data.

Predicts $M_h \approx$ 118 GeV = too light.

Impact of the discovery $M_h = 125.5$ GeV in the MSSM

In the decoupling limit:

$$\begin{aligned} M_h^2 &= m_Z^2 \cos^2(2\beta) \\ &\quad + \frac{3}{4\pi^2} y_t^2 m_t^2 \left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \sin^2(2\theta_{\tilde{t}}) F_1 - \sin^4(2\theta_{\tilde{t}}) F_2 \right] + \ldots \end{aligned}$$

where F_1 and F_2 are certain positive functions of $m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2}$.

To get $M_h = 125.5$ GeV, need

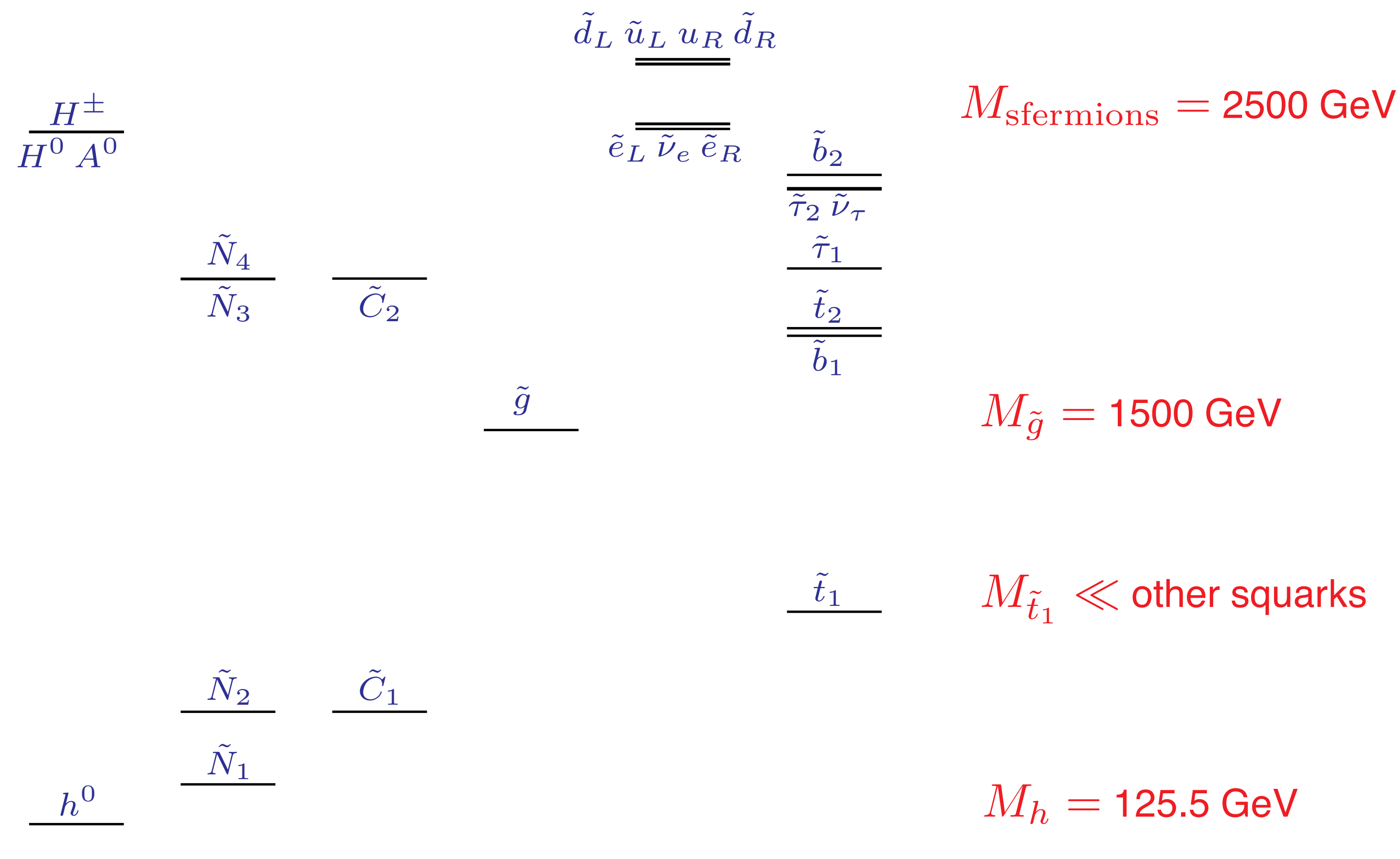
- heavy top squarks $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gg m_t^2$,

  and/or

- large stop mixing $\sin(2\theta_{\tilde{t}})$, in which case $\Big[\ldots\Big] \lesssim \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + 3$

The level-repulsion associated with large stop mixing suggests that one of the stop masses is much lighter than the other.

So what's left for mSUGRA? Here's a typical model that survives:



$M_{1/2} =$ 600 GeV, $m_0 =$ 2500 GeV, $A_0 = -$5000 GeV, $\tan\beta = 30.$

This model survives all LHC constraints . . . for now. (More on this later.)

To accomplish this, moved the squark masses out of reach.

Computer programs can generate the superpartner mass spectrum for you, given a choice of SUSY-breaking model parameters. Some publically available ones:

- `SOFTSUSY` (Allanach)
- `SuSpect` (Djouadi, Kneur, Moultaka)
- `SPheno` (Porod)
- `ISASUSY` (Paige, Protopopescu, Baer, Tata)

These can be interfaced, through an agreed set of conventions called the SUSY Les Houches Accords (SLHA) to programs that produce cross-sections, decay rates, and Monte Carlo events: `MadGraph/MadEvent`, `Pythia`, `ISAJET`, `HERWIG`, `WHIZARD`, `SUSYGEN`, `SDECAY`, `HDECAY`, `GRACE`, `CompHEP`, `CalcHEP`, `PROSPINO`, ...

The can also be interfaced to programs that compute the abundance of dark matter and dark matter detection signals: `micrOMEGAs`, `DarkSUSY`, `ISAReD`.

SUSY signatures at colliders

I will concentrate mostly on models with conserved R-parity and a neutralino LSP dark matter candidate $(\tilde{N}_1)$. Recall:

- The most important interactions for producing sparticles are gauge interactions, and interactions related to gauge interactions by SUSY.
- The production rate is known, up to mixing of sparticles, because of SUSY prediction of couplings.
- Two sparticles produced in each event (but not always with opposite momenta!)
- The LSPs are neutral and extremely weakly interacting, so they carry away energy and momentum.
- At hadron colliders, the component of the momentum along the beam is unknown, so only the energy component in particles transverse to the beam is observable. So one sees "missing transverse energy", $E_T^{\rm miss}$.

Superpartner decays:

1) Neutralino decays

2) Chargino decays

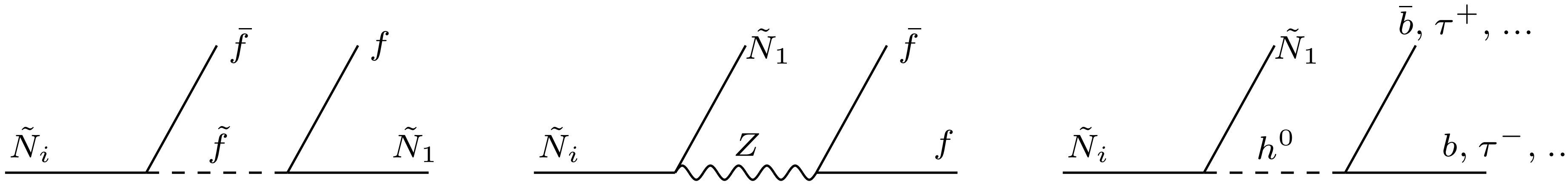
3) Gluino decays

4) Squark decays (especially stops)

5) Slepton decays

1) Neutralino Decays

If R-parity is conserved and $\tilde{N}_1$ is the LSP, then it cannot decay. For the others, the decays are of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, Z boson, or Higgs boson h^0) might be on-shell, if that two-body decay is kinematically allowed.

In general, the visible decays are either:

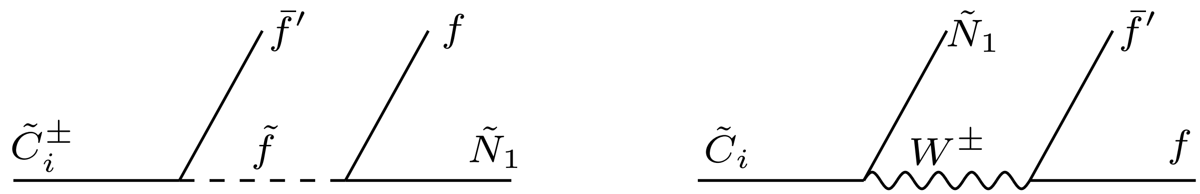
$$\tilde{N}_i \to Q\bar{Q}\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$

$$\tilde{N}_i \to \ell^+\ell^-\tilde{N}_1 \qquad \text{(seen in detector as } \ell^+\ell^- + \not{E}\text{)}$$

Some SUSY signals rely on leptons in the final state. This is more likely if sleptons are relatively light. If $\tilde{N}_i \to \tilde{N}_1 h^0$ is kinematically open, then it often dominates. Can one use the known Higgs mass to enhance the signal?

2) Chargino Decays

Charginos $\tilde{C}_i$ have decays of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, or W boson) might be on-shell, if that two-body decay is kinematically allowed.

In general, the decays are either:

$$\tilde{C}_i^{\pm} \rightarrow Q\bar{Q}'\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$

$$\tilde{C}_i^{\pm} \rightarrow \ell^{\pm}\nu\tilde{N}_1 \qquad \text{(seen in detector as } \ell^{\pm} + \not{E}\text{)}$$

Again, leptons in final state are more likely if sleptons are relatively light.

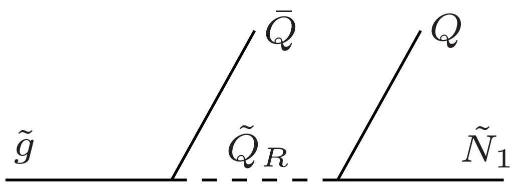
For both neutralinos and charginos, a relatively light, mixed $\tilde{\tau}_1$ can lead to enhanced τ's in the final state. This is increasingly important for larger $\tan\beta$. Tau identification may be a crucial limiting factor for experimental SUSY.

3) Gluino Decays

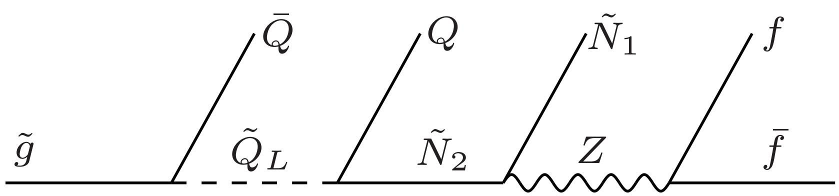
The gluino can only decay through squarks, either on-shell (if allowed) or virtual.

If $m_{\tilde{t}_1} \ll$ other squark masses, top quarks are plentiful in these decays.

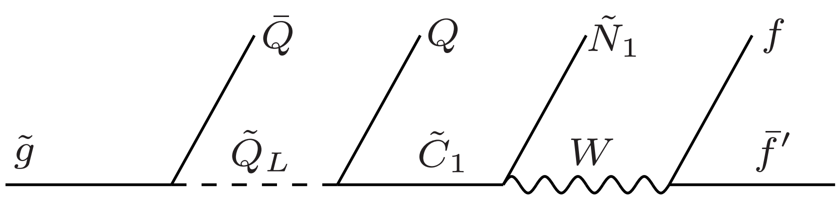
For example:



$jj + \not{E}$ or $t\bar{t} + \not{E}$



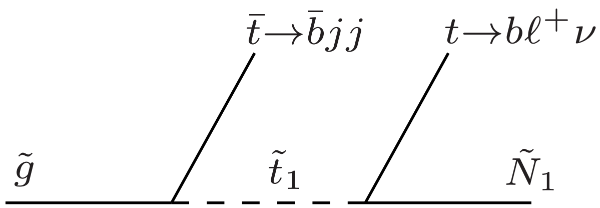
$jjjj + \not{E}$ or $jj\ell^+\ell^- + \not{E}$ or
$t\bar{t}jj + \not{E}$ or $t\bar{t}\ell^+\ell^- + \not{E}$



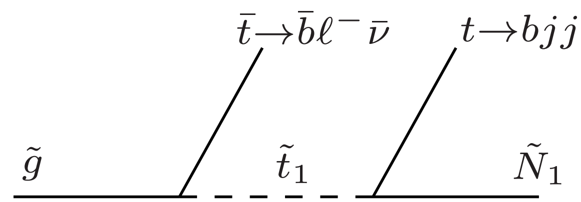
$jjjj + \not{E}$ or $jj\ell^{\pm} + \not{E}$ or
$t\bar{t}jj + \not{E}$ or $t\bar{t}\ell^{\pm} + \not{E}$

The possible signatures of gluinos and squarks can be numerous and complicated because of these and other **cascade decays**.

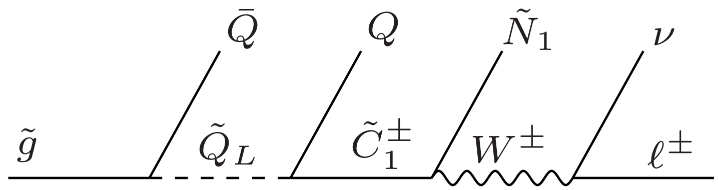
An important feature of gluino decays with one lepton, for example:



or



or



or . . .

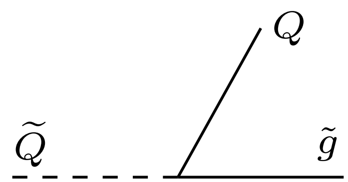
The lepton has either charge with equal probability. (The gluino does not “know” about electric charge.) So, when two gluinos are produced, probability 0.5 to have **same-charge leptons**, and probability 0.5 to have opposite-charge leptons.

$$\text{(SUSY)} \to \ell^\pm \ell'^\pm + \text{ jets } + E_T^{\text{miss}}$$

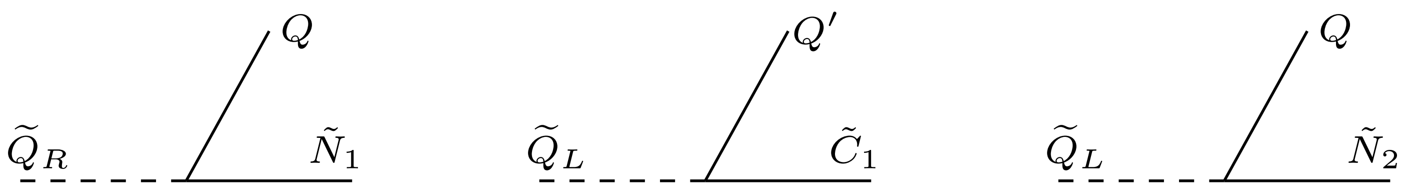
Same-charge lepton signals are important at the LHC, because Standard Model backgrounds are much smaller. Note lepton flavors are uncorrelated. Event may also have 2 or 4 taggable b jets.

4) Squark Decays

If a decay $\tilde{Q} \to Q\tilde{g}$ is kinematically allowed, it will always dominate, because the squark-quark-gluino vertex has QCD strength:



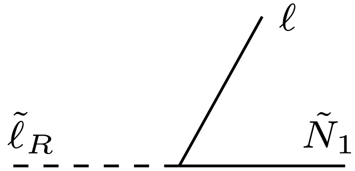
Otherwise, right-handed squarks prefer to decay directly to a bino-like LSP, while left-handed squarks prefer to decay to a wino-like $\tilde{C}_1$ or $\tilde{N}_2$:



If a top squark is light, then the decays $\tilde{t}_1 \to t\tilde{g}$ and $\tilde{t}_1 \to t\tilde{N}_1$ may not be kinematically allowed, and it may decay only into charginos: $\tilde{t}_1 \to b\tilde{C}_1$. If those decays are also closed, it has $\tilde{t}_1 \to bW\tilde{N}_1$. If even that is closed, it has only a suppressed flavor-changing decay $\tilde{t}_1 \to c\tilde{N}_1$ or 4-body decay $\tilde{t}_1 \to bf\bar{f}'\tilde{N}_1$.

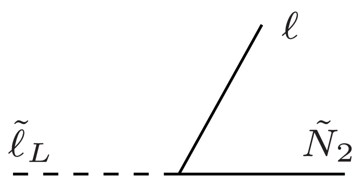
5) Slepton Decays

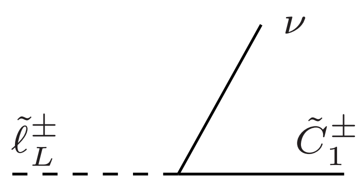
When $\tilde{N}_1$ is the LSP and has a large bino content, the sleptons $\tilde{e}_R$, $\tilde{\mu}_R$ (and often $\tilde{\tau}_1$ and $\tilde{\tau}_2$) prefer the direct two-body decays with strength proportional to g'^2:

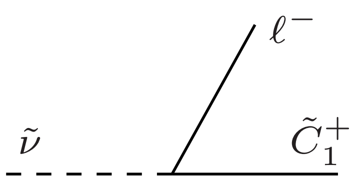


(seen in detector as $\ell^{\pm} + \not{E}$)

However, the left-handed sleptons $\tilde{e}_L$, $\tilde{\mu}_L$, $\tilde{\nu}$ have no coupling to the bino component of $\tilde{N}_1$, so they often decay preferentially through $\tilde{N}_2$ or $\tilde{C}_1$, which have a large wino content, with strength proportional to g^2:







with $\tilde{N}_2$ and $\tilde{C}_1$ decaying as before.

Lecture 4: Experimental searches for supersymmetry

- SUSY search strategies
- Results of LHC searches
- The remaining SUSY parameter space; where SUSY might be hiding
- Impact of the Higgs discovery on SUSY
- Future searches
- Why I'm still optimistic about SUSY at the LHC

SUSY Limits from LEP2 e^+e^- collisions up to $\sqrt{s} = 208$ GeV

The CERN LEP2 collider had the capability of producing all sparticle-antisparticle pairs, except for the gluino:

$$e^+e^- \to \tilde{\ell}^+\tilde{\ell}^-,\ \tilde{C}_1^+\tilde{C}_1^-,\ \tilde{N}_1\tilde{N}_2,\ \tilde{N}_2\tilde{N}_2,\ \gamma\tilde{N}_1\tilde{N}_1,\ \tilde{Q}\tilde{Q}^*$$

Exclusions for charged sparticles are typically close to the kinematic limit, except when mass difference are small. For example, at 95% CL:

$$m_{\tilde{C}_1} > 103 \text{ GeV} \qquad (m_{\tilde{C}_1} - m_{\tilde{N}_1} > 3 \text{ GeV or } < 100 \text{ MeV})$$

$$m_{\tilde{C}_1} > 92 \text{ GeV} \qquad (\text{any heavier than } \tilde{N}_1)$$

and

$$m_{\tilde{e}_R} > 100 \text{ GeV} \qquad (m_{\tilde{e}_R} - m_{\tilde{N}_1} > 5 \text{ GeV})$$

See `http://lepsusy.web.cern.ch/lepsusy/`

for detailed results. These are still the strongest published limits in some cases!

Fermilab Tevatron limits on SUSY have been mostly overtaken by LHC limits, because they are both hadron colliders and 8 TeV $>$ 1.96 TeV.

An exception is the case of light top squarks, which I will show a little later.

The LHC vs. Supersymmetric Models, 2010-2013

Many SUSY models have been eliminated just by the discovery of a Standard Model-like Higgs boson with $M_h =$ 125.5 GeV. As discussed earlier, getting h to be this heavy seems to require top squarks that are heavy and/or highly mixed.

However, **non-minimal** SUSY models, can easily get $M_h = 125.5$ GeV by adding to the h^4 coupling in the effective potential. This makes the potential steeper, increasing M_h^2. Two examples:

- MSSM plus an extra singlet that couples to the Higgs.

  Next-to-Minimal Supersymmetric Standard Model (NMSSM) adds to h^4 coupling at tree-level.

  For a review, see Ellwanger, Teixeira, Hugonie 0910.1785.

- MSSM plus vector-like quarks that couple to the Higgs but get their mass mostly from bare couplings.

  One-loop effective action gives a positive radiative correction to h^4 coupling.

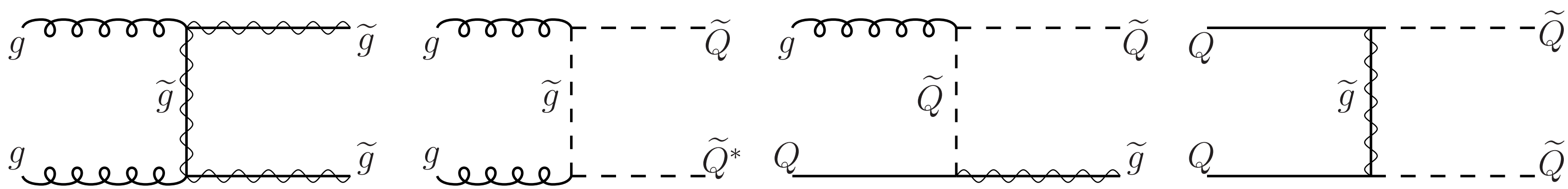
  Moroi and Okada MPLA 7, 187, (1992), PLB 295, 73, (1992), Babu, Gogaladze and Kolda hep-ph/0410085, Babu, Gogoladze, Rehman, Shafi hep-ph/0807.3055, SPM arXiv:0910.2732.

LHC Signals for SUSY in pp collisions

The LHC is a gluon-gluon and gluon-quark collider, to first approximation. The dominant production cross-sections are:

$$pp \to \tilde{g}\tilde{g}, \;\; \tilde{g}\tilde{Q}, \;\; \tilde{Q}\tilde{Q}^*, \;\; \tilde{Q}\tilde{Q}$$

Some sample production diagrams:



After decays, get high-p_T jets, possibly leptons from the decays, and large $E_T^{\rm miss}$ from $\tilde{N}_1$ LSPs escaping detector.

Backgrounds come from QCD jets with mis-measured energies, and $t\bar{t}$, and $W+$ jets, and $Z+$ jets, and WW, and ZZ, and WZ.

Some useful discovery discriminants:

- **Missing transverse energy**: $E_T^{\text{miss}} = |\sum_{\text{visible objects}} \vec{p}_T|$
- **Effective mass**: $M_{\text{eff}} = E_T^{\text{miss}} + \sum_i p_T^{\text{jet},i} + \sum_i p_T^{\text{lepton},i}$.

  If SUSY discovered, also gives a rough mass measurement.
- $H_T = \sum_i p_T^{\text{jet},i}$
- More complicated quantities α_T, M_{T2}, razor variables. Used more often by CMS than by ATLAS.

Warning: precise standard definitions of M_{eff} and H_T do not exist! The criteria for which and how many jets contribute to the sums varies greatly depending on the analysis.

Typical ATLAS cuts for a jets + $E_T^{\rm miss}$ signal (with early data):

- $E_T^{\rm miss} > 150$ GeV or more
- p_T of leading jet > 100 GeV or more
- $\geq$1 other jets with $p_T > 20$ GeV or more
- $M_{\rm eff} > 500$ GeV or more
- $\Delta\phi(\vec{p}_{T,{\rm missing}}\ ,\ \vec{p}_{T,{\rm jet}}) > 0.2$
- Number of leptons?
- Number of b-tagged jets?

triggers

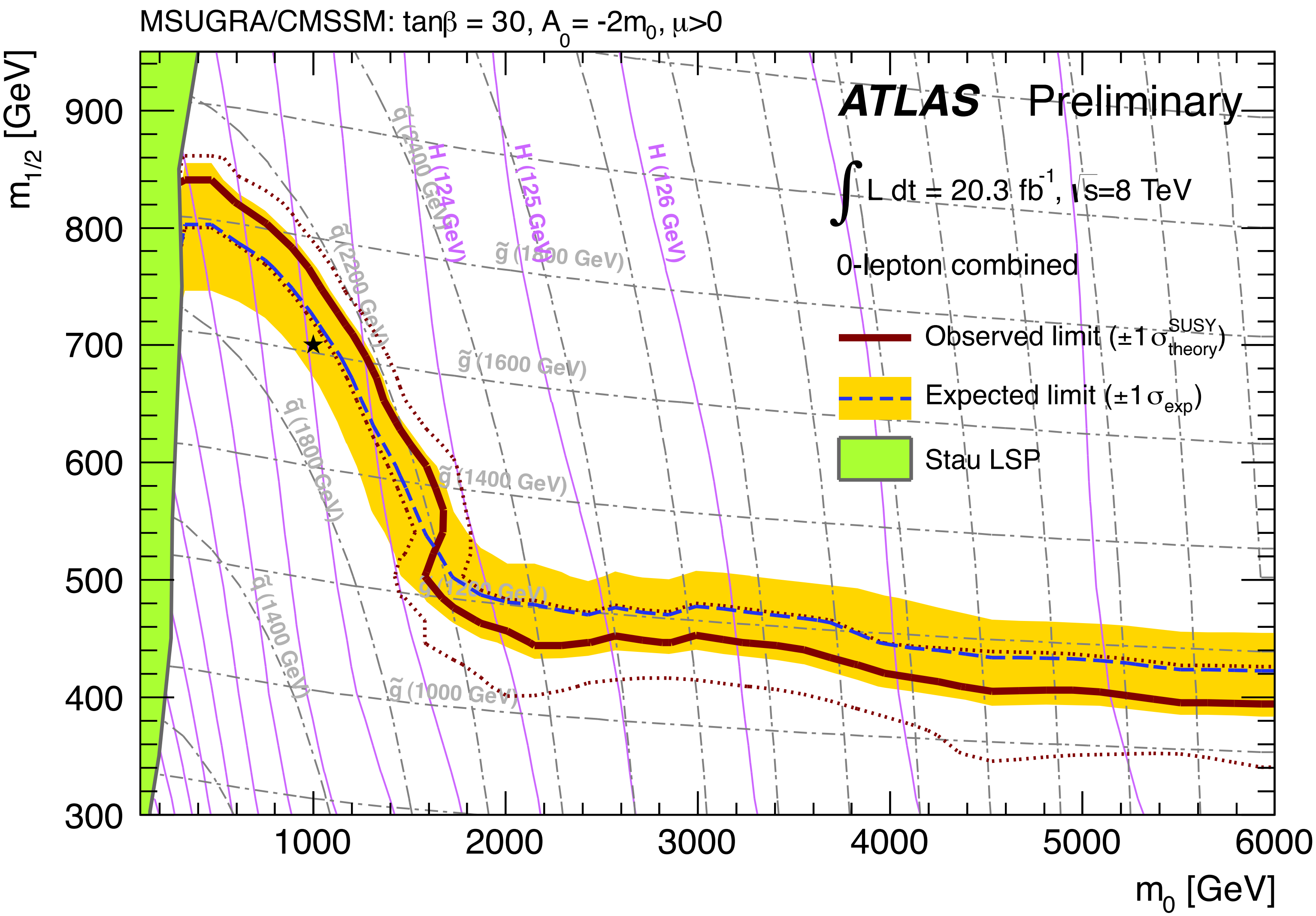
avoids backgrounds from mismeasured jet energies

depends on model being searched for

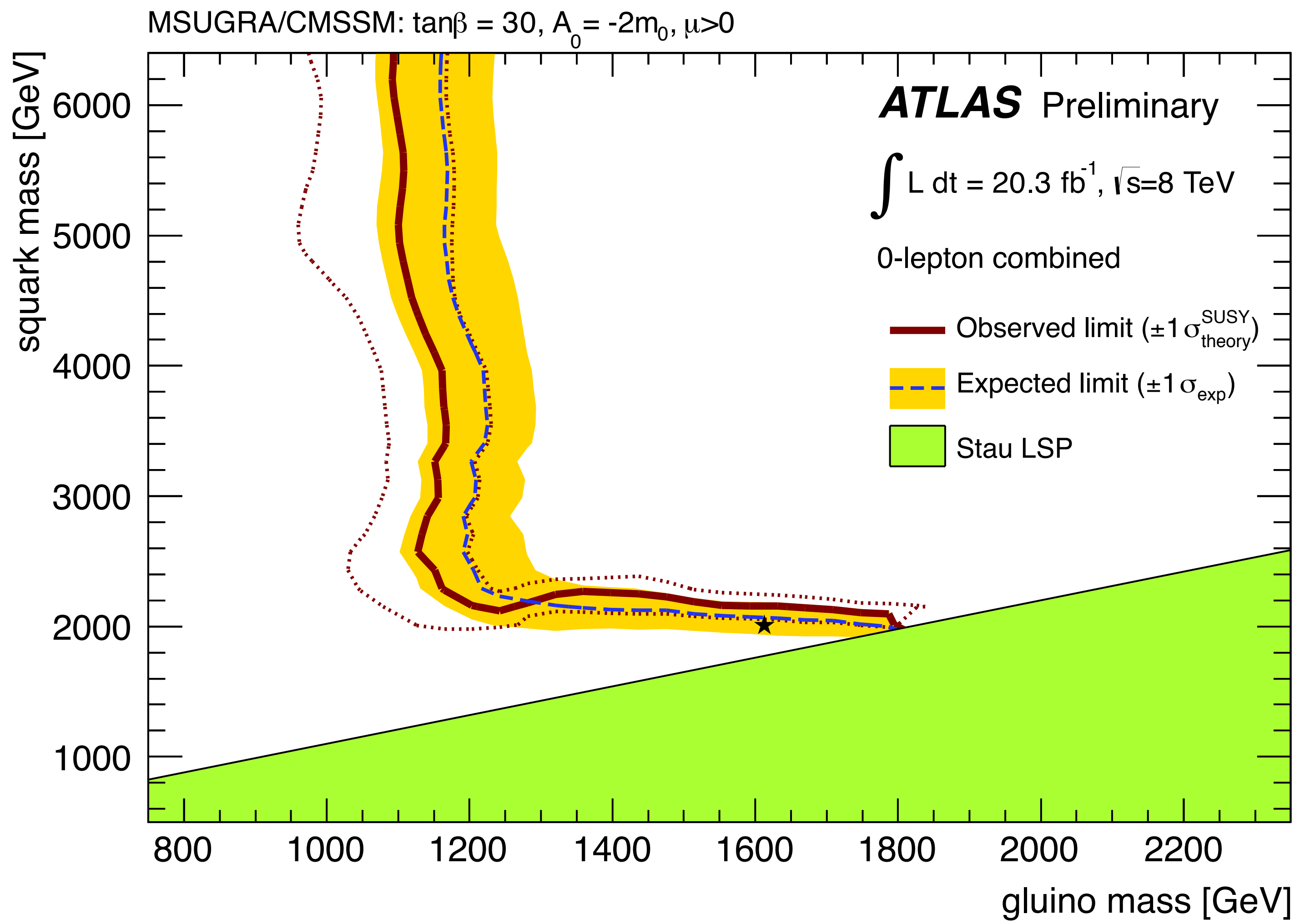
"or more" means: with larger data samples, these cuts are made much stronger.

ATLAS limit on mSUGRA/CMSSM, from ATLAS-CONF-2013-047.

Note $M_h = 124, 125, 126$ GeV contours.



A better way of showing the same limits: $M_{\tilde{Q}}$ vs. $M_{\tilde{g}}$ plane.

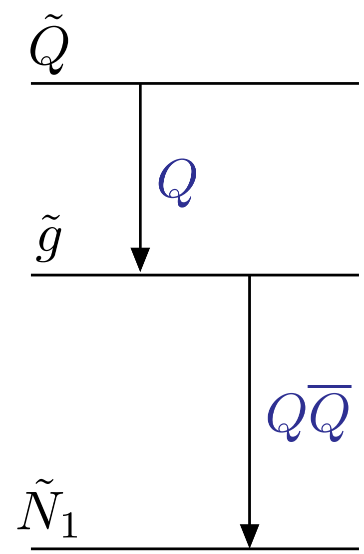


Roughly: $M_{\tilde{g}} \gtrsim 1100$ GeV and $M_{\tilde{Q}} \gtrsim 2100$ GeV in mSUGRA/CMSSM.

The interpretation of these limits for other models can be tricky, because of the complication of multiple production processes and decay modes.

Many or most recent searches use **Simplified Models** instead. These just take a small subset of the particles in SUSY, and assume 100% decay branching ratios.

A squark/gluino/neutralino simplified model:

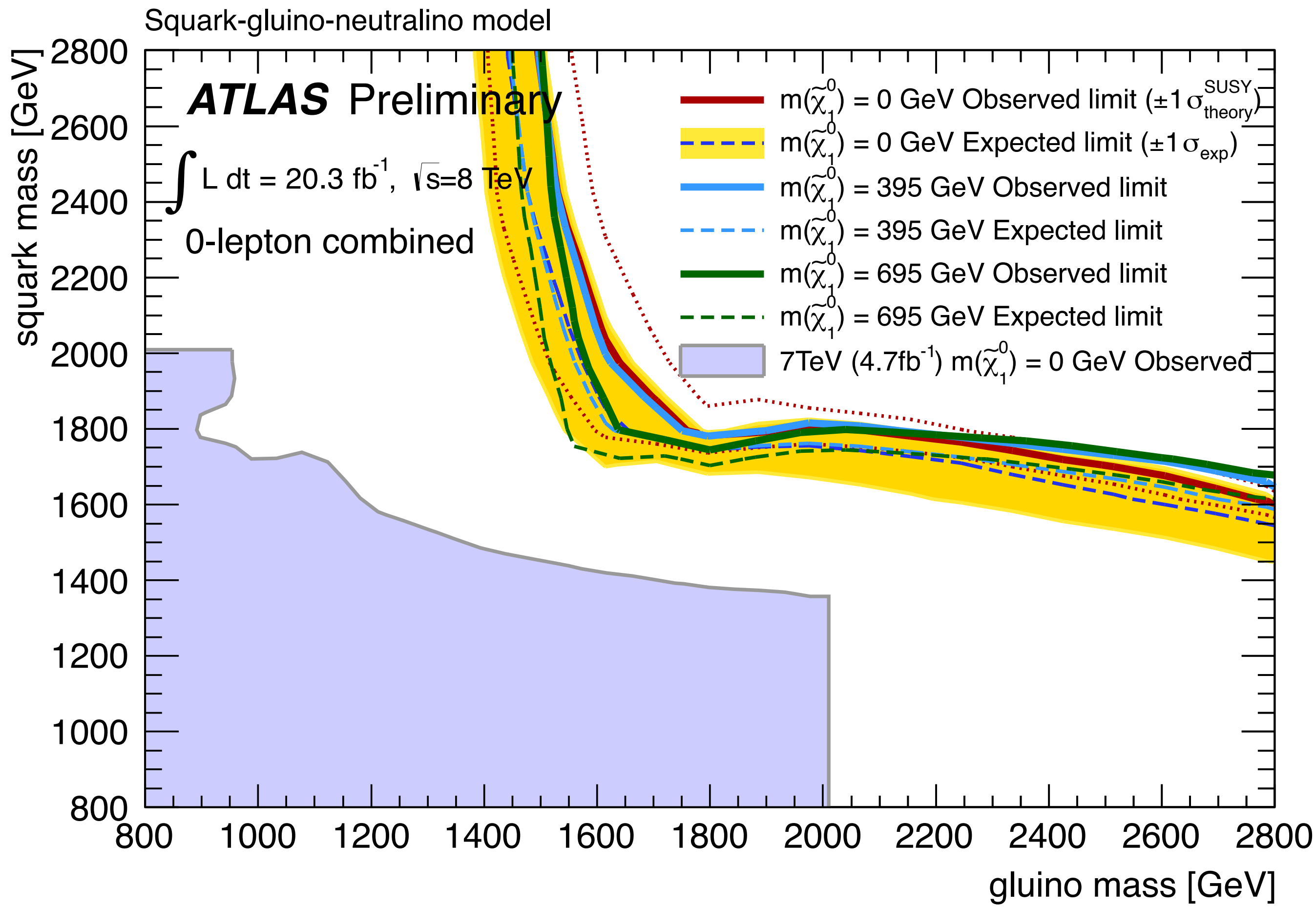


Production rates are determined by QCD couplings.

For more examples of SUSY-motivated Simplified Models, see hep-ph/0703088 and 1105.2838.

Search results can be re-interpreted in terms of your favorite model, even non-SUSY, if the cuts, efficiencies, and assumptions are made public.

For simplified model with $\tilde{g}$, $\tilde{Q}$, and $\tilde{N}_1$ only (no $\tilde{C}_1$, $\tilde{N}_2$, or $\tilde{t}_1$), the limits are stronger for gluinos, and weaker for squarks:



$M_{\tilde{g}} \gtrsim 1500$ GeV and $M_{\tilde{Q}} \gtrsim 1600$ GeV in simplified model.

Some other specialized searches

Like-Charge Dileptons + $E_T^{\rm miss}$

Exploit the fact mentioned earlier that gluinos decay into leptons of either charge:

$$pp \to \tilde{g}\tilde{g} \to \text{(jets)} + \ell^\pm \ell^\pm + E_T^{\rm miss}.$$

Multi-b-jets + $E_T^{\rm miss}$

Produce gluons that decay into bottom quark and bottom squark:

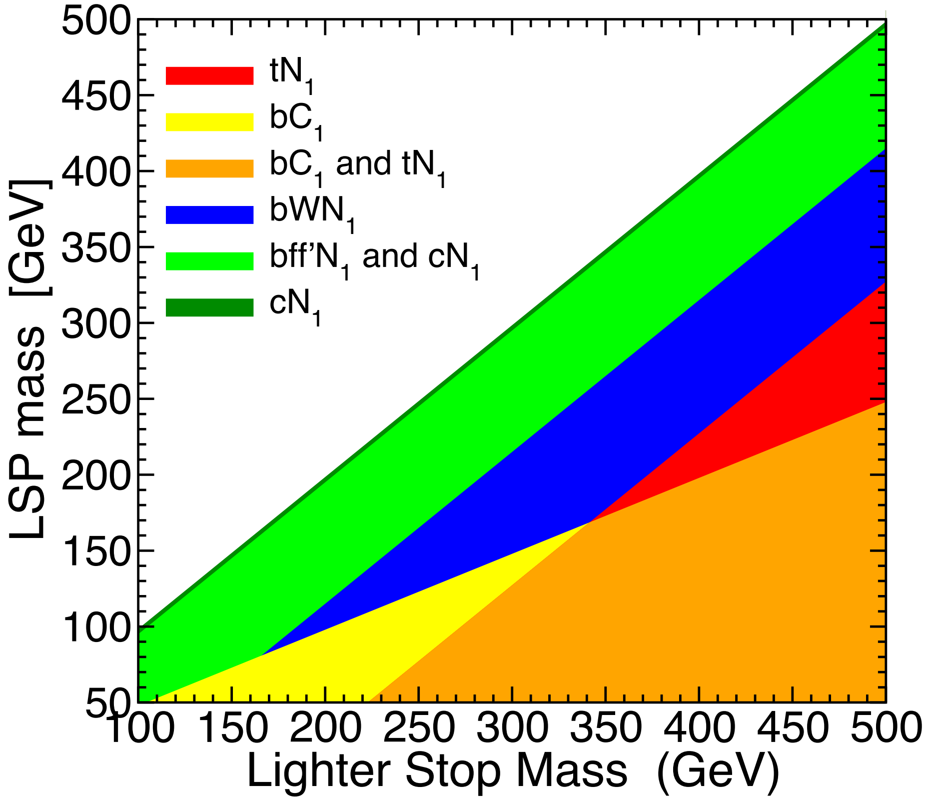
$$pp \to \tilde{g}\tilde{g} \to (t\tilde{t}_1)(\bar{t}\tilde{t}_1^*) \to (t\bar{t}\tilde{N}_1)(t\bar{t}\tilde{N}_1) \to bbbb + \text{jets} + \text{leptons} + E_T^{\rm miss}.$$

Light Top Squarks

Top squarks with $m_{\tilde{t}_1} <$ Min$[m_{\tilde{N}_1} + m_b + m_W, m_{\tilde{C}_1} + m_b, m_b + m_{\tilde{\nu}}]$ have only suppressed flavor-violating decays:

$$\begin{aligned} pp &\to \tilde{t}_1\tilde{t}_1^* \to (c\tilde{N}_1)(\bar{c}\tilde{N}_1) \to jj + E_T^{\rm miss} \\ pp &\to \tilde{t}_1\tilde{t}_1^* \to (bW^*\tilde{N}_1)(\bar{b}W^*\tilde{N}_1) \to b\bar{b} + jjjj + E_T^{\rm miss}, \text{ etc.} \end{aligned}$$

Kinematically allowed decays of lighter top squark $\tilde{t}_1$, for unified gaugino masses (including mSUGRA) $M_1 = 0.5 M_2$ at the TeV scale:



Red, yellow, orange = 2-body

Blue = 3-body

Green = competition between 2-body flavor violating, 4-body decays

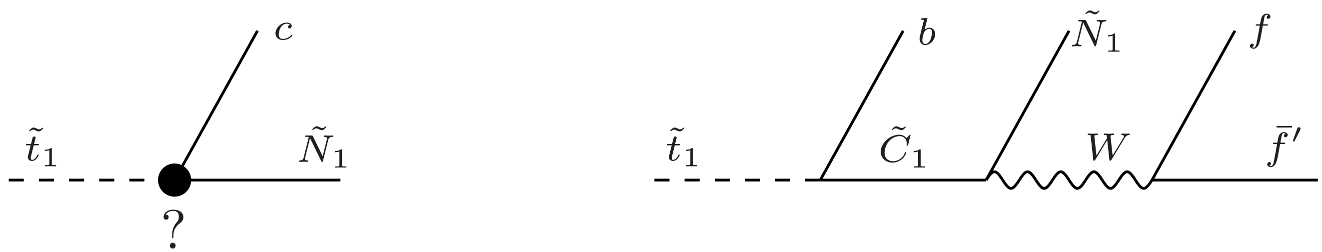
An aside: in green and blue regions, the top squark hadronizes before it decays, and forms bound states.

Which decay wins in the light green region, 5 GeV $< M_{\tilde{t}_1} - M_{\tilde{N}_1} <$ 85 GeV?

$\tilde{t}_1 \to c\tilde{N}_1$ 2-body, flavor violating (wins for ΔM small)

$\tilde{t}_1 \to b f \bar{f}' \tilde{N}_1$ 4-body suppressed (wins for $\Delta M \sim 85$ GeV)

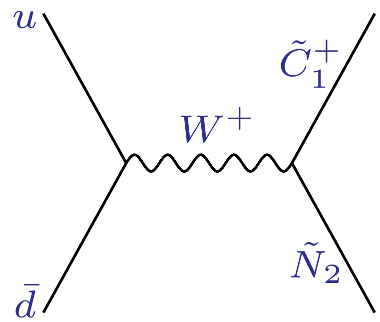
But, 2-body decay depends on unknown coupling:



Nobody, including your favorite model and event simulation programs, knows for sure which one wins. Both decays must be searched for if both are kinematically allowed.

Predictions rely on **assumptions**, like “Minimal Flavor Violation”.

Electroweak SUSY production at Hadron Colliders



$$pp \to C_1^+ \tilde{N}_2$$

This can lead to the classic **trilepton** SUSY signal if:

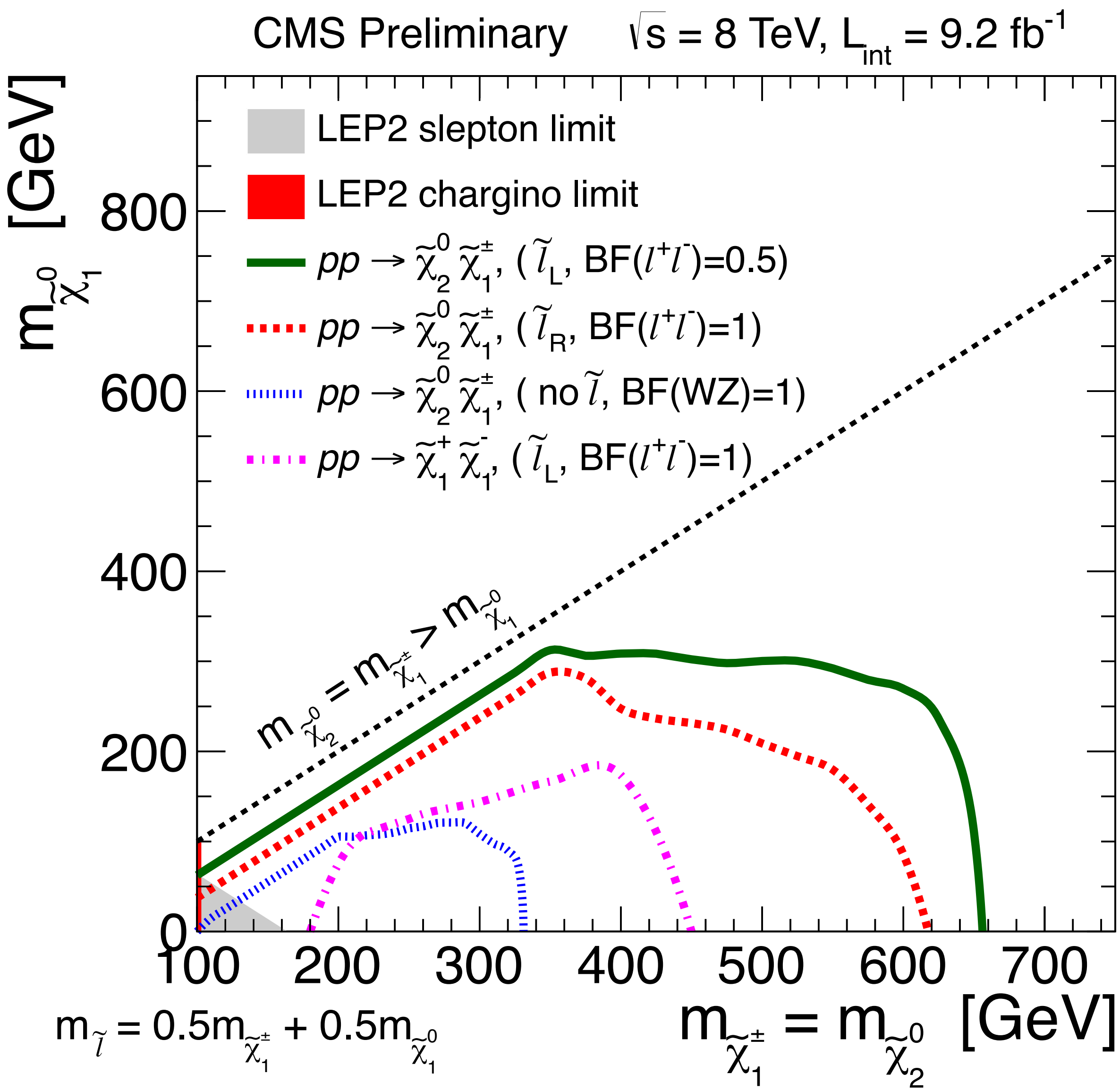
$$\tilde{N}_2 \to \ell^+ \ell^- \tilde{N}_1, \qquad \tilde{C}_1^\pm \to \ell^\pm \nu \tilde{N}_1$$

With no hard jets in the event, and three identified leptons, the Standard Model backgrounds are small.

However, recall that the decays

$$\tilde{N}_2 \to Z \tilde{N}_1, \qquad \tilde{N}_2 \to h \tilde{N}_1 \qquad \tilde{C}_1 \to W \tilde{N}_1$$

will dominate if they are allowed, and have much smaller branching ratios to leptons in the final state.



CMS-PAS-SUS-12-022

The blue dashed line is the limit if sleptons are much heavier than charginos and neutralinos. No limit if LSP mass is over 100 GeV.

Why has SUSY not been discovered yet?

- Superpartners (up, down squarks) are too heavy to produce
  - “Natural SUSY”
  - 100-1000 TeV scale SUSY
  - Split SUSY
- R-parity violation
- Superpartner mass spectrum is compressed
- Superpartner decays are stealthy
- SUSY isn’t there at all (no solution to the hierarchy problem?)

Natural SUSY

Why did we think superpartners should be light?

Minimizing the Higgs potential, we find:

$$M_Z^2 = -2(|\mu|^2 + m_{H_u}^2) + \mathcal{O}(1/\tan^2\beta) + \text{loop corrections}$$

So avoiding fine-tuning just suggests that **Higgsinos should be light** $\mu \sim M_Z$.

Other superpartners should be light only if their masses are correlated with, or feed into, $m_{H_u}^2$.

Corrections from loop diagrams give:

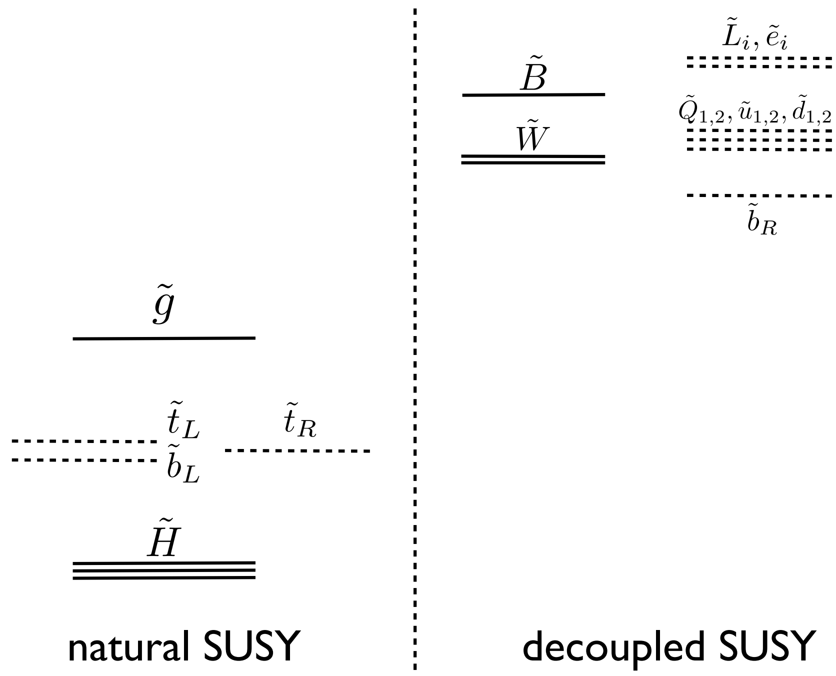
$$\Delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\ln(\Lambda/\text{TeV}) - \frac{\alpha_S y_t^2}{\pi^3} M_{\tilde{g}}^2 \ln^2(\Lambda/\text{TeV}) + \text{small.}$$

So we conclude that the **top squarks and gluino should also not be too heavy**.

But, **the other superpartners have no "reason" to be at the 1 TeV scale!**

Natural SUSY (continued)

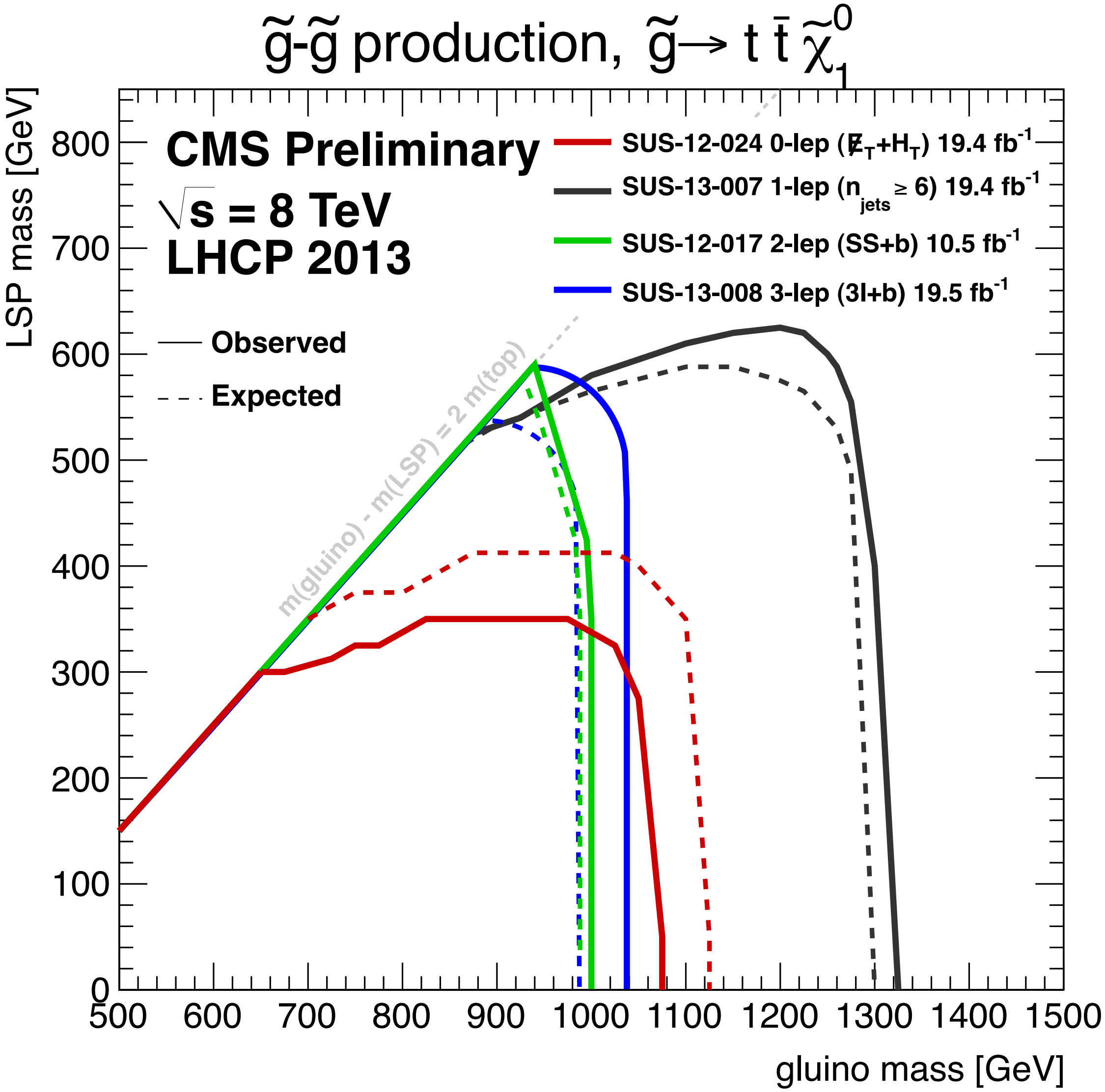
This is actually an ancient idea. From a recent paper Papucci, Ruderman, Weiler 1110.6926 that nicely emphasizing the issues:



We should expect to see $\tilde{t}_1$, $\tilde{g}$ and maybe $\tilde{t}_2$, $\tilde{b}_1$ first at LHC. (Higgsinos are not strongly produced, so harder to see even if lighter.)

So, let's look at limits on the top squarks and the gluino. . .

- gluino pair production with top-squark mediated decays
- direct top squark pair production
- stoponium?

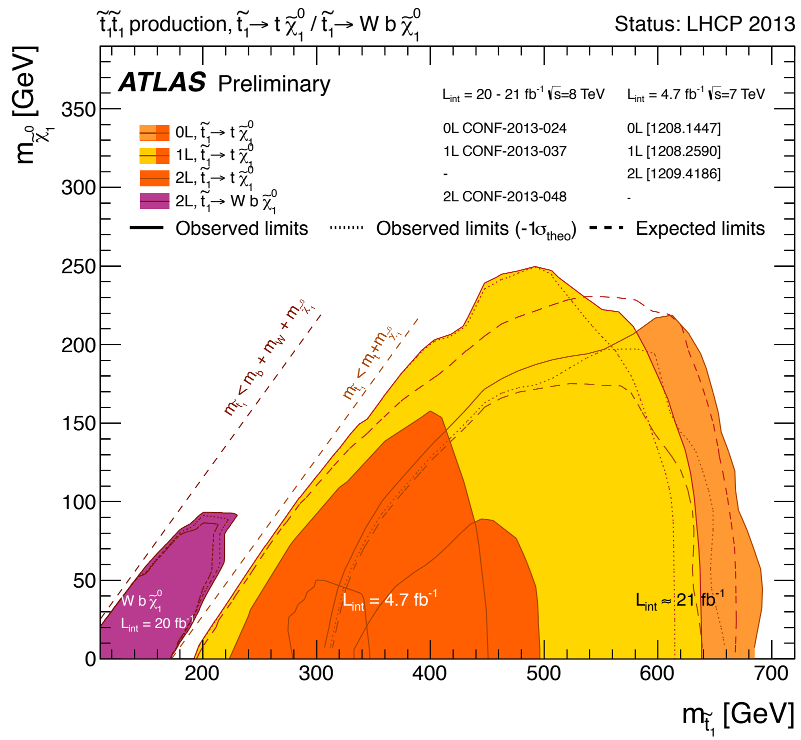


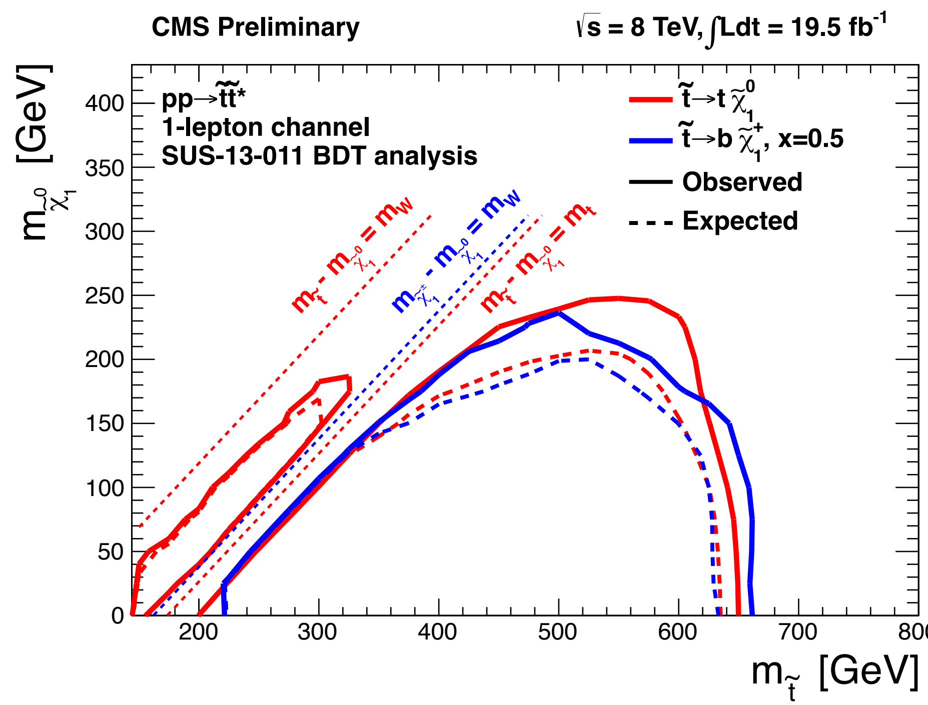
CMS gluino pair production.

Signal with 1 lepton and 6 jets + $E_T^{\rm miss}$ is the strongest.

Sets a strong limit if $M_{\tilde{N}_1} < 600$ GeV. Then the 1-lepton search gives $M_{\tilde{g}} > 1300$ GeV.

Limits on direct top-squark production:

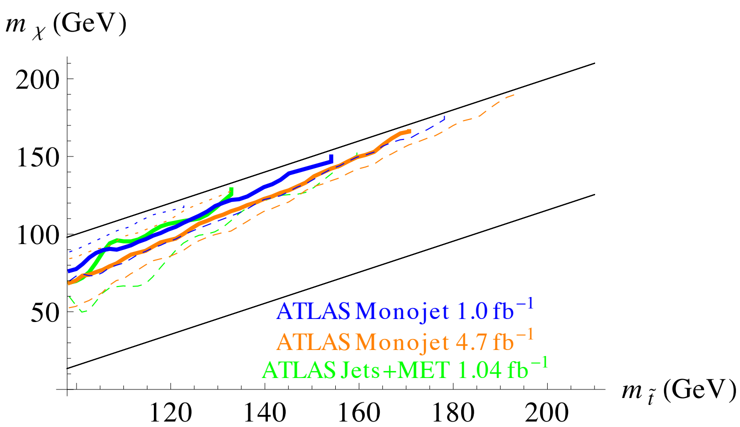




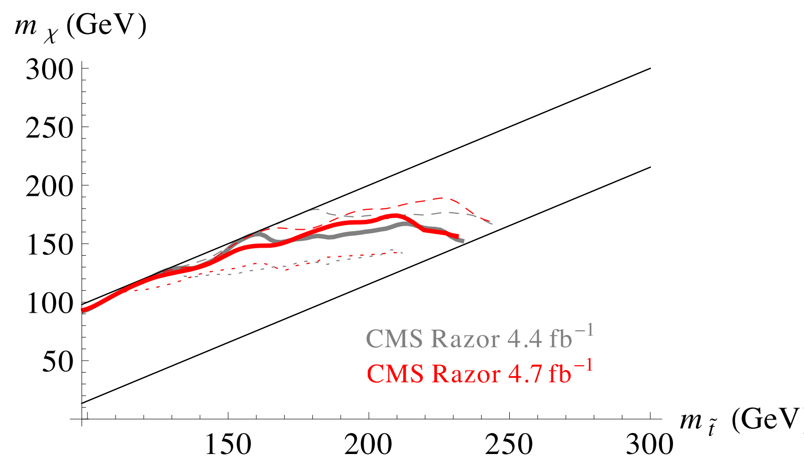
Note that the case of small $m_{\tilde{t}_1} - m_{\tilde{N}_1} < m_W + m_b$ is not yet covered by LHC published limits. Top squark decay products are very soft.

However, a group of theorists (Kriska, Kumar, Morrissey 1212.4856) have claimed that it should be possible to set such a limit with existing LHC data, by straightforward re-interpretation of published limits on **other** models from both ATLAS and CMS.

$\tilde{t}_1 \to c\tilde{N}_1$



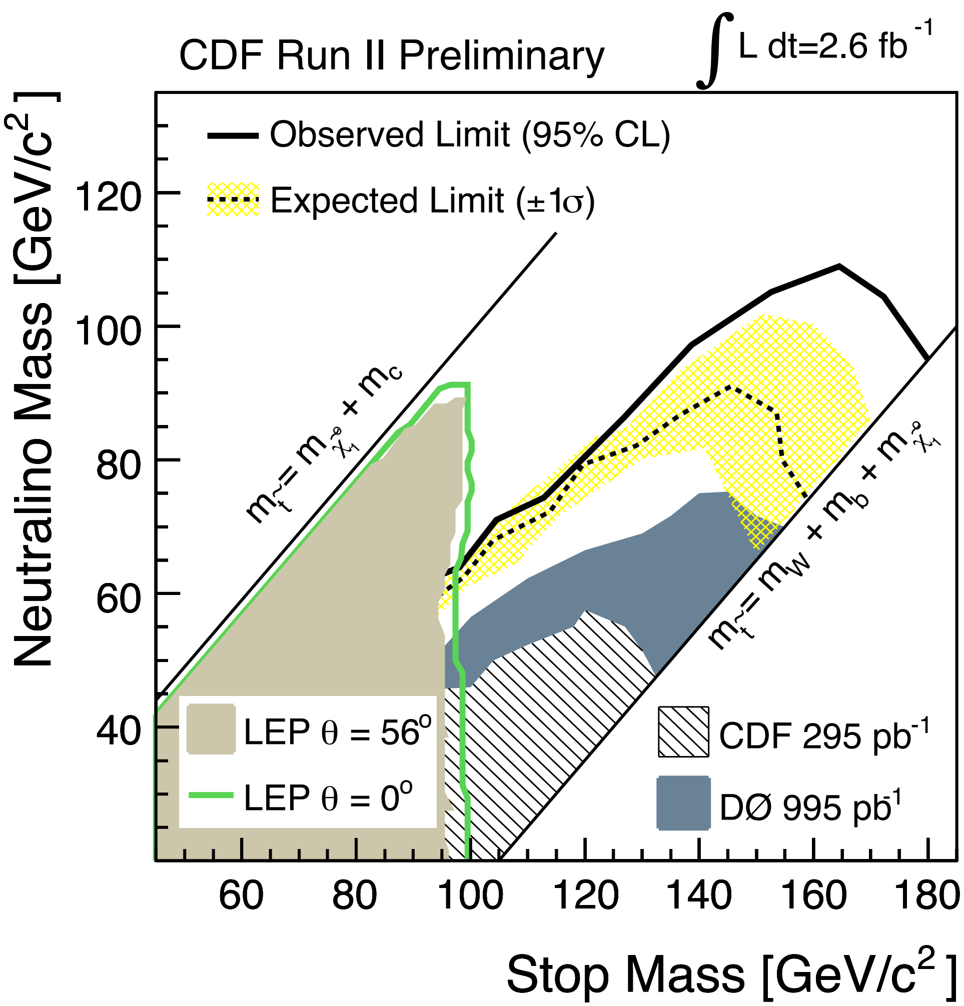
$\tilde{t}_1 \to bW^{(*)}\tilde{N}_1$



So far, ATLAS and CMS have not published their own analyses; will they confirm?

This is a game that is increasingly being played by theorists. There are too many models for experimentalists to cover them all! But, if they provide enough information about their searches, theorists can re-interpret the limits in other models.

Light top squarks at the Fermilab Tevatron CDF



Top-squark pair production:
$p\overline{p} \rightarrow \tilde{t}_1 \tilde{t}_1^*$

If $M_{\tilde{t}_1} - M_{\tilde{N}_1} < M_W + M_b$, then look for $\tilde{t}_1 \rightarrow c\tilde{N}_1$.

This is a difficult region for LHC because of backgrounds.

But in this case, the top-squark can form bound states like…

Stoponium

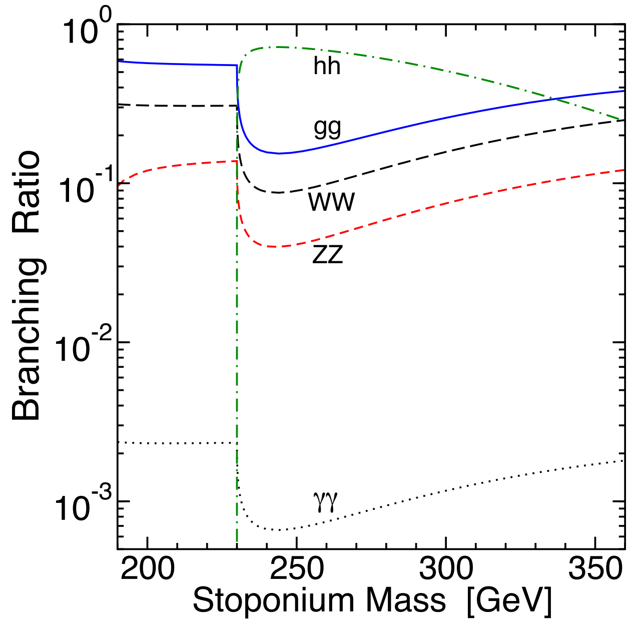
A spin-0 $\tilde{t}_1 \tilde{t}_1^*$ bound state, forms if both of the decays

$$\tilde{t}_1 \to b\tilde{C}_1 \qquad \text{and} \qquad \tilde{t}_1 \to t\tilde{N}_1$$

are not kinematically allowed. In that case, the decay width of the top squark is much smaller than the stoponium binding energy, so stoponium does form.

Typical branching fractions:

(SPM hep-ph/0801.0237)



- stoponium $\to \gamma\gamma$ is very rare, but gives a clean peak like the Higgs. Could provide a precision mass measurement if detected.
- stoponium $\to hh$ could have larger branching ratio, but backgrounds are larger. Look for $hh \to bb\gamma\gamma$ resonance.

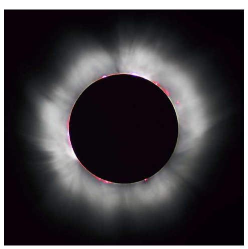
Both will require high luminosity.

__100-1000 TeV scale SUSY__   J.D. Wells, hep-ph/0411041, . . .

Maybe we're emphasizing the wrong naturalness problem? If all superpartners have masses above 100 TeV, then they decouple from flavor violating effects. So it is "natural", from the point of view of flavor-changing neutral current and CP violation data, to assume that the superpartners are out of reach of the LHC.

We then have $m^2_{H_u}$ fine-tuned to be $-|\mu|^2$, in order to get M_Z^2 correct.
Fine tuning is of order 1 part in 10^6.

But, small numbers sometimes do happen in Nature for no obvious reason!

- Electron Yukawa coupling is 3×10^{-6}. Why?
- 

  $$\frac{R_{\rm sun}}{D_{\rm sun}} = \frac{6.955\times 10^8 \text{ m}}{(1.496\pm 0.025)\times 10^{11} \text{ m}} = 0.00465 \pm 0.00008$$

  $$\frac{R_{\rm moon}}{D_{\rm moon}} = \frac{1.738\times 10^6 \text{ m}}{(3.844\pm 0.214)\times 10^{8} \text{ m}} = 0.00452 \pm 0.00028$$

  Why?

So maybe we will see **no** SUSY particles at the LHC? 

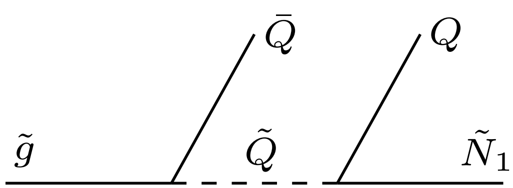
Split SUSY

Arkani-Hamed, Dimopoulos, Giudice, Romanino hep-th/0405159, hep-ph/0406088, hep-ph/0409232

Take the Higgsinos and gauginos at the TeV scale, but everything else near the Planck or GUT scale. Accept extreme fine-tuning?!

The gauge couplings still unify near 2×10^{16} GeV.

Predicts long-lived gluinos at the LHC. Recall the gluino can only decay through squarks:

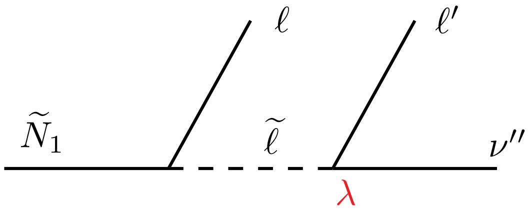


Suppression due to $M_{\tilde{Q}} \gg$ 1 TeV.

Look for gluino bound states moving through the detector, with anomalous energy deposition, late decays, or long time-of-flight. The usual signatures for SUSY won't work here.

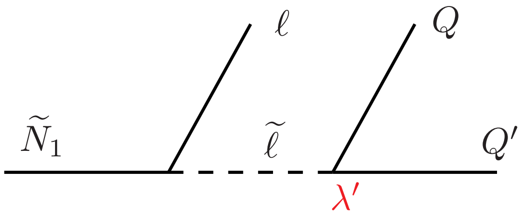
Can R-parity violation hide SUSY at the LHC?

The LSP can decay, so get less $E_T^{\rm miss}$, or maybe none. Some possibilities:



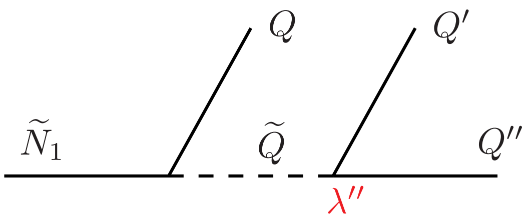
$\tilde{N}_1 \to \ell\ell' + E_T^{\rm miss}$

Lepton-rich or τ-rich, and $E_T^{\rm miss}$.



$\tilde{N}_1 \to \ell jj$

Same-sign leptons or taus, and jets

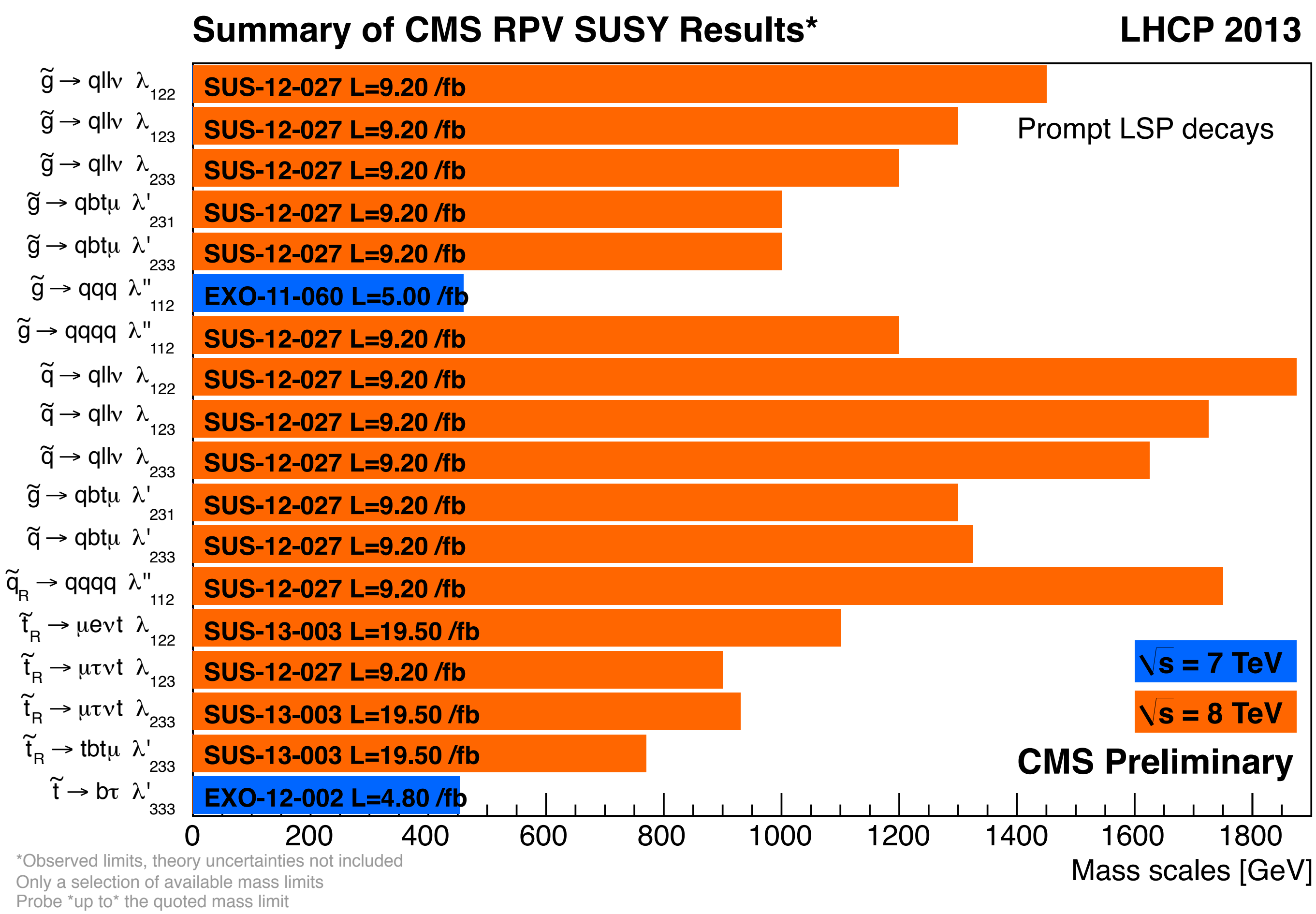


$\tilde{N}_1 \to jjj$

No leptons, no $E_T^{\rm miss}$. Difficult!

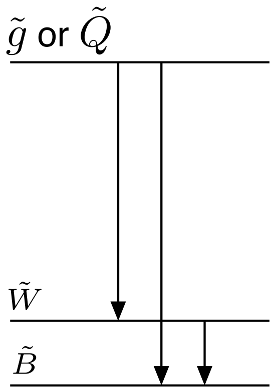
There are many possibilities, each with different signatures and flavor structure.

$\not R$-parity violating results from CMS:

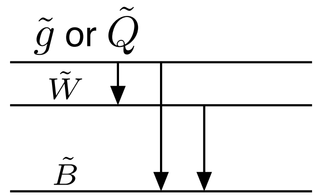


Warning: each of these limits replies on very special and favorable assumptions! General limits on $\not R$-parity violating SUSY are much weaker.

What happens if the superpartner mass spectrum is more compressed?



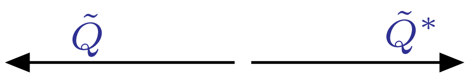
replaced by:



Less visible energy: smaller jet p_T, $M_{\rm eff}$, and $E_T^{\rm miss}$. Signal looks more like QCD, $t\bar{t}$, W+jets, and Z+jets backgrounds.

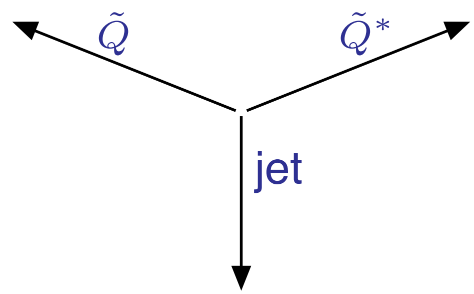
Radiation of additional QCD jets important; supplies transverse kick.

Leading order production



$E_T^{\rm miss}$ small, tends to cancel.

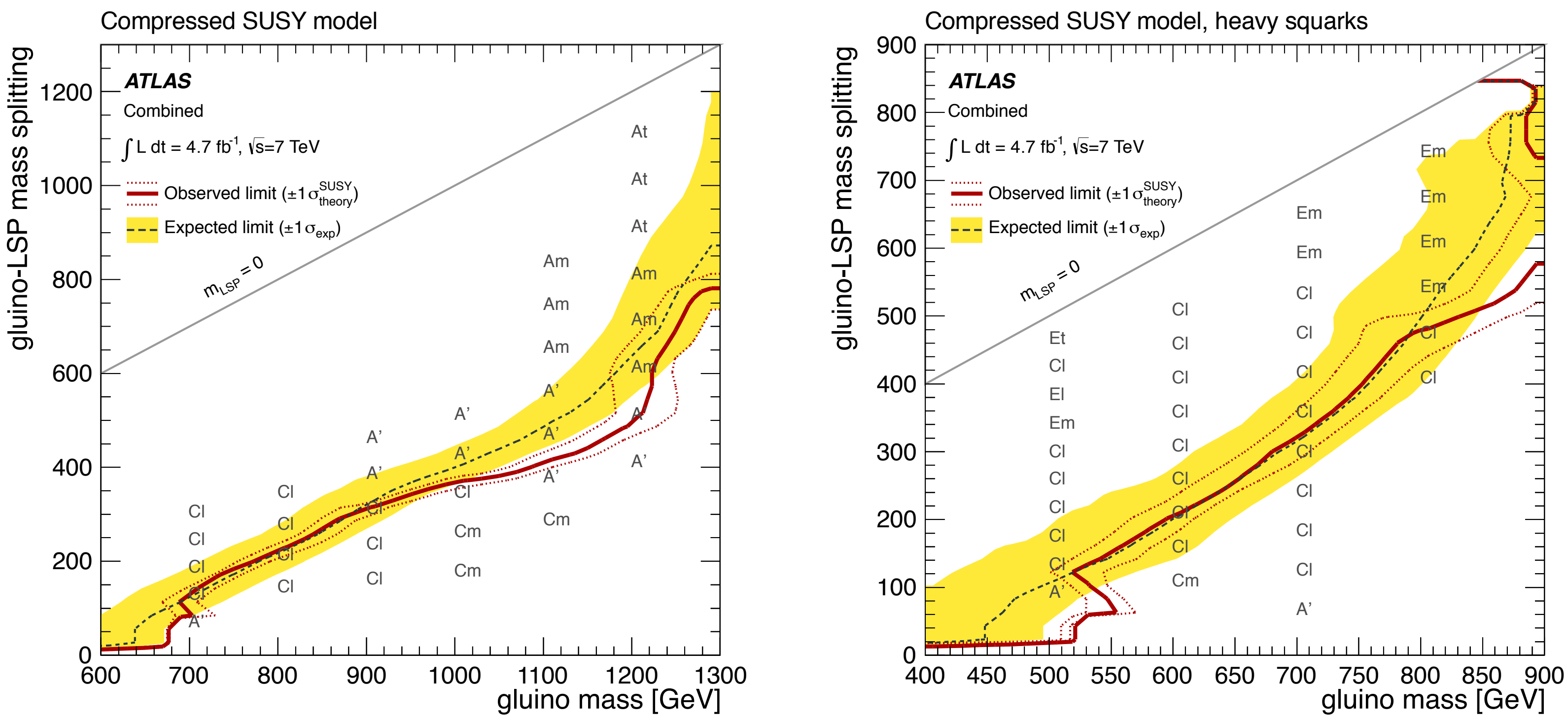
With extra radiated jet kick



Extra jet can pass triggers, cuts. Heavy LSP neutralino carries away more $E_T^{\rm miss}$.

Compressed SUSY: the limits from jets + $E_T^{\rm miss}$ get weaker.

(ATLAS 1208.0949)



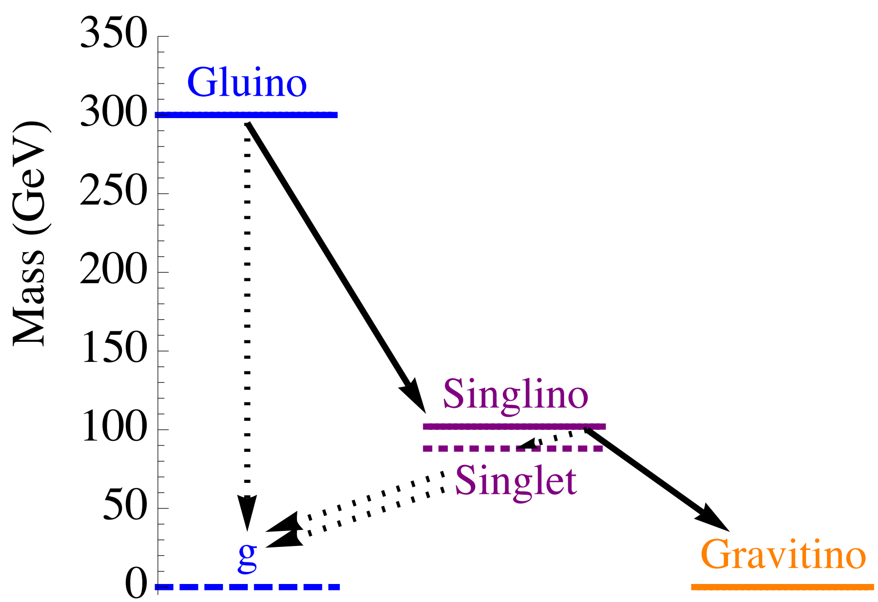
Even for very small $M_{\tilde g} - M_{\tilde N_1}$, still get limits.

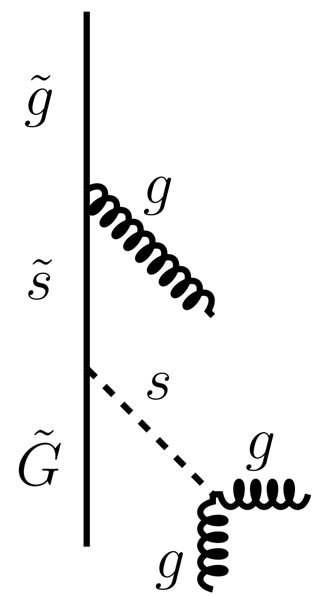
Need to keep the $M_{\rm eff}$ cut low to see signal events from compressed SUSY.

Stealth Supersymmetry   Fan, Reece, Ruderman 1105.5135, 1201.4875

Maybe the superpartners decay through intermediate closely spaced states in such a way that most of the energy goes into **visible** Standard Model particles.

One of many examples:





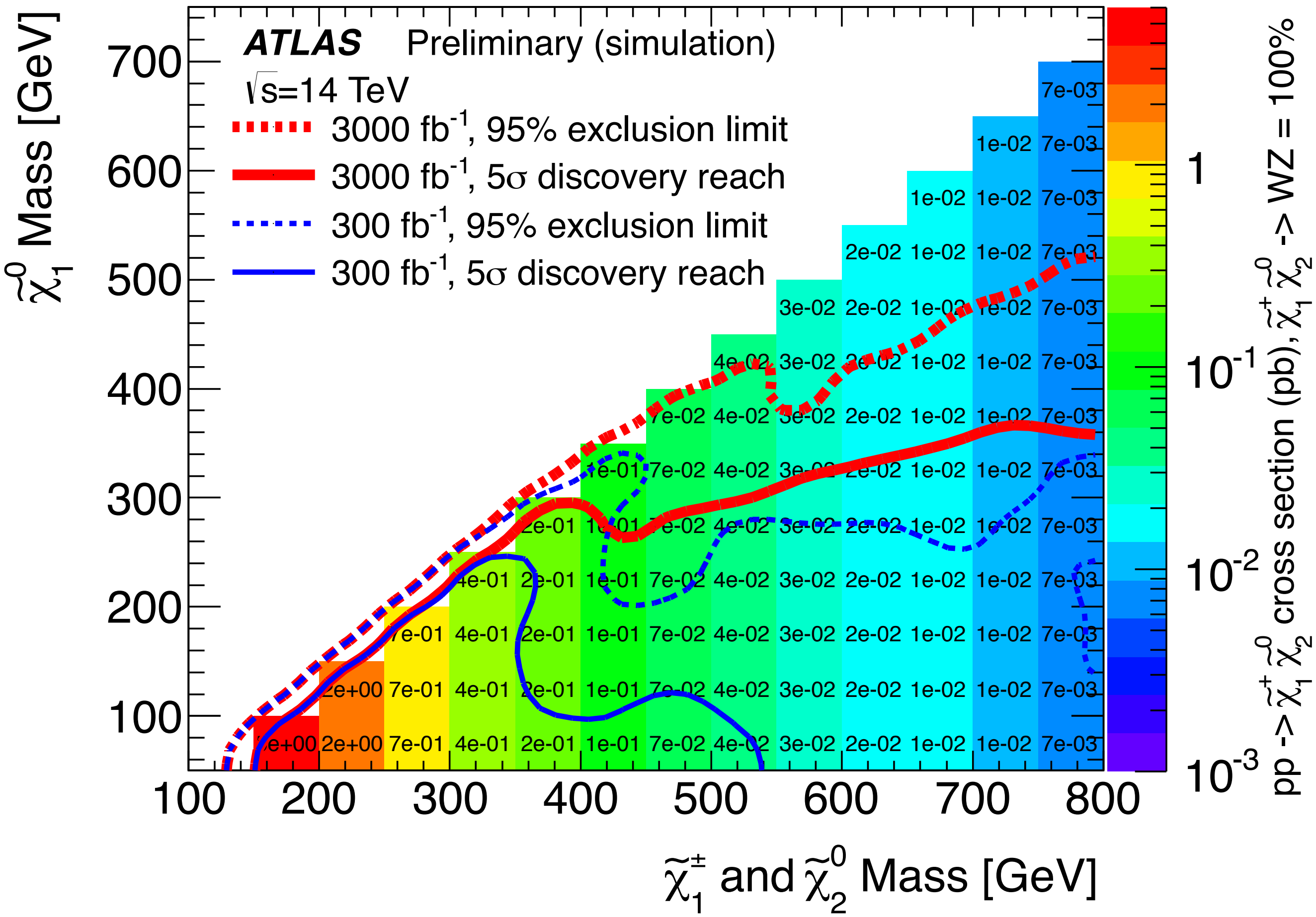
Very small $E_T^{\rm miss}$. Look instead for displaced vertices in decays, or resonances.

The LHC will eventually reach $\sqrt{s} = 13$ or 14 TeV, with much higher luminosity than today.

A couple of sample projections:

- Neutralino/chargino electroweak SUSY production
- Direct top-squark pair production

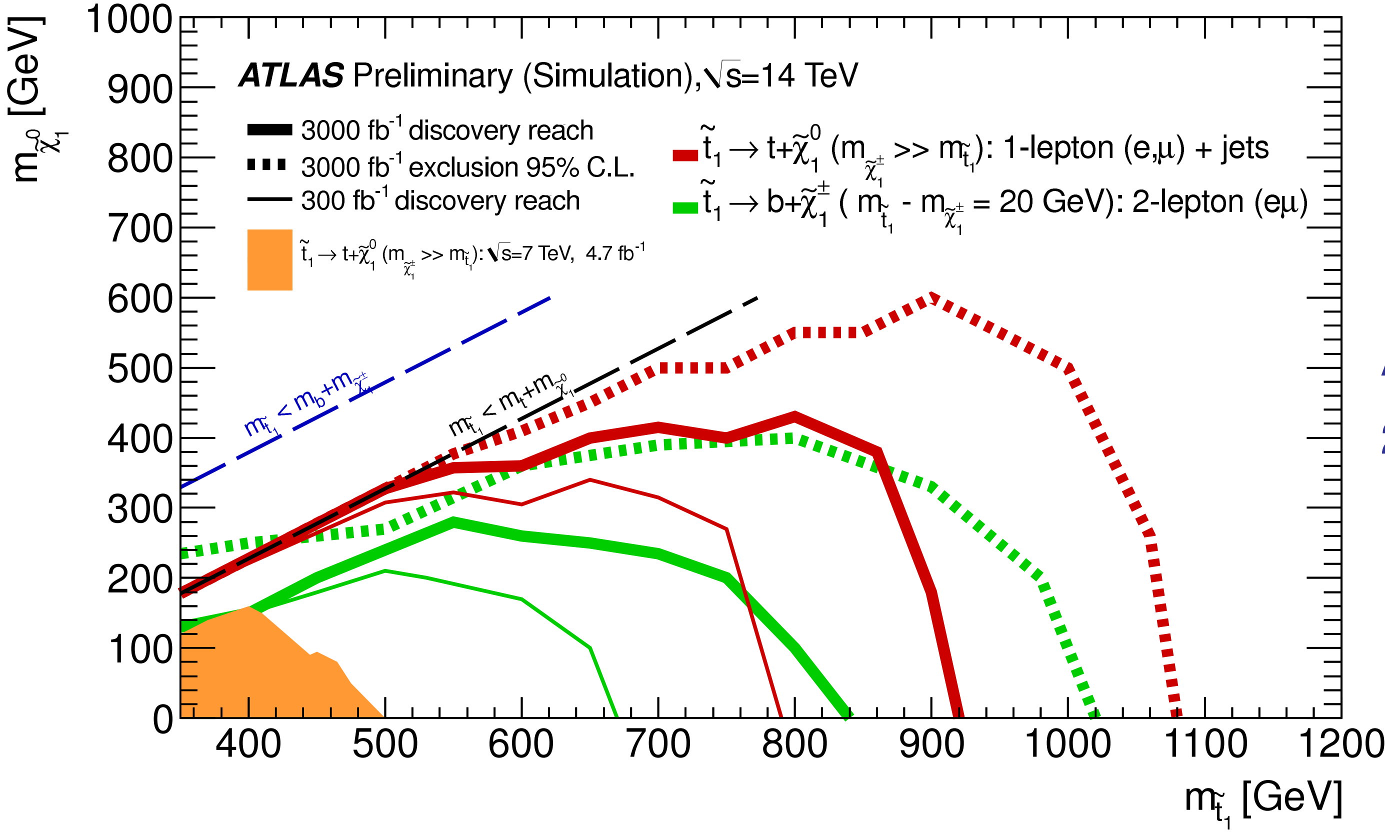
Neutralino/chargino **projected** limits, simulation only. ATL-PHYS-PUB-2013-002



Branching ratios BR$(\tilde{C}_1 \to W\tilde{N}_1) = 1$ and BR$(\tilde{N}_2 \to Z\tilde{N}_1) = 1$

may be optimistic! Expect BR$(\tilde{N}_2 \to h\tilde{N}_1) = 0.9$ instead.

Direct top-squark pair production **projected** limits, simulation only.



ATL-PHYS-PUB-2013-002

The reach in $M_{\tilde{t}_1}$ grows **very** slowly with luminosity!?

High luminosity (3000 fb^{-1}) needed just to start excluding $M_{\tilde{t}_1} > 1000$ GeV?

Important things I've completely skipped include:

- Superfields and Superspace
- Fayet-Iliopoulos (D-term) breaking of SUSY
- Gauge Mediated SUSY Breaking (GMSB)
- Anomaly Mediated SUSY Breaking (AMSB)
- Supersymmetric GUTs
- Supergravity
- Dirac gauginos
- Other LSP candidates (axino, gravitino, singlino) and cosmological issues
- Searches for the extended Higgs sector $(H^0, A^0, H^\pm)$
- Models with extra chiral supermultiplets (NMSSM, vector-like quarks, ...)
- Detection of SUSY dark matter
- Anomalous magnetic moment of the muon
- The μ problem and possible solutions
- Constraints from flavor physics, including BR$(B_s \to \mu^+\mu^-)$
- Sophisticated kinematic variables and methods for searches
- baryogenesis in SUSY

Why I am still optimistic about the discovery of SUSY at the LHC:

The hierarchy problem discussed at the beginning of this lecture is still there, unsolved. There are good hints for several mass scales in fundamental physics well above the weak scale:

- Unification: quantization of $U(1)_Y$ weak hypercharge, fermions fit neatly into $SU(5)$ or $SO(10)$ multiplets. Mass scale has to be very high because of proton decay constraints.

- Neutrino masses: large mass scale in the see-saw mechanism.

- Solution of the strong CP problem: Peccei-Quinn breaking scale f_a must be between 10^{10} GeV $< f_a < 10^{12}$ GeV.

- Baryogenesis: not explainable within the Standard Model alone.

- Dark matter: not explainable within the Standard Model alone.

All of the **other** models (technicolor, topcolor, large extra dimensions, . . .) that have been proposed to explain the hierarchy problem are doing no better than SUSY is. In fact some are now **completely** dead.

My best guess is that the superpartners are still out there, and some of them will eventually be found at the LHC.

Thank you for your attention!