ICTP School
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Higgs Physics

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EXERCISES

Basic calcultaional rules

1. Vertices and propagators
2. Dirac algebra
3. Cross sections and decay widths
4. Calculation of loop integrals.

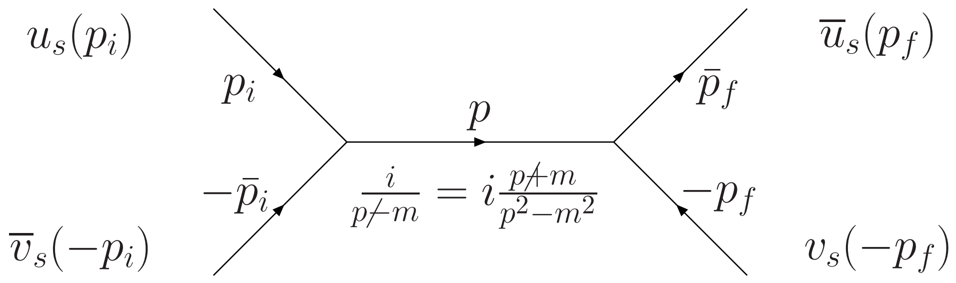
Exercise 1: Higgs boson production and decay mechanisms.

1. Higgs decays into fermions and gauge bosons
2. Higgs production in e^+e^- collisions
3. Higgs production in hadronic collisions
4. Higgs contributions to radiative corrections.

Basic Calculational Rules

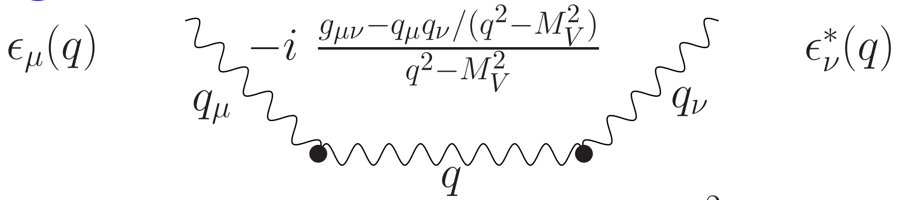
1. Vertices and propagators

Rules for fermions:



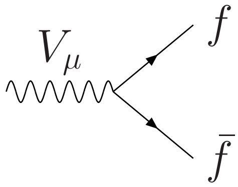
$$\Sigma_s\, u_s(p)\, \bar{u}_s(p) = p\!\!\!/ + m \quad , \quad \Sigma_s\, v_s(p)\, \bar{v}_s(p) = p\!\!\!/ - m$$

Rules for gauge bosons:



$$\Sigma_{\rm pol} = \epsilon^*_\nu \epsilon_\mu = -(g_{\mu\nu} - q_\mu q_\mu/M_V^2)$$

For the photon, discard everything which is longitudinal $(q^\mu q^\nu)$ above. Note that the trasversality of the photon implies: $\epsilon_\mu \cdot q^\mu = 0$.



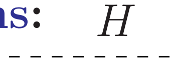
$$-ie\gamma_\mu(v_f - a_f\gamma_5)$$

$$Z\,:\; v_f = (2I_f^3 - 4e_f s_W^2)/(4s_W c_W)\;,\; a_f = 2I_f^3/(4s_W c_W)$$

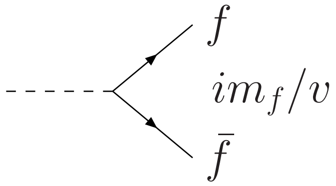
$$W\,:\; v_f = a_f = 1/(2\sqrt{2}s_W)$$

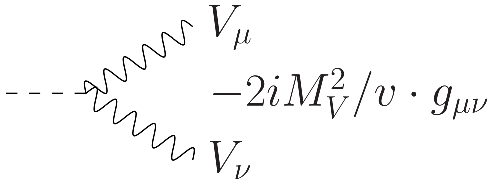
$$\gamma\,:\; v_f = e_f\;,\; a_f = 0$$

Rules for Higgs bosons:



$i/(p^2 - M_H^2)$





2. Diracology: contractions and traces of γ matrices

Basic relations:

$$\{\gamma_\mu\,,\,\gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu} \quad and \quad \not{p} = p_\mu\gamma^\mu$$

$$\gamma_5 = \frac{i}{4!}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3 \text{ and } \{\gamma_\mu,\gamma_5\} = 0$$

$$\mathrm{Tr}(\mathbf{1}) = 4\ ,\ \ \mathrm{Tr}(\gamma_\mu) = 0\ ,\ \ \mathrm{Tr}(\gamma_5) = 0$$

$$\mathrm{Tr}(A_1A_2) = \mathrm{Tr}(A_2A_1)\ ,\ \ \mathrm{Tr}(A_1A_2\cdots A_N) = \mathrm{Tr}(A_2\cdots A_NA_1)$$

Contractions of γ matrices

$$\begin{aligned}
\gamma_\mu\gamma^\nu &= 2g^\nu_\mu - \gamma_\mu\gamma^\nu \Rightarrow \gamma^\mu\gamma_\mu = \delta^\mu_\mu = 4\\
\gamma^\mu\gamma_\nu\gamma_\mu &= \gamma^\mu(2g_{\mu\nu} - \gamma_\mu\gamma_\nu) = 2\gamma_\nu - 4\gamma_\nu = -2\gamma_\nu\\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)(2g^\rho_\mu - \gamma_\mu\gamma^\rho)\\
&= 4g^{\nu\rho} - 2\gamma^\nu\gamma^\rho - 2\gamma^\nu\gamma^\rho + 4\gamma^\nu\gamma^\rho = 4g^{\nu\rho}
\end{aligned}$$

Traces of γ matrices:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = \mathrm{Tr}(2g^{\mu\nu} - \gamma^\mu\gamma^\nu) = 2g^{\mu\nu}\mathrm{Tr}(1) - \mathrm{Tr}(\gamma^\mu\gamma^\nu) \Rightarrow \mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$

Trace of an odd number n of γ matrices (using $\gamma_5^2 = 1$):

$$\mathrm{Tr}(\gamma^{\mu_1}\cdot\gamma^{\mu_n}) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5\gamma^5) = (-1)\,\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^5\gamma^{\mu_n}\gamma^5)$$

$$= (-1)^n\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5) = -\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5)$$

$$\Rightarrow \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_5) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = \frac{1}{4}\mathrm{Tr}(\gamma^\alpha\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5) = (1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5\gamma^\alpha)$$

$$= -(1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma^\alpha\gamma_5) = -\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = 0$$

Using the same tricks as above, proof the trace of 4 γ matrices:

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\sigma\rho}$$

3. Cross sections and decay widths

The differential cross section for a $2 \times n$ process $i_1 i_2 \to f_1 \cdots f_n$ is

$$\mathrm{d}\sigma = \frac{|M(i_1 i_2 \to f_1..f_n)|^2}{4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} \left(\Pi_n \frac{d^3 p_f}{(2\pi)^3 2 e_f} \right) (2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_f)\, S$$

- **In the amplitude squared $|M|^2$, one has to average (sum) on degrees of freedom (polarisation, color) of initial (final) particles.**
- **There is a symmetry factor $S = 1/n!$ for n identical particles.**
- **The flux factor is $2(p_1+p_2)^2 = 2s$ for $2 \to n$ process with $m_1 = m_2 = 0$. It is $2M$ for the decay of a particle with a mass M ($1 \to n$ process).**

Calculation of phase–space for a two–body process $a + b \to f_1 + f_2$:

$$\mathrm{dPS2} = \frac{1}{16\pi^2} \frac{d^3 p_1}{e_1} \frac{d^3 p_2}{e_2}\, \delta^4(p_a + p_b - p_1 - p_2)$$

$$\int \frac{d^3 p_2}{e_2} \delta^4(p_a + p_b - p_1 - p_2) = \frac{1}{e_2} \delta(e_a + e_b - e_1 - e_2)$$

$$\text{with :} \quad |\vec{p}_2| = |\vec{p}_a + \vec{p}_b - \vec{p}_2| \ \text{and} \ e_2^2 = |\vec{p}_2|^2 + m_2^2$$

$$\text{and} \quad \mathrm{d}^3 p_1 = \mathrm{d}\Omega\, |p_1|^2\, \mathrm{d}|p_1| \ \text{with} \ e_1^2 = |\vec{p}_1|^2 + m_1^2$$

In the c.m. frame: $w = e_a + e_b,\ w' = e_1 + e_2 = (m_2^2 + p^2)^{\frac{1}{2}} (m_1^2 + p^2)^{\frac{1}{2}}$:

$$\frac{\mathrm{d}w'}{\mathrm{d}p} = p \left(\frac{1}{e_1} + \frac{1}{e_2} \right) \Rightarrow \mathrm{d}w' = p\mathrm{d}p \left(\frac{1}{e_1} + \frac{1}{e_2} \right) = e_1 \mathrm{d}e_1 \frac{e_1 + e_2}{e_1 e_2}$$

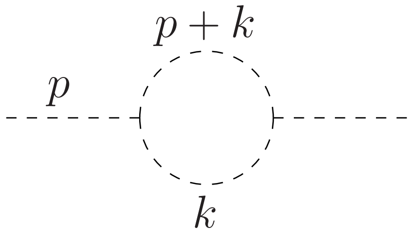
$$= \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{e_1 \mathrm{d}e_1}{e_1 e_2} \delta(w - w') = \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{\mathrm{d}w'}{w'} \delta(w - w') \Rightarrow \frac{\mathrm{d}\Omega}{16\pi^2} \frac{|p|}{\sqrt{s}}$$

(for the last equality, the integral over $\mathrm{d}w'$ has been performed). The differential cross section for a two body process is then:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2s} \times \Sigma |M(i_1 i_2 \to f_1 f_2)|^2 \times \frac{1}{16\pi^2} \left(\frac{|p|}{\sqrt{s}} \right) \times S$$

Note that $|p| = \frac{1}{2}\sqrt{s}\lambda = \frac{1}{2}\sqrt{s}[1 - m_1^2/s - m_2^2/s)^2 - 4m_1^2 m_2^2/s^2]^{\frac{1}{2}}$.

4. Calculation of loop integrals



$$= -i\Pi(p^2)$$

- **Measure of loop integral over internal momentum:** $\int \mathrm{d}^4k/(2\pi)^4$**.**

(For fermion loops: take trace and factor (-1) **for Fermi stats).**

$$-i\Gamma = (ig)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{(p+k)^2 - m^2} \frac{i}{k^2 - m^2}$$

$$\Rightarrow \Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2}$$

- **Symmetrize the integrand using:** $1/ab = \int_0^1 \mathrm{d}x/[a + (b-a)x]^2$

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + 2pkx + p^2x - m^2)^2}$$

- **Shift variable** $k \to k' = k + px$ **(integrand becomes** k^2 **symmetric)**

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + p^2x(1-x) - m^2)^2}$$

- **Wick rotation** $k_0 \to ik_0$ **to go to Euclidean space (**$k^2 \to -k^2$**)**

$$\Gamma = -g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 - p^2x(1-x) + m^2)^2}$$

- **Polar coordinates for d**^4k**:** $\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^\infty dk^2\, k^2\, F(k^2)$

$$\Gamma = -\frac{g^2}{16\pi^2} \int_0^1 \mathrm{dx} \int_0^\infty y\mathrm{d}y \frac{1}{(y - p^2x(1-x) + m^2)^2}$$

- **Perform the integrals over the variables** y **and** x**:**

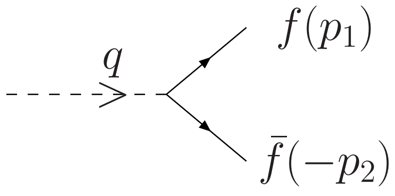
– **If integral divergent: cut-off at the energy** Λ **(**$\int_0^{\Lambda^2} \mathrm{d}k^2$**).**

– **Eventually, use the on–shell mass relation** $p^2 = m^2$**.**

Higgs production and decay mechanisms

1. Higgs bosons decays

1.1 Decays into fermions: $H \to f\bar{f}$



$$-iM = \bar{u}^{s_1}(p_1)(im_f/v)v^{s_2}(-p_2)$$

$$+iM^\dagger = \bar{v}^{s_2}(-p_2)(-im_f/v)u^{s_1}(p_1)$$

$$\sum_{s_1,s_2} MM^\dagger \; = \; N_c \left(\frac{m_f}{v}\right)^2 \sum_{s_1,s_2} \bar{v}^{s_2}(-p_2)\, u^{s_1}(p_1)\bar{u}^{s_1}(p_1)v^{s_2}(-p_2)$$

with $N_c = 3(1)$ for quarks (leptons). Only one polarisation for H.

$$\begin{aligned}(v/m_f)^2/N_c \times \Sigma|M|^2 &= \mathrm{Tr}(\not{p}_1 + m)(-\not{p}_2 - m) \\ &= \mathrm{Tr}(\gamma_\mu p_1^\mu + m)(-\gamma_\nu p_2^\nu - m) \\ &= -p_1^\mu p_2^\nu \mathrm{Tr}(\gamma_\mu\gamma_\nu) - m^2\mathrm{Tr}(1) \\ &= -4p_1.p_2 - 4m^2\end{aligned}$$

Using $q^2 = (p_1 - p_2)^2 = 2m_f^2 - 2p_1 \cdot p_2 = M_H^2$ and defining the velocity of the final fermions $\beta_f = 2|p_f|/M_H = (1 - 4m_f^2/M_H^2)^{1/2}$

$$\Rightarrow \Sigma|M|^2 = N_c\,(m_f/v)^2\,2(M_H^2 - 4m_f^2) = 2N_c\,(m_f^2/v^2)\,M_H^2\beta_f^2$$

The differential decay width is then simply given by:

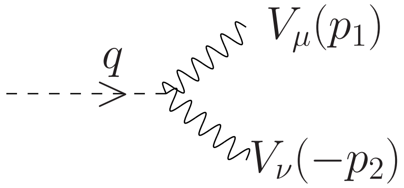
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{2M_H} \times \Sigma|M|^2 \times \frac{1}{32\pi^2} \times \frac{2|p_f|}{M_H}$$

Integrating over $\mathrm{d}\Omega = \mathrm{d}\phi \mathrm{d}\cos\theta$ (and since there is no angular dependence, $\int \mathrm{d}\Omega = 4\pi$), one obtains the partial decay width:

$$\Gamma(H \to f\bar{f}) = N_c \frac{m_f^2}{v^2} \frac{M_H}{8\pi} \beta_f^3$$

H **decays dominantly into heaviest fermion and width $\propto M_H$.**

1.2 Decays into massive gauge bosons



$$-iM = \epsilon^*_\mu(p_1)\,(-2iM_V^2/v\, g^{\mu\nu})\,\epsilon^*_\nu(-p_2)$$

$$+iM^\dagger = \epsilon_{\mu'}(p_1)\,(-2iM_V^2/v\, g^{\mu'\nu'})\,\epsilon_{\nu'}(-p_2)$$

$$\sum_{\rm pol} |M|^2 = \frac{4M_V^4}{v^2}\, g^{\mu\nu}\, g^{\mu'\nu'} \sum_{\rm pol} \epsilon^*_\mu(p_1)\epsilon_{\mu'}(p_1) \sum_{\rm pol} \epsilon^*_\nu(-p_2)\epsilon_{\nu'}(-p_2)$$

$$\begin{aligned}(v^2/4M_V^4)\Sigma &= g^{\mu\nu}\, g^{\mu'\nu'}\,(g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g_{\nu\nu'} - p_{2\nu}p_{2\nu'}/M_V^2)\\ &= (g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g^{\mu\mu'} - p_2^\mu p_2^{\mu'}/M_V^2)\\ &= 4 - p_1^2/M_V^2 - p_2^2/M_V^2 + (p_1\cdot p_2)^2/M_V^4\\ &= (M_H^4/4M_V^4)\,[1 - 4M_V^2/M_H^2 + 12M_V^4/M_H^4]\end{aligned}$$

The differential decay width, $\frac{{\rm d}\Gamma}{{\rm d}\Omega} = \frac{1}{2M_H} \times |M|^2 \times \frac{1}{32\pi^2}\frac{2|p_V|}{M_H} \times S$**, with** $S = \delta_V = \frac{1}{2}$ **for two identical final** Z **bosons. This finally gives** $(\int {\rm d}\Omega = 4\pi)$**:**

$$\Gamma(H\to VV) = \frac{\delta_V M_H^3}{16\pi v^2}\left(1 - \frac{4M_V^2}{M_H^2}\right)^{1/2}\left(1 - 4\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right)$$

The dependence on M_V **is hidden, since** $v \equiv 2M_W/g_2 = 2M_Z c_W/g_2$**. For large enough** M_H **[recall that** $H \to f\bar{f} \propto M_H$**], one has:**

$$\Gamma(H\to VV) \simeq \delta_V M_H^3/(8\pi v^2) \Rightarrow \Gamma(H\to WW) \simeq 2\Gamma(H\to ZZ)$$

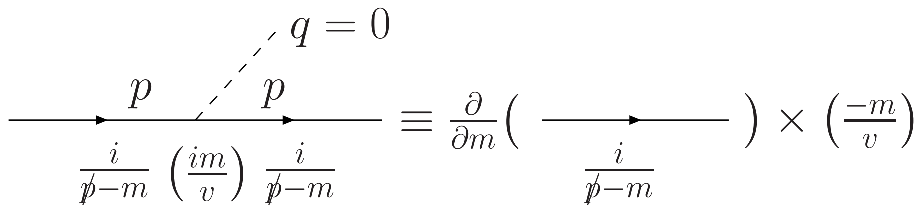
The decay widths grows like M_H^3 **i.e. is very large for** $M_H \gg M_V$**. For small** M_H**, one (two)** V **bosons can be off–shell, the width is**

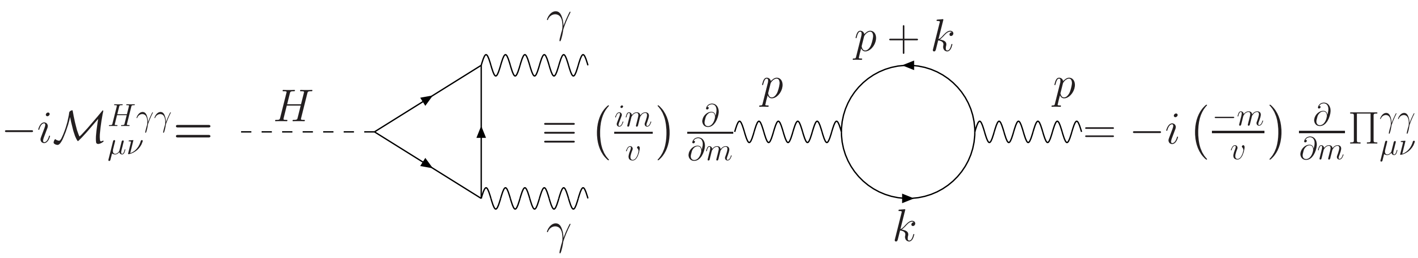
$$\Gamma = \frac{\Gamma_0}{\pi^2}\int_0^{M_H^2} \frac{{\rm d}q_1^2 M_V\Gamma_V}{(q_1^2 - M_V^2)^2 + M_V^2\Gamma_V^2}\int_0^{M_H^2 - q_1^2} \frac{{\rm d}q_2^2 M_V\Gamma_V}{(q_2^2 - M_V^2)^2 + M_V^2\Gamma_V^2}$$

$$\Gamma_0 = \frac{\delta_V M_H^3}{8\pi v^2}\lambda^{1/2}\left(\lambda - \frac{12q_1^2q_2^2}{M_H^4}\right)\ ,\ \lambda = \left(1 - \frac{q_1^2}{M_H^2} - \frac{q_1^2}{M_H^2}\right)^2 - \frac{4q_1^2q_2^2}{M_H^4}$$

1.3 Decays into photons and gluons: $H \to \gamma\gamma, gg$

H **does not couple to massless particles at tree-level: loop induced. We have vertex diagrams with fermion (top) and** W **exchange for** $H \to \gamma\gamma(Z\gamma)$; **only top for** $H \to gg$: **calculation complicated. However it is simple if** H **momentum is small (i.e.** $M_H \ll M_{\rm loop}$**):**





Let's calculate the derivative of the fermionic photon self-energy:

$$-i\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = N_c \int \frac{d^4k}{(2\pi)^4}(-1){\rm Tr}(-iee_f\gamma_\mu)\frac{i}{\not k - m}(-iee_f\gamma_\nu)\frac{i}{\not p + \not k - m}$$

$$\Pi^{\gamma\gamma}_{\mu\nu}(p) = -i\, N_c\, e^2 e_f^2 \int \frac{d^4k}{(2\pi)^4} \frac{{\rm Tr}\gamma_\mu(\not k + m)\gamma_\nu(\not p + \not k + m)}{[(p+k)^2 - m^2](k^2 - m^2)}$$

Using the rules for Diracology and loop integral calculations:

$$D = \int_0^1 \frac{{\rm d}x}{(k^2 + 2pkx + p^2x - m^2)^2} = \int_0^1 \frac{{\rm d}x}{[(k+px)^2 + p^2x(1-x) - m^2]^2}$$

$$\begin{aligned} N &= {\rm Tr}[\gamma_\mu \not k \gamma_\nu(\not p + \not k) + m^2\gamma_\mu\gamma_\nu] = k^\rho (k+p)^\sigma {\rm Tr}[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma + m^2\gamma_\mu\gamma_\nu] \\ &= 4[2k_\mu k_\nu + (m^2 - k^2 - p.k)g_{\mu\nu}] \end{aligned}$$

Shift $k \to k + px$, **Wick rotation** $k_0 \to ik_0$ **for Euclidean space** $\Rightarrow$ $k^2 \to -k^2$ **and sym. integrand with** $\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^\infty dy y F(y)$

[also use of symmetry relation $\int d^4k(k_\mu k_\nu) = \frac{1}{4}g_{\mu\nu}\int d^4k(k^2)$]:

$$\Pi^{\gamma\gamma}_{\mu\nu}(p) = -iN_c\, e_f^2\, e^2 \times 4 \times \pi^2 \times \frac{i}{16\pi^4} \times \int_0^1 \mathrm{d}x \int_0^\infty y dy \frac{[\frac{1}{2}k^2 + m^2 - x(1-x)p^2]g_{\mu\nu} + 2x(1-x)[g_{\mu\nu}p^2 - p_\mu p_\nu]}{[y + m^2 - p^2x(1-x)]^2}$$

Because of gauge invariance, photon is transverse ($\propto g_{\mu\nu}p^2 - p_\mu p_\nu$): the first term ($\propto g_{\mu\nu}$ should vanish* and we are left with:

$$\Pi^{\gamma\gamma}_{\mu\nu}(p) = \frac{N_c e_f^2 e^2}{4\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu)\int_0^1 \mathrm{d}x \int_0^\infty y dy \frac{2x(1-x)}{[y + m^2 - p^2x(1-x)]^2}$$

We can now calculate the $H\gamma\gamma$ vertex [photons to symmetrize $\rightarrow 2$; they are on–shell and $p_{1,2} \neq p$ but $p^2 = p_1 \cdot p_2 = \frac{1}{2}M_H^2$]

$$\mathcal{M}^{H\gamma\gamma}_{\mu\nu} = -2\frac{m}{v}\frac{\partial}{\partial m}\Pi^{\gamma\gamma}_{\mu\nu}(p_1, p_2) = -\frac{4m^2}{v}\frac{\partial}{\partial m^2}\Pi^{\gamma\gamma}_{\mu\nu}(p_1, p_2)$$

$$= -\frac{2m^2}{v}\frac{N_c e_f^2 e^2}{\pi^2}(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})\int_0^1 \mathrm{d}x \int_0^\infty \frac{-2x(1-x)y dy}{[y + m^2 - p^2x(1-x)]^3}$$

Inside the integral, we can suppose $m^2 \gg p^2(M_H^2)$ and integrate over x and y [$\int x(1-x)dx = 1/6$ and $\int y/(y+m^2)^3 dy = 1/2m^2$]

$$\mathcal{M}^{H\gamma\gamma}_{\mu\nu} = \frac{2}{3v}N_c\, e_f^2\, \frac{\alpha}{\pi}\,(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})$$

Now we use the same machinery as for decays into gauge bosons:

$$|\mathcal{M}|^2 = \frac{4}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{\pi^2}\,\frac{M_H^4}{2}\,\Sigma|g^{\mu\nu}\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)|^2 = \frac{2M_H^4}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{\pi^2}$$

Integrating over phase space (with factor $\frac{1}{2}$ for identical photons):

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{M_H^3}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{16\pi^3}$$

*This statement is not trivial to prove and we will come back to this discussion later on.

Several remarks to be made:

- **The amplitude was of course finite (no tree level contribution)!**
- **The approximation $m_f \gg M_H$ is in practice good up to $M_H \sim 2m_f$!**
- **Only tops contribute, other f have negligible Yukawa coupling.**
- **Infinitely heavy fermions do not decouple from the amplitude: a way to count the number of heavy particles coupling to the H!**
- **There are also contributions from W bosons. Also in the limit $M_H \ll M_W$ (valid for $M_H \lesssim 140$ GeV), one has:**

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{9v^2}\frac{\alpha^2}{16\pi^3}\left|\Sigma_f N_c e_f^2 - \frac{21}{4}\right|^2$$

- **The W contribution is larger (~ 4) than the t quark contribution and the interference of the two is destructive.**
- **With the same calculation, one can get the amplitude for $H \to Z\gamma$. Only difference, Zff, ZWW couplings and M_Z in phase space. Here again, the W contr. is much ($\gtrsim 10$) larger than that of top.**

- **The calculation holds also for gluons if we make the changes: $Q_e e \to g_s T_a$ which means $\alpha \to \alpha_s$ and $N_c^2 \to |\mathrm{Tr}(T_a T_a)|^2 = |\frac{1}{2}\delta_{ab}|^2 = 2$:**

$$\Gamma(H \to gg) = \frac{M_H^3}{9v^2}\frac{\alpha_s^2}{8\pi^3}$$

Decay width and branching ratios:

The total decay width of the Higgs is the sum of partial widths:

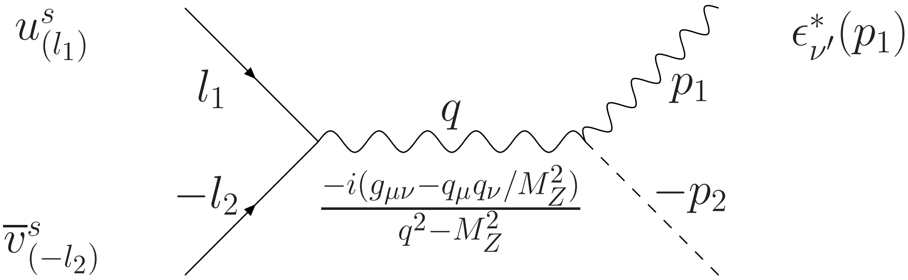
$$\Gamma_{\rm tot}(H \to \mathrm{all}) = \Sigma_f \Gamma(H \to f\bar{f}) + \Sigma_V \Gamma(H \to VV)$$

and the branching ratio for Higgs decay into a given final state is:

$$\mathrm{BR}(H \to X) = \Gamma(H \to X)/\Gamma_{\rm tot}(H \to \mathrm{all})$$

2. Higgs bosons production in e^+e^- Collisions

2.1 The Higgs–strahlung process:



$$-iM = \overline{v}^s_{(-l_2)}(-ie)\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)}\,\frac{-i(g^{\mu\nu} - q^\mu q^\nu/M_Z^2)}{q^2 - M_Z^2}\Big(\frac{-2iM_Z^2}{v}g_{\nu\nu'}\Big)\epsilon^*_{\nu'}$$

First thing to use for simplification is Dirac equation $\not{l}u(l) = m_e \sim 0$:

$$\overline{v}^s_{(-l_2)}\gamma_\mu q^\mu u^s_{(l_1)} = \overline{v}^s_{(-l_2)}[-\not{l}_2 + \not{l}_1]u^s_{(l_1)} = 2m_e \sim 0 \Rightarrow q_\mu q_\nu \to 0$$

where m_e **is supposed to be much smaller than** $\sqrt{s} = \sqrt{q^2}$**. Then:**

$$|M|^2 = \frac{4e^2M_Z^4v^{-2}}{(q^2 - M_Z^2)^2}\epsilon^\nu_{(p_1)}\epsilon^{*\mu}_{(p_1)}\overline{v}^s_{(-l_2)}\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)}\overline{u}^s_{(l_1)}\gamma_\nu(v_e - a_e\gamma_5)v^s_{(-l_2)}$$

Average over polarizations of $e^\pm$ **and sum on those of photon:**

$$\frac{1}{4}\Sigma|M|^2 = \frac{k}{4}\mathrm{Tr}\,\not{l}_1(v_e - a_e\gamma_5)\gamma_\mu(-\not{l}_2)(v_e - a_e\gamma_5)\gamma_\nu\Big(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{M_Z^2}\Big)$$

$$-\mathrm{Tr} = (v_e^2 + a_e^2)\mathrm{Tr}\,\not{l}_1\gamma_\mu\,\not{l}_2\gamma_\nu - 2a_ev_e\mathrm{Tr}\,\not{l}_1\gamma_\mu\,\not{l}_2\gamma_\nu\gamma_5$$

$$= 4(v_e^2 + a_e^2)[l_{1\mu}l_{2\nu} + l_{2\mu}l_{1\nu} - l_1.l_2g_{\mu\nu}] - 8ia_ev_e\,l_1^\alpha l_2^\beta\epsilon_{\alpha\mu\mu\beta\nu}$$

$$\frac{1}{4}\Sigma|M|^2 = k(v_e^2 + a_e^2)\left[2(l_1.l_2) - 2\frac{(l_1.p_1)(l_2.p_1)}{M_Z^2} - 4(l_1.l_2) + (l_1.l_2)\frac{p_1^2}{M_Z^2}\right]$$

$$= k(v_e^2 + a_e^2)\left[-(l_1.l_2) - 2(l_1.p_1)(l_2.p_1)/M_Z^2\right]$$

where we have used the fact that $(\epsilon_{\alpha\mu\beta\nu})\,g^{\mu\nu} - p_1^\mu p_1^\nu$ **is (anti)symmetric. In the c.m. frame, one has (with** $E_{1,2}^2 = M_{Z,H}^2 + |p|^2$**) and** $|p| = \sqrt{s}/2\lambda$**):**

$$l_{1,2} = \frac{\sqrt{s}}{2}(1, 0, 0, \pm 1) \text{ and } p_{1,2} = (E_{Z,H}, 0, \pm|p|\sin\theta, \pm|p|\cos\theta)$$

$$\Rightarrow k(v_e^2+a_e^2)\left[\frac{s}{2}+\frac{s(E_Z^2-|p|^2\cos^2\theta)}{2M_Z^2}\right]=k(v_e^2+a_e^2)\frac{s^2}{M_Z^2}\left[\frac{M_Z^2}{s}+\frac{\lambda^2\sin^2\theta}{8}\right]$$

The differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta\mathrm{d}\phi}=\frac{1}{2s}\left[\frac{4e^2M_Z^4(v_e^2+a_e^2)s}{v^2(s-M_Z^2)^2M_Z^2}\left(\frac{M_Z^2}{s}+\frac{1}{8}\lambda^2\sin^2\theta\right)\right]\frac{\lambda}{32\pi^2}$$

with $\int \mathrm{d}\phi = 2\pi$ **and** $\int \sin^2\theta \mathrm{d}\cos\theta = 4/3$ **one gets the cross section**

$$\sigma(e^+e^-\to HZ)=\frac{\alpha M_Z^2}{12v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda(\lambda^2+12M_Z^2/s)$$

A few remarks:

- **The cross section drops like** $1/s$ **at high–energies (typical of an** s**–channel process). The maximum is reached at** $\sqrt{s}=M_Z+\sqrt{2}M_H$**.**
- **At the maximum LEP2 energy,** $\sqrt{s}=209$ **GeV, the cross section for** $M_H=(100)\,115$ **GeV is given by (using the fact that** $\sigma_0=4\pi\alpha^2(0)/3=86.8$ **nb with** $\alpha(0)=1/137, \alpha(s)\simeq 1/128$ **and** $\sin^2\theta_W=0.232$**):**

$$\sigma=0.42\ (0.16)\ \text{pb for } M_H=100\ (115)\ \text{GeV}$$

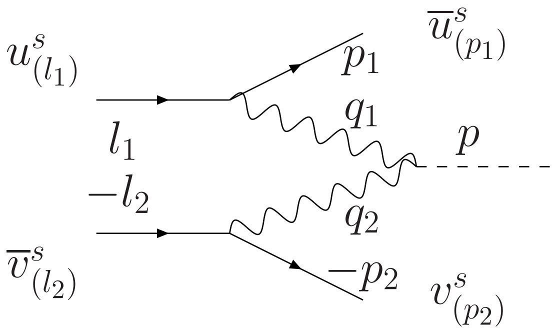
If we have an integrated luminosity of $\int\mathcal{L}\sim 100$ **pb**$^{-1}$**, this means that we have** $N=\sigma\times\int\mathcal{L}\sim 42(16)$ **Higgs boson events.**

- **Since for** $M_H\sim 100$ **GeV, BR**$(H\to b\bar{b})\sim 90\%$**, the signal is** $e^+e^-\to ZH\to Zb\bar{b}$ **and the main background is** $e^+e^-\to ZZ\to Z\bar{b}b$**.**
- **At high energies** $s\gg M_Z^2$**, one has a differential cross section**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\simeq\frac{3}{4\sigma}\sin^2\theta\ \text{with}\ \sigma\simeq\frac{\alpha M_Z^2}{12v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda^3$$

the behaviour in $\sin^2\theta$ **of the angular distribution and in** λ^3 **of the total cross section is typical for the production of two spin–zero particles (here, the** Z **boson is almost a Goldstone boson).**

2.2 The vector boson fusion mechanism:



$$M = \frac{i(-ie)^2(-i)^2(-2iM_V^2/v)}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)} g_{\mu'\nu'} \times \begin{array}{l} \bar{u}^s_{(p_1)}\gamma_\mu(v - a\gamma_5)u^s_{(l_1)} \left(g^{\mu\mu'} - \frac{q_1^\mu q_1^{\mu'}}{M_V^2} \right) \\ \bar{v}^s_{(-p_2)}\gamma_\nu(v - a\gamma_5)\bar{v}^s_{(l_2)} \left(g^{\nu\nu'} - \frac{q_2^\nu q_2^{\nu'}}{M_V^2} \right) \end{array}$$

Using the relations $q_1^\mu \gamma_\mu = \not{q} = \not{l}_1 - \not{p}_1 \propto m_e \sim 0$ **and** $g^{\nu\nu'}g^{\mu\mu'}g_{\mu'\nu'} = g^{\mu\nu}$**:**

$$\begin{aligned} |M|^2 &= \frac{4e^4 M_V^4/v^2}{D_1^2 D_2^2} \times \begin{array}{l} \bar{u}^s_{(p_1)}\gamma_\mu(v - a\gamma_5)u^s_{(l_1)}.\bar{v}^s_{(-l_2)}\gamma_\nu(v - a\gamma_5)v^s_{(-p_2)} \\ u^s_{(l_1)}\gamma^\mu(v - a\gamma_5)u^s_{(p_1)}.\bar{v}^s_{(-p_2)}\gamma^\nu(v - a\gamma_5)v^s_{(-l_2)} \end{array} \\ &= \frac{4e^4 M_V^4/v^2}{D_1^2 D_2^2} \times \begin{array}{l} \mathrm{Tr}\ \not{l}_1\gamma_\nu(v - a\gamma_5)\ \not{p}_1\gamma_\mu(v - a\gamma_5) \\ \mathrm{Tr}\ \not{l}_2\gamma^\nu(v - a\gamma_5)\ \not{p}_2\gamma^\mu(v - a\gamma_5) \end{array} \\ &= \frac{4e^4 M_V^4/v^2}{D_1^2 D_2^2} \times \begin{array}{l} (v^2 + a^2)\mathrm{Tr}\ \not{l}_1\gamma_\nu\ \not{p}_1\gamma_\mu - 2va\mathrm{Tr}\ \not{l}_1\gamma_\nu\ \not{p}_1\gamma_\mu\gamma_5 \\ (v^2 + a^2)\mathrm{Tr}\ \not{l}_2\gamma^\nu\ \not{p}_2\gamma^\mu - 2va\mathrm{Tr}\ \not{l}_2\gamma^\nu\ \not{p}_2\gamma^\mu\gamma_5 \end{array} \end{aligned}$$

Performing the trace and product using $\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\alpha'\beta'} = \delta^\alpha_{\alpha'}\delta^\beta_{\beta'} - \delta^\alpha_{\beta'}\delta^\beta_{\alpha'}$

$$\frac{1}{4}|M|^2 = \frac{32e^4 M_V^4/v^2}{D_1^2 D_2^2} \times [g_S(l_1.p_2)(l_2.p_1) + g_A(l_1.l_2)(p_1.p_2)]$$

with $g_S = (v^2 + a^2)^2 + 4a^2v^2$ and $g_A = (v^2 + a^2)^2 - 4a^2v^2$

Let's write the momenta of the particles in a convenient way:

$$l_1 = (E, 0, 0, E)\ , p_1 = (\sqrt{x_1^2E^2 + p_{T1}^2}, p_{T1}\sin\theta_1, p_{T1}\cos\theta_1, x_1E)$$

$$l_2 = (E, 0, 0, -E)\ , p_2 = (\sqrt{x_2^2E^2 + p_{T2}^2}, p_{T2}\sin\theta_1, p_{T1}\cos\theta_1, -x_2E)$$

and assume high energies $s \gg M_V^2$ so that $p_{T1,T2}/E$ are rather small:

$$l_i.p_i \sim p_{Ti}^2/2x_i \ , \ l_1.p_2 \sim 2E^2 x_2 \ , \ l_2.p_1 \sim 2E^2 x_1 \ , \ p_1.p_2 \sim 2E^2 x_1 x_2$$

hold, together with $2l_1.l_2 = s$ and the Higgs momentum squared:

$$M_H^2 = (q - p_1 - p_2)^2 = s - 2q.p_1 - 2q.p_2 + 2p_1.p_2 = s(1-x_1)(1-x_2)$$

Using these products, one has then for the amplitude squared:

$$\begin{aligned} \frac{1}{4}|M|^2 &= \frac{32 e^4 M_V^4}{v^2} \times \frac{4E^4(g_S + g_A)x_1 x_2}{(p_{T1}^2/x_1 + M_W^2)^2 (p_{T2}^2/x_2 + M_W^2)^2} \\ &= \frac{8 e^4 M_V^4}{v^2} \times \frac{(g_S + g_A)s^2 x_1^3 x_2^3}{(p_{T1}^2 + x_1 M_W^2)^2 (p_{T2}^2 + x_2 M_W^2)^2} \end{aligned}$$

Let us now deal with the three body phase space:

$$\mathrm{dPS3} = \frac{1}{(2\pi)^5} \frac{\mathrm{d}^3 p_1}{2E_1} \frac{\mathrm{d}^3 p_2}{2E_2} \frac{\mathrm{d}^3 p}{2E_H} \, \delta^4(q - p_1 - p_2 - p)$$

Defining $\tau_H = M_H^2/s$ and using the known relation for δ functions:

$$\int \frac{\mathrm{d}^3 p}{2E_H} = \int \mathrm{d}^4 p \, \delta(p^2 - M_H^2) = \int \mathrm{d}^4 p \, \delta[s(1-x_1)(1-x_2) - s\tau_H]$$

and decomposing the momenta along the 3 directions, one obtains:

$$\mathrm{dPS3} \ = \ \frac{1}{(2\pi)^5} \frac{\mathrm{d}(x_1 E)}{2x_1 E} \mathrm{d}^2 p_{T1} \frac{\mathrm{d}(x_2 E)}{2x_2 E} \mathrm{d}^2 p_{T2} \, \delta[s(1-x_1)(1-x_2) - s\tau_H]$$

Noting that $\int \mathrm{d}p_{Ti}^2/(p_{Ti}^2 + x_i M_V^2)^2 = \pi \int_0^\infty \mathrm{d}p^2/(p^2 + x_i M_V^2)^2 = \pi^2/(x_i M_V^2)$ and using $M_W = ev/(2s_W)$, the differential cross section is given by:

$$\begin{aligned} \mathrm{d}\sigma &= \frac{1}{2s} \times \left(\frac{8 e^6 M_V^4}{4 M_W^2 s_W^2} \right) (g_S + g_A) s^2 x_1^3 x_2^3 \times \frac{1}{(2\pi)^5} \frac{\mathrm{d}x_1}{2x_1} \frac{\mathrm{d}x_2}{2x_2} \frac{\pi^2}{x_1 x_2 M_V^4} \, \delta \\ \sigma &= \frac{\alpha^3}{2 M_W^2 s_W^2} (g_S + g_A) \int \mathrm{d}x_1 \int \mathrm{d}x_2 x_1 x_2 \, s\delta[s(1-x_1)(1-x_2) - s\tau_H] \end{aligned}$$

Now perform the integrals using $\int \delta[f(x)] = |f'(x)|^{-1}_{x=x_0}$ **with** $f(x_0) = 0$

$$\int \mathrm{d}x_1 \int \mathrm{d}x_2 \cdots = \int_0^{1-\tau_H} \mathrm{d}x_1 x_1 \left(1 - \frac{\tau_H}{1-x_1}\right) s \frac{1}{s(1-x_1)}$$
$$= \int_0^{1-\tau_H} \mathrm{d}x_1 \left[-1 + \frac{1+\tau_H}{1-x_1} + \frac{\tau_H}{(1-x_1)^2}\right] = (1+\tau_H)\log\frac{1}{\tau_H} - 2(1-\tau_H)$$

where the boundary conditions are obtained by requiring that $p_{1Z} = p_{2Z} = x_{1,2}E = 0 \Rightarrow x_1 = 0$ **and** $x_2 = 1-\tau_H/(1-x_1) = 0 \rightarrow x_1 = 1-\tau_H$**. Collecting all results, one obtains then the total cross section*****:**

$$\sigma = \frac{\alpha^3}{2M_W^2 s_W^2}(g_S + g_A)\left[(1+\tau_H)\log\frac{1}{\tau_H} - 2(1-\tau_H)\right]$$

Let us now make a few remarks:

- **The cross section rises as** $\log(s/M_H^2)$**: small at low** $\sqrt{s}$ **and large at high** $\sqrt{s}$**. Dominant Higgs production process for** $s \gg M_H^2$**.**
- **This approximation is good only within a factor of 2 and works better at higher energies. It can be obtained in an easier way using the effective longitudinal vector boson approximation.**
- **In the case of** WW **fusion,** $g_s = 8/(2\sqrt{2})^4 = 1/8$ **and** $g_A = 0$**, one has:**

$$\sigma(e^+e^- \rightarrow H\bar{\nu}\nu) \;=\; \frac{\alpha^3}{16M_W^2 s_W^2}\left[(1+\tau_H)\log\frac{s}{M_H^2} - 2(1-\tau_H)\right]$$

- **At LEP2 energies,** $\sqrt{s} \sim 200$ **GeV, the cross section is** $\sigma \sim 5(2) \cdot 10^{-3}$ **pb for** $M_H = 100(115)$ **GeV, i.e. less than one event for** $\int \mathcal{L} = 100$ **pb**$^{-1}$**. This process is not very useful for Higgs searches at LEP2.**
- **For** ZZ **fusion with** $s_W^2 \sim 1/4$**,** $g_s \sim g_A \sim a_e^4 \sim 1/(16 \times 9)$**: the cross section** $\sigma(e^+e^- \rightarrow e^+e^-H)$ **is** ~ 9 **times smaller than for** WW **fusion.**

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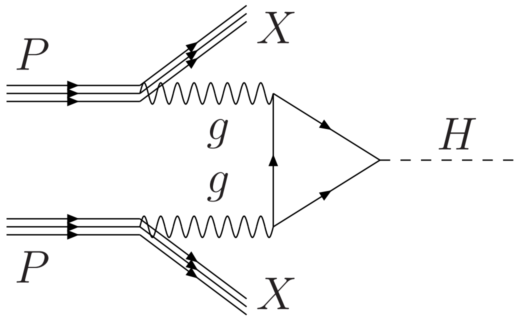
*This calculation, including details is done in: G. Altarelli, B. Mele and F. Pitolli, Nucl. Phys. B287 (1987) 205.

3. Higgs bosons production in hadronic Collisions

3.1 The gluon–gluon fusion process*



The cross section of the subprocess, $gg \to H$, is given by:

$$\mathrm{d}\hat{\sigma} = \frac{1}{2\hat{s}} \times \frac{1}{2\cdot 8} \times \frac{1}{2\cdot 8} |\mathcal{M}_{Hgg}|^2 \frac{\mathrm{d}^3 p_H}{(2\pi)^3 2E_H} (2\pi^4)\delta^4(q - p_H)$$

Using the fact that $\int \mathrm{d}^3 p_H/(2E_H) = \int \mathrm{d}^4 p_H \delta(p_H^2 - M_H^2)$ and that $|M_{Hgg}|^2 = 32\pi M_H \Gamma(H \to gg)$ calculated before, one obtains for $\hat{\sigma}$:

$$\hat{\sigma} = \frac{\pi^2 M_H}{8\hat{s}} \Gamma(H \to gg)\, \delta(\hat{s} - M_H^2)$$

Convolute with gluon densities to obtain the total cross section

$$\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{\pi^2 M_H}{8\hat{s}} \Gamma(H \to gg) g(x_1) g(x_2) \delta(\hat{s} - M_H^2)$$

with $\hat{s} = s x_1 x_2$, implying $\hat{s} - M_H^2 = s(x_1 x_2 - \tau_H)$ with $\tau_H = M_H^2/s$:

$$\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{\pi^2}{8 M_H} \Gamma(H \to gg)\, g(x_1) g(x_2) \delta[s(x_1 x_2 - \tau_H)]$$

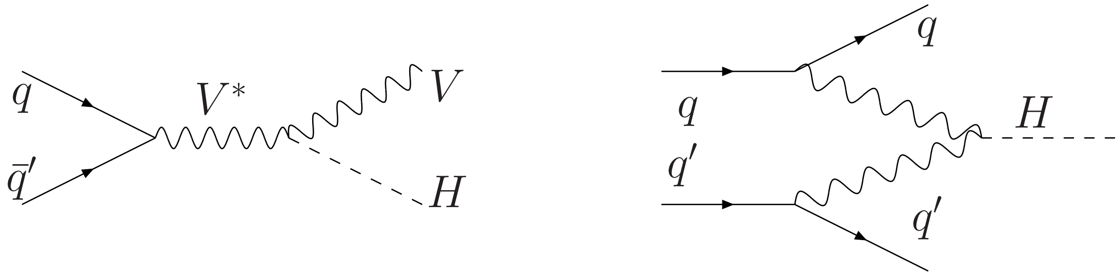
We perform the integral on x_2 $[\int \delta[f(x)] = |f'(x)|^{-1}_{x=x_0}$ with $f(x_0) = 0]$

$$\sigma = \frac{\pi^2}{8 M_H^3} \Gamma(H \to gg)\, \tau_H \int_{\tau_H}^1 \frac{\mathrm{d}x}{x} g(x) g(x/\tau_H) = \frac{1}{576 v^2} \frac{\alpha_s^2}{\pi} \tau_H \frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau_H}$$

where the integration bounds are $x_1^{\max} = 1, x_1^{\min} = x_1$(for $x_2 = 1) = \tau$.
At LHC, gg luminosity is large and $gg \to H$ dominant process!

*Calculation to be checked!!!

3.2 The Higgs strahlung and vector boson fusion process



The cross sections for these processes are the same as in e^+e^- collisions, provided that the following changes are performed:
– The total energy $\sqrt{s}$ is replaced by the subprocess energy $\hat{s}$.
– The average over the quark colors is made: factor $\frac{1}{3} \cdot \frac{1}{3}$.
– In the bremsstrahlung process, possibility of $q\bar{q}' \to W^* \to WH$.
– The couplings of the electrons are replaced by those of quarks:
in $q\bar{q} \to VH$: $a_e^2 + v_e^2 \to a_q^2 + v_q^2$.
in $qq \to Hqq$: $g_{S,A} \to [(v^2 + a^2)(v'^2 + a'^2) \pm 4(av)(a'v')$.

The cross sections for a given initial state, are given by:

$$\begin{aligned} \sigma(q\bar{q}' \to HV) &= \frac{1}{9}\frac{\alpha M_V^2}{12v^2}\frac{v_q^2 + a_q^2}{\hat{s}(1 - M_V^2/\hat{s})^2}\hat{\lambda}(\hat{\lambda}^2 + 12M_V^2/\hat{s}) \\ \sigma(qq \to qqH) &= \frac{1}{9}\frac{\alpha^3}{2M_W^2 s_W^2}(g_S + g_A)\left[(1 + \hat{\tau}_H)\log\frac{1}{\hat{\tau}_H} - 2(1 - \hat{\tau}_H)\right] \end{aligned}$$

Summing over all possibilities for quark/antiquark initial states and folding with the proper densities, the total cross sections are:

$$\sigma[pp \to H + X] = \sum_{q,q'} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 f_q(x_1) f_{q'}(x_2)\, \hat{\sigma}[qq' \to H + X]$$

Remarks:
- At LHC, $qq \to Hqq$ is the dominant process but not as $gg \to H$.
- The cross section for $q\bar{q} \to HV$ is OK for low M_H; $\sigma(HW) \sim 2\sigma(HZ)$.
- At Tevatron, Higgs–strahlung (esp. $q\bar{q}' \to HW$) more important.