BRIEF HISTORY OF OUR UNIVERSE
STANDARD COSMOLOGICAL MODEL

FOUNDATIONS : COSMOLOGICAL PRINCIPLE
GENERAL RELATIVITY
STD. MODEL OF PARTICLE PHYSICS
+ EXTENSIONS: DARK MATTER, INFLATION, DARK ENERGY

4 AGES : INFLATION
RADIATION
MATTER
DARK ENERGY

BASIC TOOLS : FRW SPACETIME $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$

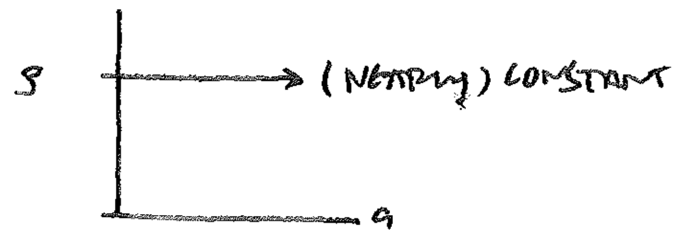
GR EQNS $$\frac{3}{8\pi G} H^2 = \sum_i \rho_i$$

$$-\frac{1}{4\pi G} \dot{H} = \sum_i (\rho_i + p_i)$$

+ MATTER EQNS : $p(\rho)$.

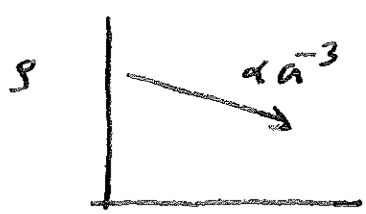
INFLATION, DARK ENERGY $\quad p \simeq -\rho$

$$\dot{\rho} = -3H(\rho + p)$$



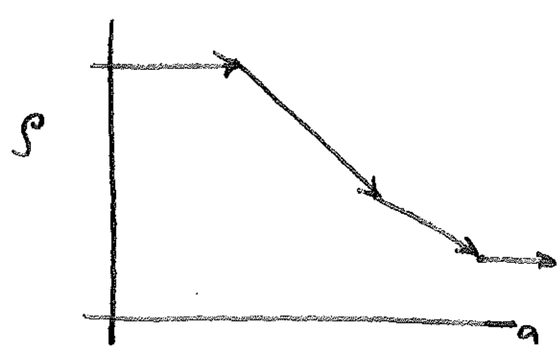
RADIATION $\quad p = \frac{1}{3}\rho$

MATTER $\quad p \ll \rho$



PUT IT TOGETHER

QUICK GLIMPSE
OF UNIVERSE



CLOSER LOOK: BRIEF HISTORY OF CURVATURE, OR TRACE

$$g^{\mu\nu} G_{\mu\nu} = -R = 8\pi G_N T$$

$$\Theta \equiv -T = \rho - 3p$$

$$\text{SO} \quad R = 8\pi G_N \Theta = 8\pi G_N (\rho - 3p)$$

WHAT IS HISTORY OF R, Θ vs. a?

ANSWER TO ~FEW % ACCURACY

5 STAGES:

1. MATTER ERA ONWARDS
2. FREE FIELD TREATMENT OF RADIATION ERA
3. QCD PHASE TRANSITION
4. ELECTROWEAK PHASE TRANSITION
5. BSM TO INFLATION, REHEATING

[SEE CALDWELL & GUBSER PRD 87 063523 (2013)]

1. MATTER ERA

$$R = \kappa \Theta = \kappa\left(\rho_b + \rho_c + (1-3w_{DE})\rho_{DE}\right)$$

$$= 3H_0^2\left[(\Omega_b+\Omega_c)\left(\frac{a_0}{a}\right)^3 + 4\Omega_{DE}\right]$$

$$\Omega_B + \Omega_c + \Omega_{DE} = 1$$

WMAP9 + BAO + H_0

$\Omega_b h^2 = 0.02266 \pm 0.00043$

$\Omega_c h^2 = 0.1157 \pm 0.0023$

$\Omega_{DE} = \Omega_\Lambda = 0.712 \pm 0.010$

$H_0 = 69.33 \pm 0.88$ km/s/Mpc

PLANCK + WP

$\Omega_b h^2 = 0.2205 \pm 0.00028$

$\Omega_c h^2 = 0.1199 \pm 0.0027$

$\Omega_{DE} = \Omega_\Lambda = 0.685 \pm 0.017$

$H_0 = 67.3 \pm 1.2$ km/s/Mpc

SUFFICES TO DESCRIBE UNIVERSE SINCE $z \lesssim 100$ (LAST 13 GYRS!)

WHILE WE'RE AT IT, EVERYONE SHOULD KNOW

$$\hbar c = 197 \text{ MeV-fm}$$
$$1 \text{ eV} = 1.16 \times 10^4 \text{ K}$$
$$\text{Mpc} = 3.08 \times 10^{22} \text{ m}$$
$$1 \text{ yr} = \pi \times 10^7 \text{ s}$$

AGE OF UNIVERSE:

$$t_u = \int dt = \int da \frac{dt}{da} = \int_a^{a_0} \frac{da'}{a' H(a')} = \int_0^z \frac{dz'}{(1+z') H(z')}$$

LUMINOSITY DISTANCE

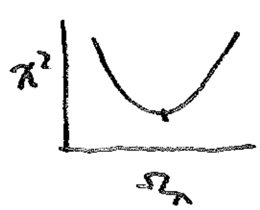
$$d_L(z) = (1+z) \int_0^z dz' / H(z')$$

OBSERVED INTENSITY = SOURCE REST FRAME LUMINOSITY / $4\pi d_L^2$

EXERCISE : DOWNLOAD SUPERNOVA DATA

$$m = 5 \log_{10}(H_0 d_L) + M \quad \text{vs.} \quad z$$

MINIMIZE χ^2 TO FIND BEST FIT Ω_Λ.



PROVE COSMIC ACCELERATION FOR YOURSELF.

2. FREE FIELD TREATMENT OF RADIATION ERA

USE STD. FORMULAE FOR A COLLECTION OF NON-INTERACTING FERMIONS AND BOSONS IN THERMAL EQUILIBRIUM

$$\rho = \sum_j \frac{g_j}{2\pi^2} \int_{m_j}^{\infty} dE \, \frac{E^2\sqrt{E^2 - m_j^2}}{e^{E/T_j} - s_j}$$

$$p = \sum_j \frac{g_j}{2\pi^2} \int_{m_j}^{\infty} dE \, \frac{(E^2 - m_j^2)^{3/2}}{e^{E/T_j} - s_j}$$

$s_j = \pm 1$ B/F.

μ ? CHEMICAL POTENTIALS ARE SMALL FOR CASES OF INTEREST

RELATIVISTIC SPECIES $T \gg m$

$$\rho_B = 3p_B = g \frac{\pi^2}{30} T^4$$

$$\rho_F = 3p_F = g \times \frac{7}{8} \frac{\pi^2}{30} T^4$$

$g = 2$ FOR PHOTONS

2×2 FOR ELECTRONS

$$\rho_{TOT} = g_* \frac{\pi^2}{30} T^4 \qquad (g_* = 2, \tfrac{7}{2} \text{ for } \gamma, e)$$

CONSIDER γ, e, ν SYSTEM

AT $T \gg \mathrm{MeV}$ $\quad \gamma, e, \nu$ IN EQUILIBRIUM

BUT $\nu - e$ INTERACTIONS SLOW DOWN

$$\sigma \sim G_F^2 T^2 , \quad n \sim T^3$$

interaction rate $\Gamma = \langle \sigma n \rangle \sim G_F^2 T^5$

expansion rate $H^2 = \frac{8\pi G}{3} \rho \sim T^4 / M_p^2$

$$\Gamma \sim H ? \quad G_F^2 T^5 \sim T^2 / M_p$$

$$T \sim \mathrm{MeV}$$

so as universe cools below MeV

ν's decouple from $\gamma - e$

yet $T_\nu = T_{\gamma e}$

* detailed result is flavor dependent

$$T_{\nu_e} \neq T_{\nu_\mu} \neq T_{\nu_\tau}$$

$$\rho_\nu = \frac{7}{8} \times 2 \times N_\nu \frac{\pi^2}{30} T_\nu^4$$

$$N_\nu \neq 3 \to 3.046$$

NEUTRINOS EVOLVE ADIABATICALLY

TOTAL ENTROPY $\mathcal{S} = a^3 s$, $s = \frac{\rho + p}{T}$

so $\mathcal{S}_i = \mathcal{S}_f$

$$\mathcal{S}_i = a_i^3 \left(1+\tfrac{1}{3}\right) \times \tfrac{7}{8} \times 2 \times N_\nu \frac{\pi^2}{30} T_{\nu,i}^3$$

$$\mathcal{S}_f = a_f^3 \left(1+\tfrac{1}{3}\right) \times \tfrac{7}{8} \times 2 \times N_\nu \frac{\pi^2}{30} T_{\nu,f}^3$$

so $T_{\nu,f} = T_{\nu,i} \times \left(\frac{a_i}{a_f}\right)$

PHOTONS & ELECTRONS

$$\mathcal{S}_i = a_i^3 \left(\tfrac{4}{3} \times 2 \times T_{\gamma,i}^3 + \tfrac{4}{3} \times \tfrac{7}{8} \times 2 \times 2 \times T_{e,i}^3 \right)$$

for $T_{e,i} > m_e$

and note $T_{\nu,i} = T_{\gamma,i} = T_{e,i}$

$$\mathcal{S}_f = a_f^3 \left(\tfrac{4}{3} \times 2 \times T_{\gamma,f}^3 + \frac{\rho_e(T_e) + p_e(T_e)}{T_e} \right)$$

or after $T_e \ll m_e$

$$= a_f^3 \left(\tfrac{4}{3} \times 2 \times T_{\gamma,f}^3 \right)$$

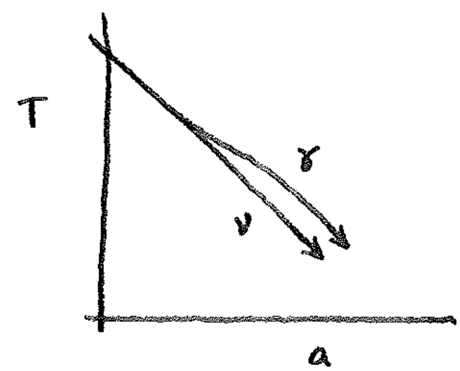
so $T_{\nu,f} = \left(\frac{4}{11}\right)^{1/3} \times T_{\gamma,f}$

SMOOTH NR TRANSITION:

$$F(T) = \int_{m_e}^{\infty} \frac{E^2\sqrt{E^2-m_e^2} + \frac{1}{3}(E^2-m_e^2)^{3/2}}{e^{E/T}+1}\, dE$$

$$\frac{a}{a_i} = \frac{T_{\gamma,i}}{T_\gamma}\left[\frac{\frac{\pi^2}{30}\times\frac{4}{3}\left(2+\frac{7}{8}\times 2\times 2\right)}{\frac{\pi^2}{30}\times\frac{4}{3}\times 2 + \frac{2\times 2}{2\pi^2}F(T_\gamma)}\right]^{1/3}$$

$$\text{or} \quad \frac{T_\nu}{T_\gamma} = \left[\ \text{''}\ \right]^{-1/3}$$



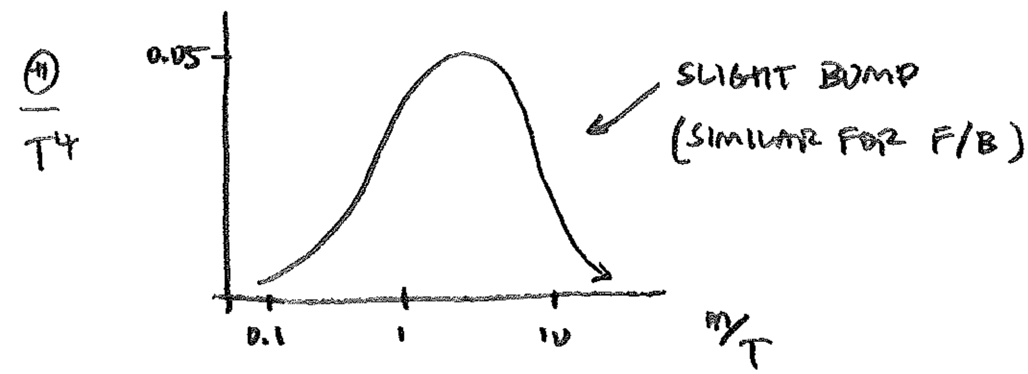
PHOTONS GAIN FROM e's RELATIVE TO ν's.

STRESS-ENERGY TRACE OF RELATIVISTIC SPECIES

$$\Theta = \rho - 3p = \begin{cases} \frac{g}{24} m^2 T^2 & F \\ \frac{g}{12} m^2 T^2 & B \end{cases}$$

SO $\rho - 3p$ NOT PRECISELY ZERO

WHEN A SPECIES BECOMES NON-RELATIVISTIC, A BUMP



THIS MEANS

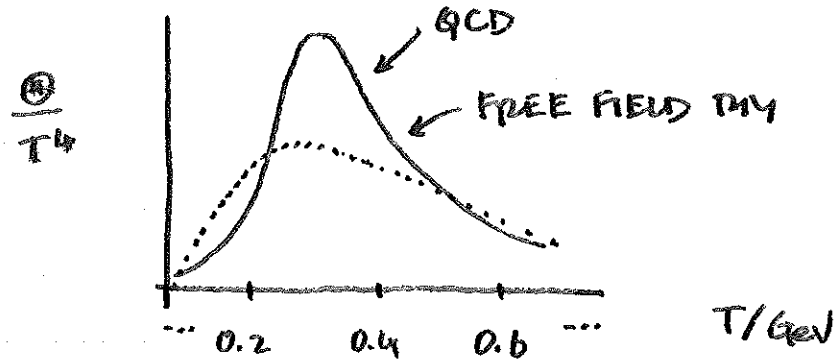
$0 < z \lesssim 2$ Θ DUE TO MATTER, DARK ENERGY

$2 \lesssim z \lesssim 10^8$ DUE TO MATTER: B & DM

$z \gtrsim 10^8$ ($T \gtrsim 10^{-4}$ GeV) DUE TO RELATIVISTIC MATTER

3. QCD PHASE TRANSITION

LATTICE QCD RESULTS : Θ / T^4



$$\frac{\Theta}{T^4} = \left[1 - \left(1 + e^{(T-c_1)/c_2}\right)^2\right]^{-1} \left(\frac{d_2}{T^2} + \frac{d_4}{T^4} \right)$$

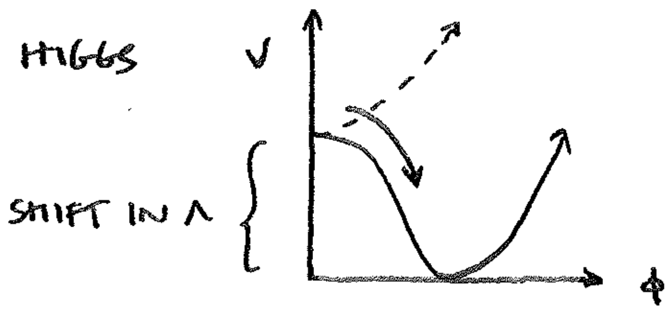
$c_1 = 193.8$ MeV $\qquad d_2 = 0.241\ \mathrm{GeV}^2$

$c_2 = 13.6$ MeV $\qquad d_4 = 0.0035\ \mathrm{GeV}^4$

Bazavov et al, PRD 80 014504 (2009).

Feature: a sharp spike in trace as quarks, gluons coalesce.

4. ELECTROWEAK PHASE TRANSITION



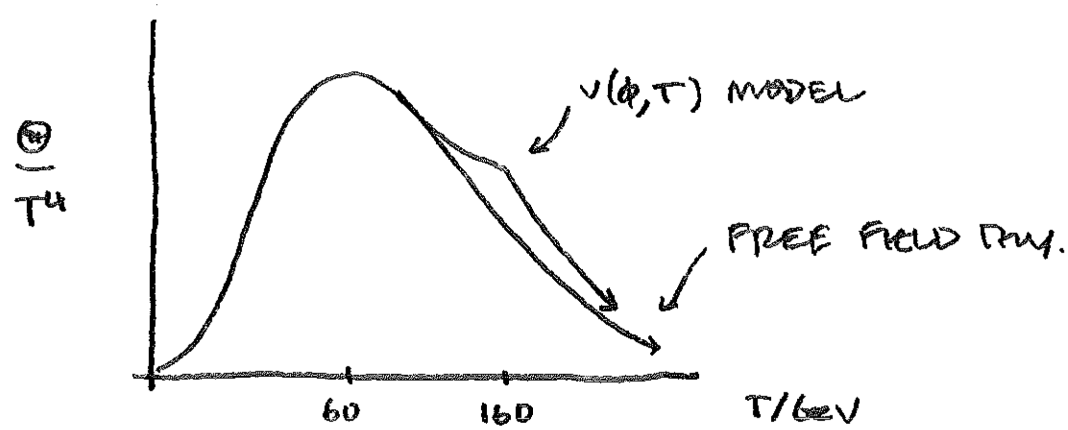
MODEL OF HIGGS, FEW GAUGE BOSONS, FERMIONS

$V(\phi, T)$: FROM THE 1-loop, thermally corrected potential

$$p = -V(\phi(T), T)$$

$$\rho = -p + T \frac{d}{dT} p$$

QUICK ANSWER:



MODEL OF EW TRANSITION

$$V(\phi,T) = V_{tree}(\phi) + V_{vac}^{(1)}(\phi,T) + V_{therm}(\phi,T)$$

$$V_{tree}(\phi) = \frac{\lambda_0}{4}\left(\phi^2 - \sigma^2\right)^2$$

$$V_{vac}^{(1)}(\phi,T) = \sum_j \frac{s_j}{64\pi^2}\left[m_j^4(\phi,T)\ln\left(\frac{m_j^2(\phi,T)}{m_j^2(\sigma,T)}\right)\right.$$

$$\left. - \frac{3}{2} m_j^4(\phi,T) + 2m_j^2(\phi,T)\, m_j^2(\sigma,T) - \frac{1}{2} m_j^4(\sigma,T)\right]$$

$$V_{therm}(\phi,T) = \sum_j \frac{g_j s_j}{2\pi^2} T^4 \int_0^\infty dx\, x^2 \ln\left(1 - s_j e^{-\sqrt{x^2 + m_j^2(\phi,T)/T^2}}\right)$$

THERMALLY CORRECTED MASSES: AT $T \gtrsim M_W$, BOSONS MIX W/ HIGGS SCALARS TO YIELD LONGITUDINAL W_L, Z_L, γ_L

eg $\quad W: \quad M^2 = M^2_{W,VAC}(\phi/\sigma)^2 \qquad g=4$

$\quad W_L: \quad M^2_{W,VAC}\left[\left(\frac{\phi}{\sigma}\right)^2 + \frac{22}{3}\left(\frac{T}{\sigma}\right)^2\right] \qquad g=2$

ASIDE. At $T \gg \sigma$

$$④ = h_0 + h_2 T^2 + \mathcal{O}(T^{-2})$$

$$h_0 = \frac{1}{2}\sigma^2 m_H^2$$

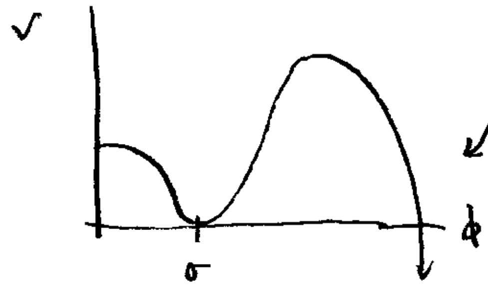
$$h_2 \simeq -m_H^2/45 \quad < 0$$

In the absence of new contributions in the range $\sigma < T \lesssim 2\,\mathrm{TeV}$, $④ < 0$.

BSM physics? DM species?

Feb 23, 2013

* THE STANDARD MODEL, ZERO TEMPERATURE, ONE-LOOP CORRECTED VACUUM POTENTIAL *



$$V(\phi) = V_{tree}(\phi) + V^{(1)}_{vac}(\phi)$$

$$V_{tree}(\phi) = \frac{\lambda_0}{4}(\phi^2 - \sigma^2)^2$$

$$V^{(1)}_{vac}(\phi) = \sum_j \frac{s_j}{64\pi^2}\Big[m_j(\phi)^4 \ln\frac{m_j(\phi)^2}{m_j(\sigma)^2} + 2m_j(\phi)^2 m_j(\sigma)^2 - \frac{3}{2}m_j(\phi)^4 - \frac{1}{2}m_j(\sigma)^4\Big]$$

$s_j = \pm 1$ for bosons, fermions

MOST IMPORTANT TO THE SUM IS THE CONTRIBUTION FROM THE TOP QUARK

$$m_t(\phi)^2 = m_{t,vac}^2 \cdot \left(\frac{\phi}{\sigma}\right)^2$$

$$m_{t,vac} \simeq 174.2 \text{ GeV}$$

AND THE HIGGS

$$m_H(\phi)^2 = \lambda_0(3\phi^2 - \sigma^2)$$

$$\lambda_0 = \frac{1}{2}\left(\frac{m_{H,vac}}{\sigma}\right)^2$$

$$m_{H,vac} \simeq 125 \text{ GeV}$$

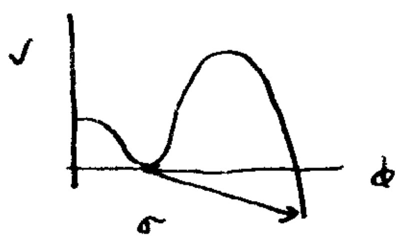
$$\sigma \simeq 246 \text{ GeV}$$

USING THESE EQUATIONS, $V(\phi)$ TURNS OVER AT $\phi \simeq 20{,}000$ GeV.

IF THERE IS NO FURTHER PHYSICS OF THE HIGGS ϕ, IS THERE A DANGER OF OUR METASTABLE VACUUM (AT $\phi = \sigma$) EVOLVING TO THE (APPARENT) TRUE VACUUM (AT $\phi \simeq 20$ TeV)?

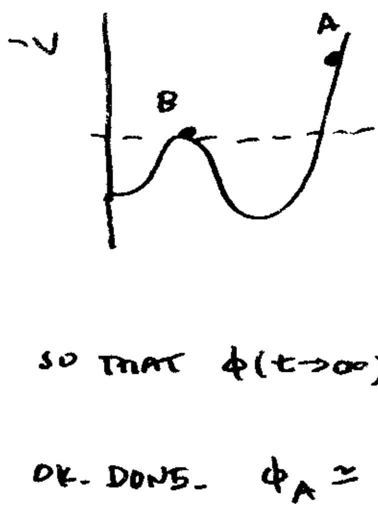
A CM, $\sqrt{s} \simeq 20$ TeV PROCESS IS NOT SUFFICIENT TO MOVE ϕ OVER TO THE TRUE VACUUM. THE REASON IS THAT WE WOULD WANT TO RUN THE COUPLINGS AND MASSES UP TO $\sqrt{s}$, AT WHICH POINT WE FIND THAT V TURNS OVER AT AN EVEN HIGHER VALUE OF ϕ!

WHAT ABOUT VACUUM TUNNELING? (See Kolb & Turner 7.1.2)



THAT'S A POSSIBILITY.

$$\Gamma = A e^{-S_E}$$



INSTANTON: SOLVE

$$\frac{d^2\phi}{dt^2} + \frac{3}{t}\frac{d\phi}{dt} = V'(\phi(t))$$

WITH $\phi(t=0) = \phi_A$

$$\frac{d\phi}{dt}(t=0) = 0$$

SO THAT $\phi(t \to \infty) = \phi_B$.

OK. DONE. $\phi_A \simeq 5.8 \times 10^6$ GeV

EVALUATE EUCLIDEAN ACTION

$$S_E = 2\pi^3 \int t^3 dt \left(\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + V(\phi(t)) \right) \simeq 504.$$

Let's define the instanton radius τ as the radius within which 90% of the Euclidean action is contained.

$$\tau = 0.0015\,\sigma^{-1}$$

Let's use $A = \tau^{-4}$

In which case, the tunneling probability per time, per volume is $\tau^{-4} e^{-S_E}$

In a Hubble volume, H_0^{-3}, in a Hubble time H_0^{-1}, the net probability is

$$P \sim (H_0 \tau)^{-4} e^{-S_E} \sim 10^{-30}$$

The probability to tunnel to the true vacuum is negligible.

5. INFLATON AND REHEATING

CONSIDER INFLATON $V = \frac{1}{2} m^2 \Psi^2$, $m \sim 10^{14}$ GeV

INFLATON DECAY WIDTH $\Gamma = 10^{10}$ GeV
INTO RADIATION

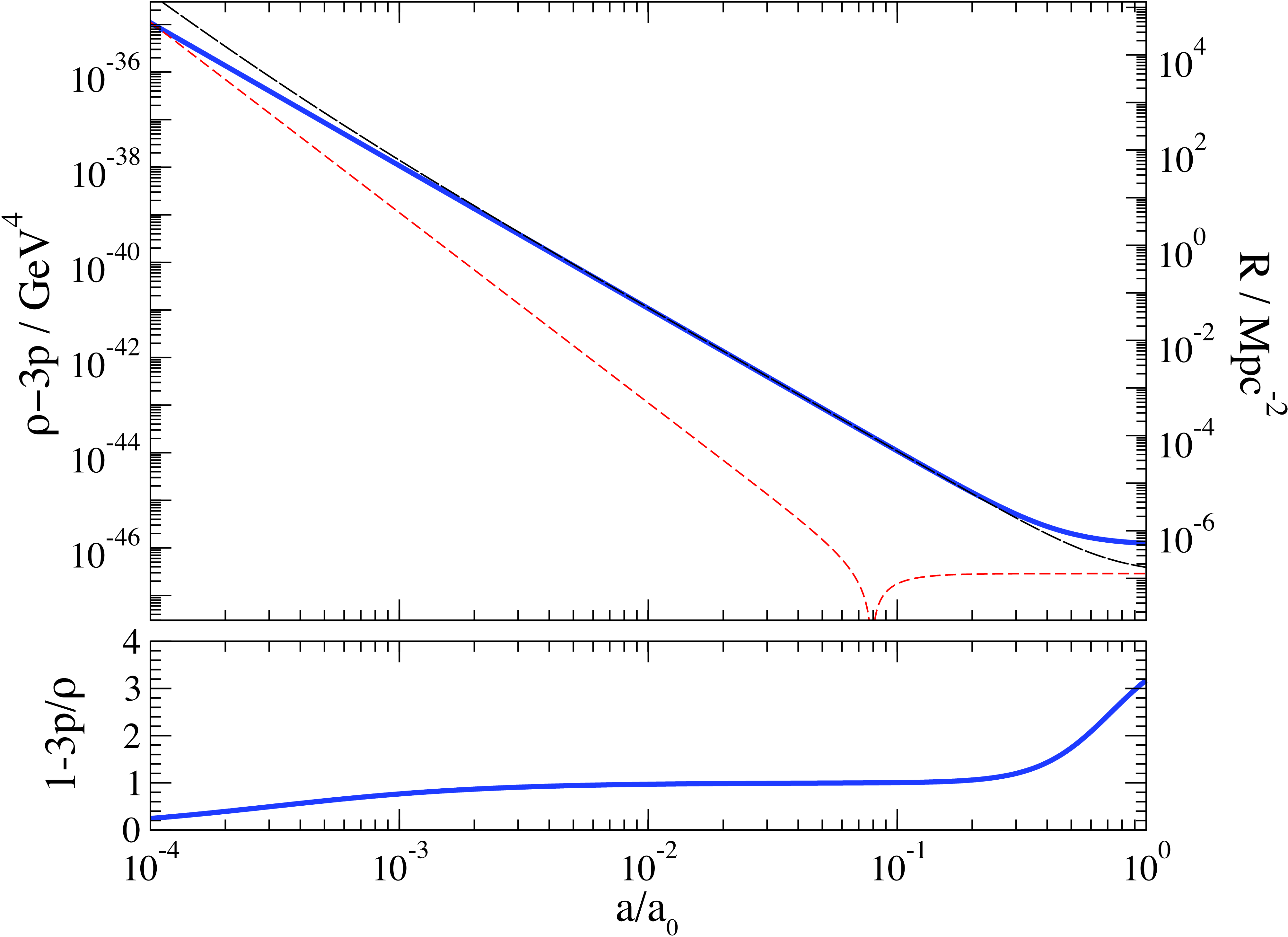
$$\ddot{\Psi} + (3H + \Gamma)\dot{\Psi} + V' = 0$$
$$\dot{\rho}_r + 4H\rho_r - \Gamma\dot{\Psi}^2 = 0$$

SIMPLISTIC, BUT REASONABLE PICTURE

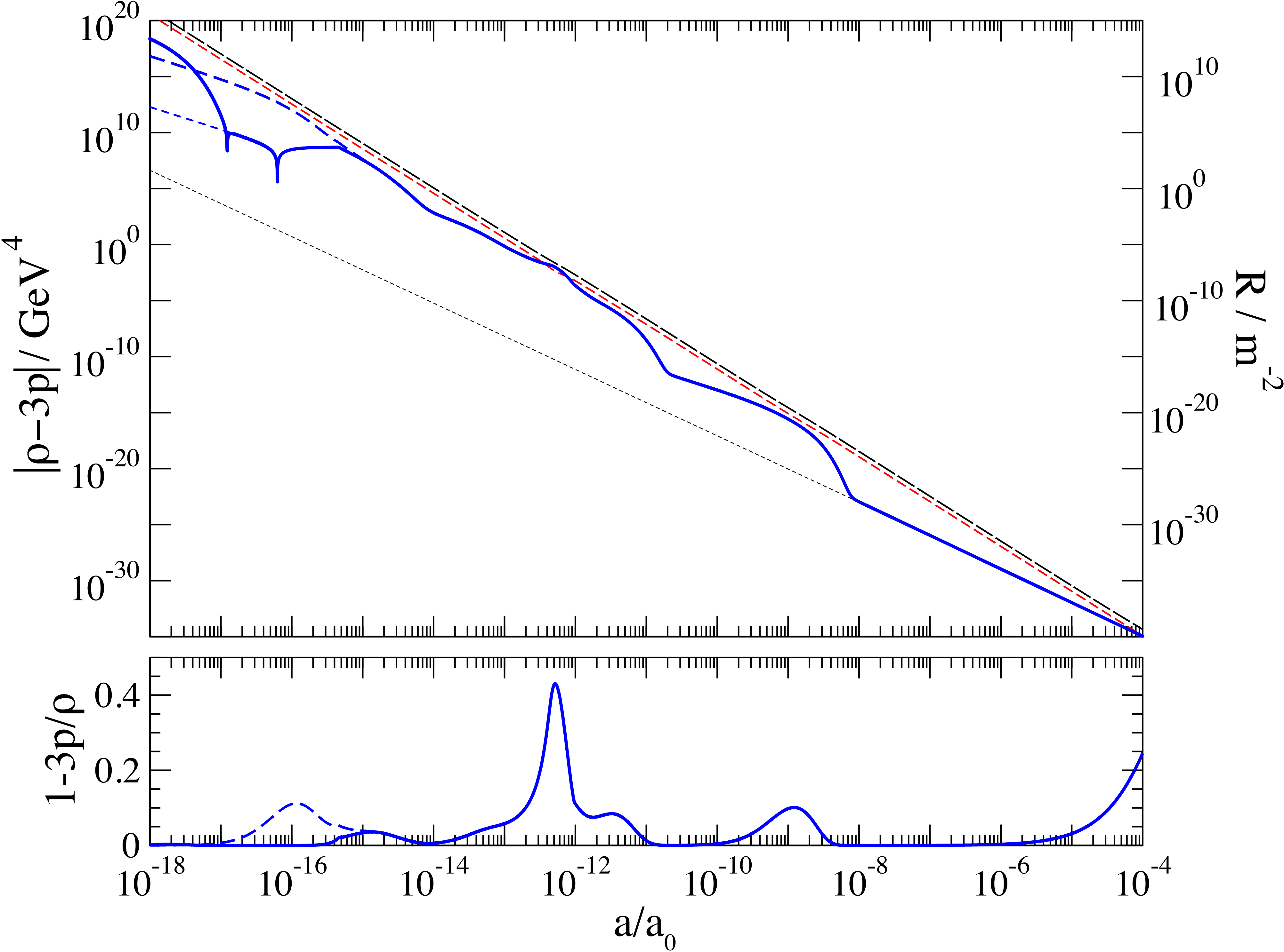
- Ψ OSCILLATES BRIEFLY AS RAD'N GROWS
- SHORT EPOCH $|p| \ll \rho$
- H DROPS AS RADIATION EPOCH BEGINS
  SENSITIVE TO DETAILS OF DECAY PRODUCTS

SEE FIGURES
FROM PAPER

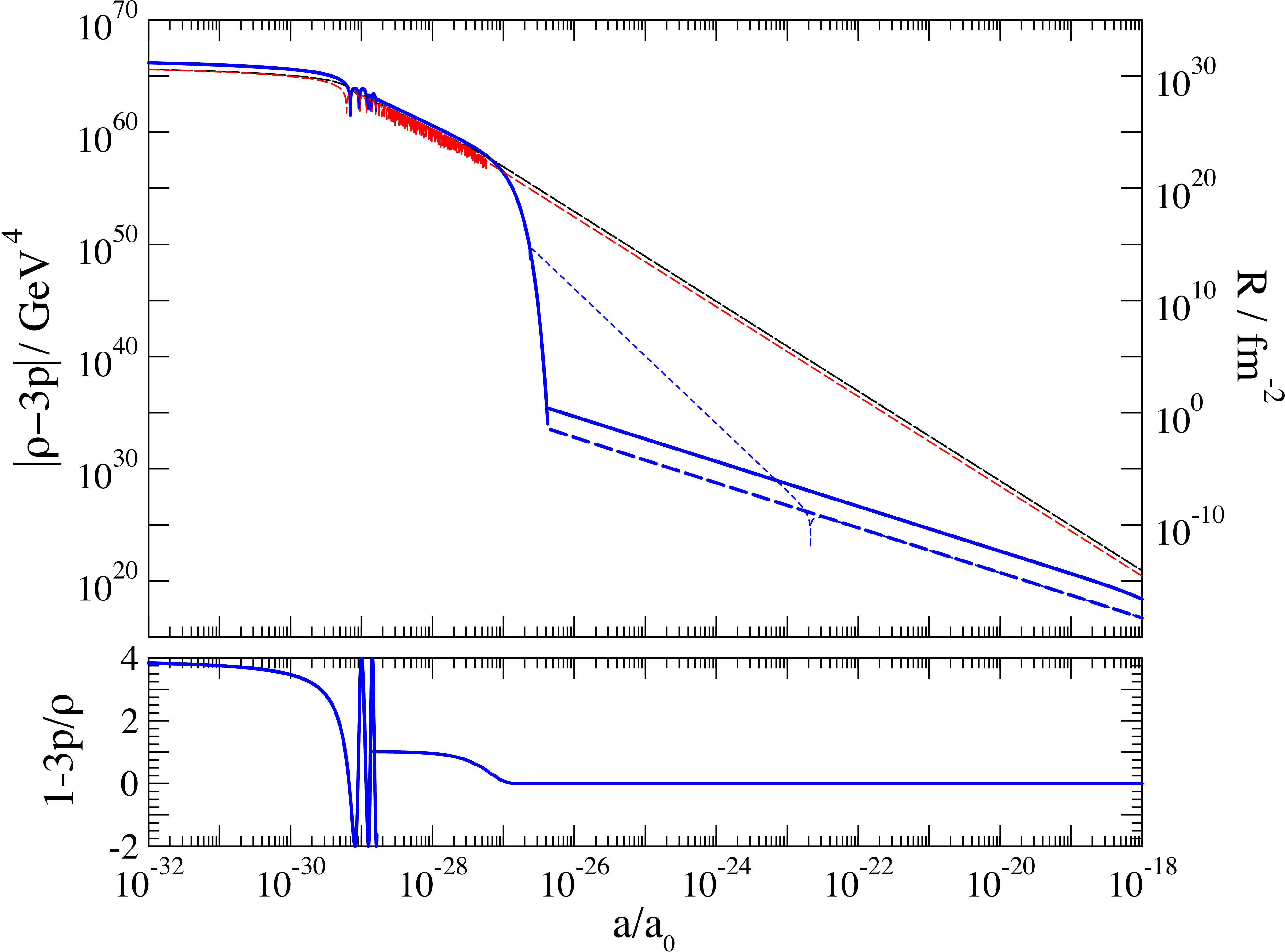
OVERVIEW OF
SCM,
particle physics
& cosmology.
OPEN QUESTIONS?



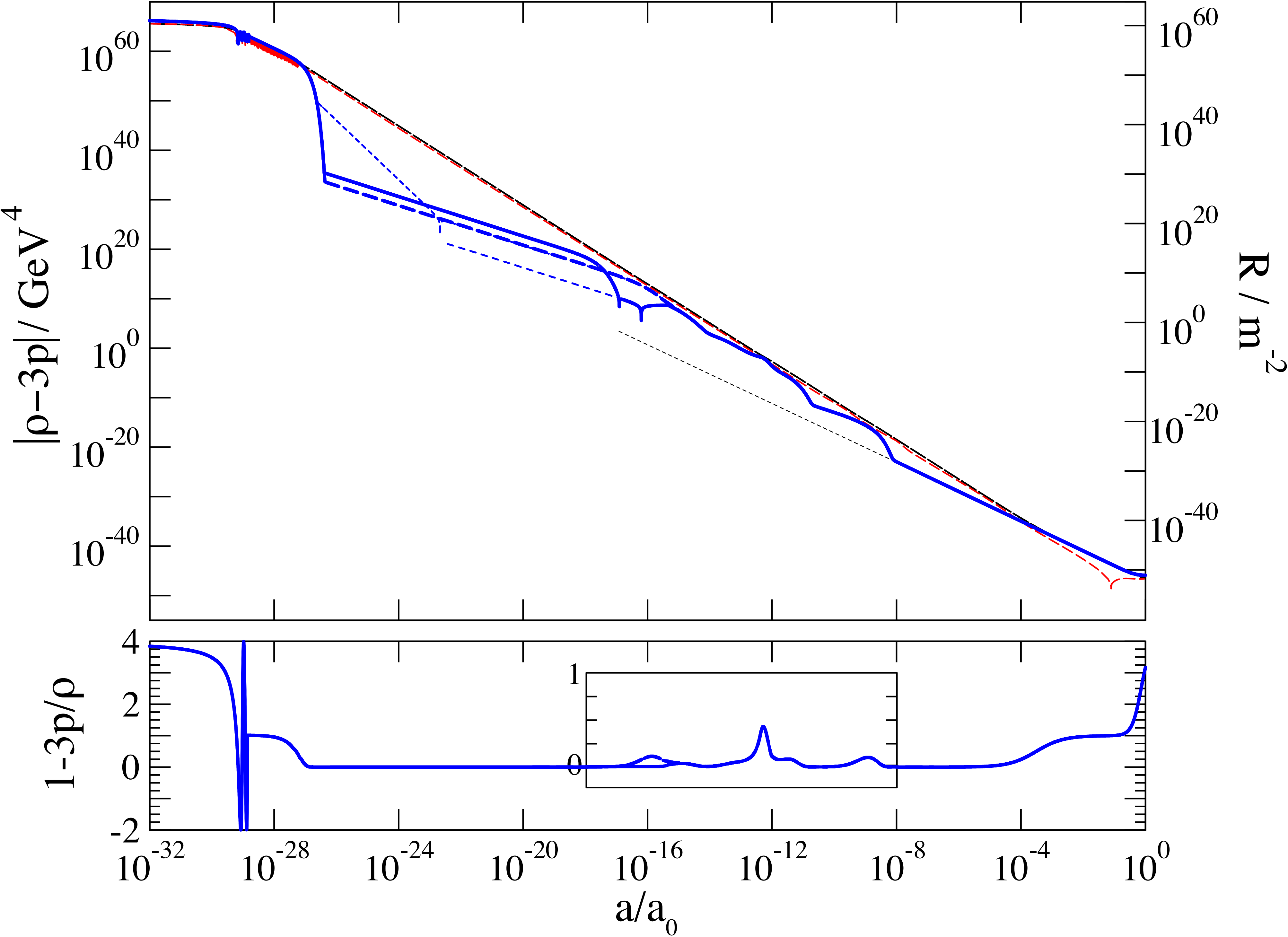
ρ−3p / GeV^4
R / Mpc^-2
1-3p/ρ
a/a_0
10^-36
10^-38
10^-40
10^-42
10^-44
10^-46
10^4
10^2
10^0
10^-2
10^-4
10^-6
0
1
2
3
4
10^-4
10^-3
10^-2
10^-1
10^0



10^{20}
10^{10}
10^{0}
10^{-10}
10^{-20}
10^{-30}
$|\rho-3p|$ / GeV^4
10^{10}
10^{0}
10^{-10}
10^{-20}
10^{-30}
R / m^{-2}
0.4
0.2
0
$1-3p/\rho$
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
a/a_0



$|\rho-3p|$/ GeV^4
10^{70}
10^{60}
10^{50}
10^{40}
10^{30}
10^{20}
R / fm^{-2}
10^{30}
10^{20}
10^{10}
10^{0}
10^{-10}
1-3p/ρ
4
2
0
-2
10^{-32}
10^{-30}
10^{-28}
10^{-26}
10^{-24}
10^{-22}
10^{-20}
10^{-18}
a/a_0



$|\rho - 3p|$ / GeV^4
R / m^{-2}
1-3p/ρ
a/a$_0$

WHAT DOES PLANCK TELL US?

VINDICATION OF SCM, AGREEMENT W/ WMAP

BUT, A FEW ITEMS ATTRACT ATTENTION

- TIGHTENED CONSTRAINTS ON INFLATION MODELS
- POWER ASYMMETRY, MULTIPOLE ALIGNMENTS
- SMALL NON-GAUSSIANITY, BROADLY
- WIGGLE ROOM FOR "DARK RADIATION"

ALL WORTH A CLOSER LOOK.

$w < -1$?

HOT TOPICS, per inspire?

Higgs, DM, ν's ...

WMAP, PLANCK, ACT, SPT, SDSS/BOSS

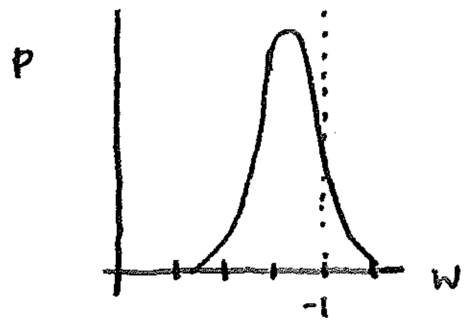
MASSIVE, MODIFIED GRAVITY

HIGGS COSMOLOGY

WHAT IS PLANCK TELLING US ABOUT DARK ENERGY?

$w < -1$ @ 2σ !?

$w = -1.1 \pm 0.15$ (2σ) w SN.



PLANCK + WP + SN (PLANCK XVI)

WMAP9 + eCMB + BAO + H_0 (1212.5225)

$w = -1.073 \pm 0.09$

PHANTOM?

NEW PHYSICS?

SYSTEMATIC?

Λ OR QUINTESSENCE?

WHAT IS H_0 TELLING US?

LOCAL MEASUREMENTS $\qquad H_0 = 73.8 \pm 2.4$ km/s/Mpc

CMB+LSS $\qquad H_0 = 67.8 \pm 0.77$ km/s/Mpc

$$\frac{\Delta H}{H} \sim 10\% \;, \qquad \sim 2.4\sigma$$

STATISTICAL FLUKE? NEGLECTED SYSTEMATIC?

IDEAS: RESULT FOR CMB+LSS IS MODEL DEPENDENT; MAKE w MORE NEGATIVE?

LOCAL UNDERDENSITY? NEED $\frac{\delta\rho}{\rho} \sim -0.3$!

LONG RANGE DE–DM INTERACTION?

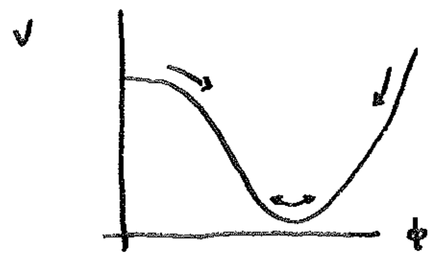
NEW GRAVITATIONAL PHYSICS?

OR ...?

see Marra et al PRL $\underline{110}$ 241305 (2013)
Salvatelli et al 1304.7119
Pettorino 1305.7457

INFLATION

a slowly rolling field



(& reheating)

... an epoch of exponential growth of the Universe, $a \propto e^{t}$, driven by a potential-energy dominated, slowly rolling scalar

EQ'NS: $\cdot = \frac{d}{dt}$ "

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$3H^2 = 8\pi G\left(\tfrac{1}{2}\dot{\phi}^2 + V\right)$$

USEFUL: $\dot{H} = -4\pi G \dot{\phi}^2$, $H' = \partial_\phi H = -4\pi G \dot{\phi}$

SLOW ROLL $\ddot{\phi} \ll 3H\dot{\phi}$, $\dot{\phi}^2 \ll V$ means inflation

THESE CONDITIONS IMPLY

$$\left(\frac{V'}{V}\right)^2 \ll 24\pi G \quad , \quad \left(\frac{V''}{V}\right) \ll 24\pi G$$

$$\epsilon_V = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 , \quad \eta_V = \frac{1}{8\pi G}\left(\frac{V''}{V}\right)$$

$\epsilon_V, |\eta_V| \ll 1 \longrightarrow$ INFLATION

FOR EXAMPLE

$$\ddot{\phi} \ll 3H\dot{\phi} \quad \text{means} \quad 3H\dot{\phi} \simeq -V'$$

$$\dot{\phi}^2 \ll V \quad \text{"} \quad 3H^2 \approx 8\pi G V$$

SO $$\frac{V'}{V} \simeq \frac{-3H\dot{\phi}}{\frac{3}{8\pi G}H^2} = -8\pi G \frac{\dot{\phi}}{\sqrt{\frac{8\pi G}{3}V}}$$

$$\frac{1}{24\pi G}\left(\frac{V'}{V}\right)^2 = \frac{\dot{\phi}^2}{V} \ll 1 \quad \checkmark$$

ALSO NOTICE THAT

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

$$= \frac{8\pi G}{3}\left[\frac{1}{2}\left(\frac{V'}{3H}\right)^2 + V\right]$$

$$= \frac{8\pi G}{3}V\left[1 + \frac{1}{2}\frac{1}{24\pi G}\left(\frac{V'}{V}\right)^2\right]$$

$$= \frac{8\pi G}{3}V\left[1 + \frac{1}{3}\epsilon_V\right] \quad \checkmark$$

SO $$w = \frac{p}{\rho} = \frac{-1 + \frac{1}{3}\epsilon_V}{1 + \frac{1}{3}\epsilon_V} \simeq -1 + \frac{2}{3}\epsilon_V$$

HOW LONG IS INFLATION ?

At scale factor a, before end of inflation at a_F

$$\frac{a}{a_F} = e^{-N} \qquad N = \text{e-foldings to end}$$

$$N = \ln \frac{a_F}{a} = \int_a^{a_F} \frac{da}{a} = \int_t^{t_F} H \, dt$$

TRADE t for ϕ TO KEEP TIME...

$$N = \int_\phi^{\phi_F} \frac{H}{\dot\phi} \, d\phi$$

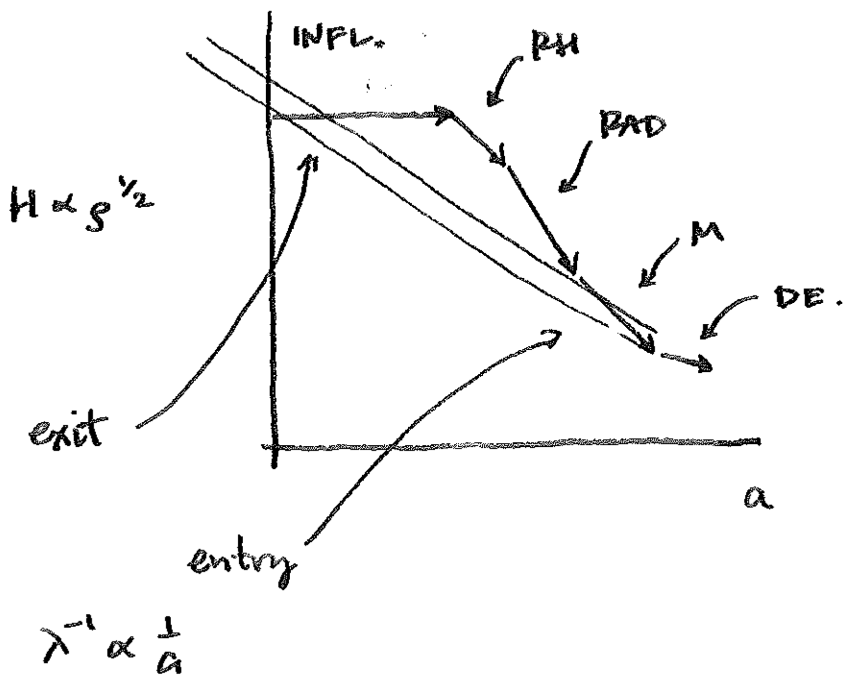
$$H' = -4\pi G \dot\phi'$$

$$\epsilon = \frac{1}{4\pi G} \left(\frac{H'}{H} \right)^2$$

$$= -\sqrt{4\pi G} \int_\phi^{\phi_F} \frac{d\phi}{\sqrt{\epsilon}} \quad \checkmark$$

Benefit of Inflation:

Causally produced density perturbation is stretched longer than Hubble radius (where amplitude gets frozen) and re-enters later.



HOW LONG?

CONSIDER A MODE OF WAVENUMBER k

AT EXIT $k = aH$ (call it $k = a_k H_k$)

SO TODAY $$\frac{k}{a_0 H_0} = \frac{a_k}{a_0} \frac{H_k}{H_0}$$

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_F} \cdot \frac{a_F}{a_{RH}} \cdot \frac{a_{RH}}{a_{EQ}} \cdot \frac{a_{EQ}}{a_0} \cdot \frac{H_k}{H_0}$$

USE $a_k / a_F = e^{-N(k)}$

$$a_F / a_{RH} = (\rho_{RH} / V_F)^{1/3}$$

$$a_{RH} / a_{EQ} = \left(\rho_{RH} / \Omega_R \rho_0 (1+z_{EQ})^4 \right)^{1/4}$$

$$a_{EQ} / a_0 = (1+z_{EQ})^{-1}$$

& $(1+z_{EQ}) = \Omega_m / \Omega_R$

$$\Omega_R = \frac{\pi^2}{30} g_* T_\gamma^4 \Big/ \frac{3}{8\pi G} H_0^2$$

$$\checkmark \quad N(k) \simeq 60 - \ln\left(\frac{k}{a_0 H_0}\right) - \ln\left(\frac{10^{16}\,\mathrm{GeV}}{V_k^{1/4}}\right) + \frac{1}{4}\ln\left(\frac{V_k}{V_F}\right) - \frac{1}{3}\ln\left(\frac{V_F^{1/4}}{\rho_{RH}^{1/4}}\right) - \ln h$$

INFLATION HOMEWORK

POWER LAW INFLATION

$$V(\phi) = M^4 \exp(-\phi/f)$$

SHOW THAT $\phi(t) = f \ln\left[8\pi G \dfrac{\epsilon^2 M^4 t^2}{(3-\epsilon)} \right]$

$$H = \frac{1}{\epsilon t}$$

$$a \propto t^{1/\epsilon} \quad \text{or} \quad \tau^{\epsilon - 1}$$

IN CONFORMAL TIME

SLOW ROLL PARAMETERS

$$\epsilon = \frac{1}{16\pi G f^2}, \quad \eta = \frac{1}{8\pi G f^2}$$

$$\delta = \frac{\ddot{H}}{2H\dot{H}} = \eta_V - \epsilon_V = -\frac{1}{16\pi G f^2}$$