



The Abdus Salam  
**International Centre  
for Theoretical Physics**



**2464-3**

**Earthquake Tectonics and Hazards on the Continents**

*17 - 28 June 2013*

**Faults have rules: scaling, friction, stress drops**

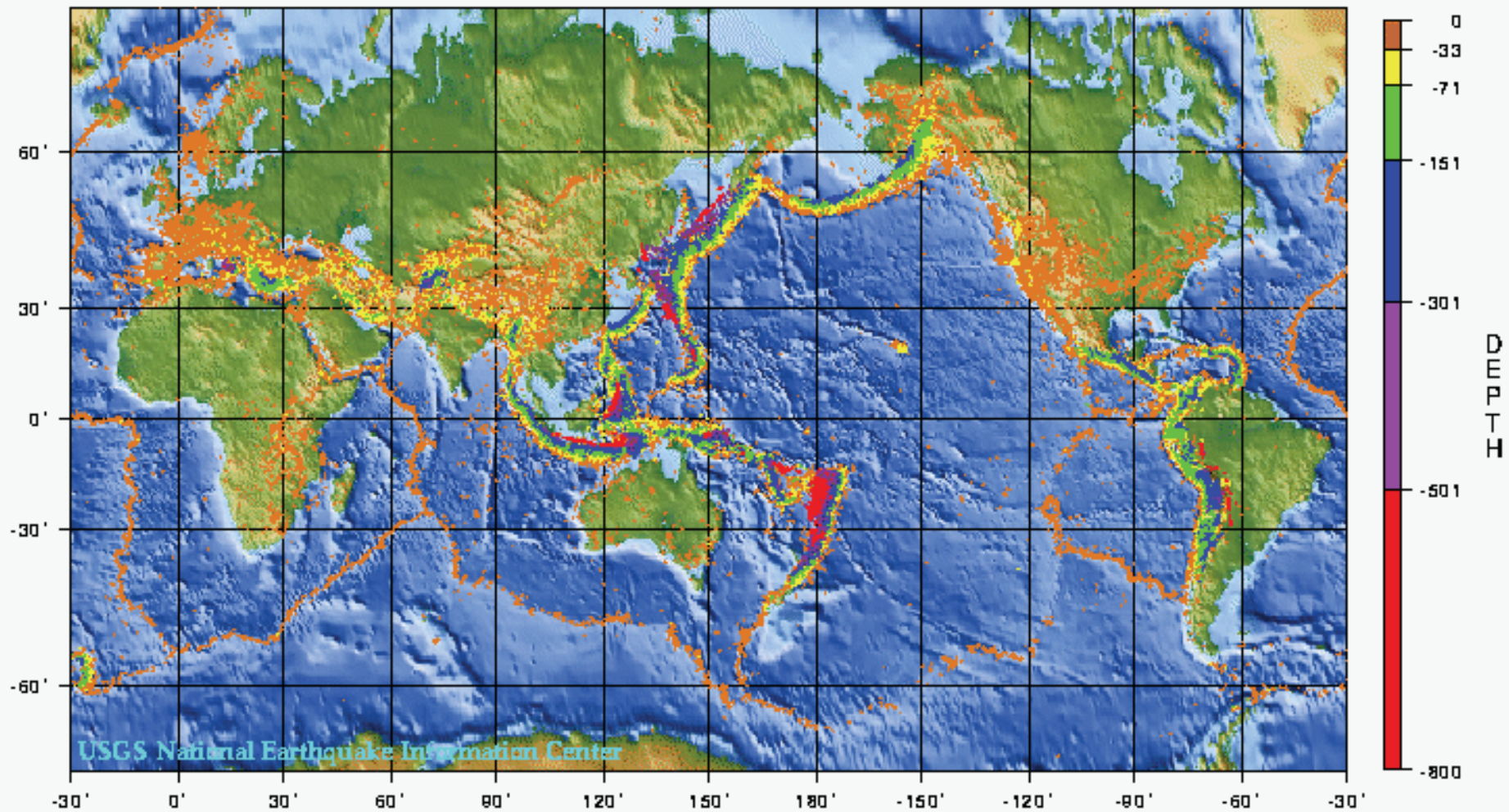
S. G. Wesnousky  
*University of Nevada  
USA*

Rules of the Game...

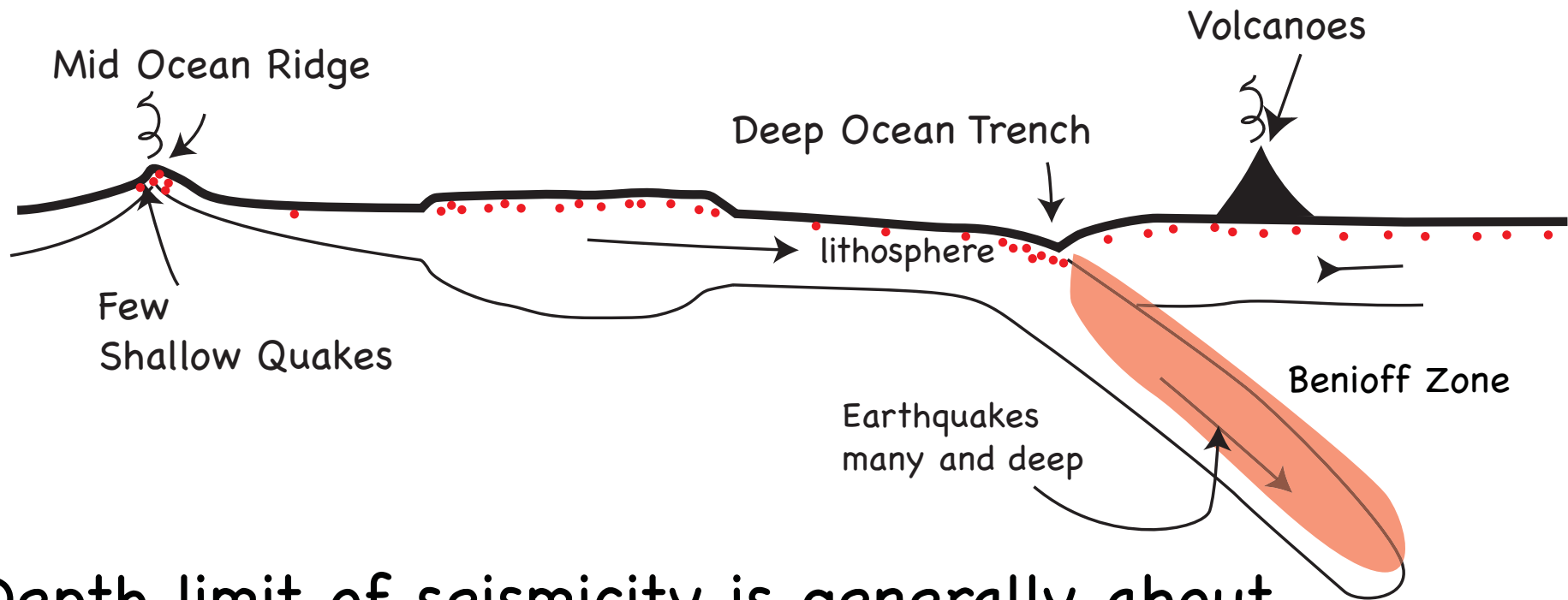
Scaling Laws....

2013-Trieste-Wesnousky

## World Seismicity: 1975 - 1995



Observation: The Depth to which Earthquakes occur in the earth is limited....

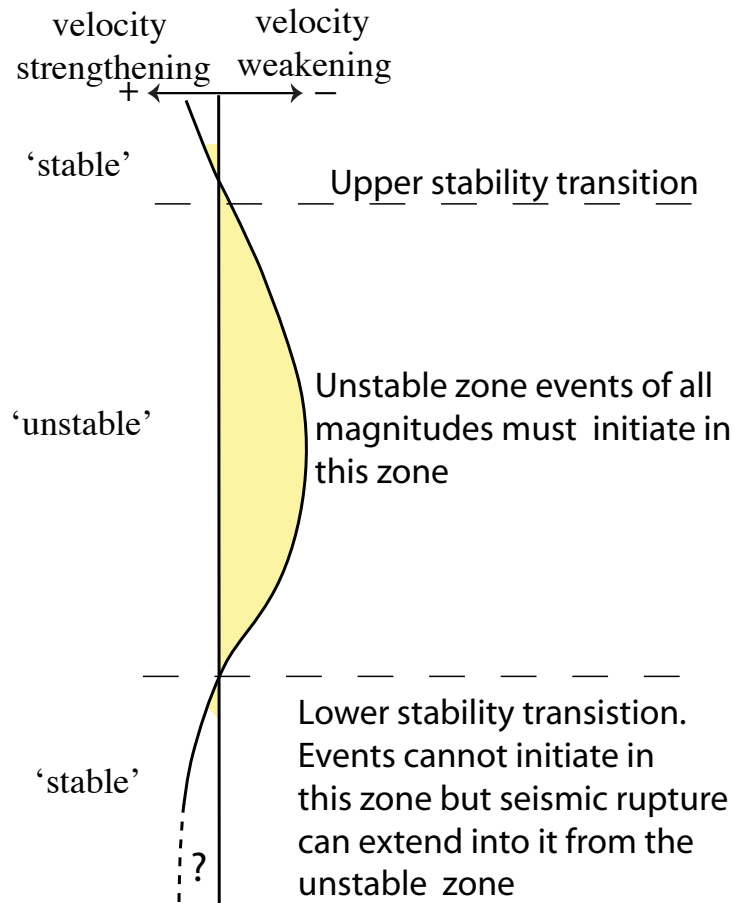


Depth limit of seismicity is generally about 10 to 20 km away from convergent plate boundaries....

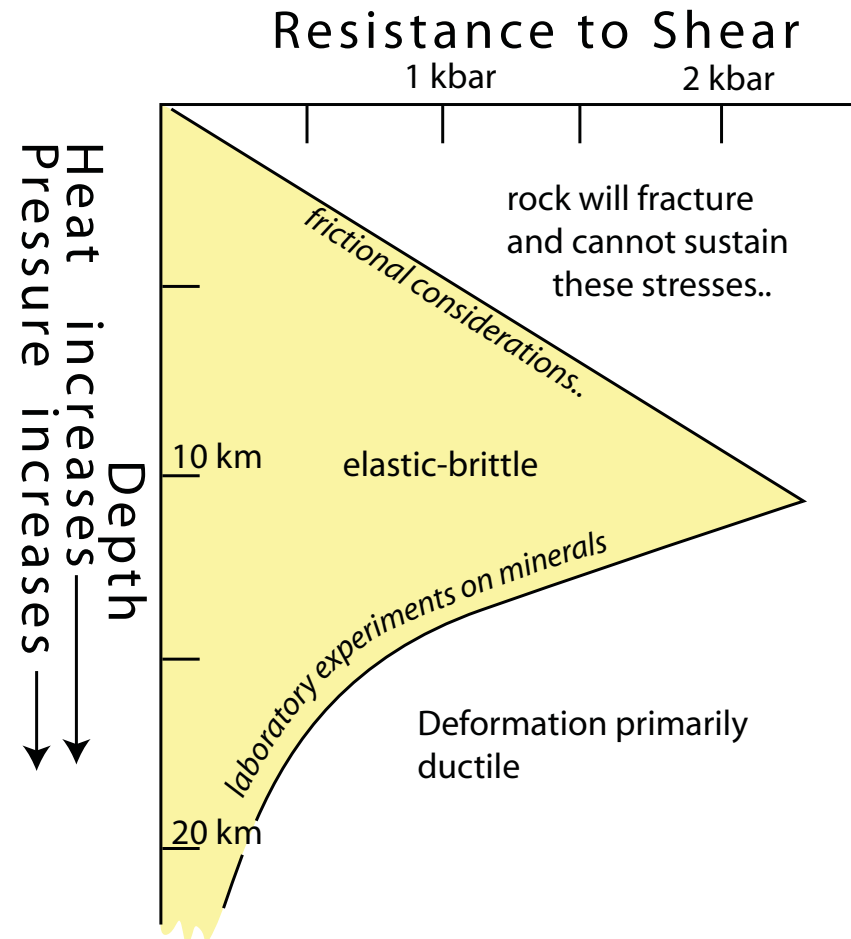
Generally attributed to heat (distribution of isotherms - presence of seismicity defining Benioff Zone is argument that old 'cool' lithosphere is being subducted)

# Rheological Explanations

## Friction rate behaviour

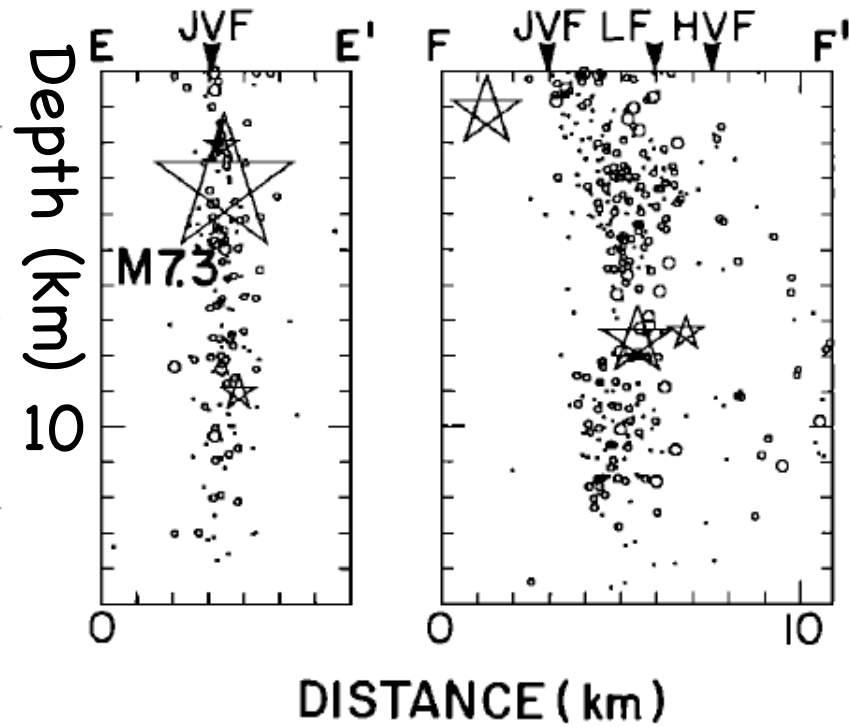
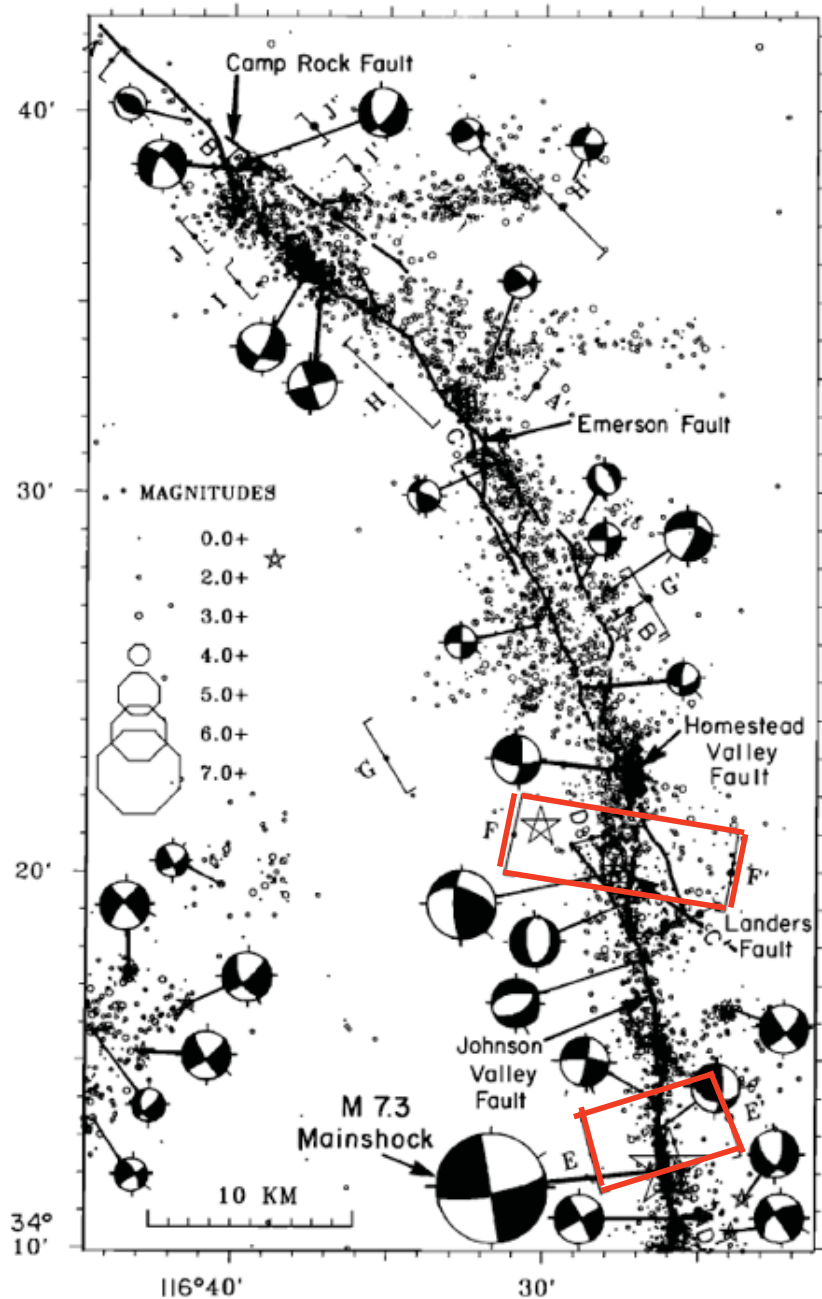


## Strength Envelope





Jun 22, 1992  
Landers

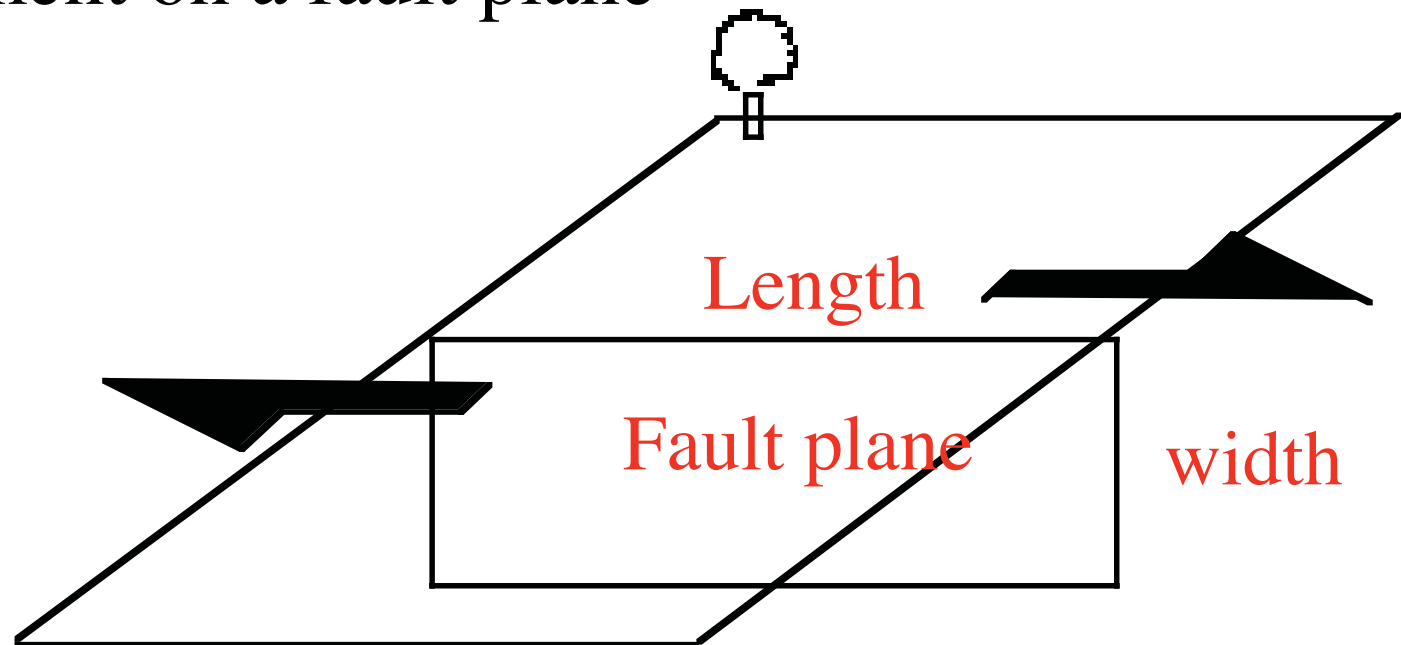


Idea of a  
'Fault Plane'  
extending to  
base of  
seismogenic layer

## Concept of a Fault Plane:

A fracture across which there has occurred a relative displacement of rock on alternate sides of the fracture.

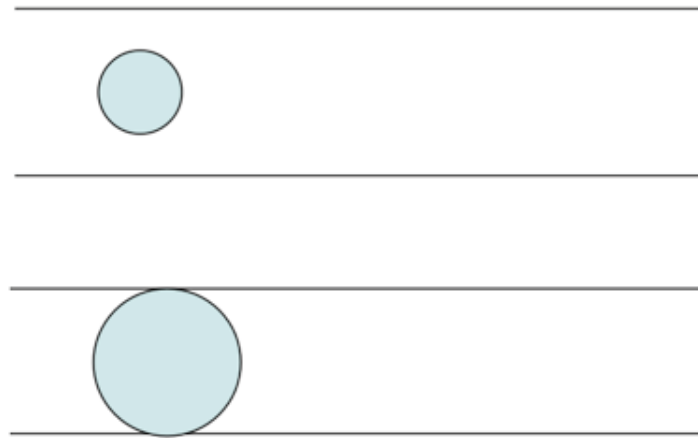
Concept of Earthquake: Sudden and discrete displacement on a fault plane





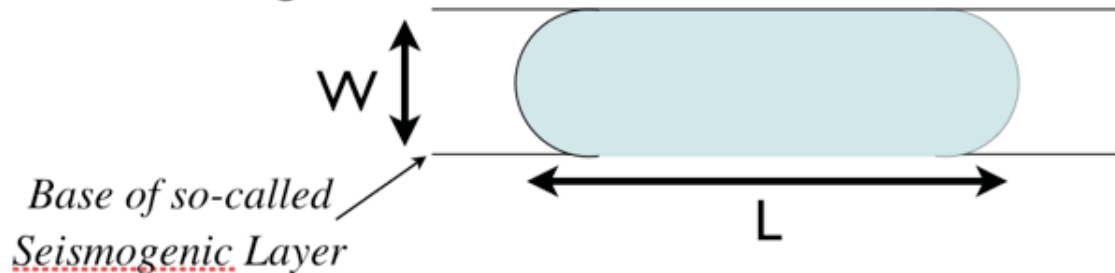
# Small (circular) versus Large (rectangular) earthquakes ruptures on faults...

Small Quake



'small'  
circular- rupture growth in  
all directions on fault  
plane

Large Quake



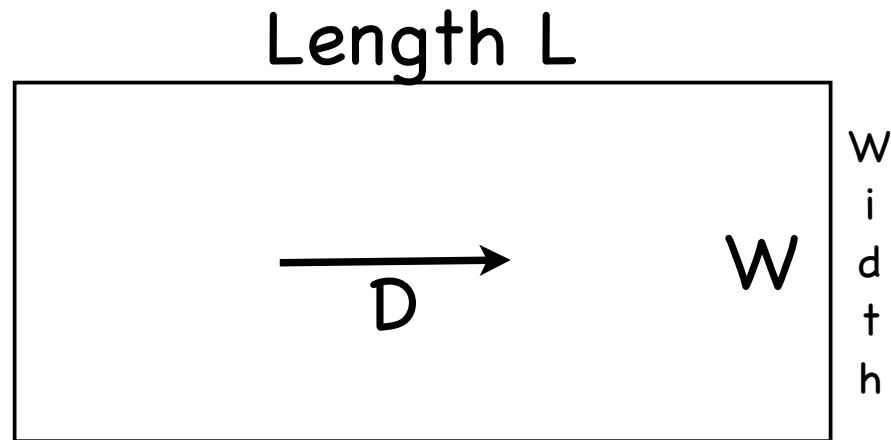
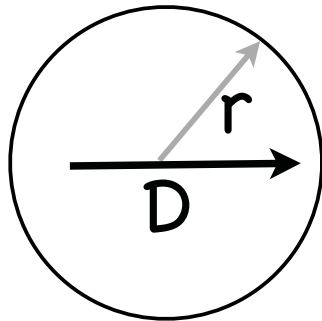
'large'  
rectangular - rupture limited  
in vertical and grows  
horizontally

“Aspect Ratio” =  $L/W$

# seismic moment $M_0$

like Magnitude  $M$  can be measured directly from a seismogram...

unlike Magnitude - it can also be related directly to physical parameters that describe the earthquake source...



$$M_0 = \mu \pi r^2 D \quad \text{or} \quad = \mu L W D$$

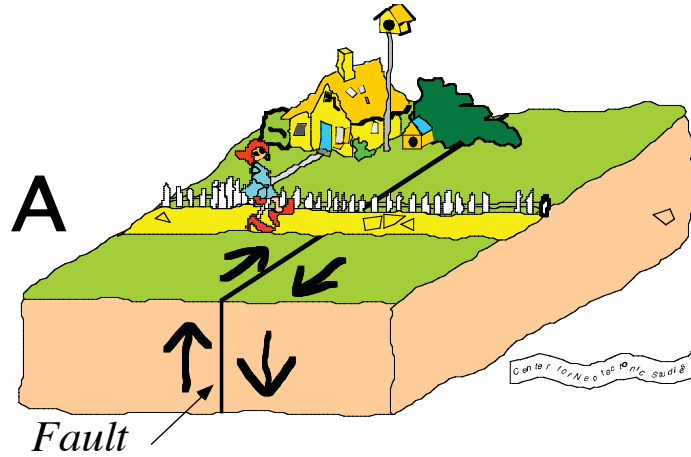
$\mu$  is crustal rigidity (proportionality constant between stress and strain)

$D$  is slip on fault plane

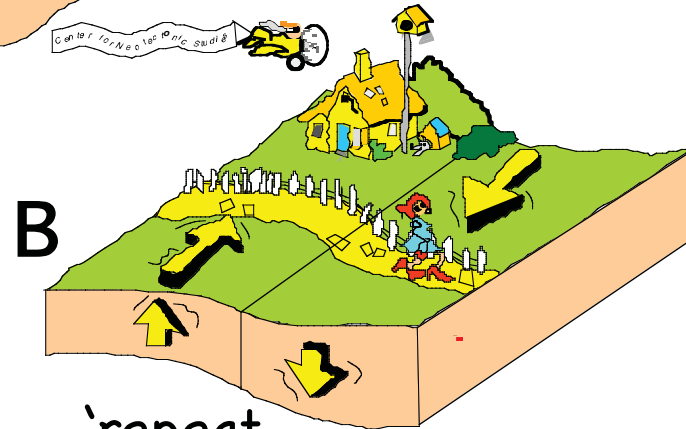
$S = \text{Area of fault plane} = \pi r^2$  for circular fault

$S = \text{Area of fault plane} = L W$  for rectangular fault

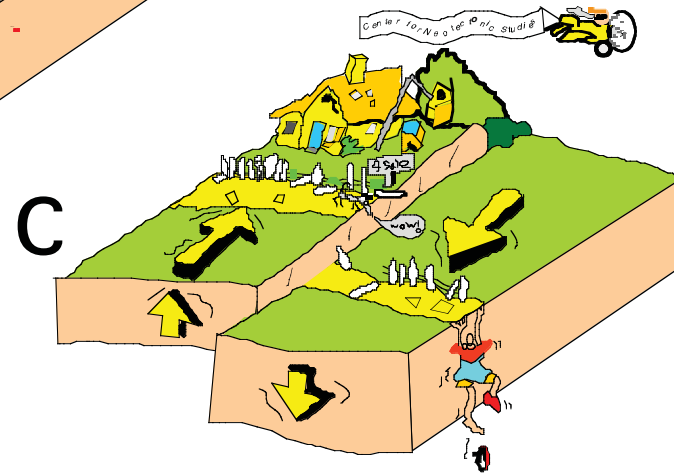
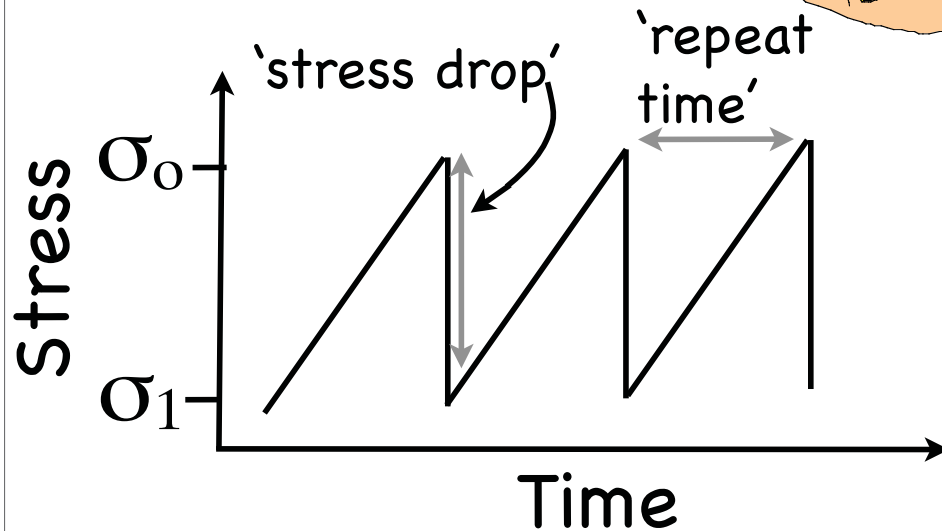
# Concept of Elastic Rebound and Stress Drop..



Stress accumulates in crust around fault during interseismic period..



When stress accumulates to level equivalent to fault strength - fault slips and crust experiences a 'stress drop'



## Summary: Physical Variables Used to Describe the Earthquake Fault Source

$L$  = fault dimension (e.g., length, width, radius)

$S$  = fault area

$D$  = average coseismic offset

$\mu$  = material property (rigidity)

$\sigma_0$  = initial stress on fault

$\sigma_1$  = final stress on fault

$\sigma$  = mean stress on fault =  $(\sigma_0 + \sigma_1)/2$

$\Delta\sigma$  =  $\sigma_0 - \sigma_1$  = stress drop on fault

$\Delta\sigma$  is proportional to strain energy released  
during quake =  $\Delta W$

$\eta \chi \Delta W$  = amount of strain energy released as  
seismic waves

## Summary: Measures of Earthquake Size from Seismology

$M$  = magnitude

$M_0$  = seismic moment

# Moment Magnitude $M_w$ and Seismic Moment $M_o$

Work done during an earthquake (strain energy drop)

$$W = \bar{\sigma} \bar{D} S \quad \left( \frac{\text{force}}{\text{area}} \times \text{distance} \times \text{area} \right)$$

$\bar{\sigma} = \text{average mean stress}$

average mean stress in terms of stress drop

$$\Delta\sigma = 2 \left( \frac{\sigma_o - \sigma_1}{2} \right) = 2\bar{\sigma} \quad \text{true if stress drop total}$$

rewriting

$$\bar{\sigma} = \frac{\Delta\sigma}{2}$$

elastic strain energy drop/release during earthquake

$$W = W_o = \frac{1}{2} \Delta\sigma \bar{D} S$$

and because  $M_o = \mu S \bar{D}$  because  $\mu = 3 - 6 \times 10^{11} \frac{\text{dyn}}{\text{cm}}$

because observations show  $\Delta\sigma = 20 - 60 \text{ bars} = 2 - 6 \times 10^7 \text{ dyn/cm}^2$

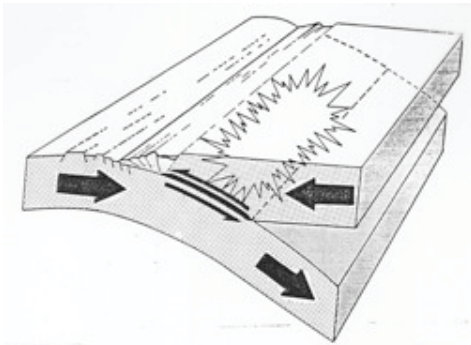
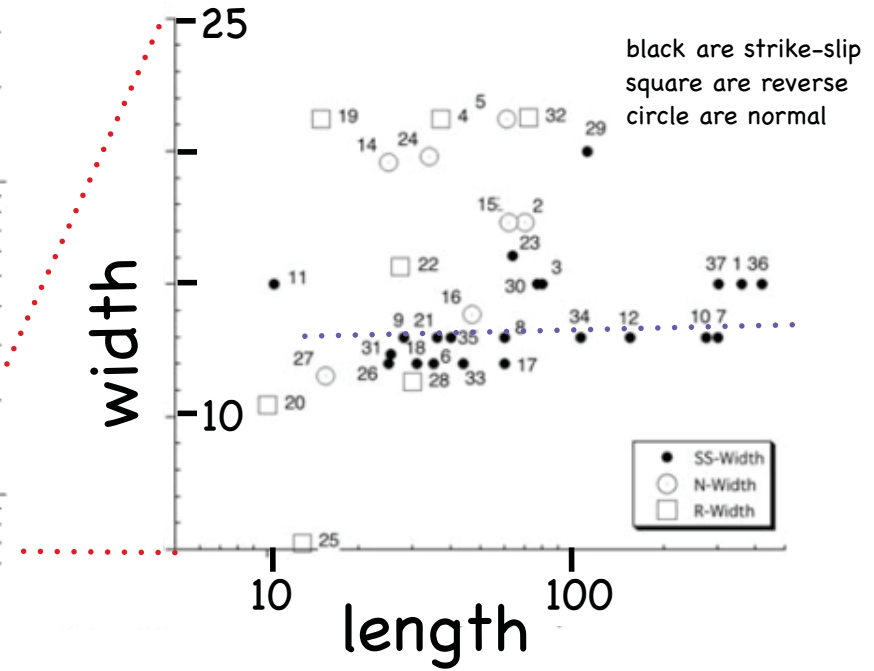
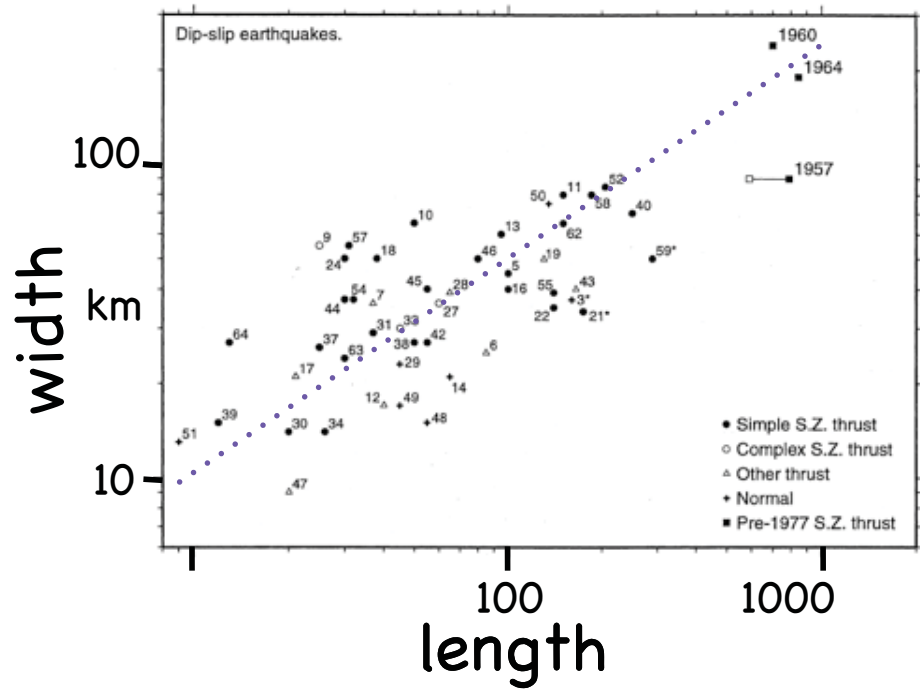
$$W = W_o = \frac{1}{2} \Delta\sigma \bar{D} S = \frac{\Delta\sigma}{2\mu} M_o = \frac{M_o}{2 \times 10^4}$$

placing this expression for work/energy into Gutenberg-Richters relationship between seismic energy and magnitude..  $\text{Log Energy} = 1.5 M + 11.8...$

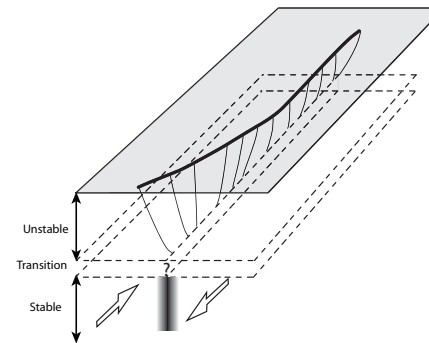
$$\text{Log}\left(\frac{M_o}{2 \times 10^4}\right) = 1.5M + 11.8$$

$$\text{Log}(M_o) = 1.5M_w + 16.1$$

# Rupture Length versus Width

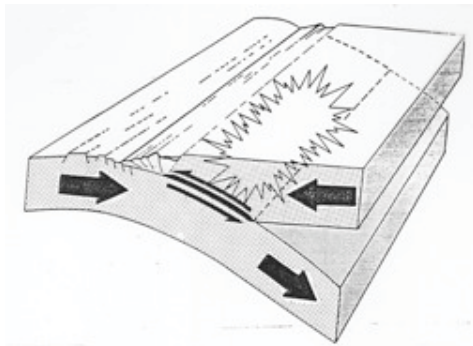
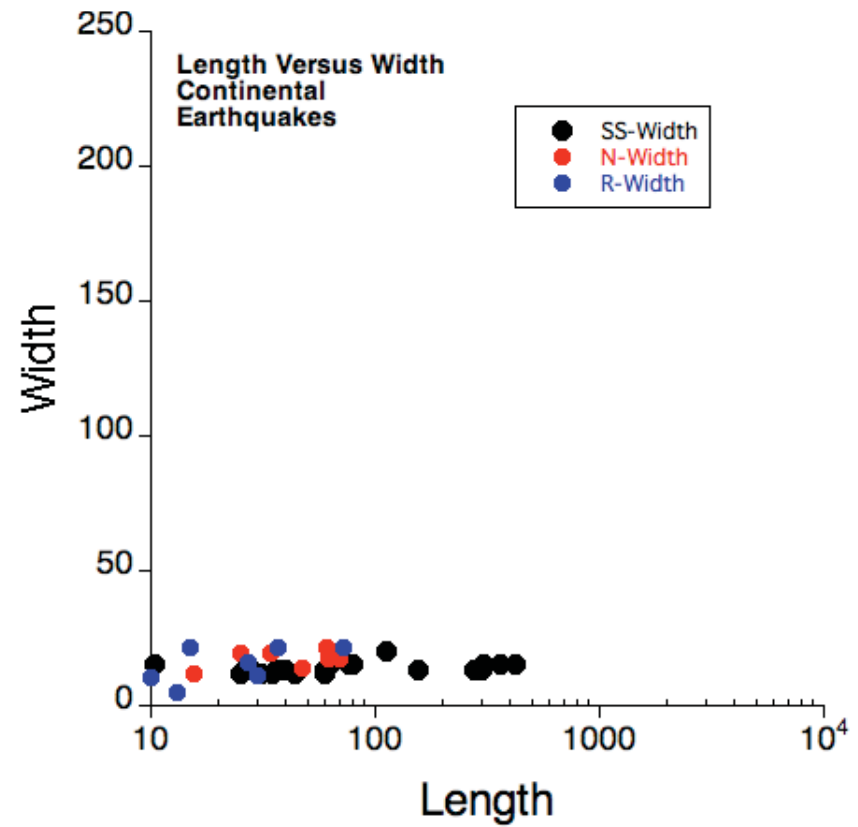
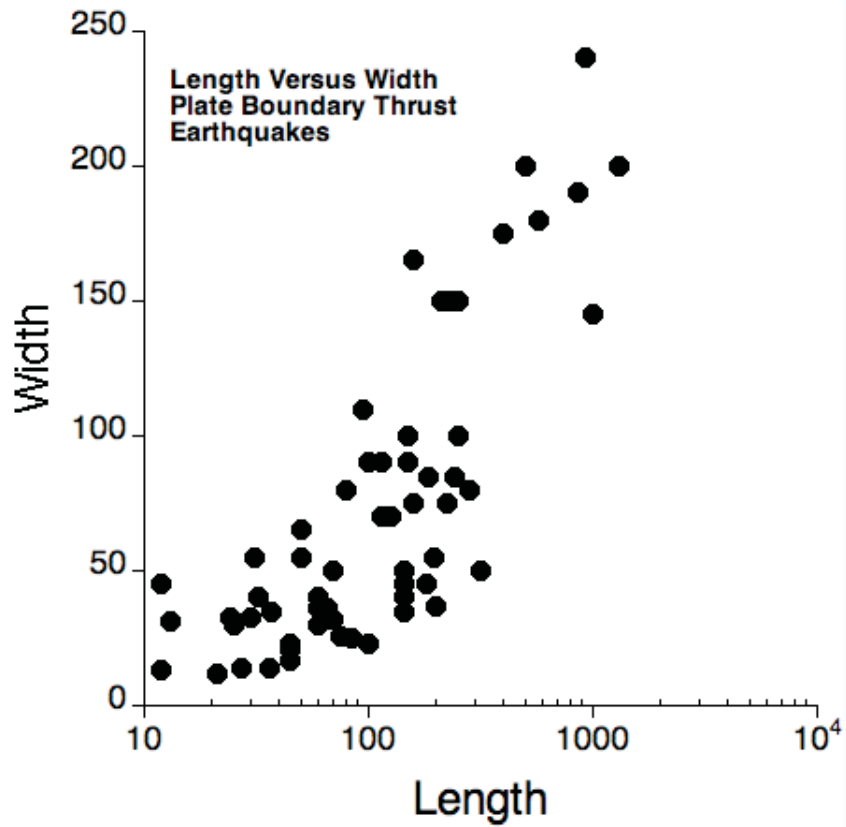


Subduction

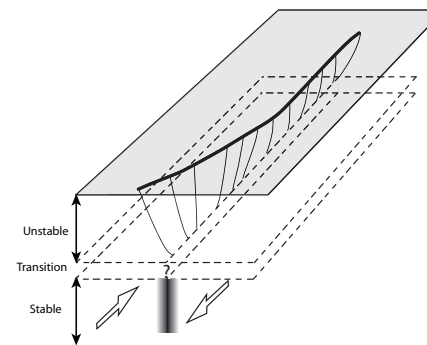


Continental

# Rupture Length versus Width



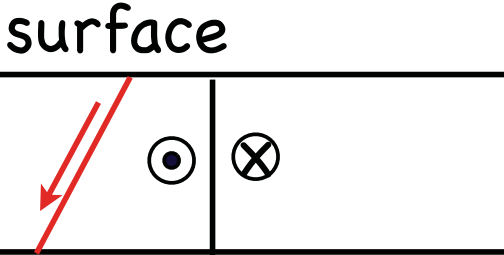
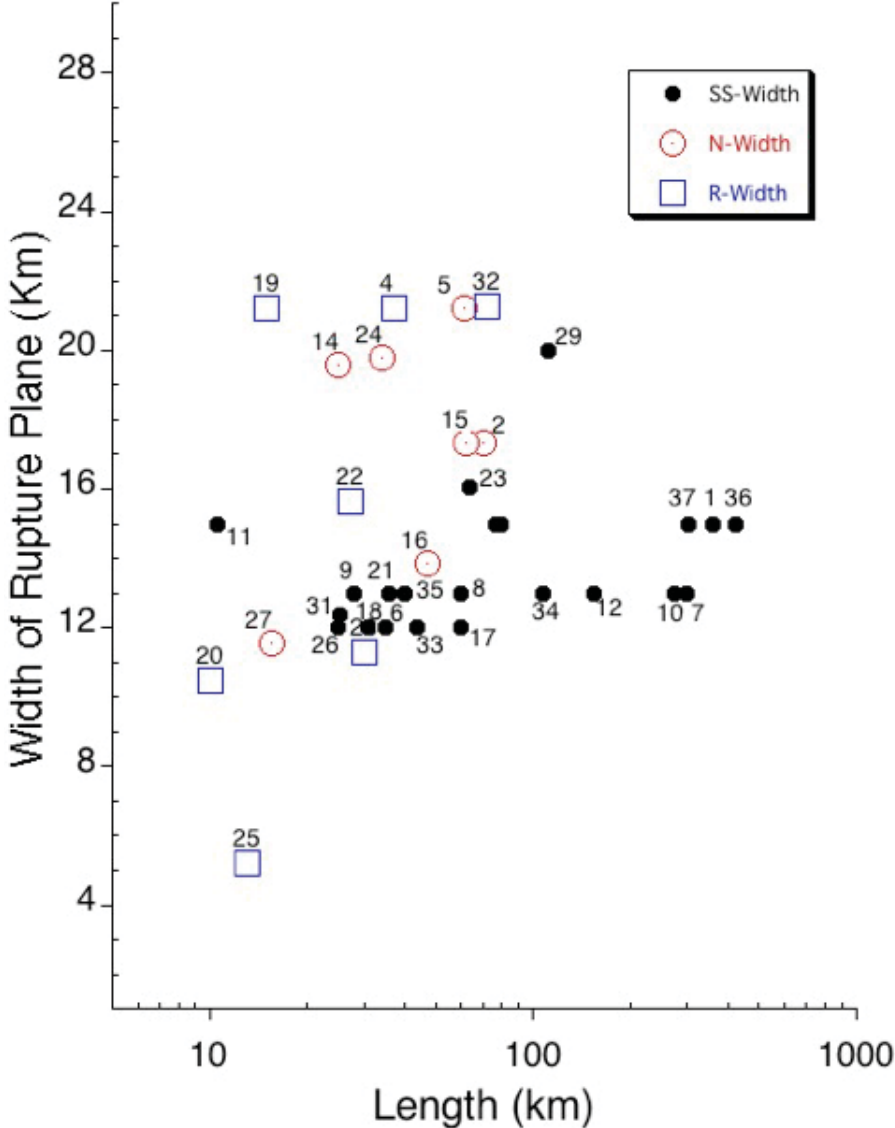
**Subduction**



**Continental**

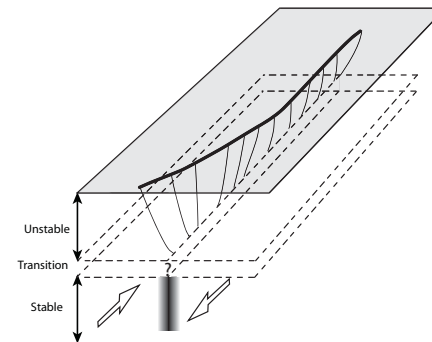
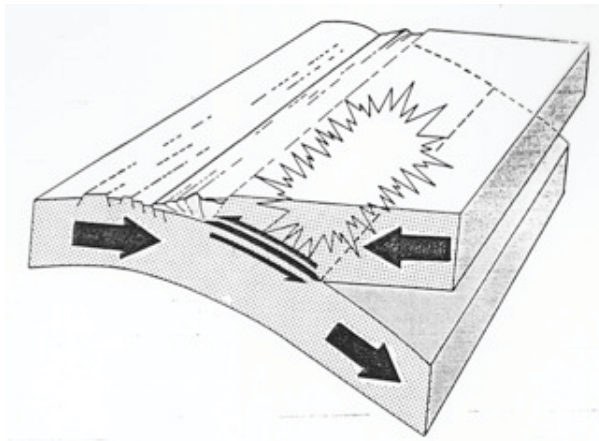
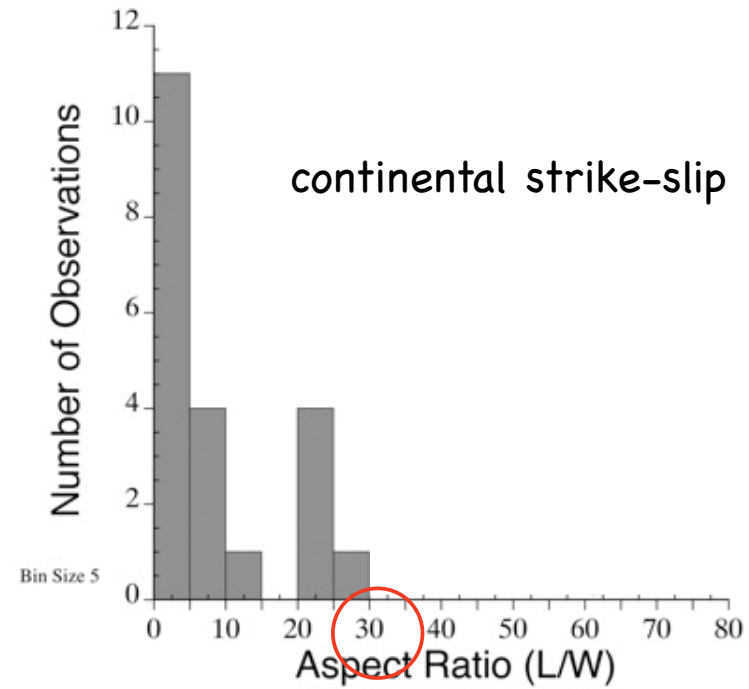
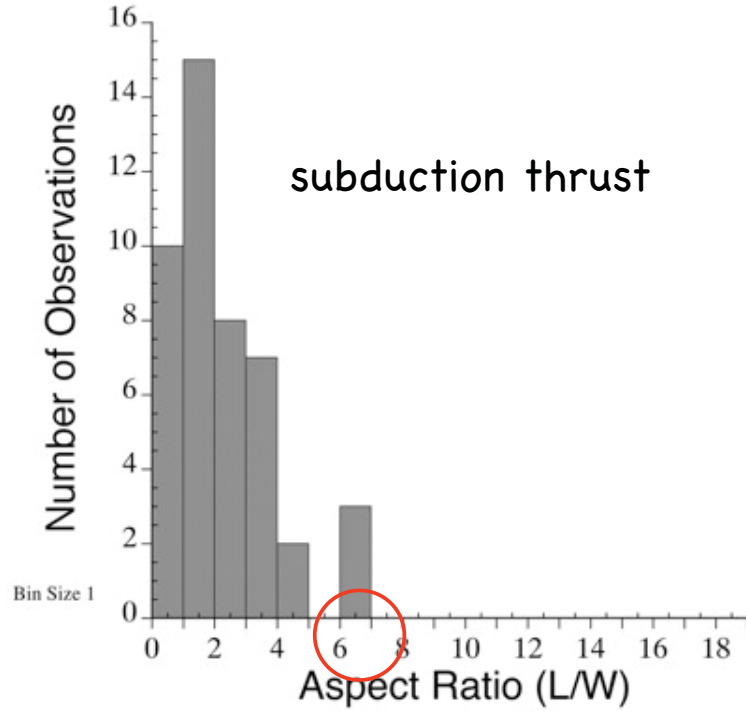


# Length vs Width Continental Earthquakes...

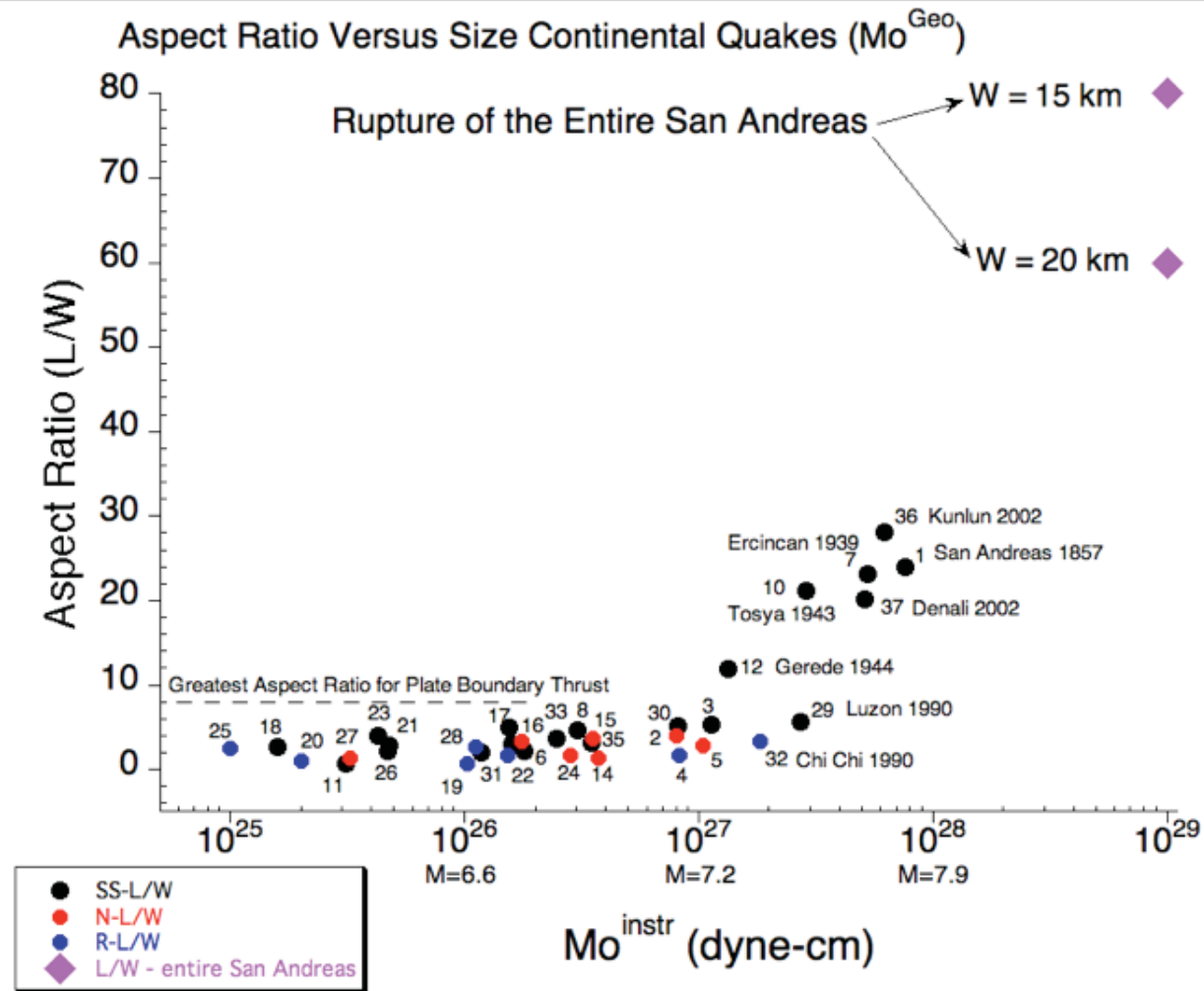
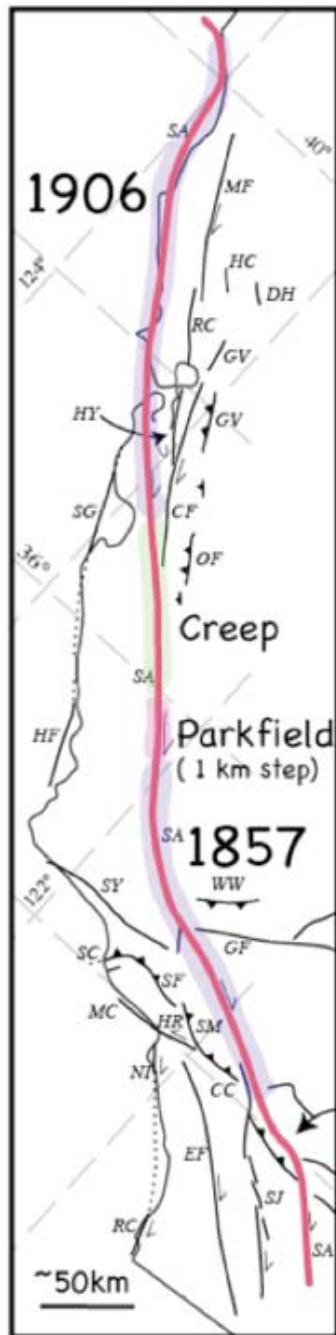


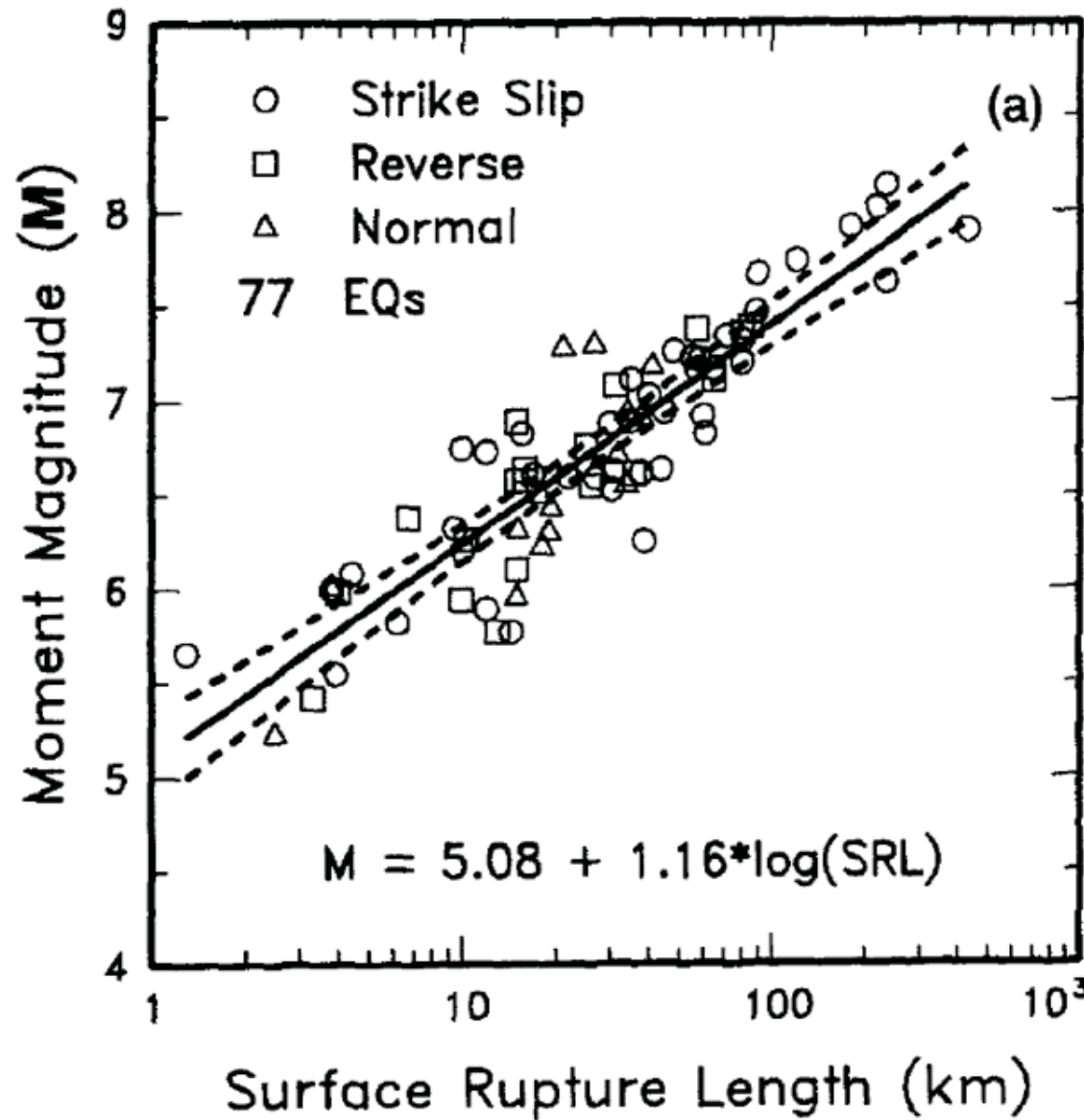
# Aspect Ratio: Rupture Length $L$ / Rupture Width $W$

histograms of total number quakes with given aspect ratio...



# Can/will entire 1200 km length of San Andreas rupture in single M8.6 quake?

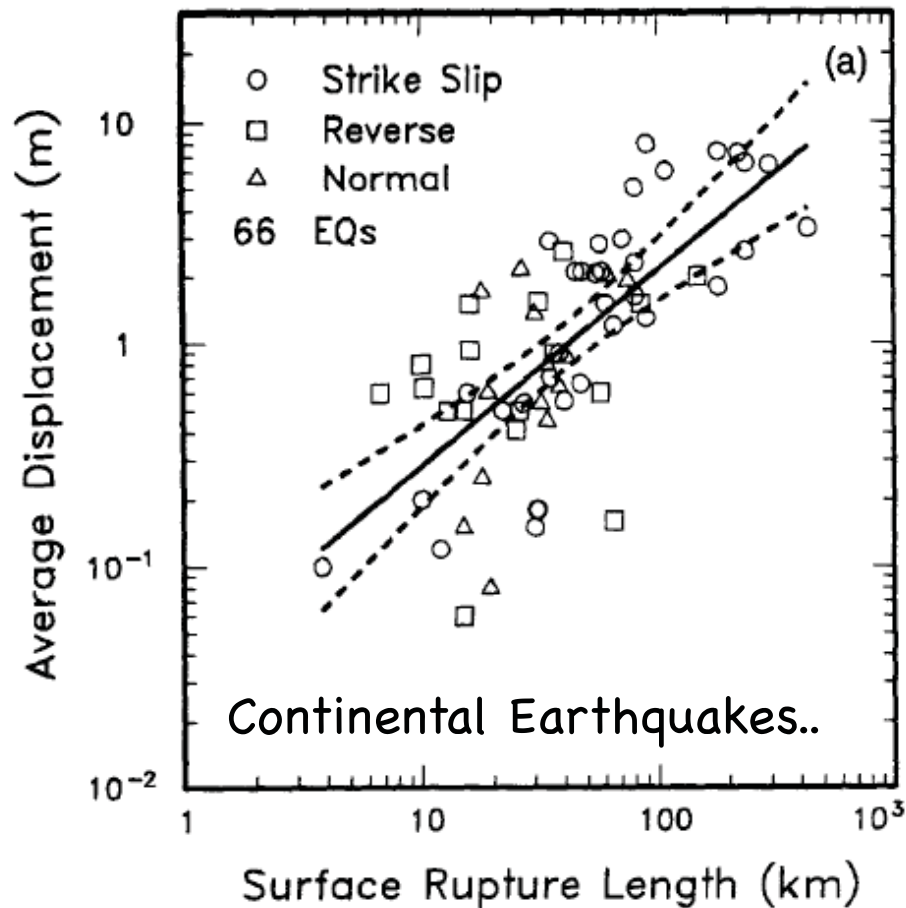




Empirical Scaling  
 Law for  
 Magnitude  
 vs  
 Rupture Length

of practical  
 importance to  
 seismic hazard

# Empirical Scaling Law for Displacement vs Rupture Length



all events

$$\text{Log}(D) = -1.43 + 0.88 \cdot \text{Log}(L)$$

just strike-slip

$$\text{Log}(D) = -1.70 + 1.04 \cdot \text{Log}(L)$$

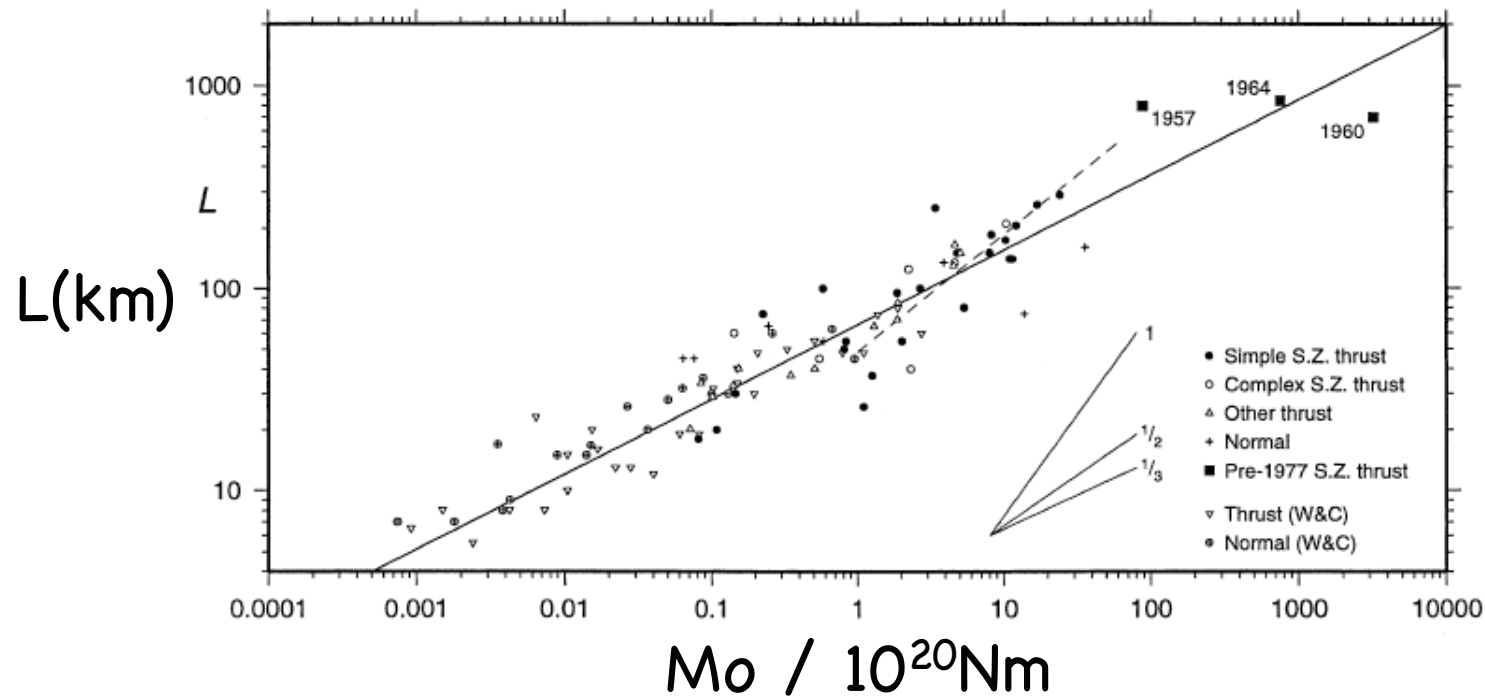
$$\text{Log}(D) = \text{Constant} + \sim 1.0 \cdot \text{Log}(L)$$

$$D = \text{Constant} * L^1$$

Slip Proportional to  
Rupture Length

$$M_0 = \mu L W D = \mu L (\alpha L) \propto \mu L^2$$

# Subduction Zone Thrusts...



all events

$$\text{Log}(L) = -5.58 + 0.4 * \text{Log}(M_0)$$

$$L = \text{Constant} * M_0^{0.4} \quad \rightarrow \quad 0.4 \approx 0.5 \quad \rightarrow \quad = \text{Constant} * M_0^{0.5}$$

$L^2 \propto M_0$  -----> Seismic Moment proportional to rupture length squared

# Seismic Moment Versus Rupture Area...

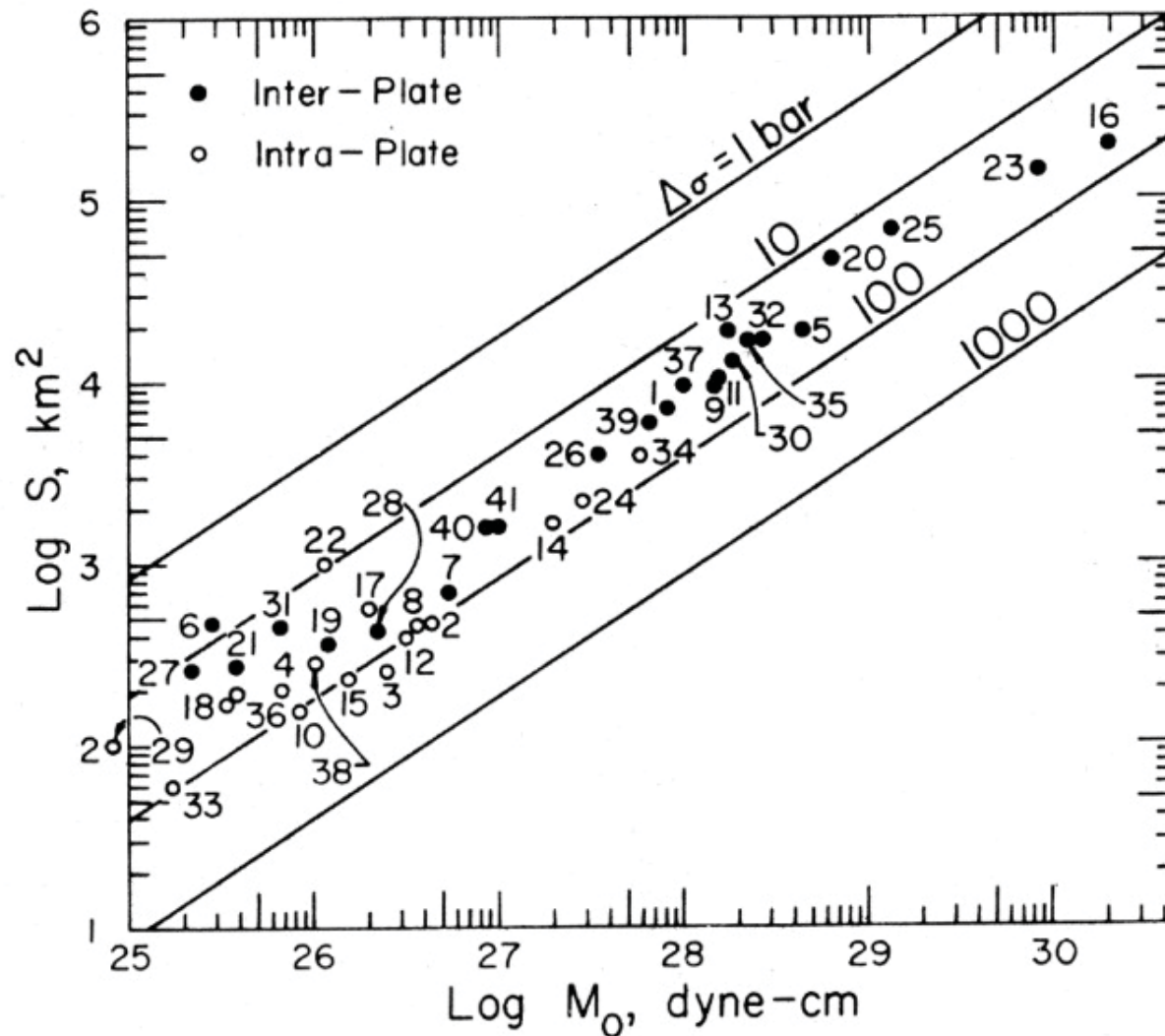


FIG. 2. Relation between  $S$  (fault surface area) and  $M_0$  (seismic moment). The straight lines give the relations for circular cracks with constant  $\Delta\sigma$  (stress drop). The numbers attached to each event correspond to those in Table 1.

General Expression  
for stress drop

$$\Delta\sigma = C\mu\frac{\bar{D}}{\hat{a}}$$

rearranging

$$\bar{D} = \frac{\hat{L}}{C\mu}\Delta\sigma$$

$\bar{D}$  = ave slip on fault plane

$\hat{a}$  = Length Dimension (smallest)

$C$  = shape factor (circular, rectangular, etc.)

=  $\frac{7\pi}{16}$ , for circular fault

Seismic Moment in terms of Stress Drop and Fault Area

$$M_0 = \mu S \bar{D} = \mu S \frac{\hat{a}}{C\mu} \Delta\sigma$$



$$M_o = \mu S \bar{D} = \mu S \frac{\hat{a}}{C_\mu} \Delta\sigma$$

*recognizing S for circular fault =  $\pi \hat{a}^2$*   
*recalling C for circular fault =  $\frac{7\pi}{16}$*

$$M_o = \mu S \bar{D} = \mu S \frac{\hat{a}}{C_\mu} \Delta\sigma = \mu \frac{\pi \hat{a}^3 16}{7\pi\mu} \Delta\sigma$$

*to make convenient expression between  $M_o$ , StressDrop, and Fault area*

$$M_o = \frac{16}{7\pi^{3/2}} \Delta\sigma S^{3/2}$$

*note  $a^3 = \frac{(\pi a^2)^{3/2}}{\pi^{3/2}} = \frac{S^{3/2}}{\pi^{3/2}}$*

*take log of each side*

$$\text{Log } M_o = (3/2) \text{Log } S + \text{Log } \frac{16\Delta\sigma}{7\pi^{3/2}}$$

$$\text{Log } M_0 = (3/2)\text{Log } S + \text{Log} \frac{16\Delta\sigma}{7\pi^{3/2}}$$

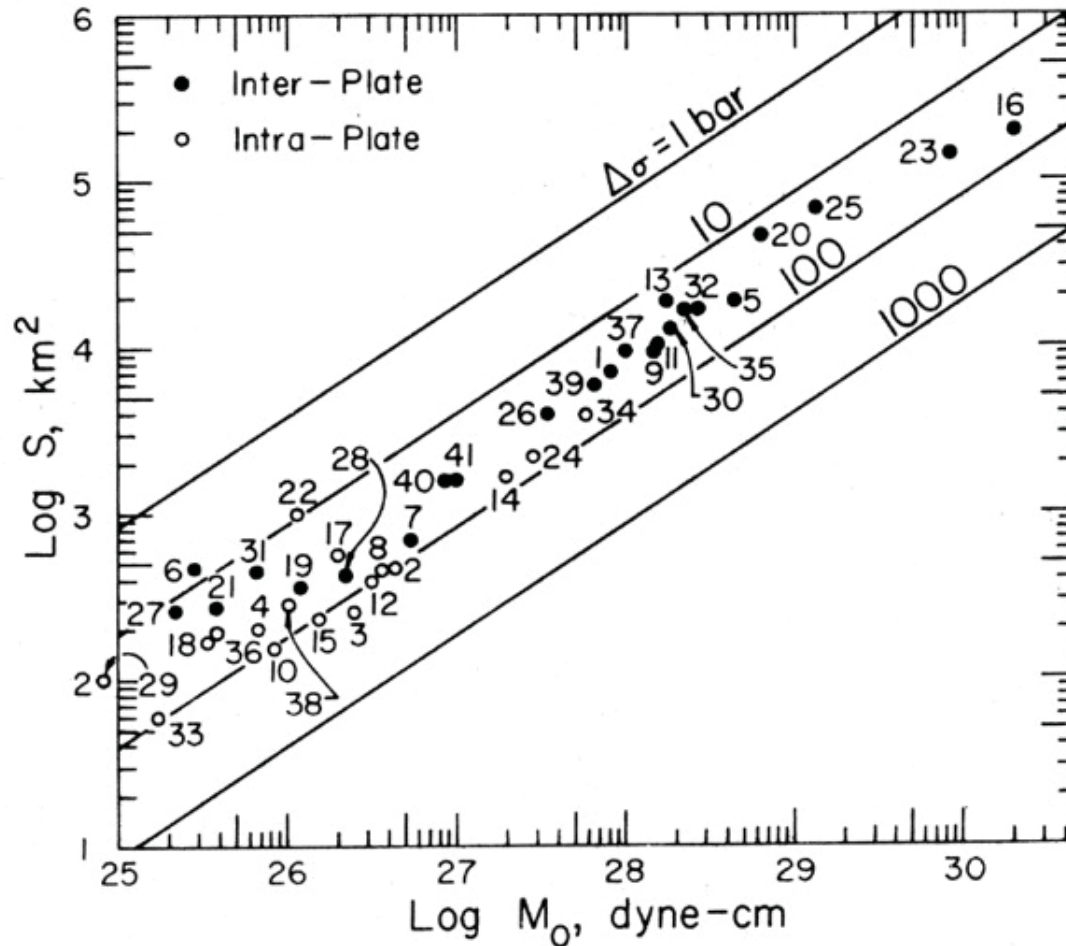


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Relationships (scaling) between fault parameters can vary between tectonic environments...

The relationship between magnitude and rupture length varies if data are separated according to how frequently the particular earthquakes are expected to occur (on the respective sections of the faults on which they occurred)

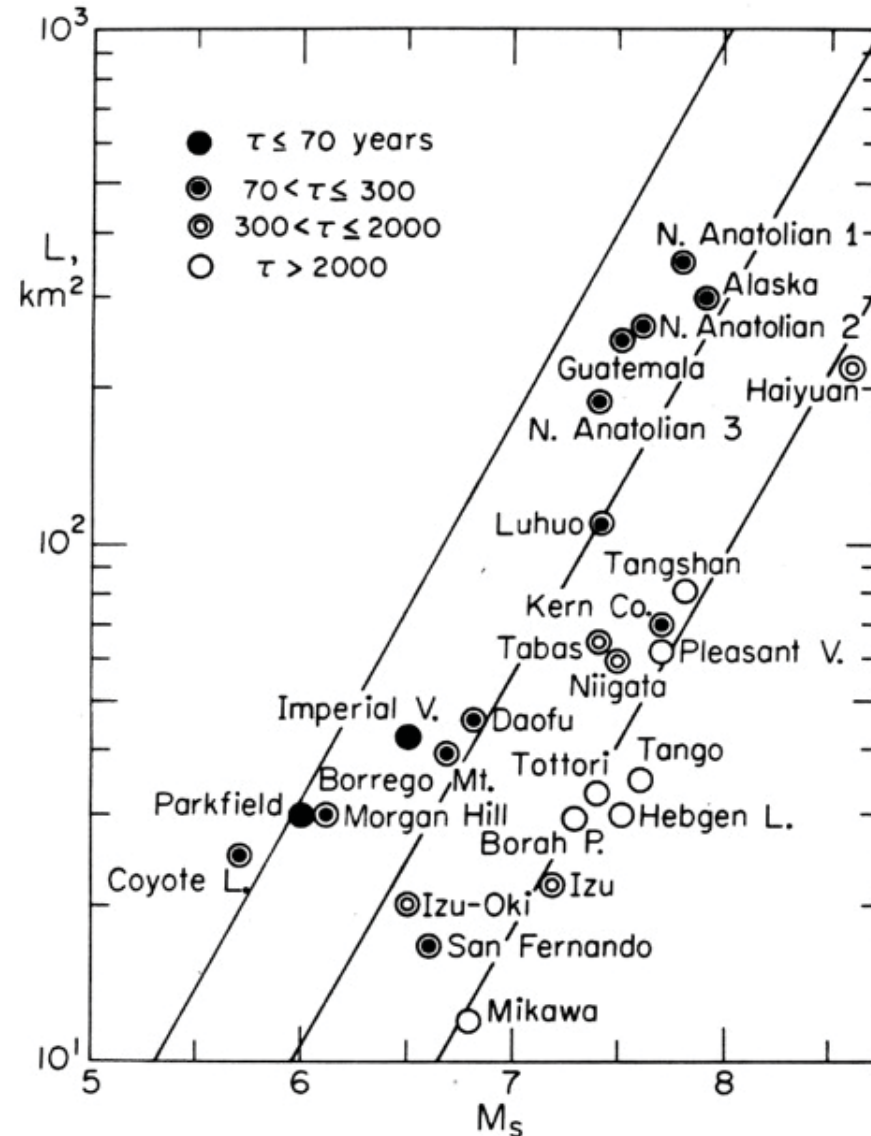


Fig. 1. The relation between the surface-wave magnitude,  $M_s$ , and the fault length,  $L$ . The solid lines indicate the trend for a constant stress drop.

Relationships (scaling) between fault parameters can vary between tectonic environments...

*to start, define average stress drop as ratio*

$$\bar{\Delta\sigma} = \frac{\int_S \Delta\sigma ds}{\int_S D ds}$$

*rewrite by recalling*  $W = \frac{1}{2}\Delta\sigma\bar{D}S$  *so*  $\Delta\sigma\bar{D}S = 2W$  ( $E_s$ )

$$\text{and } M_o = \mu S\bar{D} \quad \text{so} \quad D = \frac{M_o}{\mu S}$$

$$\bar{\Delta\sigma} = \frac{2\mu E_s}{M_o} = \frac{2E_s}{S\bar{D}} = \frac{2 \times 10^{1.5M_s+11.8}}{LWD}$$

*a relationship between stress drop, magnitude, fault dimensions, and offset*

$$\Delta\sigma = \frac{2 \times 10^{1.5M_s+11.8}}{LWD}$$

*Assume like suggested in earlier observations that width  $W$  is constant and  $D \propto L$*

$$L^2 D \propto \frac{2 \times 10^{1.5M_s+11.8}}{\Delta\sigma}$$

*taking Log of both sides...*

$$\text{Log}L \propto \frac{1.5}{2}M_s - \frac{1}{2}\text{Log}\Delta\sigma$$

*a relationship between rupture length, magnitude, and stress drop*

$$\text{Log}L \propto \frac{1.5}{2}M_s - \frac{1}{2}\text{Log}\Delta\sigma$$

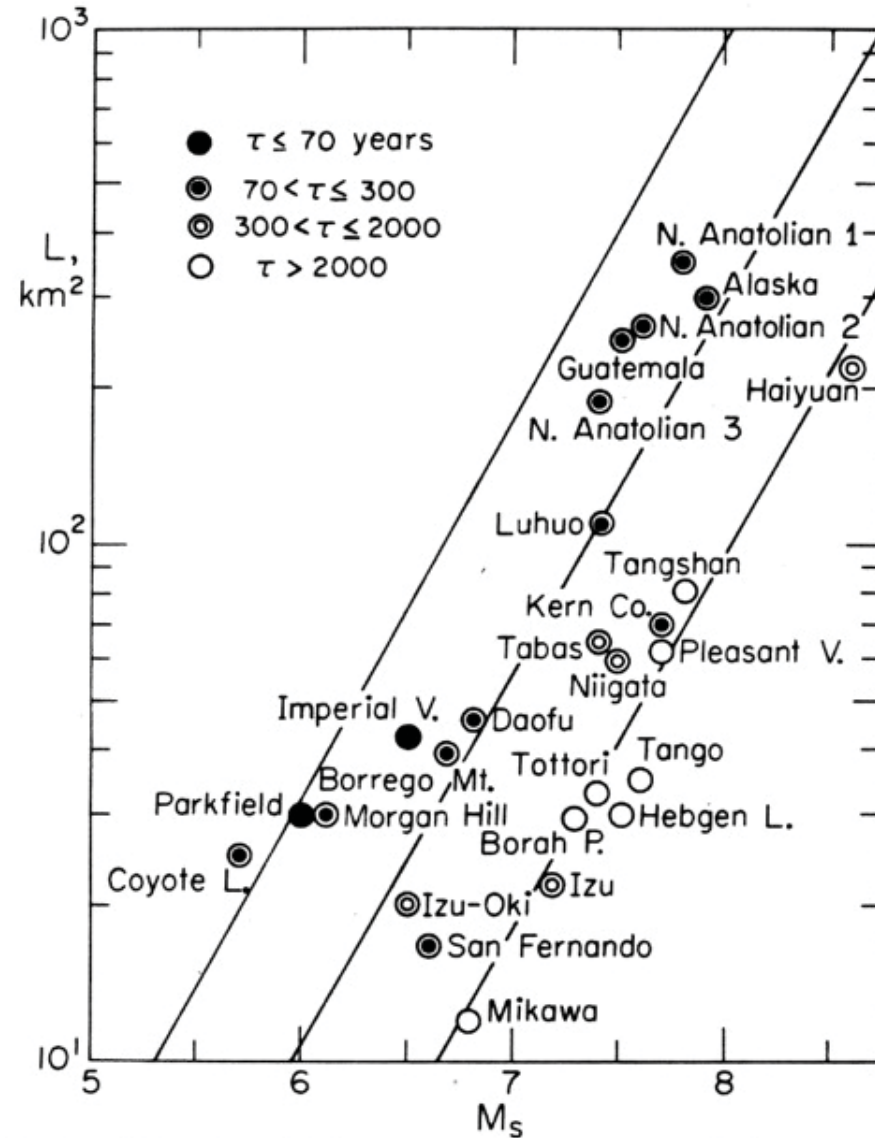
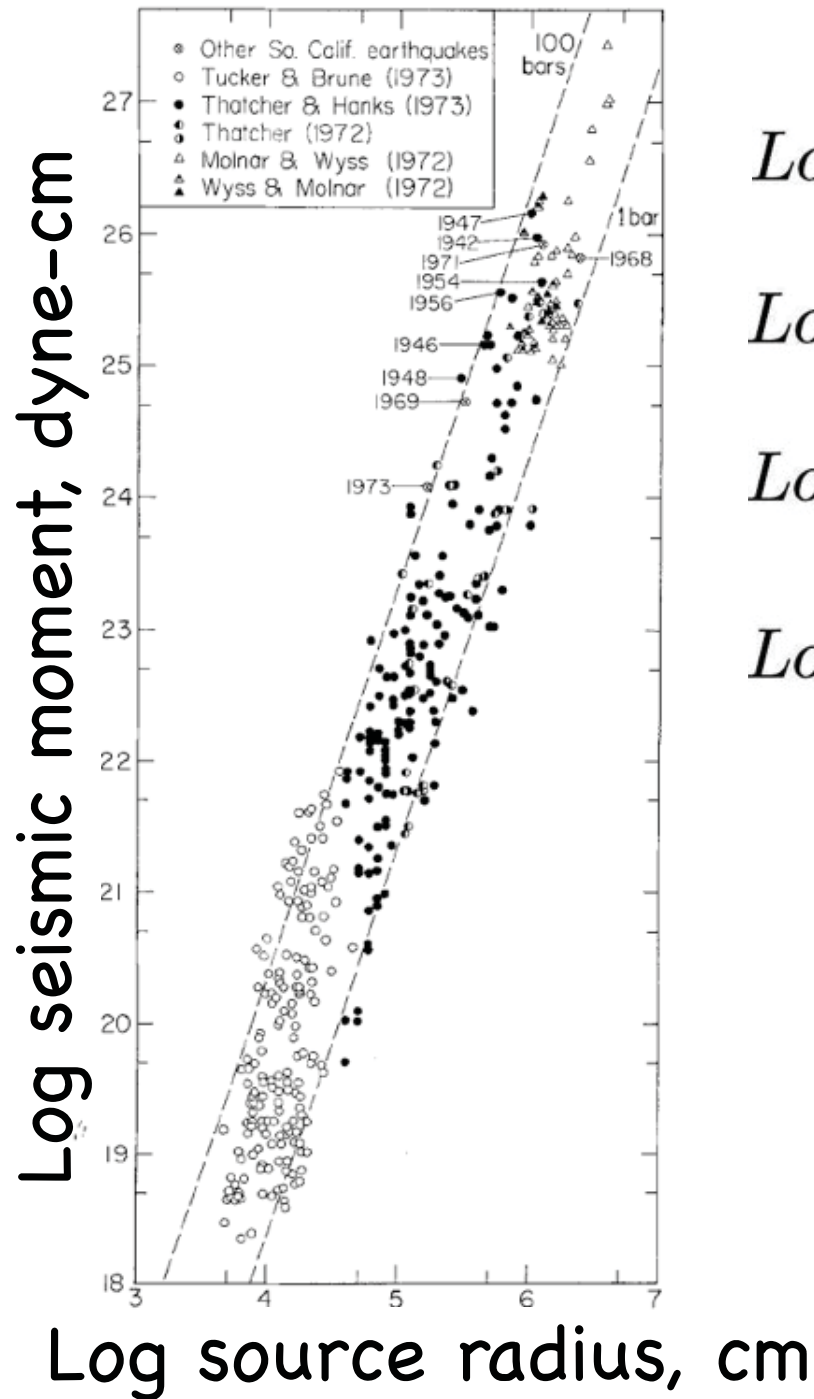


Fig. 1. The relation between the surface-wave magnitude,  $M_s$ , and the fault length,  $L$ . The solid lines indicate the trend for a constant stress drop.



$$\text{Log } M_0 = (3/2)\text{Log}S + \text{Log} \frac{16\Delta\sigma}{7\pi^{3/2}}$$

$$\text{Log } M_0 \propto (3/2)\text{Log}\hat{a}^2$$

$$\text{Log } M_0 \propto \text{Log}\hat{a}^3$$

$$\text{Log } M_0 \propto \text{Log}\pi\hat{a}^3 = 3\text{Log}\pi a$$