Discrete Symmetry and the μ term in G_2-MSSM

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G_2 Manifold 101

- 7-D Manifold with G_2 holonomy
  - 11D M theory $\xrightarrow{G_2 \text{ Manifold}}$ 4D N=1 SUSY
- Invariant 3-form

$$\varphi = e^{123} + e^{145} + e^{167} + e^{246} - e^{257} - e^{347} - e^{356} = \sum_{N=1}^{b_3} s^N \beta_N$$

Moduli (Scalar Fields): s^N

- Volume

$$V_7 = \frac{1}{7}\int_M \varphi \wedge *_\varphi \varphi = \frac{3}{7}\sum_{i=1}^{b_3} s_i \tau_i \qquad \tau_i = \frac{\partial V_7(s_i)}{\partial s_i}$$

- "Angular" variable

$$a_i = \frac{s_i \tau_i}{V_7} \qquad \sum_i a_i = \frac{7}{3}$$

Moduli Stabilization

- Kahler Potential

$$K_0 = -3\ln V_X = -3\ln\left(\prod_i s_i^{a_i}\right) \qquad K_{\text{matter}} = \frac{1}{V_X}\bar{\phi}\phi$$

- Gaugino condensation

$$W_{\text{NP}} = A_1 \phi^a e^{-b_1 V_Q^{\mathbb{C}}} + A_2 e^{-b_2 V_Q^{\mathbb{C}}} \qquad V_Q = \sum_i N_i s_i,\ N_i \in \mathbb{N}$$

$$SU(P+1),\ \ SU(Q):\ \ b_1 = \frac{2\pi}{P},\ \ b_2 = \frac{2\pi}{Q},\ \ a = -\frac{2}{P}$$

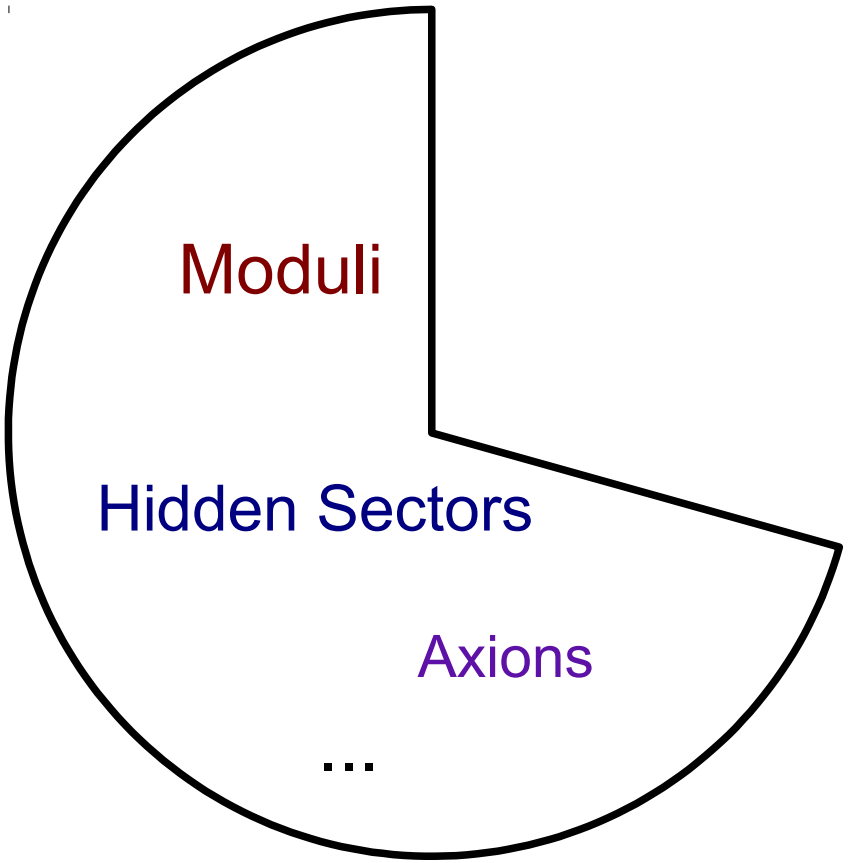
$$V_Q \gg 1, W_{\text{NP}} \ll 1$$

- Moduli VEV and F-Terms

$$s_i = \frac{3}{7}\frac{a_i}{N_i} V_Q + O\left(\frac{1}{P_{\text{eff}}}\right),\ \ V_Q \approx \frac{1}{b_1 - b_2}\ln\left(\frac{A_1 b_1 \phi^a}{A_2 b_2}\right)$$

$$F_i \approx 0 + O\left(\frac{1}{P_{\text{eff}}}\right), \qquad F_\phi \approx m_{3/2}$$

M theory on G_2 manifold



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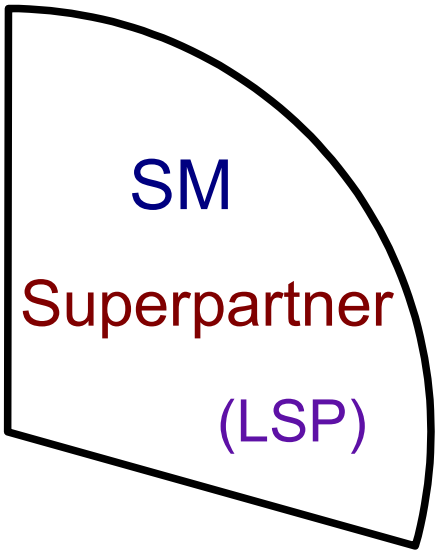
4D Low Energy Spectrum



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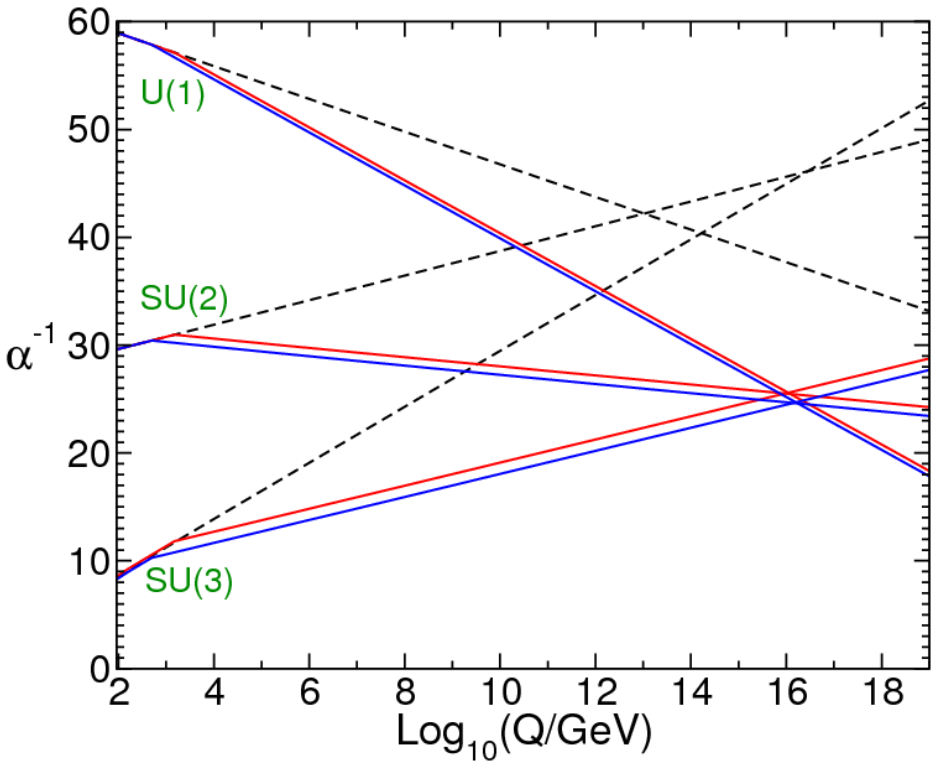
SU(5) GUT



S. Martin, hep-ph/9709356

- Matter Fields
  - $\bar{5}_M = (\bar{3},1)_{D^c} + (1,2)_L$
  - $10_M = (3,2)_Q + (\bar{3},1)_{U^c} + (1,1)_{E^c}$
- Higgs Fields
  - $5_{H_U} = (3,1)_{T_U} + (1,2)_{H_U}$
  - $\bar{5}_{H_D} = (\bar{3},1)_{T_D} + (1,2)_{H_D}$
- Wilson Line → MSSM

$\pi_1(M)$ finite

- Superpotential

$$W \supset \{5_{H_U} 10_M 10_M, \bar{5}_{H_D} \bar{5}_M 10_M, 5_{H_U} \bar{5}_{H_D}, \ldots\}$$

GUT Scale Inputs

$$m_{3/2} = e^{K/2}|W|$$

$$F^{\phi} F_{\phi} \approx 3|W|^2$$

- Scalars

$$m_{\alpha\beta}^2 = m_{3/2}^2$$

- (Normalized) Trilinears

$$A_{\alpha} = \sqrt{3}\, m_{3/2} + \text{corrections} \approx 1.4\, m_{3/2}$$

- Gauginos

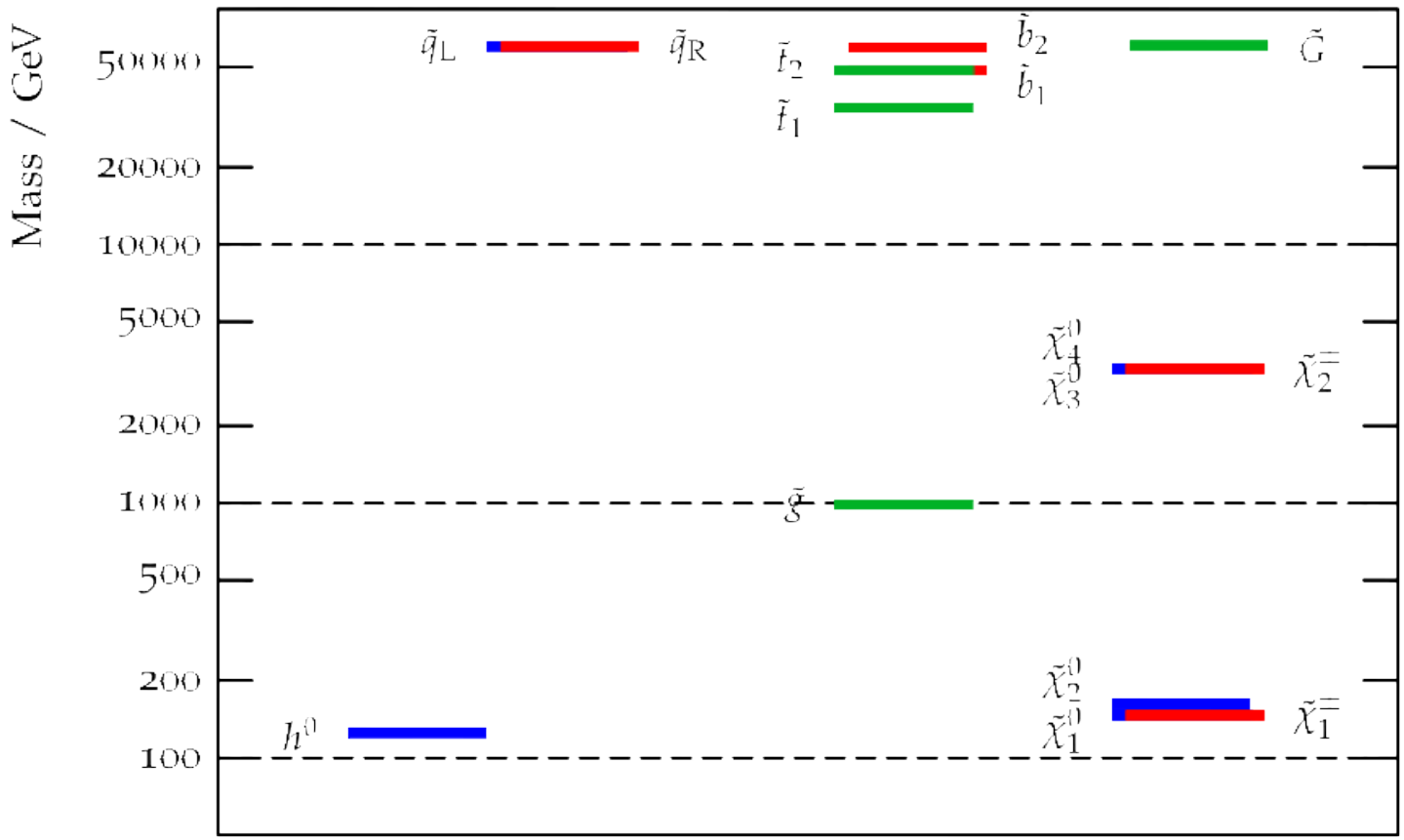
$$F_i = 0 + O\left(\frac{1}{P_{\text{eff}}}\right), \qquad M_a = O\left(\frac{1}{P_{\text{eff}}}\right) m_{3/2} + \text{Anomaly Mediation}$$

B. S. Acharya, G. Kane and P. Kumar, arXiv: 1204.2795

- Doublet-Triplet Splitting

$$W \supset m_T T_U T_D + \mu H_U H_D, \qquad m_T \sim M_{\text{GUT}},\ m_H \equiv \mu < m_{3/2}$$

G_2-MSSM


Mass / GeV
50000
20000
10000
5000
2000
1000
500
200
100
$\tilde{q}_L$
$\tilde{q}_R$
$\tilde{t}_2$
$\tilde{t}_1$
$\tilde{b}_2$
$\tilde{b}_1$
$\tilde{G}$
$\tilde{\chi}_4^0$
$\tilde{\chi}_3^0$
$\tilde{\chi}_2^\pm$
$\tilde{g}$
$\tilde{\chi}_2^0$
$\tilde{\chi}_1^0$
$\tilde{\chi}_1^\pm$
h^0

Why?

- Doublet-Triplet Splitting (SU(5) GUT)

$$W \supset m_T T_U T_D + \mu H_U H_D,$$

$$m_T \sim M_{\mathrm{GUT}}, \quad m_H \equiv \mu < m_{3/2}$$

How?

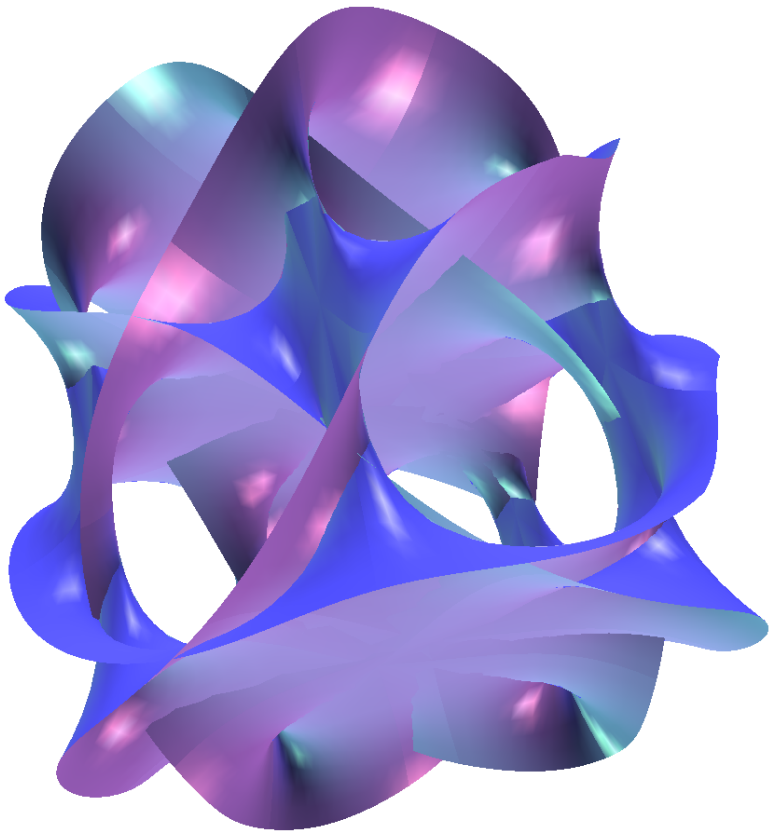
- Geometric

  Discrete Symmetry

  - Approximate
    - Size of μ

Discrete Symmetry

$K3 \rightarrow M \rightarrow Q$



- Gauge Symmetry $(SU(5))$
  - Wilson Line $\rightarrow SU(3)\times SU(2)\times U(1)$
- Discrete Symmetry (Z_N)
  - Fixed points S_1: $10^i_M, \bar{5}^j_M, 5_H$
  - Fixed points S_2: $\bar{5}_H, 10^k_M, \bar{5}^l_M$
  - Doublet-Triplet Splitting

E. Witten, arXiv:hep-ph/0201018.

Discrete Charge

- Wilson Line

$$W = \mathrm{diag}\left(e^{2i\rho}, e^{2i\rho}, e^{2i\rho}, e^{-3i\rho}, e^{-3i\rho}\right)$$

$$\left(q_{Q^i}, q_{U^i}, q_{E^i}\right) = \left(q^i_{10}, q^i_{10}, q^i_{10}\right) + \epsilon^i_{10}\left(1, -4, 6\right)$$

$$\left(q_{D^i}, q_{L^i}\right) = \left(q^i_{\bar{5}}, q^i_{\bar{5}}\right) + \epsilon^i_{\bar{5}}\left(2, -3\right)$$

$$\left(q_{T_U}, q_{H_U}\right) = \left(q_{5_H}, q_{5_H}\right) + \epsilon_{5_H}\left(-2, 3\right)$$

$$\left(q_{T_D}, q_{H_D}\right) = \left(q_{\bar{5}_H}, q_{\bar{5}_H}\right) + \epsilon_{\bar{5}_H}\left(2, -3\right)$$

Moduli Fields

- Invariant 3-form

$$\varphi \rightarrow \varphi = \sum_N s_N \beta_N : \quad \beta_N \rightarrow \beta'_N, \quad s_N \rightarrow s'_N$$

- Permutation Symmetry: $\{ s_i \rightarrow s_j \}$
  - Cyclic Symmetry (C_4/Z_4):

$$s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4 \rightarrow s_1$$

$$q(s_1 + s_2 + s_3 + s_4) = 0$$
$$q(s_1 - i\, s_2 - s_3 + i s_4) = 1$$
$$q(s_1 - s_2 + s_3 - s_4) = 2$$
$$q(s_1 + i\, s_2 - s_3 - i s_4) = 3$$

Approximate Discrete Symmetry

Allow Coupling	Constraint
$10_M 10_M H_U$	$2q_{10}+q_{5_H}=0$
$10_M \bar{5}_M H_D$	$q_{10}+q_{\bar{5}}+q_{H_D}=0$
$H_D H_D \bar{5}_M \bar{5}_M$	$2q_{H_D}+2q_{\bar{5}}=0$
$T_U T_D$	$q_{5_H}+q_{T_D}=0$

Forbid Coupling	Constraint
$H_U H_D$	$q_{5_H}+q_{H_D}\neq 0$
$10_M 10_M 10_M \bar{5}_M$	$3q_{10}+q_{\bar{5}}\neq 0$
$5_H \bar{5}_M$	$q_{5_H}+q_{\bar{5}}\neq 0$
$10_M \bar{5}_M \bar{5}_M$	$q_{10}+2q_{\bar{5}}\neq 0$

Anomaly Coefficients

$$A_3 = \frac{1}{2}\sum_{i=1}^{3}\left(2\, q_{Q_L^i} + q_{U_R^i} + q_{D_R^i}\right) + \frac{1}{2}\left(q_{T_U} + q_{T_D}\right)$$

$$A_2 = \frac{1}{2}\sum_{i=1}^{3}\left(3\, q_{Q_L^i} + q_{L^i}\right) + \frac{1}{2}\left(q_{H_U} + q_{H_D}\right)$$

$$A_1 = \frac{3}{5}\sum_{i=1}^{3}\left(\frac{1}{6} q_{Q_L^i} + \frac{4}{3} q_{U_R^i} + \frac{1}{3} q_{D_R^i} + \frac{1}{2} q_L^i + q_E^i\right)$$

$$+ \frac{3}{10}\left(q_{H_U} + q_{H_D}\right) + \frac{1}{5}\left(q_{T_U} + q_{T_D}\right)$$

$$A_1 = A_2 = A_3 = 0 \mod N/2$$

Anomaly-Free Z_4 Symmetry

q_{Q^1}	q_{Q^2}	q_{Q^3}	q_{U^1}	q_{U^2}	q_{U^3}	q_{E^1}	q_{E^2}	q_{E^3}
3	3	3	3	3	3	3	3	3

q_{D^1}	q_{D^2}	q_{D^3}	q_{L^1}	q_{L^2}	q_{L^3}
3	1	1	1	1	1

q_{H_U}	q_{H_D}	q_{T_U}	q_{T_D}
2	0	0	0

$$q(H_U H_D) = 2$$

$$Y_{H_D D Q} = \begin{pmatrix} 0 & 0 & 0 \\ x & x & x \\ x & x & x \end{pmatrix}$$

Down-type & Lepton Yukawa Matrix

- Down-type & Leptonic Yukawa $\bar{5}_H \bar{5}_M 10_M$

$$\mathrm{Det}(Y) = \sum_{\sigma \in S_3} \mathrm{sgn}(\sigma) \prod Y_{i,\sigma_i} \qquad \sum_{i=1}^{3} \epsilon_{10}^{i} = \sum_{i=1}^{3} \epsilon_{\bar{5}}^{i}$$

- Anomaly Cancellation $A_2 = A_3$

$$\sum_{i=1}^{3} \epsilon_{10}^{i} - \sum_{i=1}^{3} \epsilon_{\bar{5}}^{i} = -\epsilon_{5_H} + \epsilon_{\bar{5}_H} \qquad \epsilon_{5_H} = \epsilon_{\bar{5}_H}$$

$$q_{H_U} + q_{H_D} = q_{T_U} + q_{T_D}$$

- Doublet-triplet Splitting $\Leftrightarrow$ Zero eigenvalue(s)

Generate μ term

- Approximate discrete symmetry

$$W \supset \mu H_U H_D \approx 0$$

- Guidice-Masiero Mechanism

$$K \supset O(1) h(s_i) H_U H_D = O(1) h(a_i) H_U H_D$$

  - Z_4 Example

$$K \supset O(1)(a_1 - a_2 + a_3 - a_4) H_U H_D \approx \bar{a} H_U H_D$$

$$\rightarrow W \supset \bar{a} m_{3/2} H_U H_D$$

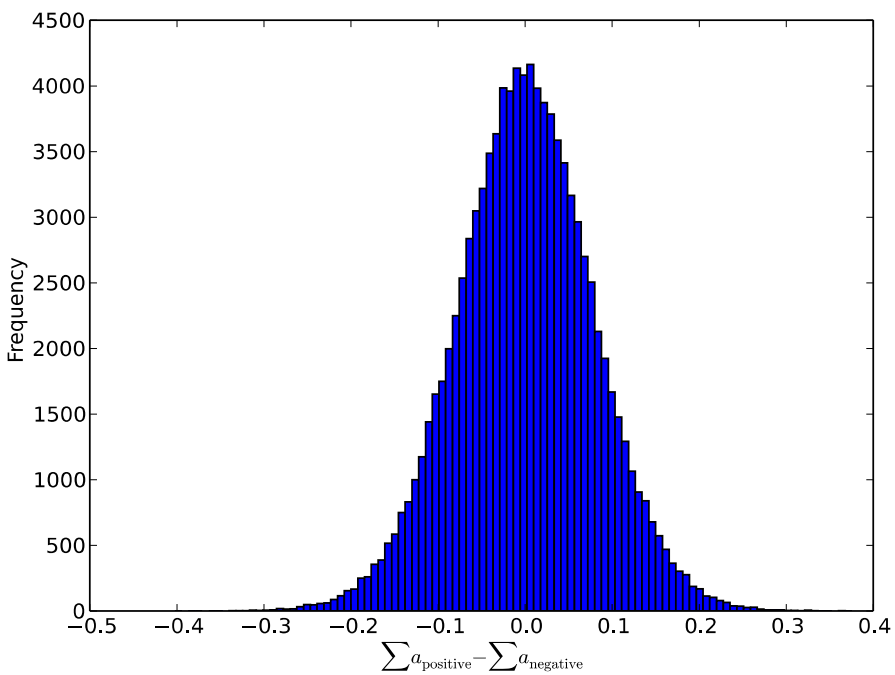
Done?

Not Yet

- Number of Moduli ~ N = *O*(100)

$$K \supset \sum{}' (a_i - a_j + a_k - a_l) H_U H_D + \sum{}'' (a_p - a_q) H_U H_D$$

$$= \left(\sum a_{\text{positive}} - \sum a_{\text{negative}} \right) H_U H_D$$



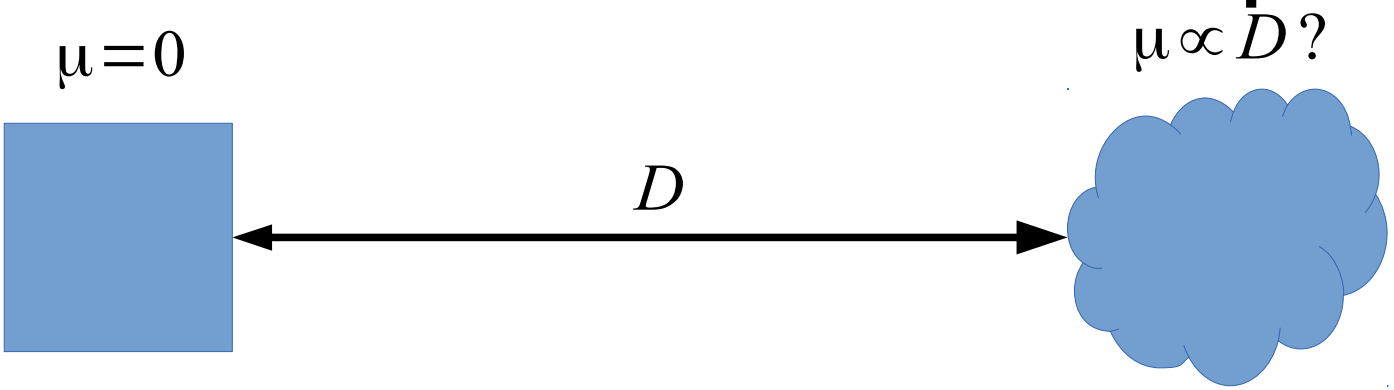
- Central Limit Theorem

$$|\mu| < O\left(\sqrt{N}\,\bar{a}\right) m_{3/2}$$

$$= O\left(\frac{1}{\sqrt{N}}\right) m_{3/2}$$

$$\left(\sum_i a_i = N\,\bar{a} = \frac{7}{3} \right)$$

Another Estimation of μ



- Distance on the moduli space(-radial direction)
  - Euclidean Metric

$$D = \sqrt{\sum \left(\left(a_i \right)_{\text{symmetric}} - a_i \right)^2} \approx \sqrt{\sum a_i^2} \approx \sqrt{N}\,\bar{a}$$

$$\mu \propto m_{3/2}$$

$$\mu = D\, m_{3/2} = O\left(\sqrt{N}\,\bar{a} \right) m_{3/2} = O\left(\frac{1}{\sqrt{N}} \right) m_{3/2}$$

Summary

- Anomaly Free Discrete Symmetry
  - Doublet-Triplet Splitting
  - Zero Eigenvalue(s) in Yukawa matrix
  - Matter Parity
- Size of the μ term
  - Guidice-Masiero
  - (Naive) Distance

$$\mu \sim O\left(\frac{1}{\sqrt{N}}\right) m_{3/2}$$