

Naturalness of Neutralino Dark Matter

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This talk is based on

arXiv:1207.4434 [P. Grothaus, M. Lindner, Y. Takanishi]

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Outline

1. Introduction

- ▶ pMSSM
- ▶ Experimental Constraints
- ▶ Fine-Tuning
- ▶ Computing Method

2. Results

- ▶ Relic Density
- ▶ Direct Detection
- ▶ $(g - 2)_\mu$

3. Conclusions

We use the phenomenological MSSM (pMSSM) with 11 parameters:

$$\tan \beta, M_1, M_2, M_3, M_A, \mu, m_{\tilde{\ell}_L}, m_{\tilde{\ell}_R}, m_{\tilde{q}_{1,2}}, m_{\tilde{q}_3}, a_0.$$

Assumption:

- ▶ LSP as neutralino:

$$\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}^0 + N_{13} \tilde{H}_d^0 + N_{14} \tilde{H}_u^0.$$

\tilde{B} bino; \tilde{W}^0 wino; \tilde{H}_d^0 down-type higgsino; \tilde{H}_u^0 up-type higgsino.

The neutralino mass matrix:

$$\begin{pmatrix} M_1 & 0 & -M_Z s_{\theta_W} c_{\beta} & M_Z s_{\theta_W} s_{\beta} \\ 0 & M_2 & M_Z c_{\theta_W} c_{\beta} & -M_Z c_{\theta_W} s_{\beta} \\ -M_Z s_{\theta_W} c_{\beta} & M_Z c_{\theta_W} c_{\beta} & 0 & -\mu \\ M_Z s_{\theta_W} s_{\beta} & -M_Z c_{\theta_W} s_{\beta} & -\mu & 0 \end{pmatrix}$$

$$s_{\theta_W} \equiv \sin \theta_W, s_{\beta} \equiv \sin \beta, c_{\theta_W} \equiv \cos \theta_W, c_{\beta} \equiv \cos \beta.$$

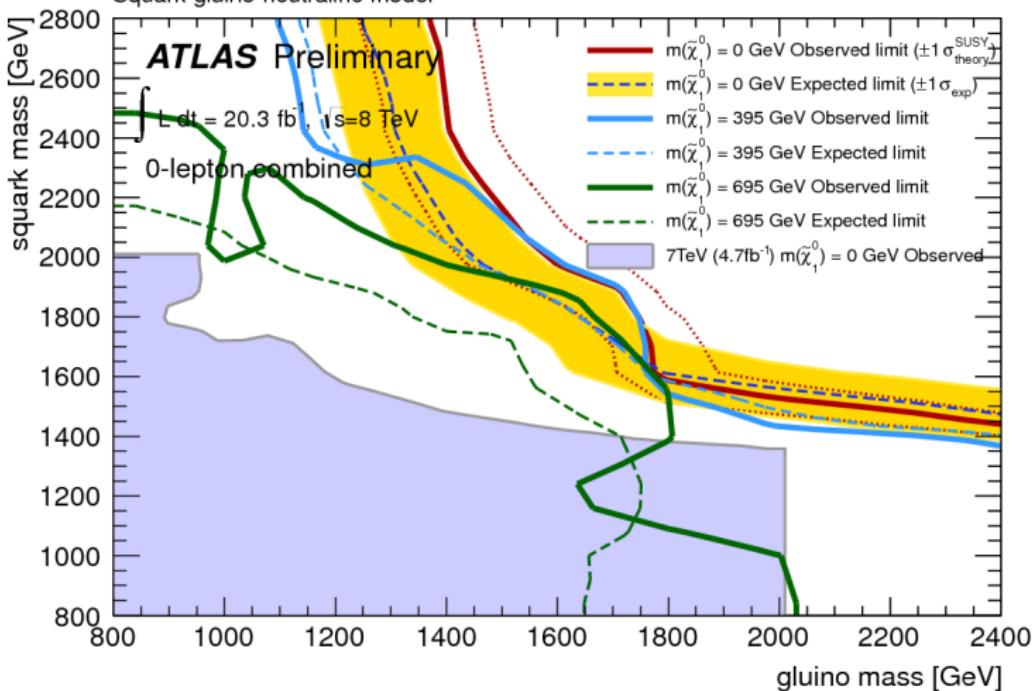
- ▶ Scan range of parameters:

$M_1 \in [10, 200]$ GeV, $M_2 \in [100, 2000]$ GeV, $M_3 \in [100, 4000]$ GeV,
 $m_A \in [90, 4000]$ GeV, $\pm\mu \in [90, 2000]$ GeV, $a_0 \in [-4.0, 4.0]$,
 $m_{\tilde{q}_{1,2}} \in [400, 4000]$ GeV, $m_{\tilde{q}_3} \in [200, 4000]$ GeV, $\tan\beta \in [2, 65]$,
 $m_{\tilde{\ell}_L} \in [100, 4000]$ GeV, $m_{\tilde{\ell}_R} \in [60, 4000]$ GeV.

- ▶ Experimental constraints used in the analysis:

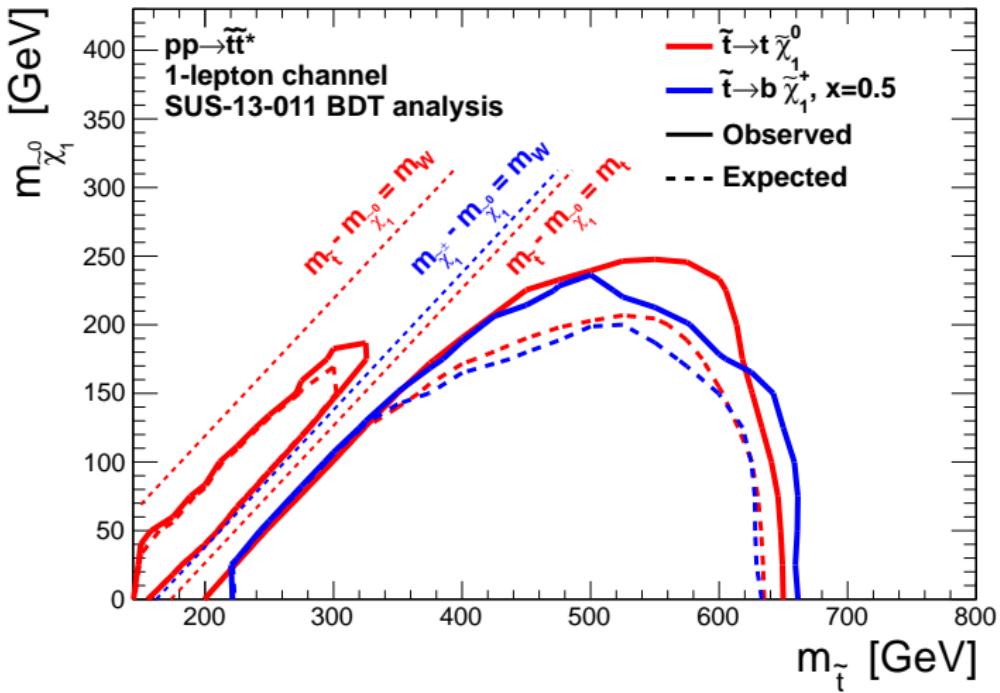
- ▶ $\Omega h^2 \in [0.089, 0.136]$
- ▶ $m_h \in (121.0, 129.0)$ GeV
- ▶ $m_{\tilde{q}_{1,2}} > 1000$ GeV $m_{\tilde{g}} > 800$ GeV
- ▶ $\text{Br}(b \rightarrow s\gamma) \in [2.89, 4.21] \times 10^{-4}$
- ▶ $\text{Br}(B_s \rightarrow \mu^+ \mu^-) < 4.5 \times 10^{-9}$
- ▶ $0.52 < R_{B\tau\nu} < 2.61$
- ▶ $0.985 < R_{l23} < 1.013$ ($K \rightarrow \mu\nu$)
- ▶ $(g-2)_\mu \in [0.34, 4.81] \times 10^{-9}$
- ▶ $\Gamma(Z \rightarrow \tilde{\chi}_1 \tilde{\chi}_1) < 3$ MeV; $\sigma(e^- e^+ \rightarrow \tilde{\chi}_1, \tilde{\chi}_{2,3}) < 100$ fb;
 $\Delta\rho < 0.002$

Squark-gluino-neutralino model



ATLAS-CONF-2013-047

CMS Preliminary

 $\sqrt{s} = 8 \text{ TeV}, \int L dt = 19.5 \text{ fb}^{-1}$ 

CMS-PAS-SUS-13-011

- ▶ Scan range of parameters:

$M_1 \in [10, 200]$ GeV, $M_2 \in [100, 2000]$ GeV, $M_3 \in [100, 4000]$ GeV,
 $m_A \in [90, 4000]$ GeV, $\pm\mu \in [90, 2000]$ GeV, $a_0 \in [-4.0, 4.0]$,
 $m_{\tilde{q}_{1,2}} \in [400, 4000]$ GeV, $m_{\tilde{q}_3} \in [200, 4000]$ GeV, $\tan\beta \in [2, 65]$,
 $m_{\tilde{\ell}_L} \in [100, 4000]$ GeV, $m_{\tilde{\ell}_R} \in [60, 4000]$ GeV.

- ▶ Experimental constraints used in the analysis:

- ▶ $\Omega h^2 \in [0.089, 0.136]$ → New Data
- ▶ $m_h \in (121.0, 129.0)$ GeV → Measured
- ▶ $m_{\tilde{q}_{1,2}} > 1000$ GeV $m_{\tilde{g}} > 800$ GeV → New Data
- ▶ $\text{Br}(b \rightarrow s\gamma) \in [2.89, 4.21] \times 10^{-4}$
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Fine-tuning definition [Barbieri, Giudice ('88)]

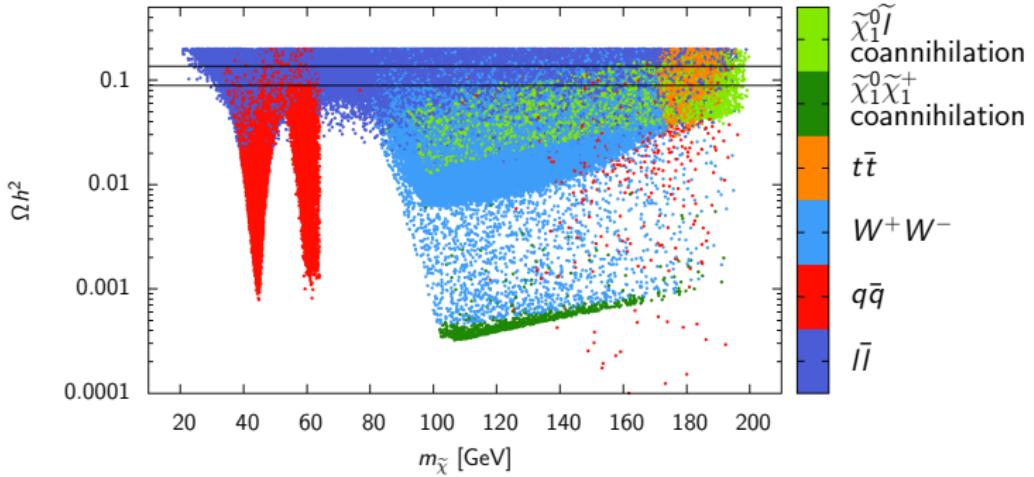
$$\Delta p_i \equiv \left| \frac{p_i}{M_Z^2} \frac{\partial M_Z^2(p_i)}{\partial p_i} \right| = \left| \frac{\partial \ln M_Z^2(p_i)}{\partial \ln p_i} \right| .$$

parameters p_i determine the Z -mass; μ , the two Higgs masses $m_{H_u}^2$, $m_{H_d}^2$ and the bilinear coupling b .

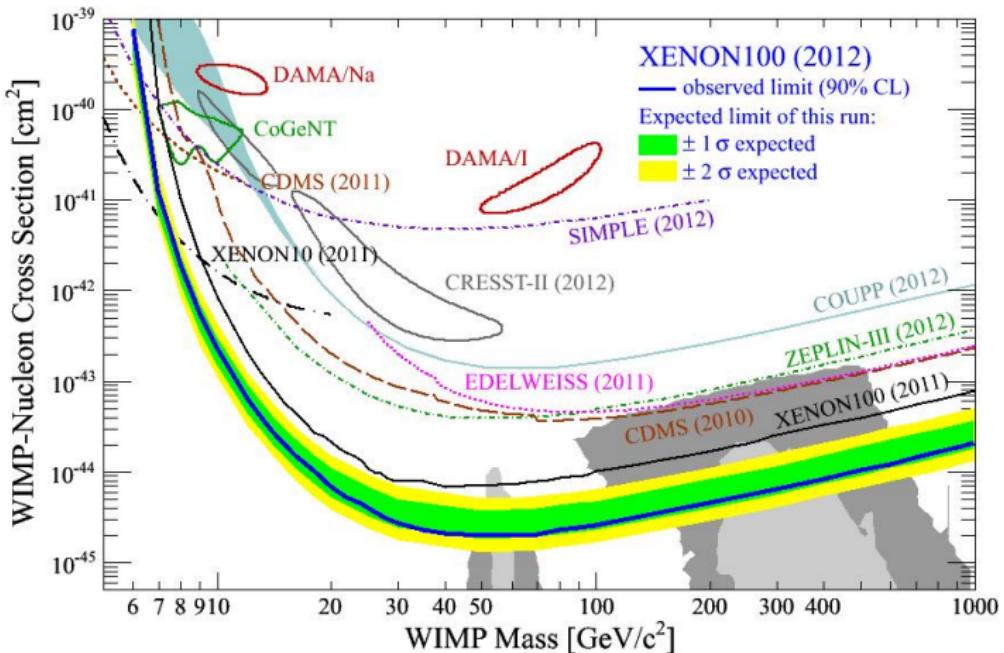
$$\Delta_{\text{tot}} \equiv \sqrt{\sum_{p_i=\mu^2, b, m_{H_u}^2, m_{H_d}^2} \{\Delta p_i\}^2}$$

Computing Method:

1. SUSY parameters are randomly chosen.
2. SUSY spectra calculations by `SuSpect`, check consistency. If not go to 1.
3. Low energy observables by `SuperIso`.
4. Relic density, Spin Independent cross sections by `micrOMEGAs`.
5. save data, go to 1.

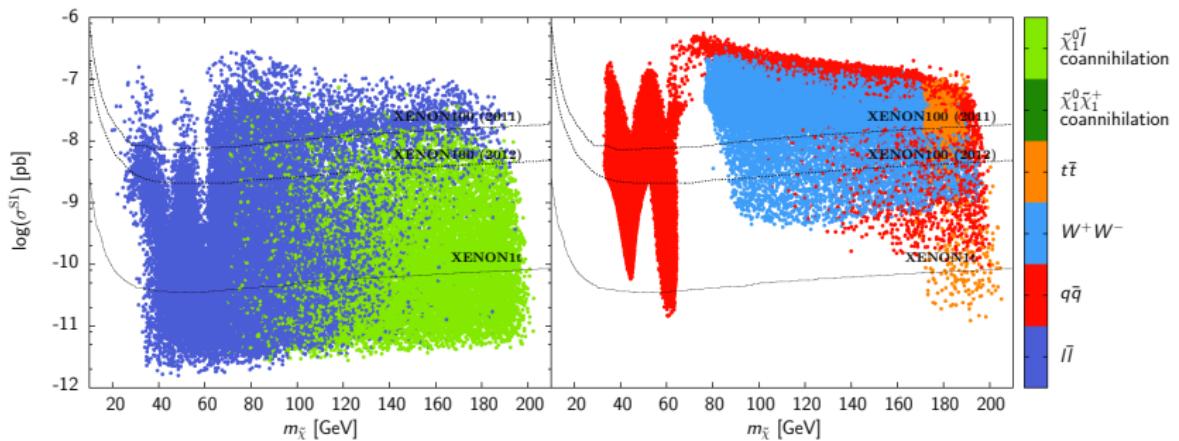


The relic density and the different (co)annihilation mechanisms.
All experimental constraints are applied. $\Omega h^2 \in [0.089, 0.136]$.

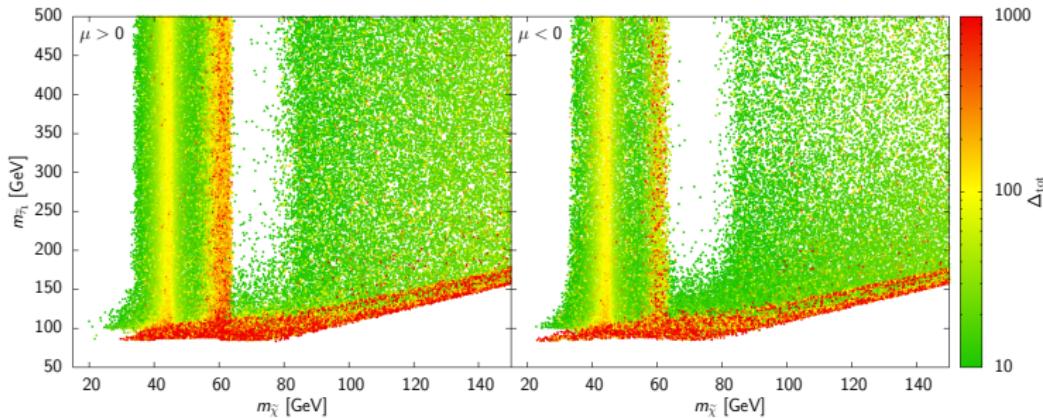


XENON 100 (2012) data

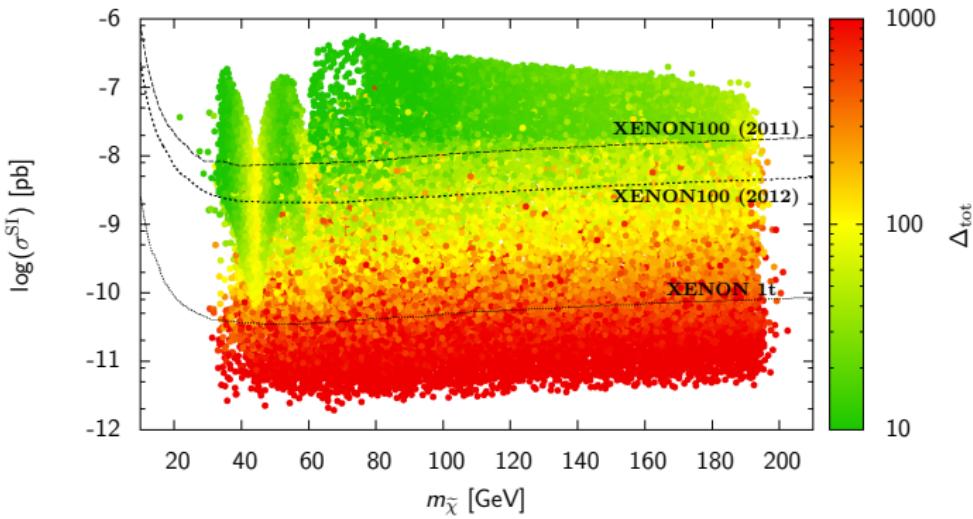
[E. Aprile *et al.* [XENON100 Collaboration], arXiv:1207.5988]



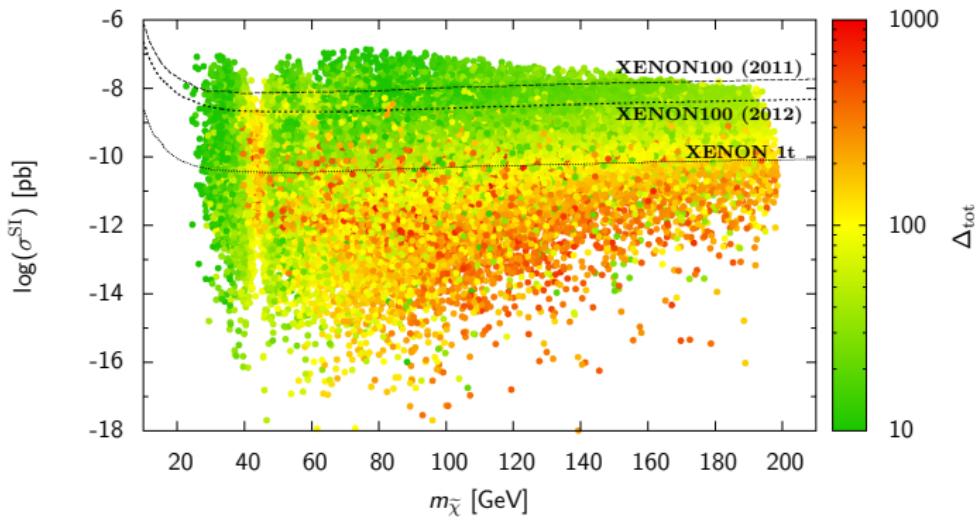
Dominant contribution to the neutralino annihilation in the $m_{\tilde{\chi}} - \sigma^{\text{SI}}$ plane ($\mu > 0$).



stau mass vs neutralino mass, and their level of fine-tuning:
Left: $\mu > 0$ and Right: $\mu < 0$.



Fine-tuning and spin indep. cross section [$\text{pb} = 10^{-36} \text{ cm}^2$].
 $\mu > 0$ with $(g - 2)_\mu$ constraint.



Fine-tuning and spin indep. cross section [$\text{pb} = 10^{-36} \text{ cm}^2$].
 $\mu < 0$ with $(g-2)_\mu$ constraint.

Why is σ^{SI} smaller for a negative μ -term?

$$\sigma^{\text{SI}} \propto \left[\frac{F_h I_h}{m_h^2} + \frac{F_H I_H}{m_H^2} \right]^2$$

where $I_{h,H}$ are functions of matrix elements and

$$F_h \equiv (-N_{11} \sin \theta_W + N_{12} \cos \theta_W) (N_{13} \sin \alpha + N_{14} \cos \alpha)$$

$$F_H \equiv (-N_{11} \sin \theta_W + N_{12} \cos \theta_W) (N_{13} \cos \alpha - N_{14} \sin \alpha)$$

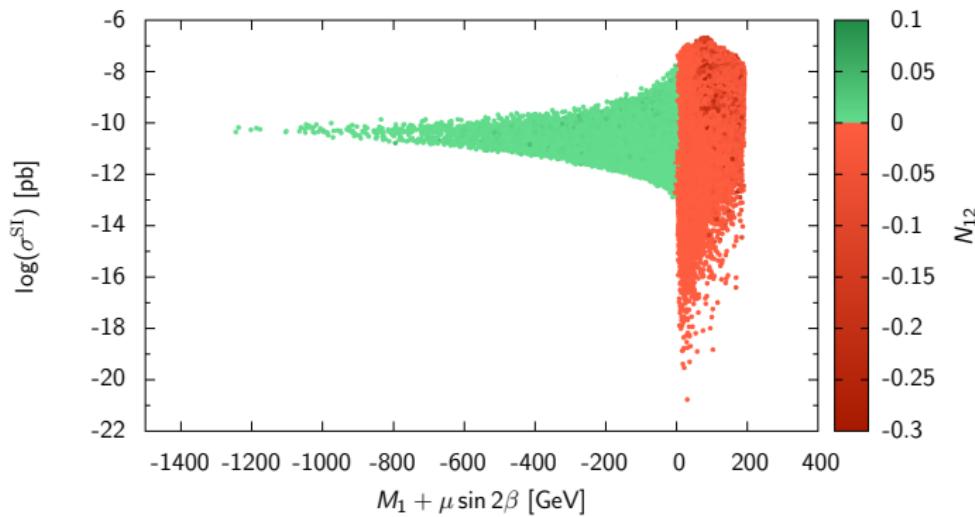
If we use approximation $(M_1 + |\mu|)^2 \gg M_Z^2$ and $[\alpha \sim \beta + \pi/2]$

$$N_{11} = \sqrt{1 - N_{12}^2 + N_{13}^2 + N_{14}^2}$$

$$N_{12} \simeq -M_Z^2 \cos \theta_W \sin \theta_W \frac{M_1 + \mu \sin 2\beta}{(M_1 - M_2)(M_1^2 - \mu^2)}$$

$$N_{13} \simeq -M_Z \sin \theta_W \frac{M_1 \cos \beta + \mu \sin \beta}{M_1^2 - \mu^2}$$

$$N_{14} \simeq M_Z \sin \theta_W \frac{M_1 \sin \beta + \mu \cos \beta}{M_1^2 - \mu^2}$$

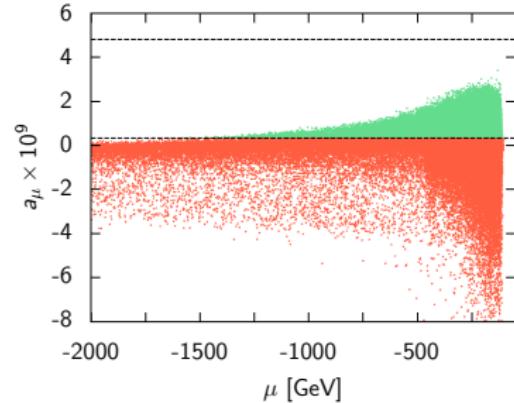
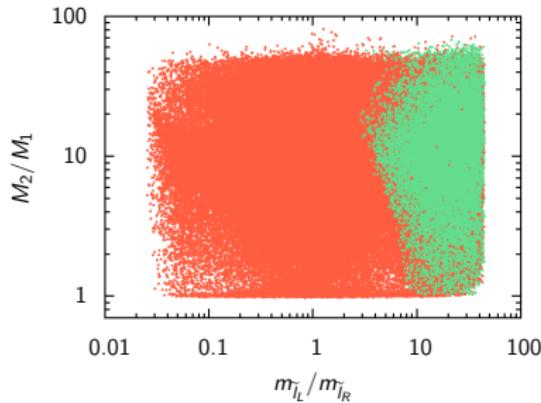


$$\sigma^{\text{SI}} \propto \left(\frac{I_H}{m_H^2} \mu \cos 2\beta + \frac{I_h}{m_h^2} (\textcolor{blue}{M}_1 + \mu \sin 2\beta) \right)^2$$

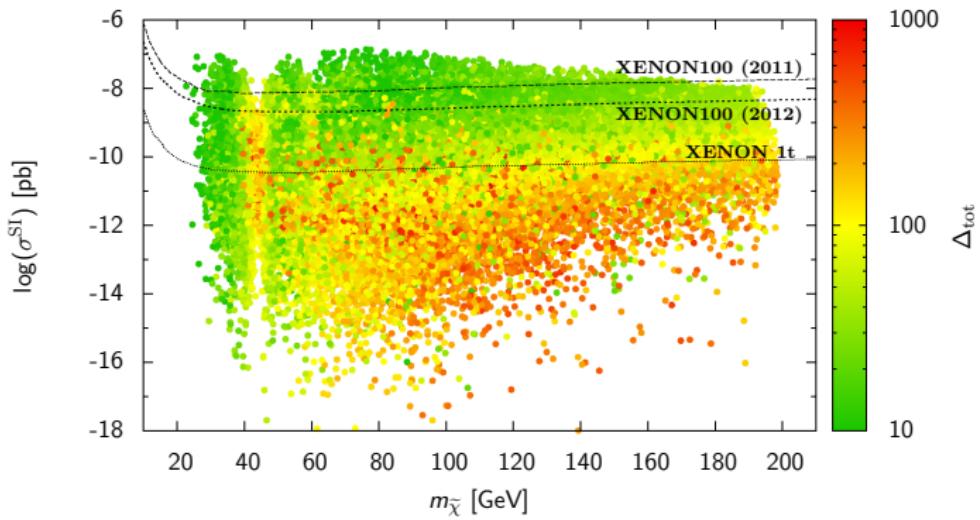
How to fulfill $(g - 2)_\mu$ with a negative μ -term?

$$\begin{aligned} a_\mu(\widetilde{W} - \widetilde{H}, \widetilde{\nu}_\mu) &= \frac{g^2}{8\pi^2} \frac{m_\mu^2 M_2 \mu \tan \beta}{m_{\widetilde{\nu}}^4} F_a \left(\frac{M_2^2}{m_{\widetilde{\nu}}^2}, \frac{\mu^2}{m_{\widetilde{\nu}}^2} \right) \\ a_\mu(\widetilde{B}, \widetilde{\mu}_L - \widetilde{\mu}_R) &= \frac{g'^2}{8\pi^2} \frac{m_\mu^2 M_1 \mu \tan \beta}{M_1^3} F_b \left(\frac{m_{\widetilde{\mu}_L}^2}{M_1^2}, \frac{m_{\widetilde{\mu}_R}^2}{M_1^2} \right) \\ a_\mu(\widetilde{B} - \widetilde{H}, \widetilde{\mu}_R) &= -\frac{g'^2}{8\pi^2} \frac{m_\mu^2 M_1 \mu \tan \beta}{m_{\widetilde{\mu}_R}^4} F_b \left(\frac{M_1^2}{m_{\widetilde{\mu}_R}^2}, \frac{\mu^2}{m_{\widetilde{\mu}_R}^2} \right) \end{aligned}$$

F_a and F_b are **positive** functions. To get positive pull for $\mu < 0$, Bino-higgsino-loop ($a_\mu(\widetilde{B} - \widetilde{H}, \widetilde{\mu}_R)$) must be dominant.



The green area satisfies $(g-2)_\mu$. $m_{\tilde{L}} \gg m_{\tilde{R}}$ and $|\mu| \ll 1.5$ TeV is necessary.



Fine-tuning and spin indep. cross section [$\text{pb} = 10^{-36} \text{ cm}^2$].
 $\mu < 0$ with $(g-2)_\mu$ constraint.

Conclusions

- ▶ We studied the pMSSM and its compatibility with cosmology, XENON 100 (2011), and (new) LHC results plus flavour physics.
- ▶ New limits on squarks and gluinos don't increase the tension on the Dark Matter sector of the pMSSM.
- ▶ Light sleptons may play a crucial role during Dark Matter freeze-out.
- ▶ A negative μ -term is favoured from a Fine-Tuning perspective and the suppression of σ^{SI} .
- ▶ There is no problem to get a positive shift in a_μ with a negative μ -term.

Backup

Fine-tuning:

$$\Delta\mu^2 = \frac{4\mu^2}{m_Z^2} \left(1 + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right) ,$$

$$\Delta b = \left(1 + \frac{m_A^2}{m_Z^2} \right) \tan^2 2\beta ,$$

$$\begin{aligned}\Delta m_{H_u}^2 &= \left| \frac{1}{2} \cos 2\beta + \frac{m_A^2}{m_Z^2} \cos^2 2\beta - \frac{\mu^2}{m_Z^2} \right| \\ &\quad \times \left(1 - \frac{1}{\cos 2\beta} + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right) ,\end{aligned}$$

$$\begin{aligned}\Delta m_{H_d}^2 &= \left| -\frac{1}{2} \cos 2\beta + \frac{m_A^2}{m_Z^2} \sin^2 2\beta - \frac{\mu^2}{m_Z^2} \right| \\ &\quad \times \left(1 + \frac{1}{\cos 2\beta} + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right) .\end{aligned}$$

$$\sigma^{\text{SI}} \simeq \frac{8G_F^2}{\pi} M_Z^2 m_{\text{red}}^2 \left[\frac{F_h I_h}{m_h^2} + \frac{F_H I_H}{m_H^2} \right]^2 ,$$

G_F is Fermi constant, m_{red} is neutralino-nucleon reduced mass.
The functions $F_{h,H}$ and $I_{h,H}$ are

$$F_h \equiv (-N_{11} \sin \theta_W + N_{12} \cos \theta_W) (N_{13} \sin \alpha + N_{14} \cos \alpha)$$

$$F_H \equiv (-N_{11} \sin \theta_W + N_{12} \cos \theta_W) (N_{13} \cos \alpha - N_{14} \sin \alpha)$$

$$I_{h,H} \equiv \sum_q k_q^{h,H} m_q \langle N | \bar{q} q | N \rangle$$

α is the mixing angle of the mass eigenstates (h and H).

The coefficients $k_q^{h,H}$:

$$k_{u\text{-type}}^h = \cos \alpha / \sin \beta , \quad k_{d\text{-type}}^h = -\sin \alpha / \cos \beta ,$$
$$k_{u\text{-type}}^H = -\sin \alpha / \sin \beta , \quad k_{d\text{-type}}^H = -\cos \alpha / \cos \beta ,$$

$$N_{12} \simeq -M_Z^2 \cos \theta_W \sin \theta_W \frac{M_1 + \mu \sin 2\beta}{(M_1 - M_2)(M_1^2 - \mu^2)}$$

$$N_{13} \simeq -M_Z \sin \theta_W \frac{M_1 \cos \beta + \mu \sin \beta}{M_1^2 - \mu^2}$$

$$N_{14} \simeq M_Z \sin \theta_W \frac{M_1 \sin \beta + \mu \cos \beta}{M_1^2 - \mu^2}$$

Using unitary condition we get

$$\sigma^{\text{SI}} \simeq \frac{8G_F^2}{\pi} m_{\text{red}}^2 \frac{M_Z^4 \sin^2 \theta_W}{(M_1^2 - \mu^2)^2} \left(\frac{I_H}{m_H^2} \mu \cos 2\beta + \frac{I_h}{m_h^2} (M_1 + \mu \sin 2\beta) \right)^2$$
$$\times (N_{11} \sin \theta_W - N_{12} \cos \theta_W)^2$$

$$(a) \ a_\mu(\tilde{W}\text{-}\tilde{H}, \tilde{\nu}_\mu) = \frac{g^2}{8\pi^2} \frac{m_\mu^2 M_2 \mu \tan \beta}{m_{\tilde{\nu}}^4} F_a \left(\frac{M_2^2}{m_{\tilde{\nu}}^2}, \frac{\mu^2}{m_{\tilde{\nu}}^2} \right)$$

$$(b) \ a_\mu(\tilde{B}, \tilde{\mu}_L\text{-}\tilde{\mu}_R) = \frac{g'^2}{8\pi^2} \frac{m_\mu^2 \mu \tan \beta}{M_1^3} F_b \left(\frac{m_{\tilde{\mu}_L}^2}{M_1^2}, \frac{m_{\tilde{\mu}_R}^2}{M_1^2} \right)$$

$$(c) \ a_\mu(\tilde{B}\text{-}\tilde{H}, \tilde{\mu}_L) = \frac{g'^2}{16\pi^2} \frac{m_\mu^2 M_1 \mu \tan \beta}{m_{\tilde{\mu}_L}^4} F_b \left(\frac{M_1^2}{m_{\tilde{\mu}_L}^2}, \frac{\mu^2}{m_{\tilde{\mu}_L}^2} \right)$$

$$(d) \ a_\mu(\tilde{W}\text{-}\tilde{H}, \tilde{\mu}_L) = -\frac{g^2}{16\pi^2} \frac{m_\mu^2 M_2 \mu \tan \beta}{m_{\tilde{\mu}_L}^4} F_b \left(\frac{M_2^2}{m_{\tilde{\mu}_L}^2}, \frac{\mu^2}{m_{\tilde{\mu}_L}^2} \right)$$

$$(e) \ a_\mu(\tilde{B}\text{-}\tilde{H}, \tilde{\mu}_R) = -\frac{g'^2}{8\pi^2} \frac{m_\mu^2 M_1 \mu \tan \beta}{m_{\tilde{\mu}_R}^4} F_b \left(\frac{M_1^2}{m_{\tilde{\mu}_R}^2}, \frac{\mu^2}{m_{\tilde{\mu}_R}^2} \right)$$

The positive defined functions $F_a(x, y)$ and $F_b(x, y)$:

$$F_a(x, y) \equiv -\frac{G_3(x) - G_3(y)}{x - y} \quad F_b(x, y) \equiv -\frac{G_4(x) - G_4(y)}{x - y}$$

which are symmetric under exchange of the two arguments.

$$G_3(x) = \frac{1}{2(x-1)^3} [(x-1)(x-3) + 2 \ln x]$$

$$G_4(x) = \frac{1}{2(x-1)^3} [(x-1)(x+1) - 2x \ln x]$$

