Quantifying phases in homogeneous twisted birefringent medium

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Introduction

.Theory of twisted birefringent material (liquid crystal) was developed by Gennes and Chandrashekhar.

.Three kinds of twist pitch exist-short, comparable and **very long** with respect to optical wavelength.

We consider **very long pitch-Maguin limit**/ GOA.

.The property of birefringence develops quantum phases.

1. Dynamical 2. Geometrical

.Phases have origin in spin or orbital angular momentum of polarized photon**.** Last decades there are many important works in the field of angular momentum of photon.

Spinorial representation of photon

We studied GP of polarized photon in connection with **helicity** in 1994.

As light gets a fixed polarization, its **helicity** also fixes. The ± helicity implies respective right/left circular polarization.

In analogy with spin system we may suggest the coordinate of polarized photon as

$$Z^{\mu} = X^{\mu} + iY^{\mu} \rightarrow X^{AA'} + \frac{i}{2}\overline{\theta}^{A}\theta^{A'},$$

$$X^{AA'} = \frac{1}{\sqrt{2}}\begin{bmatrix} x^0 - x^1 & x^2 + ix^3 \\ x^2 - ix^3 & x^0 + x^1 \end{bmatrix} --- (1), \; |Y^{\mu}|^2 = 0$$

From the twistor equation, we can define the **helicity** operator for massless spinor

$$\overline{S} = -\overline{\theta}^{A}\theta^{A'}\overline{\pi}_{A}\pi_{A'} = -\varepsilon\overline{\varepsilon}$$

Photon with helicity $+1$ as massless spinor of helicity $+1/2$.

The commutation relation of Linear momentum and conserved angular momentum becomes

$$[p_i, p_j] = i\mu\varepsilon_{ijk}\frac{x^k}{r^3},$$

$$\overleftarrow{J} = \overleftarrow{r} \, x \, \overleftarrow{p} - \mu \overleftarrow{r} ----(2)$$

$$J^2 = L^2 - \mu^2$$

$$\mu = 0, \pm 1/2, \pm 1, \ldots$$

μ is eigenvalue of $i\hbar \, {}^{\delta}/_{\delta\chi}$ and m is the eigenvalue of $i\hbar \, {}^{\delta}/_{\delta\theta}$

The function Φ(Zμ) is holomorphic in anisotropic space and apart from r,Θ and Φ the variable χ is needed to specify **helicity.** Thus the anisotropic sphere with variable (Θ,Φ,χ) is extended.

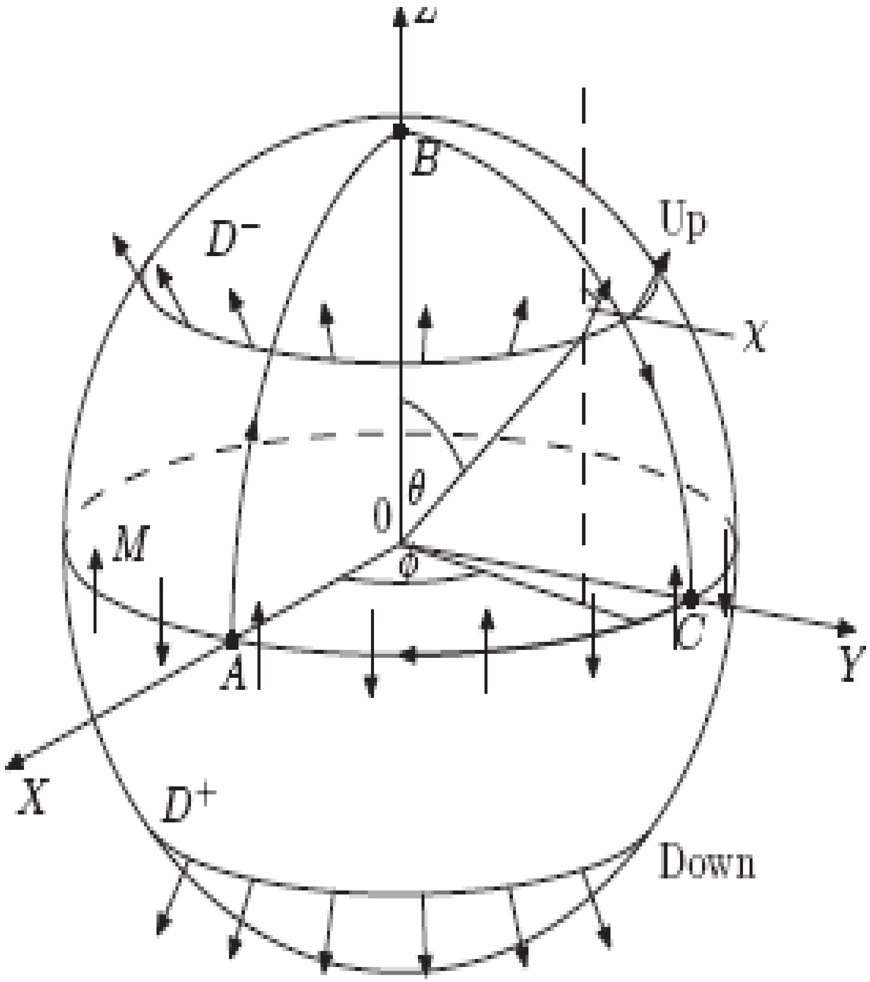
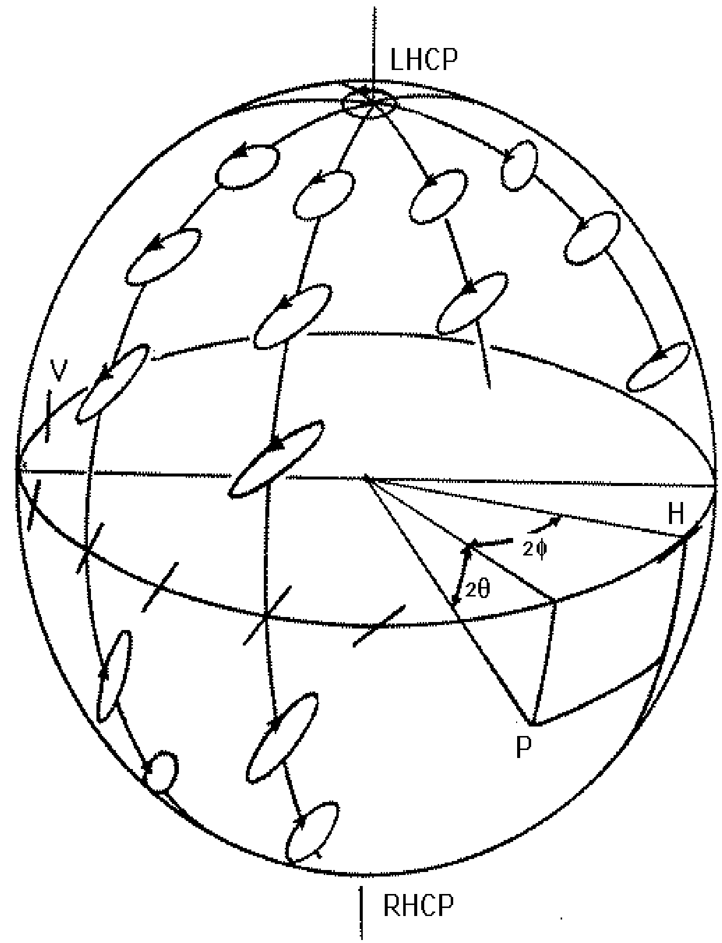


Figure 1. Geometrical interpretation of the quantized spinor.



- Incorporating μ the spherical harmonics $Y_l^{m,\mu}$
- In view of this the polarized photon may be represented in two ways

$$\varepsilon = \begin{pmatrix} Y_{1/2}^{1/2,1/2} \\ Y_{1/2}^{1/2,-1/2} \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2} e^{i(\phi-\chi)/2} \\ \cos\frac{\theta}{2} e^{i(\phi+\chi)/2} \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} e^{i\chi} \end{pmatrix} --- (3)$$

$DB\,(1997\ ,2006\ )$

$$\varepsilon = \begin{pmatrix} Y_{1/2}^{-1/2,1/2} \\ Y_{1/2}^{-1/2,-1/2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} e^{i(\phi+\chi)/2} \\ \sin\frac{\theta}{2} e^{-i(\phi-\chi)/2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} e^{i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix} --- (4)$$

$DB\,(1994\ )$

Method

The birefringence and dichroism of a medium can be described by differential matrix N. We followed the methods of Jones and Berry to find M and N.

$$N = \frac{dM}{dz} M^{-1} = \frac{dM}{d\theta}\frac{d\theta}{dz} M^{-1} \qquad ; N = \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix}$$

In a laminated crystal eight constants are paired to specify the four matrix elements of N by Linear and circular birefringence and dichroism .

- We study the pure birefringent material

$$N = \begin{bmatrix} 0 & -\tau - i\rho \\ \tau + i\rho & 0 \end{bmatrix} = \begin{bmatrix} 0 & n_2 \\ n_3 & 0 \end{bmatrix}$$

The evolution ray vector shows exchange of optical power

$$\frac{d\xi_1}{dz} = n_2 \xi_2, \frac{d\xi_2}{dz} = n_3 \xi_1,$$

The evolution of ray is the spatial change of vector as it passes through the crystal

$$\frac{d\overleftarrow{p}}{dz} = \overleftarrow{\Omega} \, x \, \overleftarrow{p},$$

$$d\alpha = \Omega \, dz$$

Here we consider long twist pitch (GOA approximation/Maguin limit)

$$d\theta = kdz$$

The twisted matrix for uniform angle of twist dependent on crystal thickness,

$$\mathrm{N}' = \mathrm{S}(\theta)\mathrm{N}_0\mathrm{S}(-\theta)$$

For angle of twist independent of crystal thickness

$$N' = N_0 - kS(\pi/2),$$

Outcome

- Here we parameterized OAM sphere by (Θ,Φ) and SAM sphere (Θ,χ). There is two to one correspondence between spin and orbital angular momentum space. The polarization matrix in SAM space

$$M(\theta,\chi)=\begin{pmatrix} -\cos\theta & \sin\theta e^{-i\chi} \\ \sin\theta e^{i\chi} & \cos\theta \end{pmatrix}---(5)$$

Here M is constructed on the sphere of Θ and Φ by M=(|ψ>< ψ|-1/2)

$$M(\theta,\phi)=\tfrac{1}{2}\begin{pmatrix} \psi_+\psi_+-1 & \psi_+\psi_- \\ \psi_-\psi_+ & \psi_+\psi_+-1 \end{pmatrix}$$

where

$$Y_1^1\approx\psi_-\psi_+\approx Y_{1/2}^{-1/2,1/2}.Y_{1/2}^{-1/2,-1/2} and Y_1^{-1}\approx\psi_+\psi_-\approx Y_{1/2}^{1/2,1/2}.Y_{1/2}^{1/2,-1/2}$$

$$Y_1^{0} \approx (\psi_+\psi_+ -1) or (1-\psi_-\psi_-)$$

$$\approx Y_{1/2}^{1/2,1/2} . Y_{1/2}^{-1/2,-1/2} - Y_{1/2}^{-1/2,1/2} . Y_{1/2}^{1/2,-1/2}$$

$$M = \tfrac{1}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & -\cos\theta \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} Y_1^{0} & Y_1^{1} \\ Y_1^{-1} & Y_1^{0} \end{pmatrix} ----(6)$$

Here the polarization matrix constructed from spinor in eq.(4) lie in the OAM space for l=1.

Higher orbital angular momentum gives the polarization matrix in different OAM sphere.

The birefringent medium becomes

$$N(\theta,\phi)=\eta\begin{pmatrix}0 & -e^{i\phi}\\ e^{-i\phi} & 0\end{pmatrix}--(7)$$

possessing circular and linear birefringence by ηcosΦ and −ηsinΦ. The eigenvalues±iη represent the internal birefringence and eigenvectors

$$\varepsilon=\begin{pmatrix}\pm ie^{i\phi}\\ 1\end{pmatrix}$$

When the angle of twist has dependence on the crystal thickness by Θ=kz, the twisted matrix N' and twisted ray becomes will be

$$N'=\begin{pmatrix}0 & -\eta e^{i\phi}+k\\ \eta e^{-i\phi}-k & 0\end{pmatrix}--(8) \qquad \varepsilon'=\begin{pmatrix}ie^{i\phi}\cos\theta+\sin\theta\\ -ie^{i\phi}\sin\theta+\cos\theta\end{pmatrix}$$

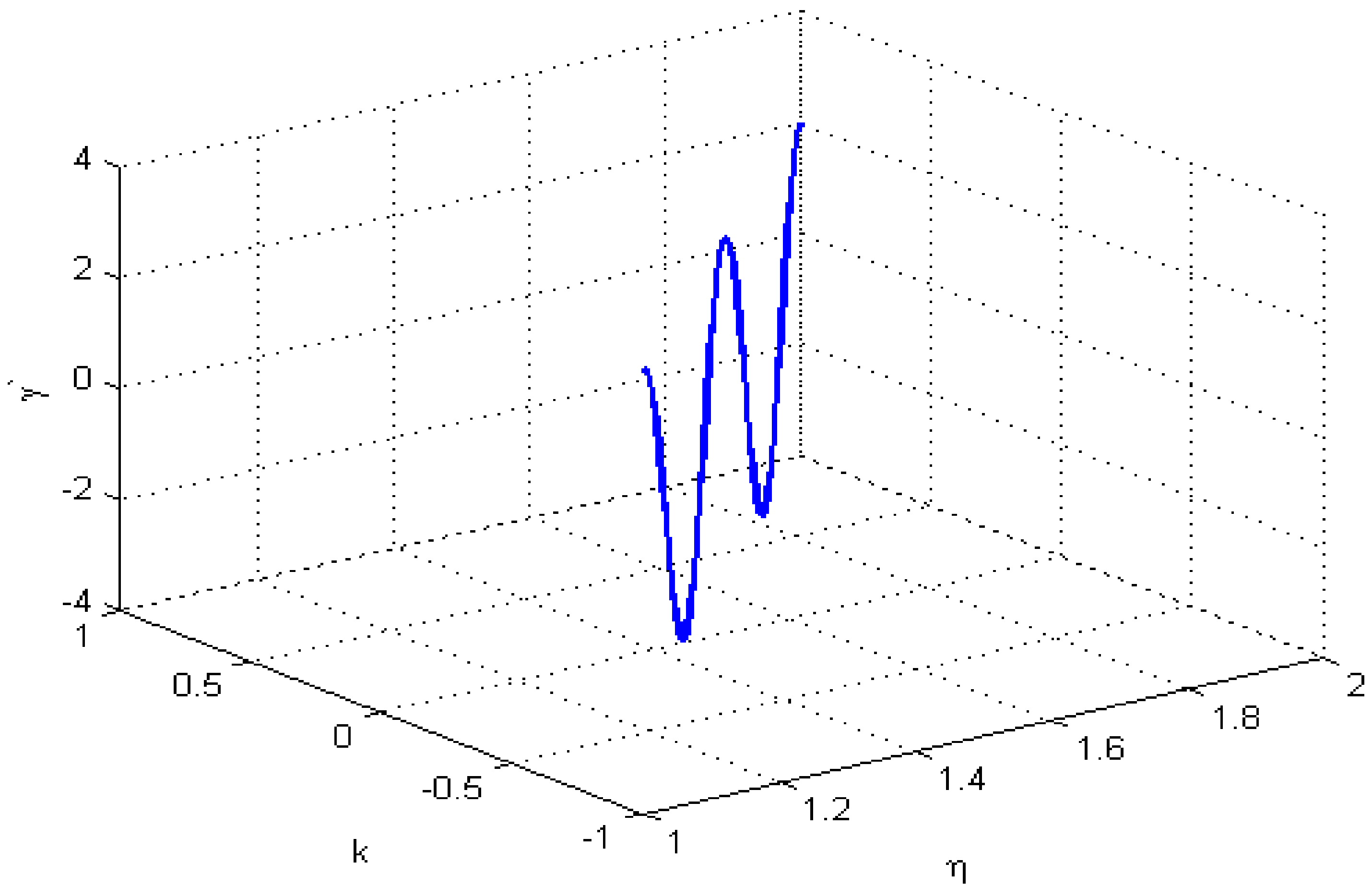
In birefringent medium (N) the dynamical phase γ varies uniformly with **η.-**

$$\gamma = \varepsilon^{*} N\varepsilon = 2i\eta --(9)$$

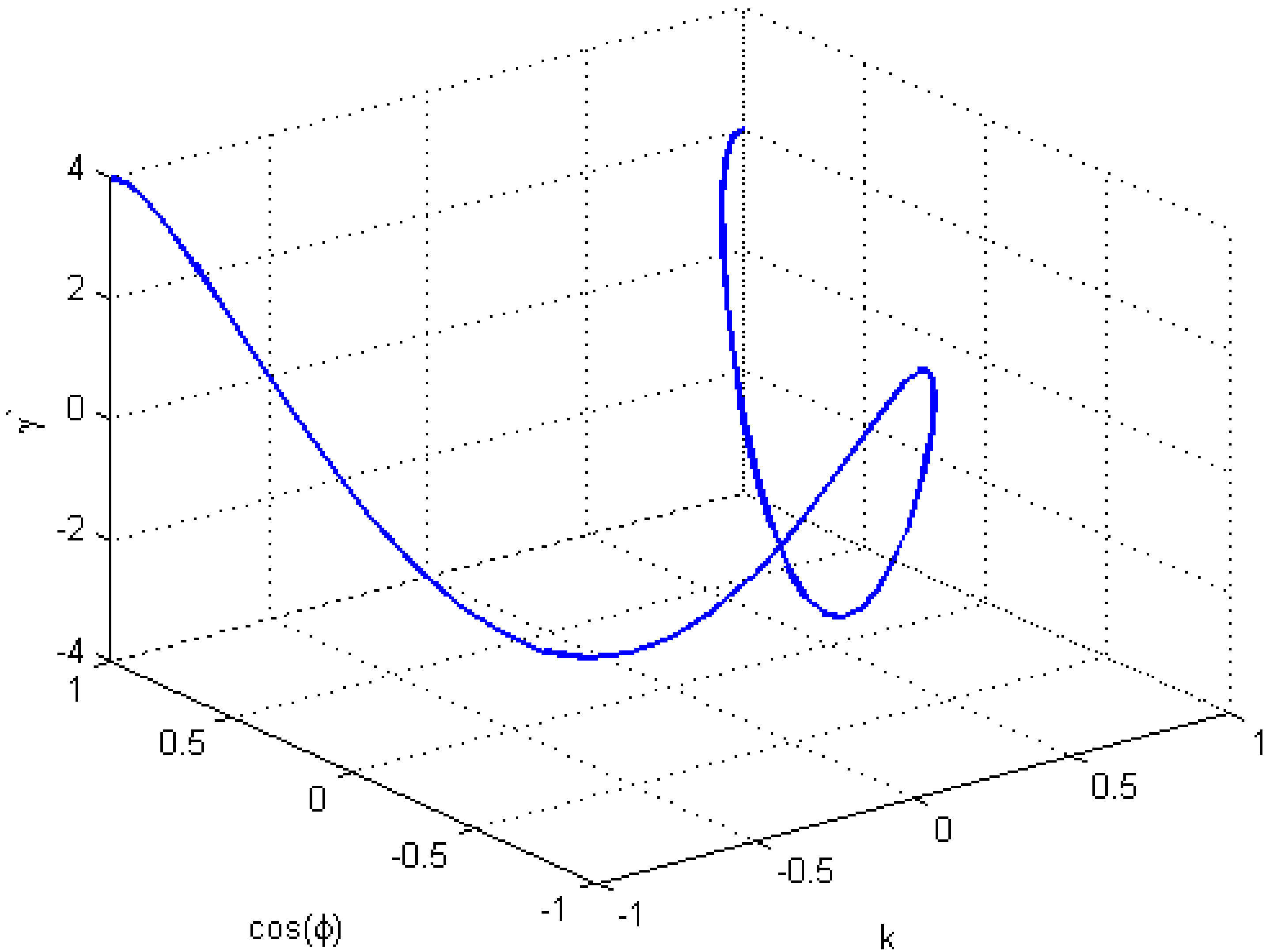
External twist for dθ=dθ' develops the phase γ' that contain both dynamical and geometric part.

$$\gamma' = \varepsilon_1^{'*} n_2^{'} \varepsilon_2^{'} + \varepsilon_2^{'*} n_3^{'} \varepsilon_1^{'} = 2i[\eta \sin^2\theta \cos 2\phi - k\cos\phi]$$

$$\gamma'_{\pi/2} = 2i[\eta\cos 2\phi - k\cos\phi] ------(10)$$



Variation of total phase γ’ with ηand k



Variation of total phase γ' with cosφ and k

The Geometric phase could be isolated by elimination **(γ’-γ)=Γ**

.

$$\Gamma = i\eta[(1-\cos 2\theta)\cos 2\phi - 2] - 2ik\cos\phi$$

At normal incidence θ=π/2, the GP will be

$$\Gamma_{\pi/2} = 2i[\eta(\cos 2\phi - 1) - k\cos\phi] --- (11)$$

At θ=0, the geometric phase becomes

$$\Gamma_1 = -2i[\eta + k\cos\phi] --- (12)$$

Graphical analysis shows that the presence of two birefringences introduces a spiral behavior in the geometric phase visualized in OAM space.

The passage of circularly polarized light also studied.

The |L> and |R> states gives rise to the dynamical phases

$$\gamma_L =< L \,|\, N \,|\, L >= -2i\eta\cos\phi$$

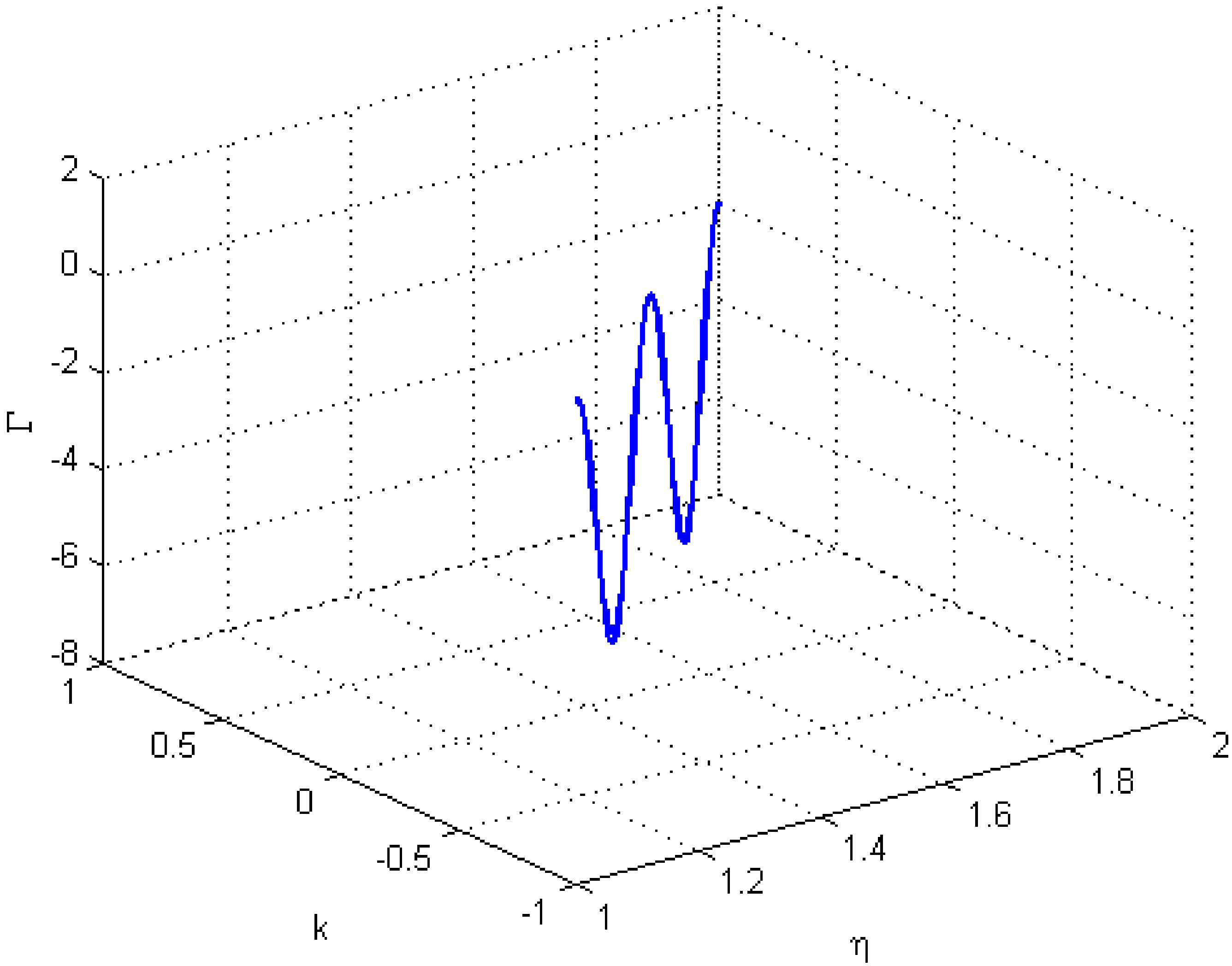
$$\gamma_R =< R \,|\, N \,|\, R >= 2i\eta\cos\phi$$

For twisted birefringent crystal the total phase γ’ for |L> becomes

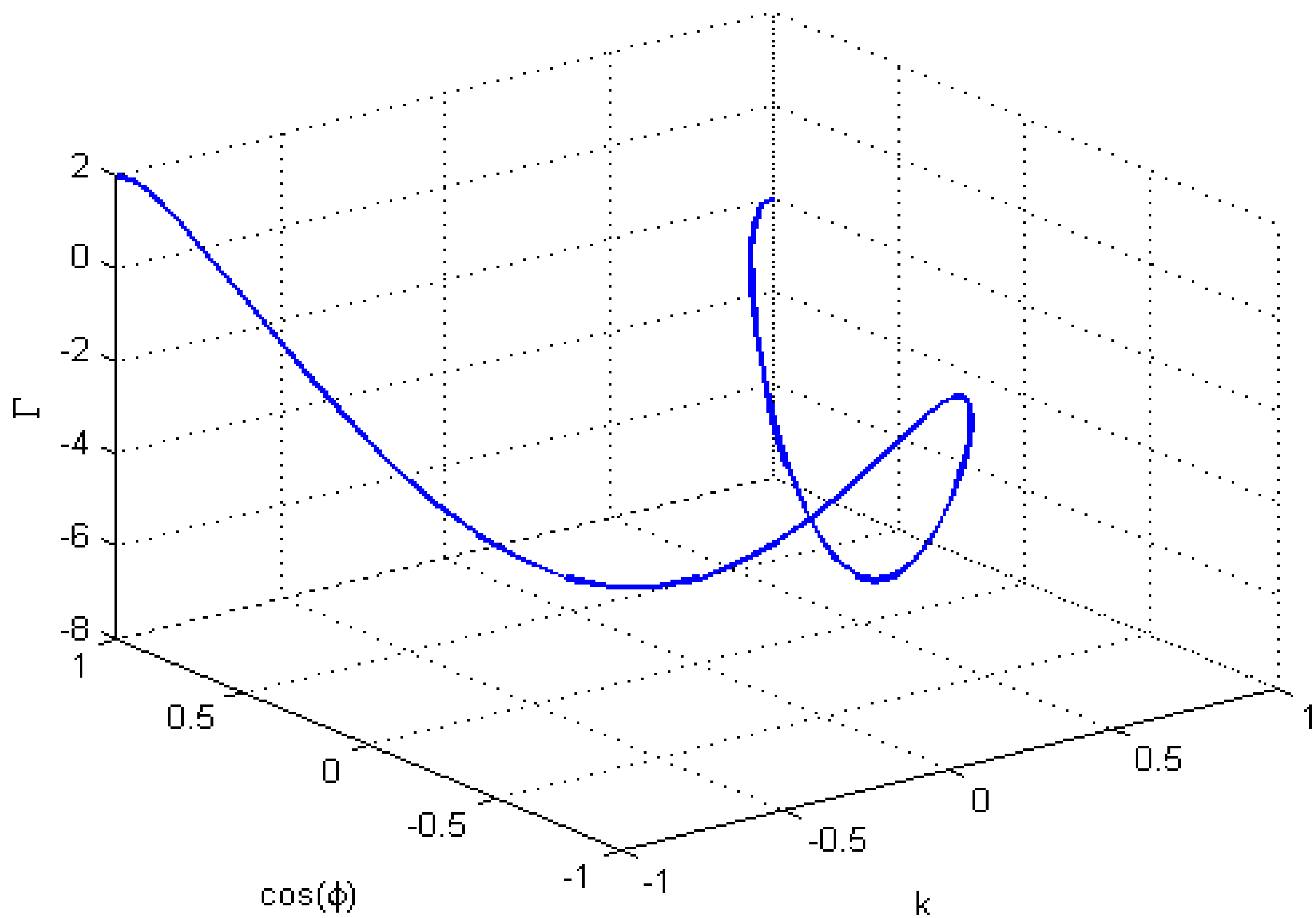
$$\gamma'_L =< L \,|\, N' |\, L >= 2ik - 2i\eta\cos\phi$$

$$\Gamma = \gamma' - \gamma = 2\mathrm{ik} ----(13)$$

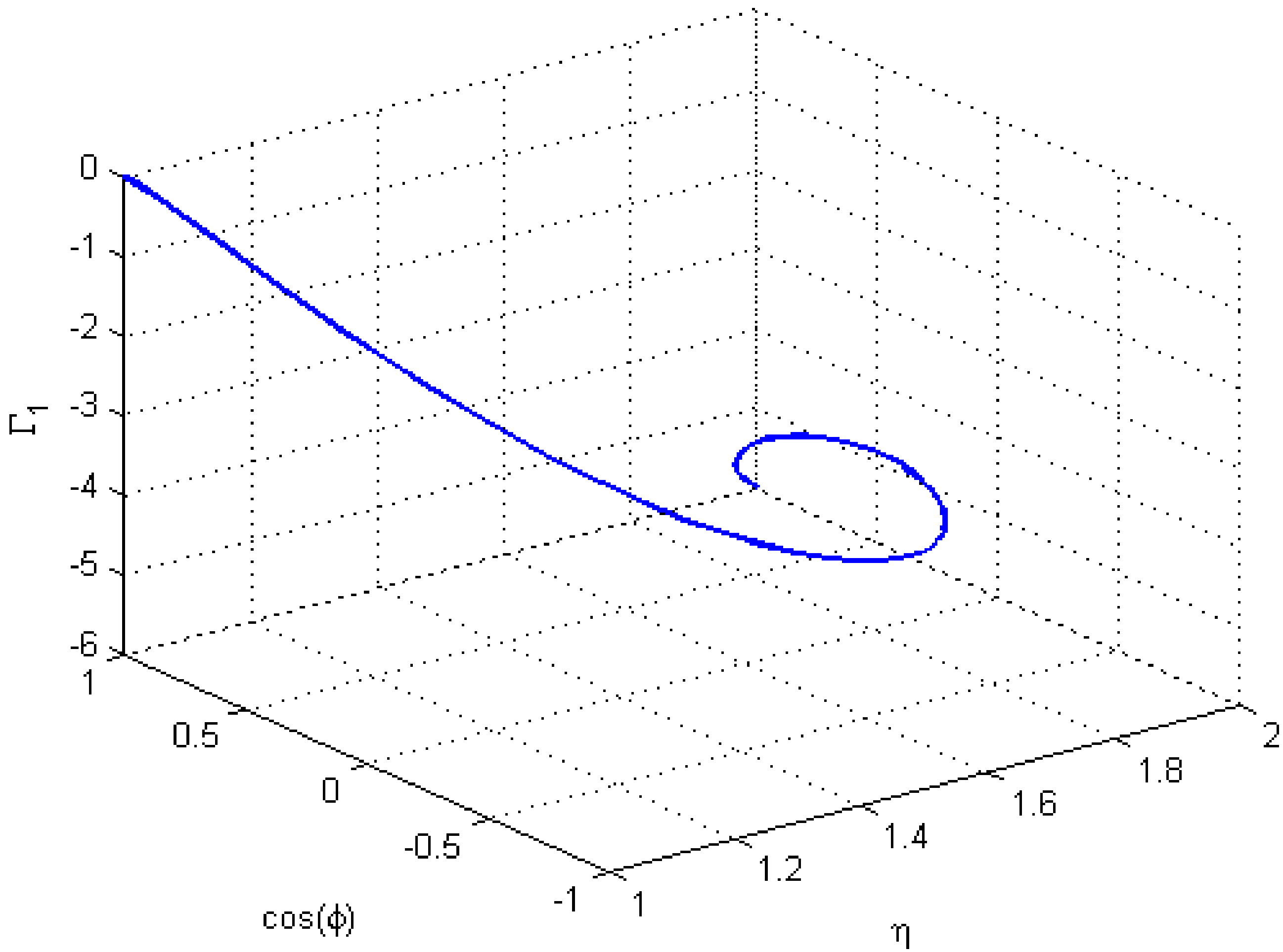
Here the geometric phase depends only on the external birefringence k of the medium.



Variation of GP-Γ with η and k



Variation of GP-Γ with η and k



Variation of GP(0) with η and φ

Future Interest

- 1.To study the twist dependent on the thickness of the medium
- in connection with STOC.

- 2.Non-uniform twist

- 3. To study the other properties of the laminated crystal.

- 4.To study the optically active medium (natural & induced) medium
- form the view point of angular momentum transfer and quantum entanglement.

- 5.This study could be fruitfully applied in Chemistry and Biology.

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Thank You