University of Glasgow | College of Science & Engineering

The Abdus Salam International Centre for Theoretical Physics (ICTP)

Light in a Twist:
Orbital Angular Momentum

Miles Padgett
Kelvin Chair of Natural Philosophy

EPSRC
CELEBRATE 350 YEARS THE ROYAL SOCIETY
DARPA
The Leverhulme Trust

The talk today

- Orbital Angular Momentum, what is it?
- What has been done with OAM
- A couple of example of what we have done and doing!

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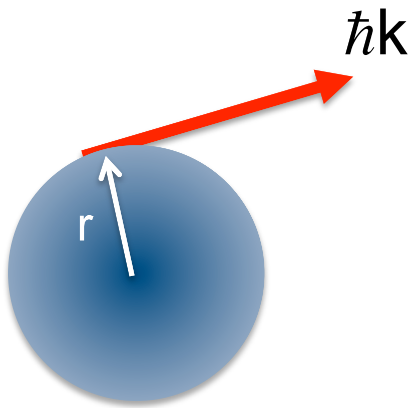
A question

- A photon carries a spin angular momentum of $\hbar$
- So how does a multi-pole transition ($\Delta J > \hbar$) conserve angular momentum?

Linear momentum at a radius exerts a torque



$\hbar k$ x r-> multipole transition

PROCEEDINGS OF THE ROYAL SOCIETY A | MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Notes on the Theory of Radiation

C. G. Darwin

Proc. R. Soc. Lond. A 1932 **136**, 36-52

Providing the lever is long enough, a fixed linear momentum can exert an arbitrary high torque

Getting started on Orbital Angular Momentum of Light

- 1992, Allen, Beijersbergen, Spreeuw and Woerdman

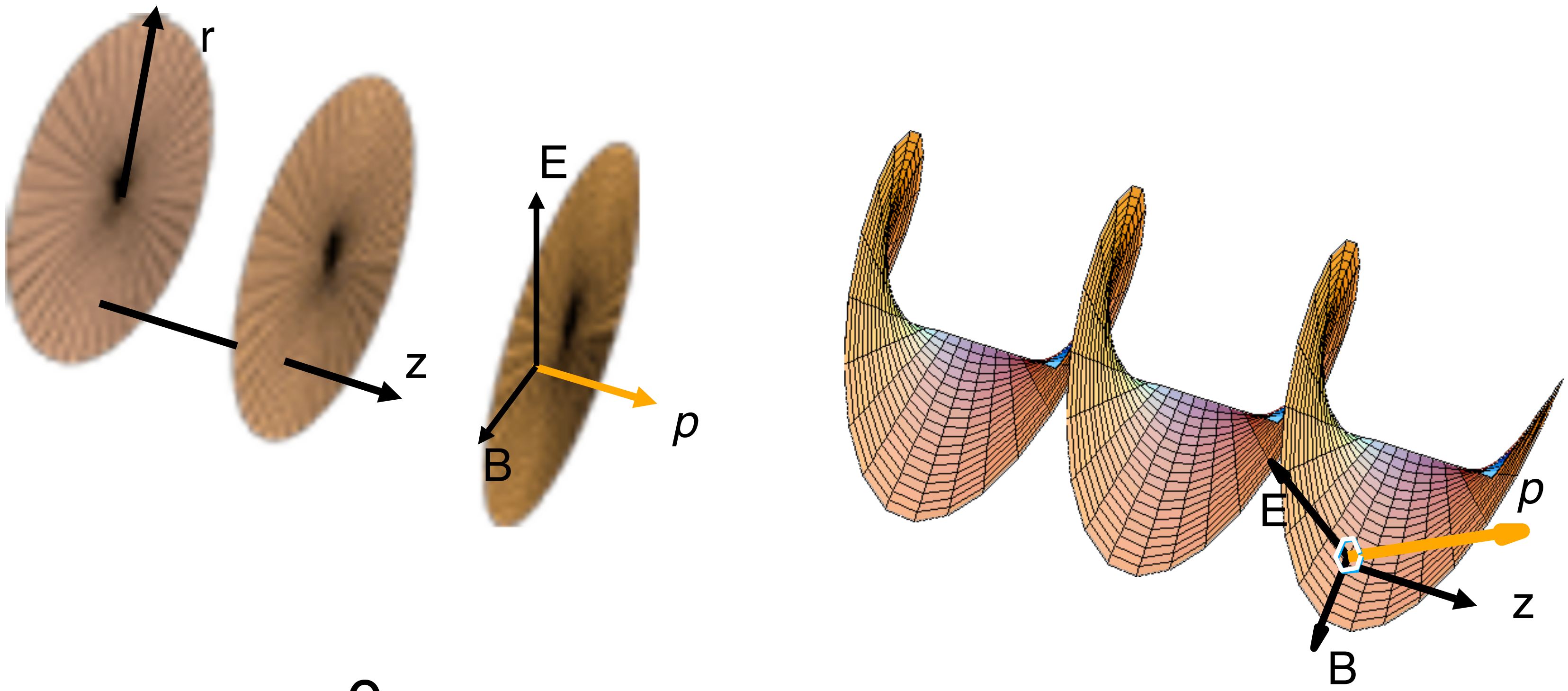
PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

- 1994, Les meets Miles at dinner…...

Orbital Angular Momentum from helical phase fronts



$p_\theta = 0$

$p_\theta \neq 0$

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Angular momentum in terms of photons

- Spin angular momentum
  - Circular polarisation
  - $\sigma\hbar$ per photon
- Orbital angular momentum
  - Helical phasefronts
  - $\ell\hbar$ per photon

$\sigma = +1$

$\sigma = -1$

$\ell = 0$ $\ell = 1$ $\ell = 2$ $\ell = 3$ etc

Optical vortices, Helical phasefronts , Angular momentum

- Intensity, $I \geq 0$
- Phase, $2\pi \geq \phi \geq 0$

  ℓ = 0, plane wave

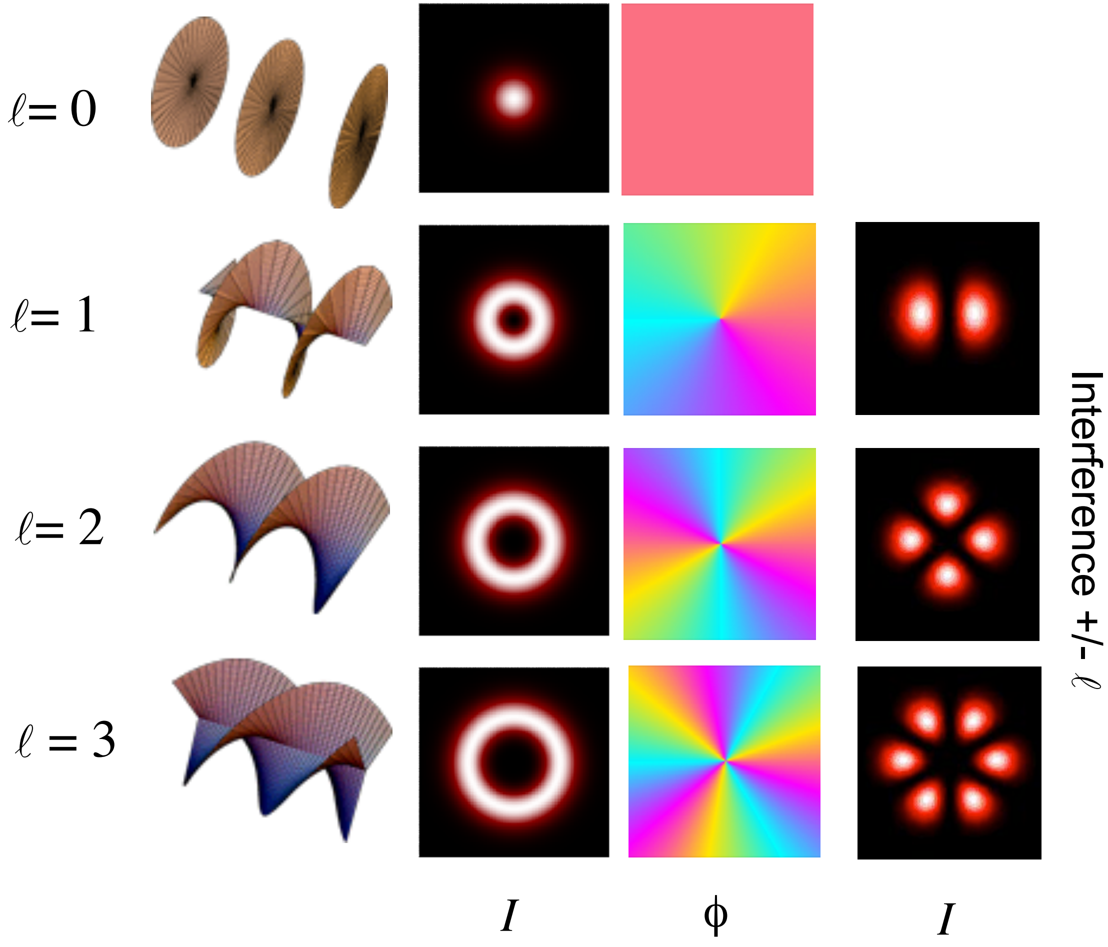
  ℓ = 1, helical wave

  ℓ = 2, double helix

  ℓ = 3, pasta fusilli

  etc.

ℓ= vortex charge



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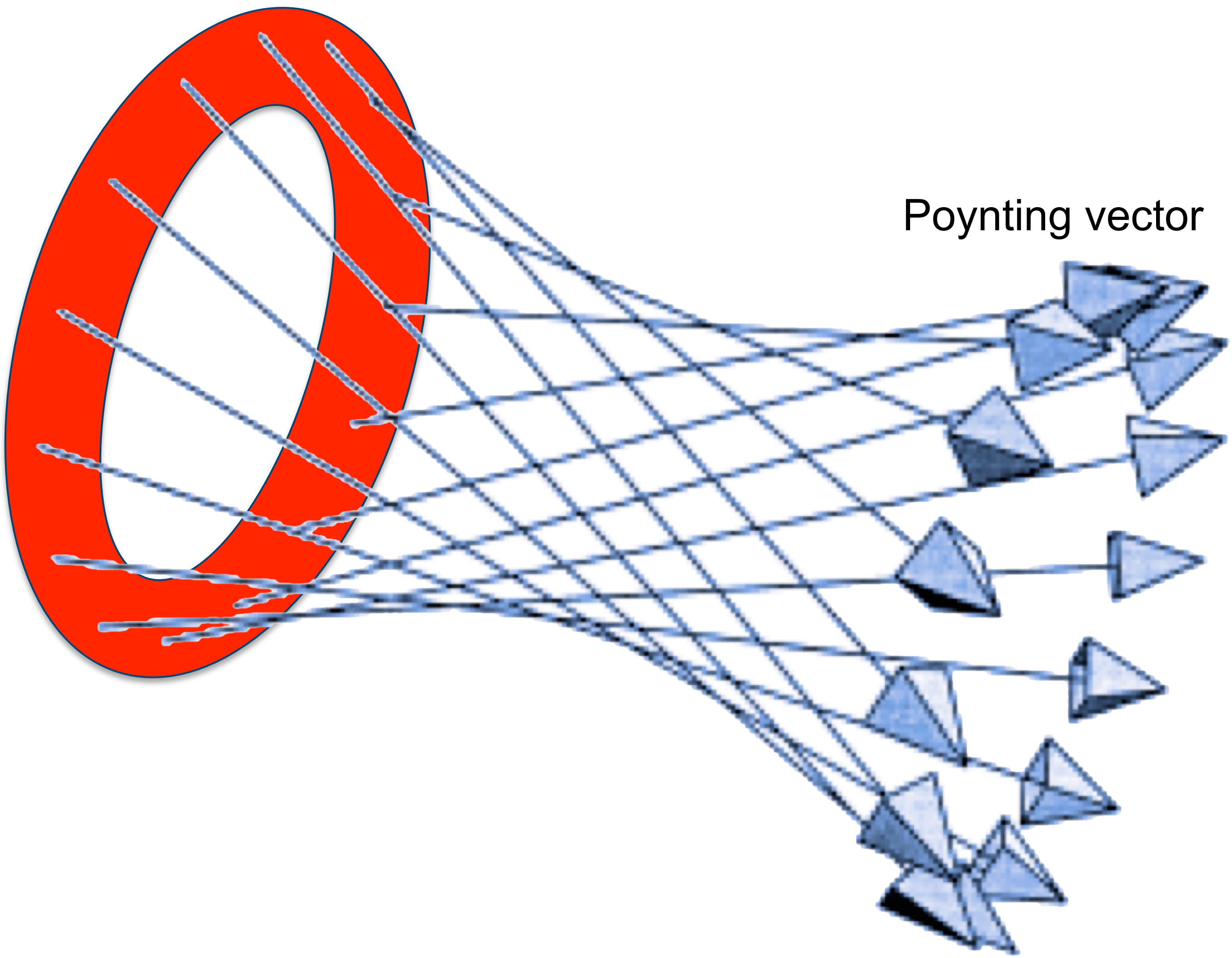
Orbital angular momentum from Skew rays



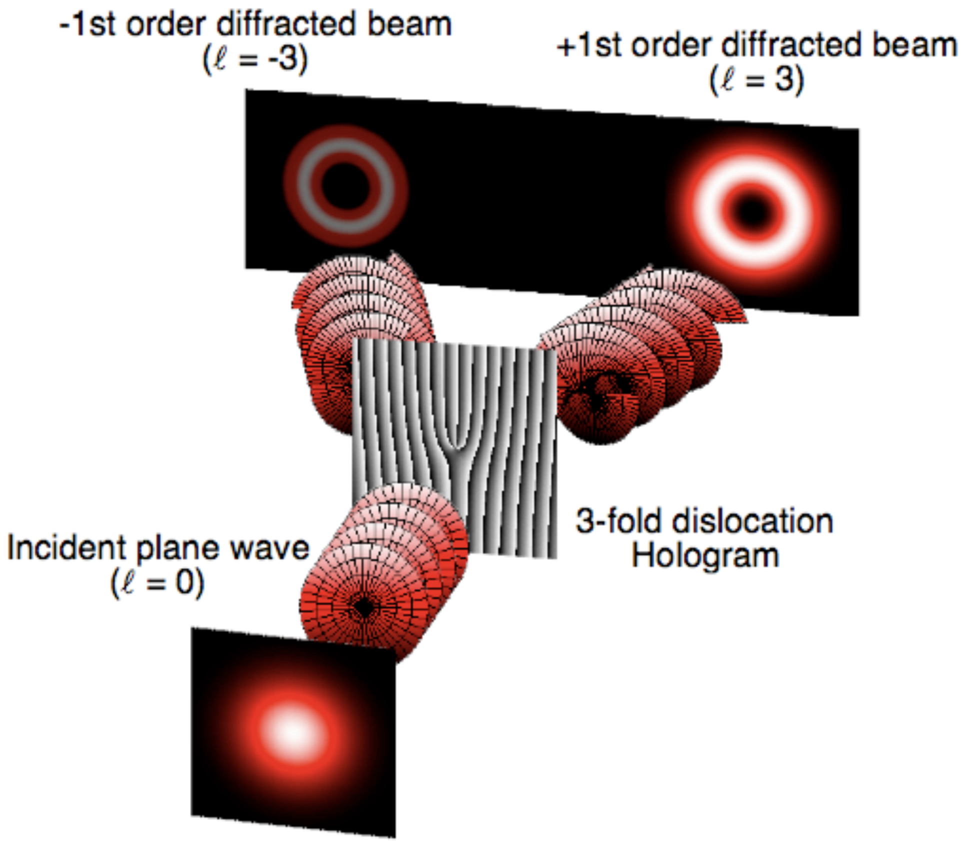
Making helical phasefronts with holograms

Screw dislocations in light wavefronts

V. YU. BAZHENOV, M. S. SOSKIN and M. V. VASNETSOV
Institute of Physics, Academy of Sciences of Ukraine,
252650 Kiev, Prospect Nauki 46, Ukraine

(Received 14 June 1991; revision received 8 January 1992)

JOURNAL OF MODERN OPTICS, 1992, VOL. 39, NO. 5, 985–990



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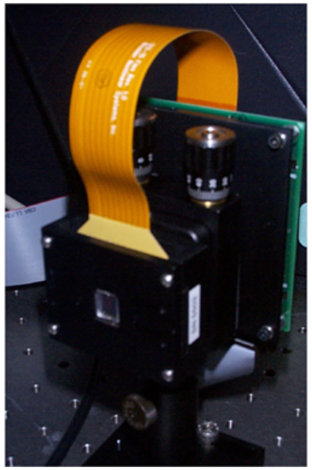
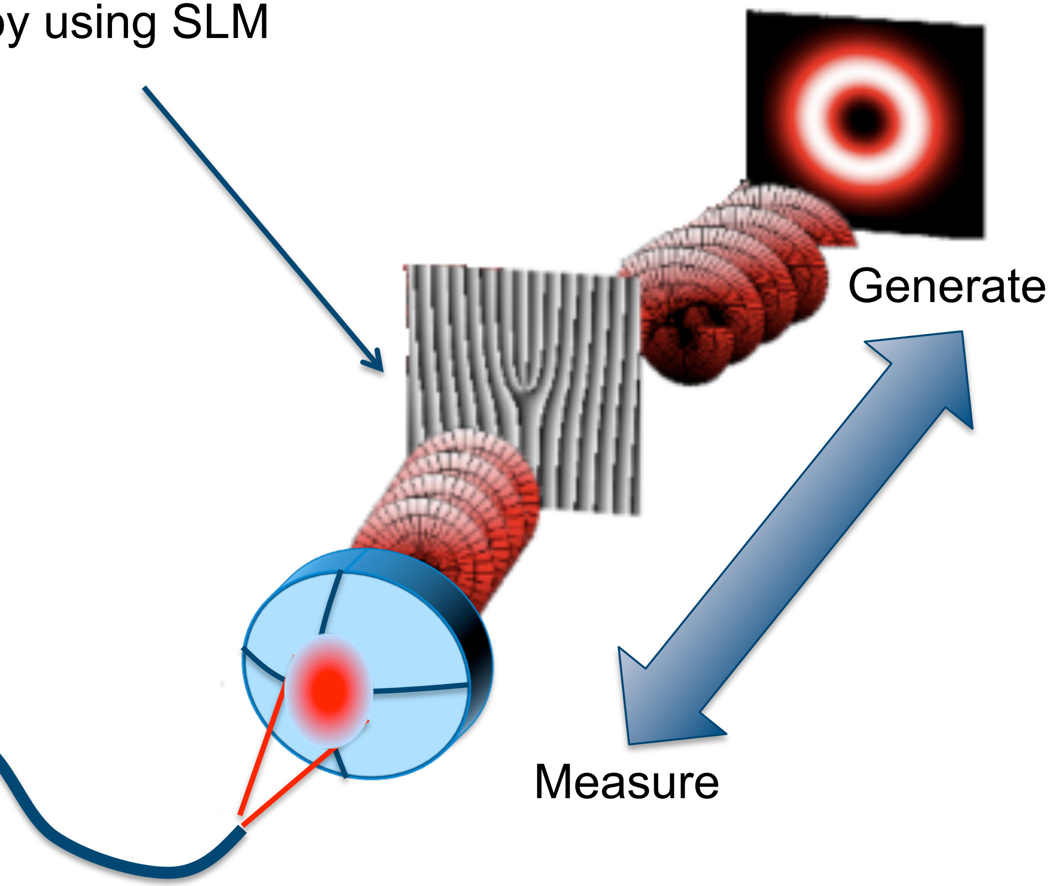
Making OR measuring phasefronts with holograms

Make interactive by using SLM

Switching time ≈5-20mSec
Efficiency ≈50%





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Richard Bowman

A gift for all the family…..

App Store › Education › Richard Bowman

iHologram

Description

iHologram creates beautiful patterns by rendering the Fraunhofer holograms used in Holographic Optical Tweezers
iPhone/iPad graphics chip. Use it to learn about diffraction and holography, or just to make pretty pictures!

Richard Bowman Web Site › **iHologram Support ›**

Free App ▾

This app is designed for both iPhone and iPad

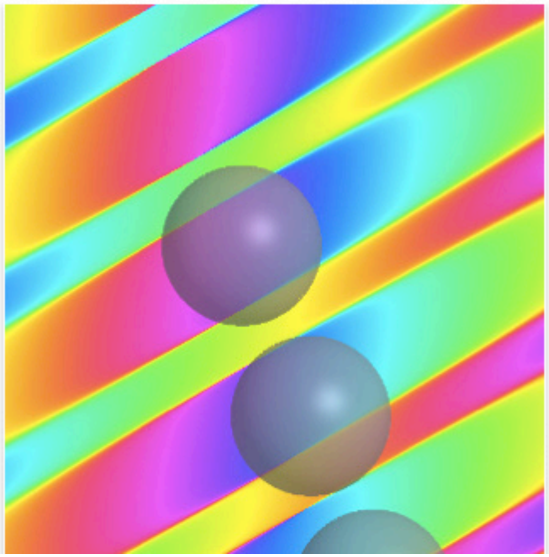
Category: Education
Released: 14 October 2010
Version: 1.0
1.0
0.2 MB
Language: English
Developer: Richard Bowman
© Richard Bowman

Rated 4+

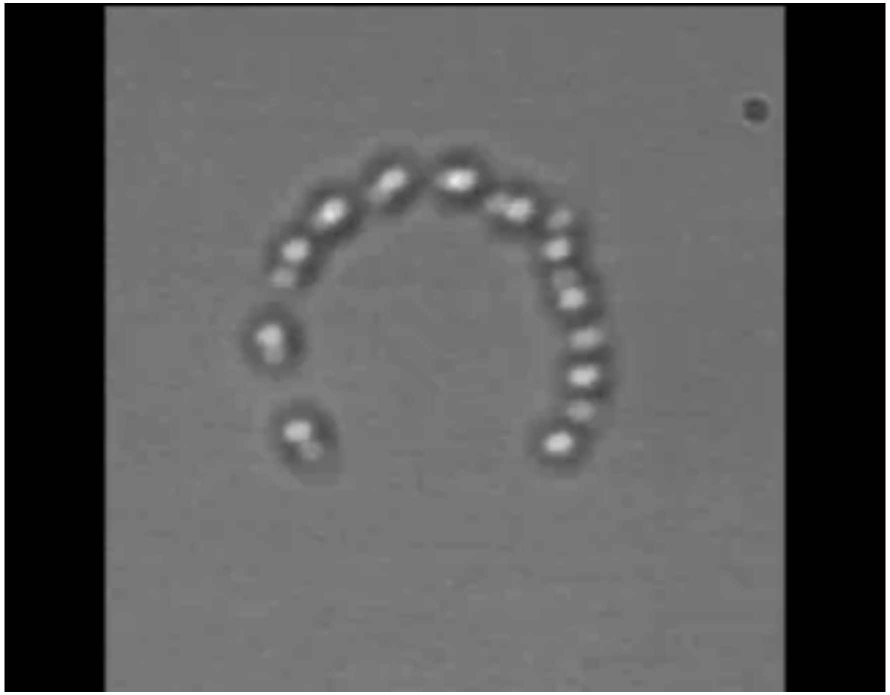
Requirements: Compatible with iPhone 3GS, iPhone 4, iPod touch (3rd generation), iPod touch (4th generation) and iPad. Requires iOS 3.2 or later.

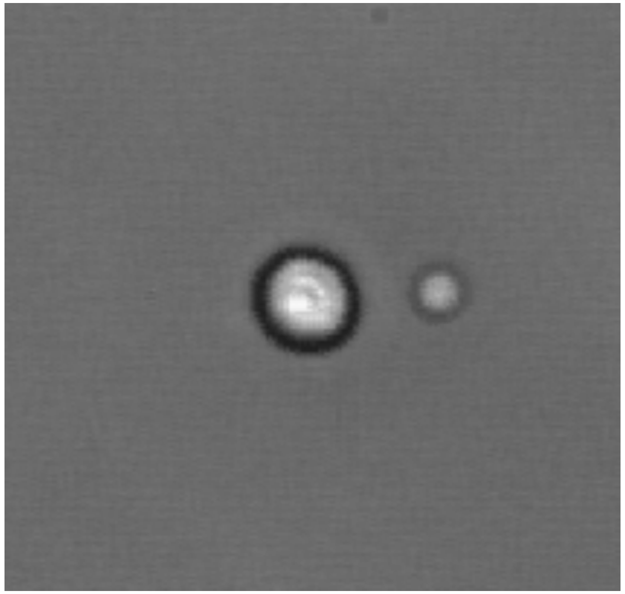
Screenshots

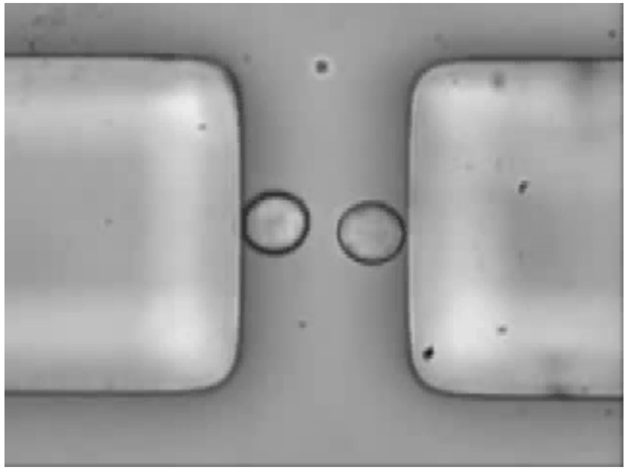
iPhone | iPad



And the point of shaping the spot is……







A double-start helix (ℓ=2)

Chambord castle (chateaux de la Loire)

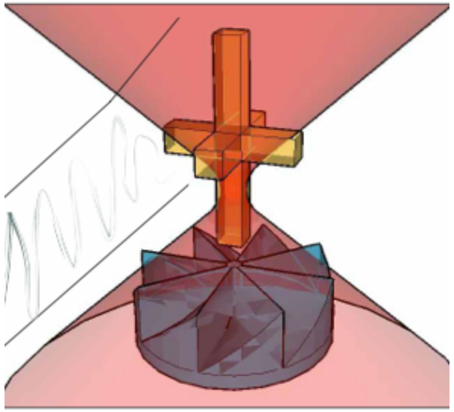
OAM in optical manipulation

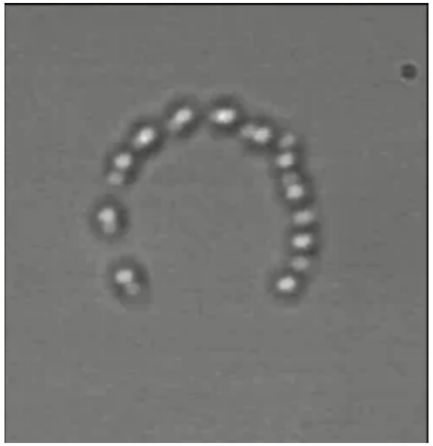
VOLUME 75, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JULY 1995

Direct Observation of Transfer of Angular Momentum to Absorptive Particles from a Laser Beam with a Phase Singularity

H. He, M. E. J. Friese, N. R. Heckenberg, and H. Rubinsztein-Dunlop
Department of Physics, The University of Queensland, Brisbane, Queensland, Australia Q4072
(Received 28 November 1994; revised manuscript received 4 April 1995)

He et al. PRL 1995





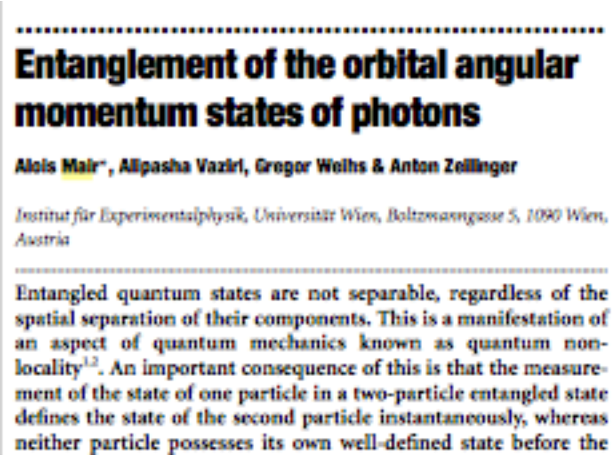
ELSEVIER
15 June 2002
Optics Communications 207 (2002) 169–175
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

Dynamic holographic optical tweezers

Jennifer E. Curtis, Brian A. Koss, David G. Grier*

Curtis et al. Opt Commun. 2002

OAM in quantum optics

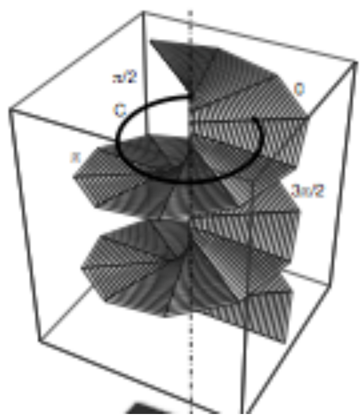


Entanglement of the orbital angular momentum states of photons

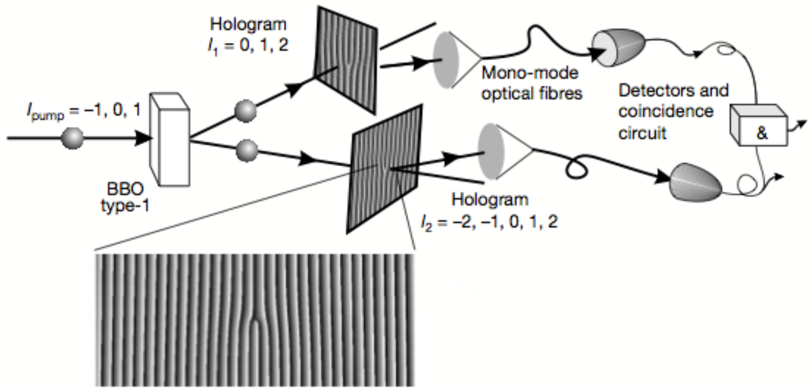
Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria

Entangled quantum states are not separable, regardless of the spatial separation of their components. This is a manifestation of an aspect of quantum mechanics known as quantum non-locality[1,2]. An important consequence of this is that the measurement of the state of one particle in a two-particle entangled state defines the state of the second particle instantaneously, whereas neither particle possesses its own well-defined state before the



Mair et al. Nature 2001



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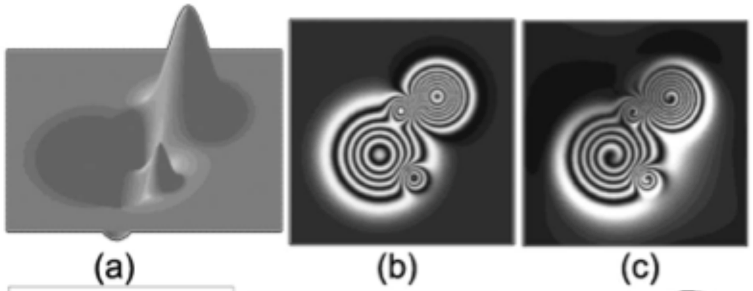
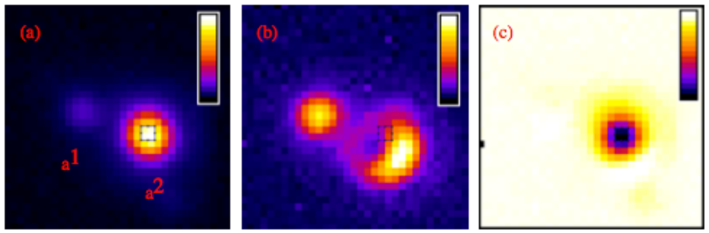
OAM in imaging

Spiral interferometry

Severin Fürhapter, Alexander Jesacher, Stefan Bernet, and Monika Ritsch-Marte
Division of Biomedical Physics, Innsbruck Medical University, Müllerstrasse 44, A-6020 Innsbruck, Austria

Fürhapter et al. Opt. Lett. 2005





Astronomical demonstration of an optical vortex coronagraph

Grover A. Swartzlander, Jr.,[1,*] Erin L. Ford,[1] Rukiah S. Abdul-Malik,[1]
Laird M. Close,[2] Mary Anne Peters,[2] David M. Palacios,[3] and Daniel W. Wilson[3]

Swartzlander et al. Opt. Express 2008

University of Glasgow

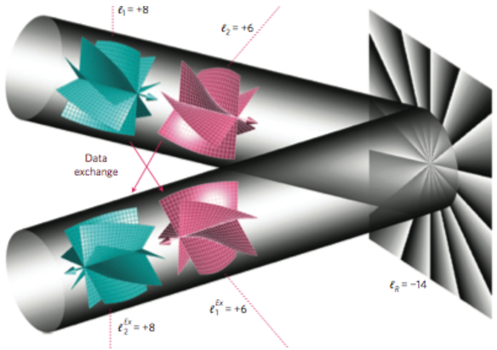
OAM in communication

New Journal of Physics
The open-access journal for physics

Encoding many channels on the same frequency through radio vorticity: first experimental test

Fabrizio Tamburini[1,2,8], Elettra Mari[3], Anna Sponselli[1], Bo Thidé[4,5], Antonio Bianchini[1] and Filippo Romanato[6,7]

Tamburini et al. New J Phys. 2012



ARTICLES
PUBLISHED ONLINE: 24 JUNE 2012 | DOI: 10.1038/NPHOTON.2012.138
nature photonics

Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang[1,2*], Jeng-Yuan Yang[1], Irfan M. Fazal[1], Nisar Ahmed[1], Yan Yan[1], Hao Huang[1], Yongxiong Ren[1], Yang Yue[1], Samuel Dolinar[3], Moshe Tur[4] and Alan E. Willner[1*]

Wang et al. Nature Photon 2012

University of Glasgow

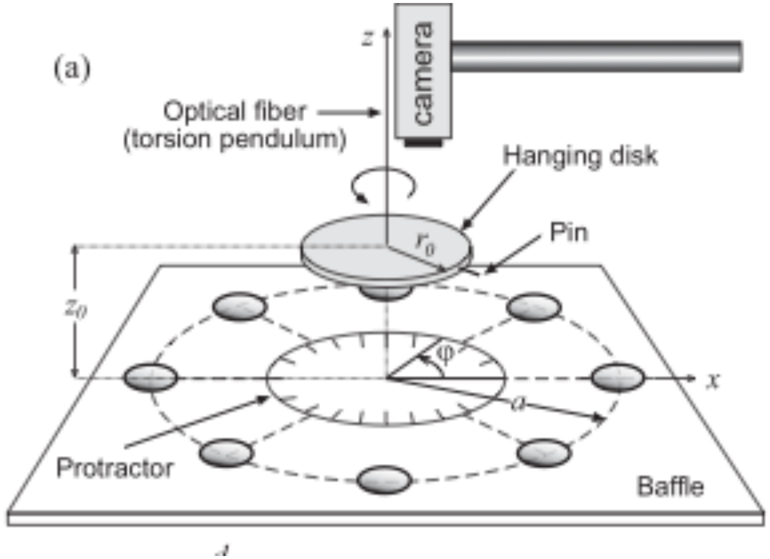
OAM in not just light

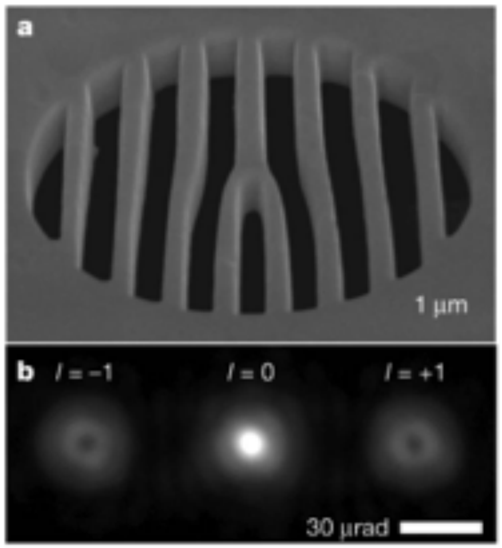
PRL **100,** 024302 (2008) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2008

Transfer of Angular Momentum to Matter from Acoustical Vortices in Free Space

Karen Volke-Sepúlveda,[1] Arturo O. Santillán,[2,*] and Ricardo R. Boullosa[2]

Volke-Sepulveda et al. PRL 2008





Vol 467|16 September 2010|**doi:10.1038/nature09366** nature

LETTERS

Production and application of electron vortex beams

J. Verbeeck[1], H. Tian[1] & P. Schattschneider[2]

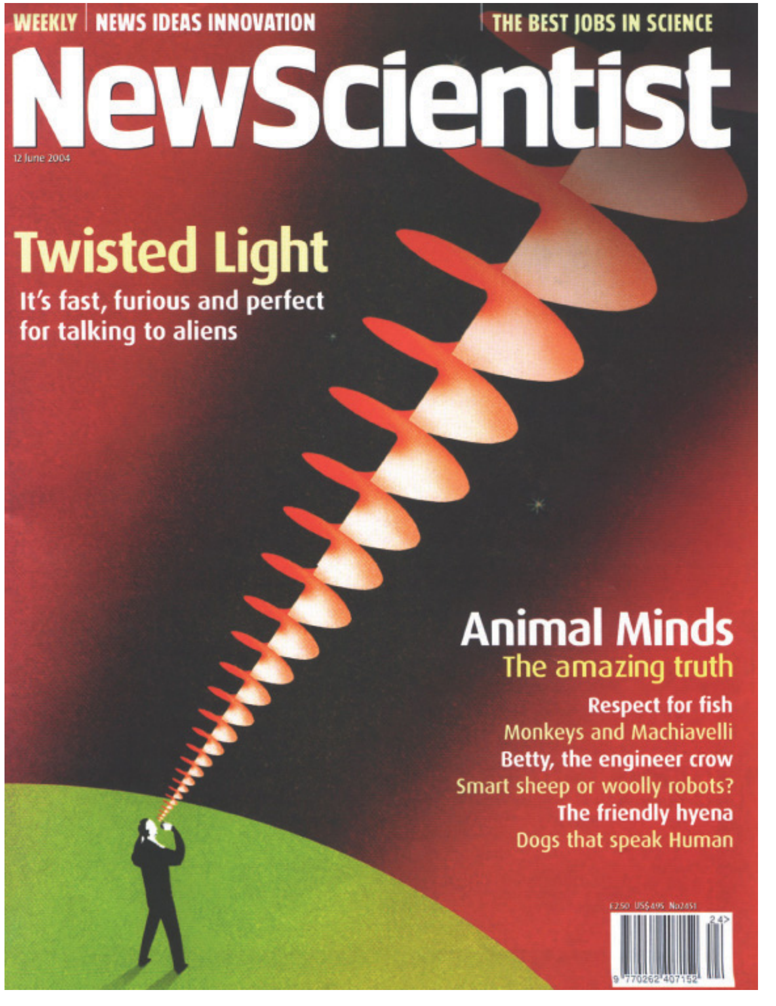
Verbeeck et al. Nature 2010

Free-space information transfer using light beams carrying orbital angular momentum

Graham Gibson, Johannes Courtial, Miles J. Padgett
Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, Scotland

1 November 2004 / Vol. 12, No. 22 / OPTICS EXPRESS 5448

The OAM communicator



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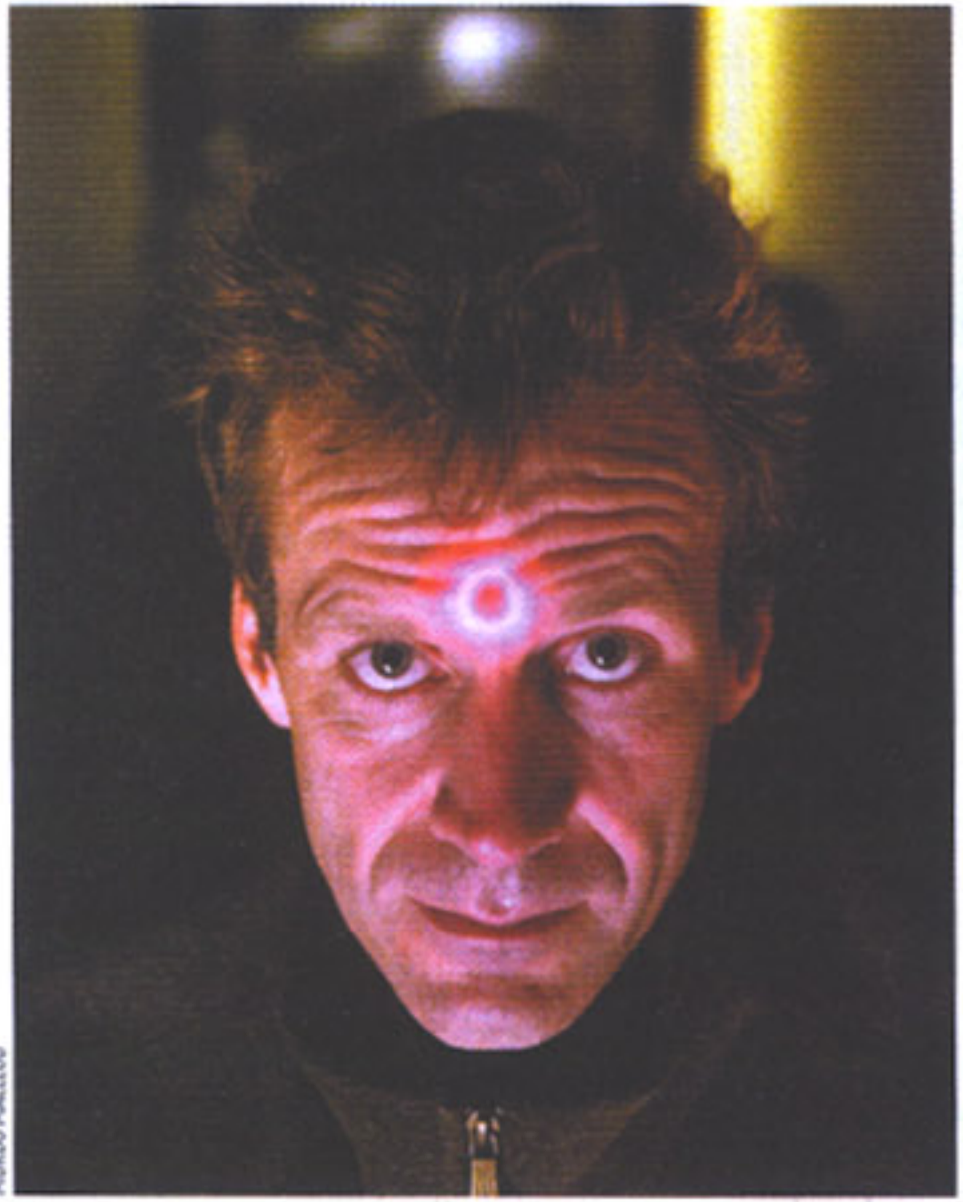
Miles Padgett's corkscrew laser beam creates a ring of light with a dark centre



University of Glasgow

Optical Vortices before Angular Momentum

Proc. R. Soc. Lond. A. **336**, 165–190 (1974)
Printed in Great Britain

Dislocations in wave trains

By J. F. Nye and M. V. Berry
H. H. Wills Physics Laboratory, University of Bristol

Quantised Singularities in the Electromagnetic Field

P. A. M. Dirac

Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, Vol. 133, No. 821. (Sep. 1, 1931), pp. 60-72.

JSTOR®

Fractality and Topology of Light's darkness

Kevin O'Holleran

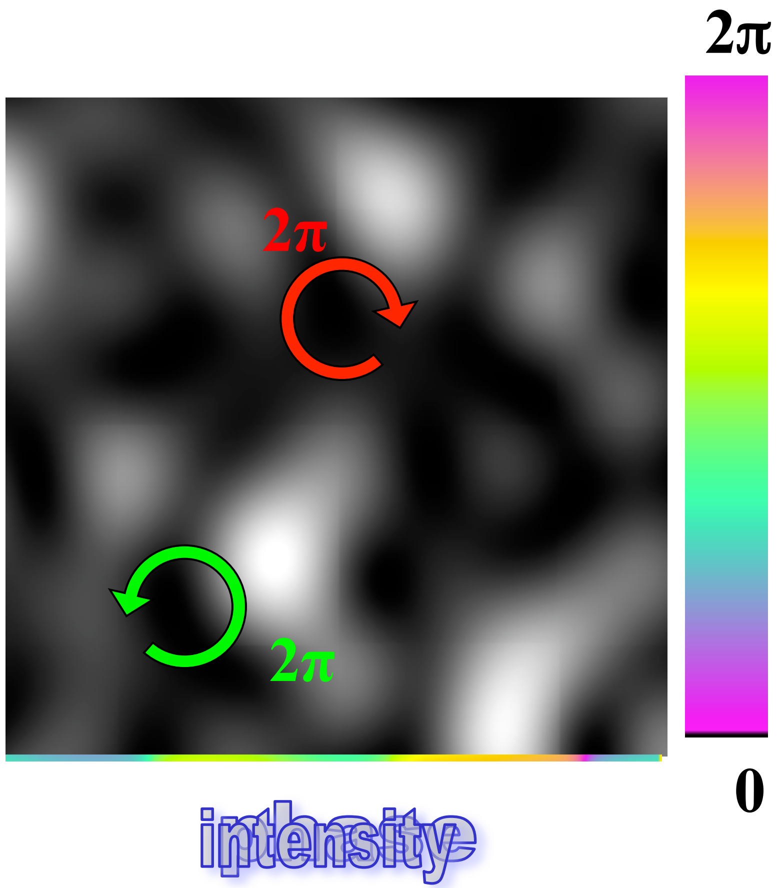
Florian Flossmann

Mark Dennis (Bristol)

University of Glasgow

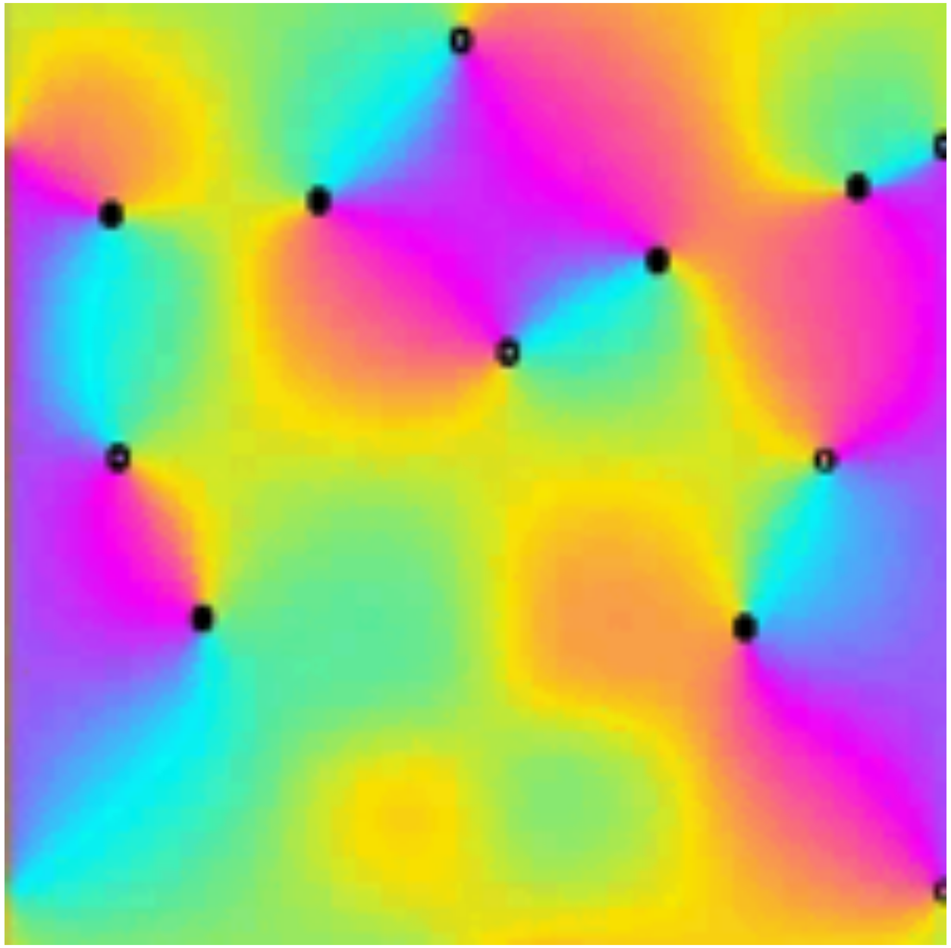
Vortices are ubiquitous in nature

- Whenever ***three*** (or more) plane waves interfere optical vortices are formed
  - Charge one vortices occur wherever there is diffraction or scattering

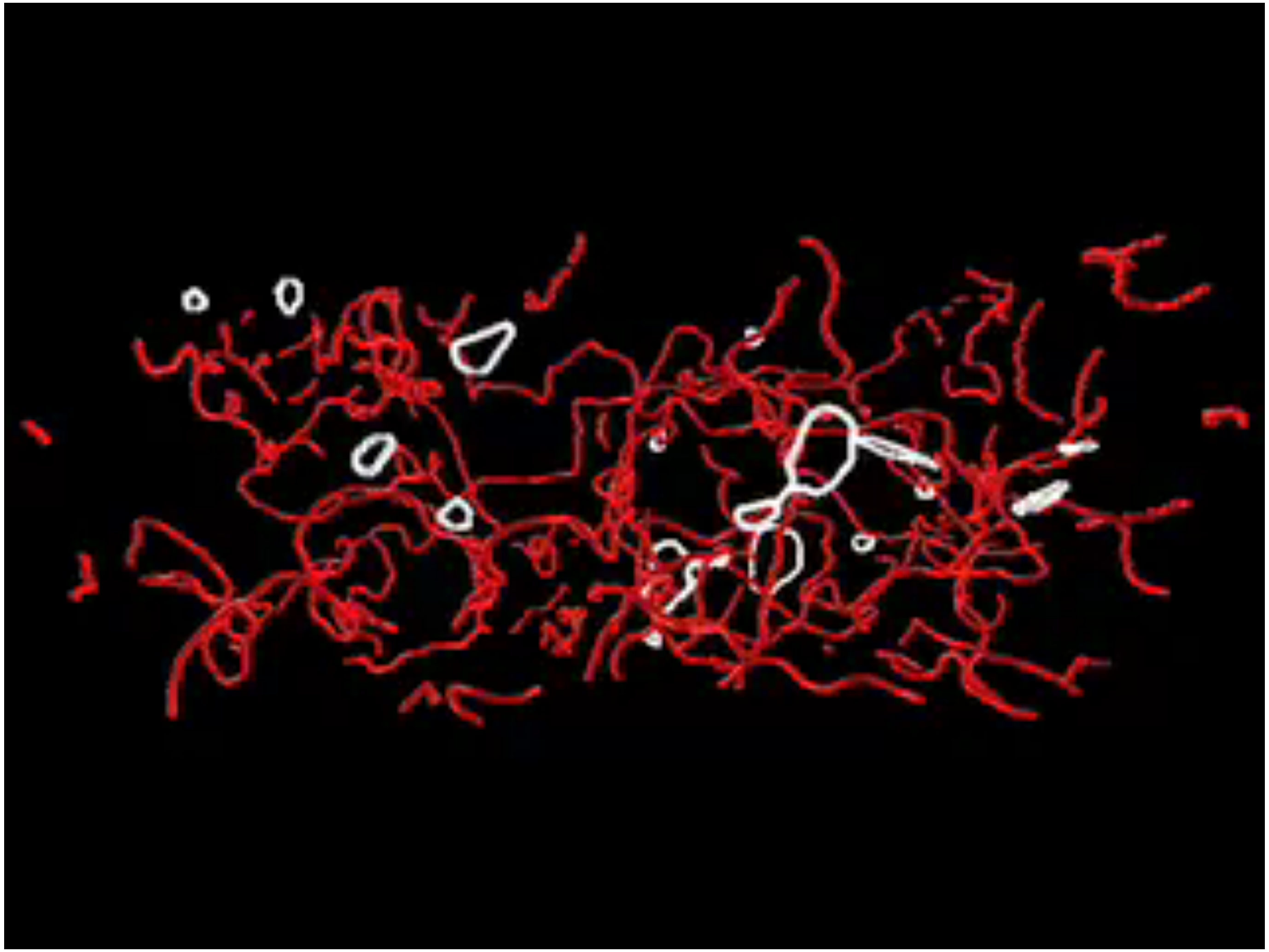


Map out the vortex position in different planes

- Either numerically or experimentally one can map the vortex positions in different planes



The tangled web of speckle



Entanglement of OAM states

Entanglement of the orbital angular momentum states of photons NATURE | VOL 412 | 19 JULY 2001 |

Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

VOLUME 93, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 30 JULY 2004

Measuring Entangled Qutrits and Their Use for Quantum Bit Commitment

N. K. Langford,* R. B. Dalton, M. D. Harvey, J. L. O'Brien, G. J. Pryde, A. Gilchrist, S. D. Bartlett, and A. G. White

University of Glasgow

Quantum entanglement with spatial light modulators

Jonathan Leach
Barry Jack
Sonja Franke-Arnold
(Glasgow)




Steve Barnett
and Alison Yao (Strathclyde)


Bob Boyd
Anand Jha (Rochester)

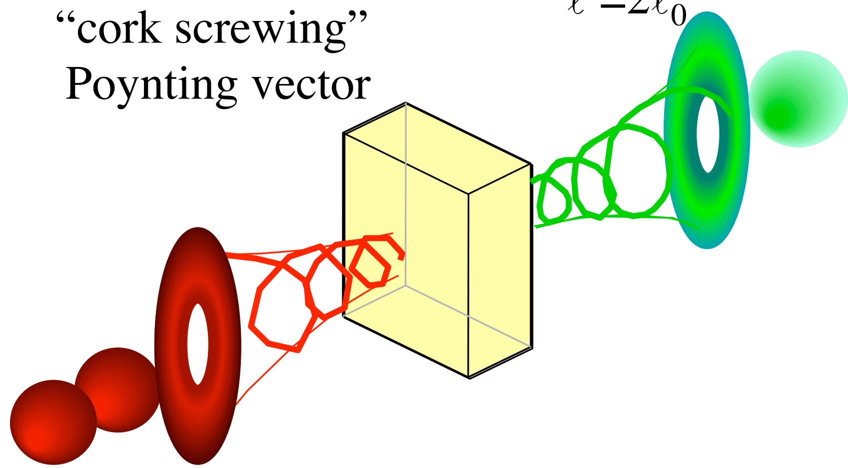

PHYSICAL REVIEW A VOLUME 56, NUMBER 5 NOVEMBER 1997

Second-harmonic generation and the conservation of orbital angular momentum with high-order Laguerre-Gaussian modes

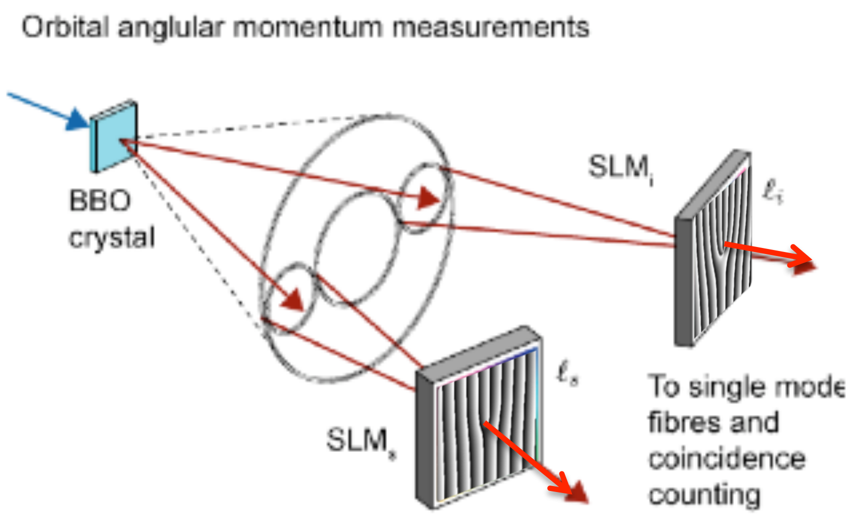
J. Courtial, K. Dholakia, L. Allen, and M. J. Padgett

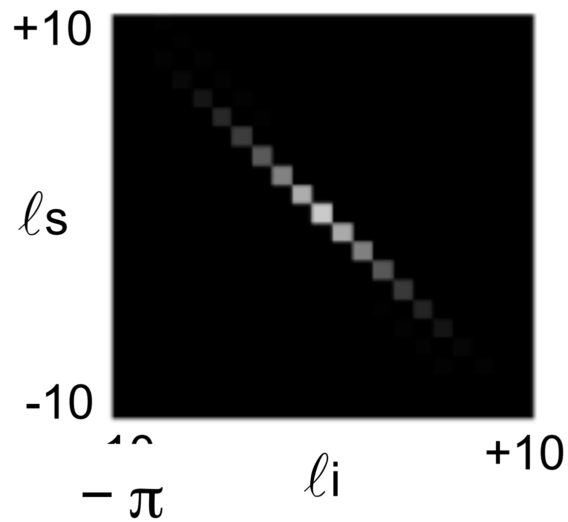
OAM in second harmonic generation

- Poynting vector "cork screws", azimuthal skew angle is
  - $\theta = \ell/kr$
- Does this upset a co-linear phase match? -No
- Frequency & ℓ-index both double
- "Path" of Poynting vector stays the same
  - phase matching maintained

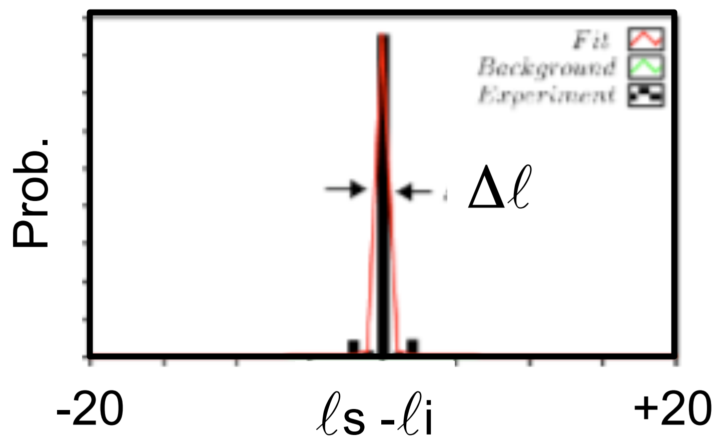


Correlations in angular momentum

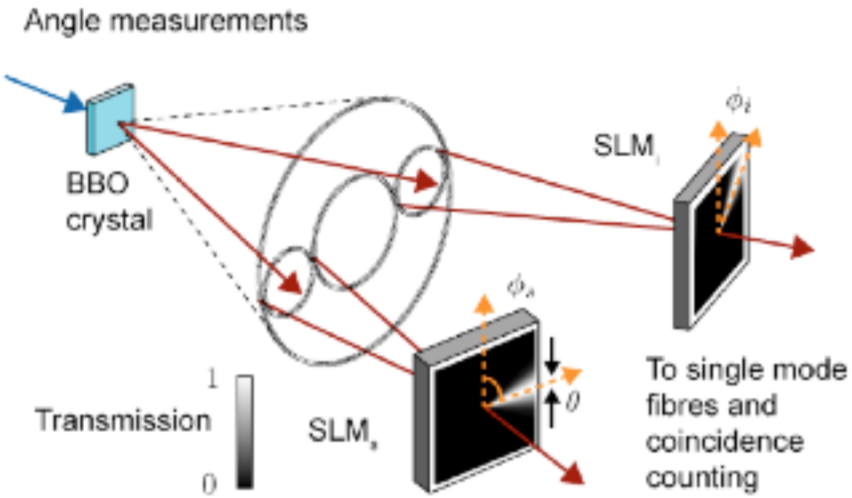


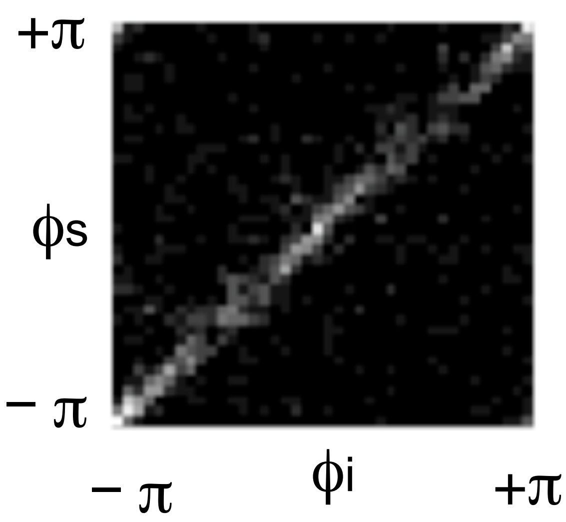


Near perfect (anti)
Correlations in angular
momentum

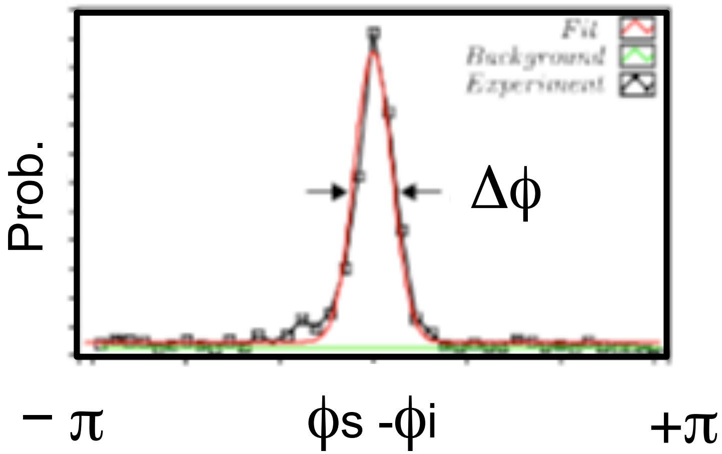


Correlations in angle

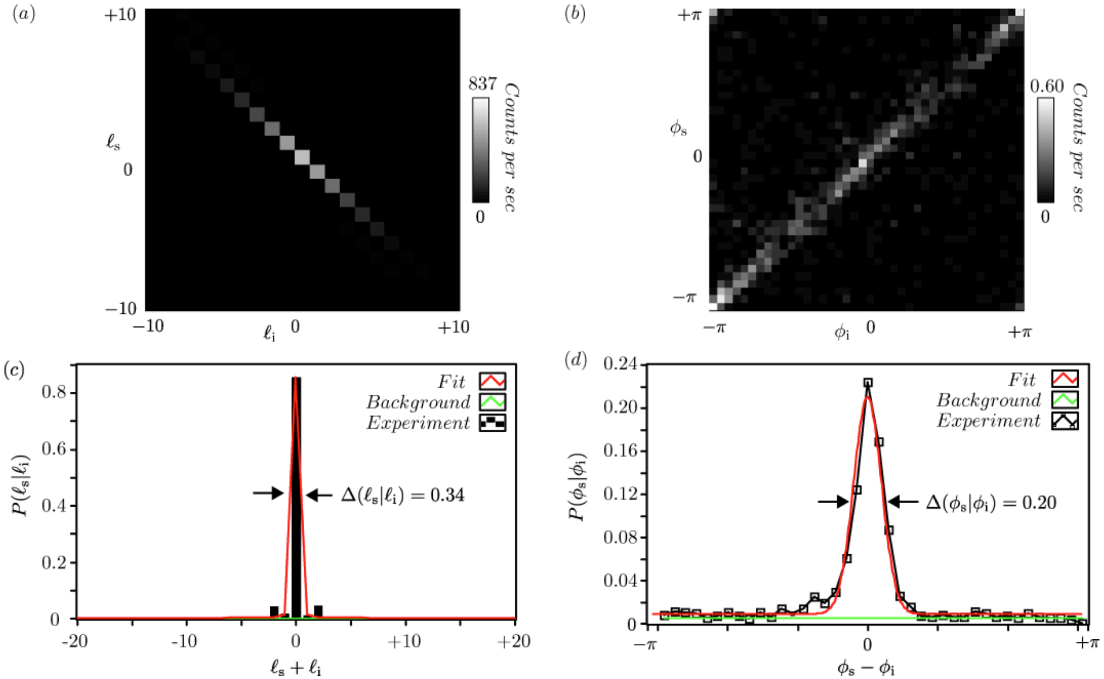




Near perfect
Correlations in angle



Angular EPR



Correlations in complimentary basis sets
-> demonstrates EPR for Angle and Angular momentum

$$\left[\Delta\left(\ell_{s}|\ell_{i}\right)\hbar\right]^2\left[\Delta\left(\phi_s|\phi_i\right)\right]^2 = 0.00475\hbar^2 << 0.25\hbar^2$$

Entanglement of OAM states

Proc. R. Soc. Lond. A. **349**, 423–439 (1976)
Printed in Great Britain

Rotary 'aether drag'

By R. V. Jones, F.R.S.
Department of Natural Philosophy, University of Aberdeen, Scotland

University of Glasgow

Optical Activity /Faraday effect for OAM

Sonja Franke-Arnold
Graham Gibson
Emma Wisniewski-Barker

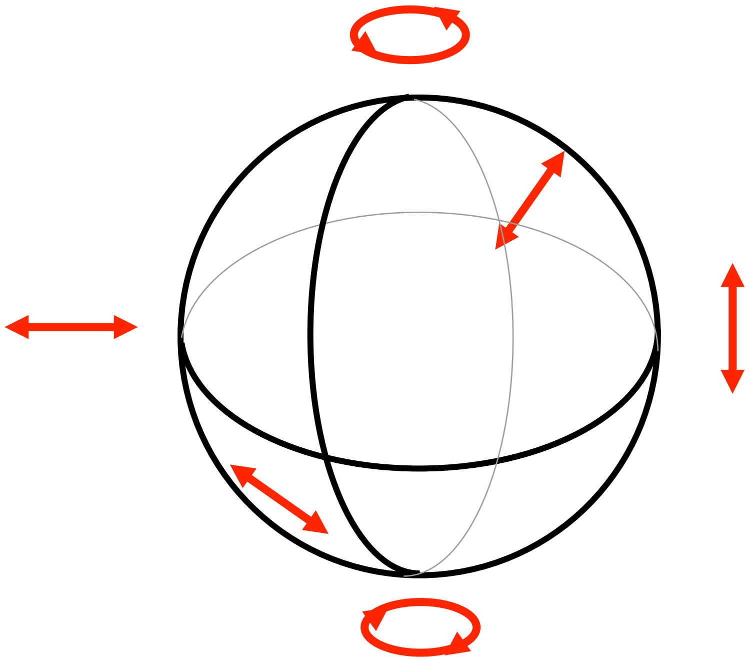


Bob Boyd

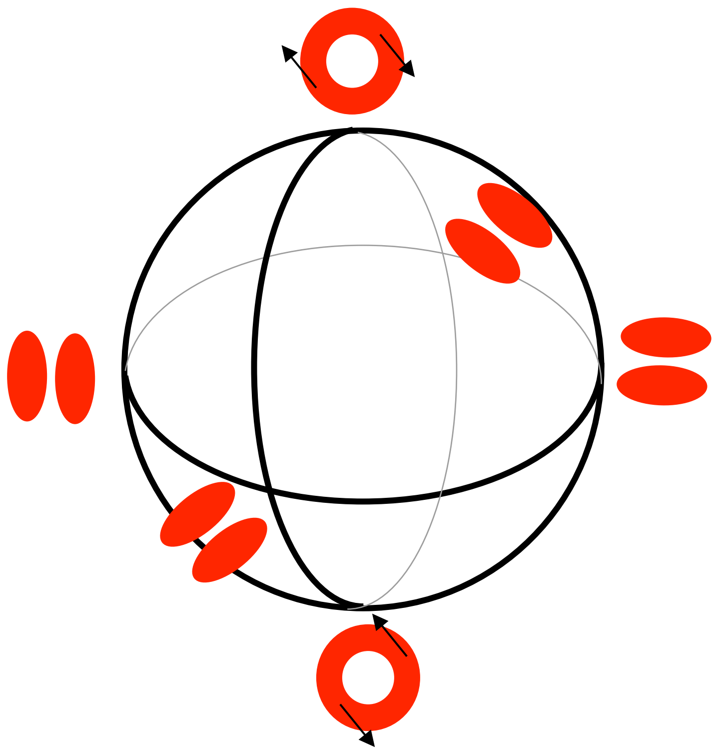
430 OPTICS LETTERS / Vol. 24, No. 7 / April 1, 1999

Poincaré-sphere equivalent for light beams containing orbital angular momentum

M. J. Padgett and J. Courtial



Poincaré Sphere



Poincaré Sphere for OAM

The (Magnetic) Faraday Effect

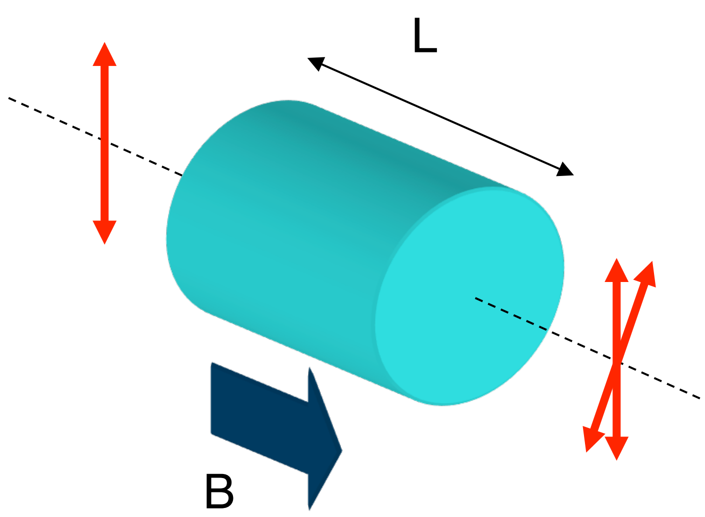
- Rotation of plane polarised light

$$\Delta\theta_{pol} = BLV$$

  - V Verdet constant

- OR treat as phase delay of circularly polarised light

$$\Delta\phi = \sigma BLV$$



$$\Delta\theta = \Delta\phi_{+\sigma,-\sigma} / \Delta\sigma$$

But the magnetic Faraday effect does NOT rotate an Image

Proc. R. Soc. Lond. A. **349**, 423–439 (1976)
Printed in Great Britain

Rotary 'aether drag'

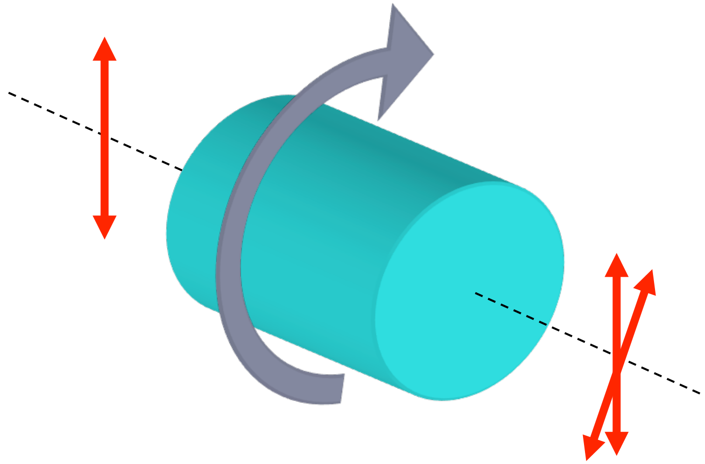
By R. V. Jones, F.R.S.
Department of Natural Philosophy, University of Aberdeen, Scotland

- Photon drag, gives Polarisation rotation

$$\Delta\theta = \frac{\Omega L}{c}\left(n_g - 1/n_\phi\right)$$

$$\Delta\phi = \frac{\sigma\Omega L}{c}\left(n_g - 1/n_\phi\right)$$

- Mechanical Faraday Effect



PHYSICAL REVIEW A VOLUME 46, NUMBER 11 1 DECEMBER 1992

Magnetic and mechanical Faraday effects

G. Nienhuis, J. P. Woerdman, and I. Kuščer*

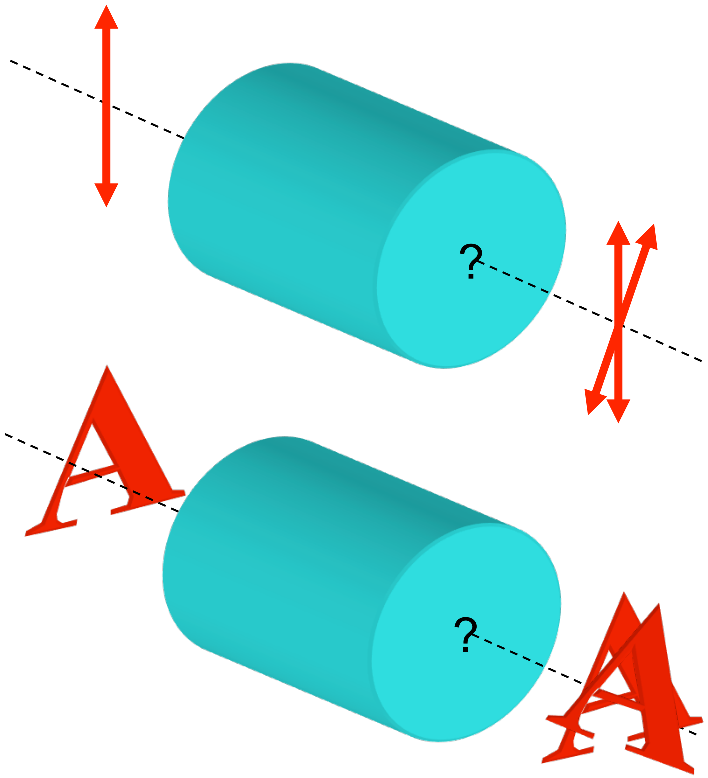
Journal of Modern Optics
Vol. 54, No. 4, 10 March 2007, 487–491

Taylor & Francis
Taylor & Francis Group

Equivalent geometric transformations for spin and orbital angular momentum of light

L. ALLEN*†‡ and MILES PADGETT†

- SAM -> Polarisation rotation
- OAM-> Image rotation
- Look through a Faraday isolator (Δθ≈45°), is the "world" rotated - NO
  - SAM and OAM are not equivalent in the Magnetic Faraday effect
  - SAM and OAM are not equivalent in the optical activitiy



Enhancing the effect…..

- Plug in “sensible numbers” and get a micro-radian rotation…
- Increase the group index to enhance the effect

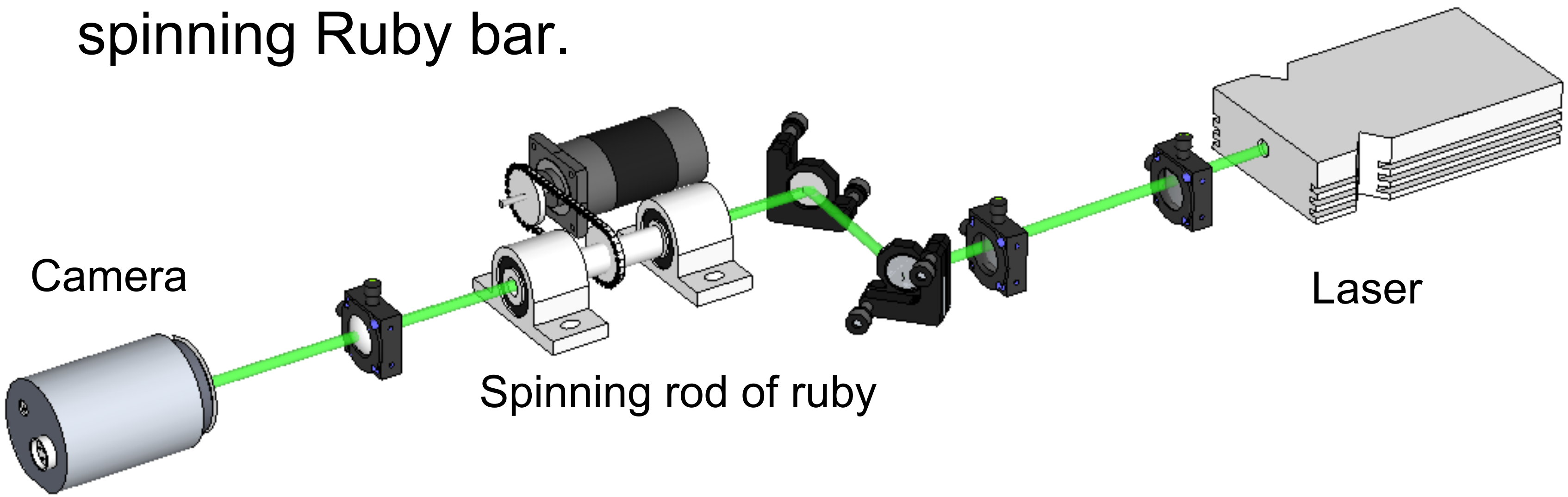
$$\Delta\theta_{image} = \frac{\Omega L}{c}\left(n_g - 1/n_\phi\right)$$

Rotary Photon Drag Enhanced by a Slow-Light Medium

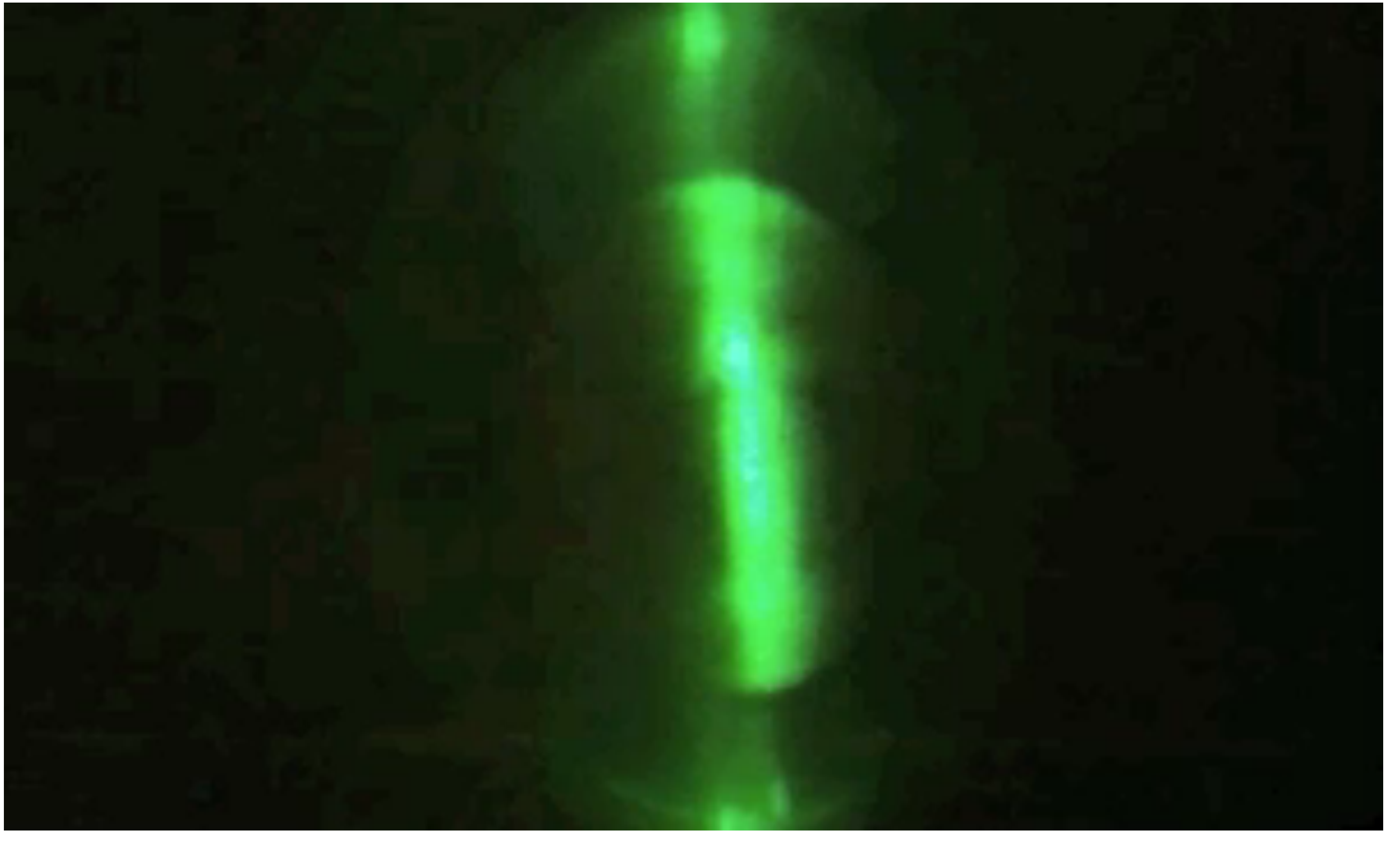
Sonja Franke-Arnold,[1*] Graham Gibson,[1] Robert W. Boyd,[2,3] Miles J. Padgett[1]

SCIENCE VOL 333 1 JULY 2011

- Shine an elliptical laser beam (≈LG, $\Delta\ell=2$) @ 532nm through a spinning Ruby bar.



≈25Hz clockwise <-> anticlockwise



A beam splitter for OAM

Martin Lavery

Gregorius Berkhout (Leiden)

Johannes Courtial

- Spin angular momentum
  - Circular polarisation
  - $\sigma\hbar$ per photon
- Orbital angular momentum
  - Helical phasefronts
  - $\ell\hbar$ per photon

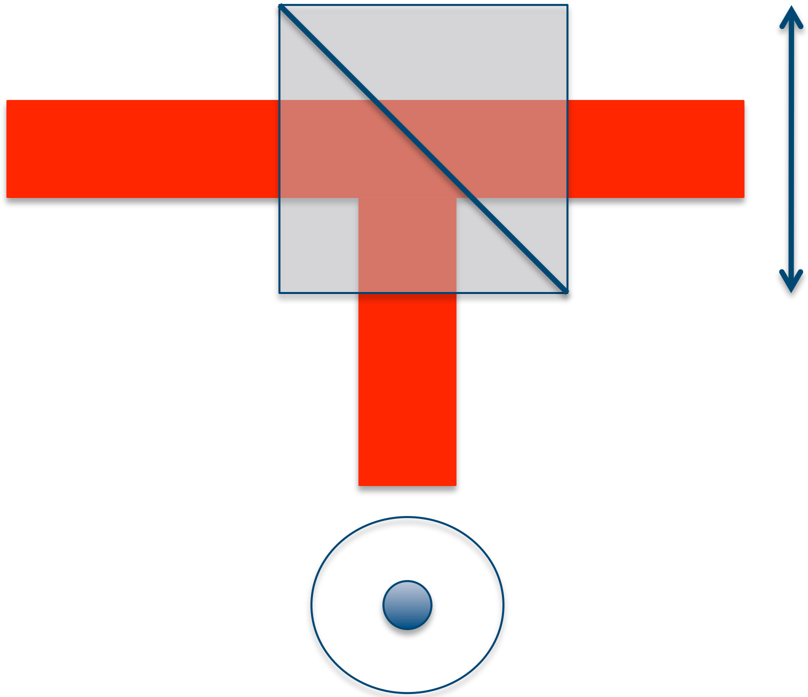
$\sigma = +1$

$\sigma = -1$

$\ell = 0$ $\ell = 1$ $\ell = 2$ $\ell = 3$ etc

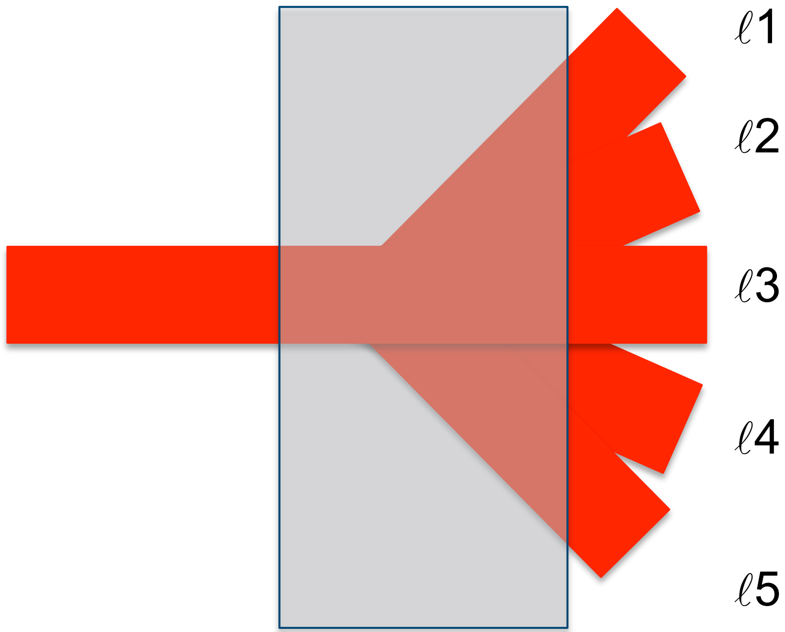
Measuring spin AM

- Polarising beam splitter give the "perfect" separation of orthogonal (linear) states
  - Use quarter waveplate to separate circular states
  - Works for classical beams AND single photons



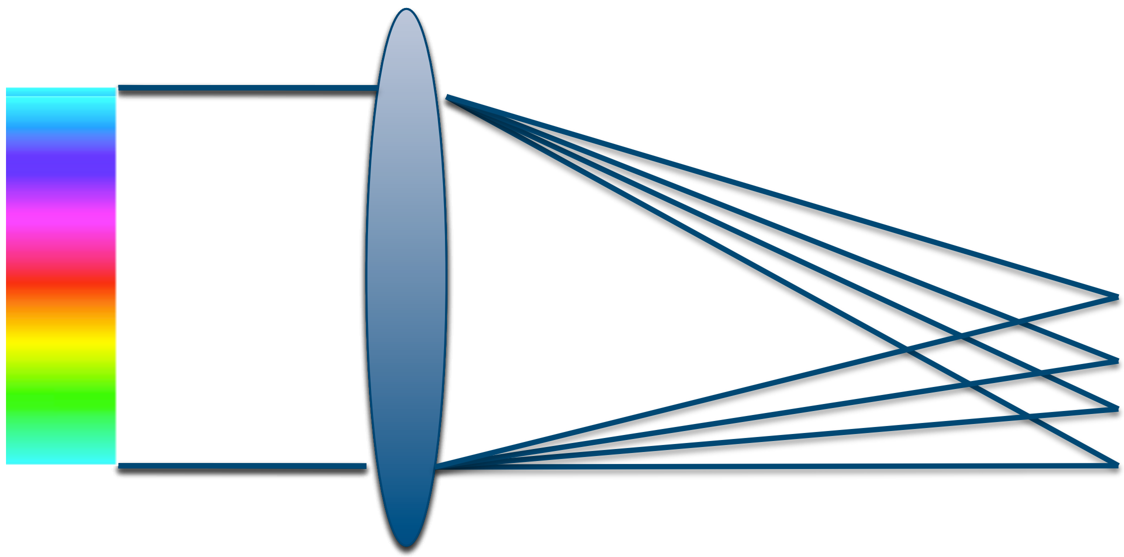
Measuring Orbital AM

- OAM beam splitter give the “perfect” separation of orthogonal states
  - But how?



It works for plane waves

- A “plane-wave” is focused by a lens
- A phase ramp of 2π displaces the spot



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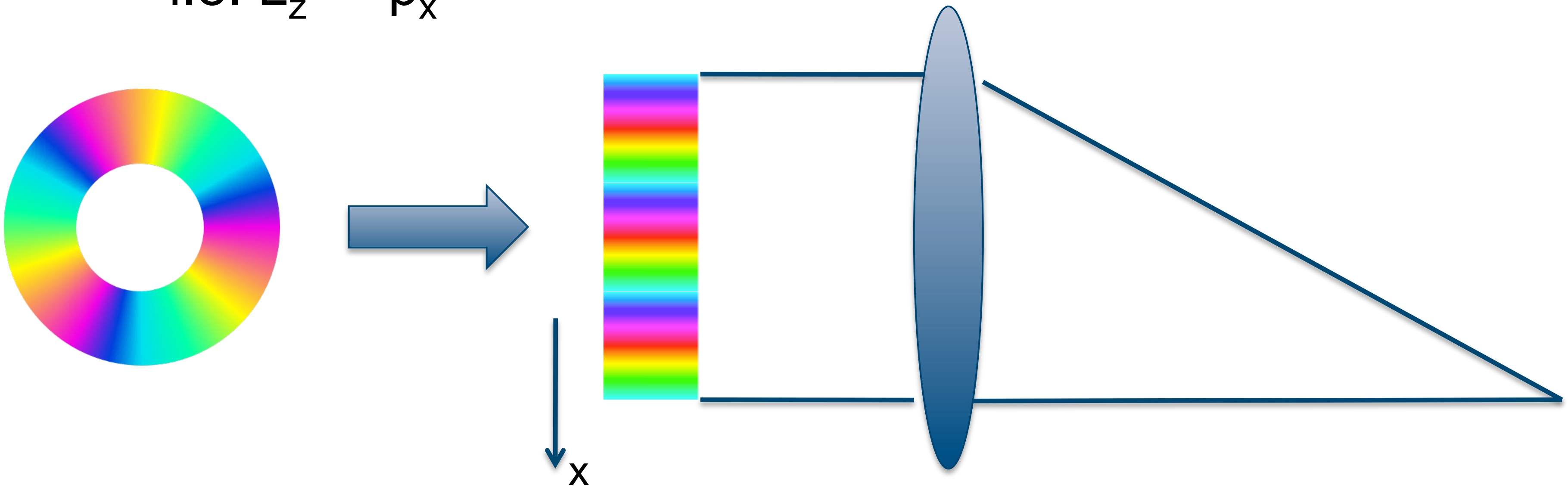
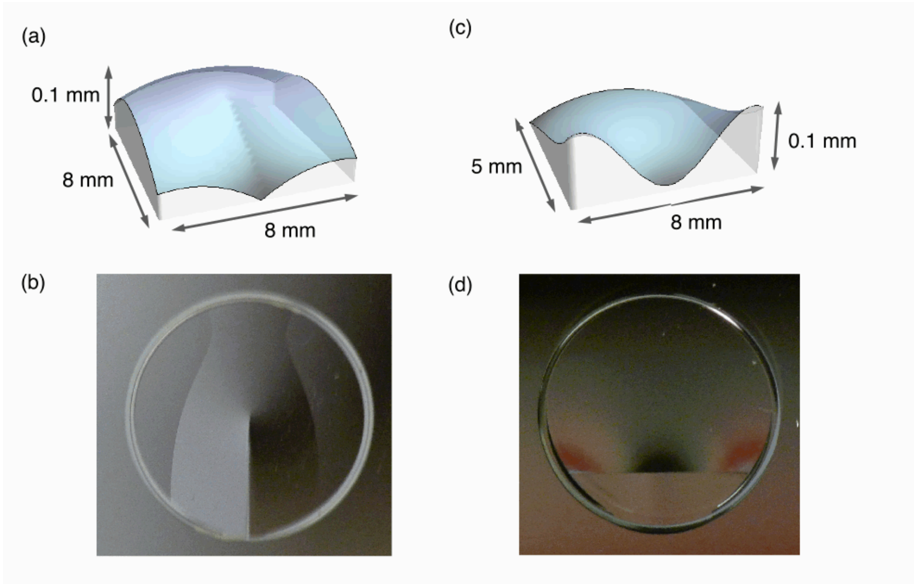
It works for plane waves

- Image transformation
  - ϕ -> x and r -> y
  - i.e. L_z -> p_x



Replacing the SLMs

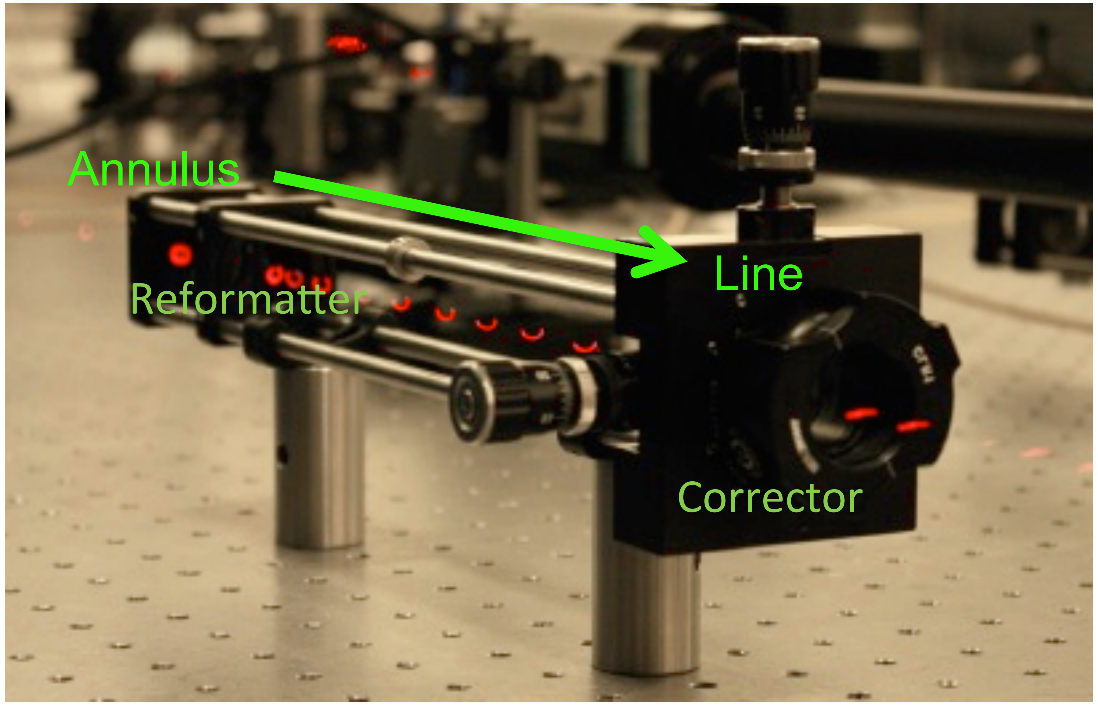
- The principle works
- But the SLMs are inefficient (≈50% x 2)
- Use bespoke optical elements (plastic)
  - Prof. David J Robertson
  - Prof. Gordon Love



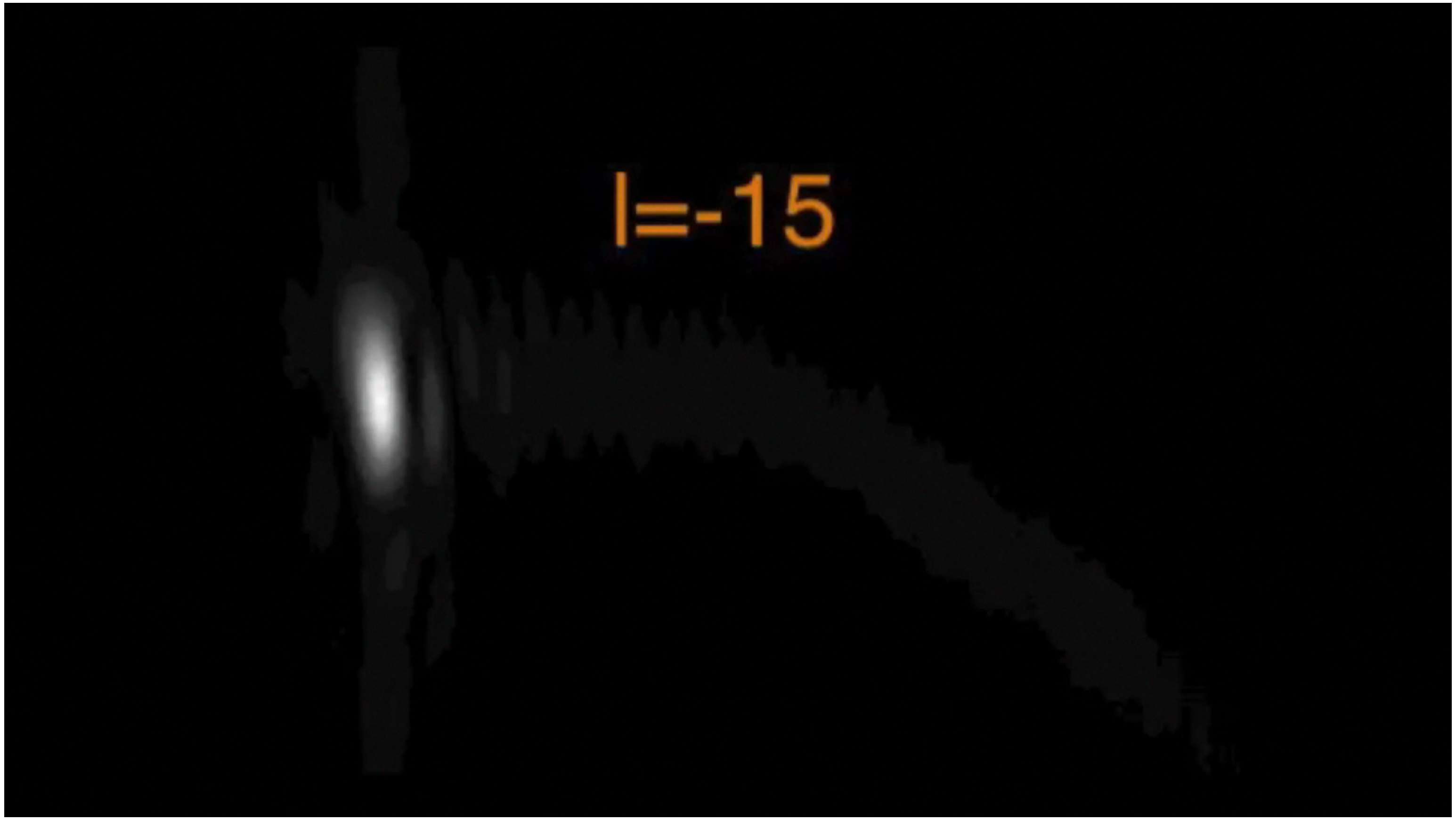
reformater phase corrector

Doughnut to hot-dog

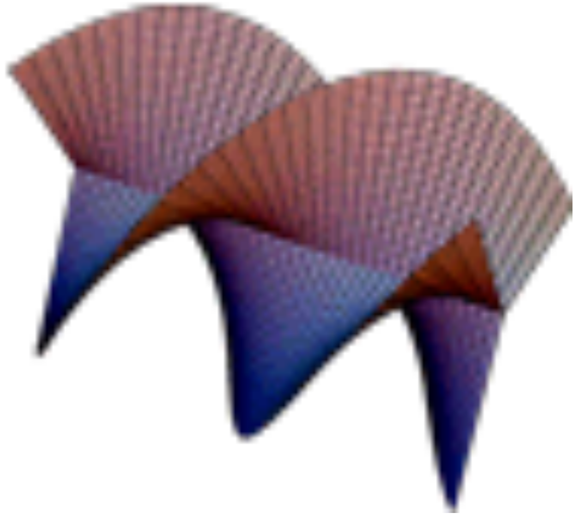
- The principle works
- But the SLMs are inefficient (≈50% x 2)
- Use bespoke optical elements (glass/ plastic)
  - Prof. David J Robertson
  - Prof. Gordon Love



The output



Evolution in time c.f. translation and rotation



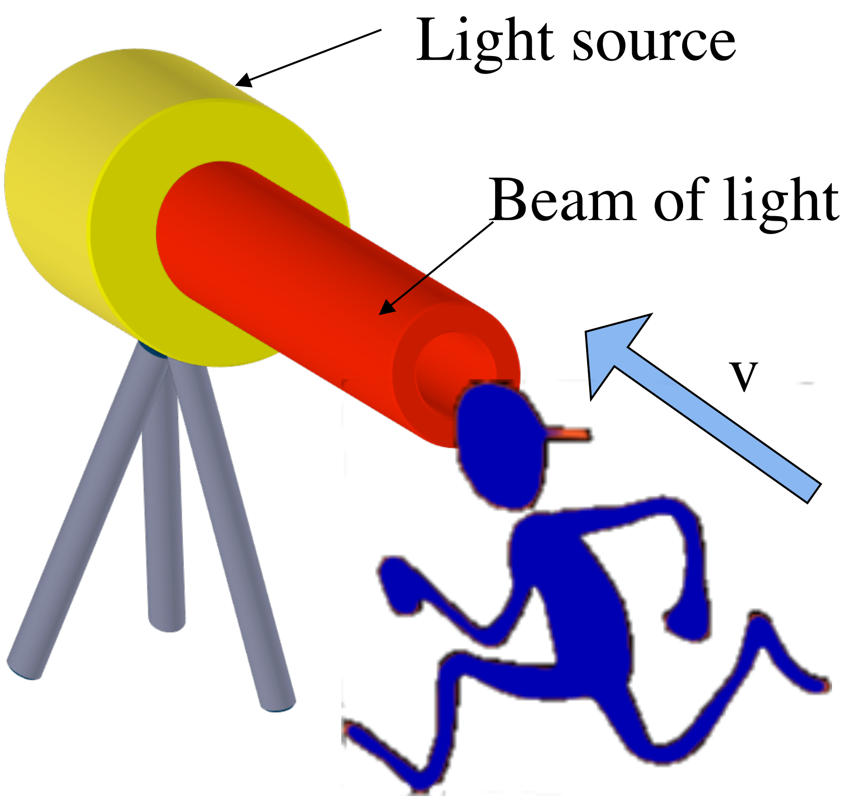
$\Phi = f(kz+\omega t)$

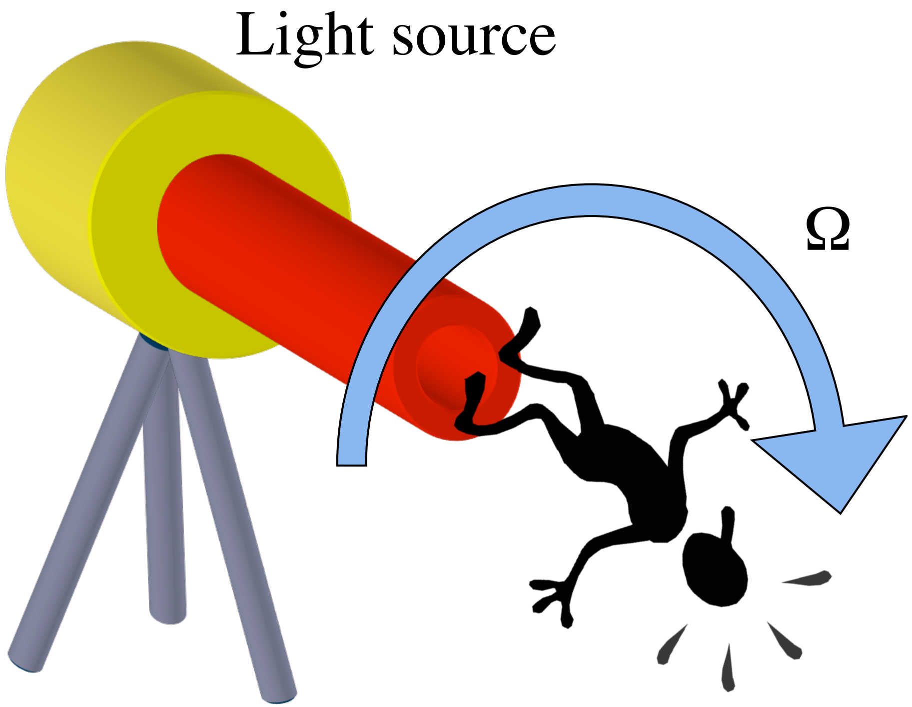
$\ell = 0$ time c.f. translation

$\Phi = f(kz+\omega t+\ell\theta)$

$\ell > 0$ time c.f. rotation

Linear vs. Rotational Doppler shifts





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VOLUME 78, NUMBER 13 PHYSICAL REVIEW LETTERS 31 MARCH 1997

Rotational Frequency Shift

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Doppler effect induced by rotating lenses

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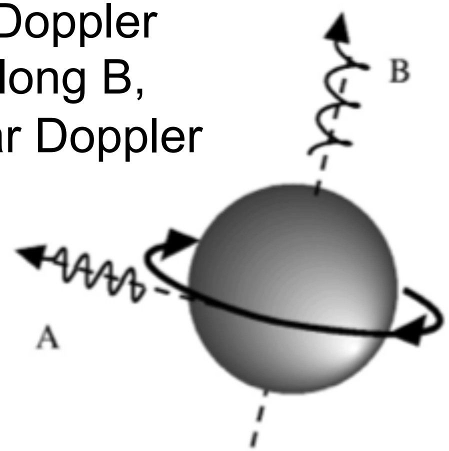
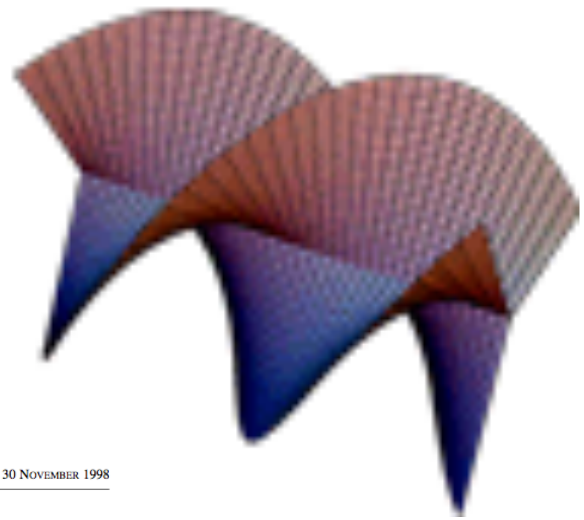
Received 8 February 1996; accepted 24 April 1996

Rotational (angular) Doppler shift

- For pure OAM states a rotation of frame between source and observer give a frequency shift
  - $\Delta\omega_\ell = \Omega\,\ell$
- Rotation of the state "looks-like" an advance in time, but…
- The rotational symmetry stays the same…..

Rotational Doppler observed along B, where linear Doppler (A) is zero





VOLUME 81, NUMBER 22 PHYSICAL REVIEW LETTERS 30 NOVEMBER 1998

Rotational Frequency Shift of a Light Beam

J. Courtial, D. A. Robertson, K. Dholakia, L. Allen, and M. J. Padgett

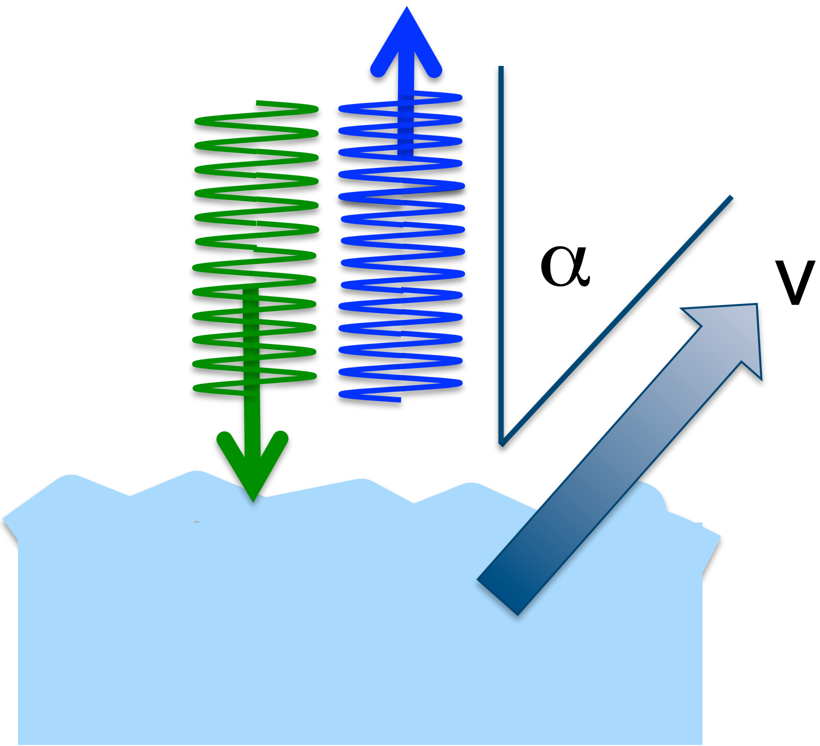
Where shall we start?

- Light scattered by a moving body is shifted in both energy ($\hbar\omega$) and linear momentum ($\hbar$k).
  - Doppler Shift

- Doppler shift is used to remotely detect the movement of a distant body

- How might we use the OAM, what might it detect?

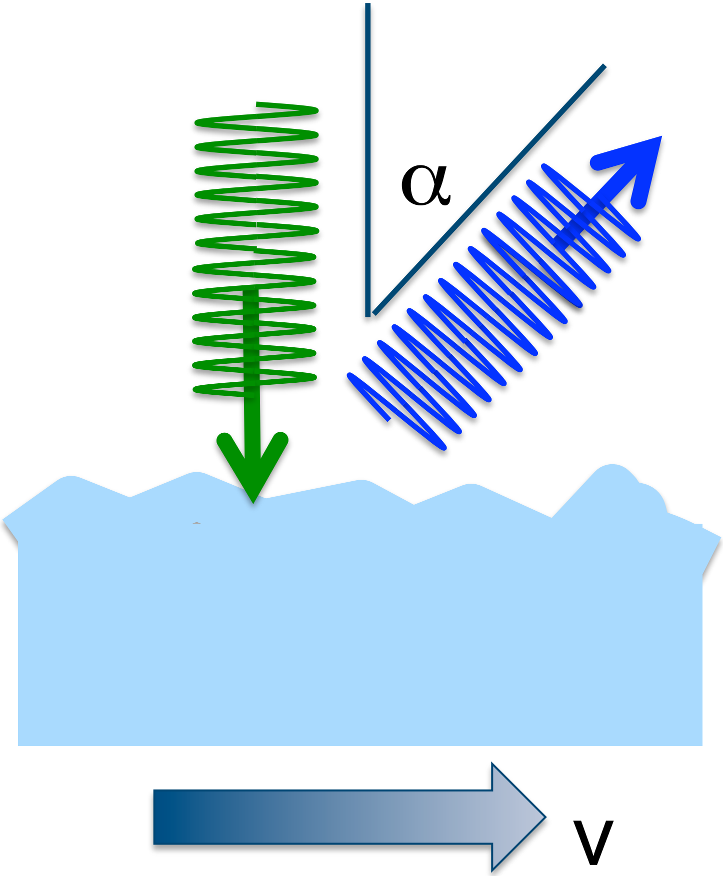
Doppler shift from a moving surface



$$\Delta\omega = 2\cos\alpha\ \omega_0 v/c$$

when $\alpha=\pi/2$, $\Delta\omega = 0$

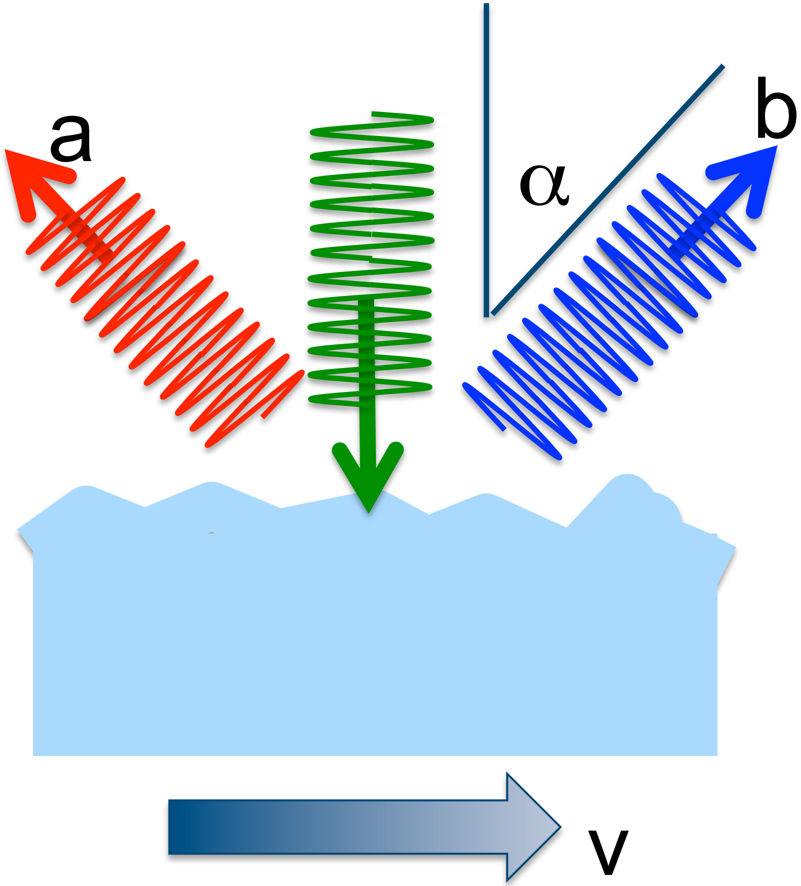
Doppler shift from translating surface

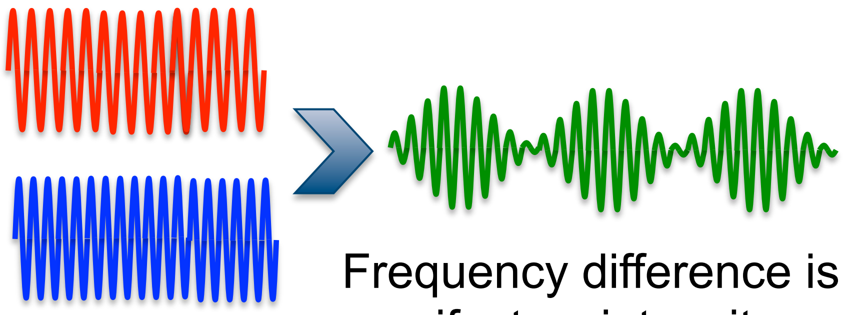


$$\Delta\omega = \sin\alpha \; \omega_0 v/c$$

Basis of Doppler velocimetry

Doppler velocimetry (frequency domain)



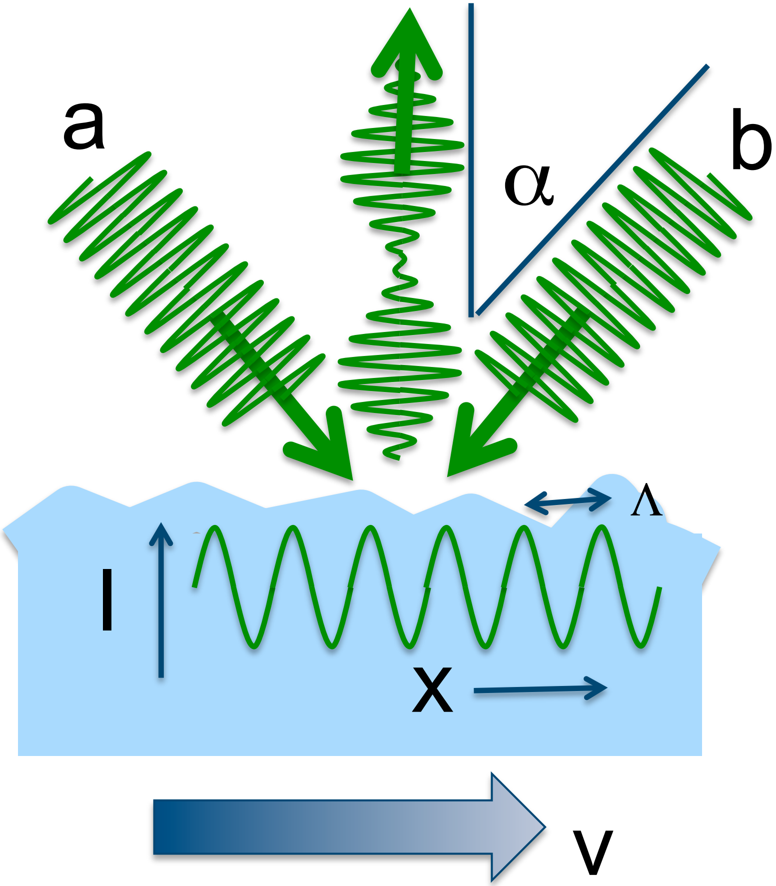


Frequency difference is manifest as intensity modulation, $\Delta\omega_{a,b}$

$$\Delta\omega = 2\sin\alpha\ \omega_0 v/c$$

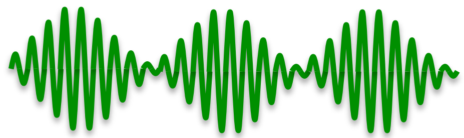
Illuminate on-axis, detect off-axis

Doppler velocimetry (time domain) – projected fringes



$$\Lambda = \lambda / 2\sin\alpha$$

Scattering centres move across the fringe pattern give intensity modulation of scattered light, $\Delta\omega_\Lambda$

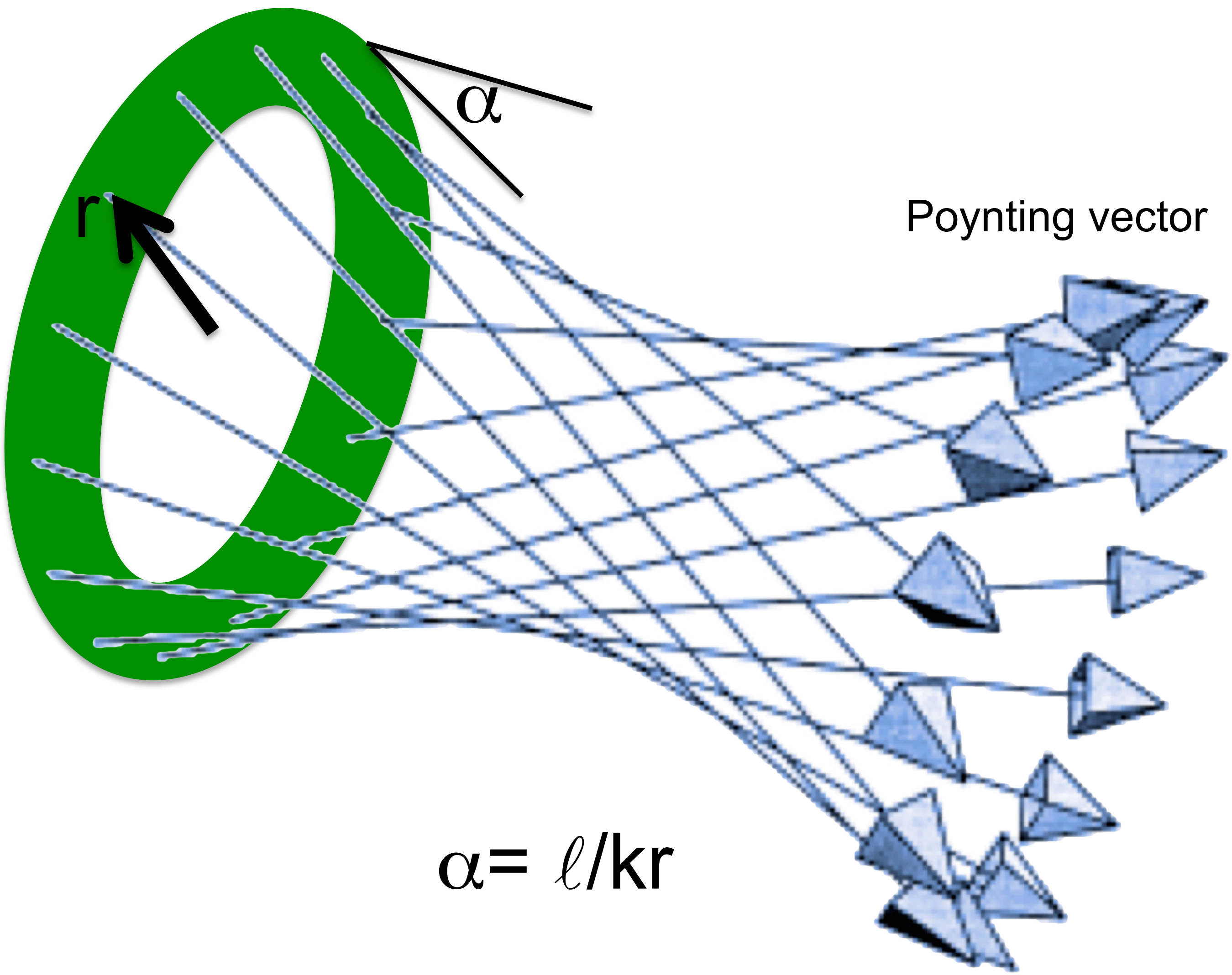


$$\Delta\omega_\Lambda = 2\pi\, v/\Lambda$$

Illuminate off-axis, detect on-axis

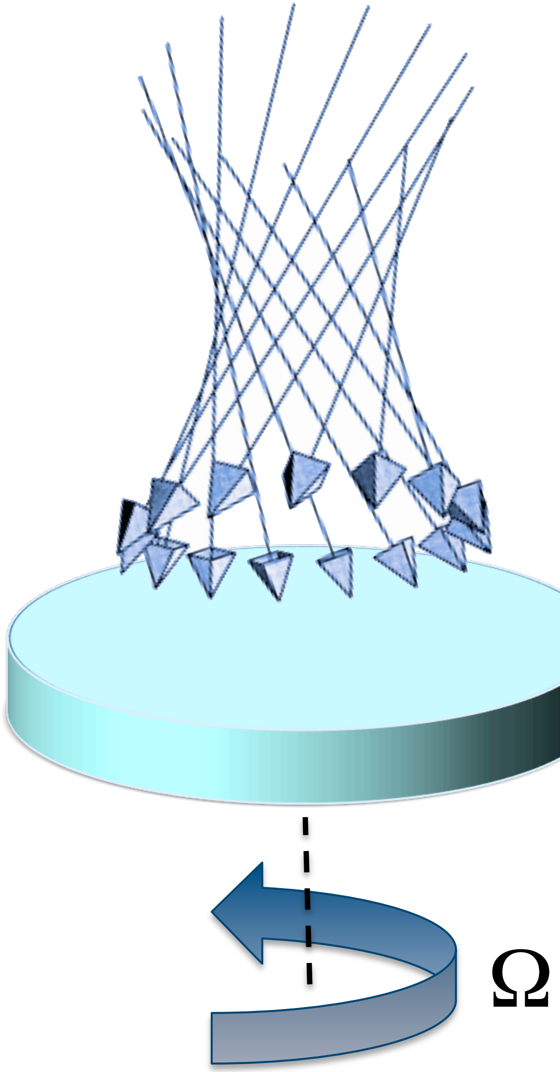
$$\Delta\omega_{a,b} = \Delta\omega_\Lambda$$

Orbital angular momentum -> Skew rays, $\alpha \neq 0$



$\alpha = \ell / kr$

Doppler shift from a SPINNING surface



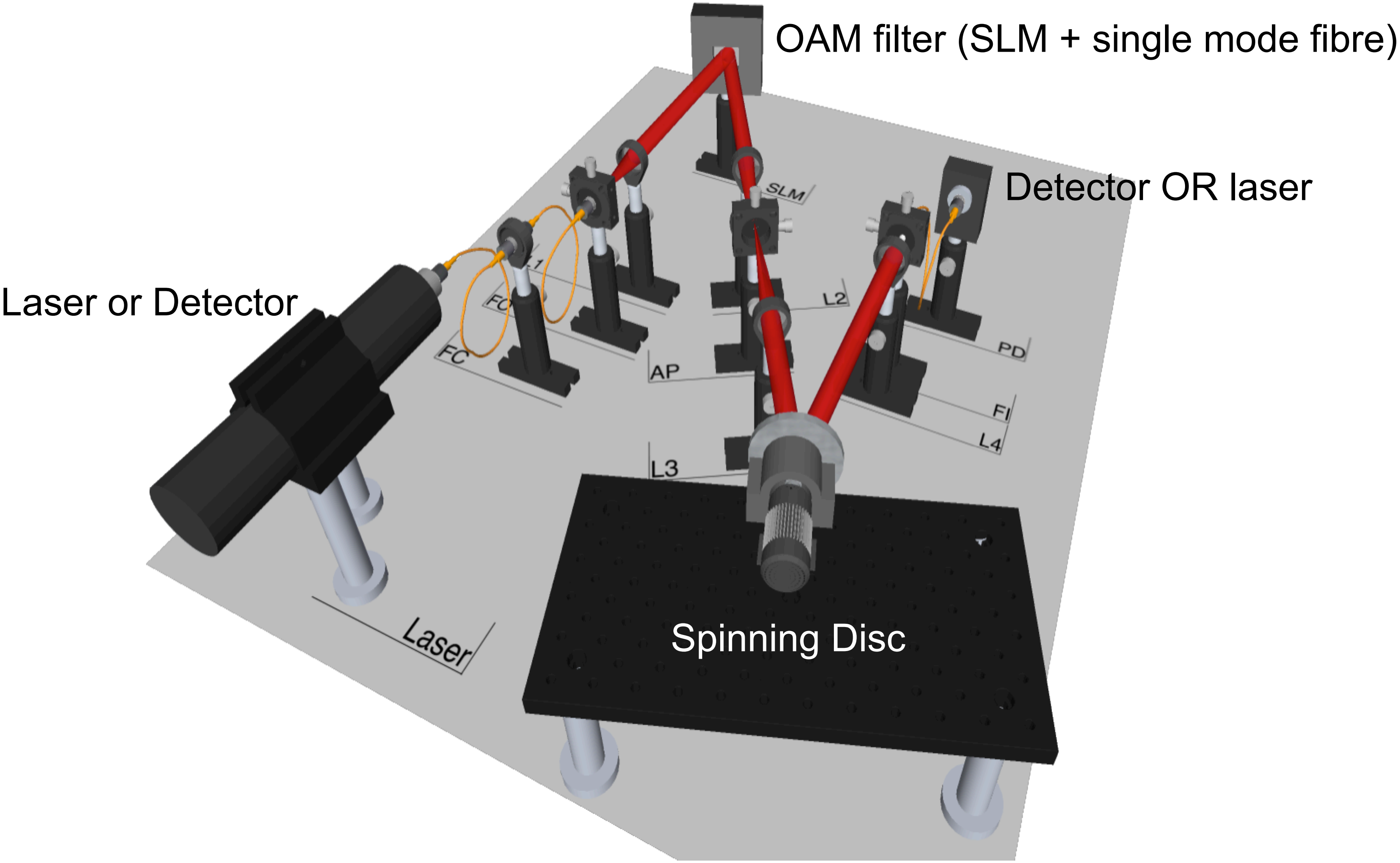
c.f. $\Delta\omega = \sin\alpha\ \omega_0 v/c$

$\sin\alpha \approx \ell/kr$ & $v = \Omega r$

$\Rightarrow \Delta\omega_{\ell} = \Omega\ \ell$

$\Delta\omega_{\ell,-\ell} = 2\Omega\ \ell$

Experimental arrangement



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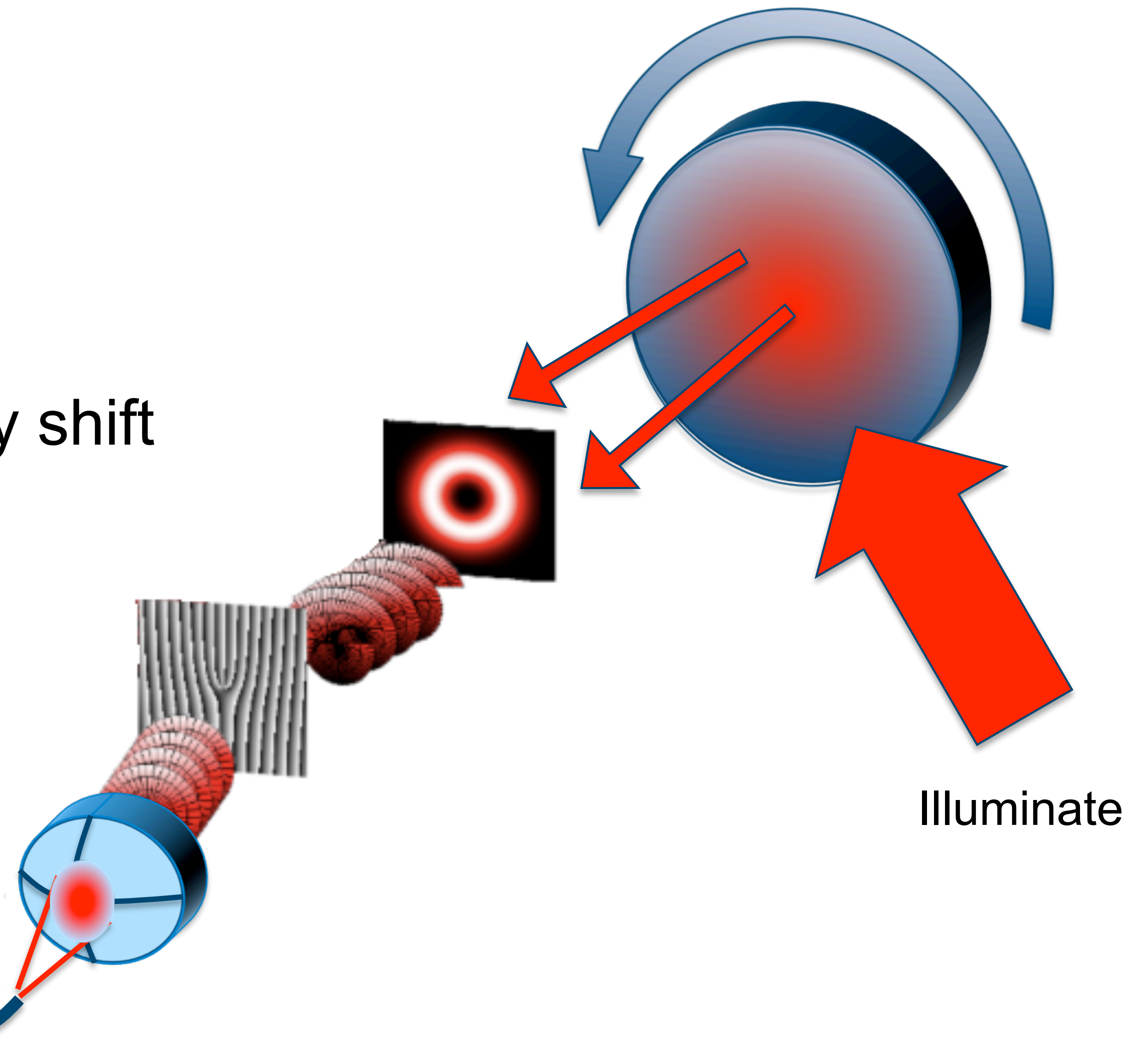
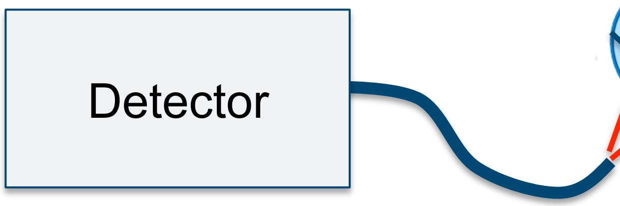
Martin Lavery

The Rotational Frequency shift of Scattered Light

Scattered Light

Observe frequency shift between +/- ℓ components





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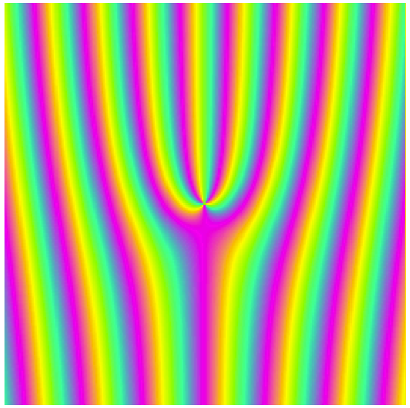
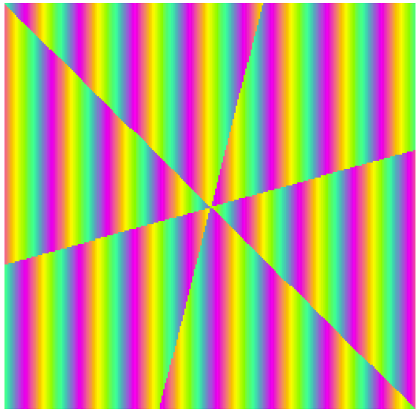
Making/Measuring OAM

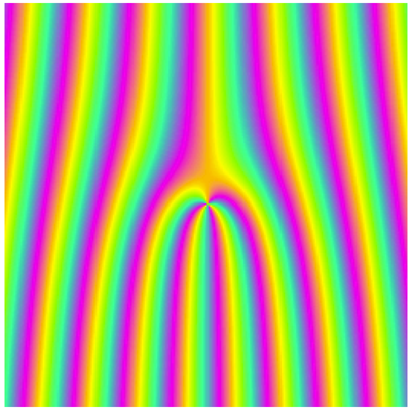
Diffraction grating (hologram) to make/ measure ℓ=3



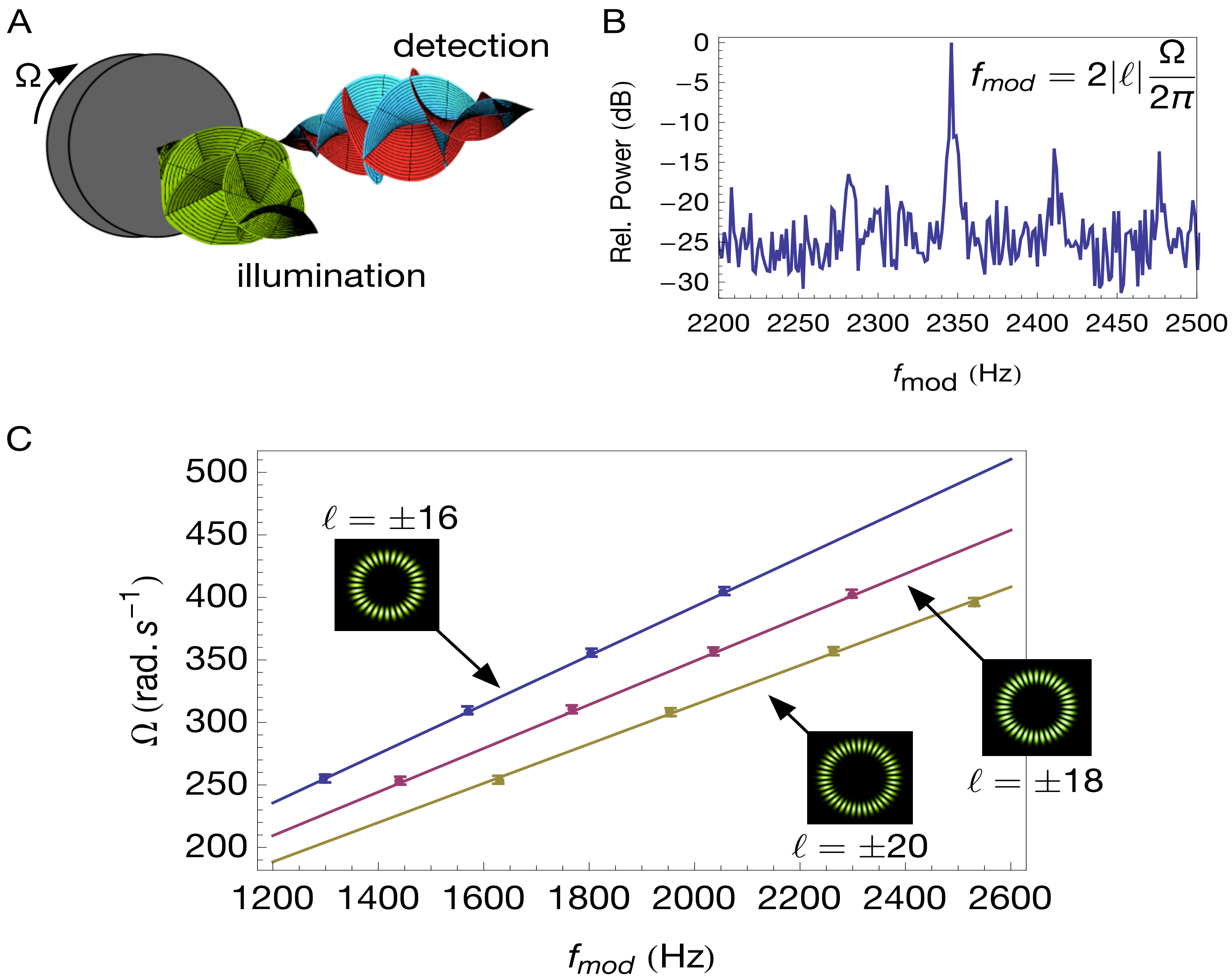


Diffraction grating (hologram) to make/ measure ℓ= +/-3

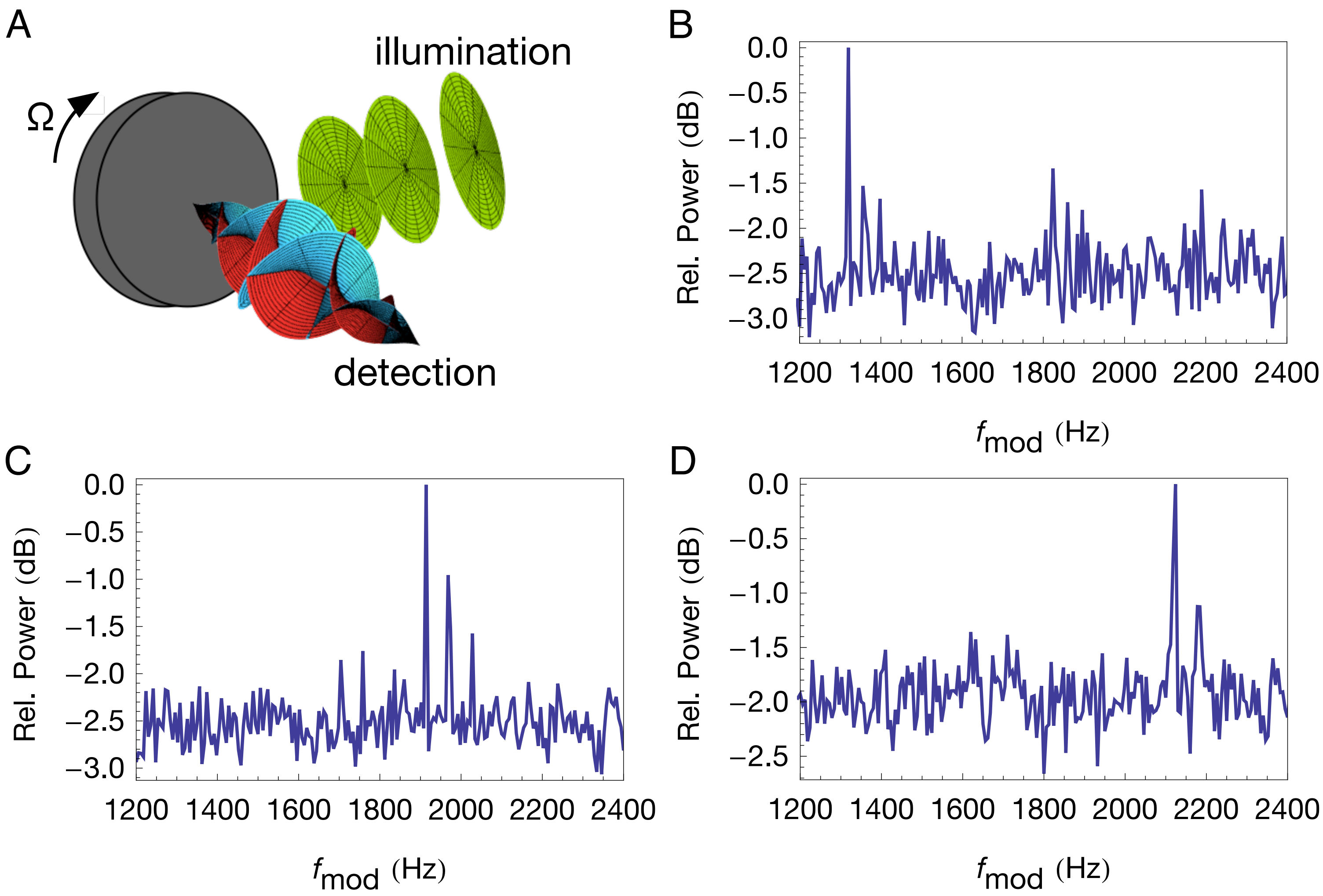
Diffraction grating (hologram) to make/ measure ℓ= -3



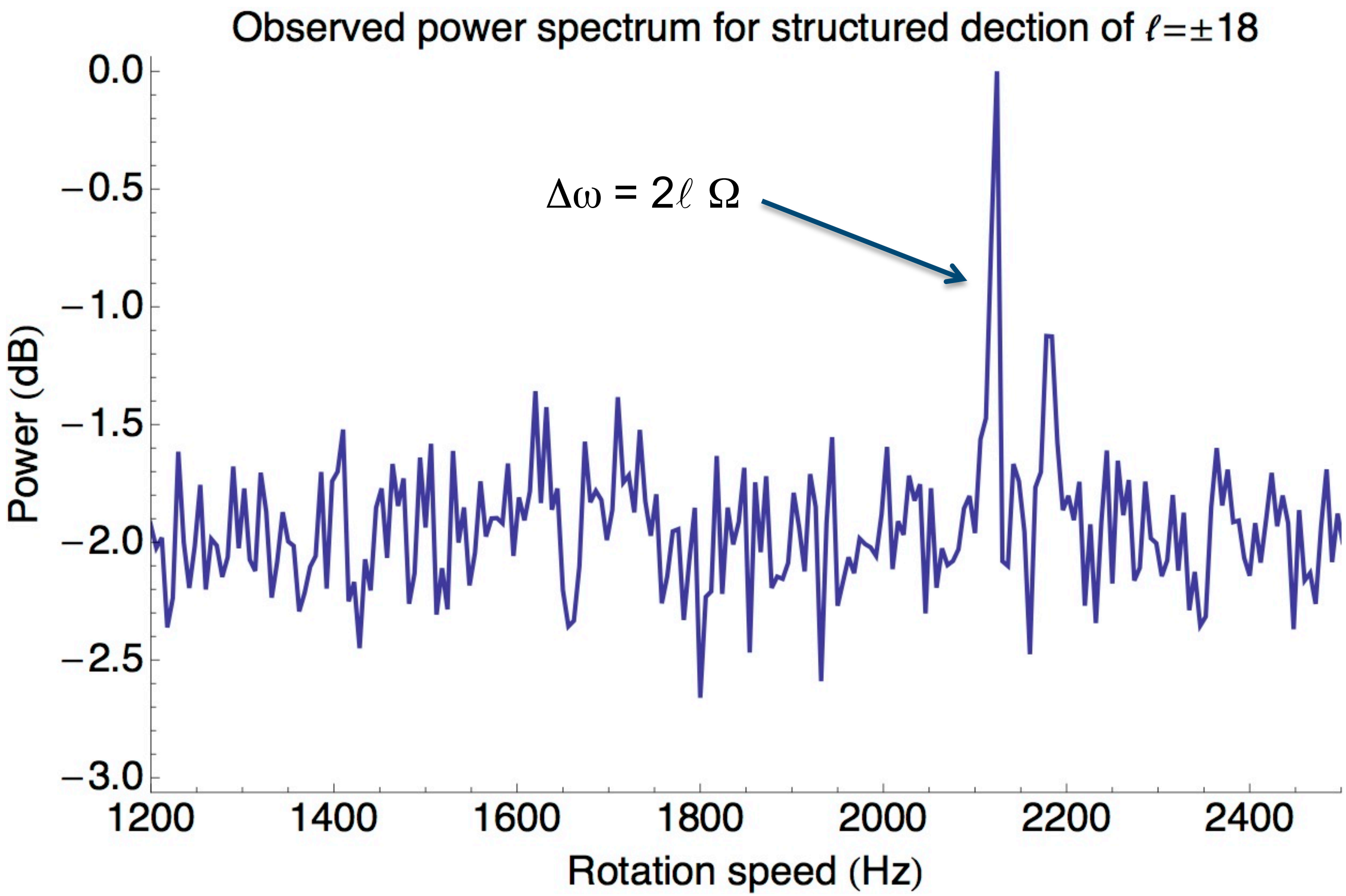
Illuminate with OAM at +/- ℓ and measure $\Delta\omega$



Illuminate with ≈ℓ=0, detect OAM at +/- ℓ and measure $\Delta\omega$

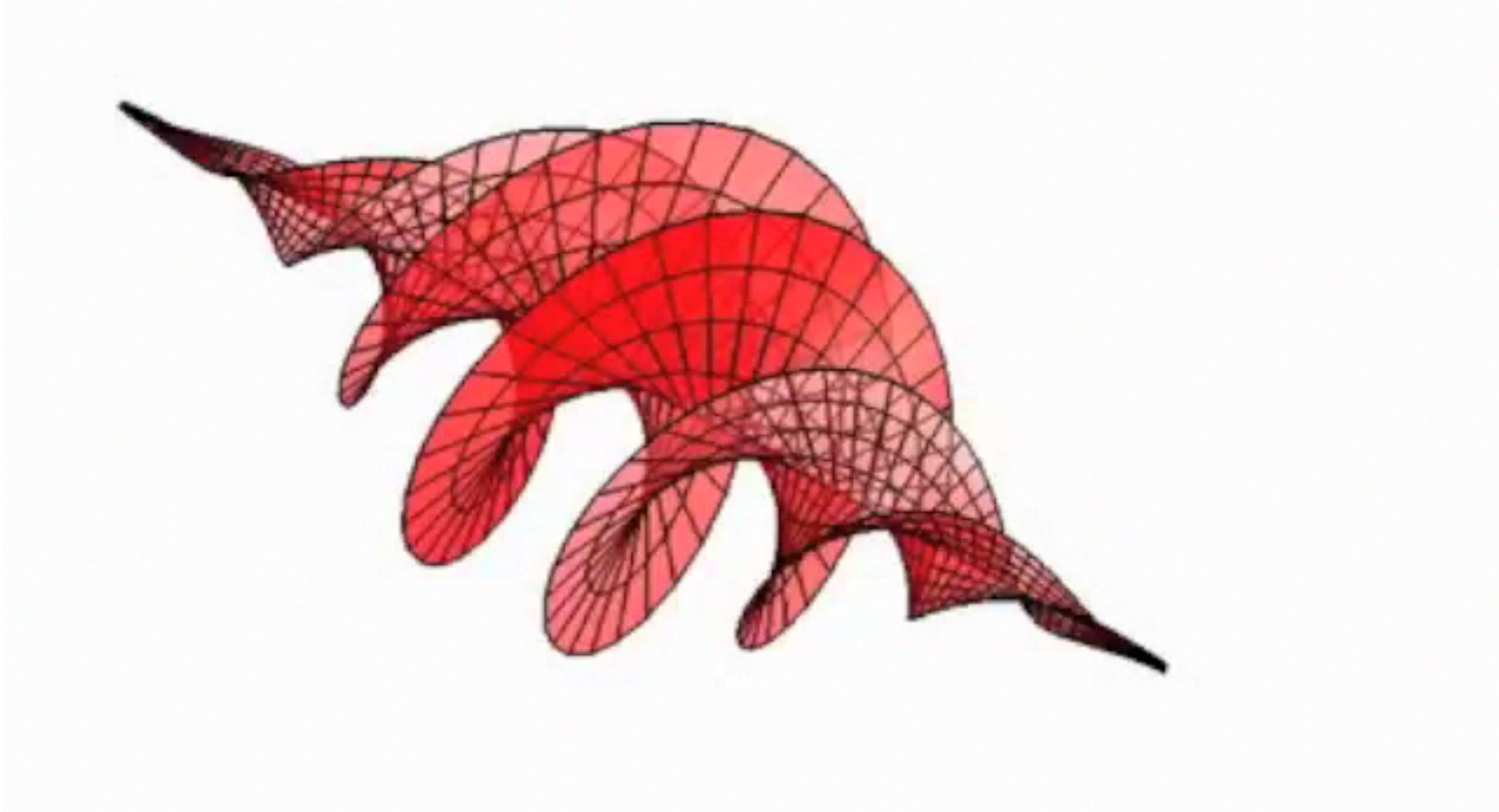


Rotational Doppler shift in scattered light



c.f. Speckle velocimetry? Albeit, in this case, angular

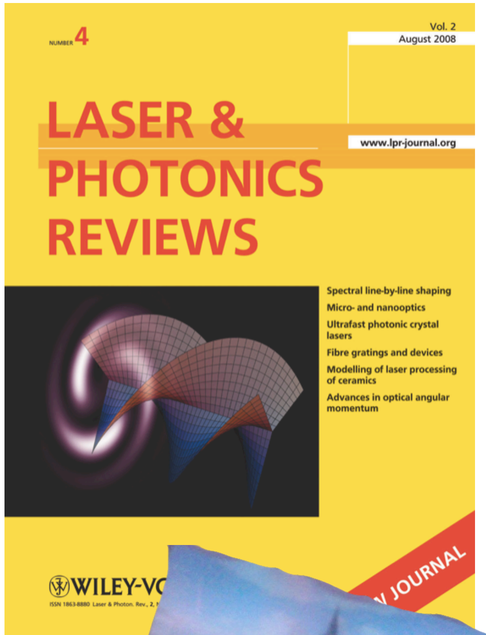
Thank you to you and my Group



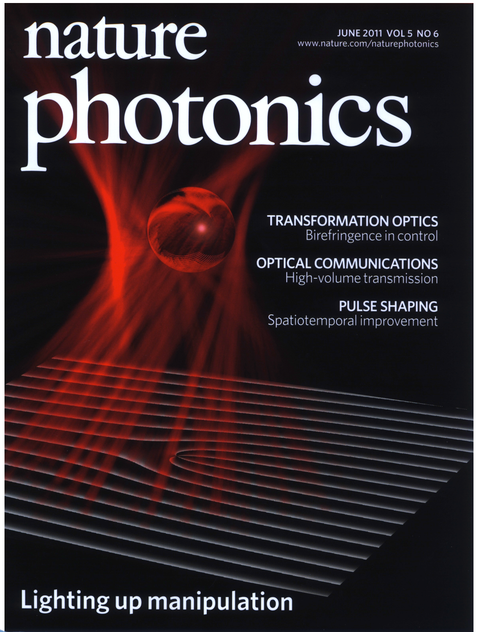
If you would like a copy of this talk please ask me

www.gla.ac.uk/schools/physics/research/groups/optics/

2008



2011



2012