Finite volume schemes for dispersive wave propagation and runup

Th. Katsaounis

Dept. of Applied Math. Univ. of Crete,
IACM, FORTH, Crete, GREECE

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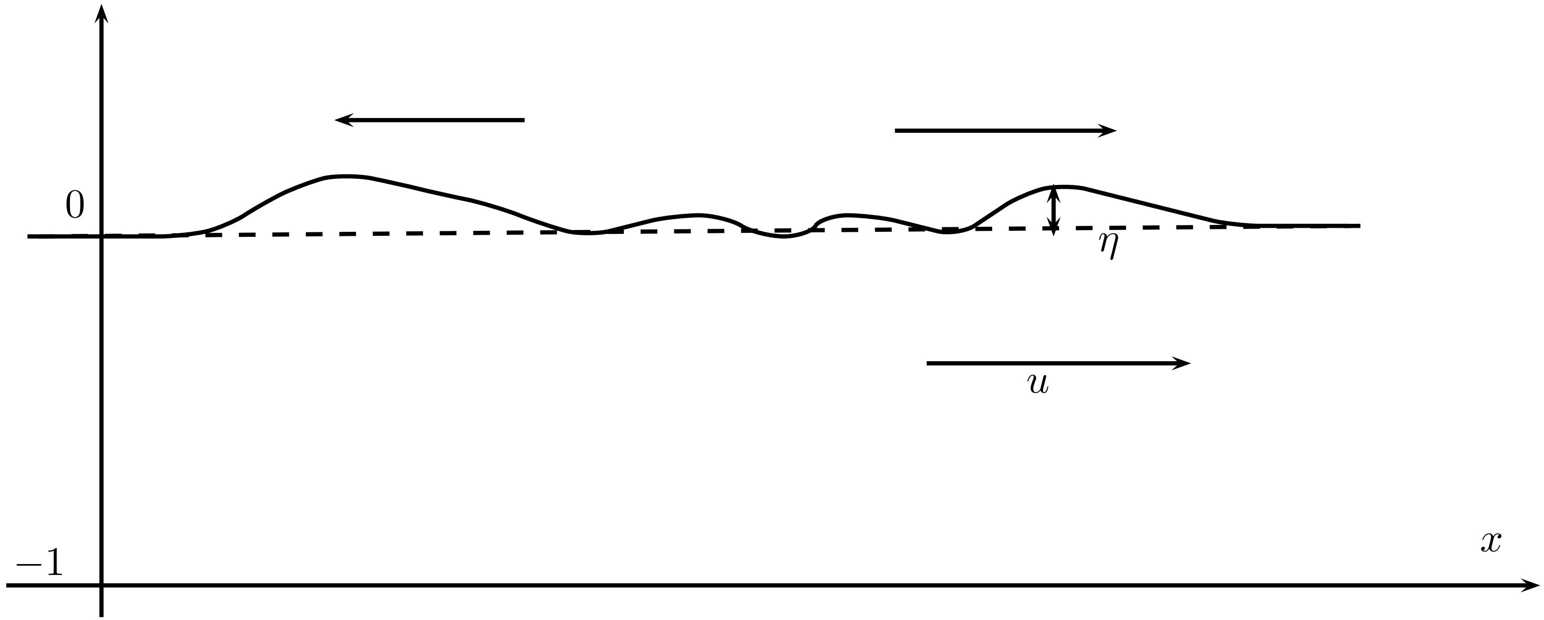
joint work with
D. Mitsotakis, UC Merced, USA , D. Dutykh, LAMA, France

Overview

- Bidirectional dispersive models (flat channel)
- A Hybrid method, FV and FD
- Numerical results
  - Convergence tests
  - Head on collision of solitary waves
  - Overtaking collision of solitary waves
- Dispersive models with variable bottom
  - A new model
  - Finite volume discretization
  - Solitary wave runup

Bidirectional models

Long dispersive waves over a flat bottom



- $\eta(x,t)$ deviation of the free surface from its rest position
- $u(x,t)$ horizontal velocity of the fluid at height z
- $z = -1 + \theta(1 + \eta(x,t))$ $0 \leq \theta \leq 1$, ($z = -1$ bottom)
- A : typical wave amplitude of fluid of depth h
- λ : typical wavelength.

Boussinesq systems

Family of Boussinesq systems introduced by Bona, Chen, Saut 2002 (nondimensional, unscaled variables)

$$\eta_t + u_x + (\epsilon)\left[(\eta u)_x + a u_{xxx} - b\eta_{xxt}\right] = 0(\epsilon^2),$$
$$u_t + \eta_x + (\epsilon)\left[u u_x + c\eta_{xxx} - d u_{xxt}\right] = 0(\epsilon^2),$$

$a = \frac{1}{2}(\theta^2 - \frac{1}{3})\nu,\ b = \frac{1}{2}(\theta^2 - \frac{1}{3})(1-\nu), c = \frac{1}{2}(1-\theta^2)\mu,\ d = \frac{1}{2}(1-\theta^2)(1-\mu),$
$0 \le \theta \le 1,\ \ \mu, \nu \in \mathbb{R}$

$\epsilon := \frac{A}{h} << 1,\ \mu^2 := \frac{h^2}{\lambda^2} << 1,\ S := \frac{\epsilon}{\mu^2} = \frac{A\lambda^2}{h^3} \sim 1$

Approximation to 2D Euler equations (irrotational free surface flow, under gravity, of an incompressible inviscid fluid in a uniform horizontal channel)

Derivation from : primitive variables or potential flow formulations

- The system describes two-way wave propagation (unlike KdV or BBM models)
- Mass conservation : $I_0 = \int \eta \, dx$.
- Hamiltonian ("energy" functional) ($b = d$):
  $I_1 = \int (\eta^2 + (1+\eta)u^2 - c\eta_x^2 - au_x^2) dx$ is invariant $t \geq 0$.
- BBM-BBM: $a = c = 0$, $b = d = 1/6 > 0$ (IVP locally well posed)
- KdV-KdV: $a = c = 1/6$, $b = d = 0$, (IVP locally well posed)
- Bona-Smith: $a = 0$, $b = d = (3\theta^2 - 1)/6 > 0$, $c = (2 - 3\theta^2)/3 < 0$, (IVP globally well posed for $2/3 < \theta^2 \leq 1$)
- Classical-Boussinesq: $a = b = c = 0$, $d = 1/3$, (IVP globally well-posed)
- Existence of solitary wave solutions

Numerical method

Conservative like form

$$(\eta - b\eta_{xx})_t + (u + \eta u)_x + a(u_{xx})_x = 0,$$
$$(u - du_{xx})_t + (\eta + \frac{1}{2}u^2)_x + c(\eta_{xx})_x = 0,$$
$$\mathbf{E}(\mathbf{w})_t + [\mathbf{F}(\mathbf{w})]_x + [\mathbf{G}(\mathbf{w})]_x = 0$$

- State variables : $\mathbf{w} = (\eta, u)^T$
- "Elliptic" operator : $\mathbf{E}(\mathbf{w}) = (\eta - b\partial_{xx}\eta, u - d\partial_{xx}u)^T$
- Advective flux : $\mathbf{F}(\mathbf{w}) = ((1+\eta)u, \eta + \frac{1}{2}u^2)^T$
- Dispersive flux : $\mathbf{G}(\mathbf{w}) = (au_{xx}, c\eta_{xx})^T$

Discretization : A hybrid method based on finite volume and finite differences

Existing Literature

Finite Differences, Continuous or Discontinuous Galerkin, Spectral Methods

FV/FD

- □ G. Bellotti, M. Brocchini, Int. J. Numer. Methods Fluids, 2001
- □ K.S. Erduran, S. Ilic, V. Kutija, Int. J. Numer. Methods Fluids, 2005
- □ F. Benkhaldoun, M. Seaid, Commun. Comput. Phys., 2008
- □ J.B. Shiach, C.G. Mingham, Coastal Eng., 2009
- □ M. Tonelli, M. Petti, Coastal Eng, 2009

A Hybrid method

- Finite Volume (FV) method for advection and dispersion
  - Numerical Fluxes for $\mathbf{F}$, $\mathbf{G}$
  - Reconstruction process for the conservative variables
- Finite Difference (FD) method for the elliptic part $\mathbf{E}$
- Time Discretization : Runge - Kutta methods which preserve the TVD property of the FV method

Finite volume schemes

$$\mathbf{E}(\mathbf{w})_t + [\mathbf{F}(\mathbf{w})]_x + [\mathbf{G}(\mathbf{w})]_x = 0$$

Semidiscrete schemes : we consider cells $C_i = [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}],\ x_i = i\Delta x,\ i \in \mathbb{Z}$

$$\frac{d}{dt}\mathcal{E}\mathbf{W}_i(t) + \frac{1}{\Delta x}\left[\mathcal{F}_{i+\frac{1}{2}} - \mathcal{F}_{i-\frac{1}{2}}\right] + \frac{1}{\Delta x}\left[\mathcal{G}_{i+\frac{1}{2}} - \mathcal{G}_{i-\frac{1}{2}}\right] = 0$$

$$\mathbf{W}_i(t) \approx \frac{1}{\Delta x}\int_{C_i} \mathbf{w}(x,t)dx, \quad \mathbf{W}_i(0) = \frac{1}{\Delta x}\int_{C_i} \mathbf{w}(x,0)dx,$$

$$\mathcal{F}_{i+\frac{1}{2}} := \mathcal{F}(\mathbf{W}^L_{i+\frac{1}{2}}, \mathbf{W}^R_{i+\frac{1}{2}}) \approx \mathbf{F}(\mathbf{w}(x_{i+\frac{1}{2}} \mp 0, t))$$

$$\mathcal{G}_{i+\frac{1}{2}} := \mathcal{G}(\mathbf{W}^L_{i+\frac{1}{2}}, \mathbf{W}^R_{i+\frac{1}{2}}) \approx \mathbf{G}(\mathbf{w}(x_{i+\frac{1}{2}} \mp 0, t))$$

$\mathbf{W}^{L,R}_{i+\frac{1}{2}} \approx \mathbf{w}(x_{i+\frac{1}{2}}, t)$ from cells $C_i,\ C_{i+1}$

$\mathcal{E}$ a finite difference discretization of $\mathbf{E}$

Finite volume - difference discretization

Advective Numerical Fluxes

1. Average flux : $\mathcal{F}^m(\mathbf{w},\mathbf{v}) = \mathbf{F}(\frac{\mathbf{w}+\mathbf{v}}{2})$

2. Central flux : (Kurganov, Tadmor 2000),
   $\mathcal{F}^{KT}(\mathbf{w},\mathbf{v}) = \frac{1}{2}\left\{[\mathbf{F}(\mathbf{w}) + \mathbf{F}(\mathbf{v})] - \mathcal{A}(\mathbf{w},\mathbf{v})\,[\mathbf{w}-\mathbf{v}]\right\}$
   $\mathcal{A}(\mathbf{w},\mathbf{v}) = \max\left[\rho\left(\partial\mathbf{F}(\mathbf{w})\right), \rho\left(\partial\mathbf{F}(\mathbf{v})\right)\right]$

3. Characteristic flux : (Ghidaglia, Kumbaro, Le Coq, 1996),
   $\mathcal{F}^{CF}(\mathbf{w},\mathbf{v}) = \frac{1}{2}\left\{[\mathbf{F}(\mathbf{w}) + \mathbf{F}(\mathbf{v})] - \mathcal{A}(\mathbf{w},\mathbf{v})\,[\mathbf{F}(\mathbf{v})-\mathbf{F}(\mathbf{w})]\right\}$
   $\mathcal{A}(\mathbf{w},\mathbf{v}) = \text{sign}\left(\partial\mathbf{F}\left(\frac{\mathbf{w}+\mathbf{v}}{2}\right)\right)$

Dispersive Flux: $\mathcal{G}(\mathbf{w},\mathbf{v}) = \mathbf{G}(\frac{\mathbf{w}+\mathbf{v}}{2})$, standard centered FD for the 2nd derv.

Reconstruction process $\mathbf{W}^{L,R}$: MUSCL(TVD2, UNO2), WENO3, Limiters

Elliptic operator $\mathcal{E}$: standard finite difference discretization

$$\mathbf{E}(\mathbf{w}) \approx \frac{\mathbf{W}_{i-1} + 10\mathbf{W}_i + \mathbf{W}_{i+1}}{12} - (b,d)\frac{\mathbf{W}_{i+1} - 2\mathbf{W}_i + \mathbf{W}_{i-1}}{\Delta x^2}$$

Fully discrete schemes

RK discretization

$$\mathcal{E}\mathbf{W}_i^{n+1} = \mathcal{E}\mathbf{W}_i^n - \frac{\Delta t}{\Delta x}\sum_{j=1}^{s} b_j \left(\mathcal{F}_{i+\frac{1}{2}}^{n,j} - \mathcal{F}_{i-\frac{1}{2}}^{n,j}\right) - \frac{\Delta t}{\Delta x}\sum_{j=1}^{s} b_j \left(\mathcal{G}_{i+\frac{1}{2}}^{n,j} - \mathcal{G}_{i-\frac{1}{2}}^{n,j}\right)$$

$$\mathcal{E}\mathbf{W}_i^{n,j} = \mathcal{E}\mathbf{W}_i^n - \frac{\Delta t}{\Delta x}\sum_{\ell=1}^{s} a_{j\ell} \left(\mathcal{F}_{i+\frac{1}{2}}^{n,\ell} - \mathcal{F}_{i-\frac{1}{2}}^{n,\ell}\right) - \frac{\Delta t}{\Delta x}\sum_{\ell=1}^{s} a_{j\ell} \left(\mathcal{G}_{i+\frac{1}{2}}^{n,\ell} - \mathcal{G}_{i-\frac{1}{2}}^{n,\ell}\right),$$

Explicit TVD Runge – Kutta :

$$\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{2} \\ \hline \frac{1}{6} & \frac{1}{6} & \frac{2}{3} & \end{array}$$

Linear system for $\mathcal{E}$: tridiagonal solve.

Numerical results

Bona-Smith system $\theta^2 = 8/10$, solitary wave solution of amplitude $\eta_0 = 1/2$ in $[-50, 50]$, $T = 200$.

$$E_h^2(k) = \|u - U^k\|_h / \|U^0\|_h, \quad \|u - U^k\|_h = \left(\sum_i \Delta x |u(x_i) - U_i^k|^2 \right)^{1/2},$$

$$E_h^\infty(k) = \|u - U^k\|_{h,\infty} / \|U^0\|_{h,\infty}, \quad \|u - U^k\|_{h,\infty} = \max_i |u(x_i) - U_i^k|,$$

(a) Average Flux(cell averages)

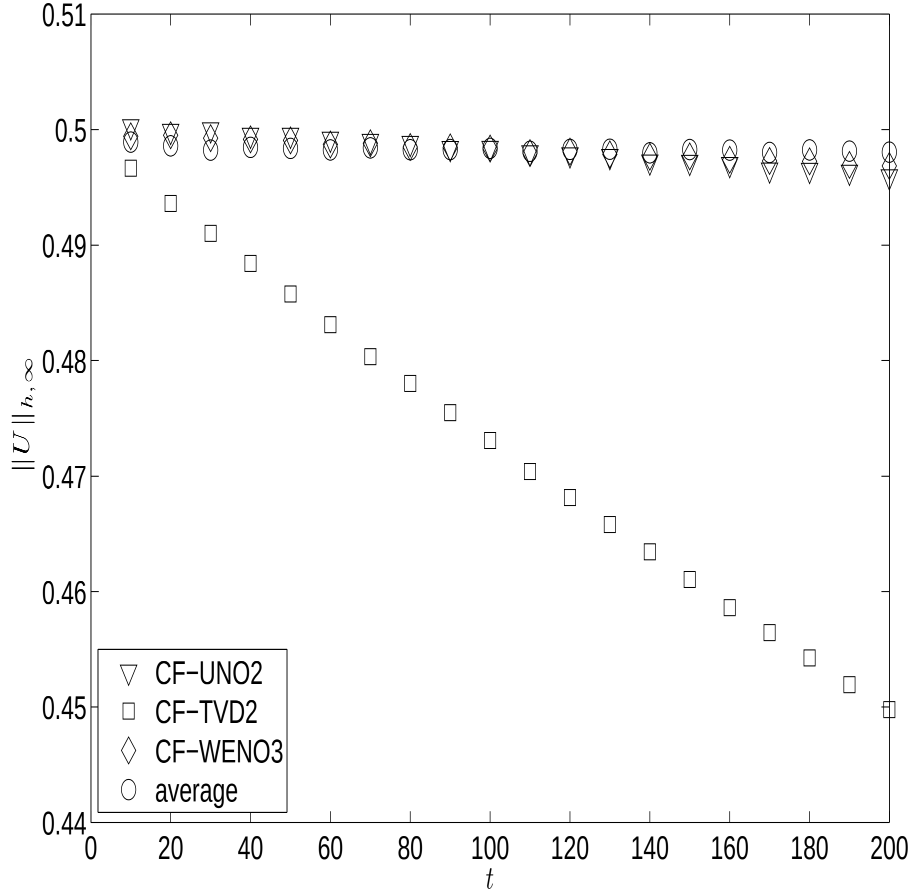
Δx	Rate(E_h^2)	Rate(E_h^∞)
0.5	1.910	1.978
0.25	1.910	1.954
0.125	1.923	1.937
0.0625	1.936	1.941
0.03125	1.946	1.948

(b) CF-TVD2 MinMod

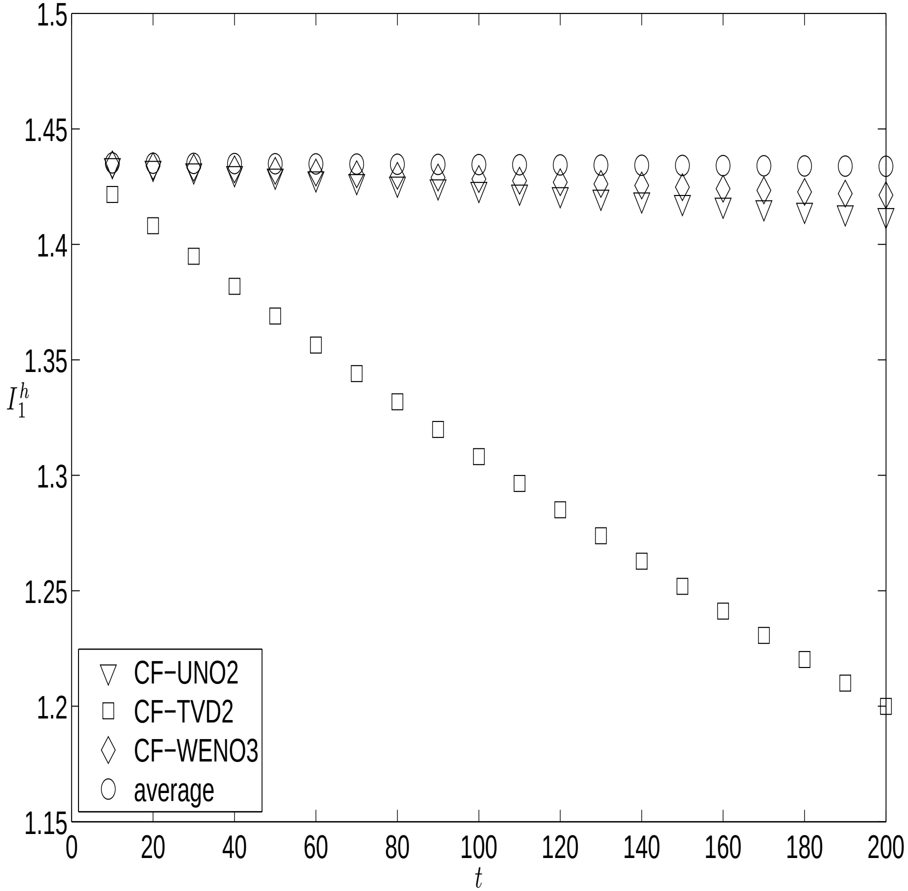
Δx	Rate(E_h^2)	Rate(E_h^∞)
0.5	2.042	2.032
0.25	2.033	2.029
0.125	2.026	2.023
0.0625	2.021	2.019
0.03125	2.017	2.016

Invariant preservation

$I_0 = \int \eta(x,t)dx = 1.932183566158$, $I_1 = \int(\eta^2 + (1+\eta)u^2 - c\eta_x^2 - au_x^2)dx$,
$\Delta x = 0.1$, $\Delta t = \Delta x/2$, $c_s = 1.2344$



(a) Evolution of η amplitude



(b) Evolution of I_1^h

Head on collision of solitary waves

- Bona-Smith system: $\theta^2 = 9/11,\ a = 0, b = d = (3\theta^2 - 1),\ c = (2 - 3\theta^2)/3$
- BBM-BBM system: $\theta^2 = 2/3$, $a = c = 0,\ b = d = 1/6$, Periodic BC's
- Periodic solitary waves in $[-300, 300]$, $c_{r,s} = 0.854m/s,\ c_{\ell,s} = 0.752m/s$
- Discretization : KT or CF , UNO2 or WENO3, $\Delta x = 0.05cm,\ \Delta t = 0.01s$
- Comparison with experimental data: (Graig, Guyenne, Hammack, Henderson, Sulem, Physics of Fluids 2006)
- Mass conservation:

BS: $I_0^h = 0.0059904310418,$ BBM-BMM: $I_0^h = 0.0059199389479$

- Very good agreement with the experimental data

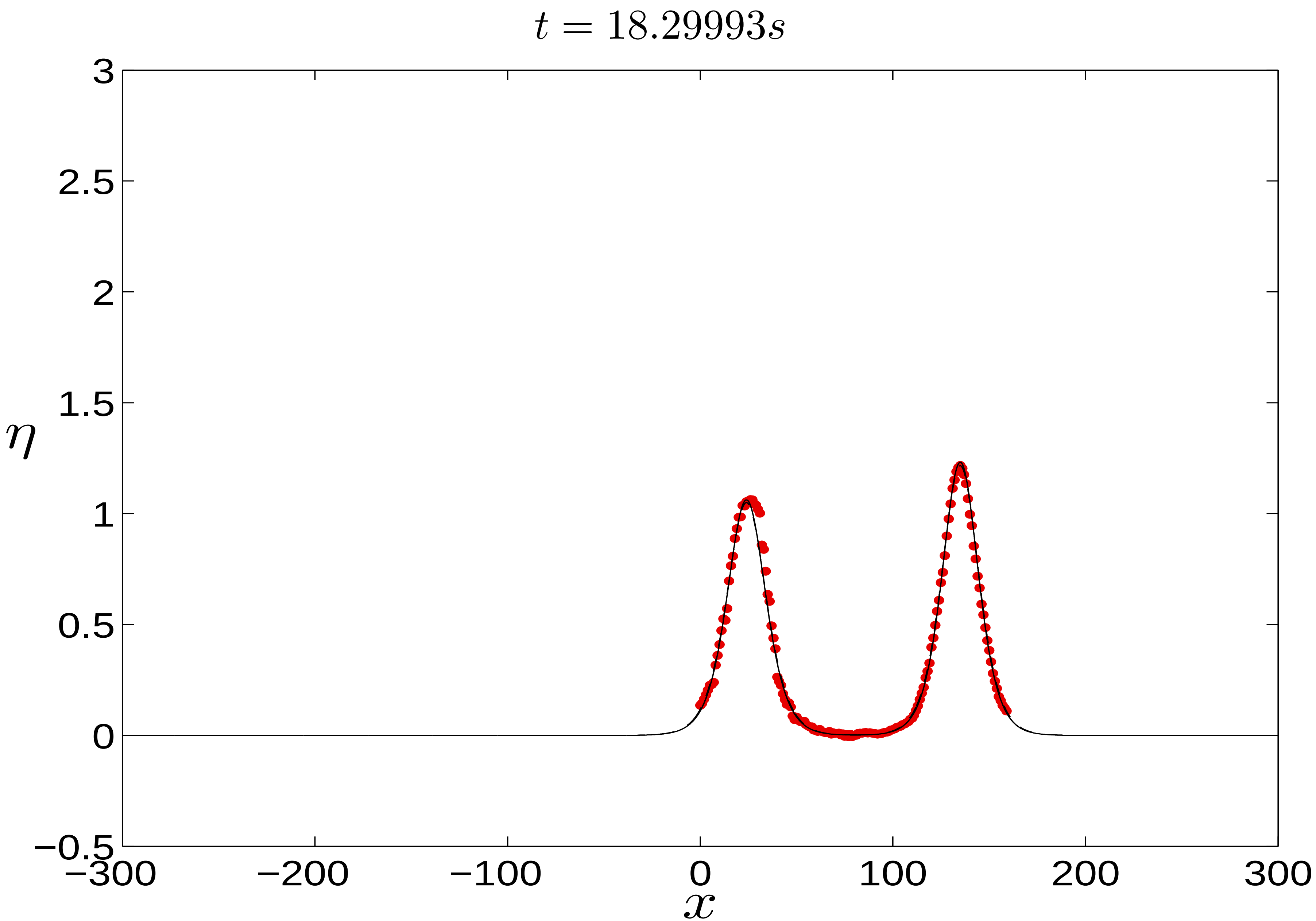
(c) Bona-Smith

flux	I_1^h
Avrg	0.000944236
UNO2	0.00094423
TVD2	0.00094
WENO3	0.00094423

(d) BBM-BBM

flux	I_1^h
Avrg	0.00092793
UNO2	0.00092793
TVD2	0.00092
WENO3	0.00092793

Head on collision



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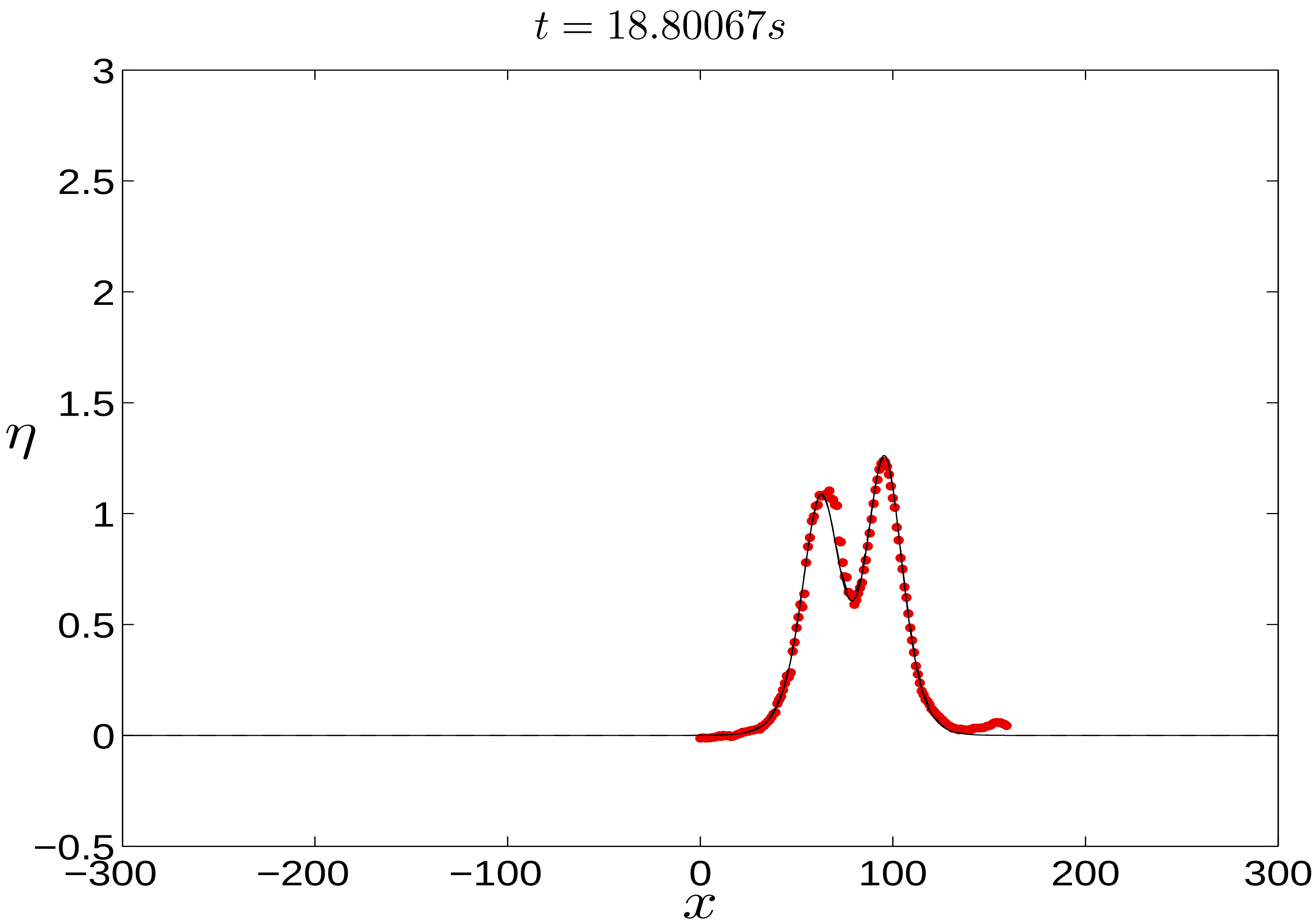

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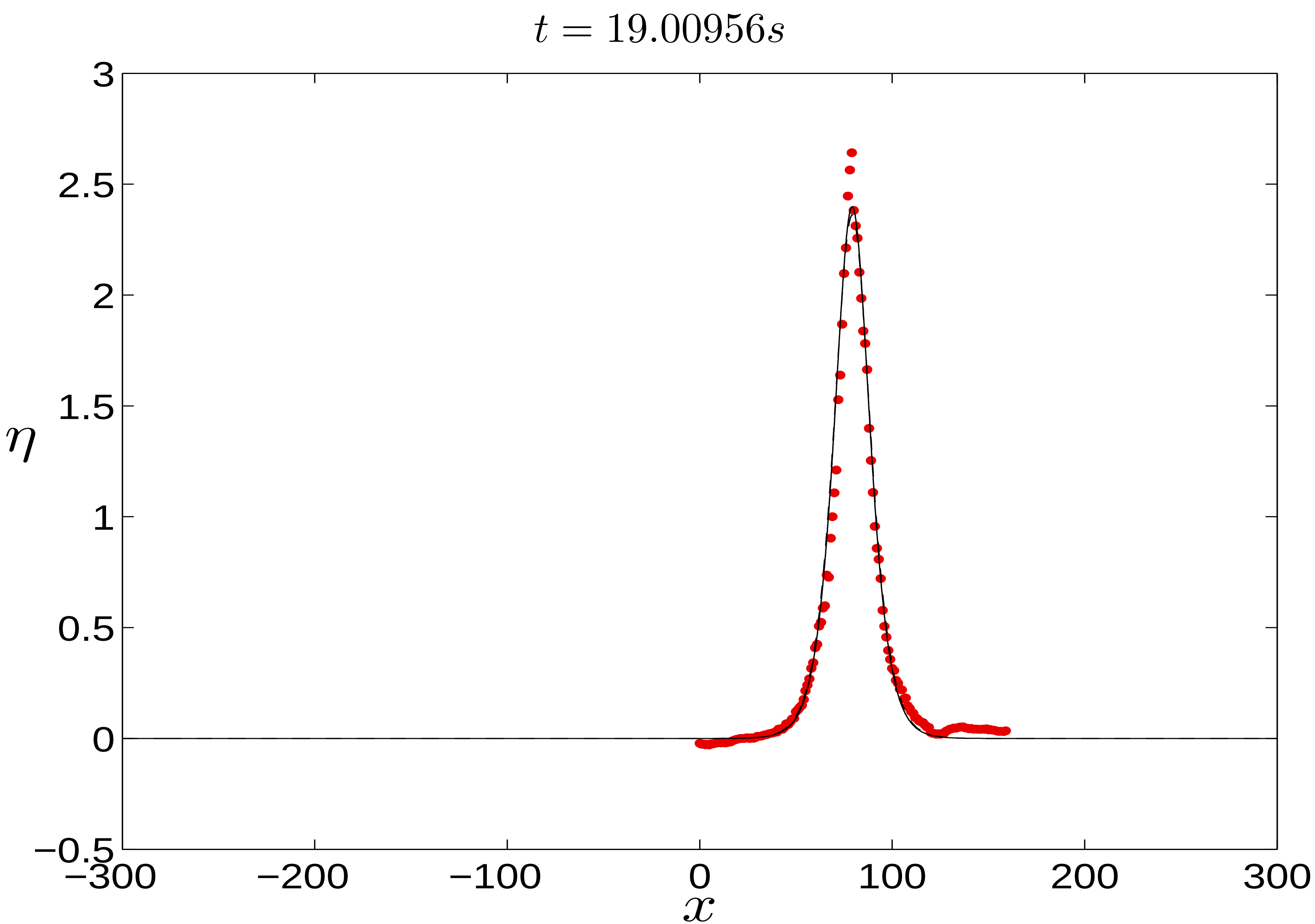

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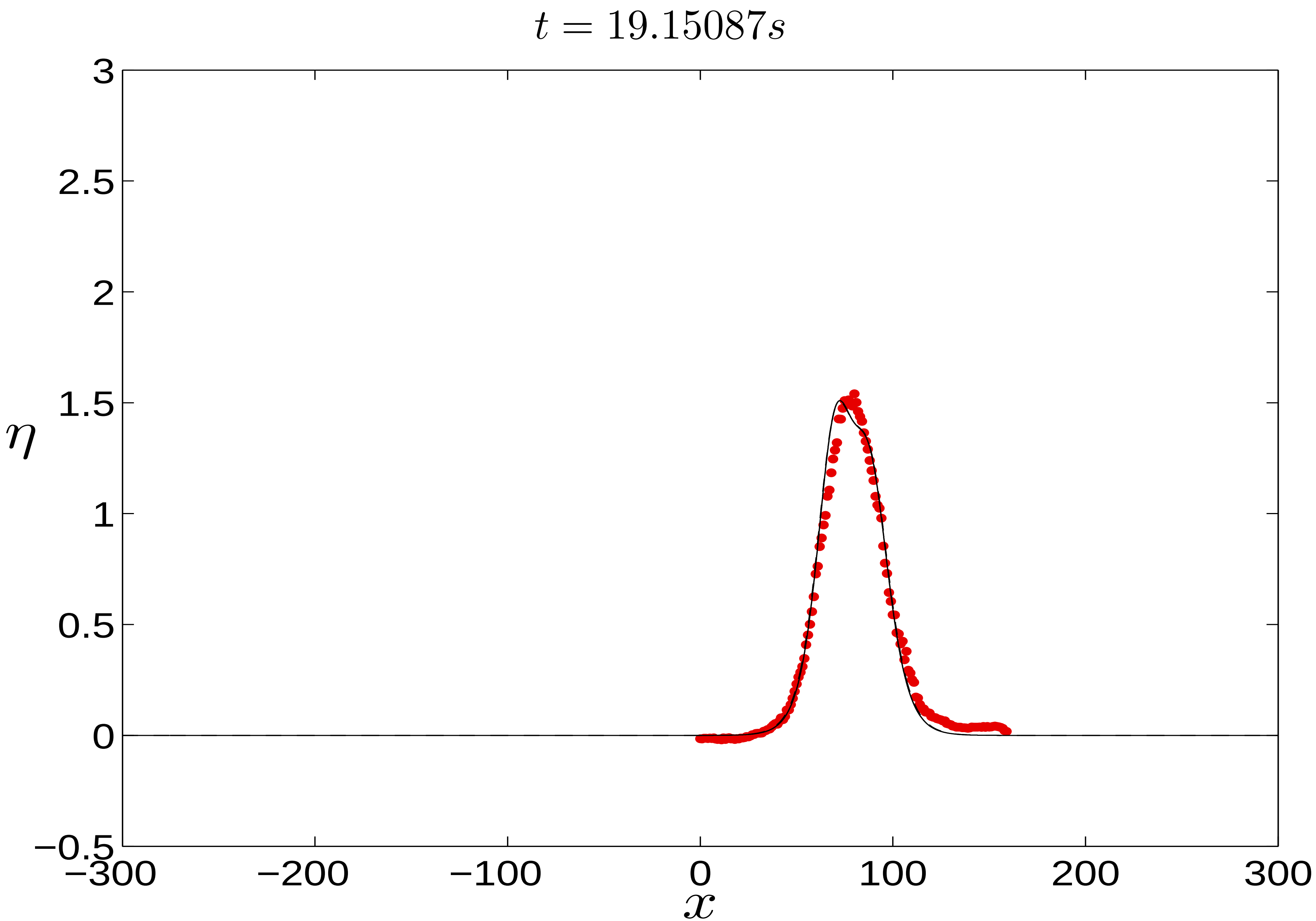

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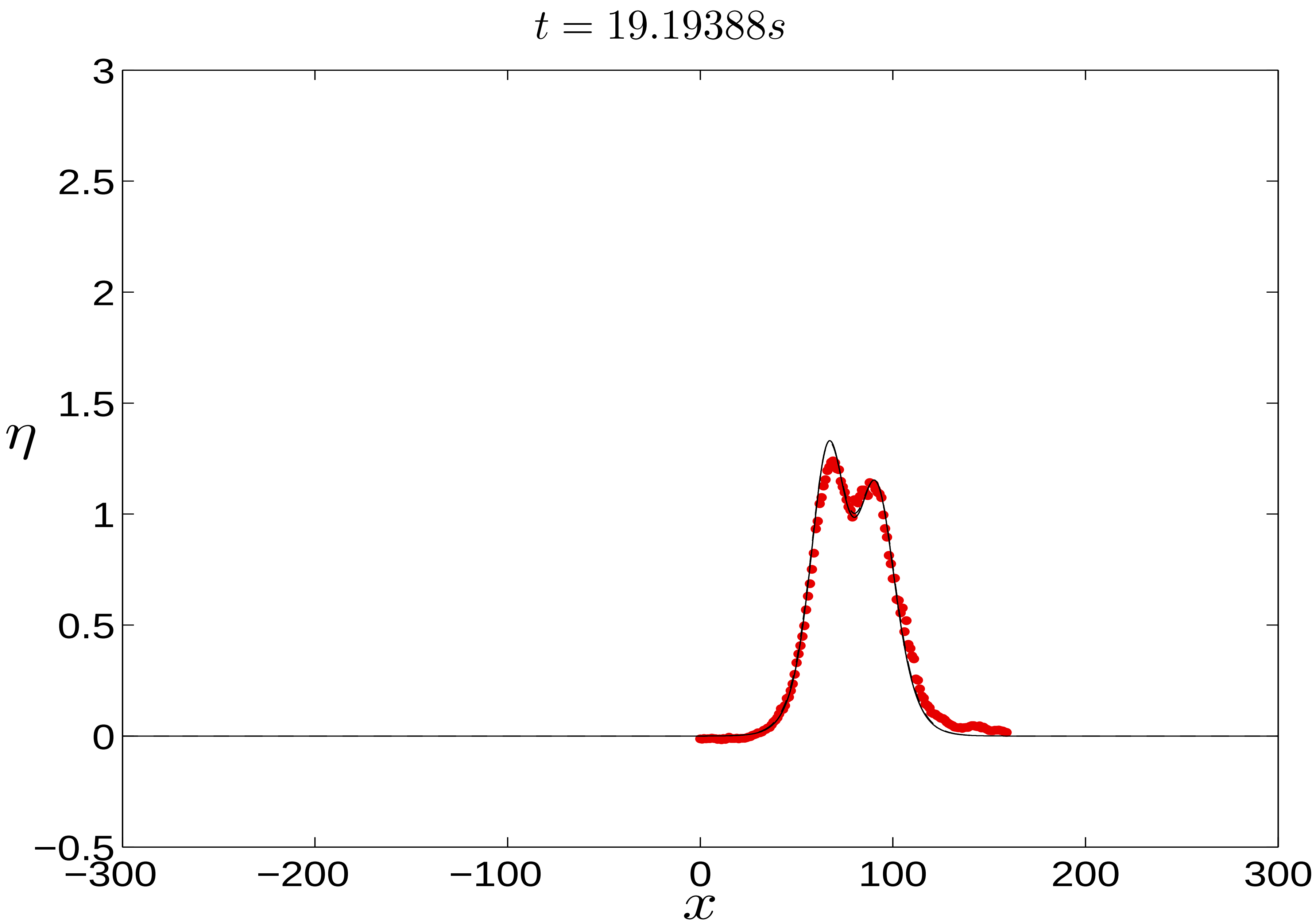

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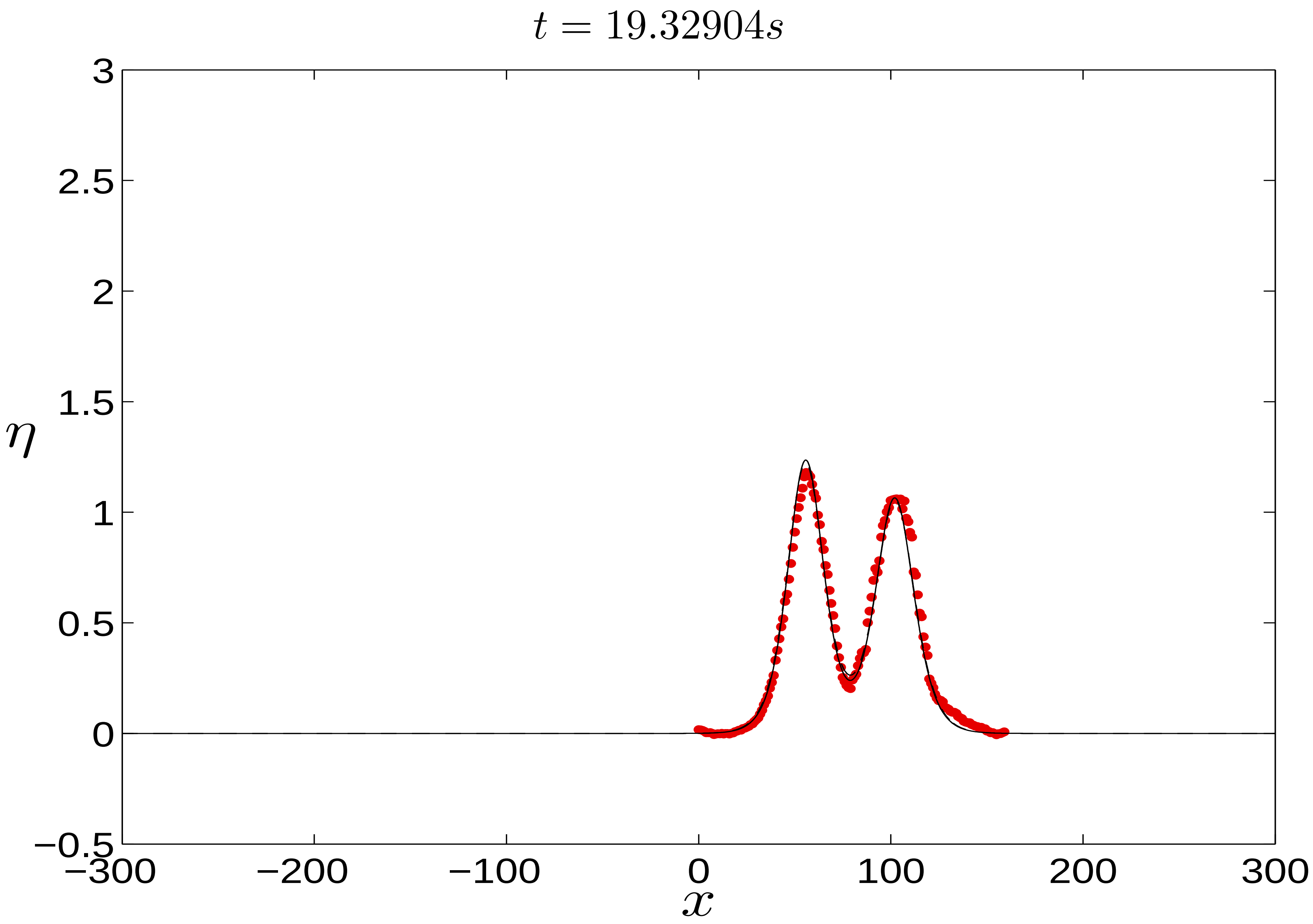

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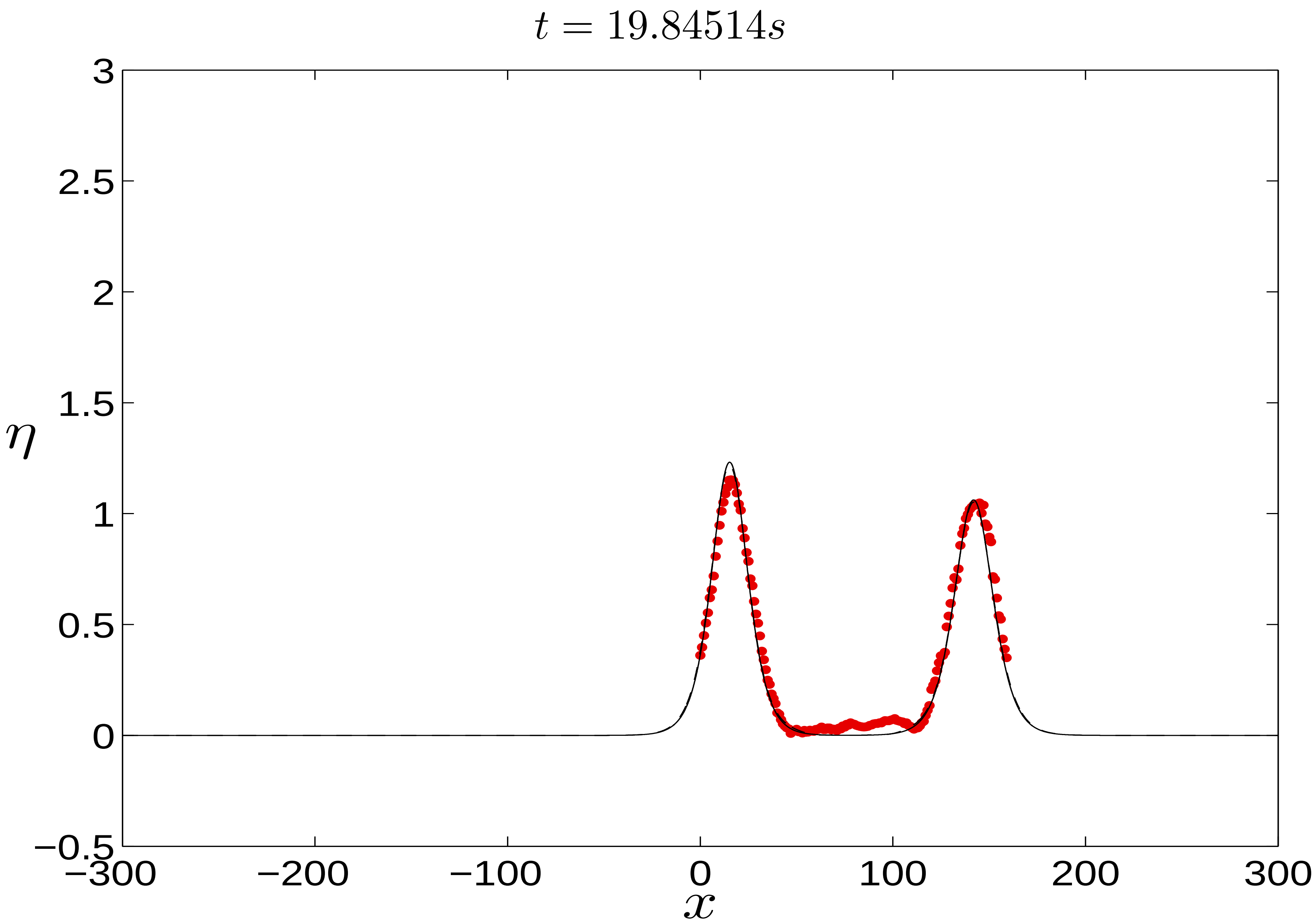

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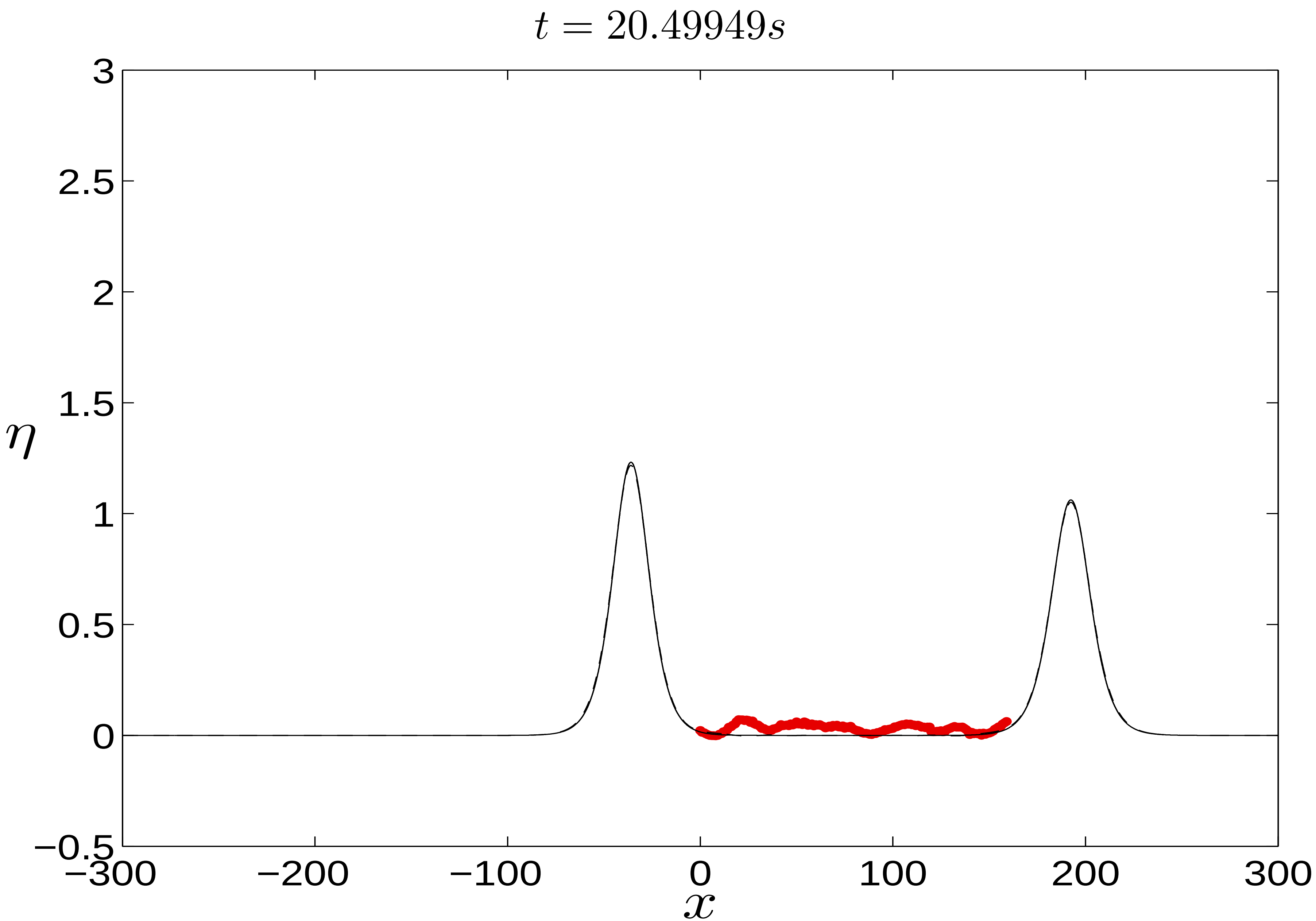


Overtaking collision of two solitary pulses

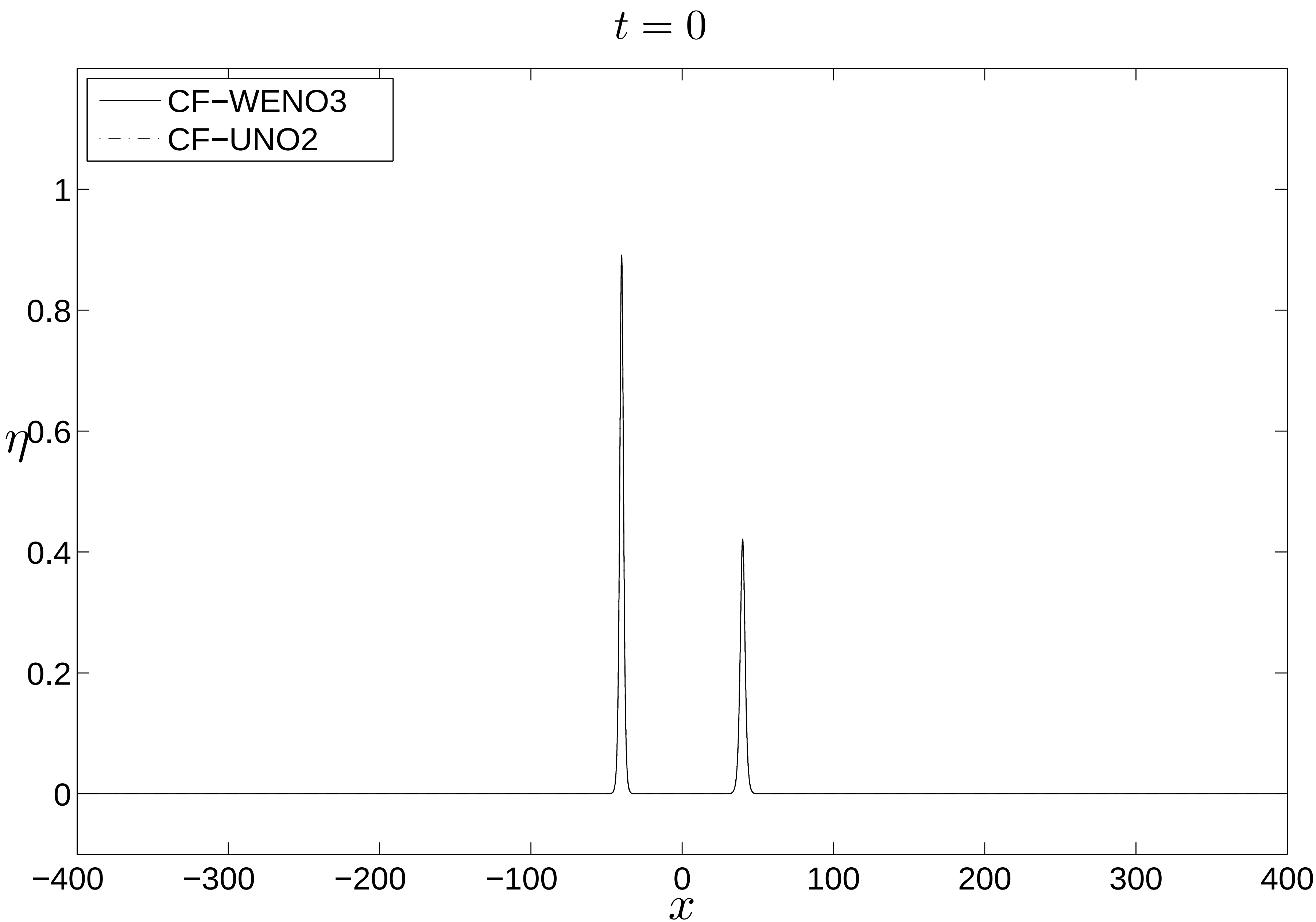
- Bona-Smith system: $\theta^2 = 9/11$, Periodic B.C's
- Periodic solitary waves in $[-400, 400]$ $c_{1,s} = 1.2, c_{2,s} = 1.4$
- Discretization : KT or CF , UNO2 or WENO3, $\Delta x = 0.01,\ \Delta t = 0.005$
- Mass conservation $I_0^h = 4.6098804880$, $I_1^h = 5.116$

- No distinction between the two discretizations,

- Mass exchange between the waves, not superposition, no merging, two different maxima.

- A new N-shape wavelet is generated during the interaction traveling in opposite direction.

Overtaking collision



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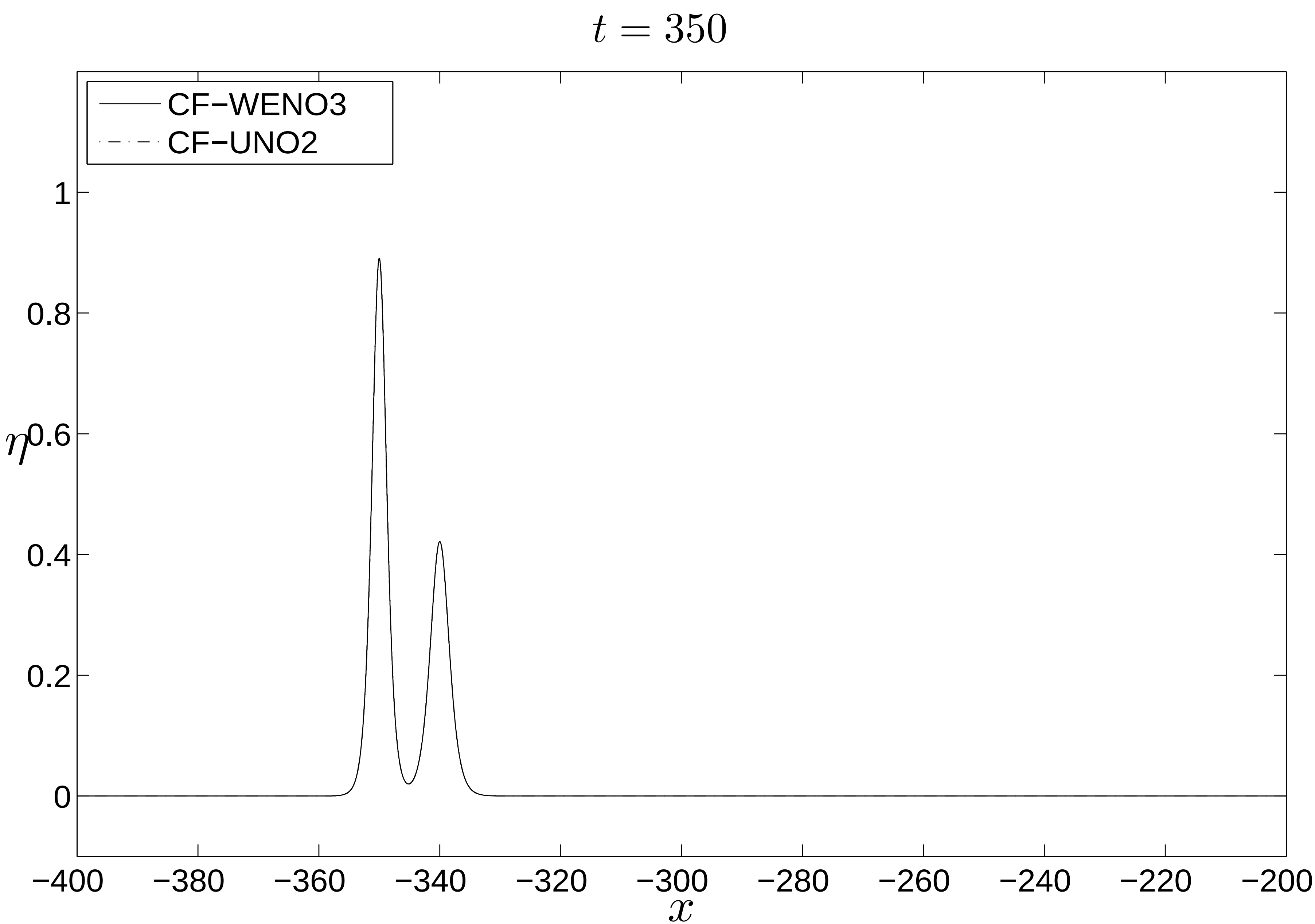
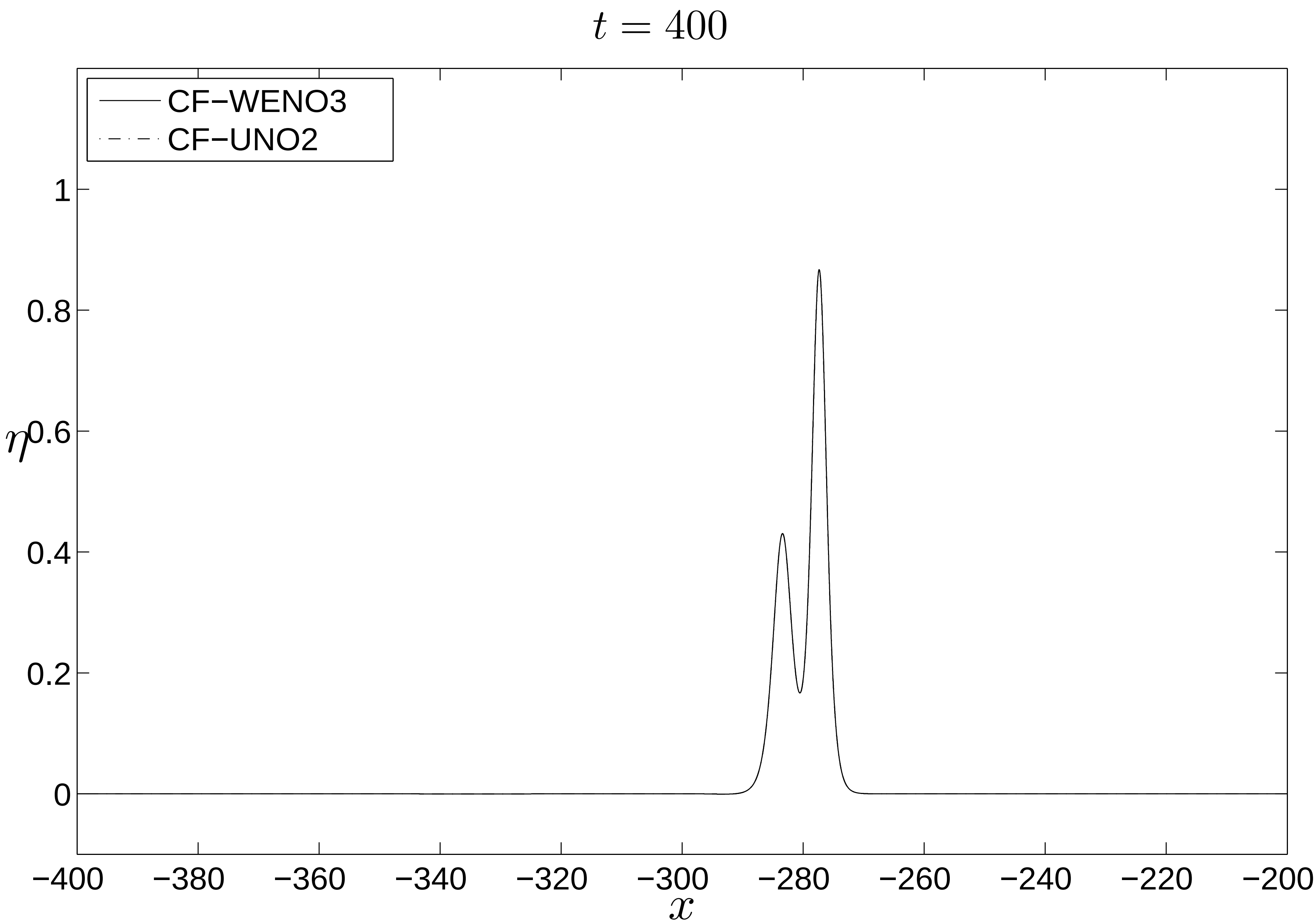


Overtaking collision



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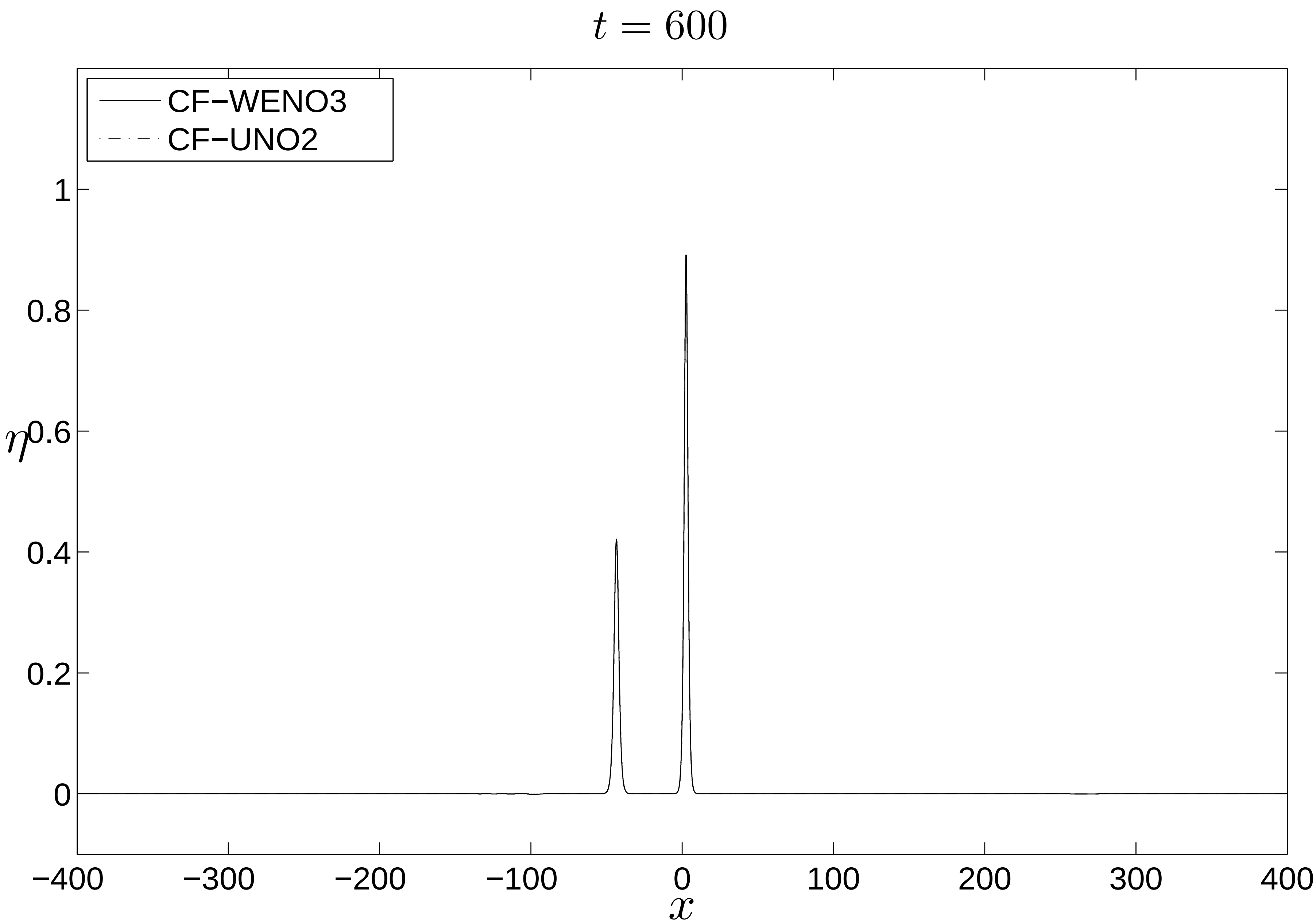

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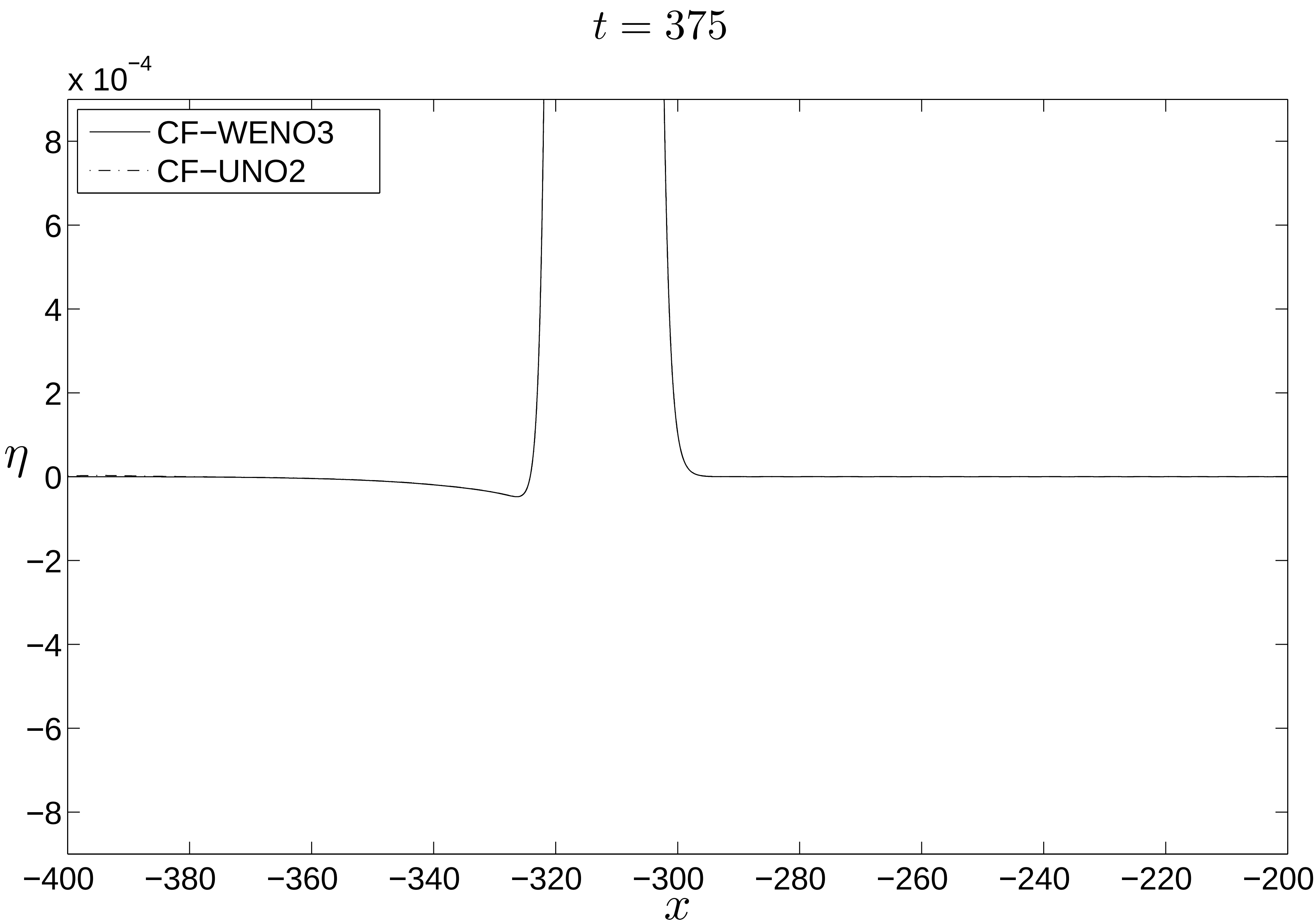

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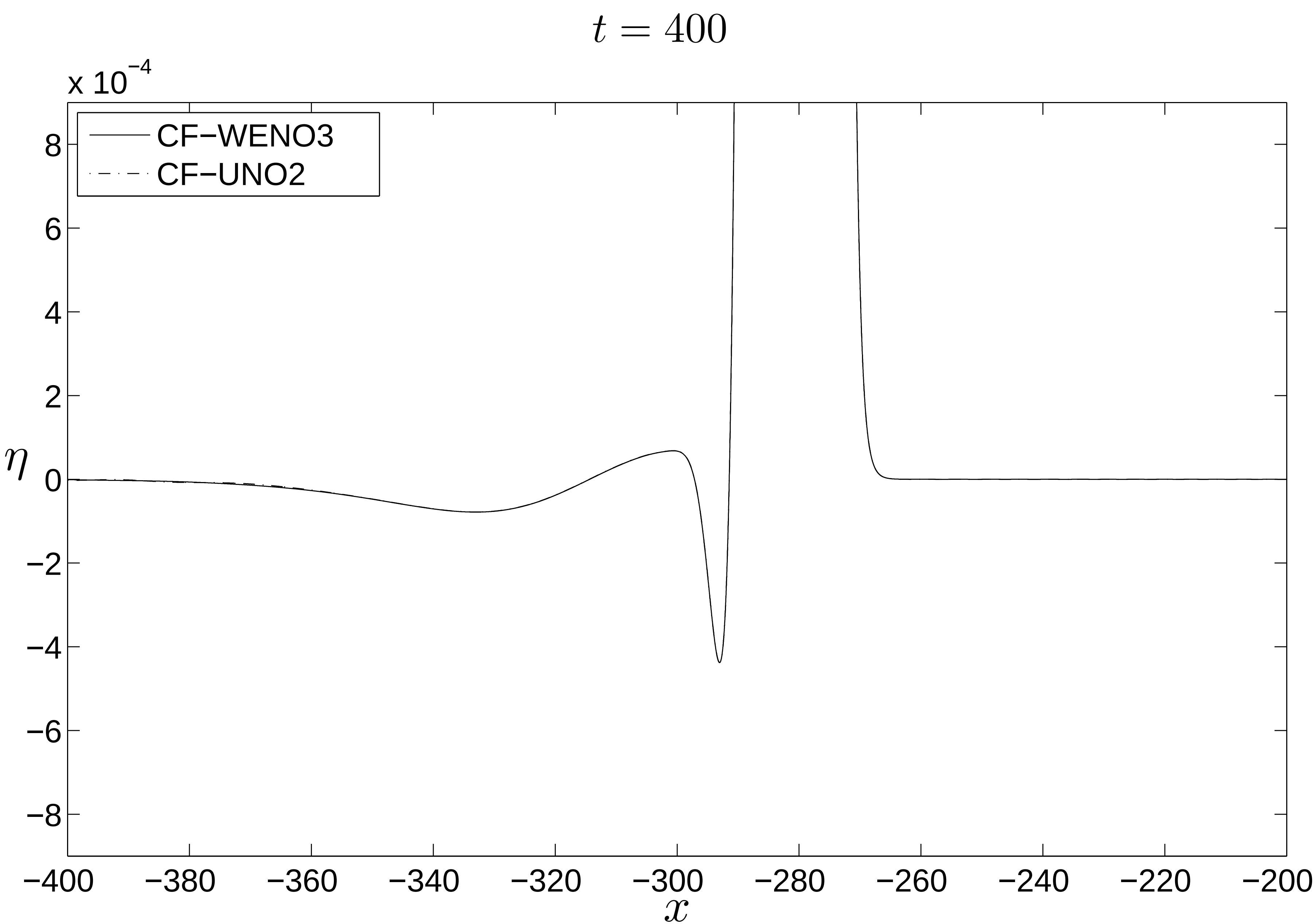

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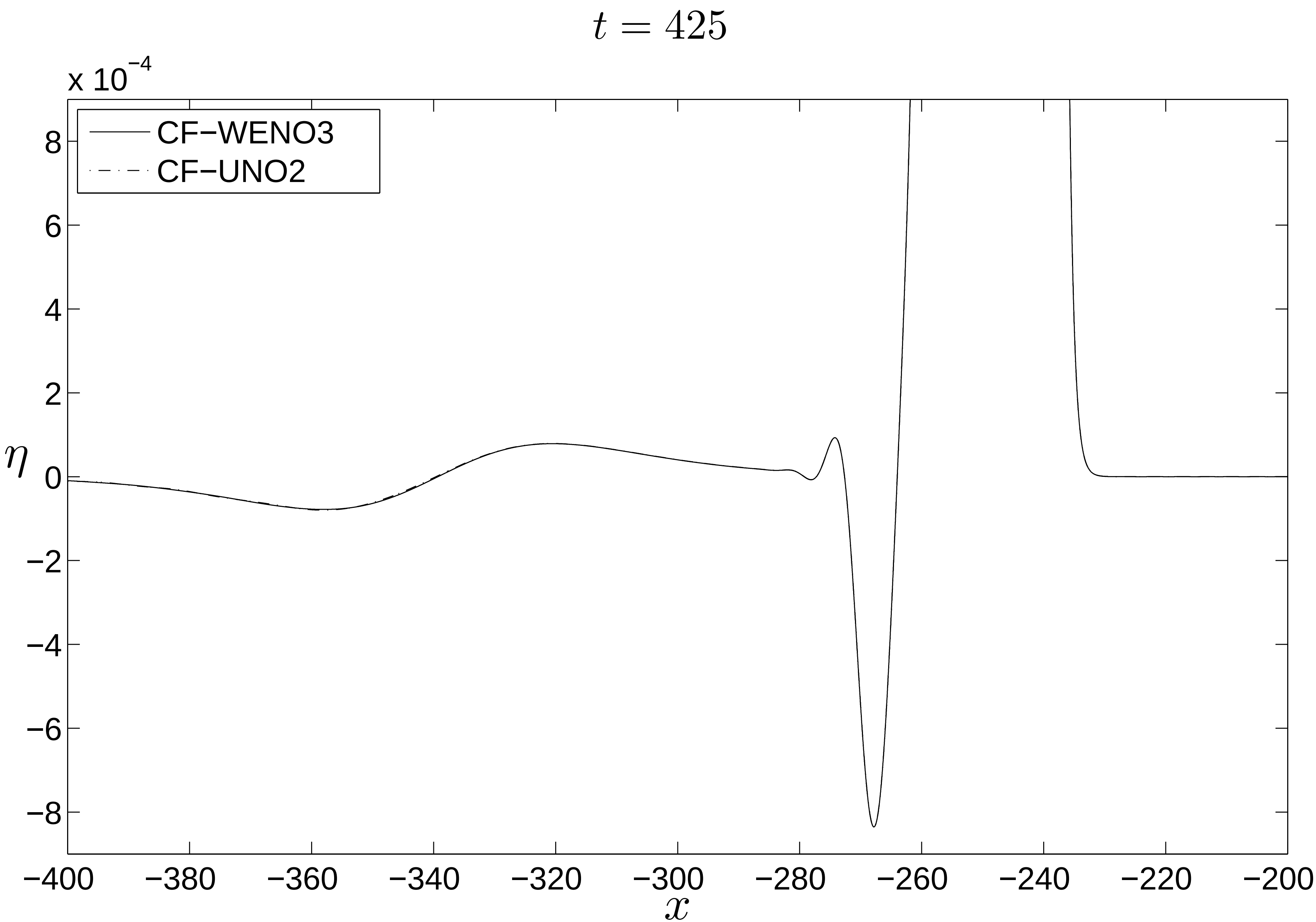

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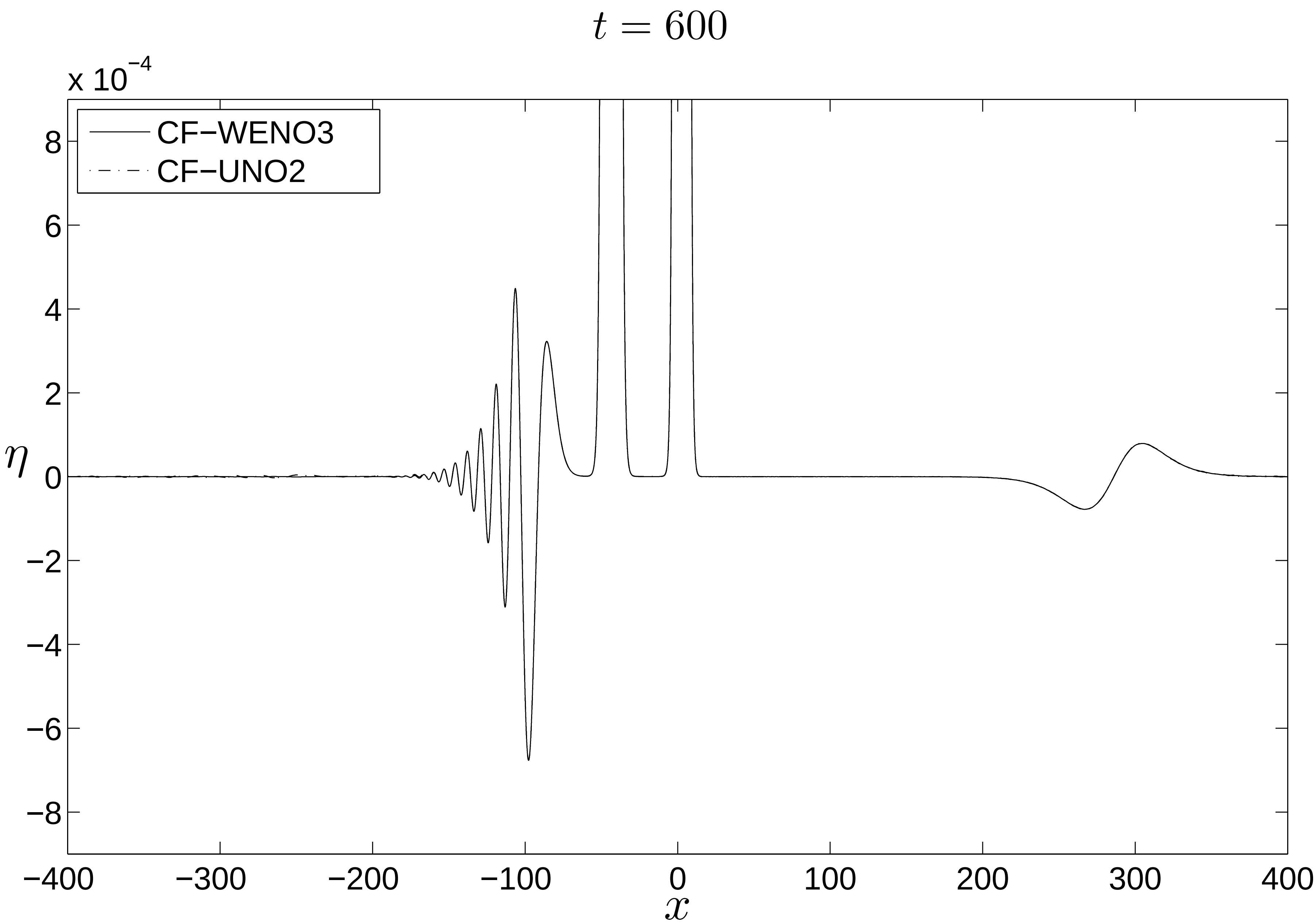

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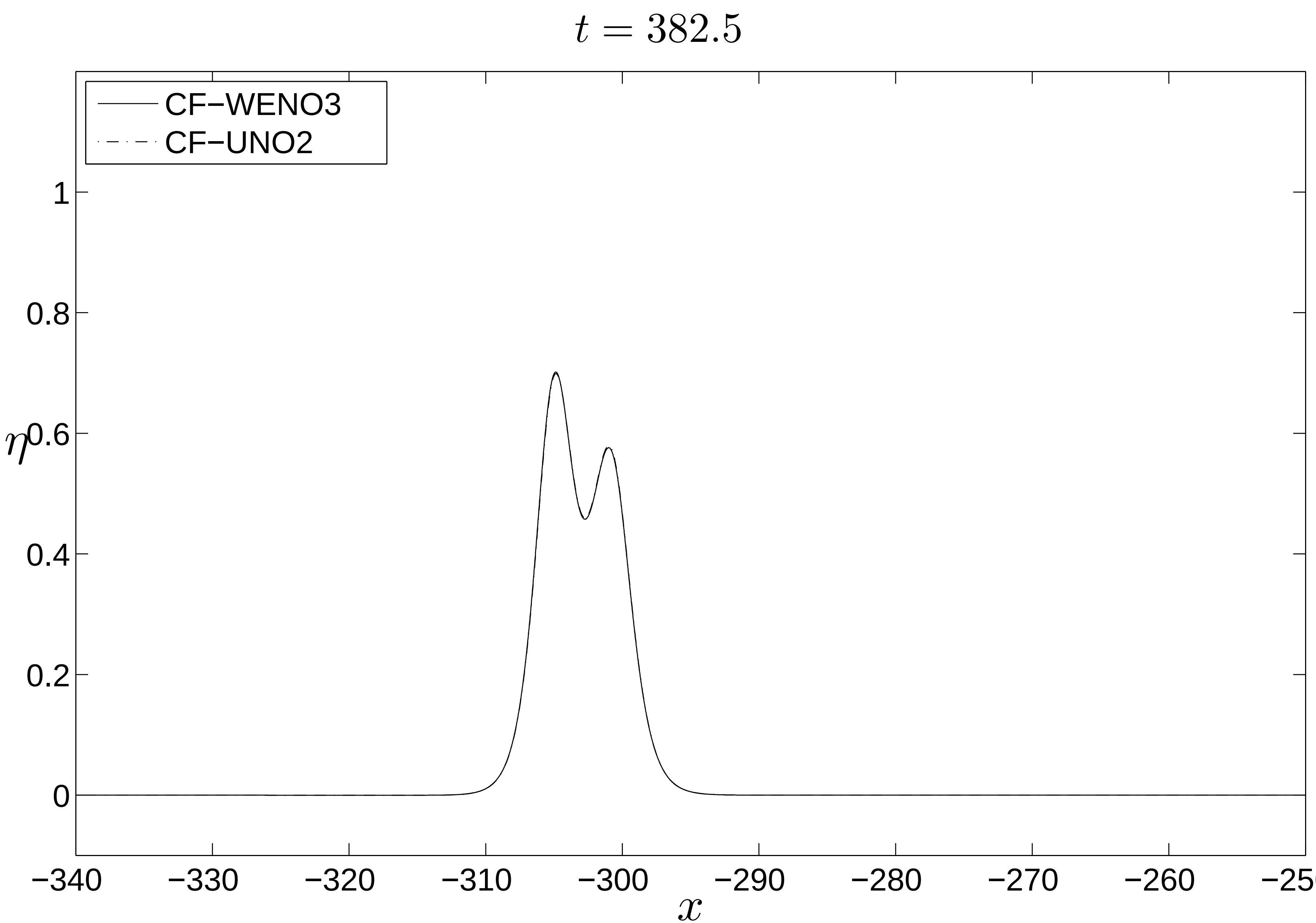

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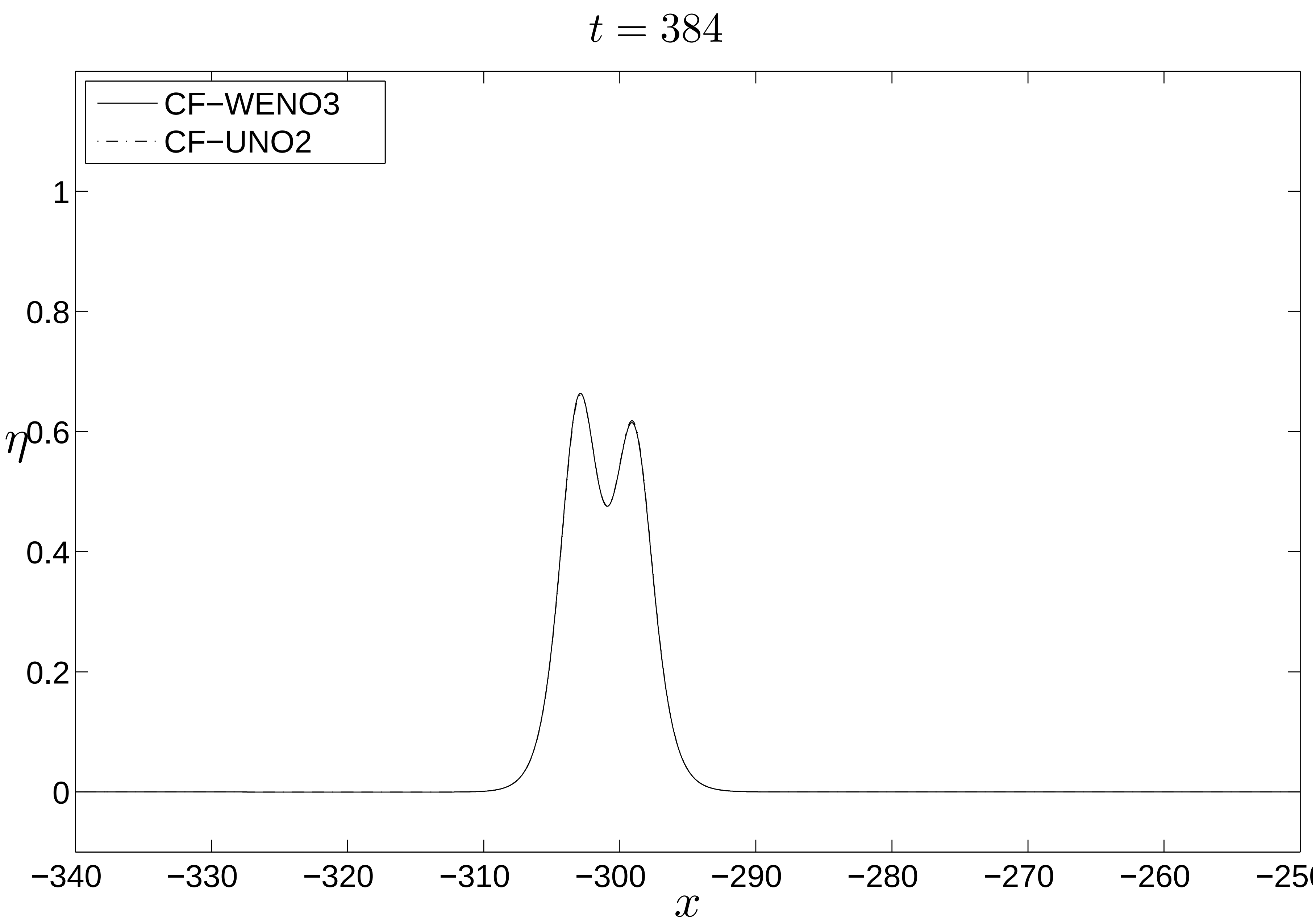

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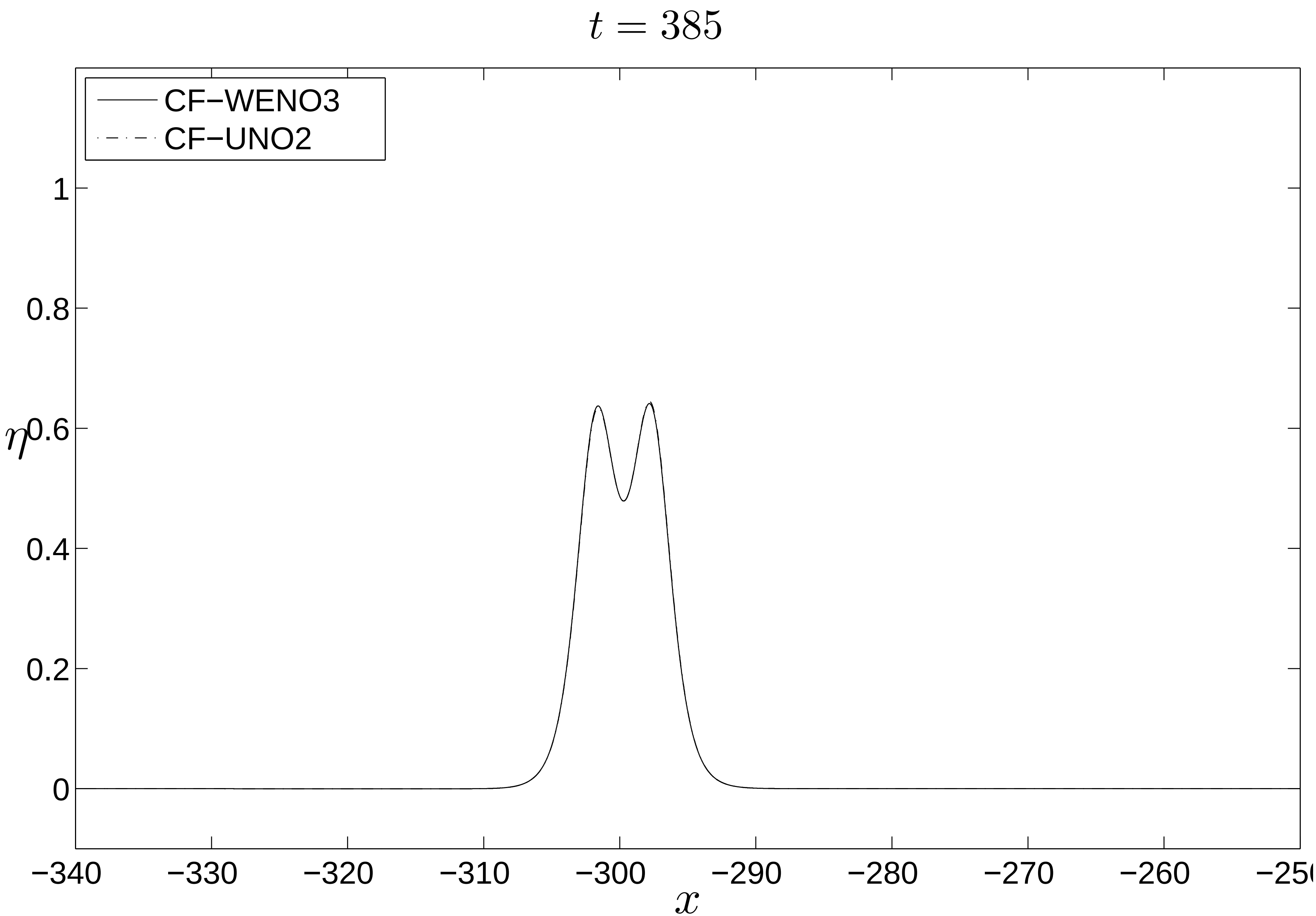

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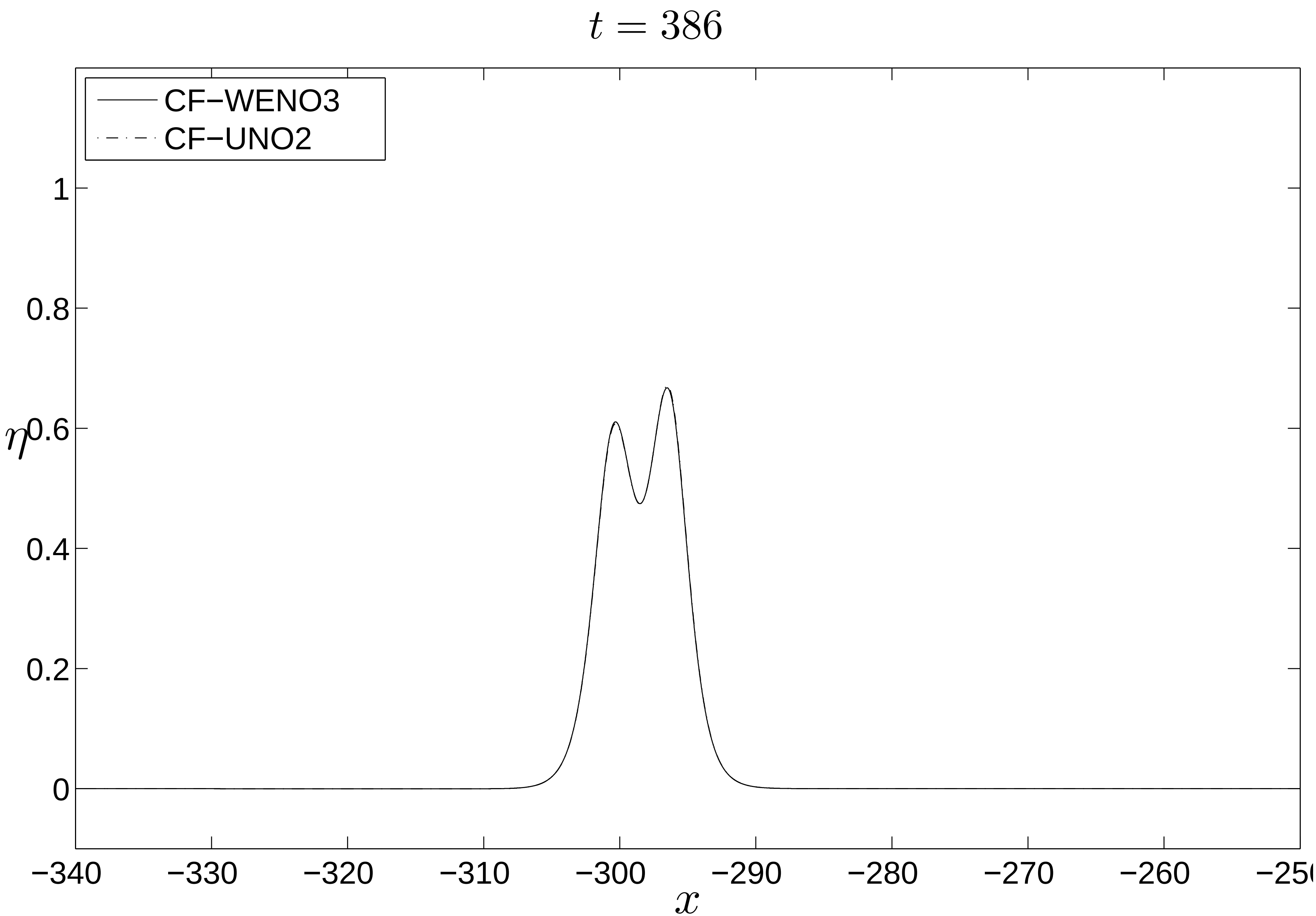
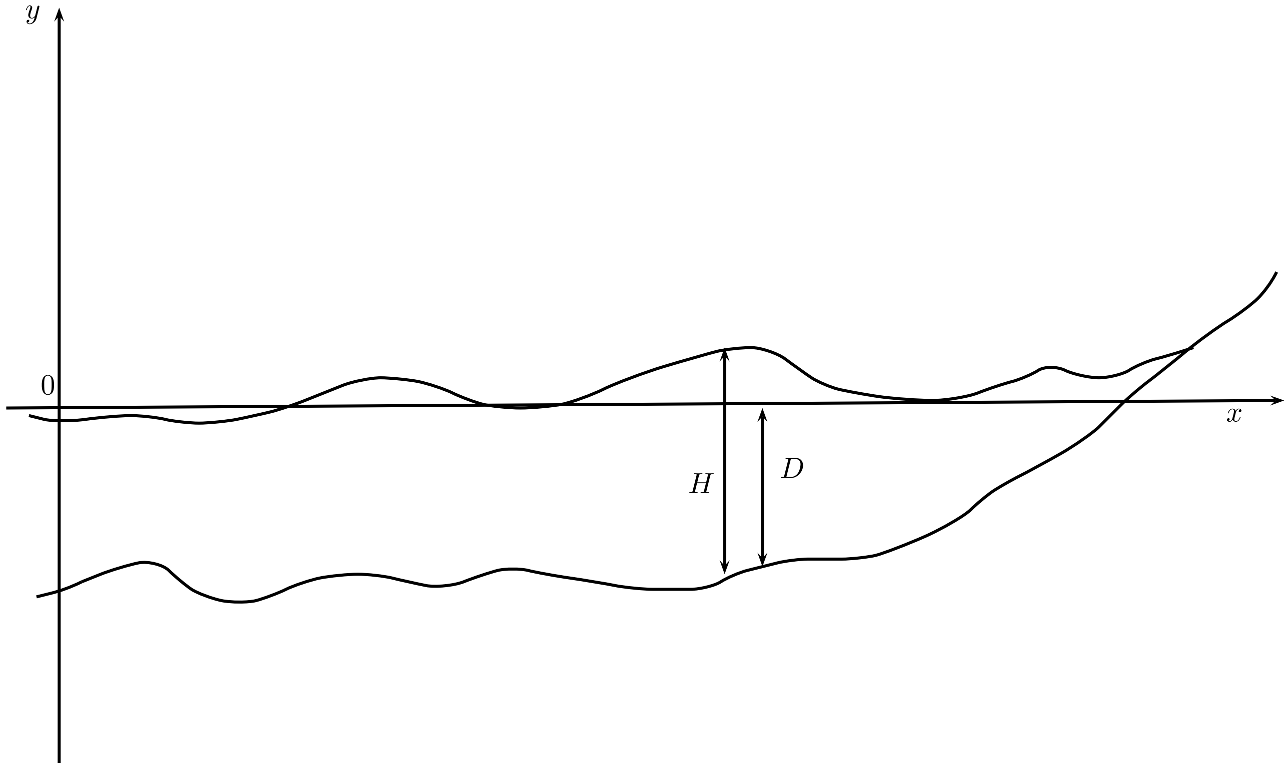


Overtaking collision - movie

Overtaking Collision

Dispersive models with variable bottom

Boussinesq system on non-flat bottom



Peregrine 1967

$$\eta_t + [(D+\eta)u]_x = 0,$$

$$u_t + g\eta_x + uu_x - \frac{D}{2}(Du)_{xxt} - \frac{D^2}{6}u_{xxt} = 0,$$

Nwogu 1993, Madsen et al. 2003

• Most of these systems break Galilean invariance and invariance under vertical translations.

• Desired properties for realistic problems like the water wave runup on non-uniform beaches.

• Complete water wave problem possesses these symmetries

We derive a new system : Peregrine's system, scale back and set $H = D + \epsilon\eta$

$$H_t + Q_x = 0,$$
$$Q_t + (Q^2/H + \frac{g}{2}H^2)_x - P(H,Q) = gHD_x.$$
$$P(H,Q) = \frac{H^2}{3}Q_{xxt} - (\frac{1}{3}H_x^2 - \frac{1}{6}HH_{xx})Q_t + \frac{1}{3}HH_xQ_{xt}$$

where $H(x,t) = \eta(x,t) + D(x)$, $Q(x,t) = H(x,t)u(x,t)$.

• Invariant under vertical translations

• Ignoring the dispersive terms P we recover Shallow Water Eqns.

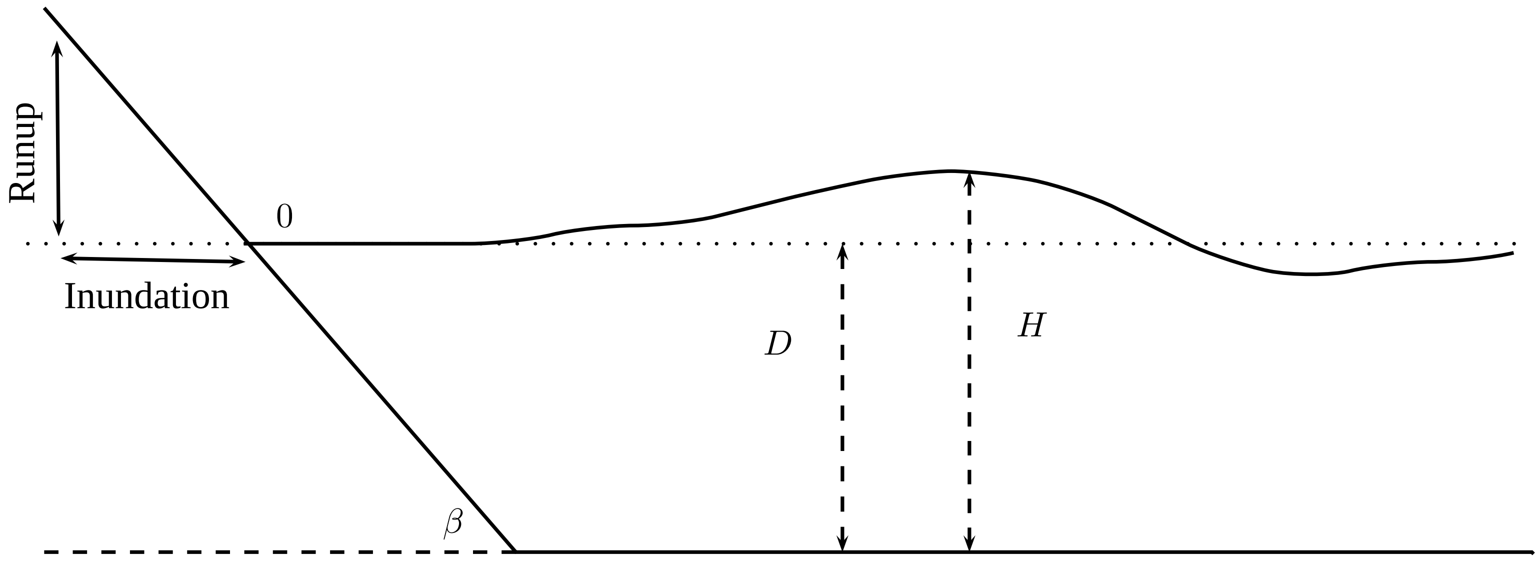
Discretization of Boussinesq system

- Advection : KT, CF + TVD2, UNO2
- Dispersion : 2nd order FD
- Source terms : Well balanced hydrostatic reconstruction (Audusse, Bouchut, Bristeau, Klein, Perthame, 2004)

$$H^L_{i+\frac{1}{2}} = \max(0, H^L_{i+\frac{1}{2}} - D^L_{i+\frac{1}{2}} + D_{i+\frac{1}{2}}),$$
$$H^R_{i+\frac{1}{2}} = \max(0, H^R_{i+\frac{1}{2}} - D^R_{i+\frac{1}{2}} + D_{i+\frac{1}{2}})$$
$$D_{i+\frac{1}{2}} = \min(D^R_{i+\frac{1}{2}}, D^L_{i+\frac{3}{2}})$$

- Reconstructed variables: H, $H - D(= \eta)$. Thus $D = H - (H - D)$
- Time stepping: Explicit TVD - RK-methods.
- Reflective BC's

Solitary wave runup - setup



- Bottom shape

$$-D(x) = \begin{cases} -x \tan\beta, & x \leq \cot\beta, \\ -1, & x > \cot\beta, \end{cases}$$

- Initial Condition: $\eta_0(x) = A_s \mathrm{sech}^2\left(\lambda(x - X_0)\right), u_0(x) = -c_s \frac{\eta_0(x)}{D_0 + \eta_0(x)}$
- Runup : height of the last dry cell.

No-specific run-up algorithm

Breaking and non-breaking solitary waves

- Experimental data by Synolakis 1987 : slope $\beta = 2.88^\circ$. Waves with amplitude $H/D > 0.045$ break
  - □ $A = 0.0185$ (Non-breaking)
  - □ $A = 0.04$ (Nearly breaking)
  - □ $A = 0.28$ (Breaking)
- Experimental data by Zelt 1991 : slope $\beta = 20^\circ$
  - □ $A = 0.12$ (Non-breaking)
  - □ $A = 0.20$ (Breaking during rundown)
- Non uniform sloping shore with a small pond

Non-breaking solitary wave

- Beach : $[-10, 70]$ Slope : $\beta = 2.88^\circ$
- Solitary wave : location $X_0 = 19.85$ Amplitude : $A_s = 0.0185$

□ Mass conservation $I_0^h = 60.3667671231$

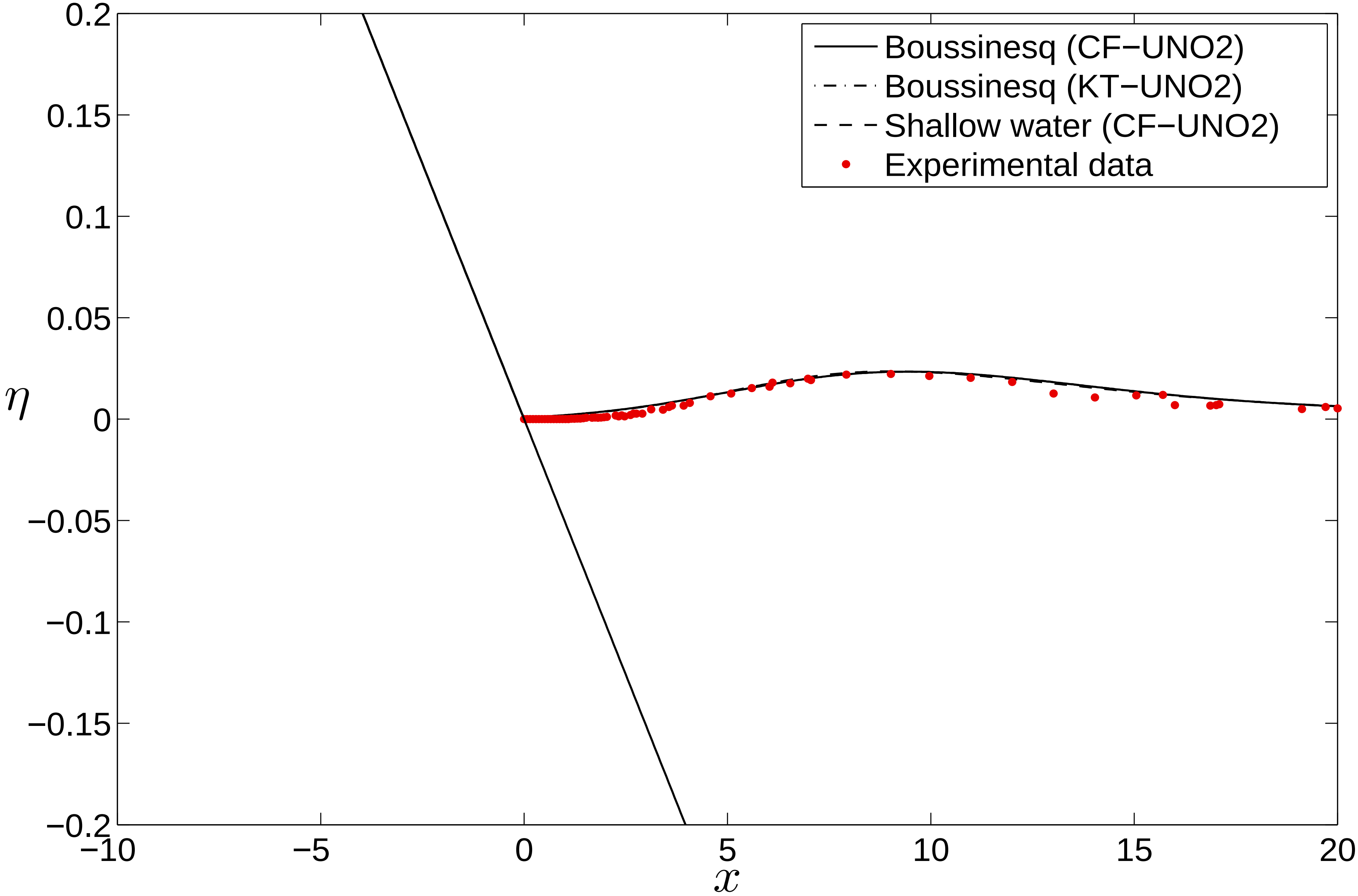
□ SW and Boussinesq converge to the same solution

□ Very good prediction of Runup and Rundown

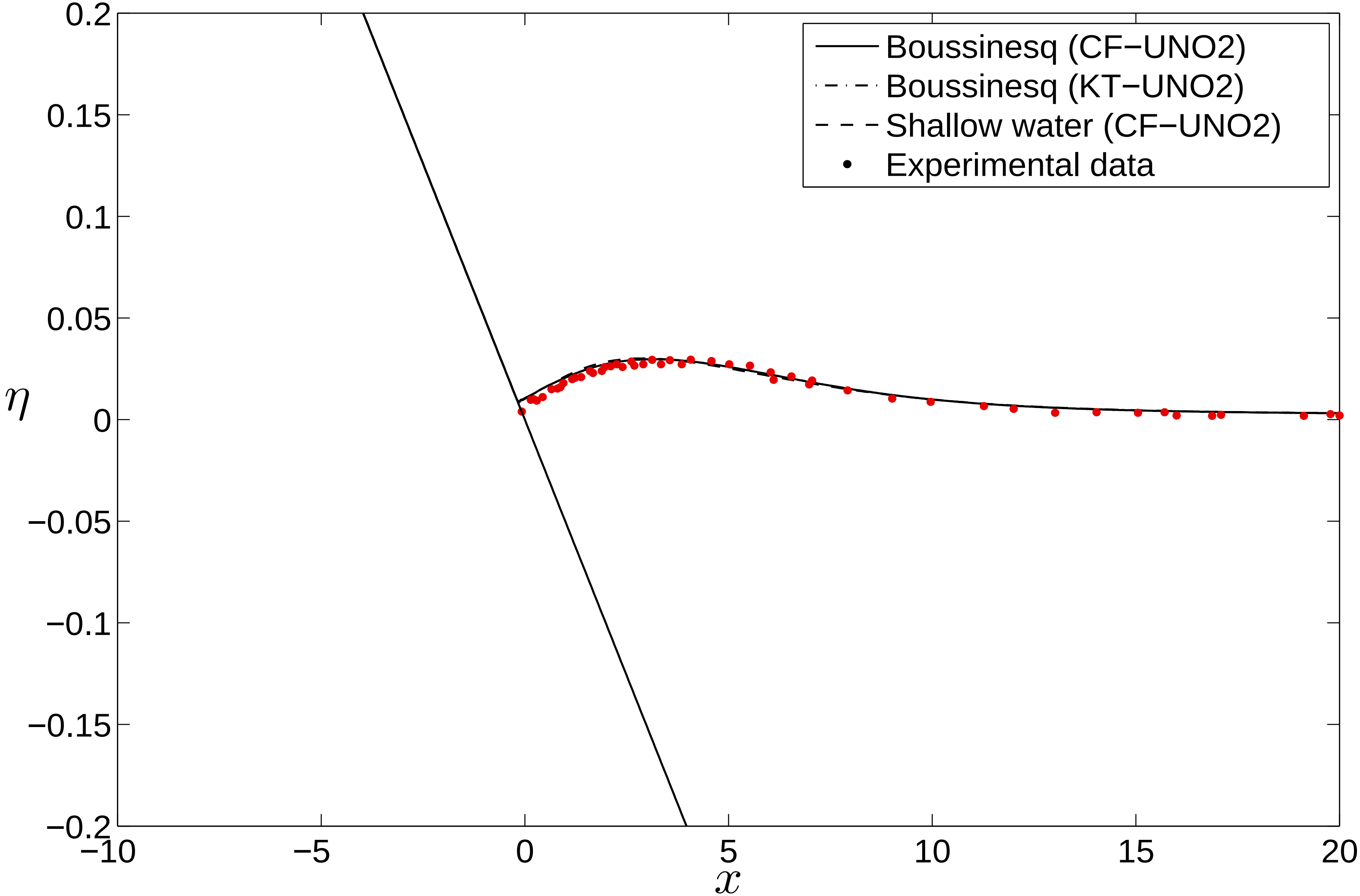
□ Max Runup(R_∞) :

Experiment : $R_\infty \approx 0.078$,

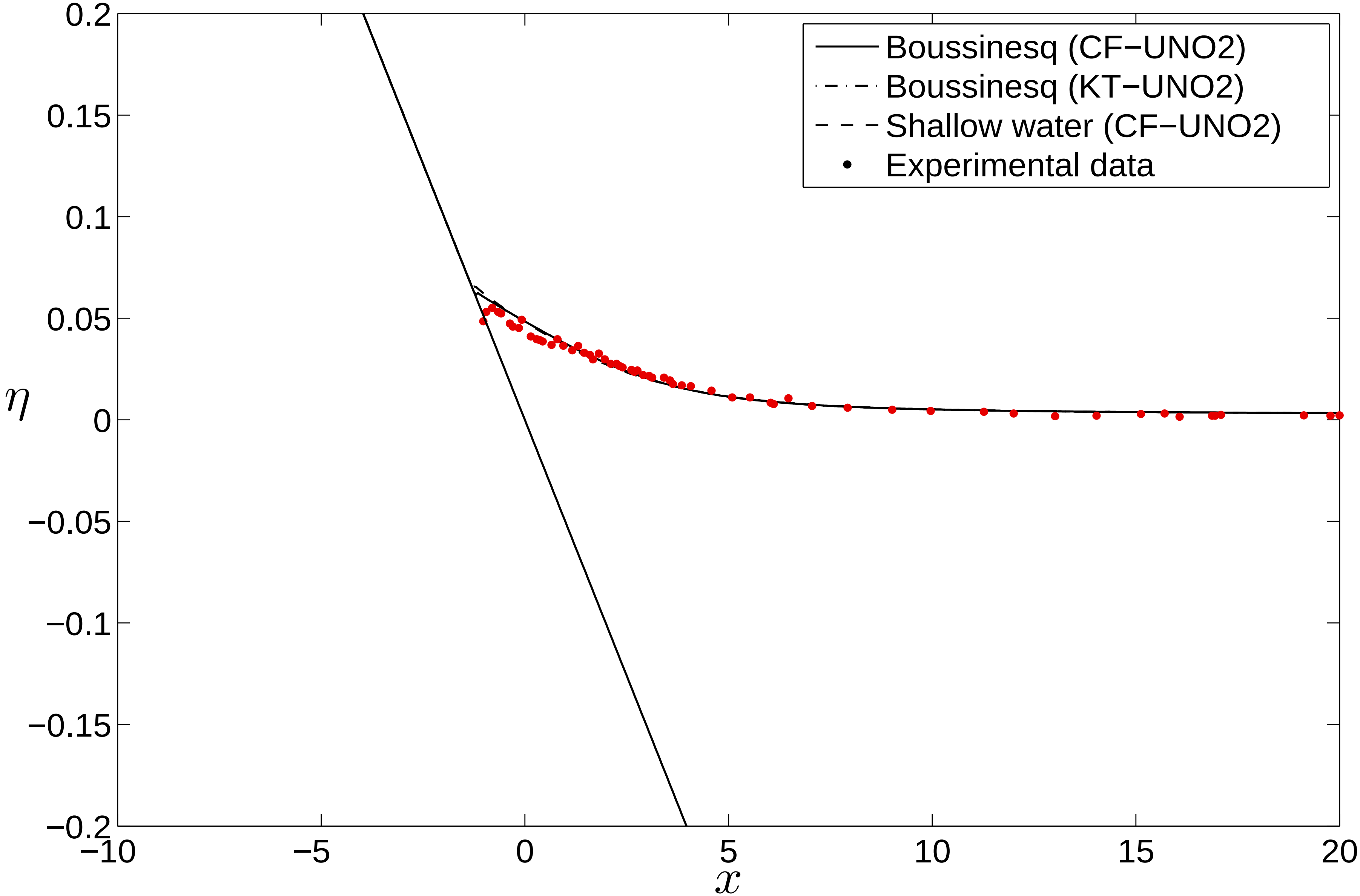
SW : $R_\infty \approx 0.088$ Boussinesq : $R_\infty \approx 0.085$


0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
η
−10
−5
0
5
10
15
20
x
Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data

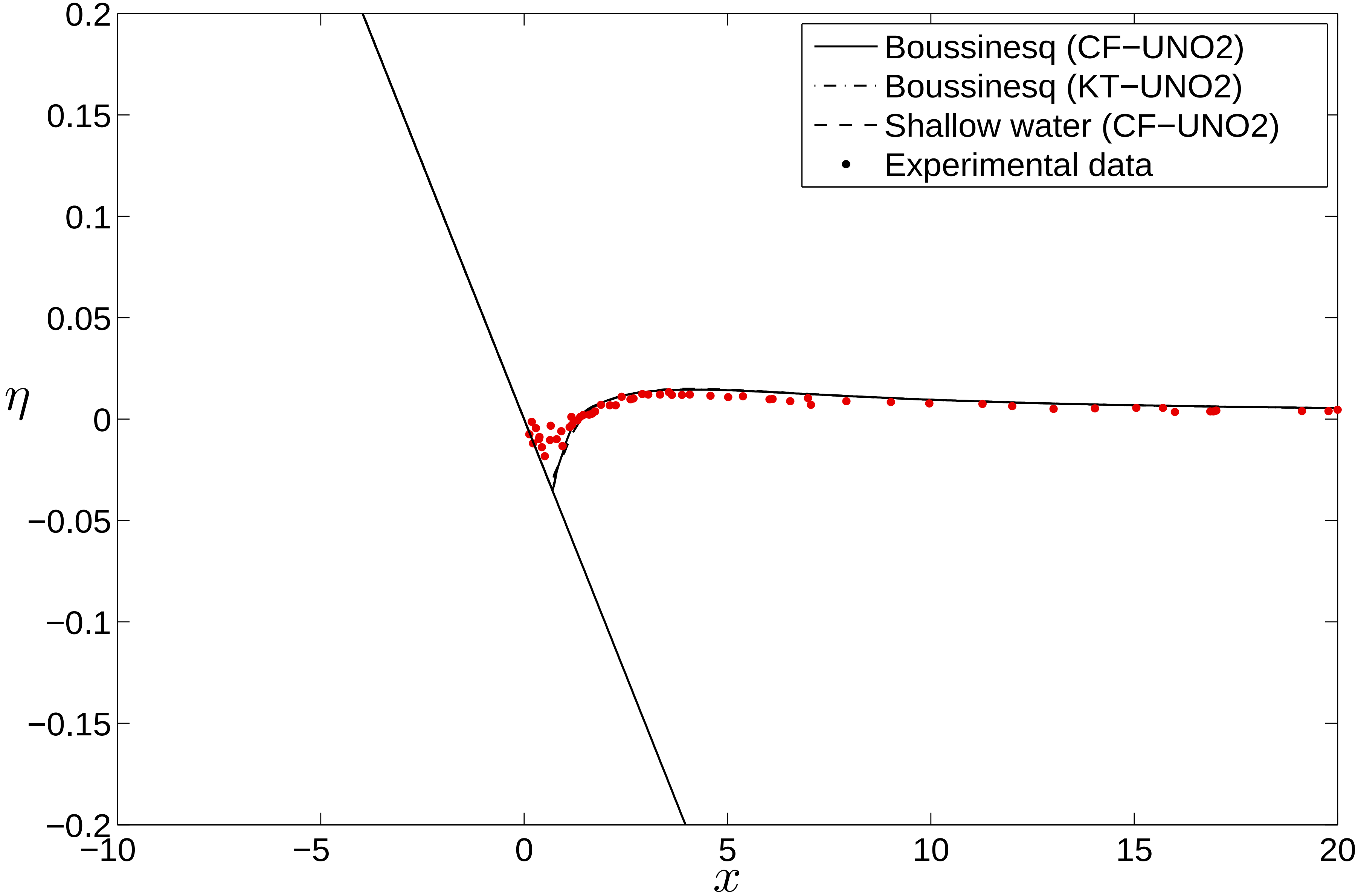
Runup : Non-breaking solitary wave



Runup : Non-breaking solitary wave



Runup : Non-breaking solitary wave



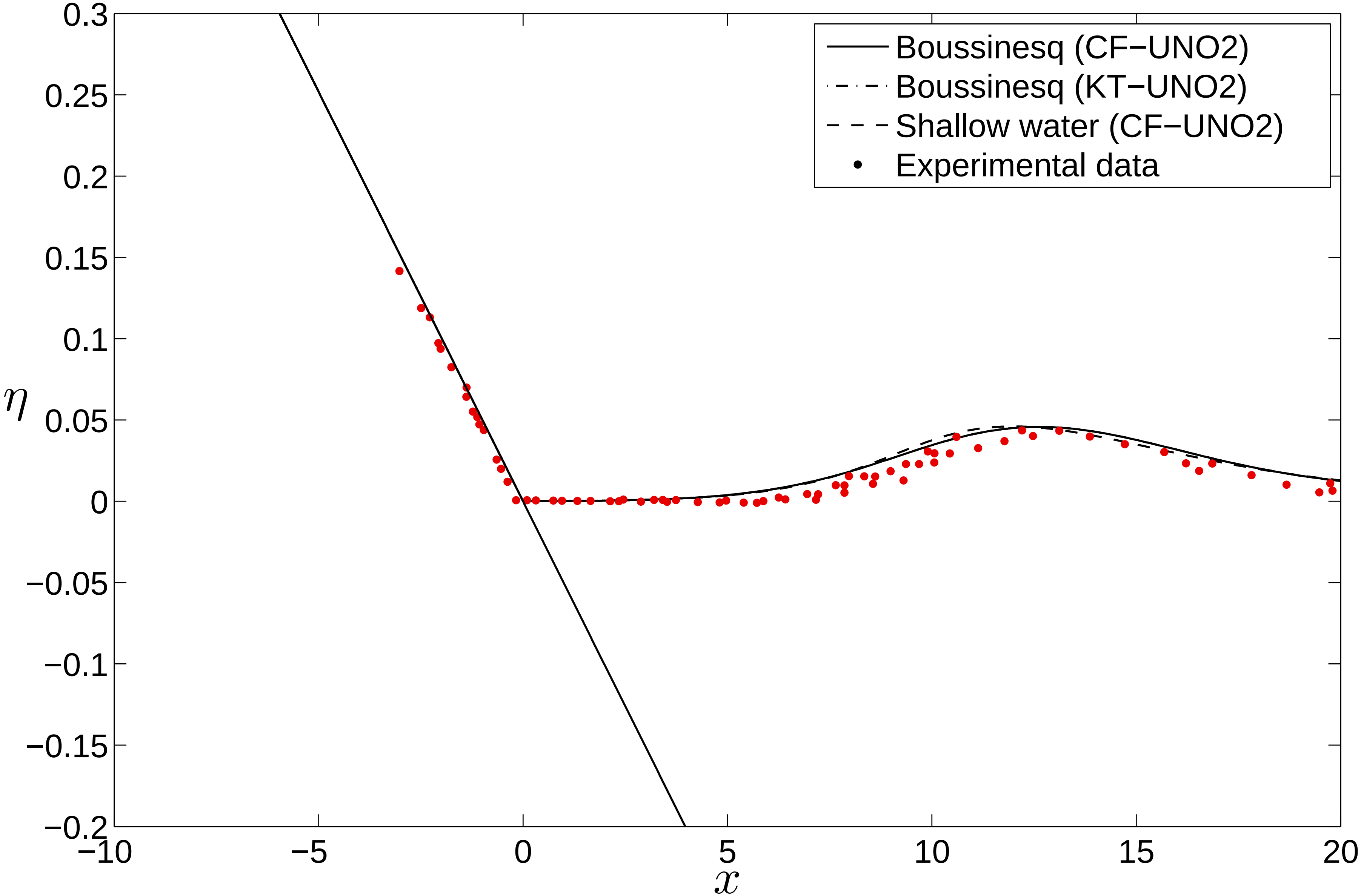
Nearly breaking solitary wave

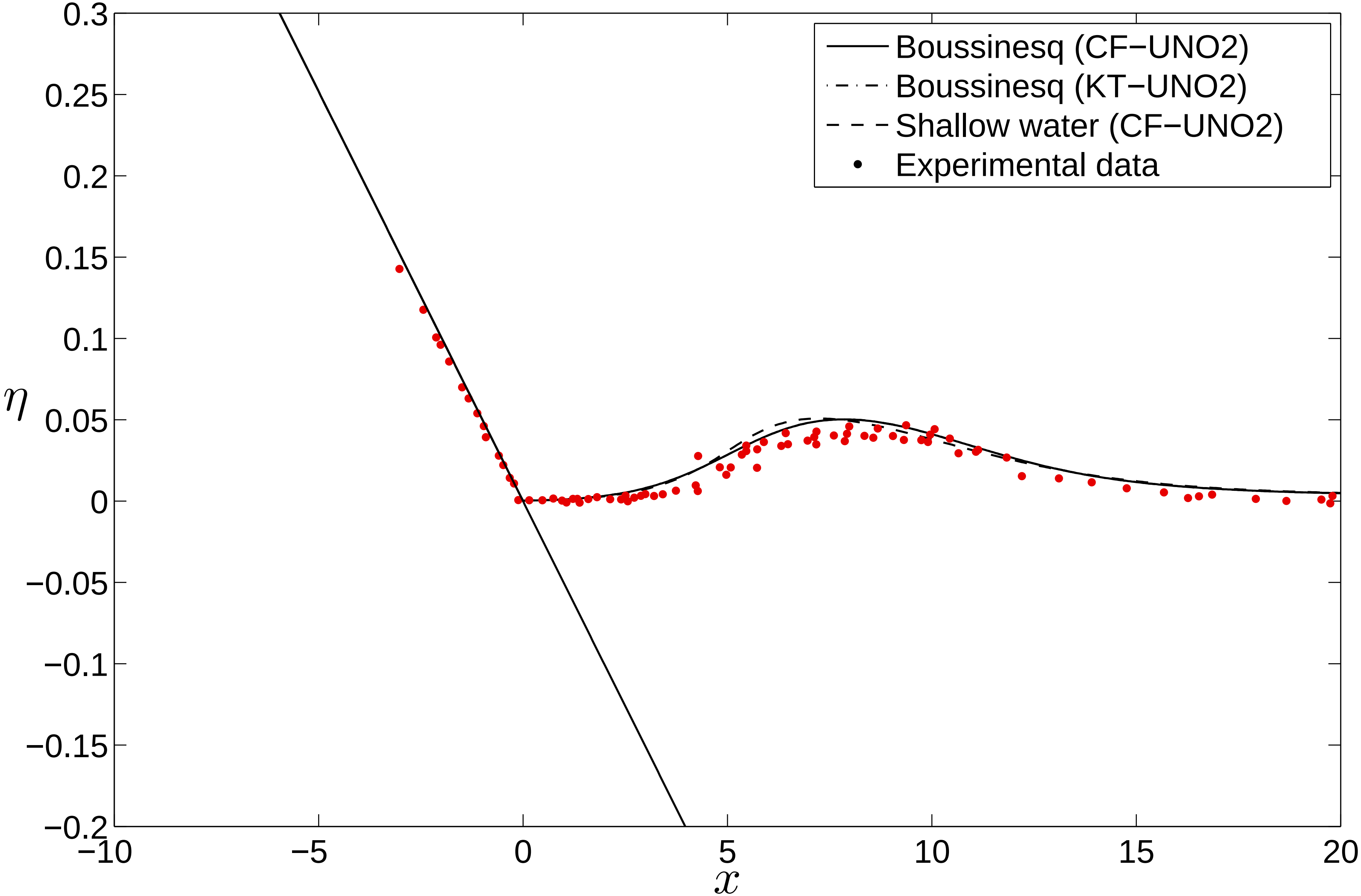
- Beach : $[-10, 70]$ Slope : $\beta = 2.88^\circ$
- Solitary wave : location $X_0 = 19.85$ Amplitude : $A_s = 0.04$
- Wave breaks during rundown
  - Mass conservation $I_0^h = 60.5210181987$
  - SW and Boussinesq converge to the same solution
  - Very good agreement with experimental data
  - Max Runup(R_∞) :

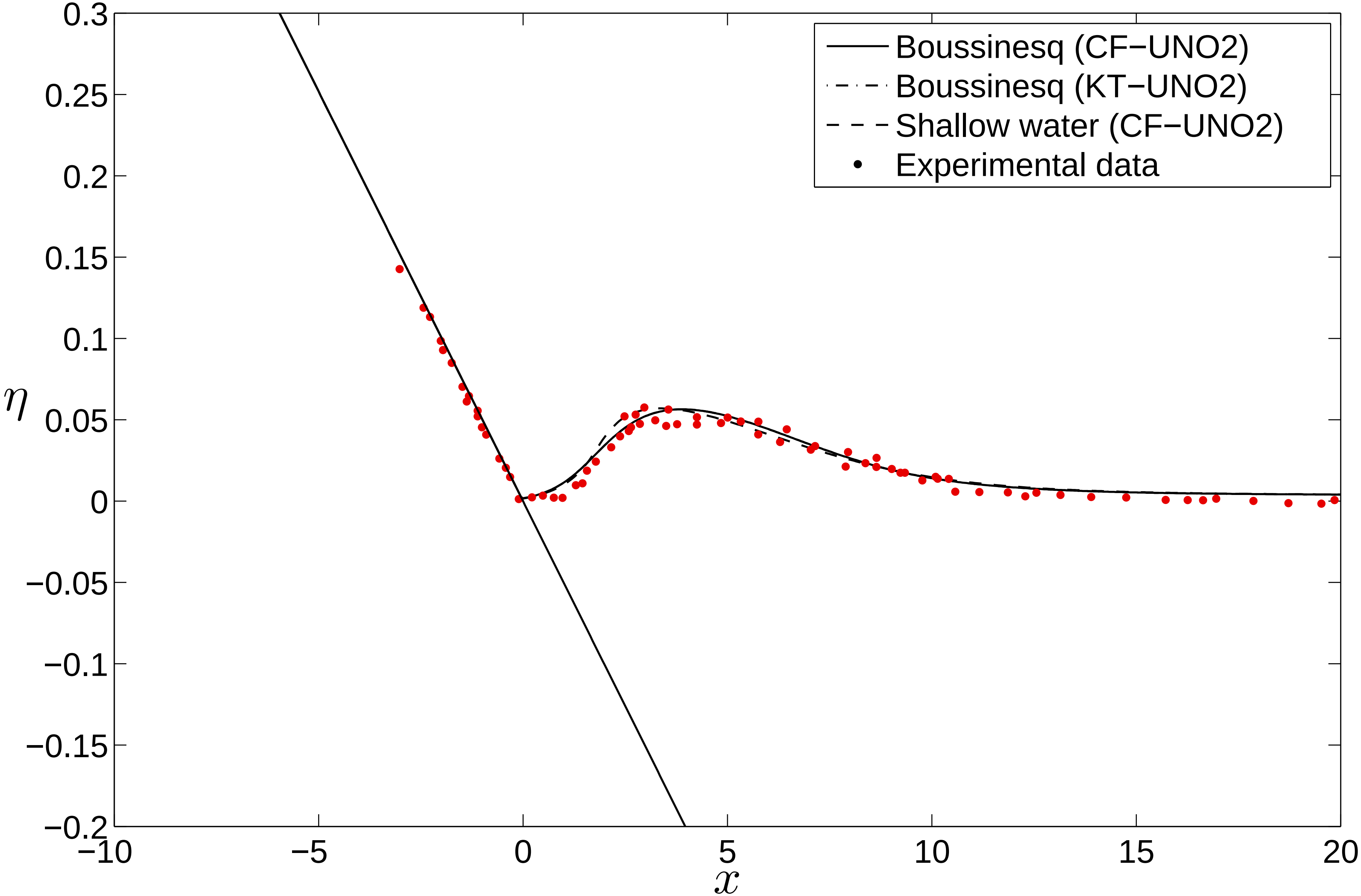
Experiment : $R_\infty \approx 0.156$

SW : $R_\infty \approx 0.21$ Boussinesq : $R_\infty \approx 0.20$

Runup : Nearly breaking solitary wave

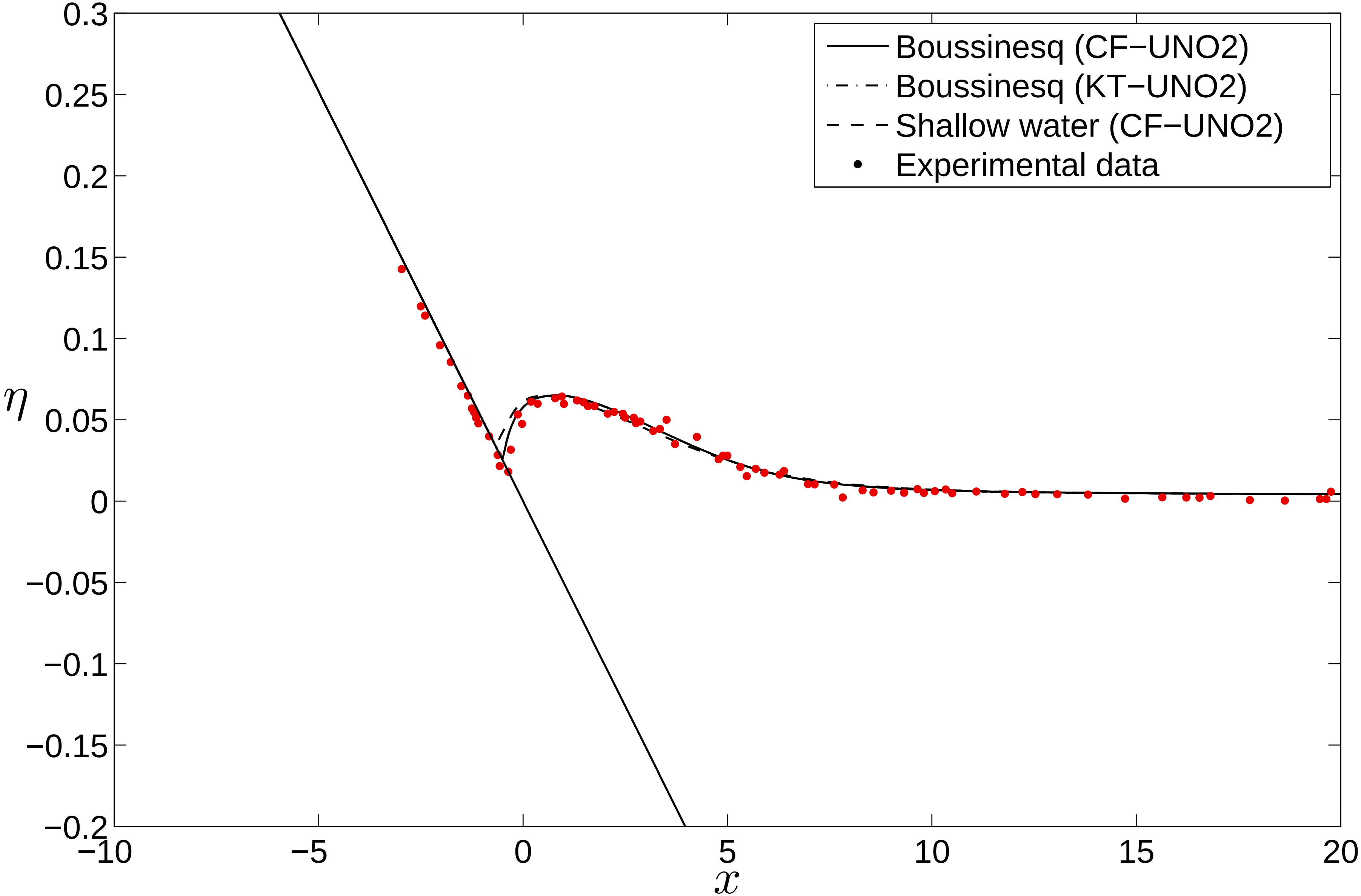


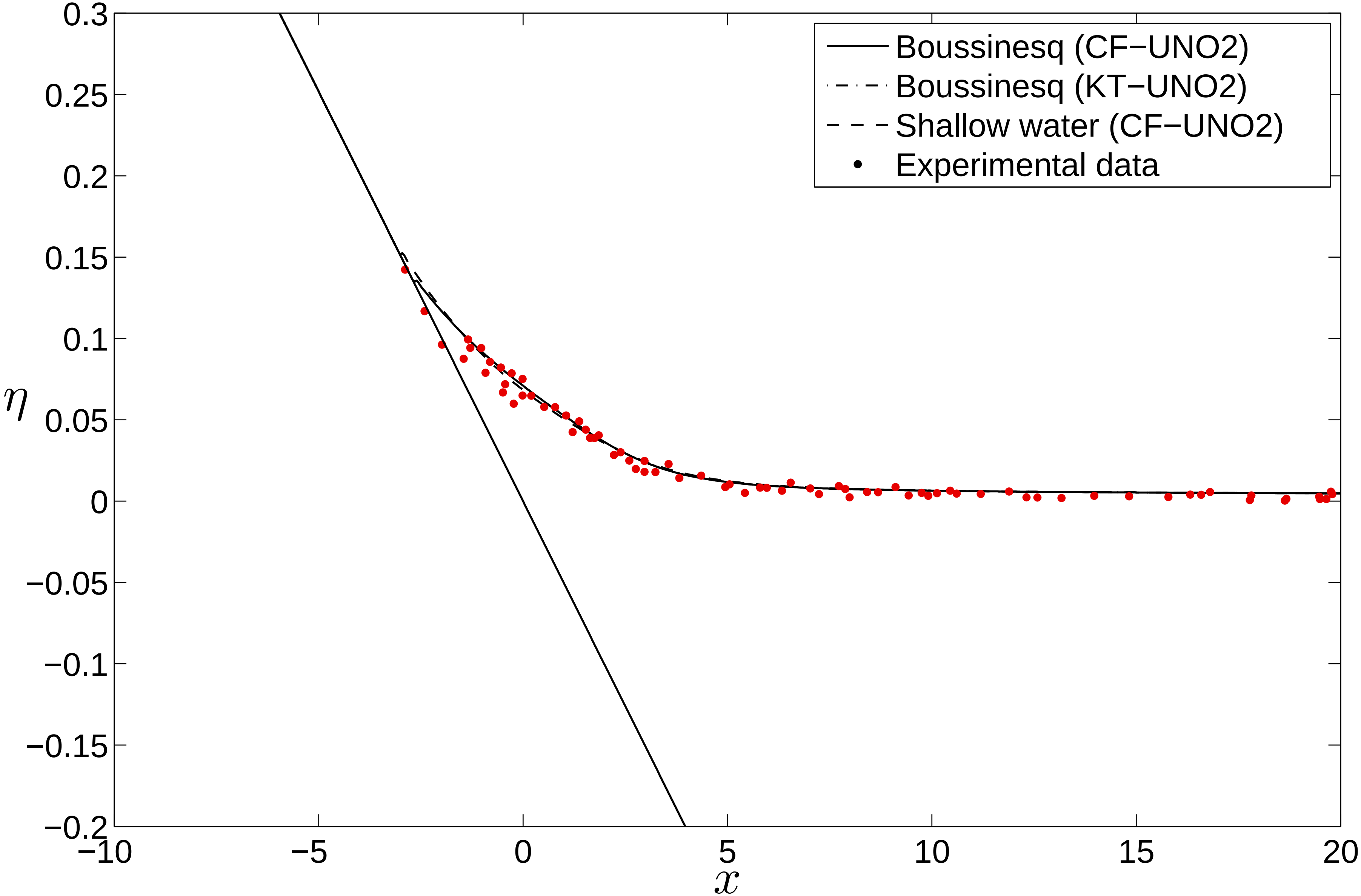

Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data
η
x
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−10
−5
0
5
10
15
20



Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−10
−5
0
5
10
15
20
η
x

Runup : Nearly breaking solitary wave




Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
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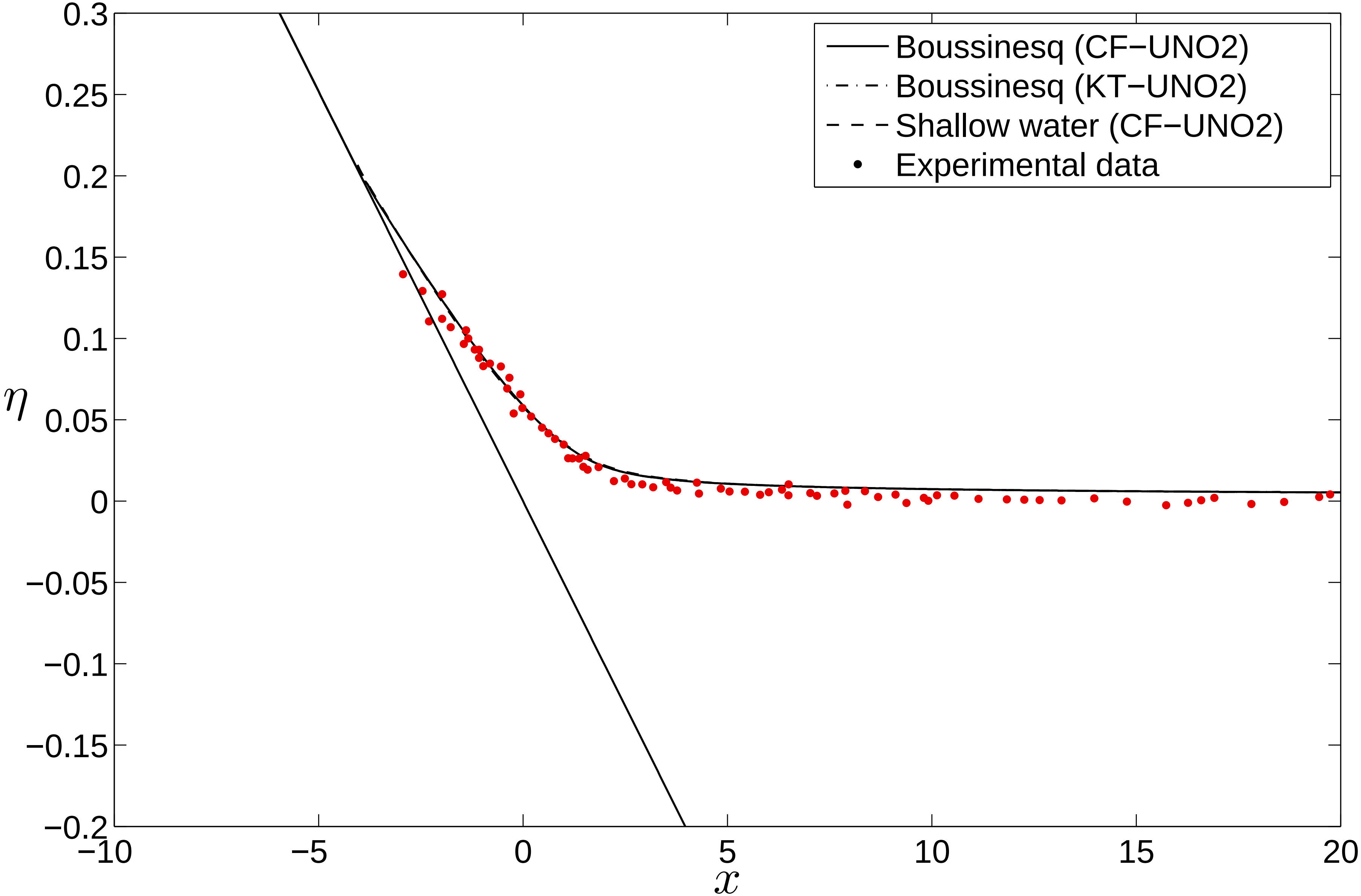
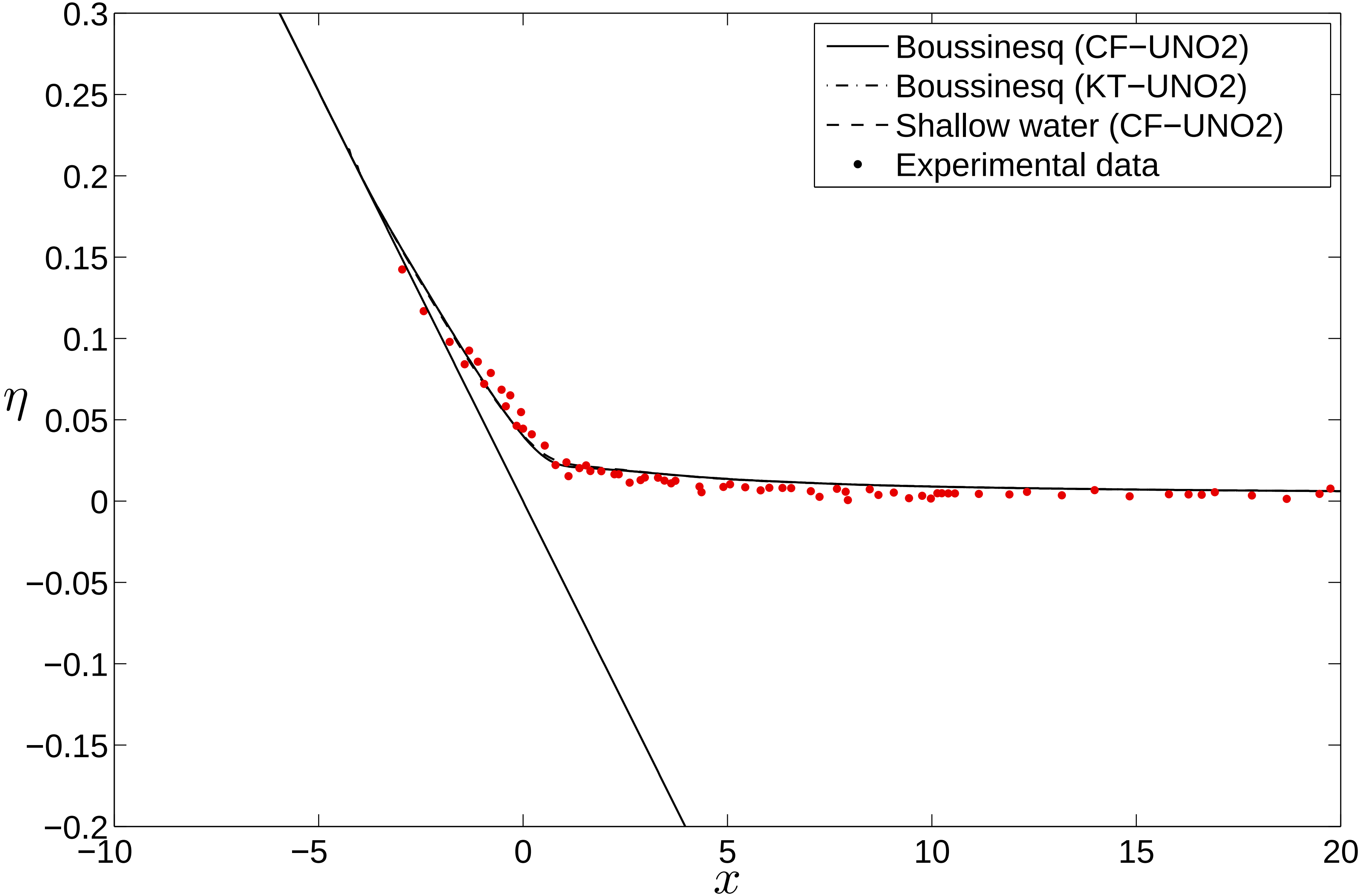
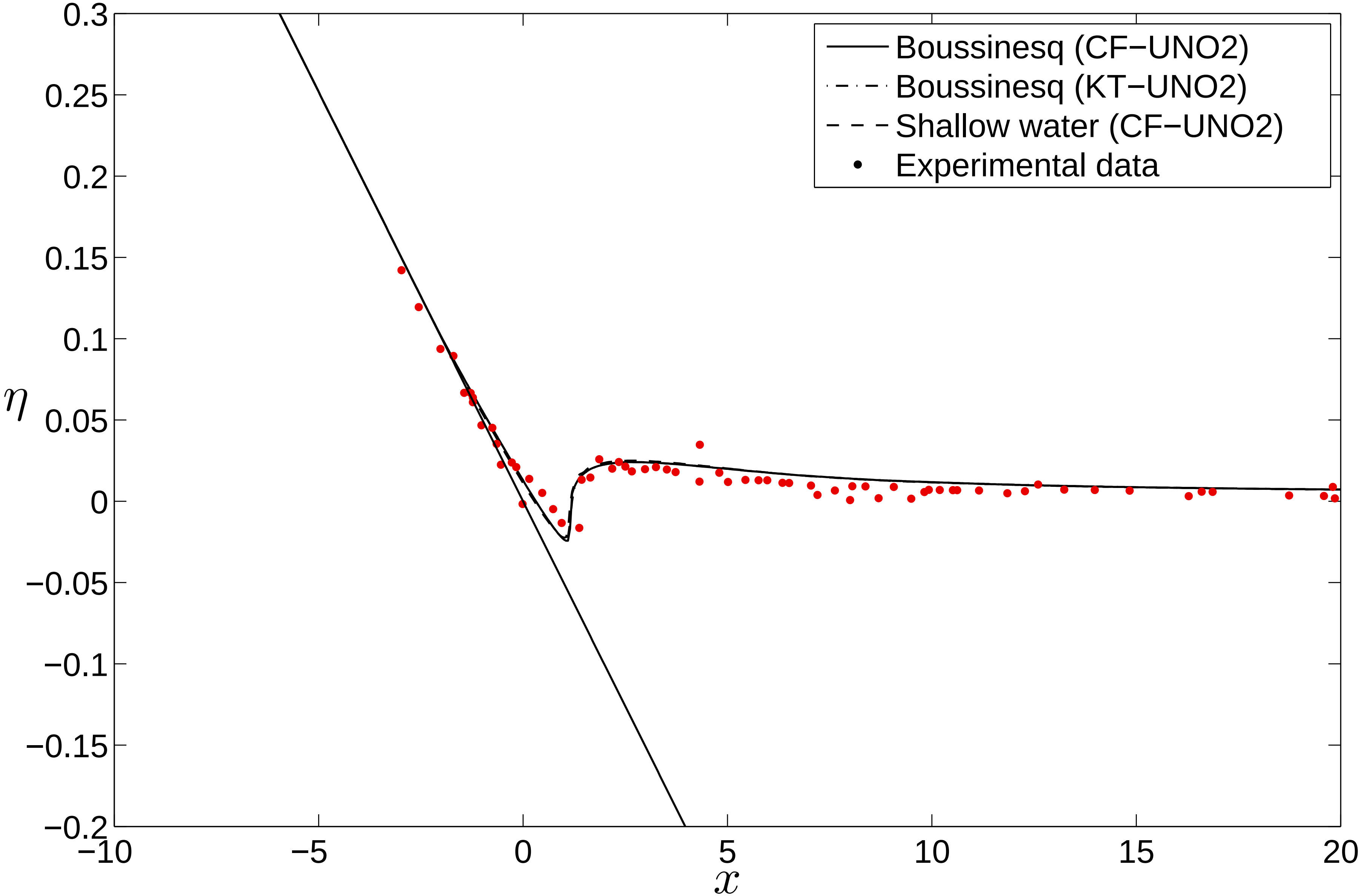
Experimental data
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
η
−10
−5
0
5
10
15
20
x

Runup : Nearly breaking solitary wave



Runup : Nearly breaking solitary wave




Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
η
−10
−5
0
5
10
15
20
x

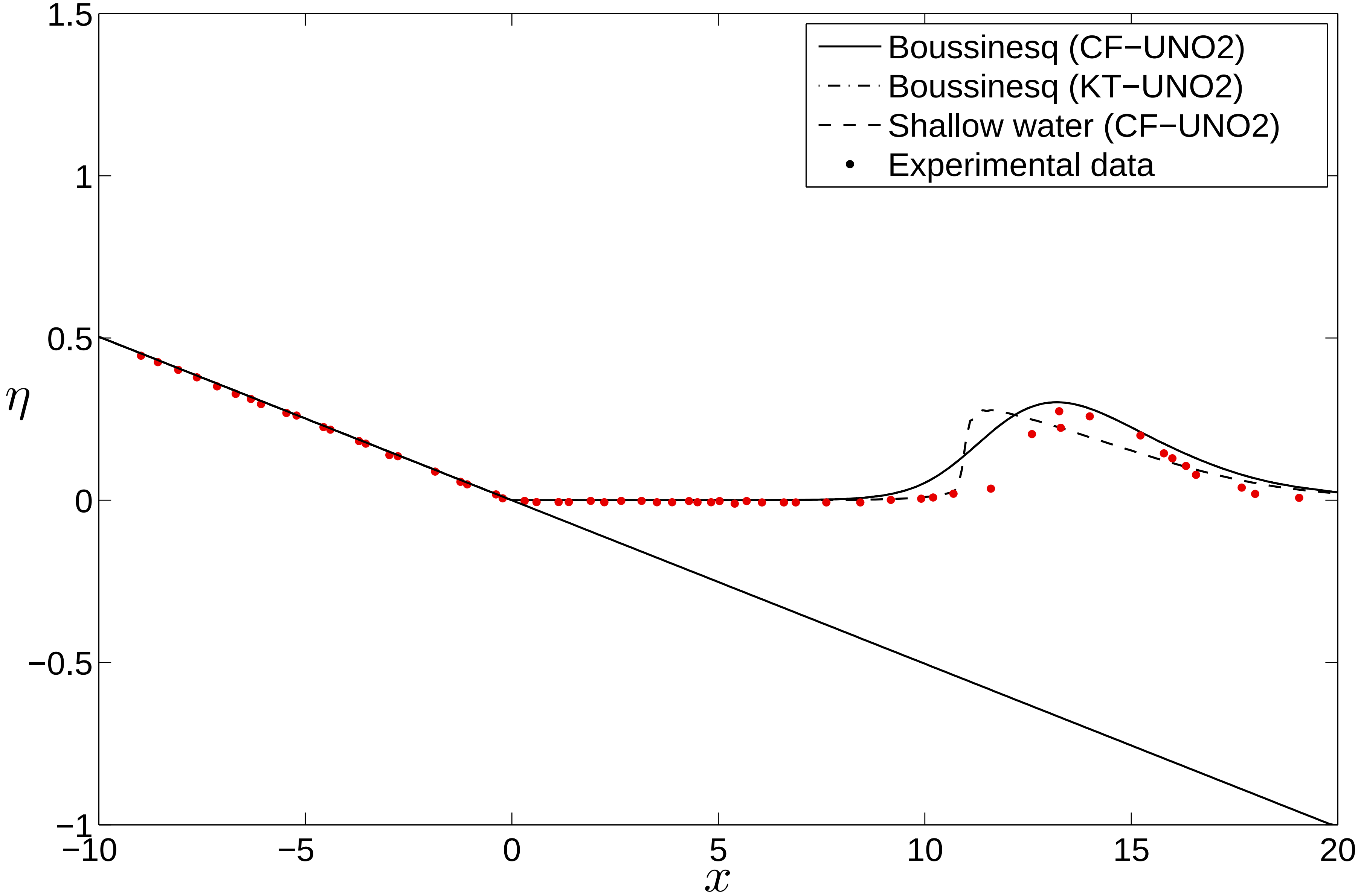
Breaking solitary wave

- Beach : $[-10, 60]$ Slope : $\beta = 2.88°$
- Solitary wave : location $X_0 = 19.85$ Amplitude : $A_s = 0.28$
- Wave breaks before reaching the beach ... Boussinesq model is not valid unless a wave breaking mechanism is included.
- Stabilization : exclude dispersion Q_{xxt} near shoreline. (no dissipative terms, no data filtering)

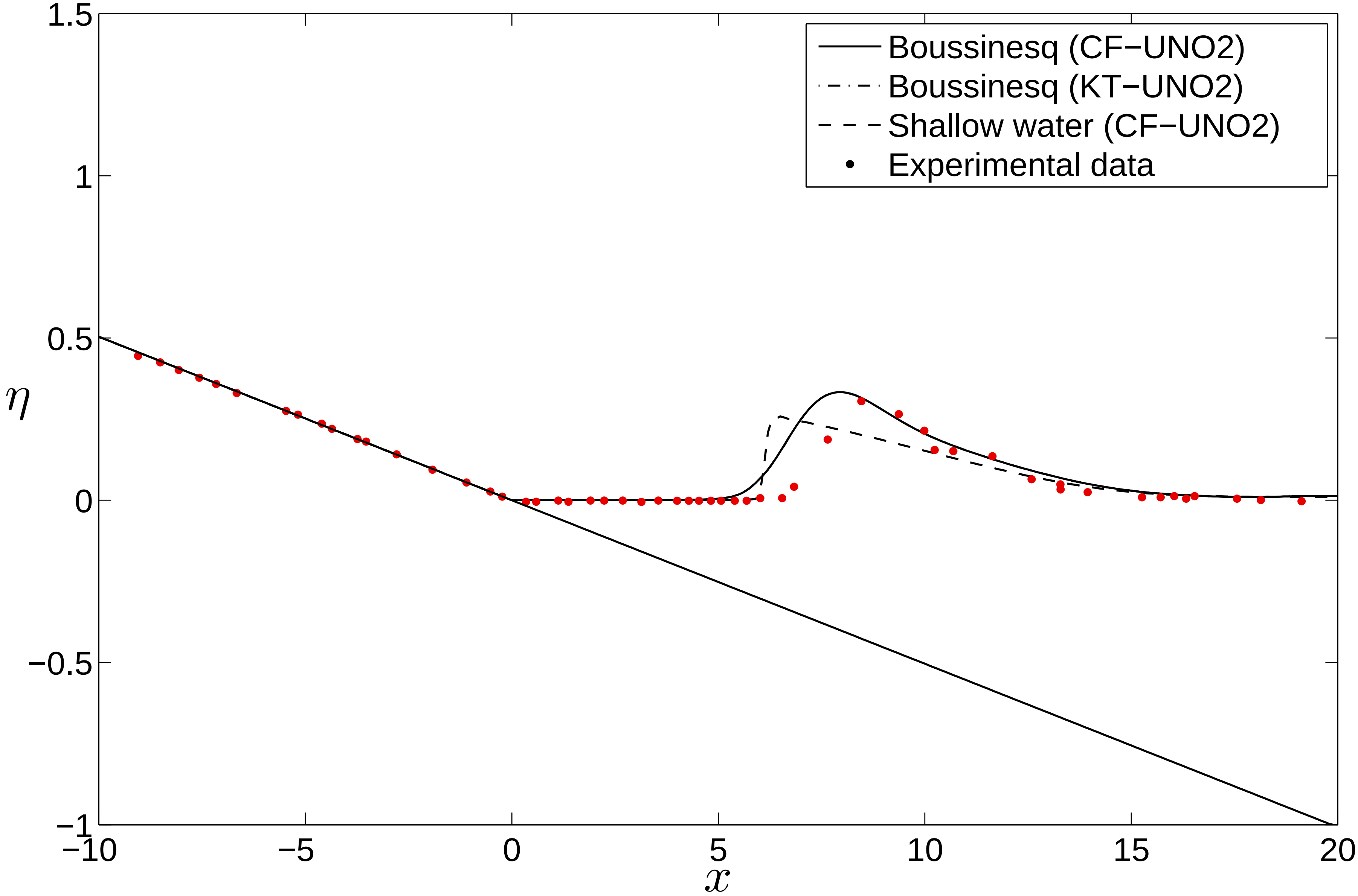
Friction is important : $F(u, H) = -c_m g \frac{u|u|}{H^{1/3}} \quad c_m = 2 \cdot 10^{-4}$

- □ Mass conservation $I_0^h = 51.7504637472$
- □ Differences between SW, Boussinesq and experimental data : amplitude and phase
- □ Runup and Rundown are approximated fairly well by both models
- □ Max Runup(R_∞) : SW + Boussinesq : $R_\infty \approx 0.47$, Theory by Synolakis 1987 $R_\infty \in [0.42, 0.53]$.

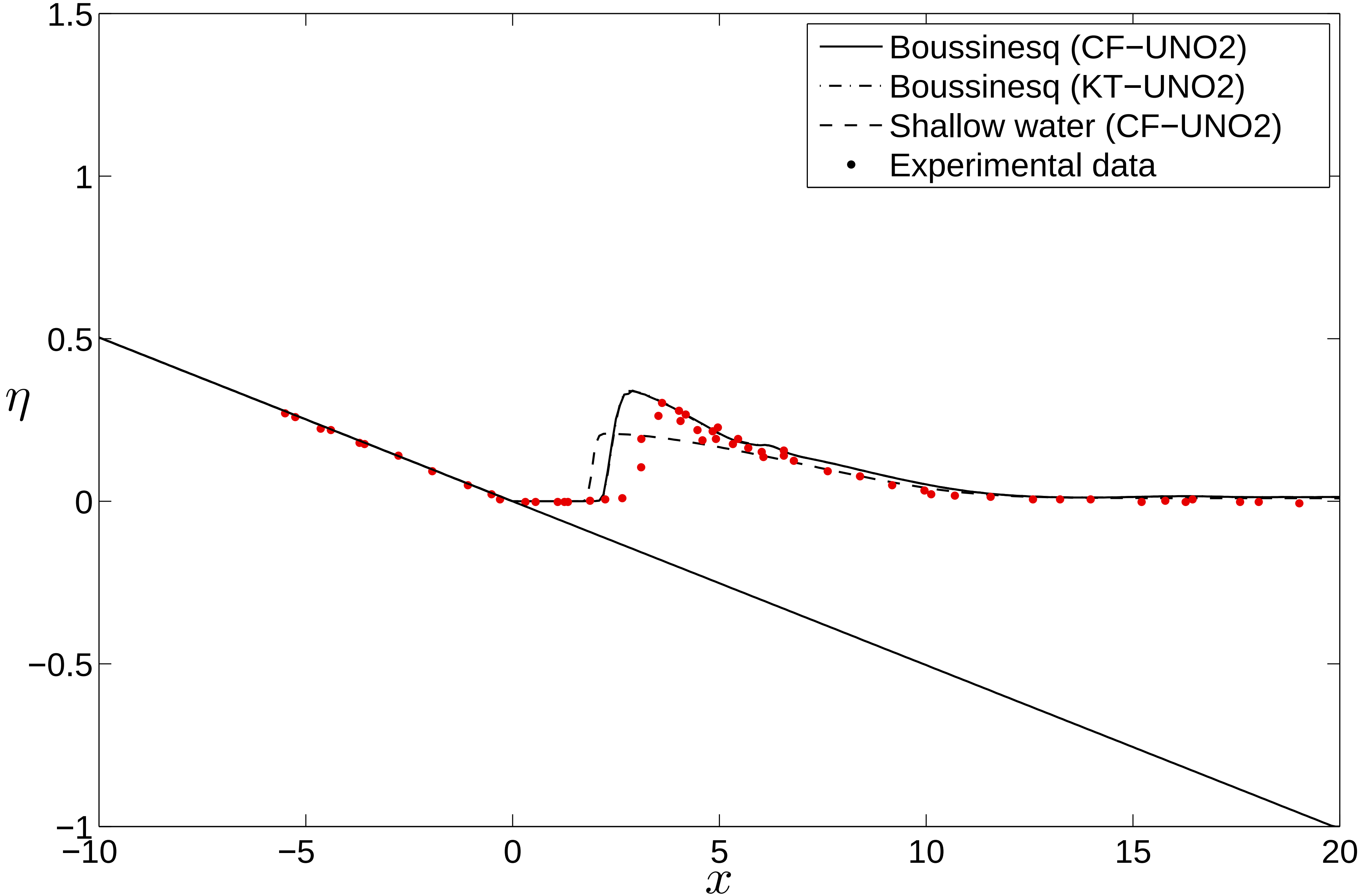
Runup : Nearly breaking solitary wave



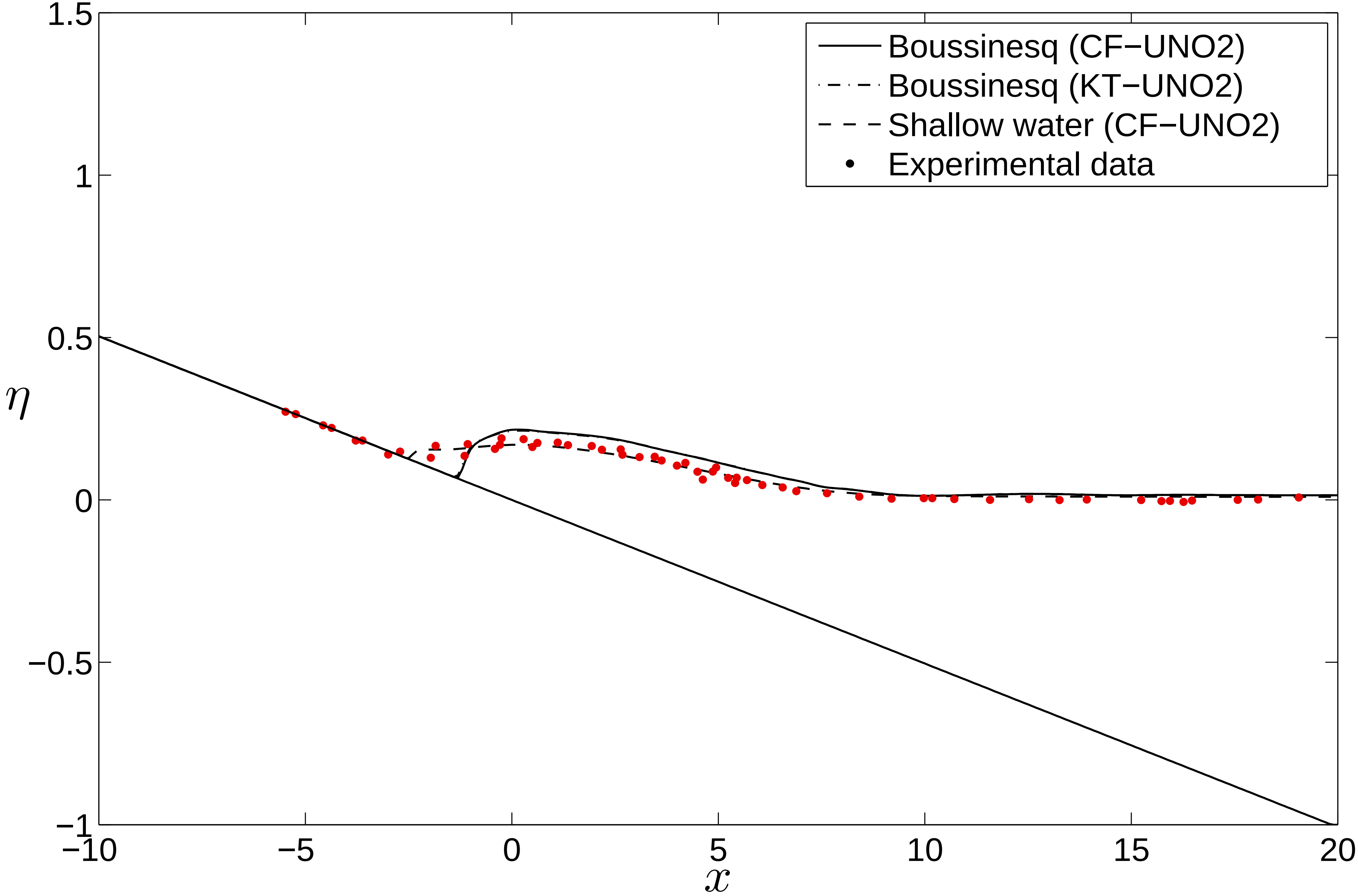
Runup : Nearly breaking solitary wave



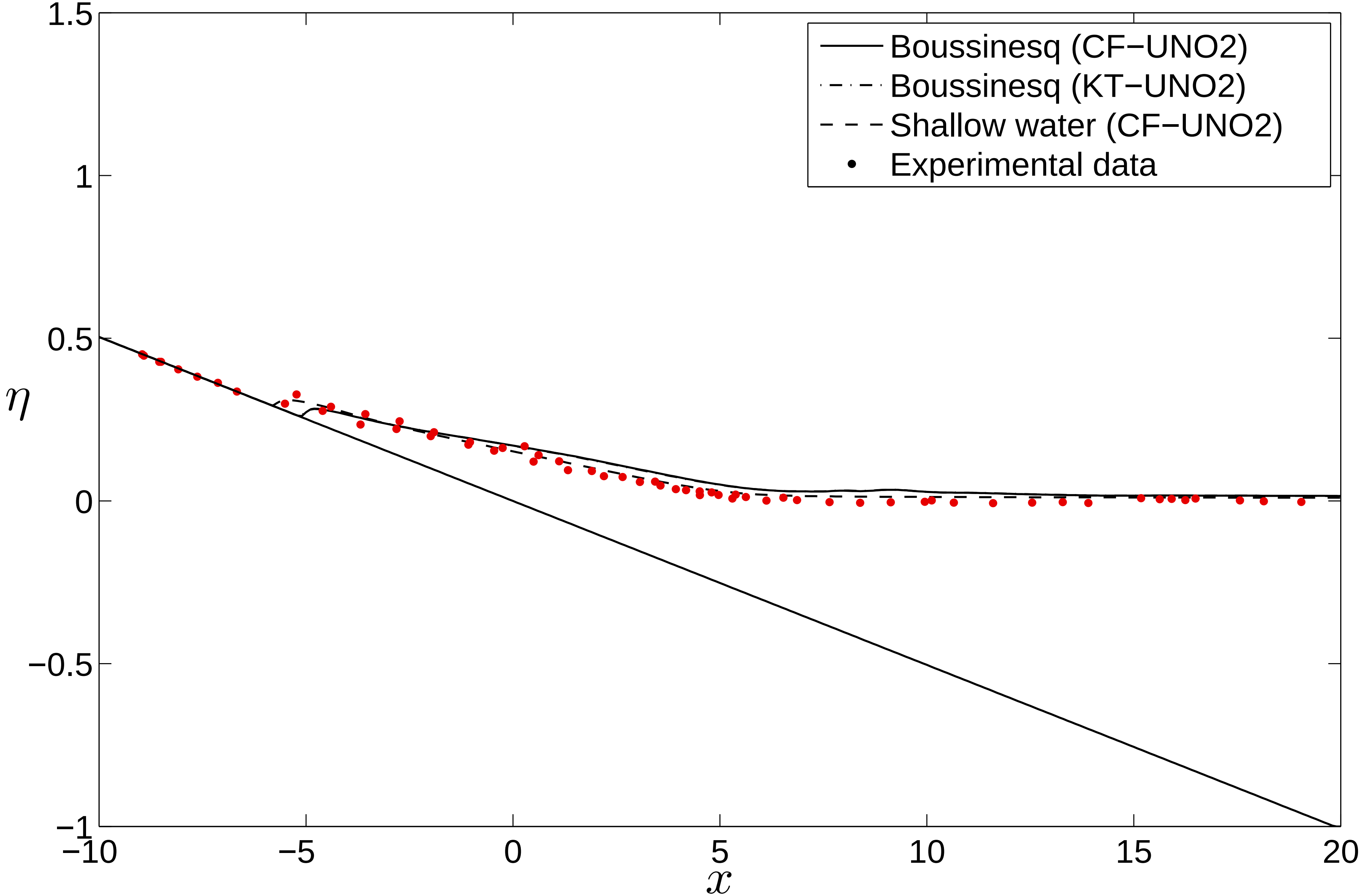
Runup : Nearly breaking solitary wave

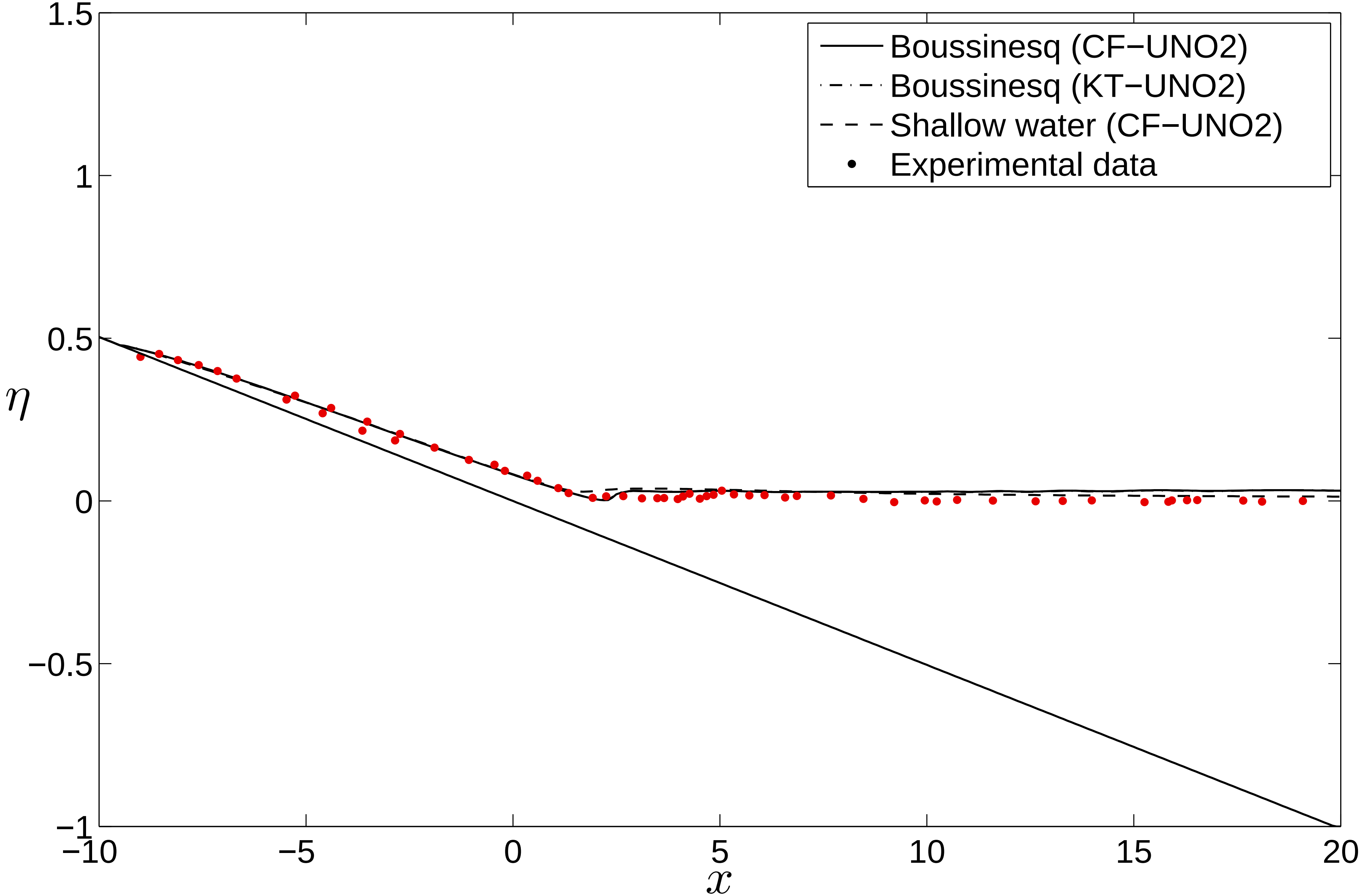


Runup : Nearly breaking solitary wave

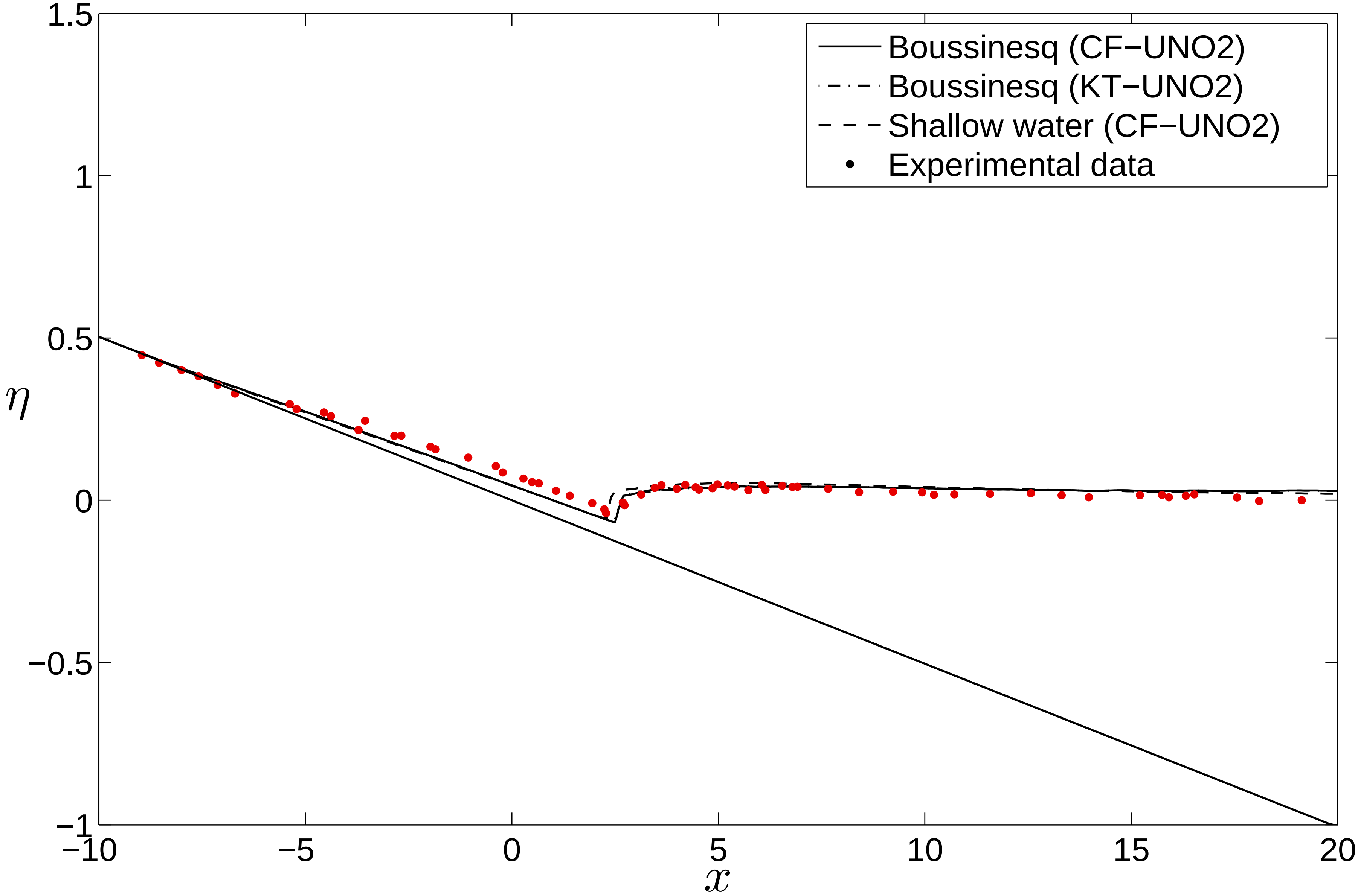


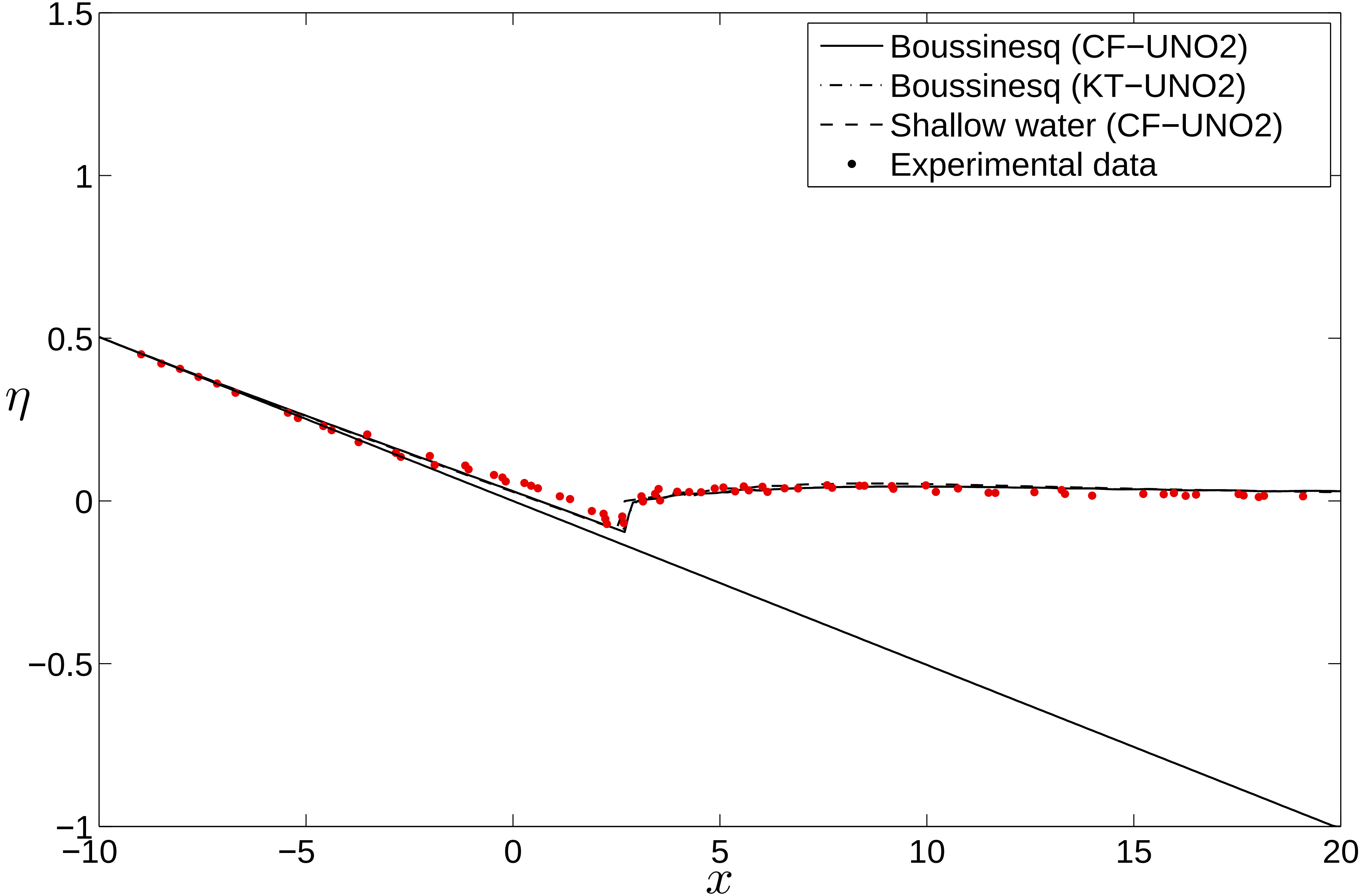
Runup : Nearly breaking solitary wave




Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data
1.5
1
0.5
0
−0.5
−1
η
−10
−5
0
5
10
15
20
x

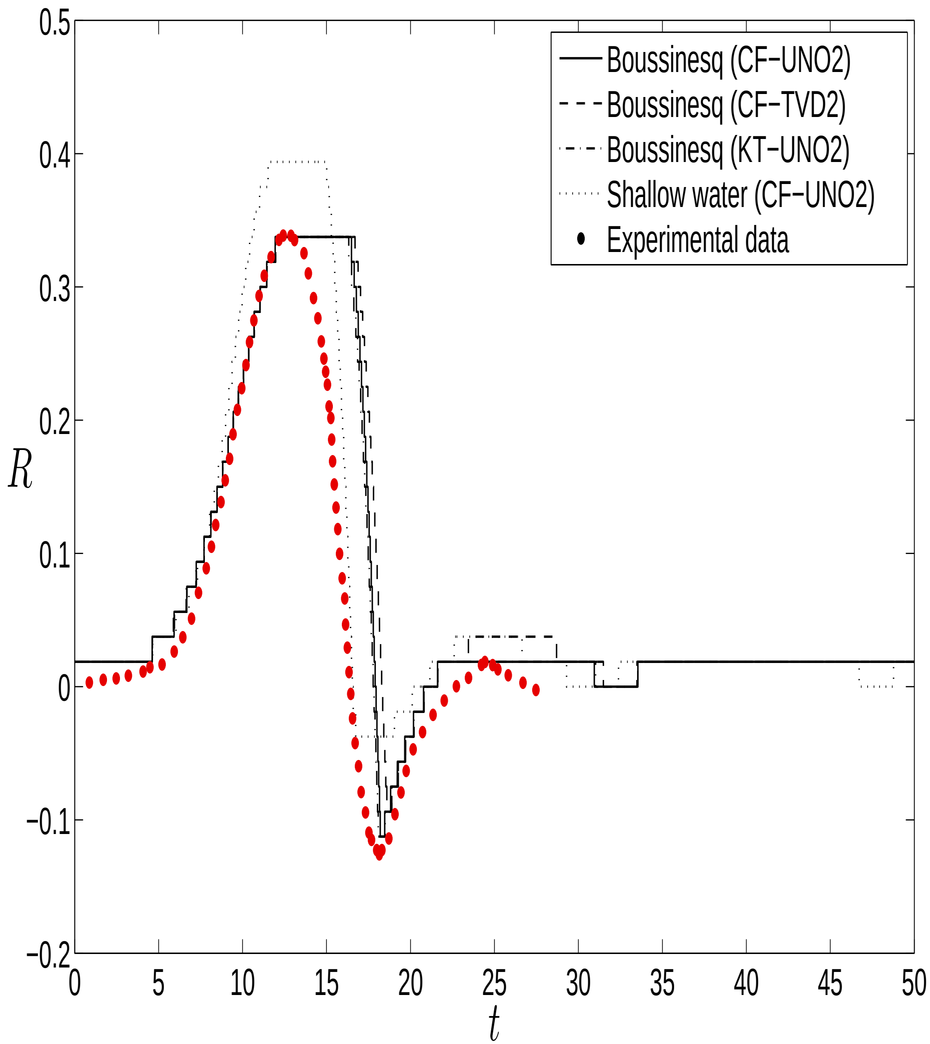
Runup : Nearly breaking solitary wave



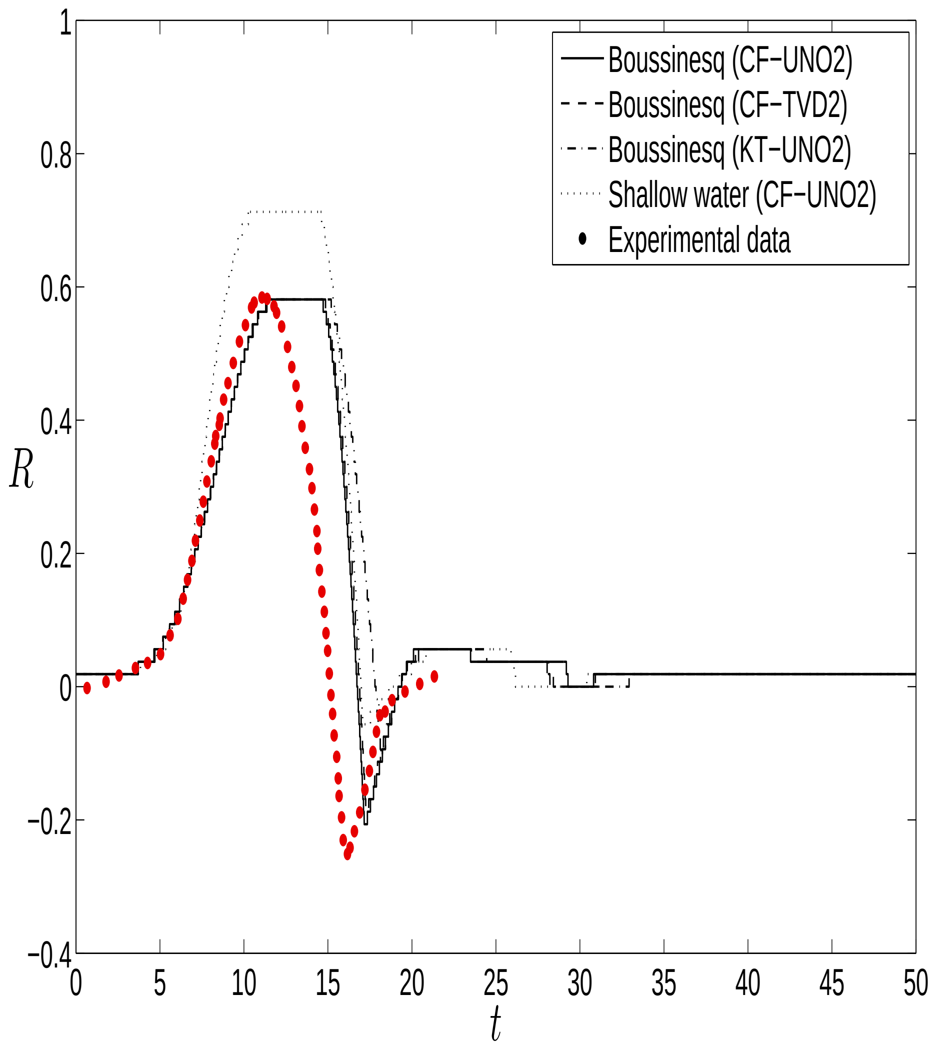

1.5
1
0.5
0
−0.5
−1
η
−10
−5
0
5
10
15
20
x
Boussinesq (CF−UNO2)
Boussinesq (KT−UNO2)
Shallow water (CF−UNO2)
Experimental data

Solitary wave on a steep slope

- Beach : $[-10, 30]$ Slope : $\beta = 20°$
- Solitary waves : location $X_0 = 8.85, X_0 = 10.62$
- Amplitudes : $A_s = 0.12$ $A_s = 0.2$
- No wave breaking during runup but the 2nd breaks during rundown.
- Friction plays no role
  - □ Mass conservation $I_0^h = 29.770808175$, $I_0^h = 29.4861671693$
  - □ Both models have a phase lag in the runup value R (last dry cell)
  - □ SW over-predicts the maximum runup and underestimates the minimum rundown
  - □ Boussinesq predicts correctly the extrema.

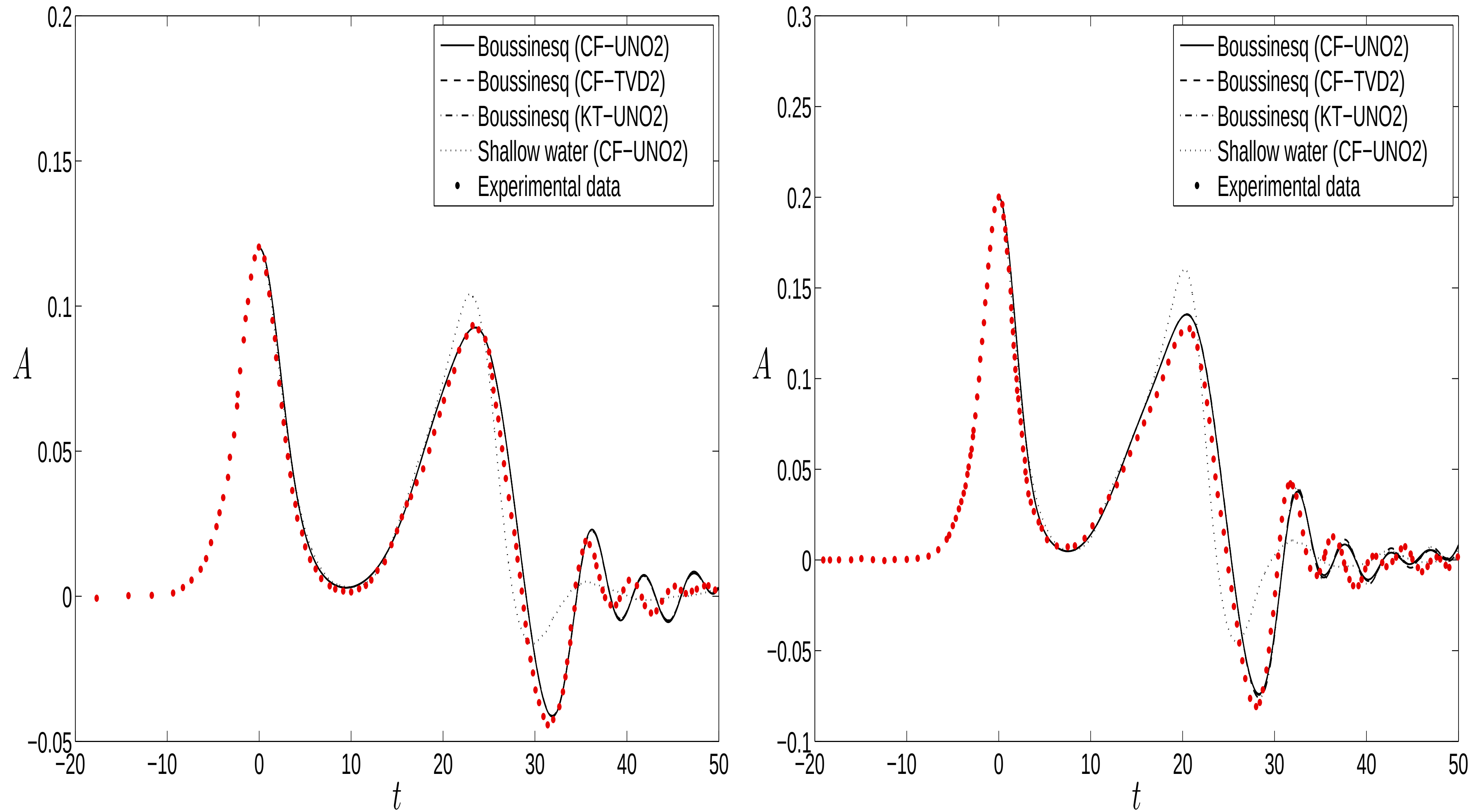


(a) $A_s = 0.12$



(b) $A_s = 0.2$

Steep slope : gauge at 8.85



(c) $A_s = 0.12$ (d) $A_s = 0.2$

Solitary wave runup on a beach with a pond

- Beach : $[-10, 50]$ Slope : $\beta = 2.88^\circ$
- Solitary wave : location $X_0 = 19.85$ Amplitude : $A_s = 0.04$
- A small pond is added over the shoreline
  - □ Mass conservation $I_0^h = 40.5198087147$
  - □ A breaking wave is reflected back .
    - SW : propagates as a shock wave.
    - Boussinesq : Airy type wave with dispersive characteristics.

Runup with pond

Sumatra tsunami

Related Publications

D.Dutykh, Th.K, D.Mitsotakis. Finite volume schemes for dispersive wave propagation and runup. Journal of Computational Physics, vol. 230, no. 8, pp. 3035-3061, 2011

D.Dutykh, Th.K, D.Mitsotakis. Finite volume methods for some unidirectional dispersive water wave models, Int. J. for Num. Methods in Fluids, vol. 71, pp. 717-736, 2013

THANK YOU