The Abdus Salam
International Centre for Theoretical Physics

2469-8

Workshop and Conference on Geometrical Aspects of Quantum States in Condensed Matter

1 - 5 July 2013

The quantum criticality of metals in two dimensions: antiferromagnetism, d-wave superconductivity and bond order HARVARD

Subir Sachdev

Harvard University
USA

The quantum criticality of metals in two dimensions: antiferromagnetism, d-wave superconductivity and bond order

ICTP, Trieste
July 2, 2013

Subir Sachdev

PHYSICS

VE RI TAS

HARVARD

Max Metlitski

Erez Berg

Rolando
La Placa

Quantum phase transitions with loss of antiferromagnetic order

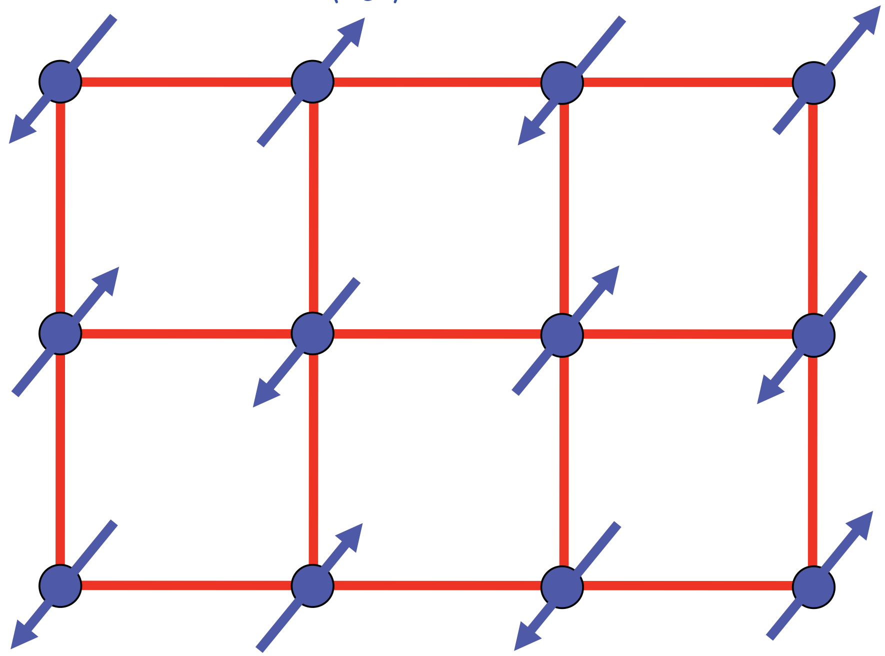
1. Dimerized insulators:
   Conformal field theory

2. Metals (at weaker coupling):
   The iron-based superconductors

3. Metals (at stronger coupling):
   The hole-doped cuprate superconductors

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

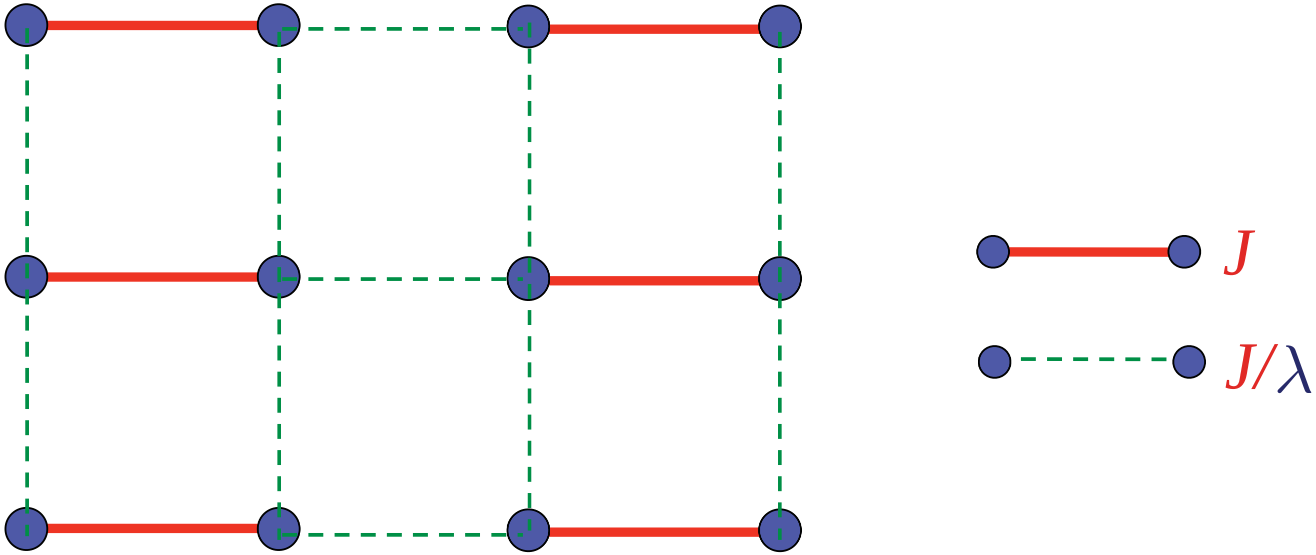
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

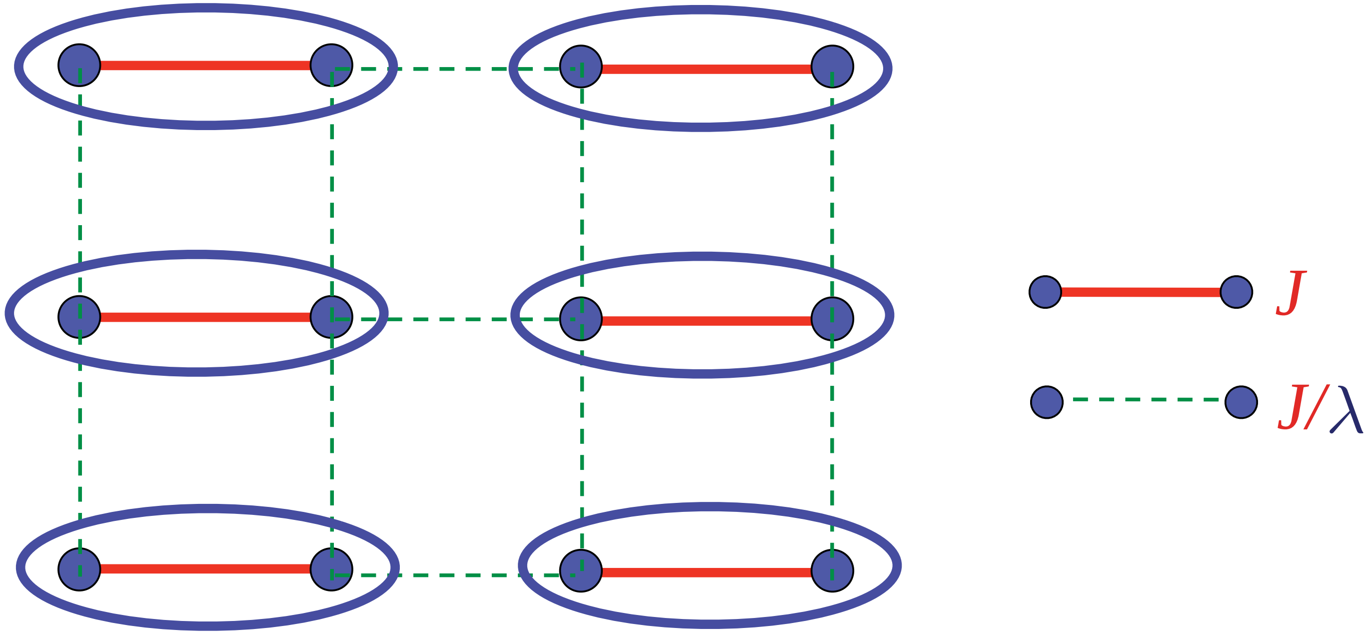
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



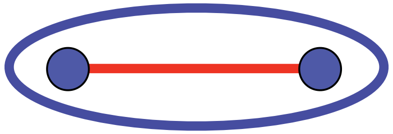
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

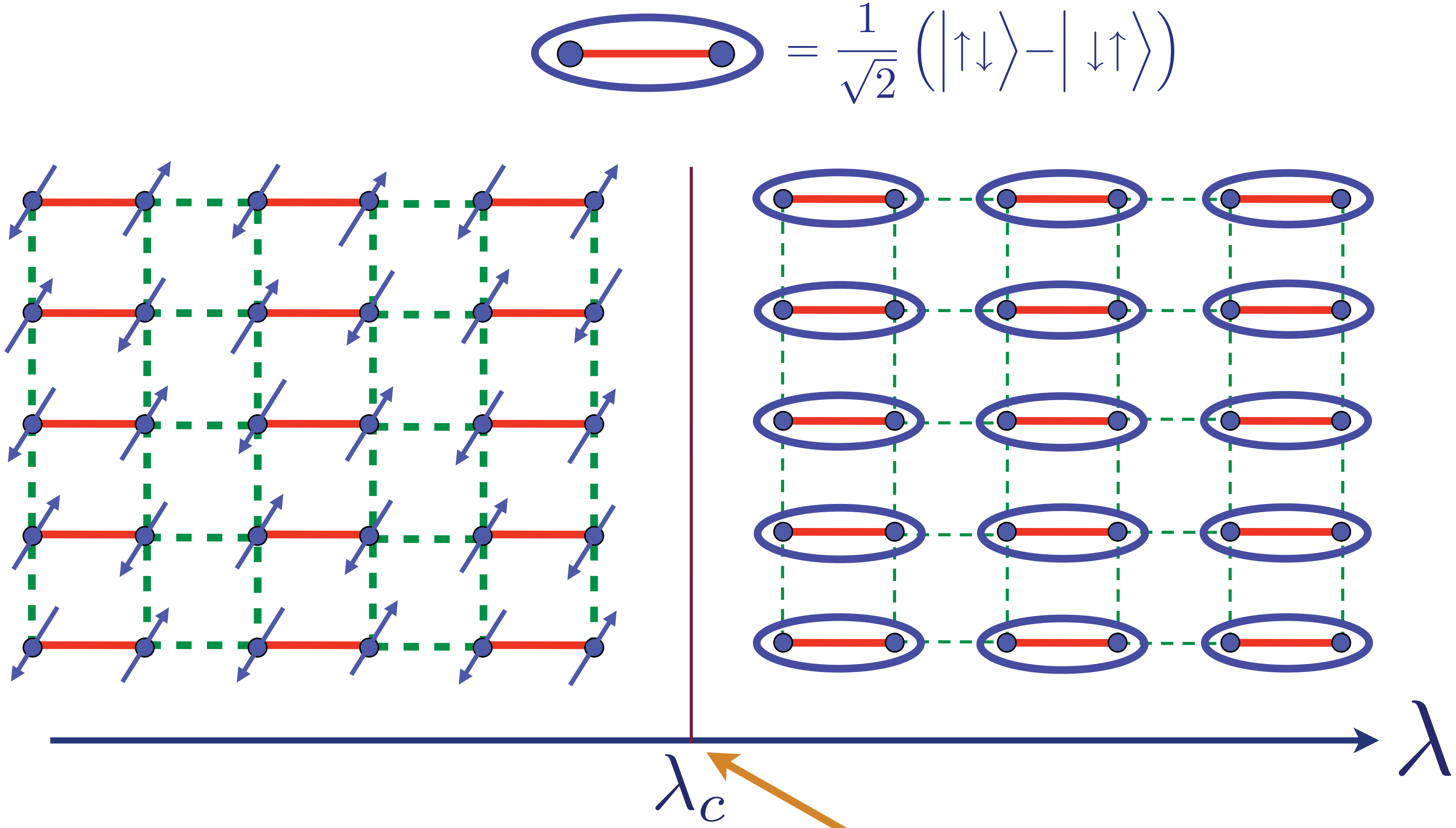
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



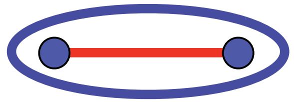
Ground state is a "quantum paramagnet" with spins locked in valence bond singlets



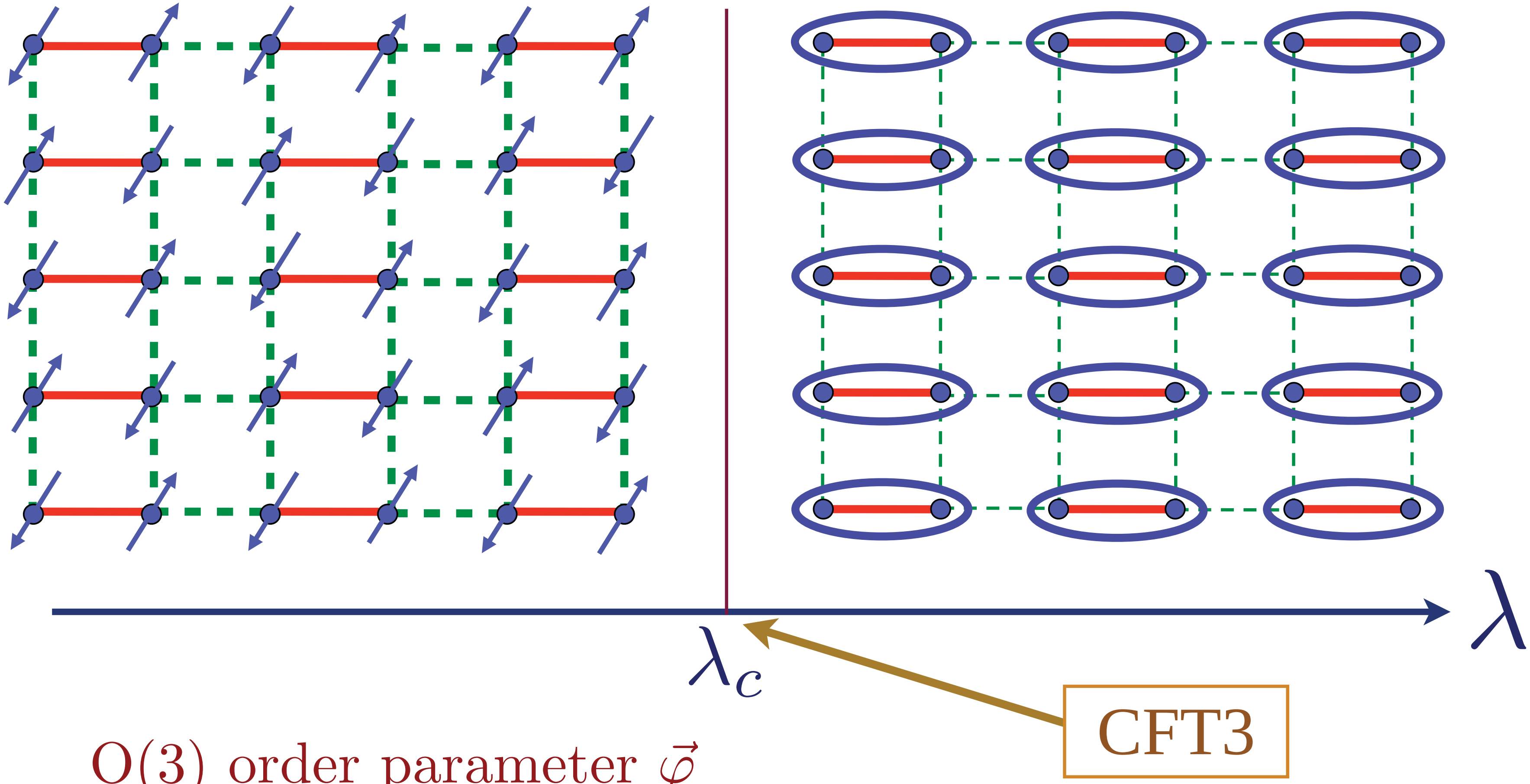
$$= \frac{1}{\sqrt{2}} \Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle \Big)$$



Quantum critical point with non-local entanglement in spin wavefunction

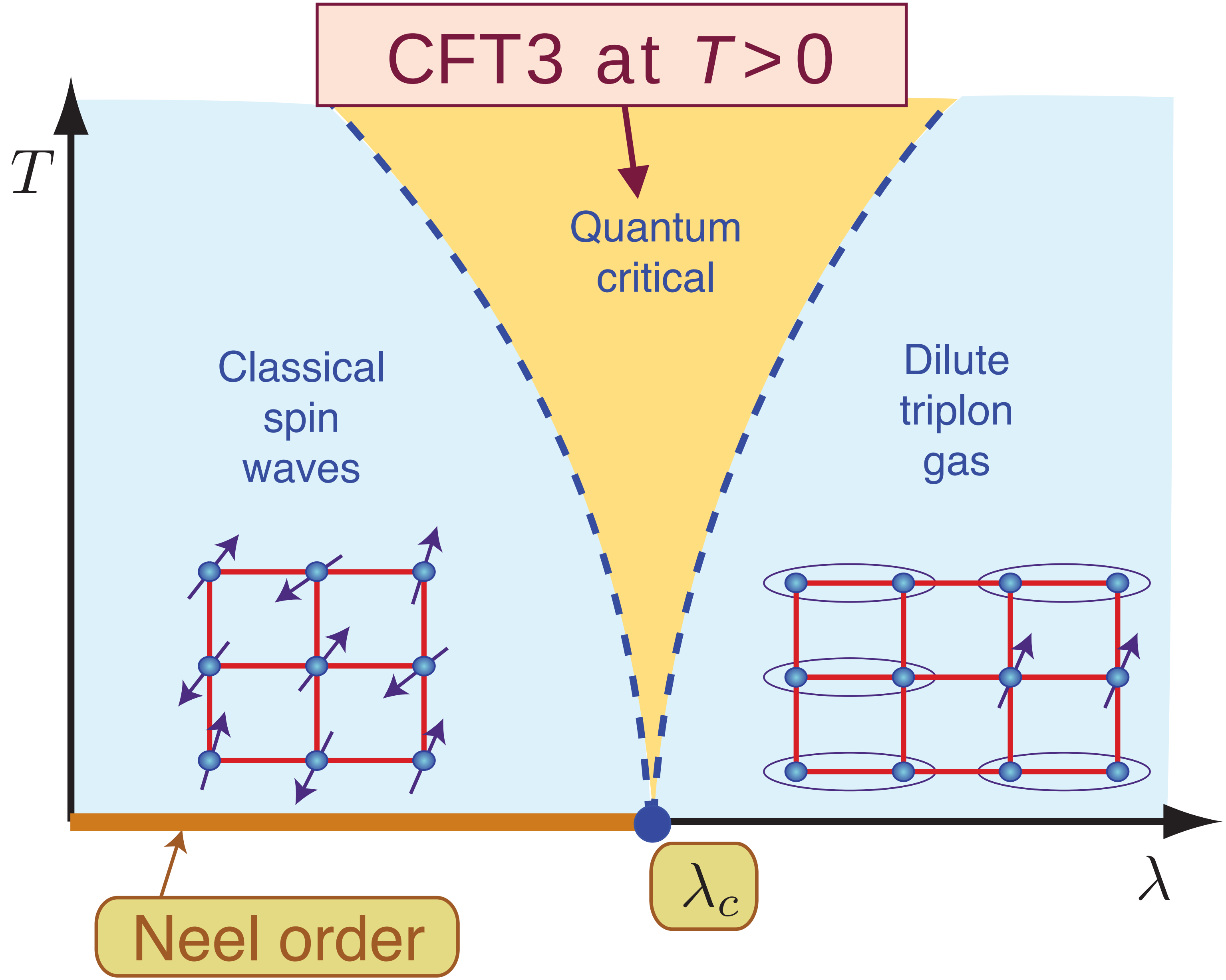


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$



S. Sachdev and
J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).

Quantum phase transitions with loss of antiferromagnetic order

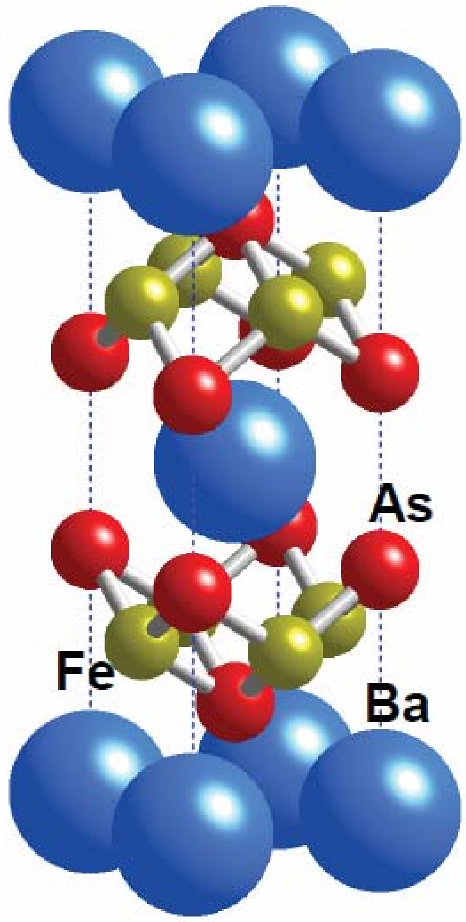
1. Dimerized insulators:
   Conformal field theory

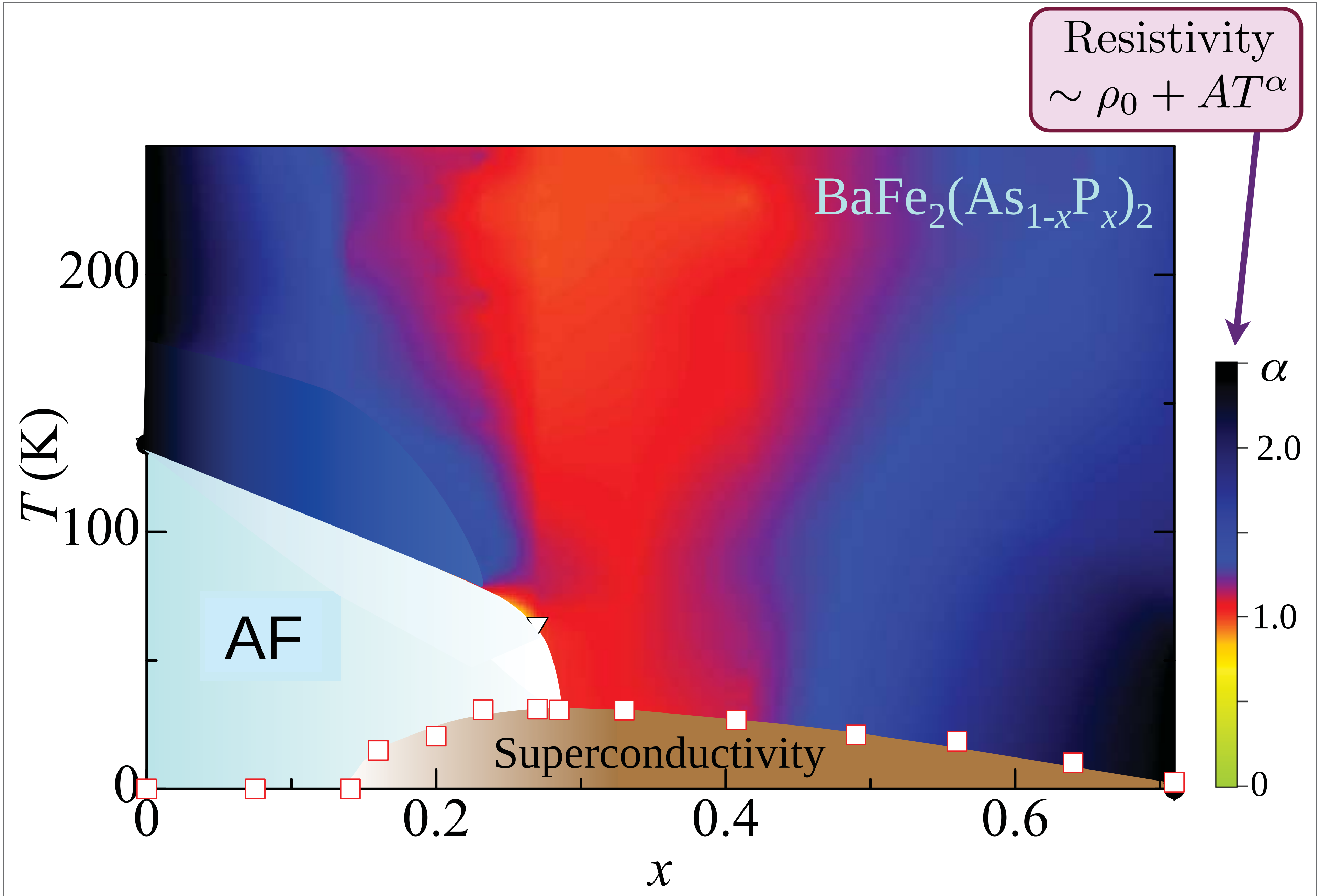
2. Metals (at weaker coupling):
   The iron-based superconductors

3. Metals (at stronger coupling):
   The hole-doped cuprate superconductors

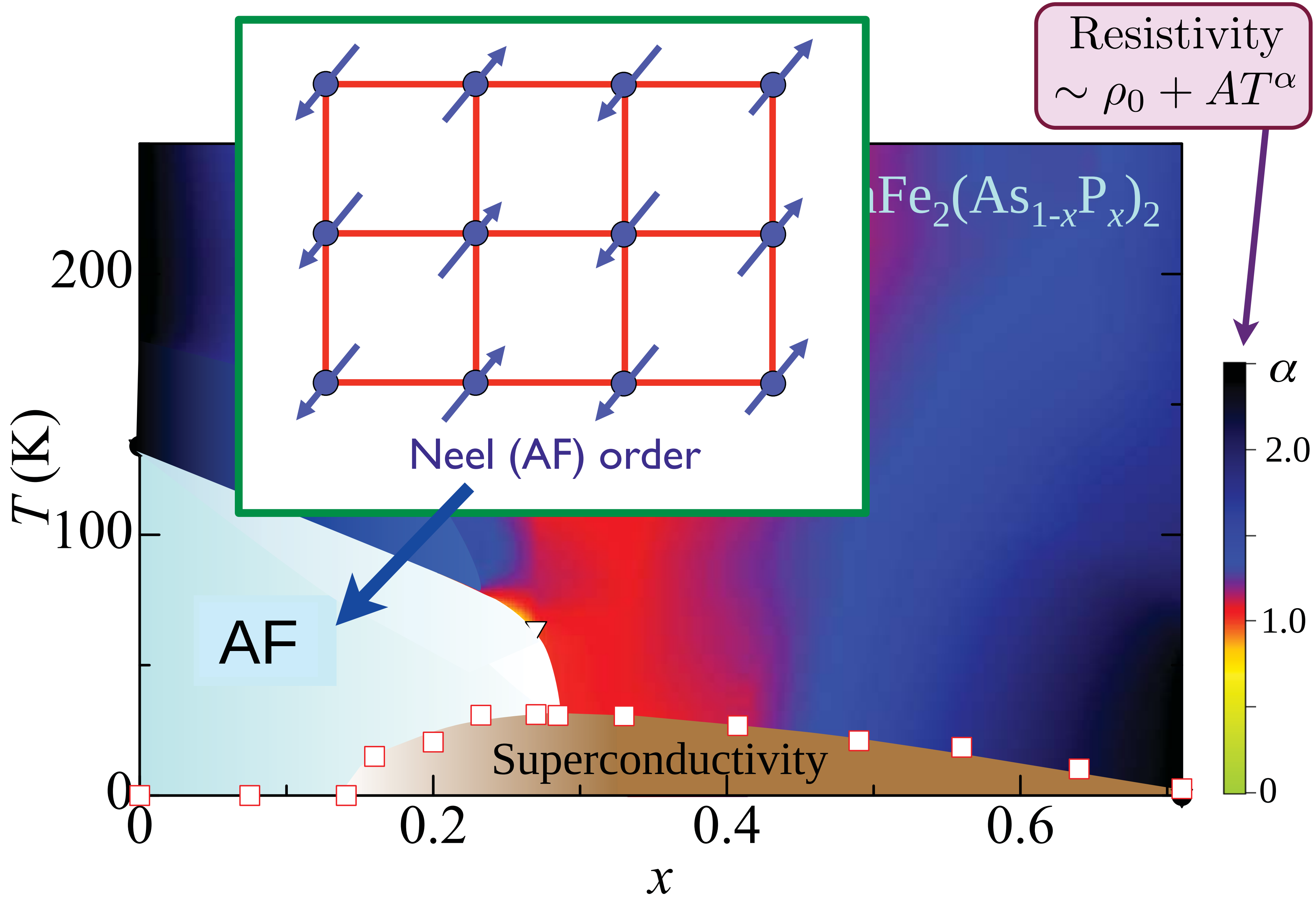
Iron pnictides:

a new class of high temperature superconductors

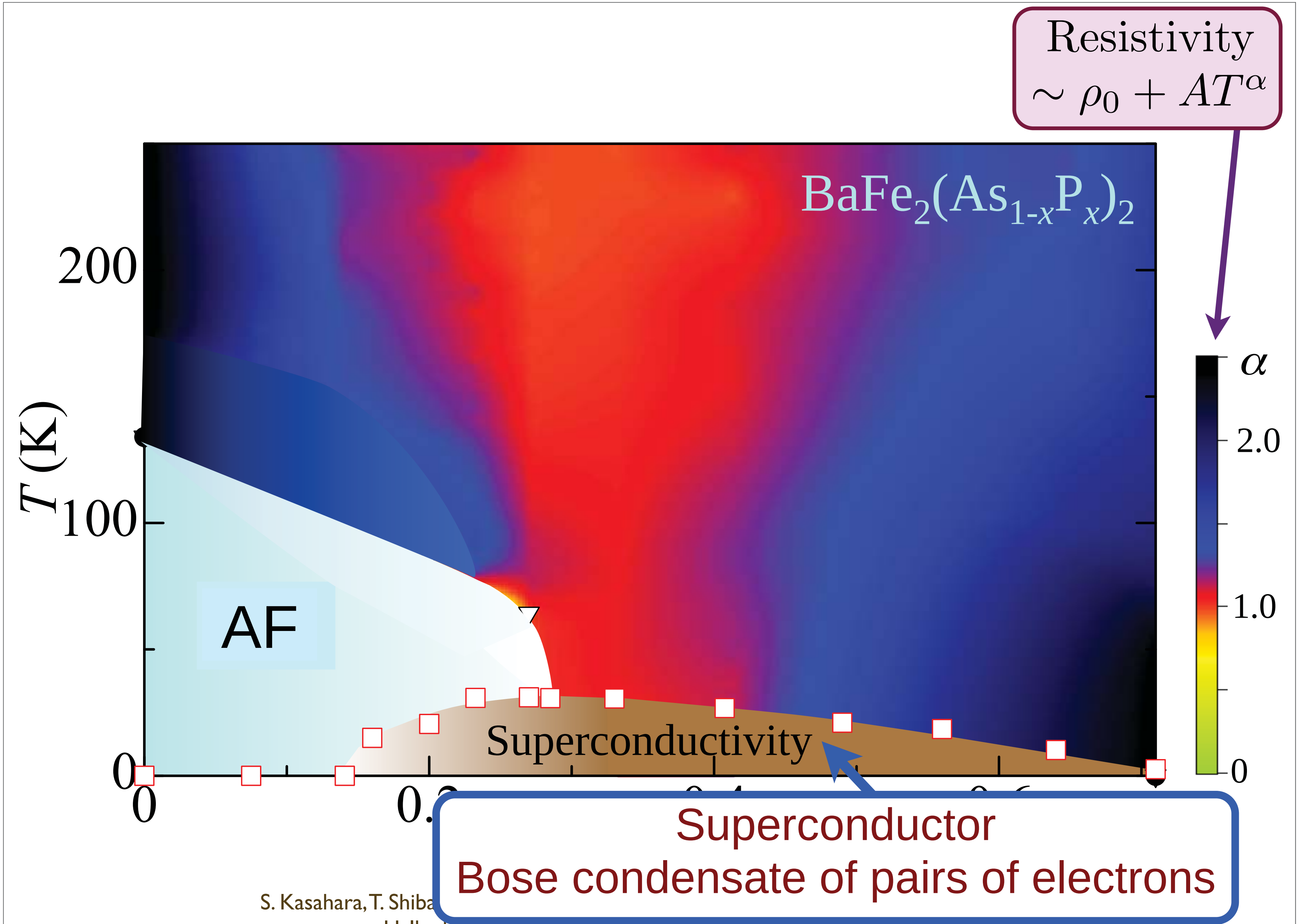




S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)



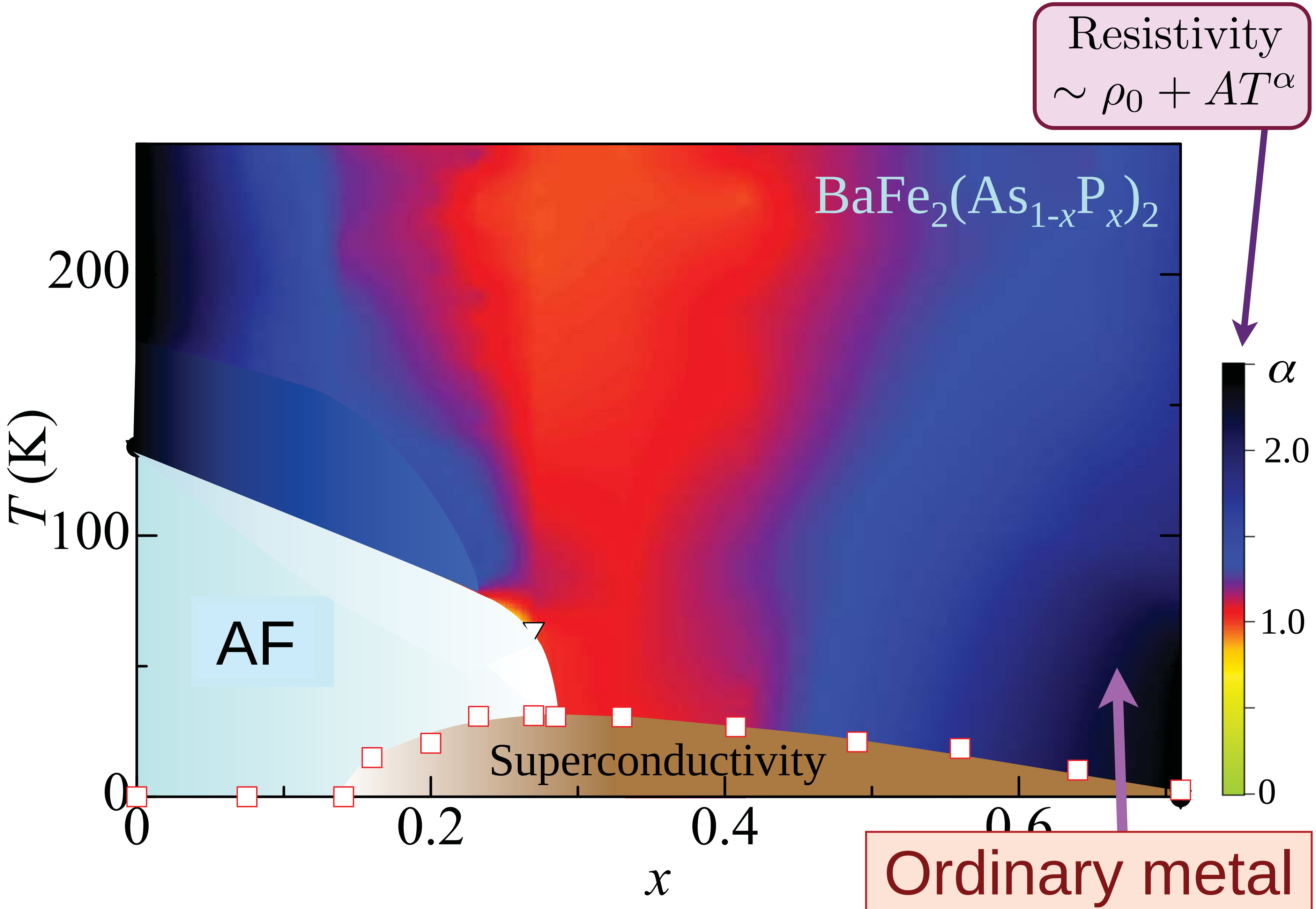
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



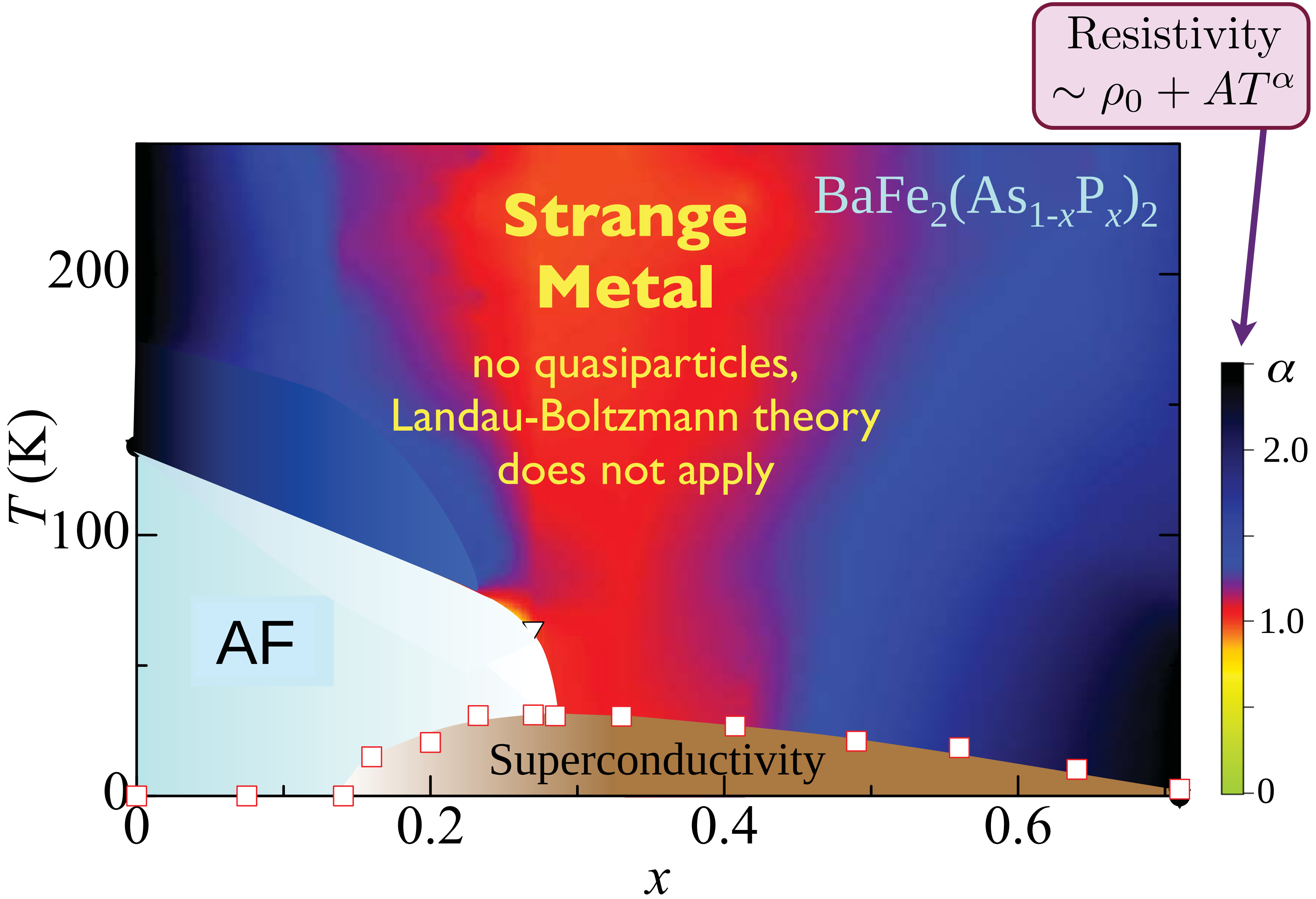
S. Kasahara, T. Shiba

H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,

Physical Review B **81**, 184519 (2010)



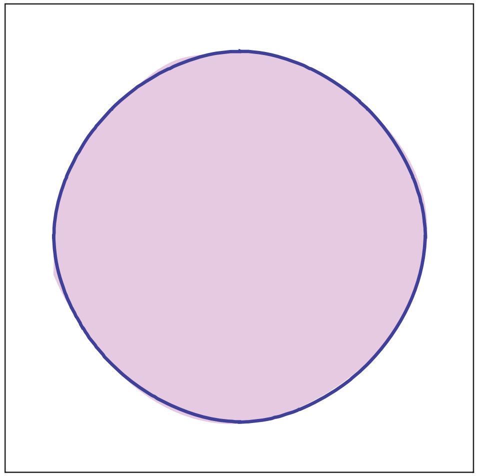
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)

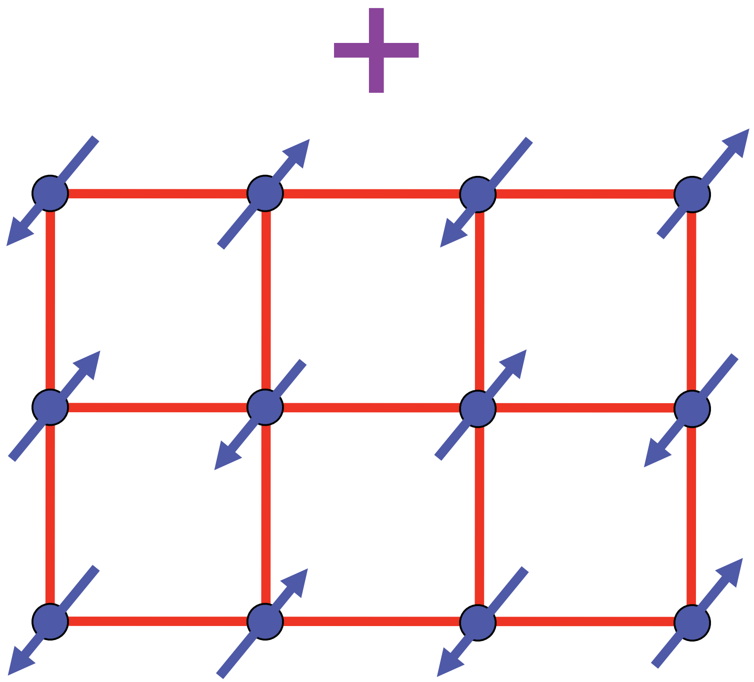


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface





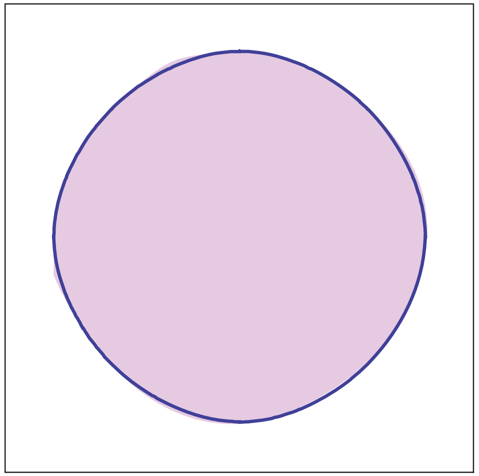
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r}, \tau) \right\rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

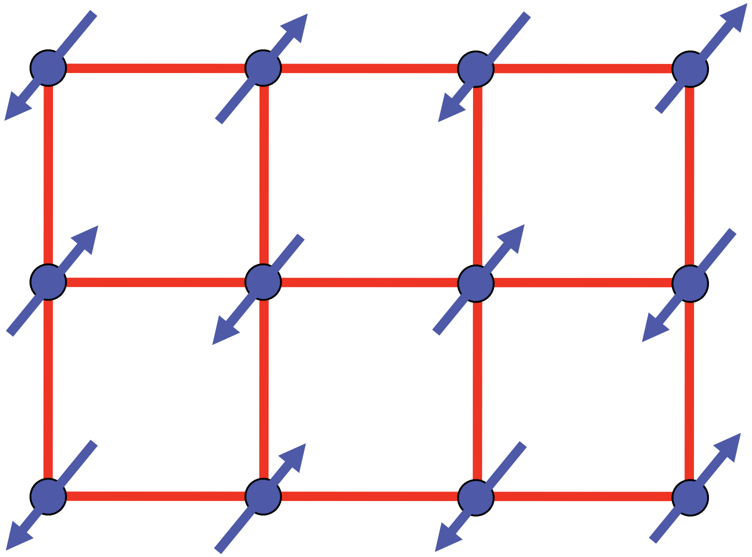
where **K** is the ordering wavevector.

Fermi surface+antiferromagnetism

$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$



+



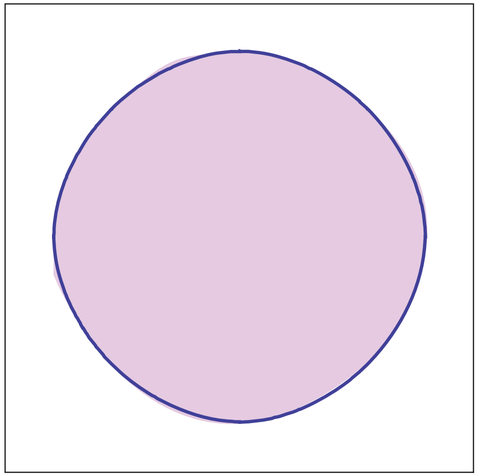
The electron spin polarization obeys

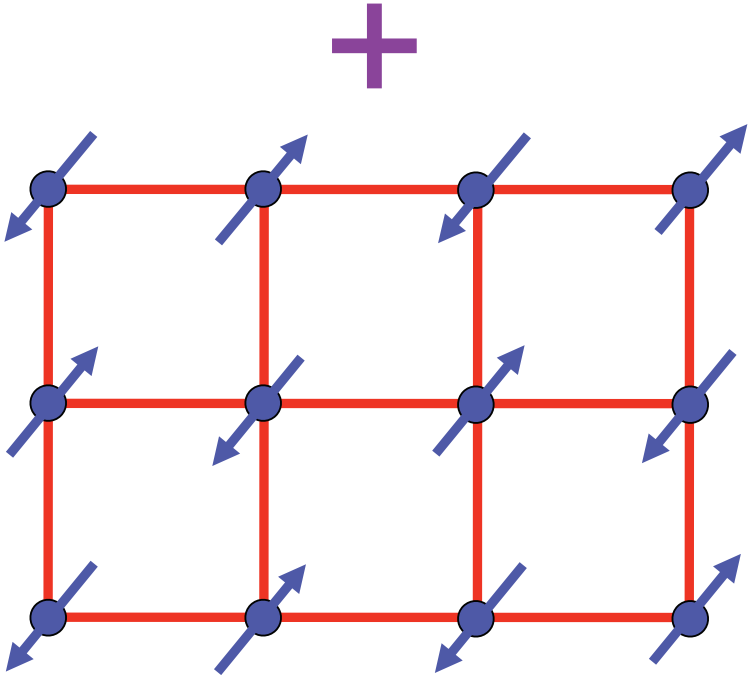
$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

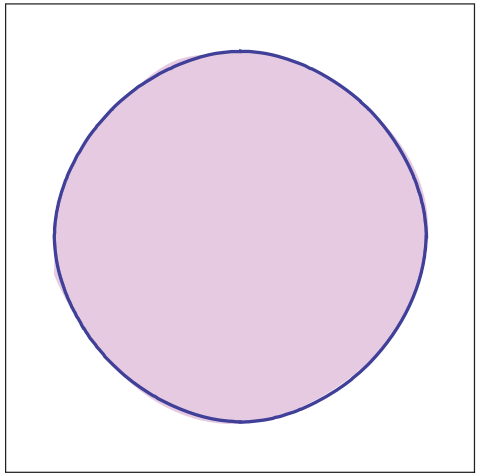




$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c_{\mathbf{k},\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices.

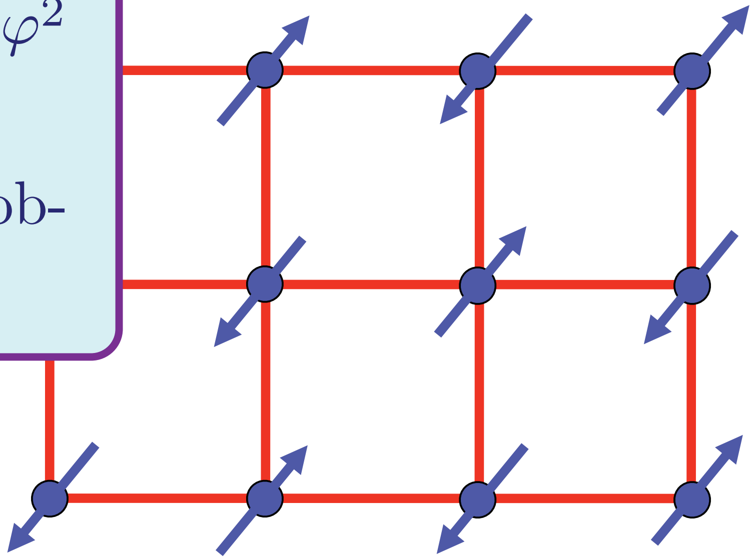
Fermi surface+antiferromagnetism



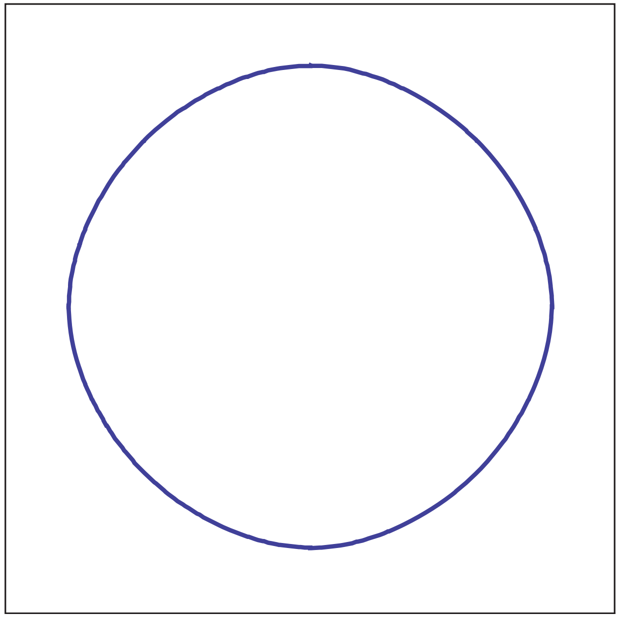
The electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ for $\vec{\varphi} \propto (0,0,1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

We now fill the lowest energy states to obtain the new Fermi surface

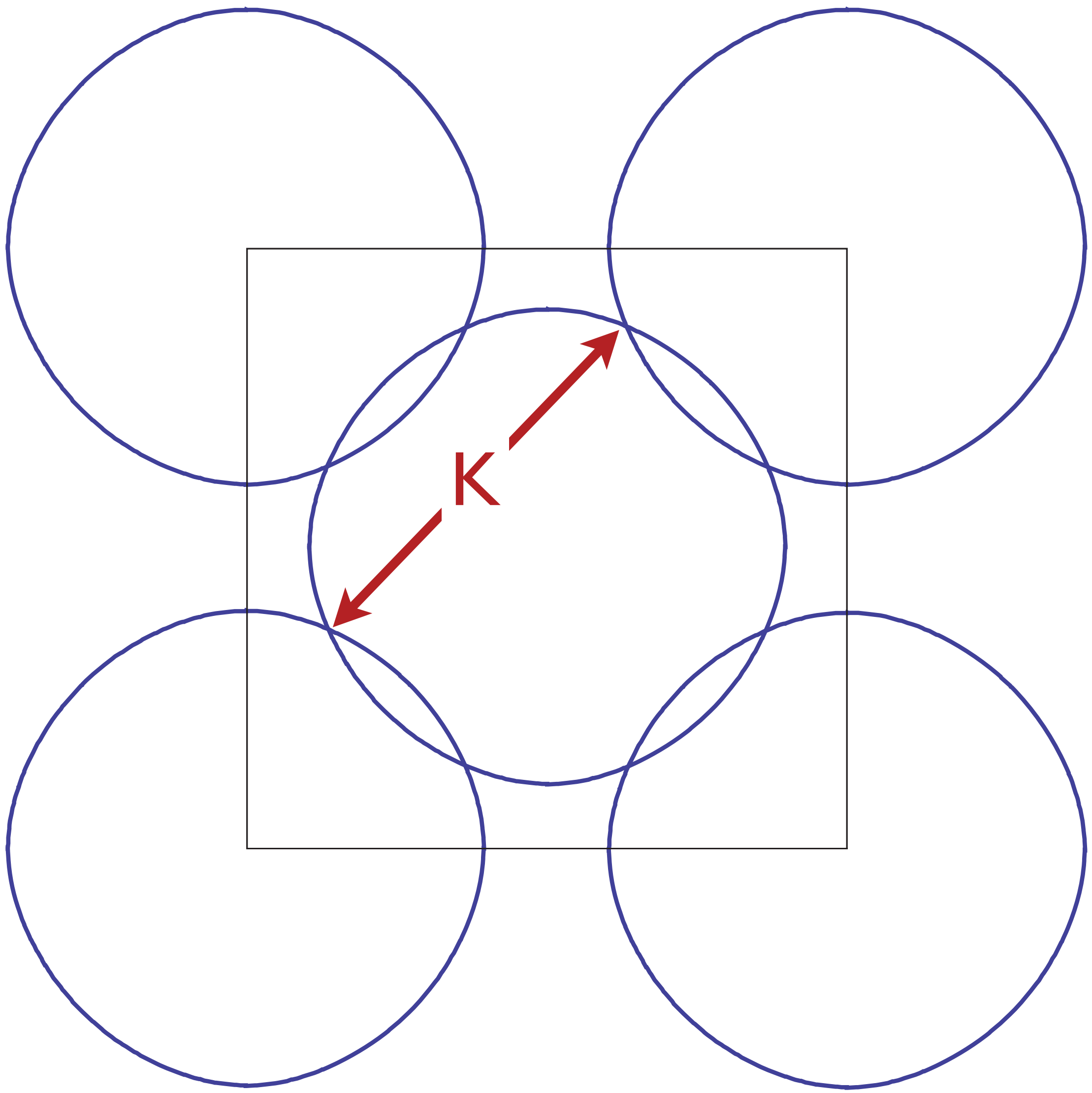


Fermi surface+antiferromagnetism



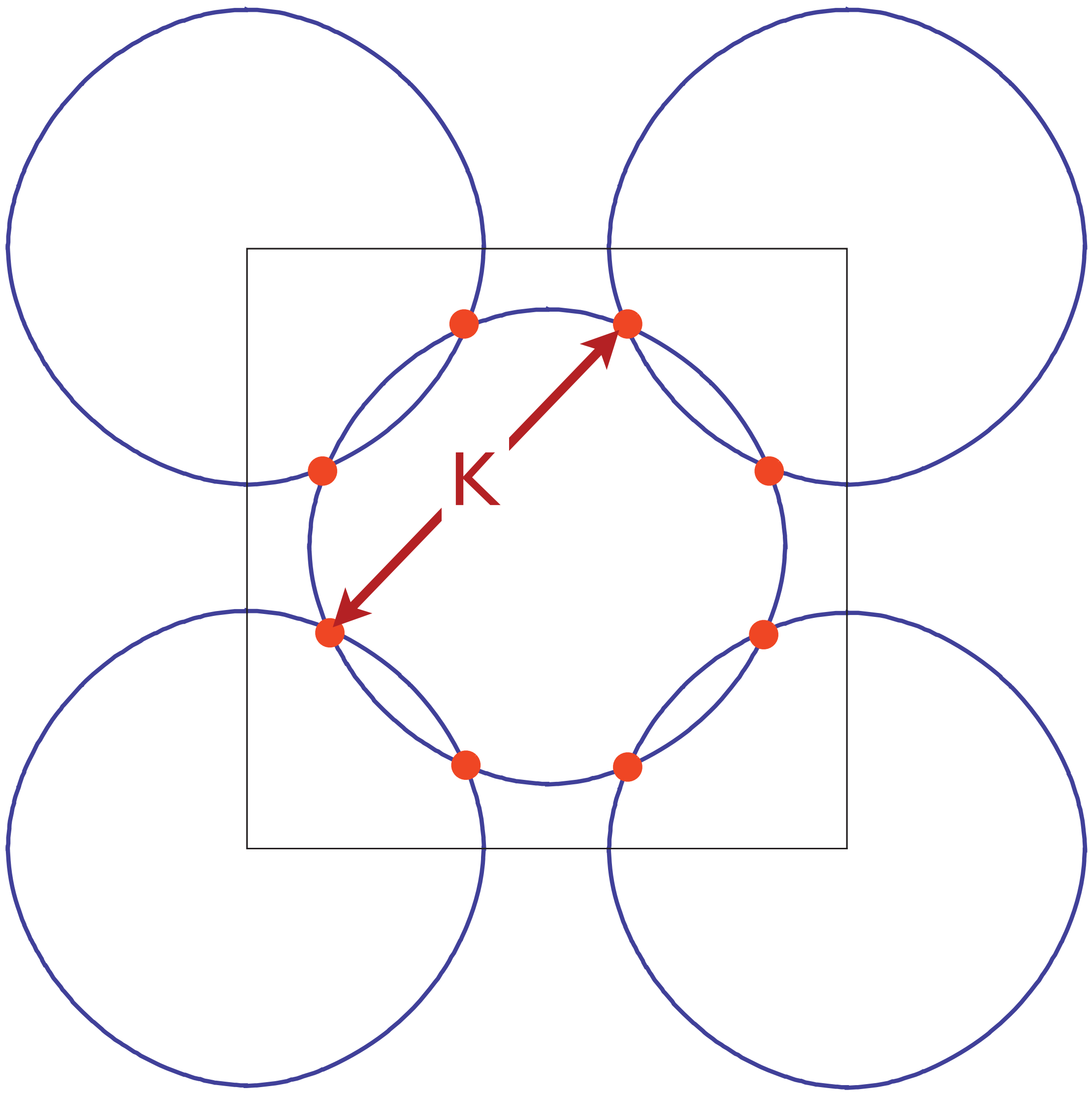
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



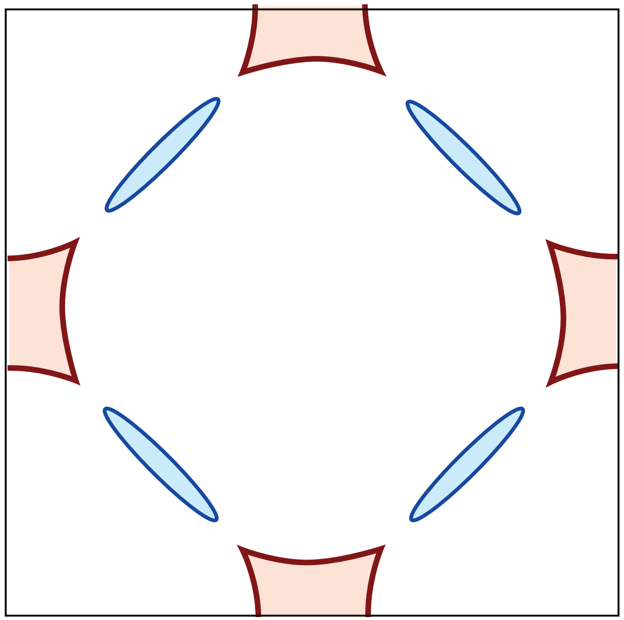
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



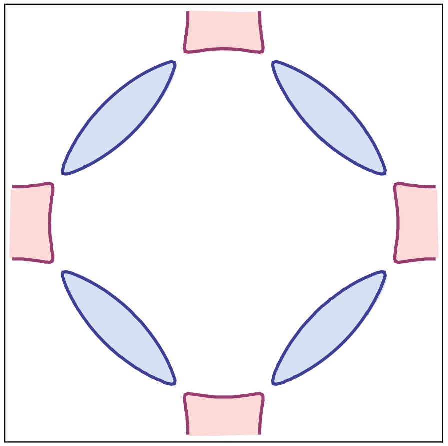
“Hot” spots

Fermi surface+antiferromagnetism



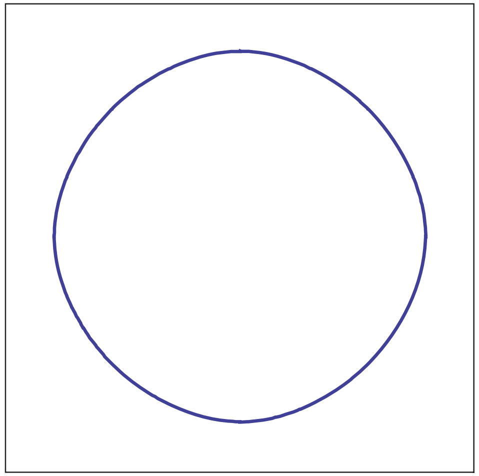
Electron and hole pockets in antiferromagnetic phase with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets

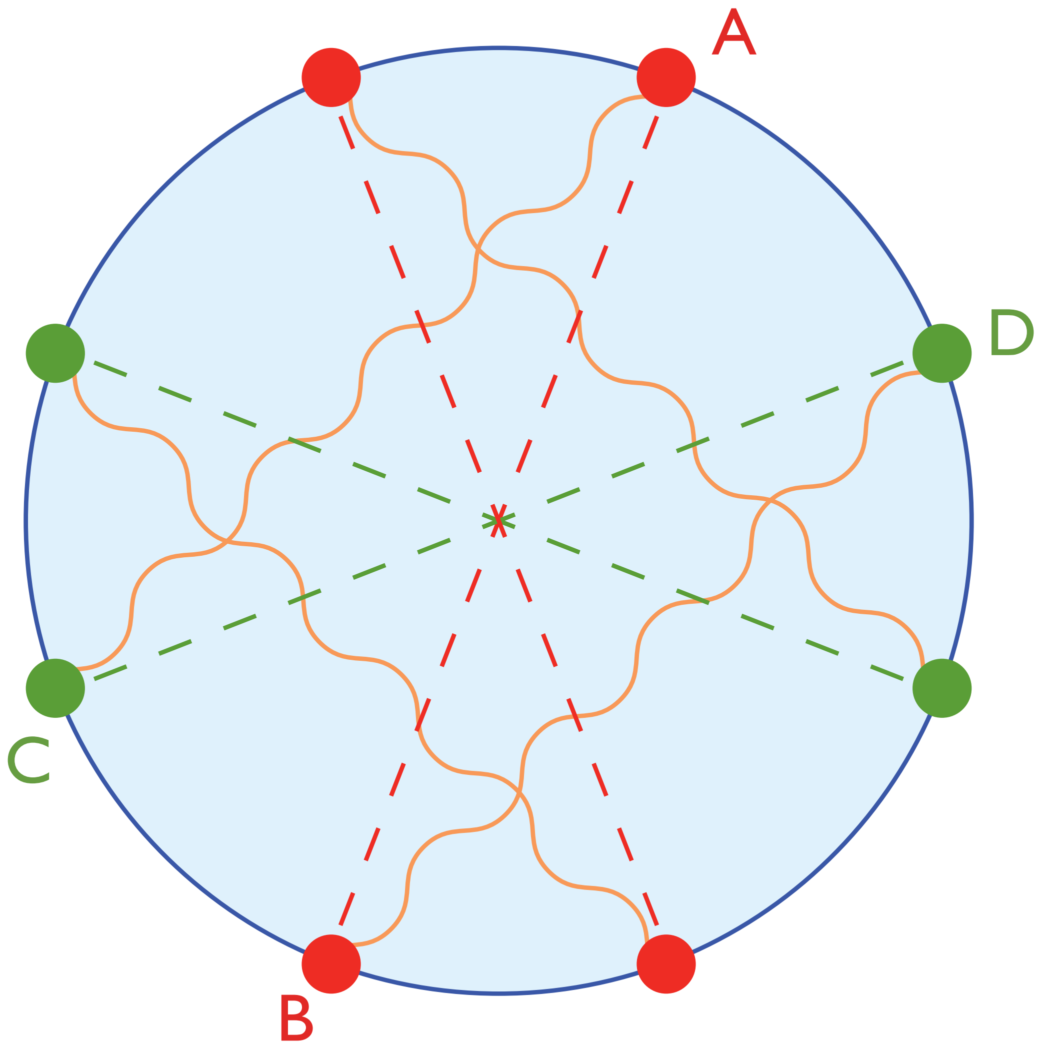


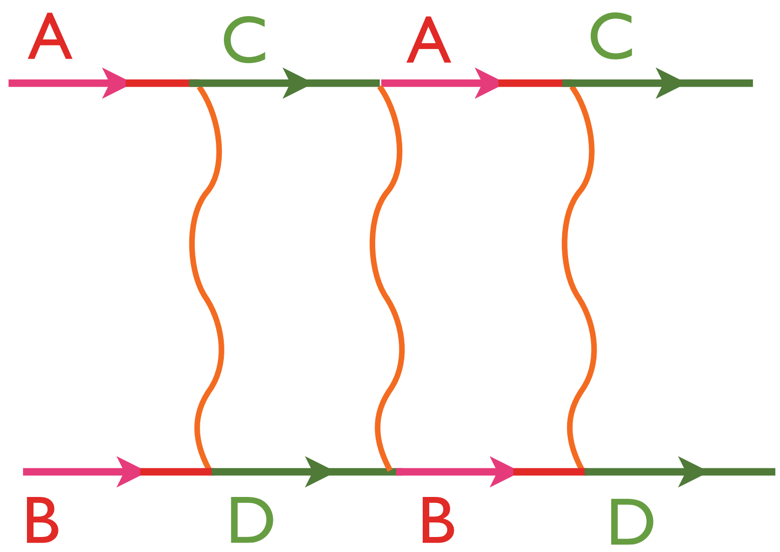
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

r

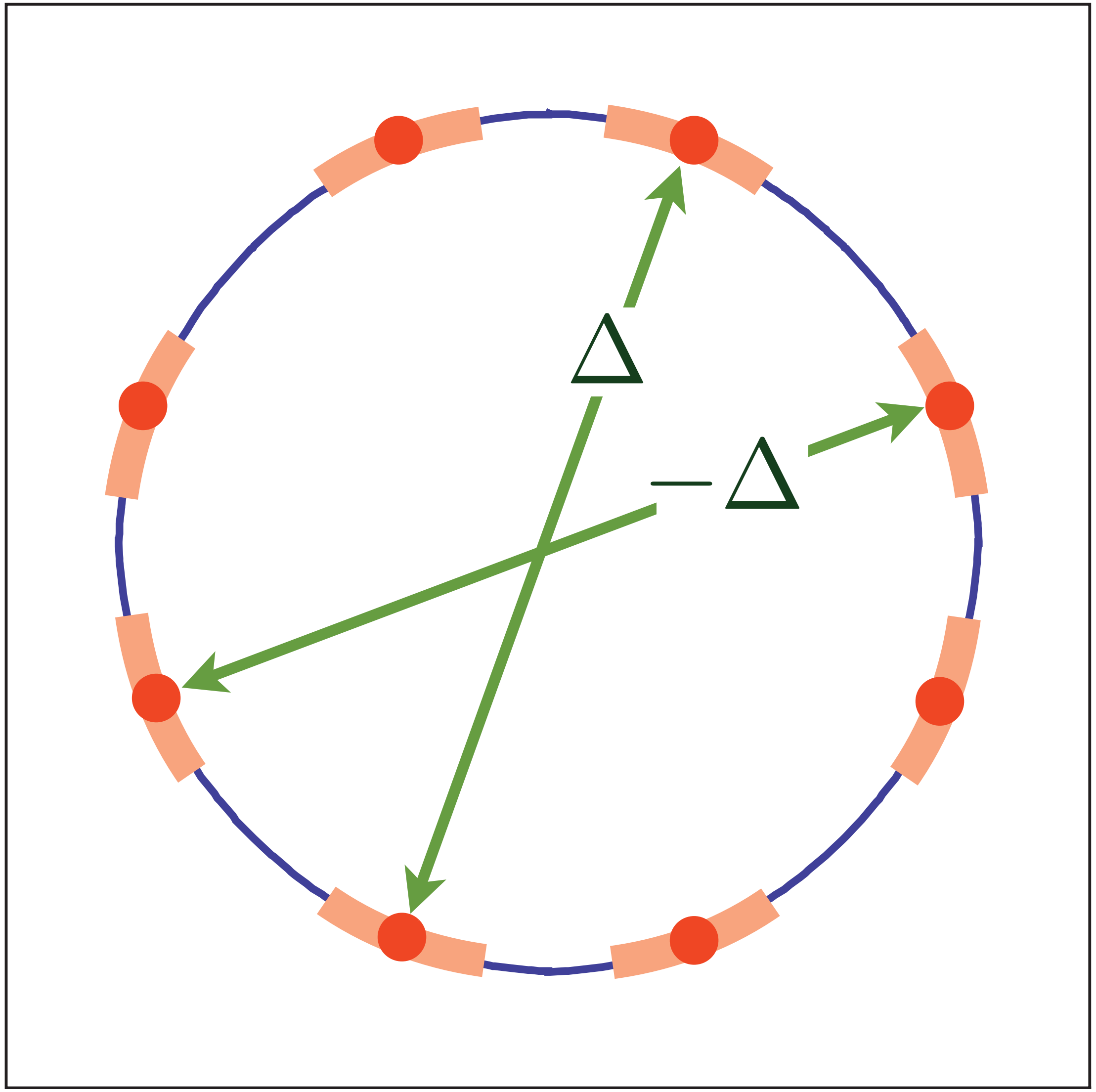
Pairing "glue" from antiferromagnetic fluctuations





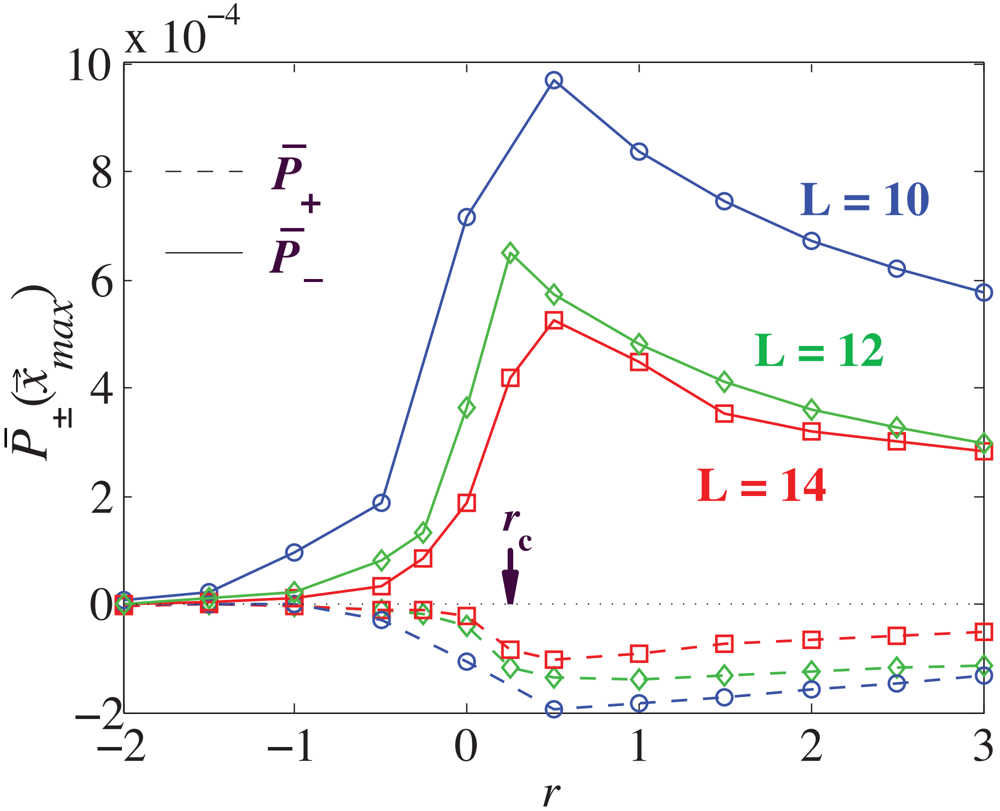
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

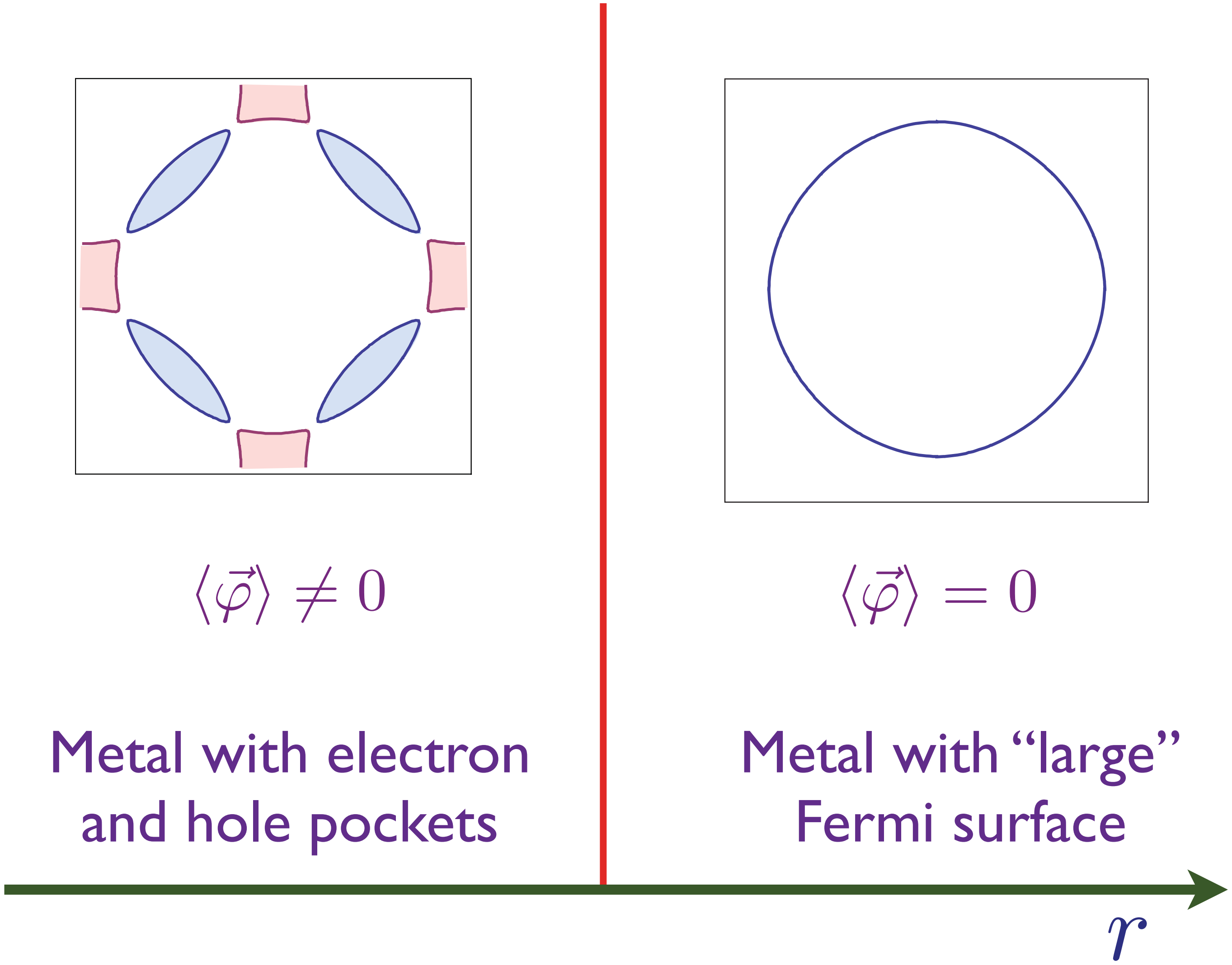
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



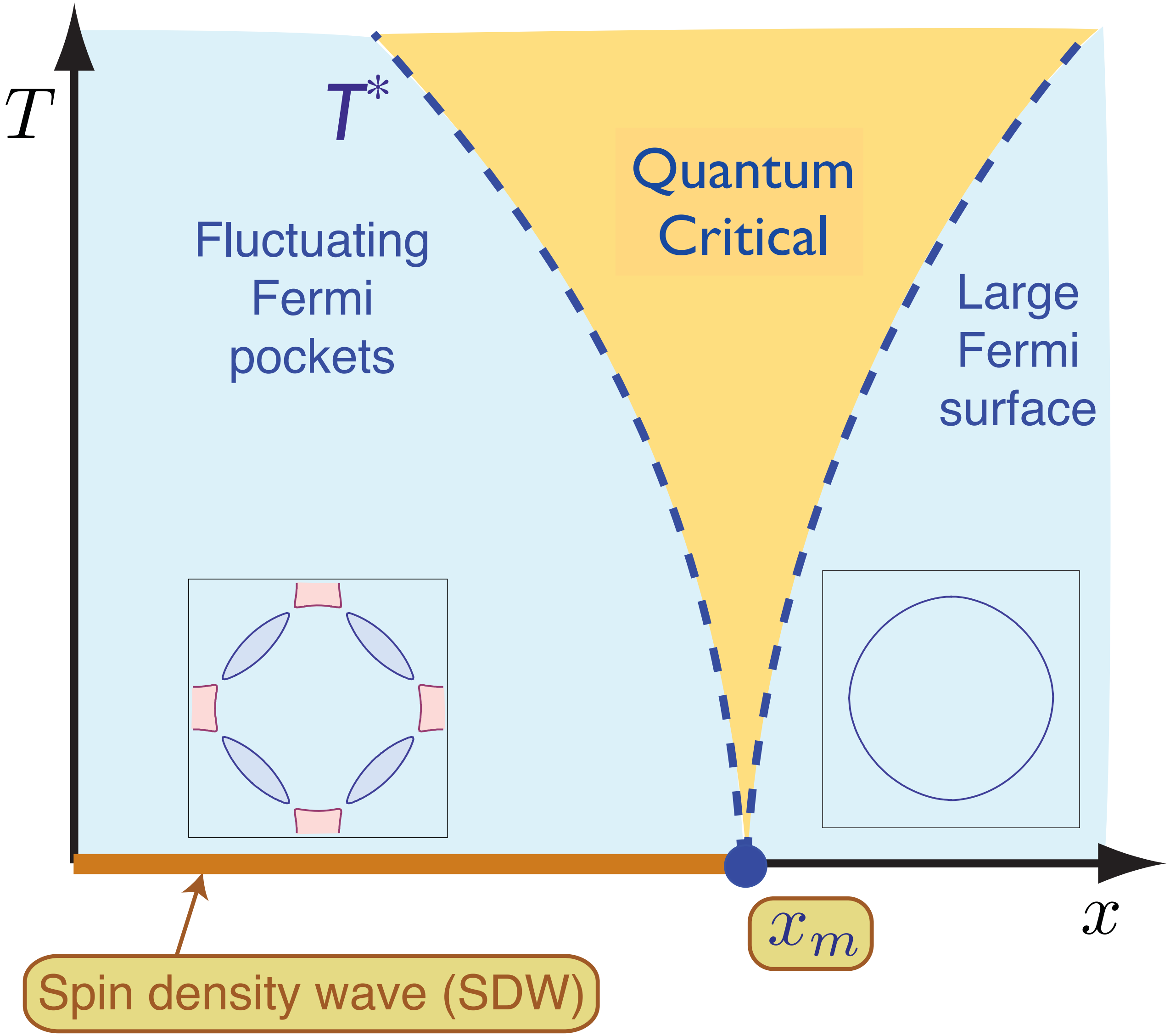
s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism

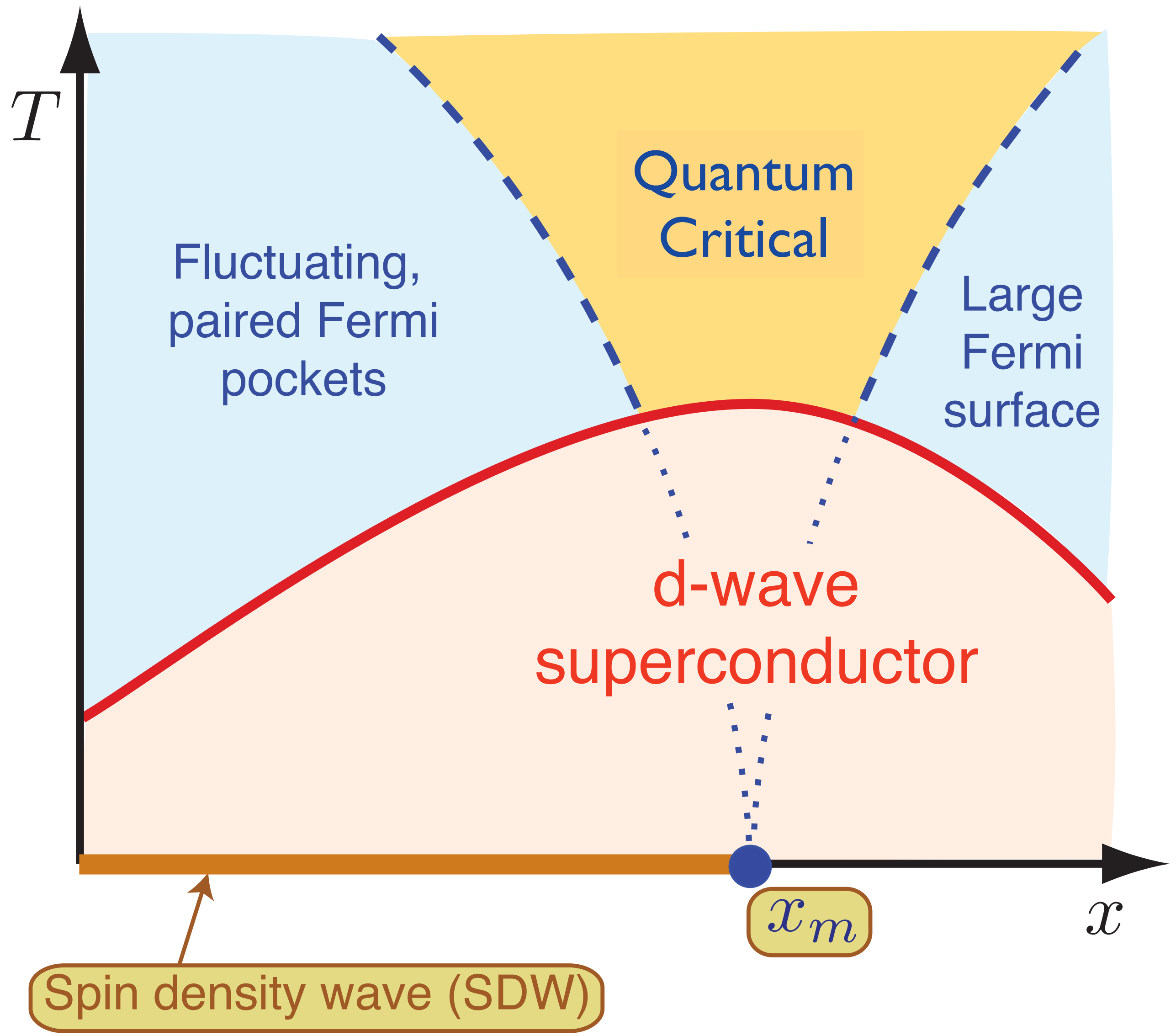


Fermi surface+antiferromagnetism

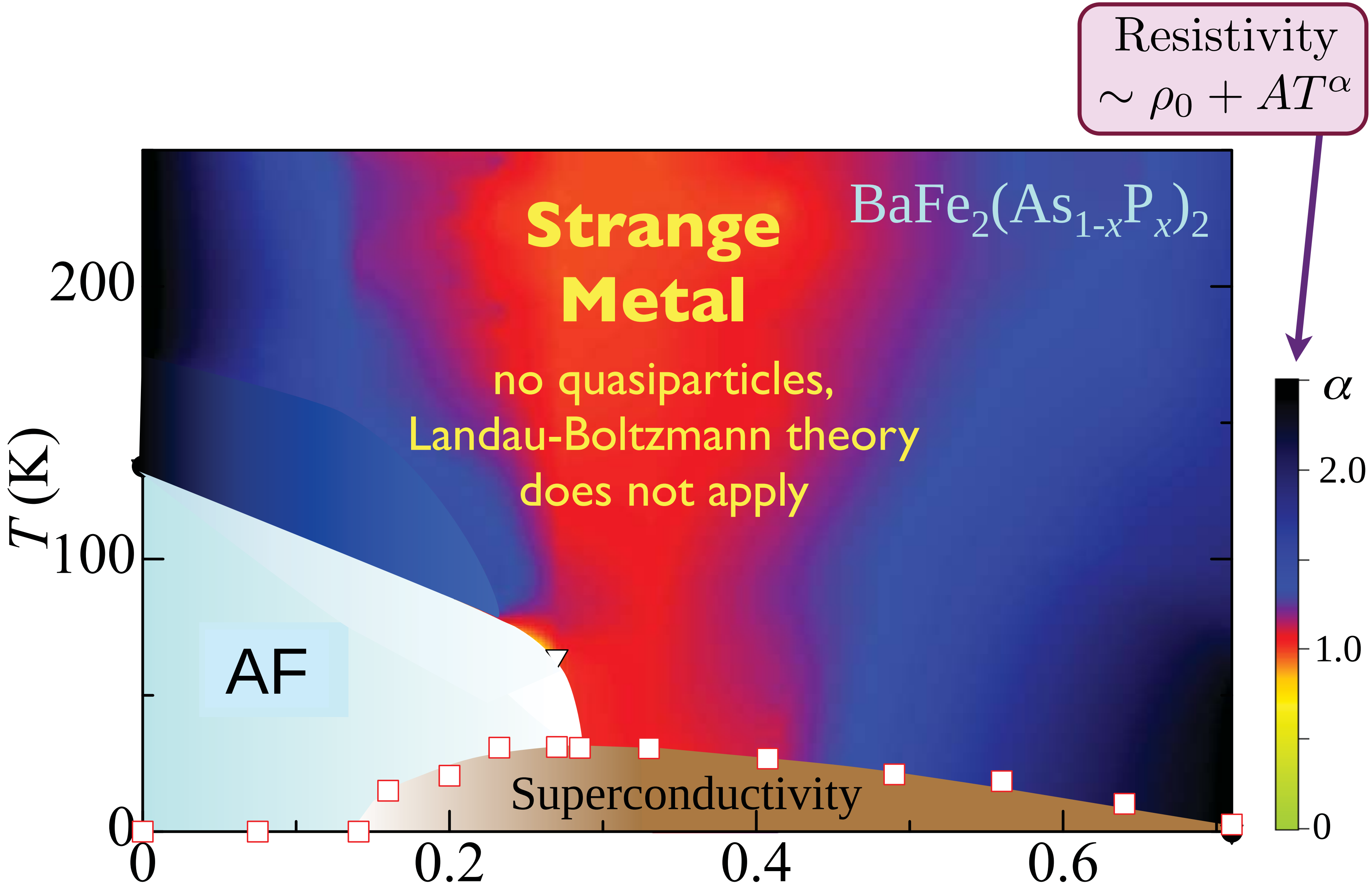


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

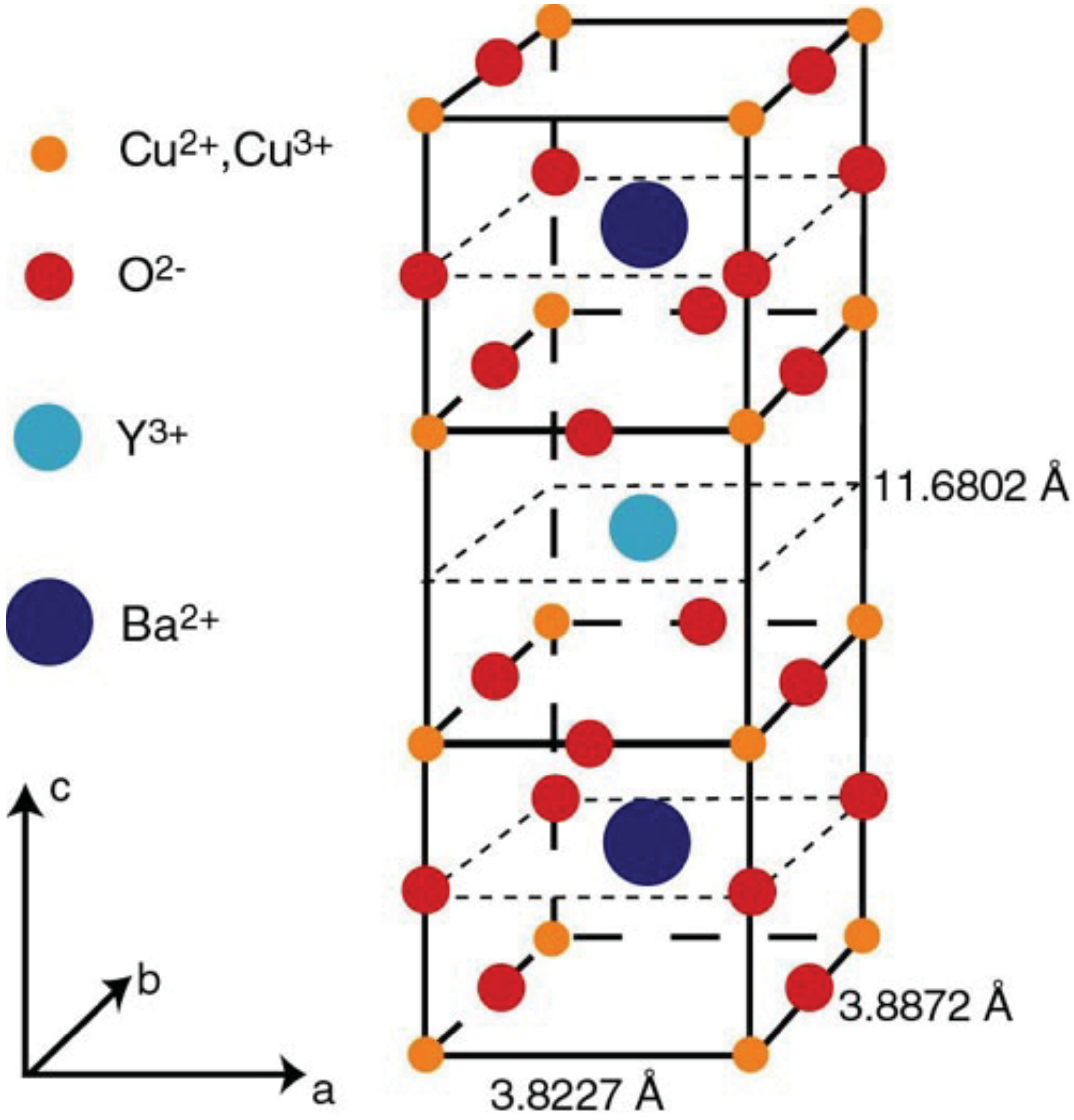
Quantum phase transitions with loss of antiferromagnetic order

1. Dimerized insulators:
   Conformal field theory

2. Metals (at weaker coupling):
   The iron-based superconductors

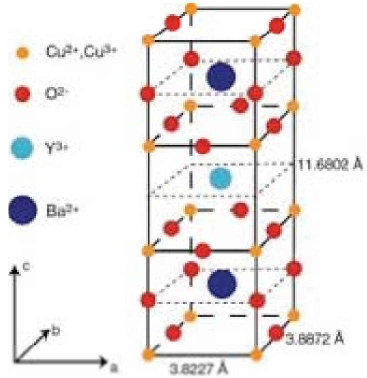
3. Metals (at stronger coupling):
   The hole-doped cuprate superconductors

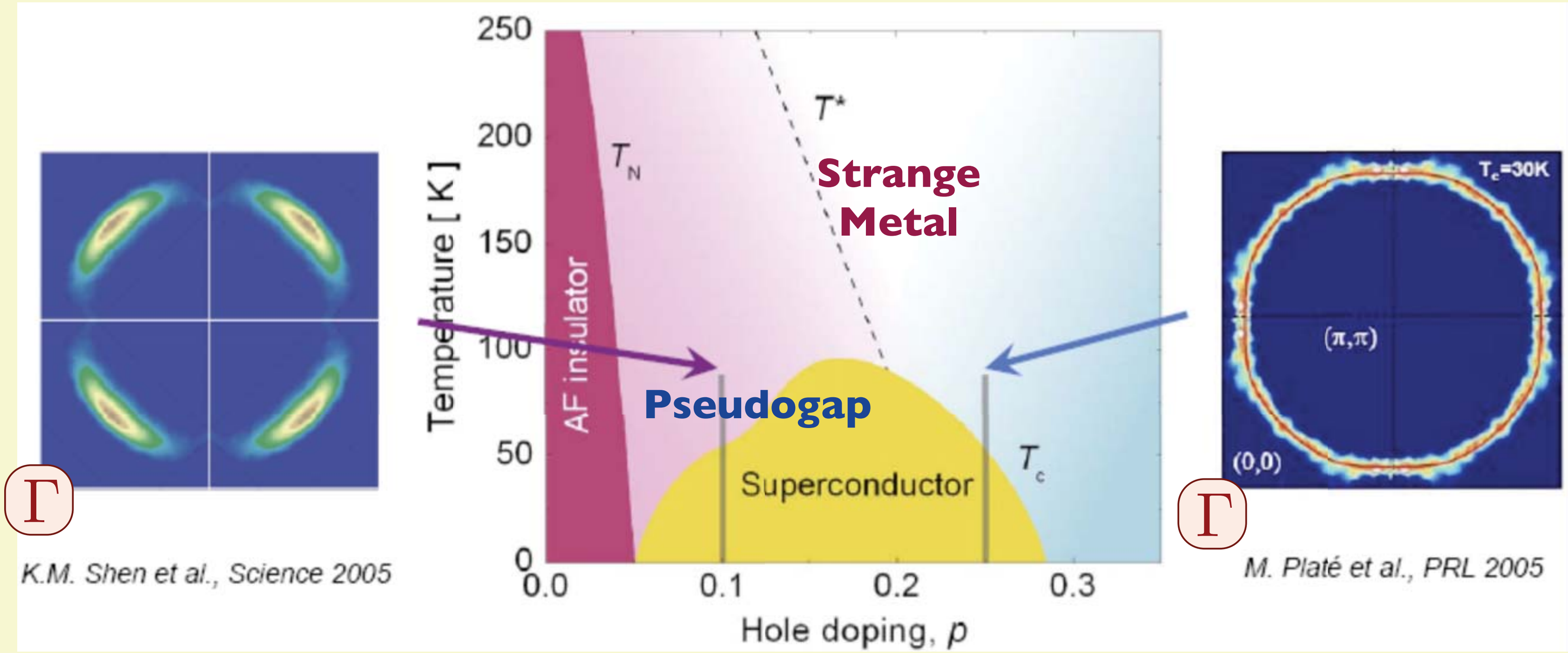
High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

What about the pseudogap ?

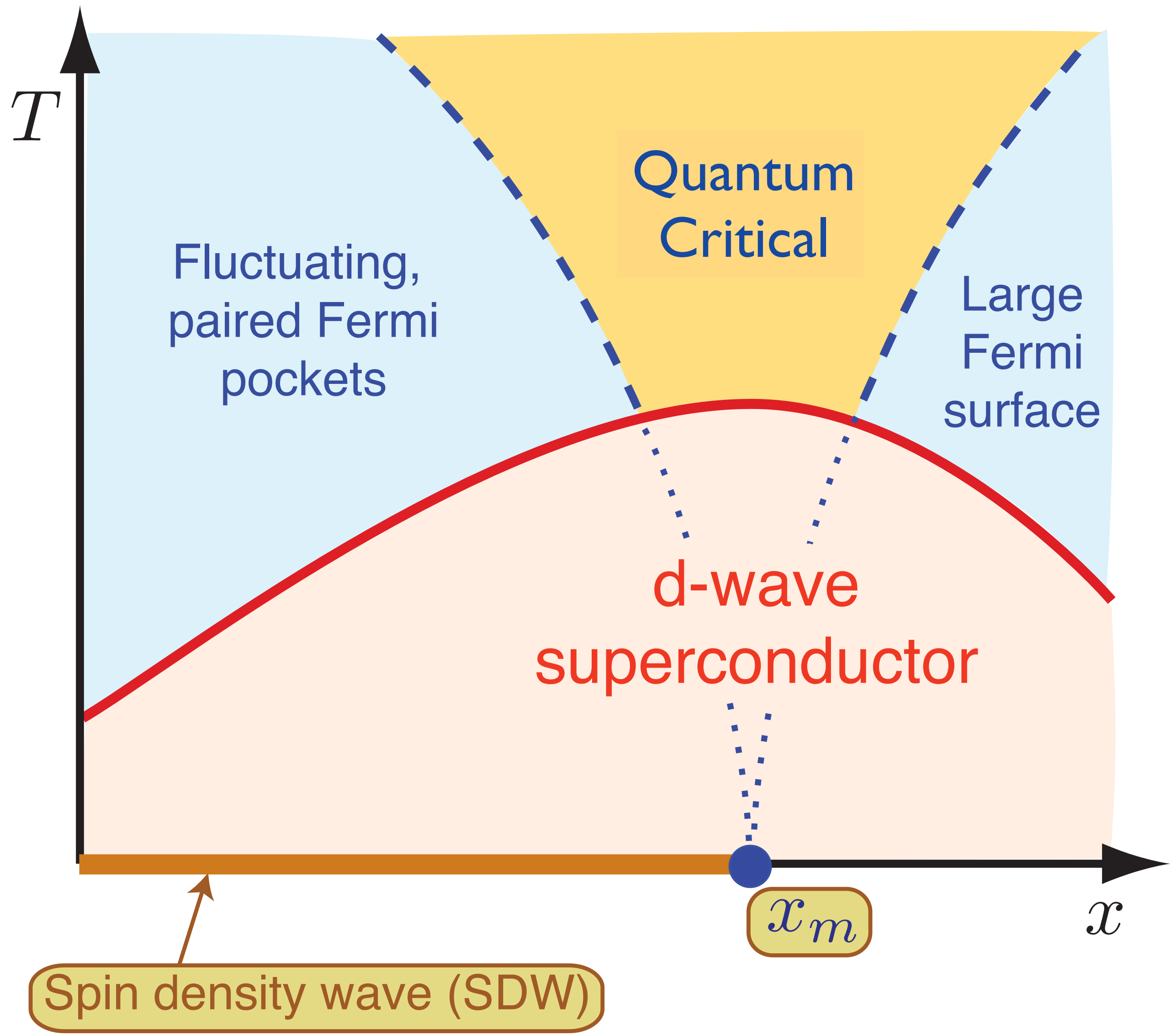




Smaller hole Fermi-pockets

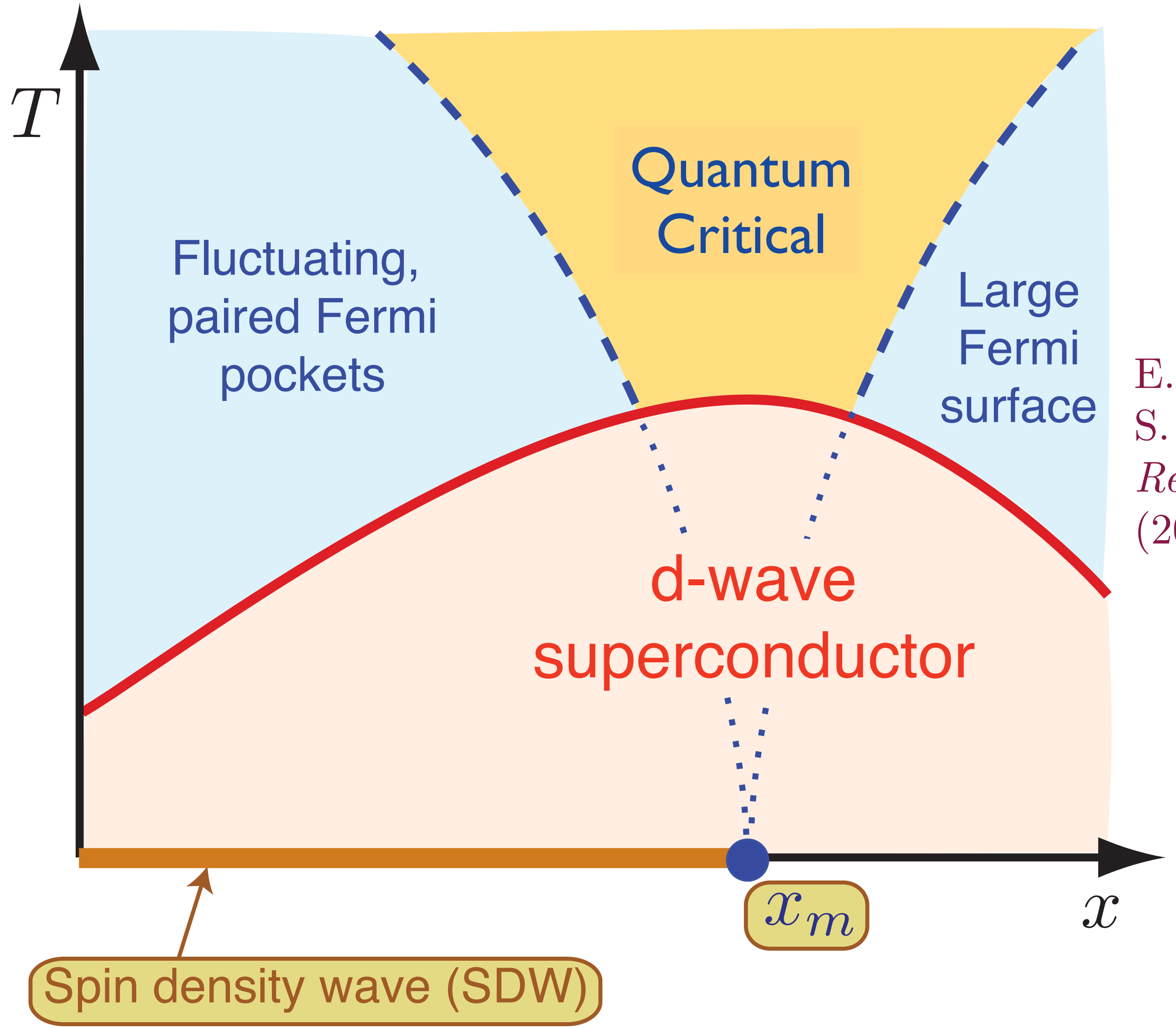
Large hole Fermi surface

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

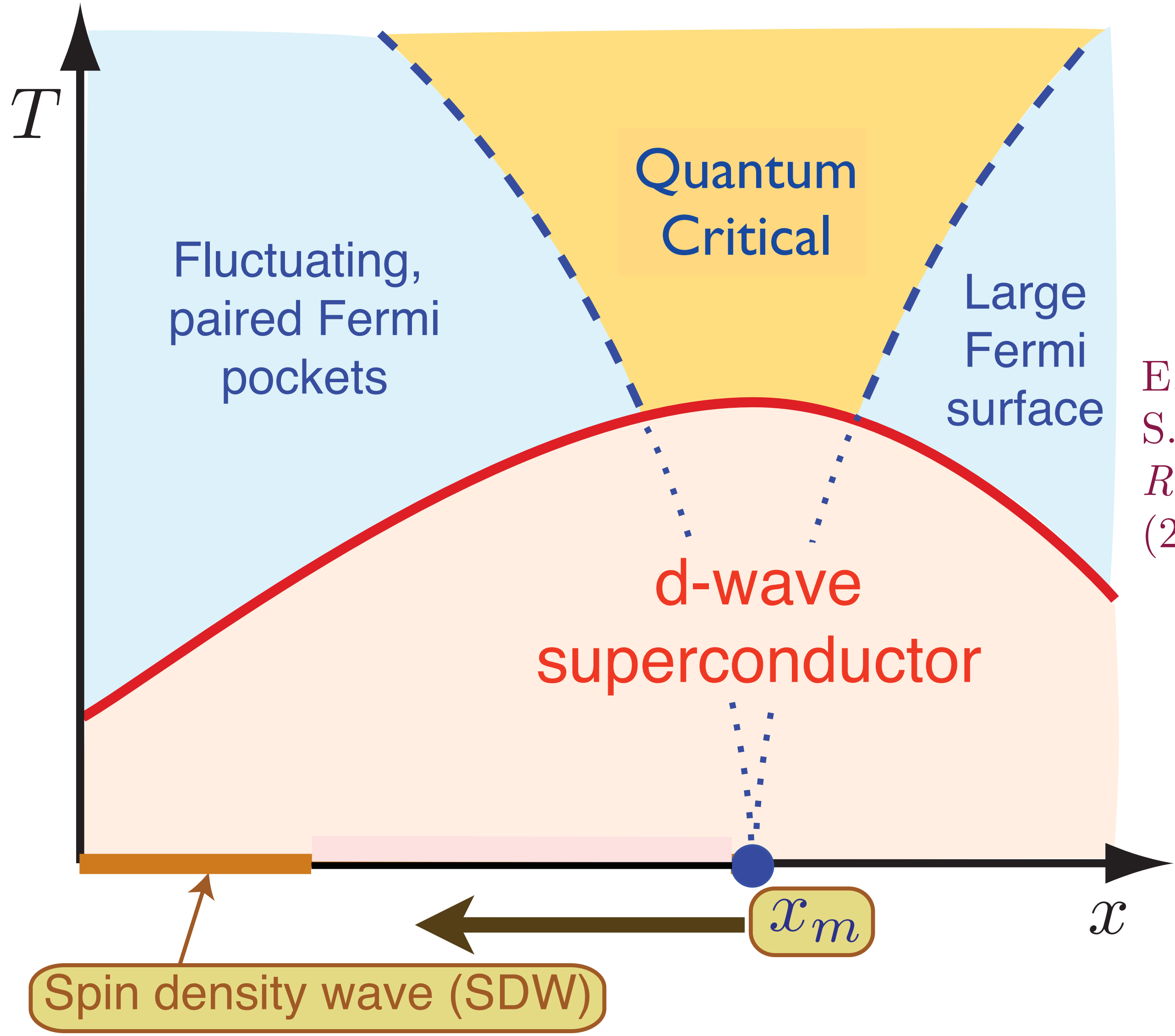
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

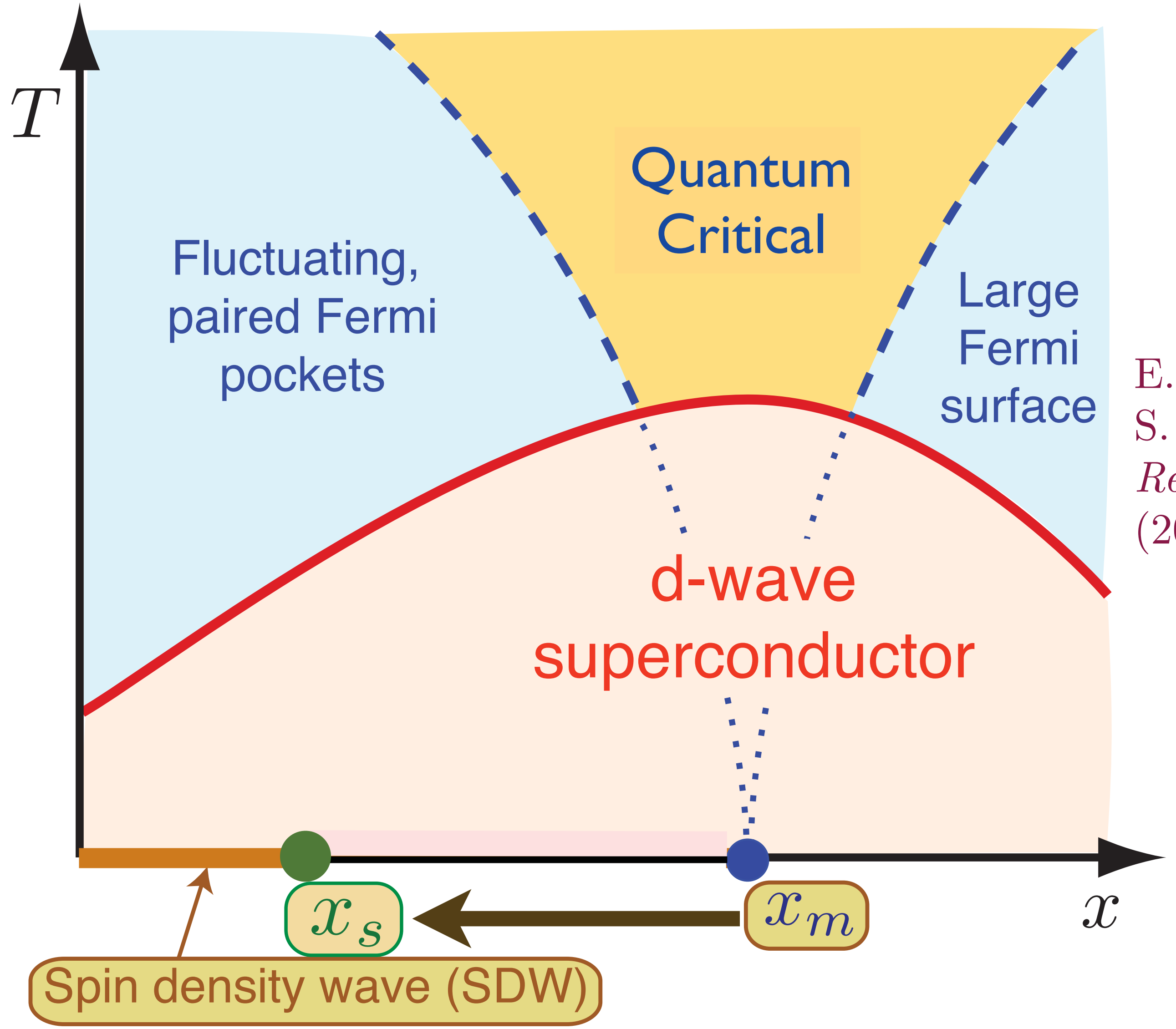
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

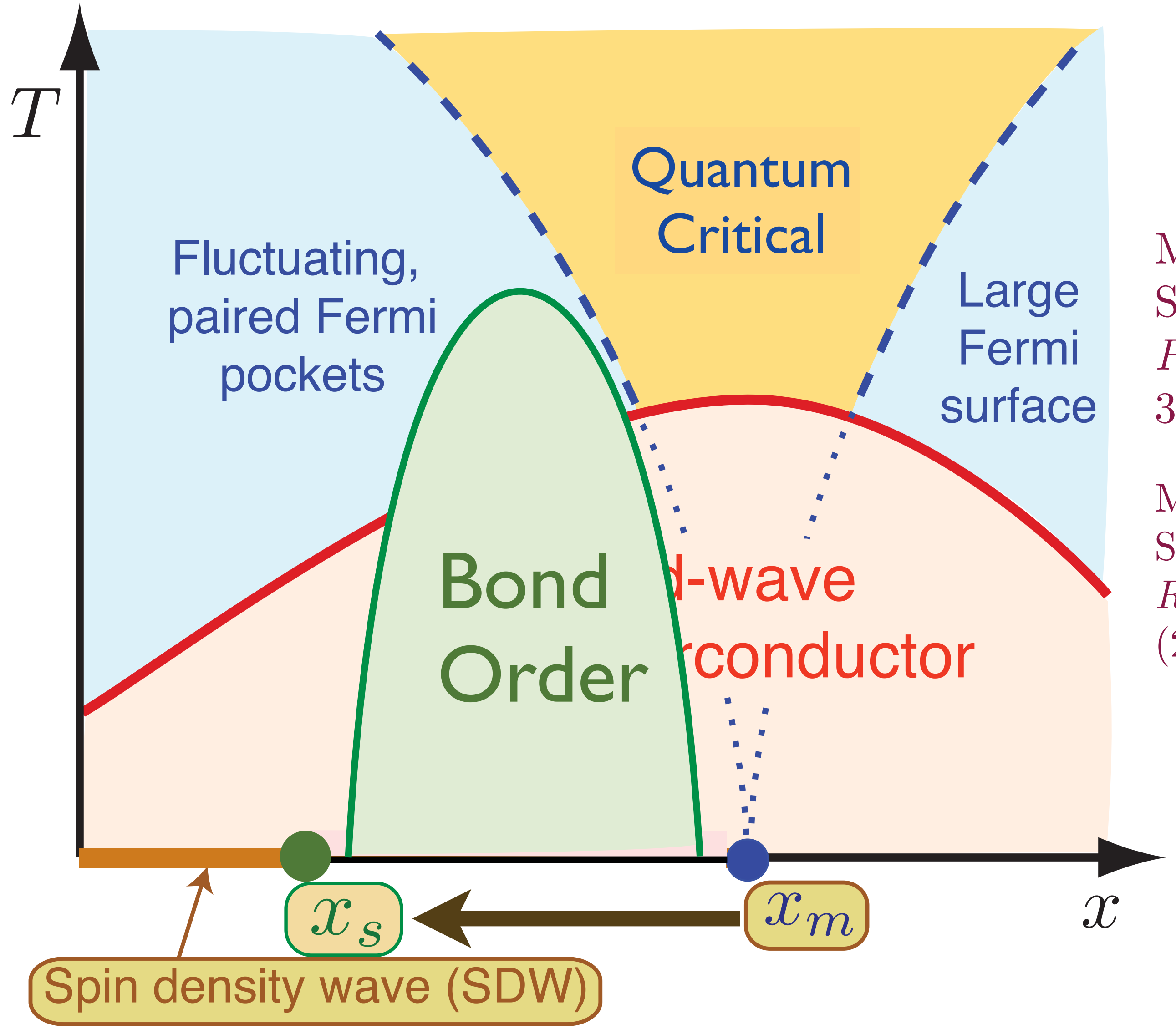
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

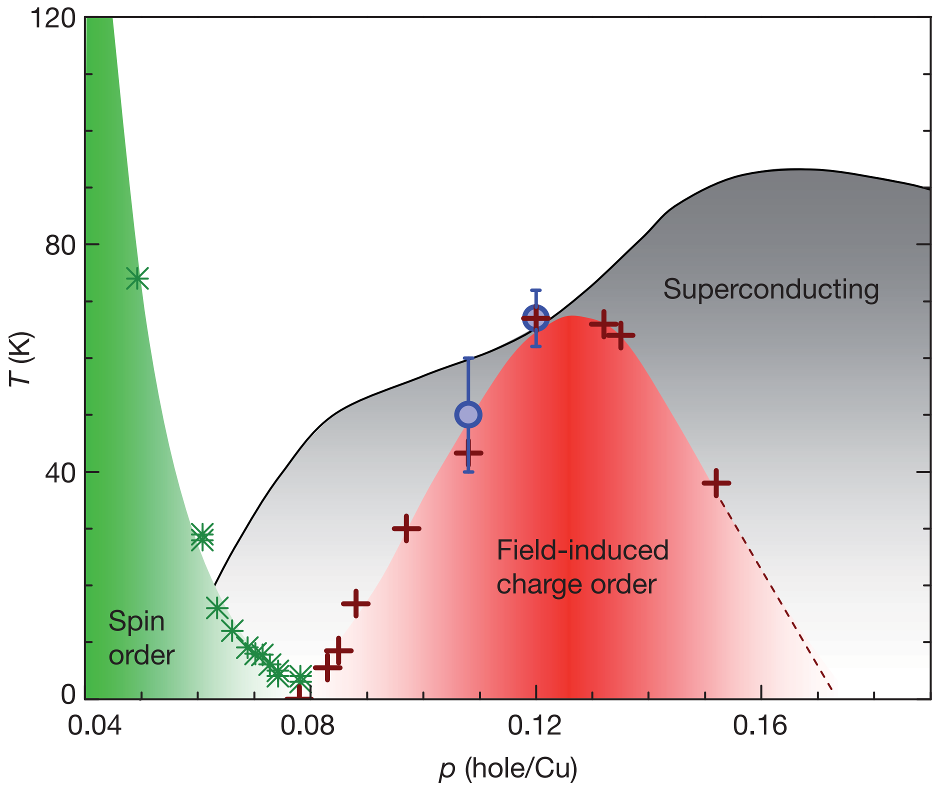
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

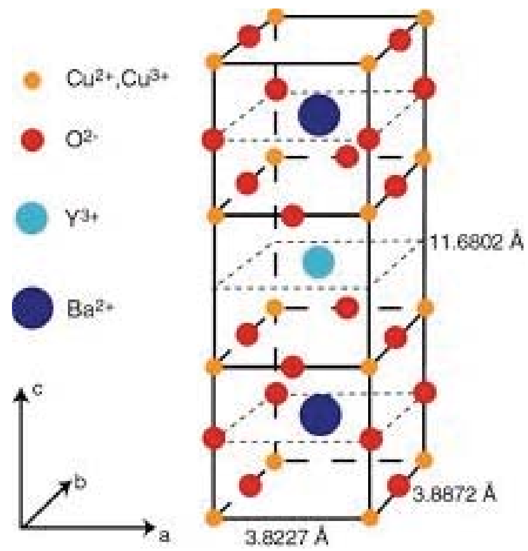
The metal has an instability to *both* d-wave superconductivity and a d-wave charge density wave (bond order).

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

- These orders form a pseudospin doublet, which is responsible for the "pseudogap" phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
K. B. Efetov, H. Meier, and C. Pépin, Nature Physics **9**, 442 (2013).
S. Sachdev and R. La Placa, Phys. Rev. Lett. in press arXiv:1303.2114

1. Pseudospin symmetry between d-wave superconductivity and bond order

Continuum field theory with exact pseudospin symmetry

2. Approximate pseudospin symmetry on the lattice t-J model

Pseudogap and bond order in the underdoped cuprates

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi^\dagger_{j\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J. It is fully broken by the electron hopping t_{ij} but does have remnant consequences in doped spin liquid states.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

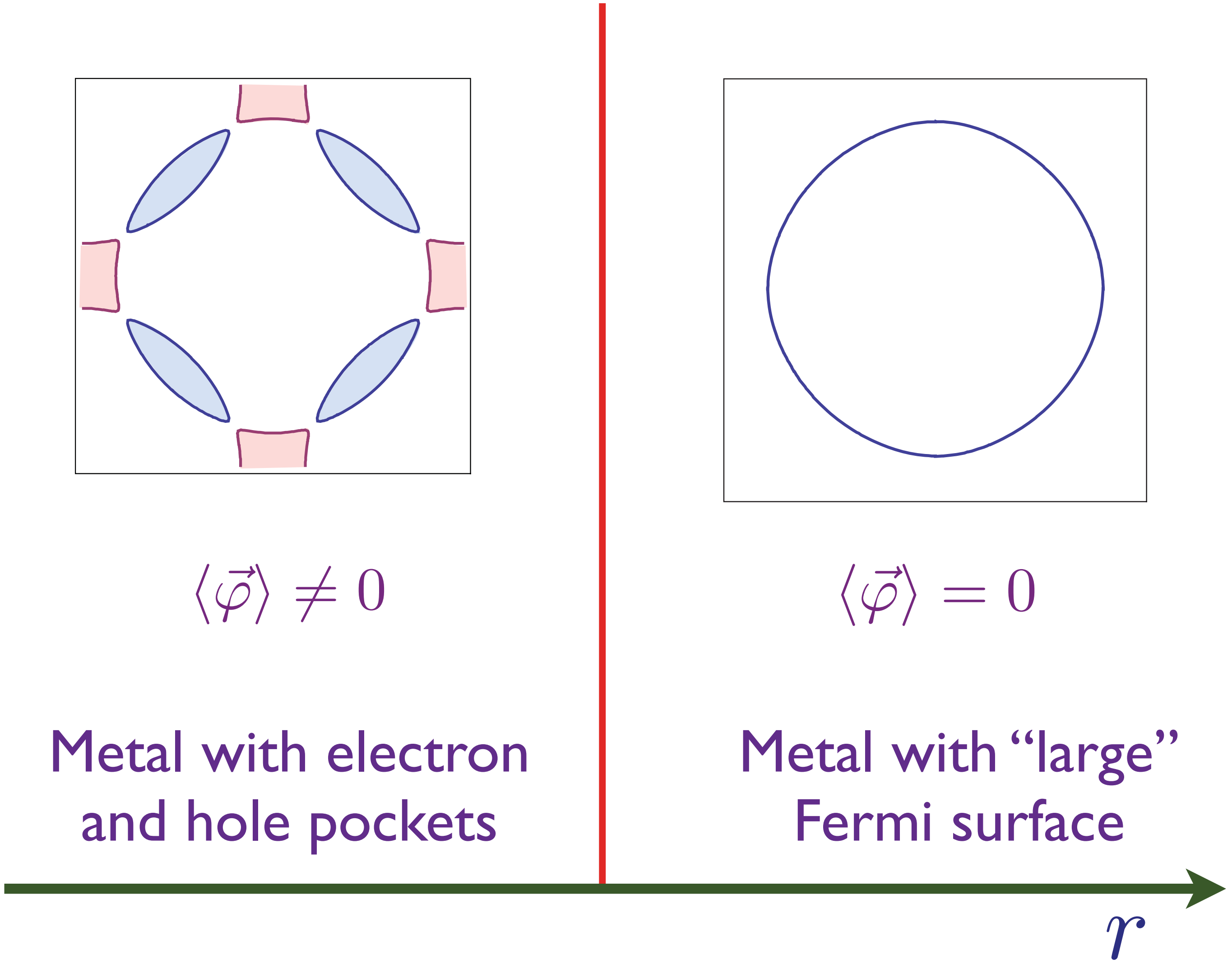
which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

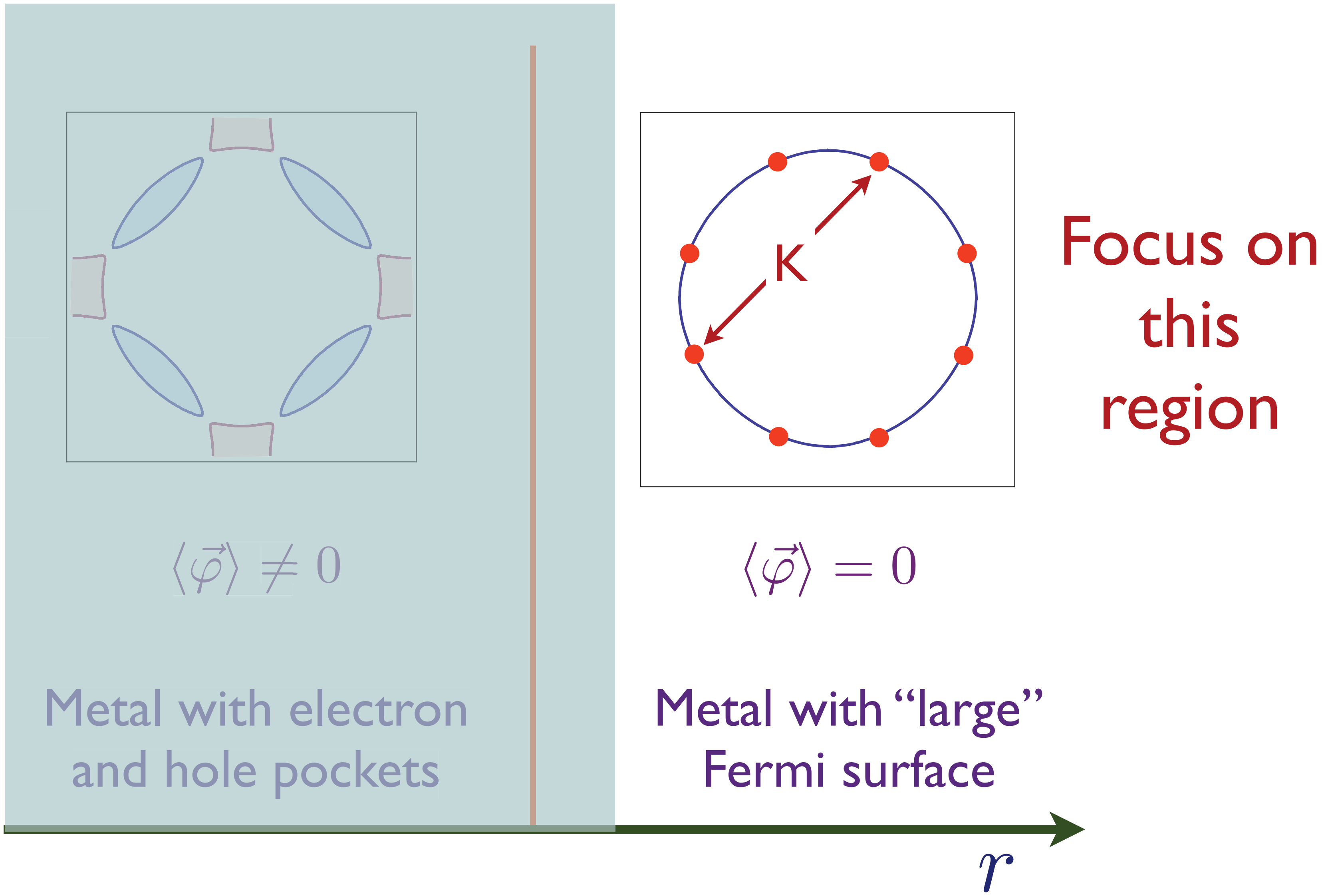
We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

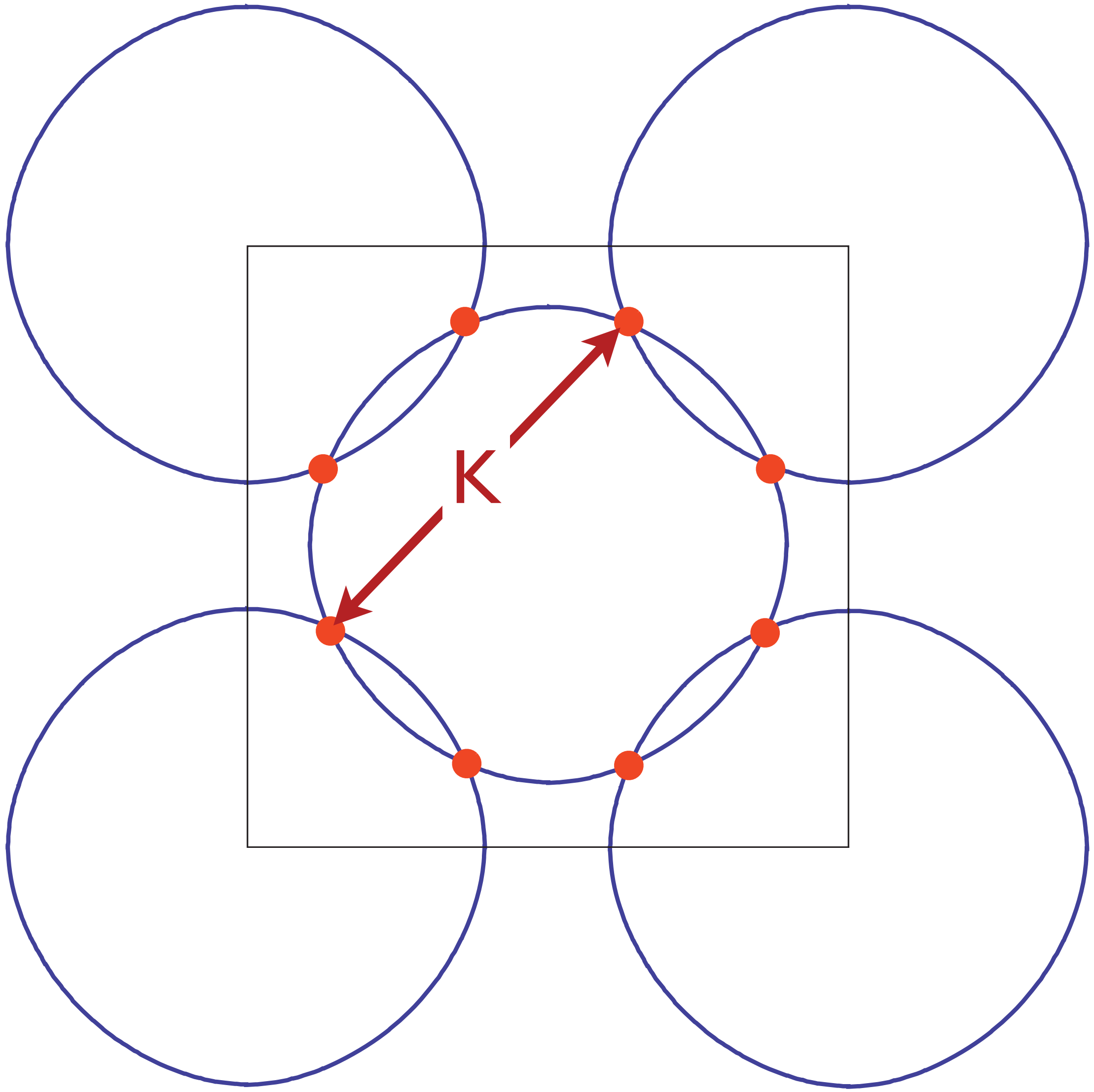
Fermi surface+antiferromagnetism



Fermi surface+antiferromagnetism

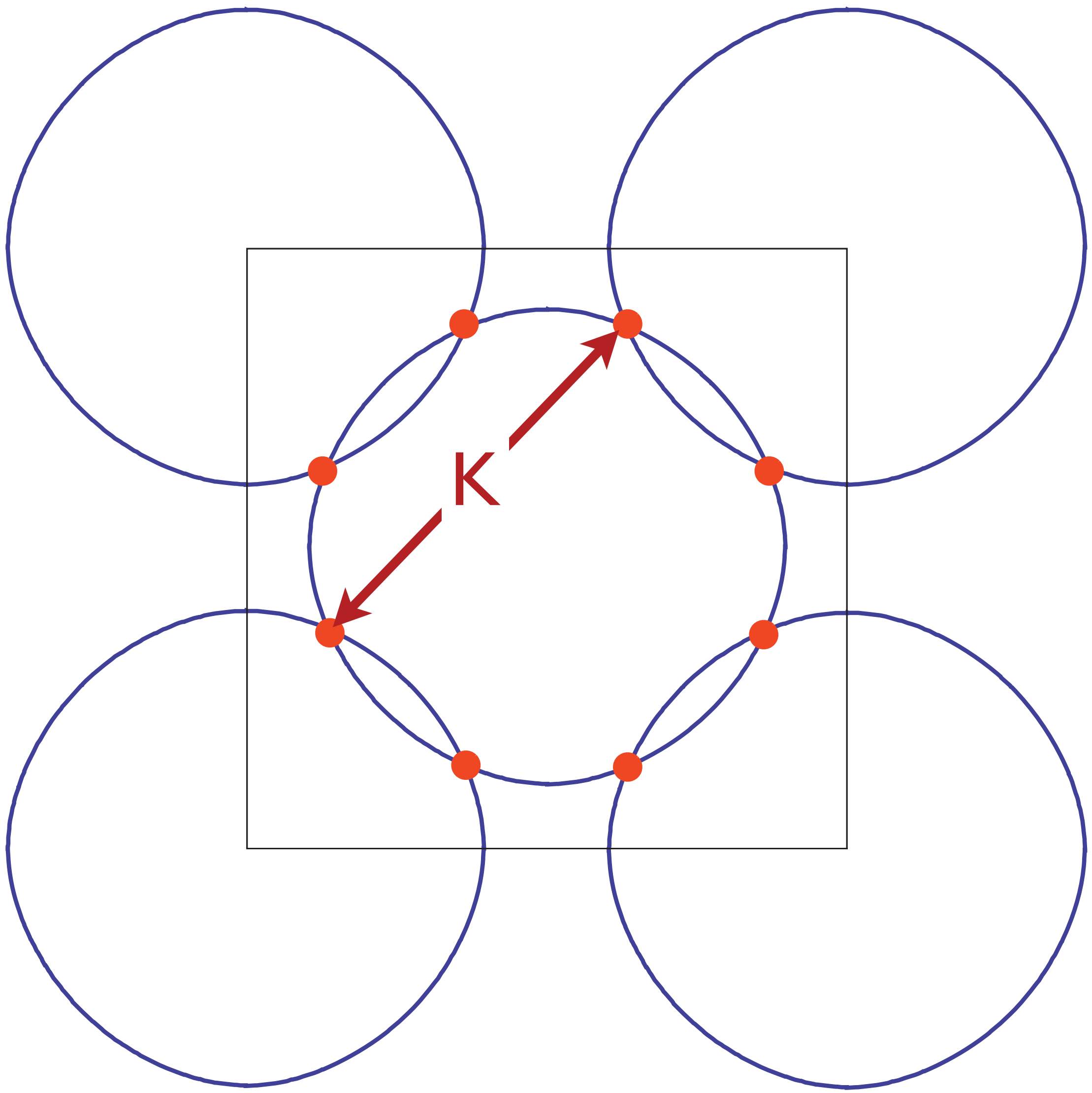


Fermi surface+antiferromagnetism



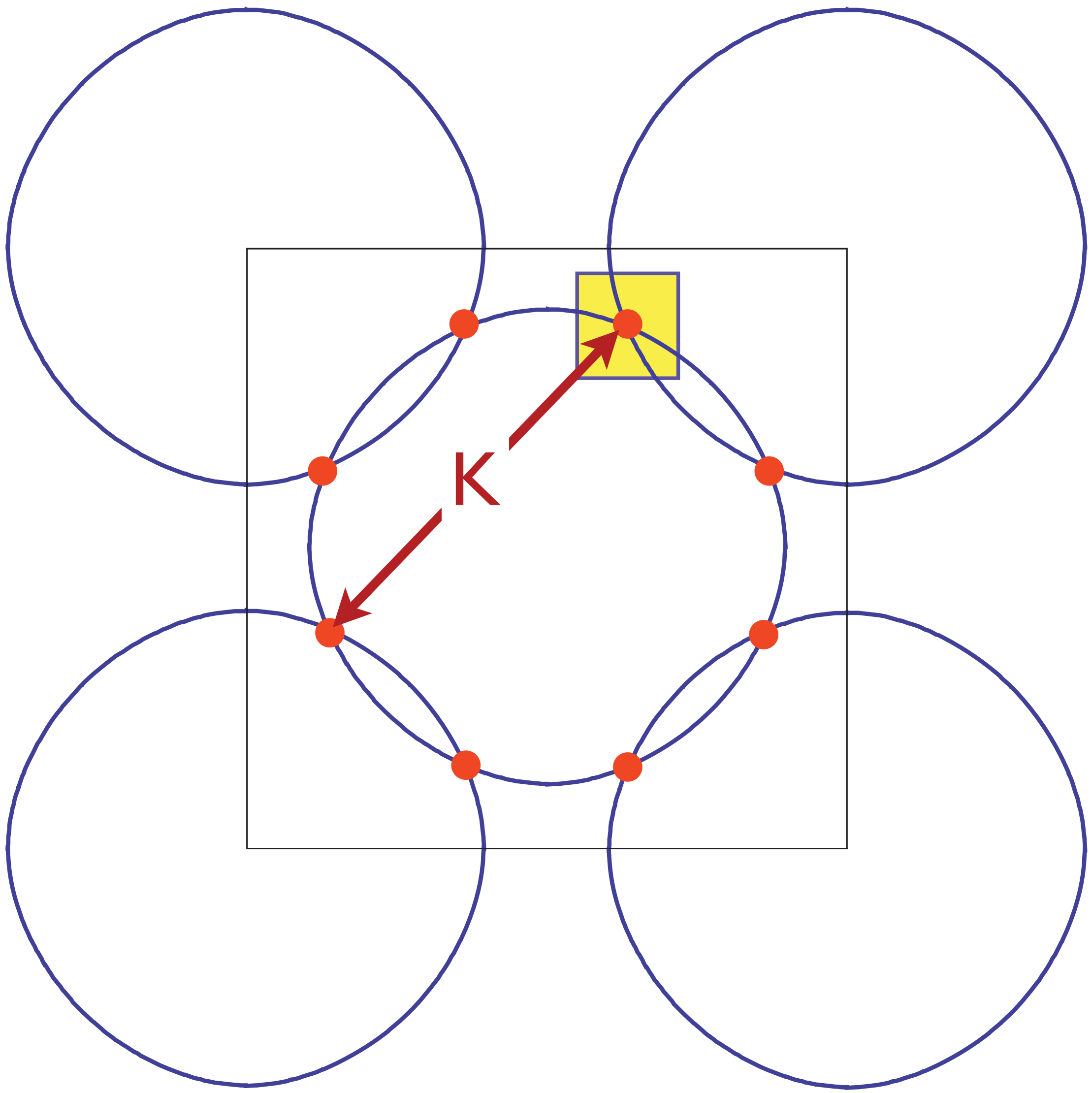
"Hot" spots

Fermi surface+antiferromagnetism



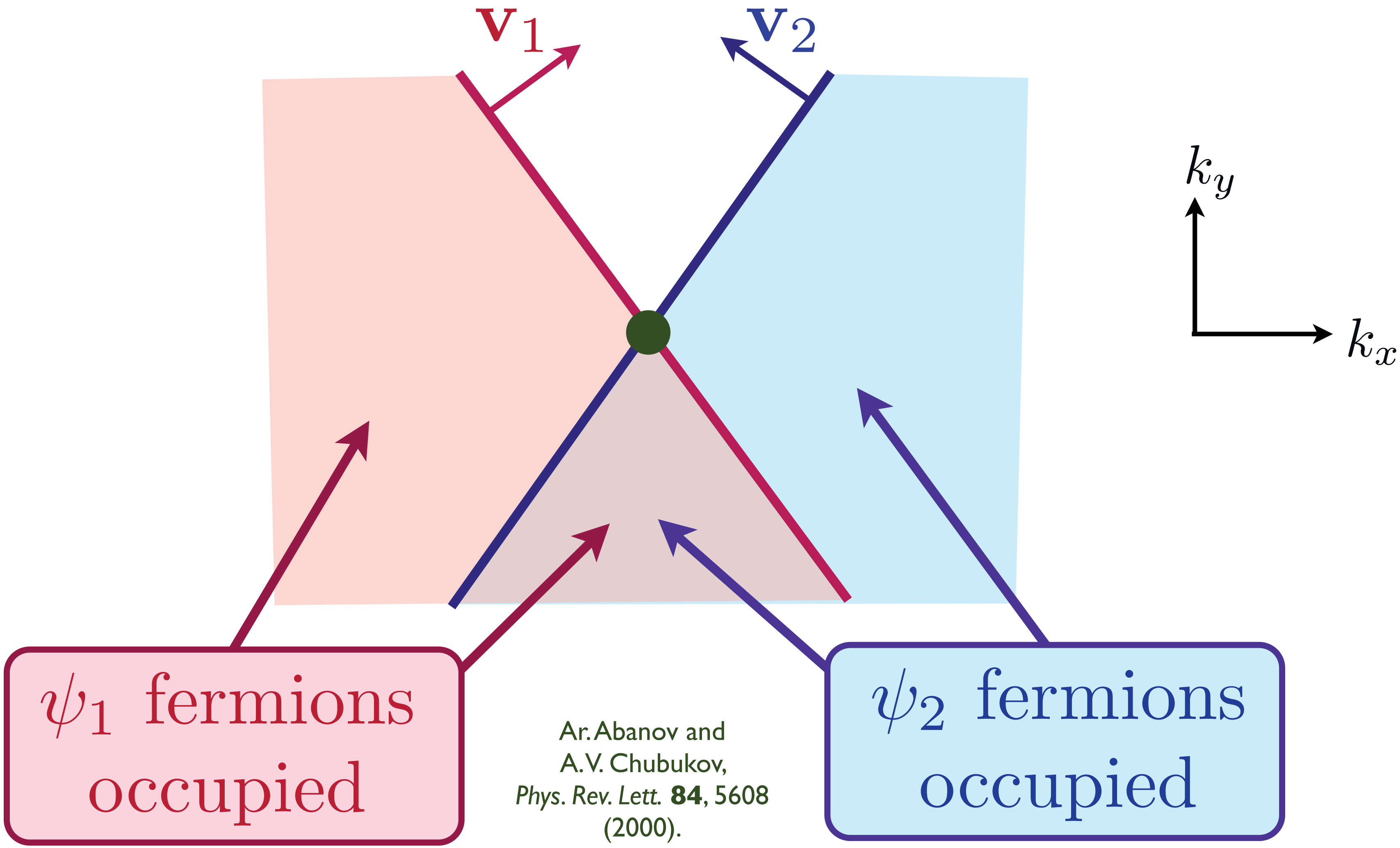
Low energy theory for critical point near hot spots

Fermi surface+antiferromagnetism



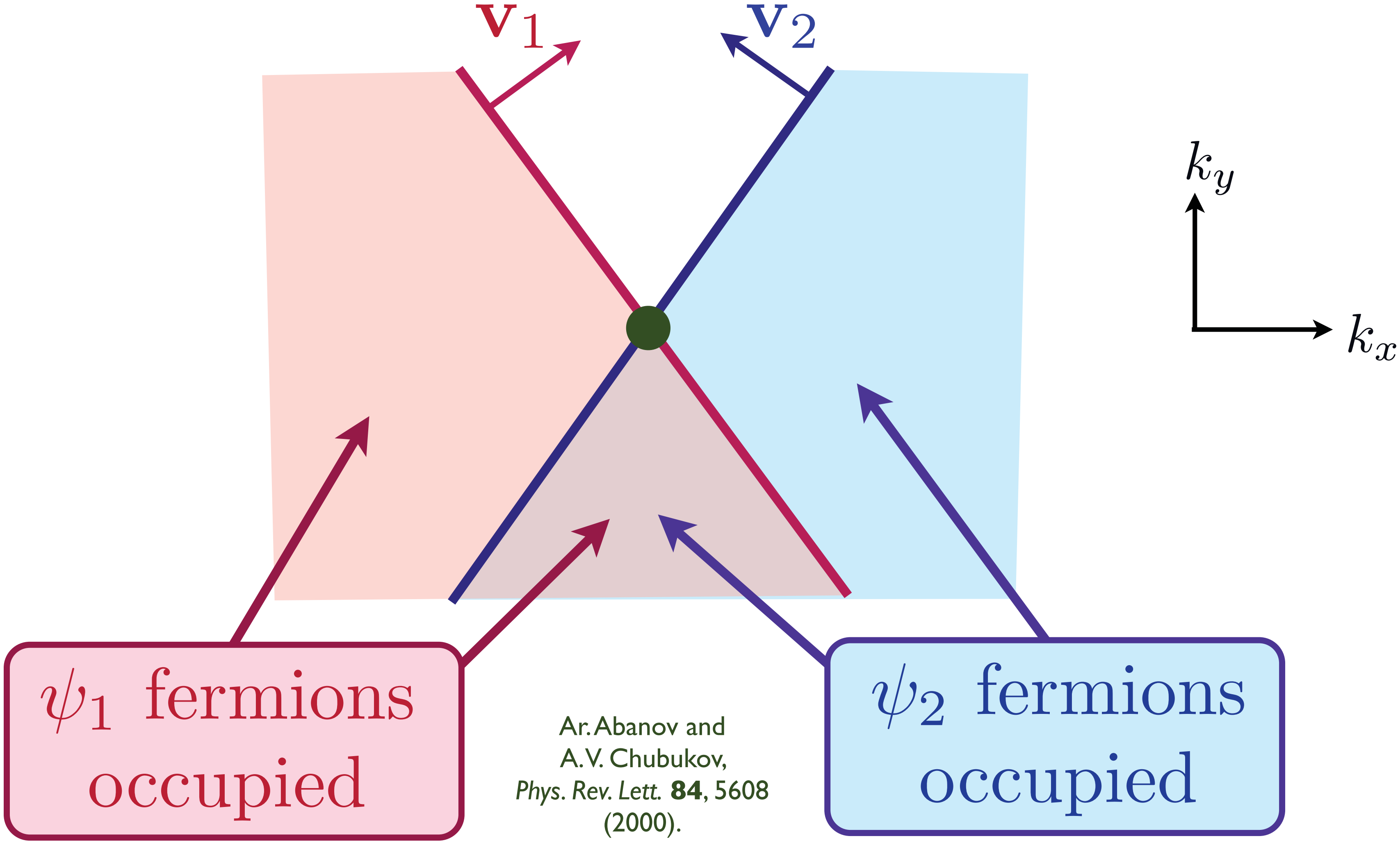
Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



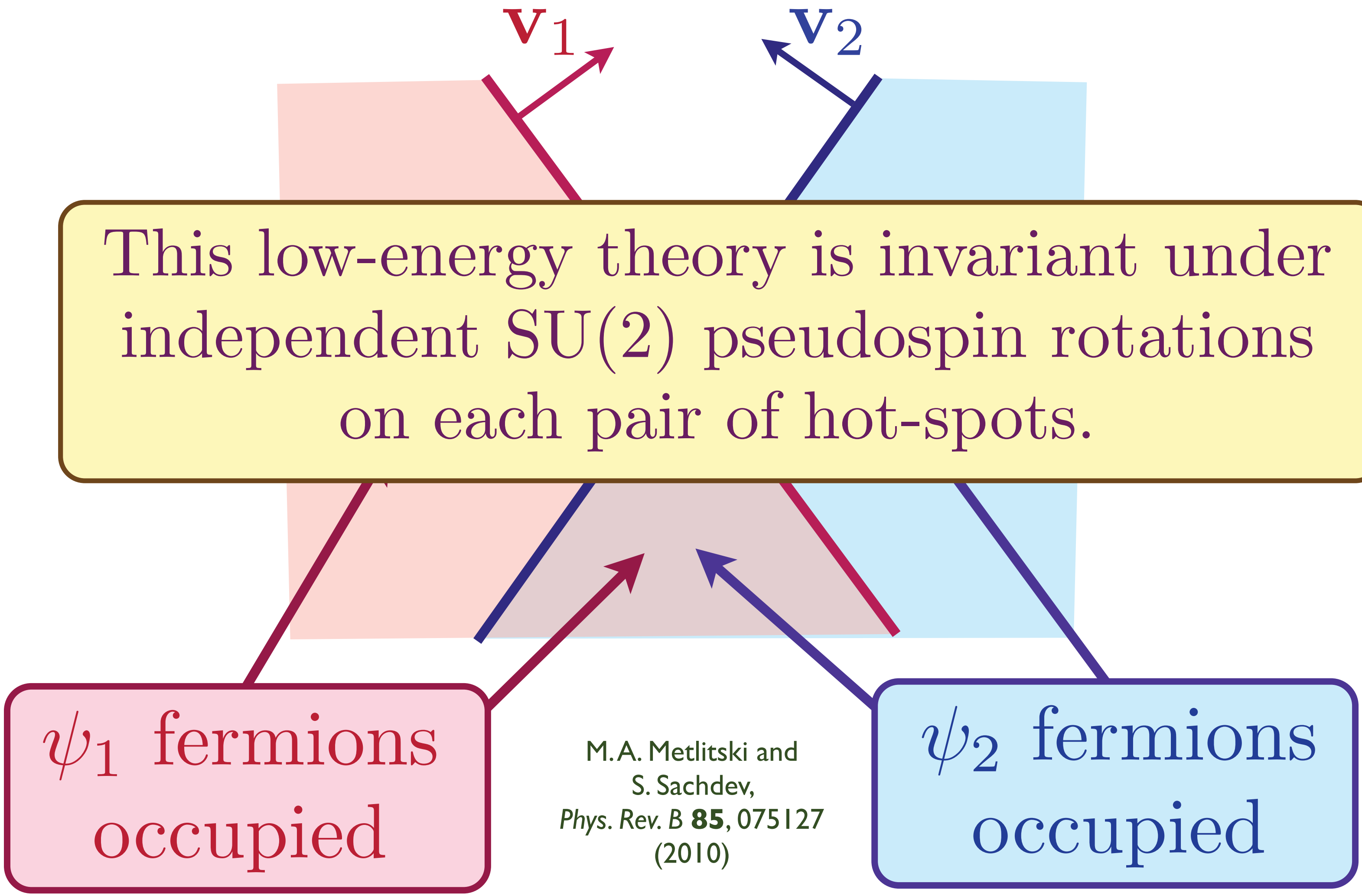
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

$$\mathcal{S} = \int d^2r d\tau \Big[\psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$
$$+ \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta}\right) \Big]$$



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

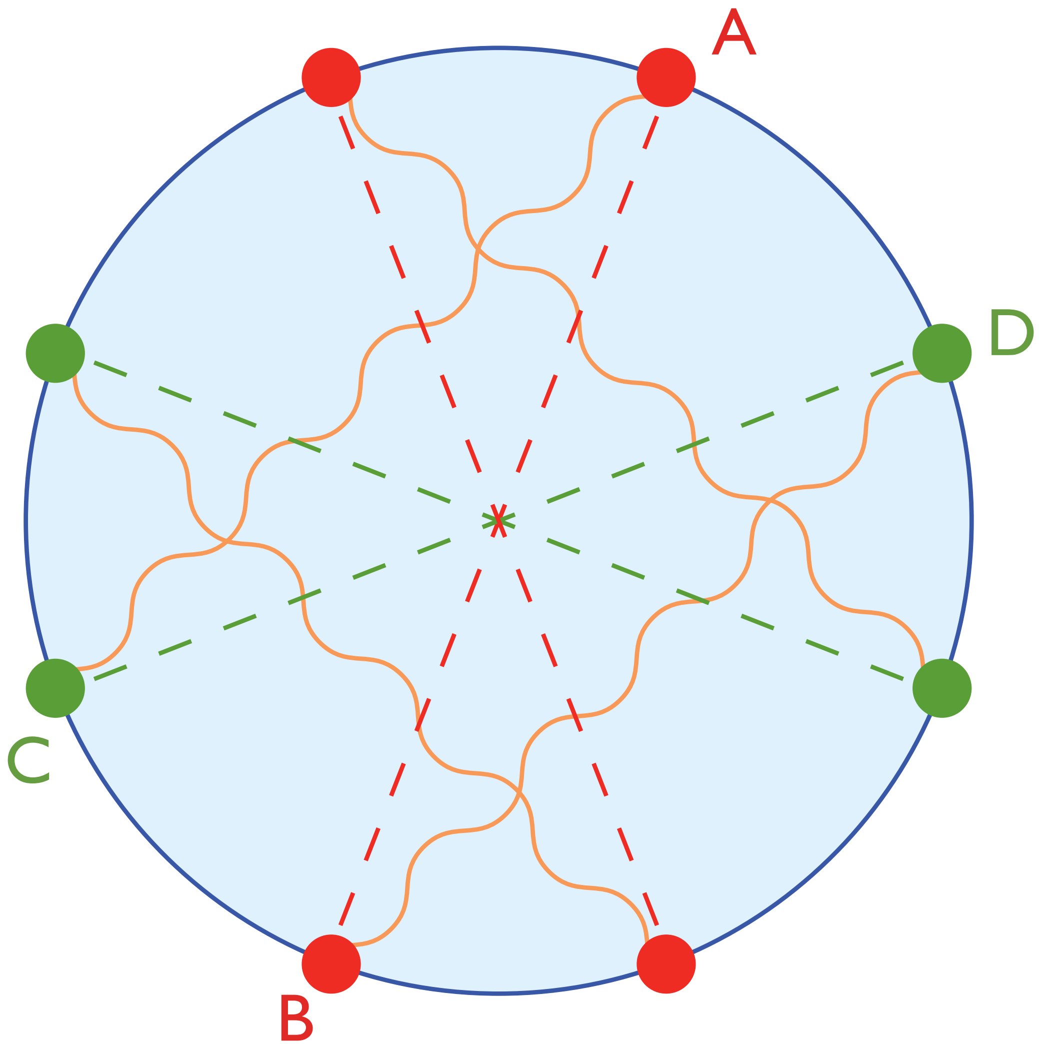
$$\mathcal{S} = \int d^2r d\tau \Big[\psi^\dagger_{1\alpha} \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r \right) \psi_{1\alpha} + \psi^\dagger_{2\alpha} \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r \right) \psi_{2\alpha} + \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi^\dagger_{2\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \Big]$$

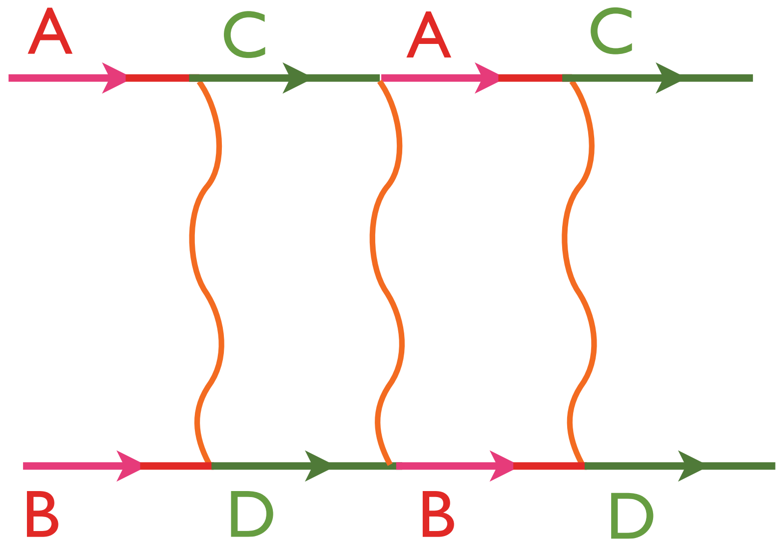


This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

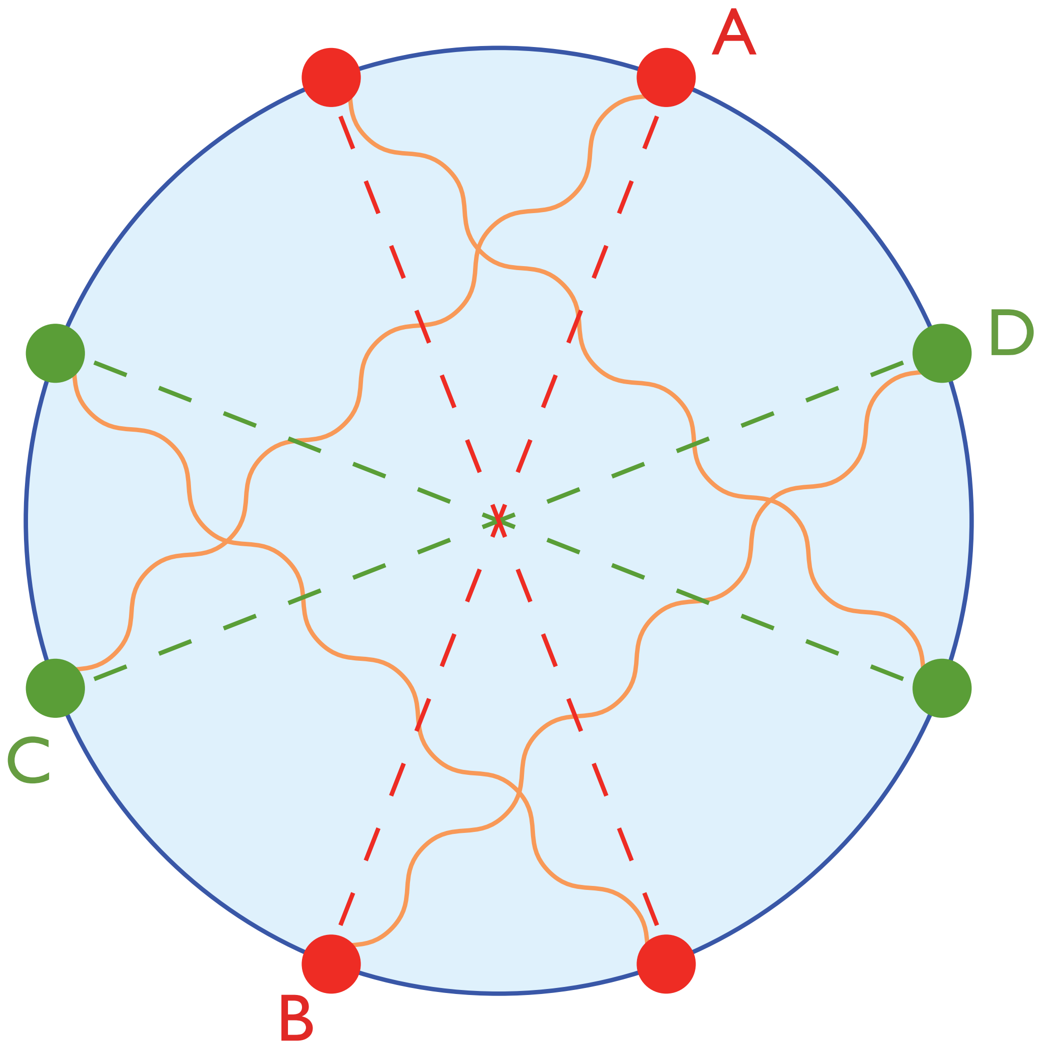
Pairing "glue" from antiferromagnetic fluctuations

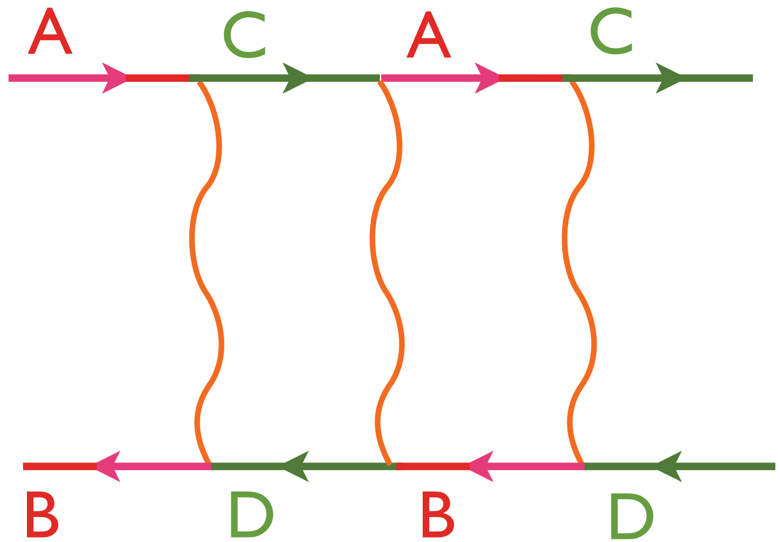




V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Same "glue" leads to particle-hole pairing

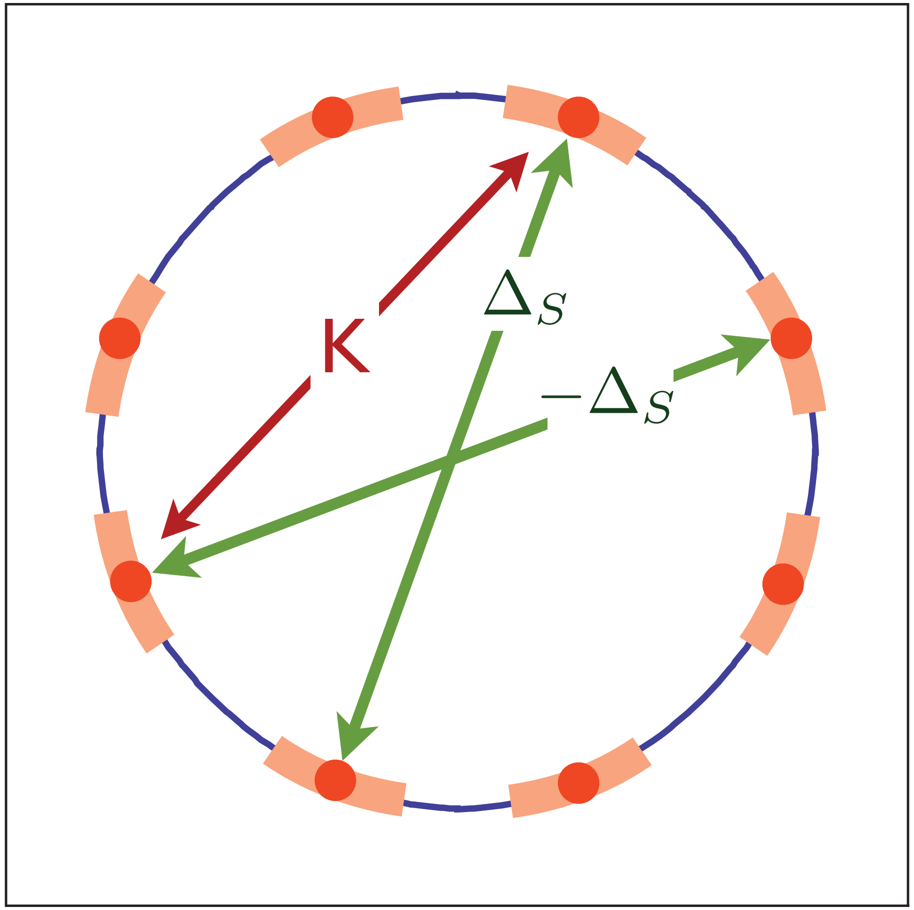




M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

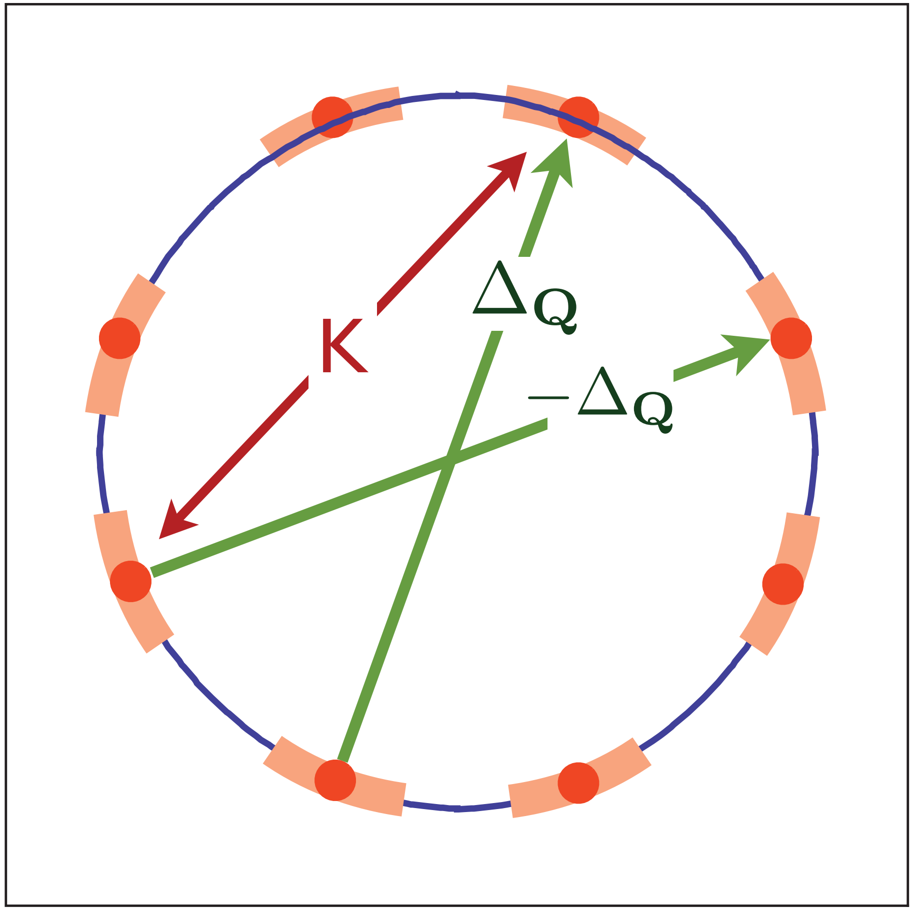


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

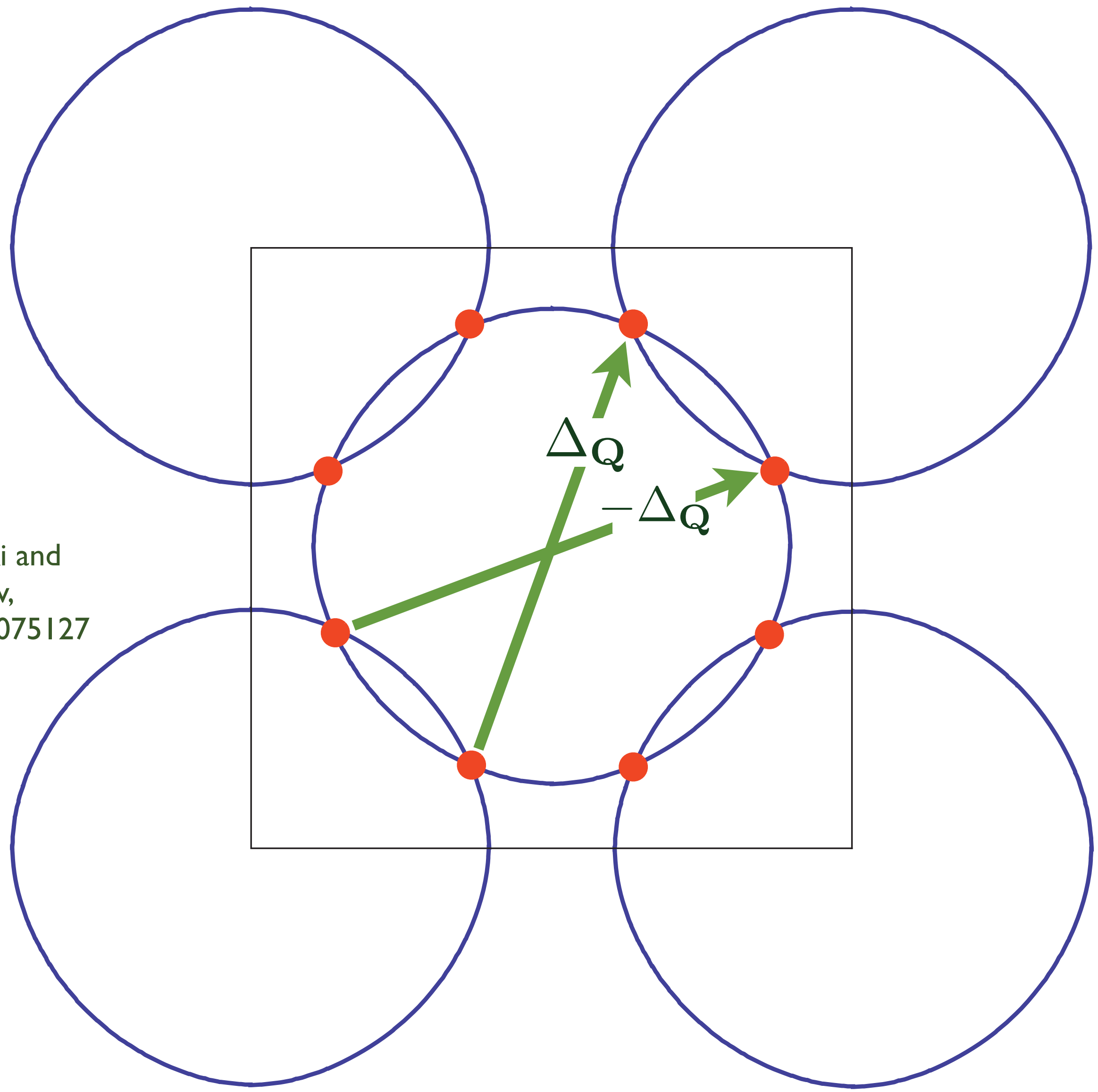
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



$\mathbf{Q}$ is '$2k_F$' wavevector

Incommensurate d-wave bond order: particle-hole pairing at *and near* hot spots, with sign-changing pairing amplitude

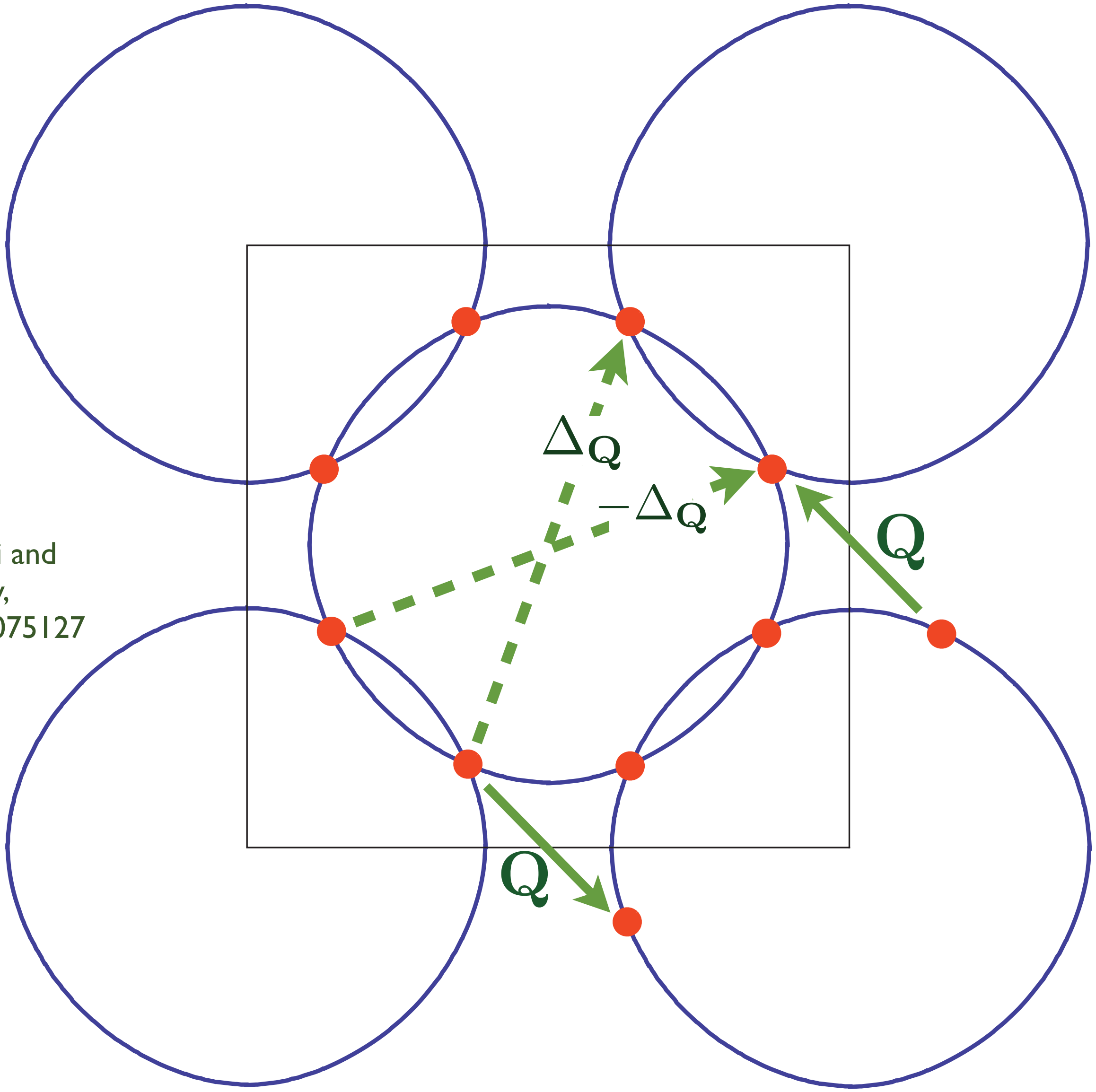
Incommensurate *d*-wave bond order



M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Incommensurate *d*-wave bond order



M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Incommensurate *d*-wave bond order

Consider modulation in an off-site "density" like variable at sites $\mathbf{r}_i$ and $\mathbf{r}_j$

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle \sim \left[\sum_{\mathbf{k}} \Delta_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

relative co-ord.

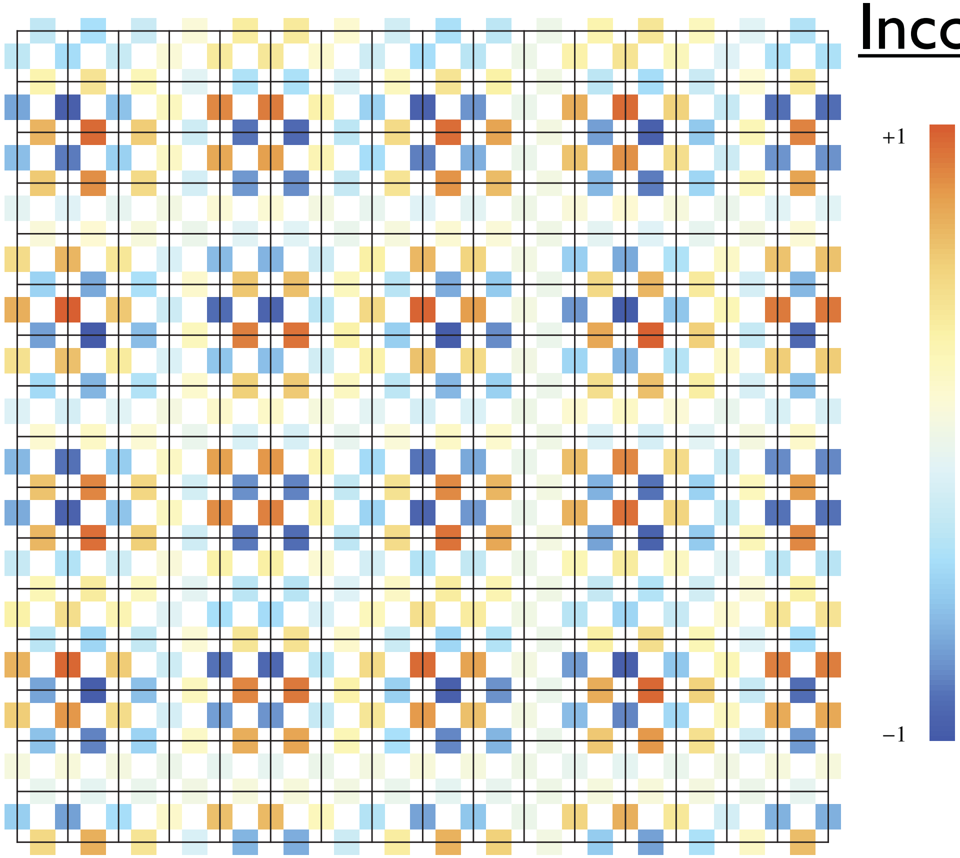
average co-ord.

The wavevector $\mathbf{Q}$ is associated with a modulation in the *average* co-ordinate $(\mathbf{r}_i - \mathbf{r}_j)/2$: this determines the wavevector of the neutron/X-ray scattering peak.
The interesting part is the dependence on the *relative* co-ordinate $\mathbf{r}_i - \mathbf{r}_j$.
Assuming time-reversal, the order parameter $\Delta_{\mathbf{Q}}(\mathbf{k})$ can always be expanded as

$$\Delta_{\mathbf{Q}}(\mathbf{k}) = c_s + c_{s'}(\cos k_x + \cos k_y) + c_d(\cos k_x - \cos k_y) + \ldots$$

The usual charge-density-wave has only $c_s \neq 0$.
The bond-ordered state we find has

$$|c_d| \gg c_s, c_{s'}, \ldots$$



Incommensurate *d*-wave bond order

"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c_{\mathbf{r}\alpha}^{\dagger} c_{\mathbf{s}\alpha} \right\rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle$$

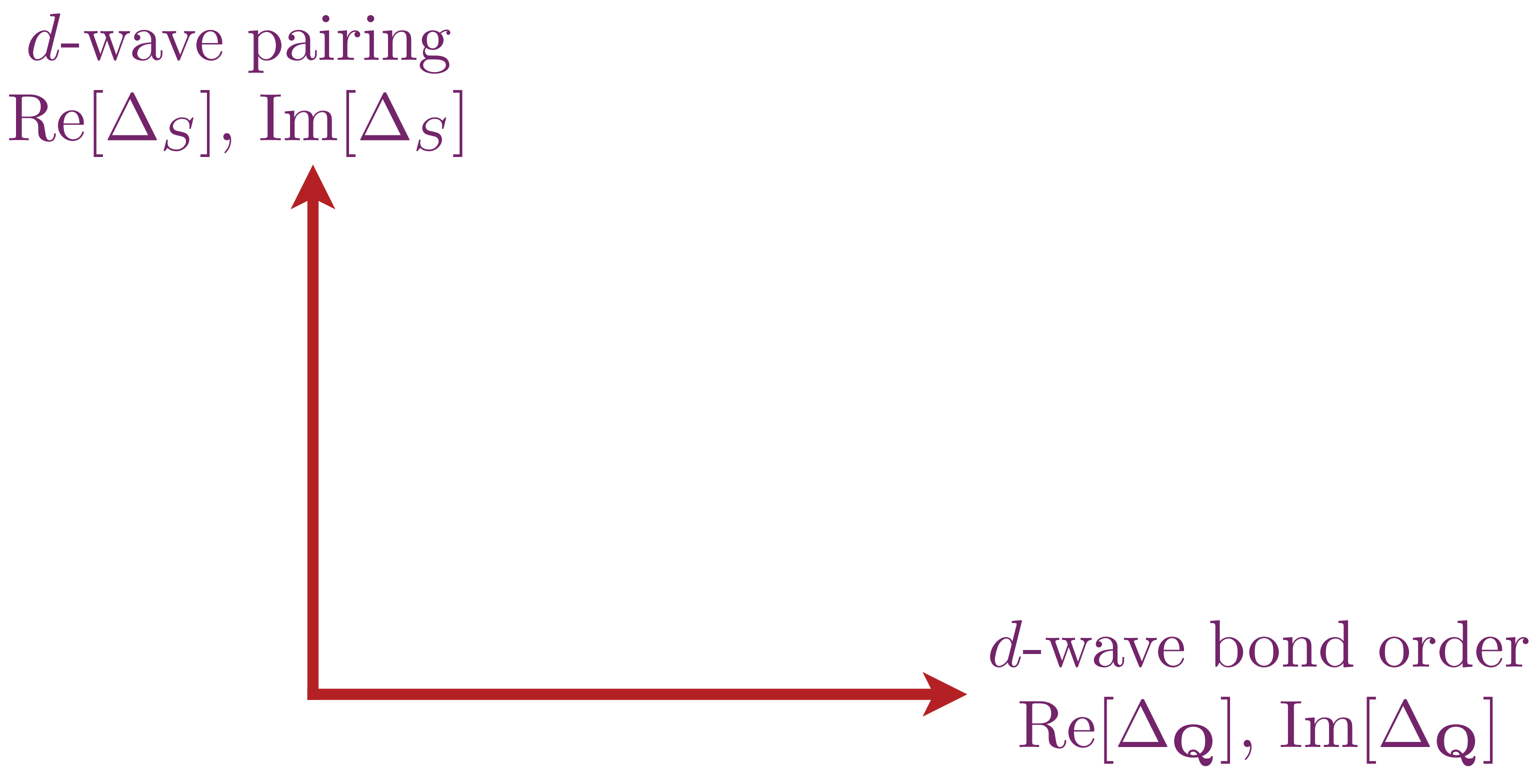
where $\mathbf{Q}$ extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\left\langle c_{\mathbf{r}\alpha}^{\dagger} c_{\mathbf{s}\alpha} \right\rangle$ is non-zero *only* when $\mathbf{r}, \mathbf{s}$ are nearest neighbors.

Order parameter for the pseudogap

4-component order parameter



- O(4) symmetry is exact in the continuum hot-spot theory for the onset of antiferromagnetism

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
$\mathrm{Re}[\Delta_S]$, $\mathrm{Im}[\Delta_S]$

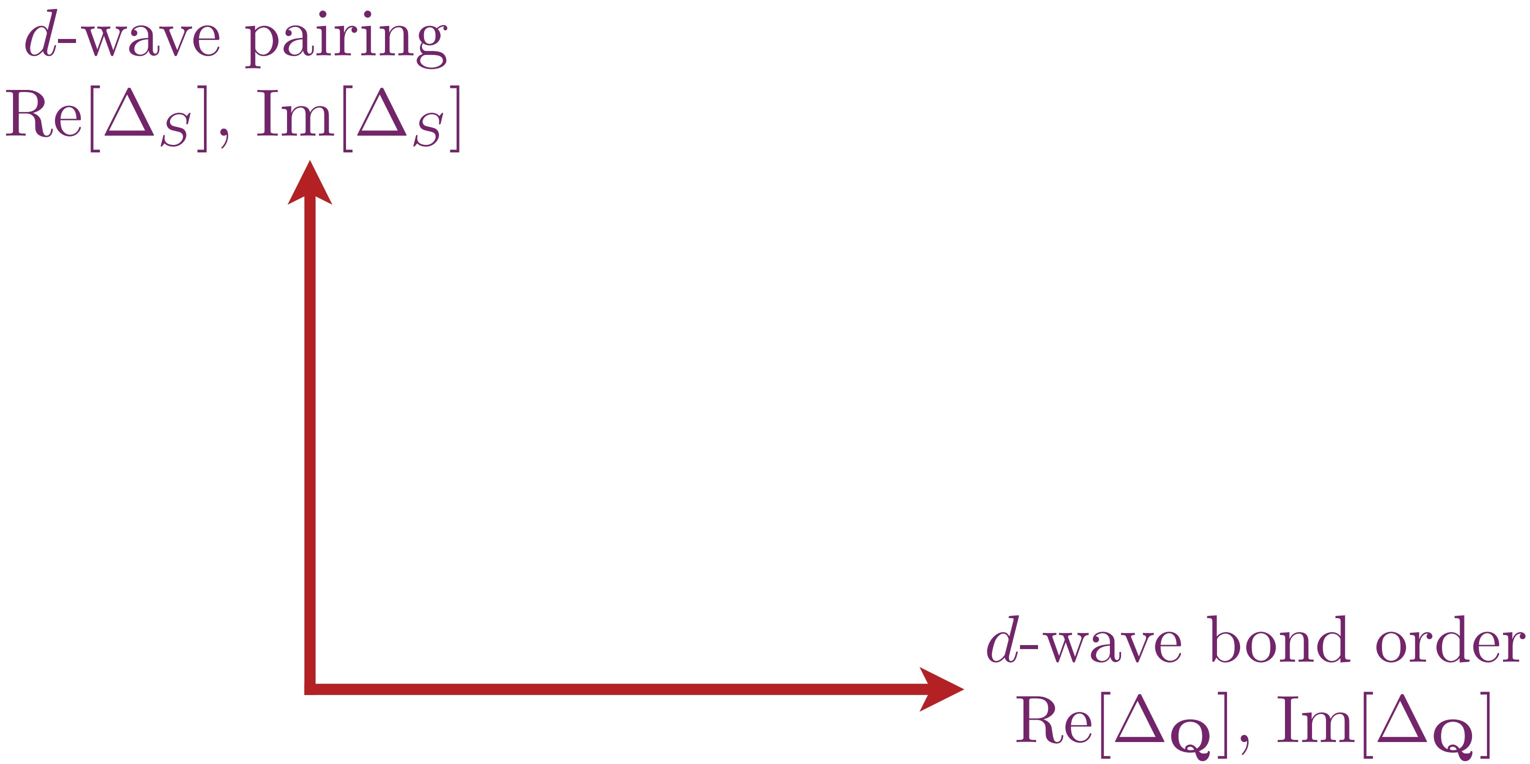
d-wave bond order
$\mathrm{Re}[\Delta_{\mathbf{Q}}]$, $\mathrm{Im}[\Delta_{\mathbf{Q}}]$

- Symmetry is "anomalous" because it requires independent rotation of left- and right-movers: it is broken by any lattice regularization of the theory.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Order parameter for the pseudogap

4-component order parameter



- Symmetry is broken by Fermi surface curvature (which prefers Δ_S) and density-density Coulomb repulsion (which prefers $\Delta_{\mathbf{Q}}$).

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
$\mathrm{Re}[\Delta_S]$, $\mathrm{Im}[\Delta_S]$

d-wave bond order
$\mathrm{Re}[\Delta_{\mathbf{Q}}]$, $\mathrm{Im}[\Delta_{\mathbf{Q}}]$

- Thermal fluctuations restore symmetry, and the pseudogap is a regime of a fluctuating O(4) order parameter (which cannot have a phase transition with O(4) symmetry).

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

1. Pseudospin symmetry between d-wave superconductivity and bond order

Continuum field theory with exact pseudospin symmetry

2. Approximate pseudospin symmetry on the lattice t-J model

Pseudogap and bond order in the underdoped cuprates

$$H_{tJ} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

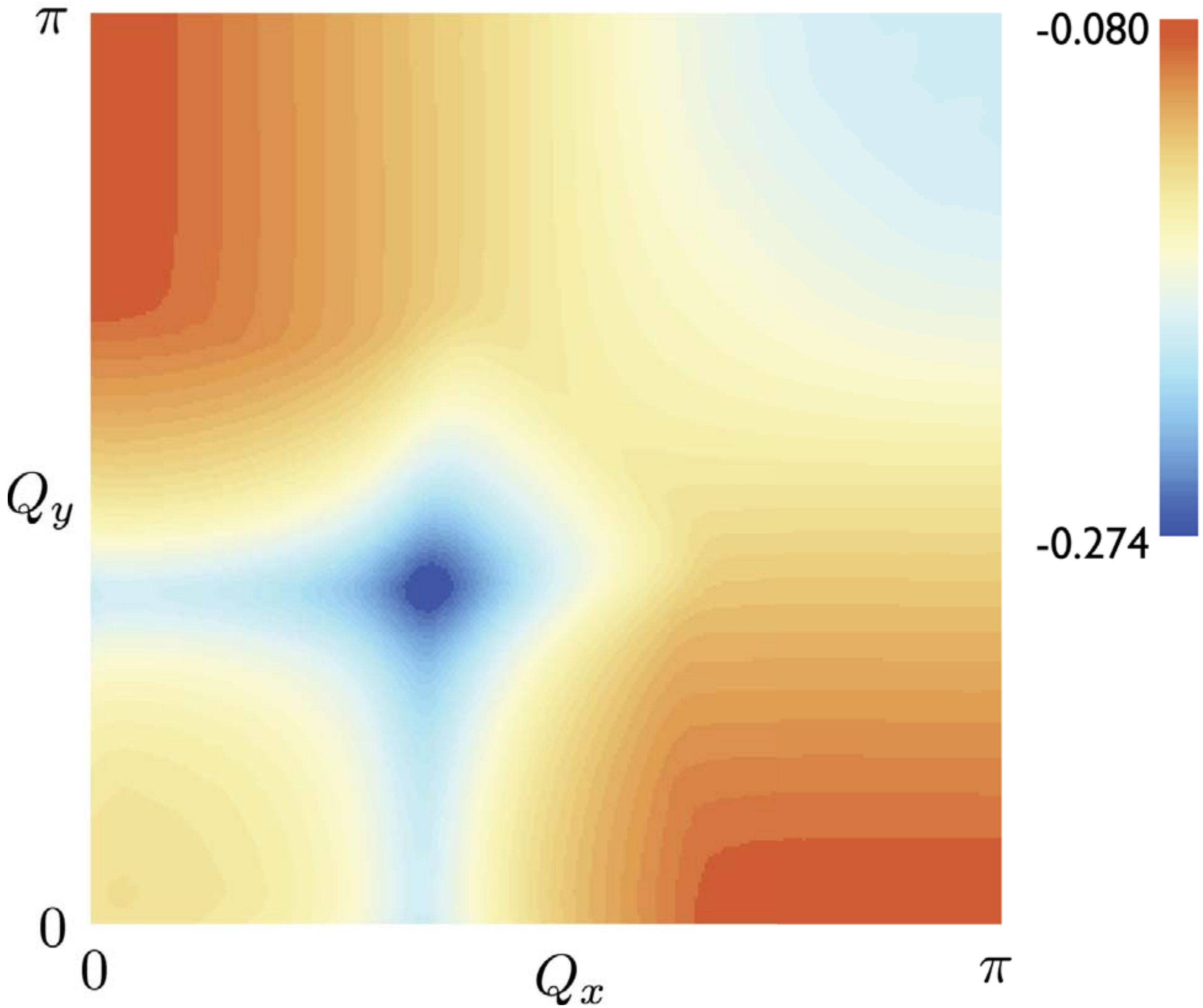
Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet charge order ($\Delta_{\mathbf{Q}}(\mathbf{k})$):

$$H_{MF} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{\mathbf{k},\mathbf{Q}} \Delta_{\mathbf{Q}}(\mathbf{k}) c^\dagger_{\mathbf{k}+\mathbf{Q}/2,\alpha} c_{\mathbf{k}-\mathbf{Q}/2,\alpha}$$

Expanding the free energy in powers of the order parameters we obtain

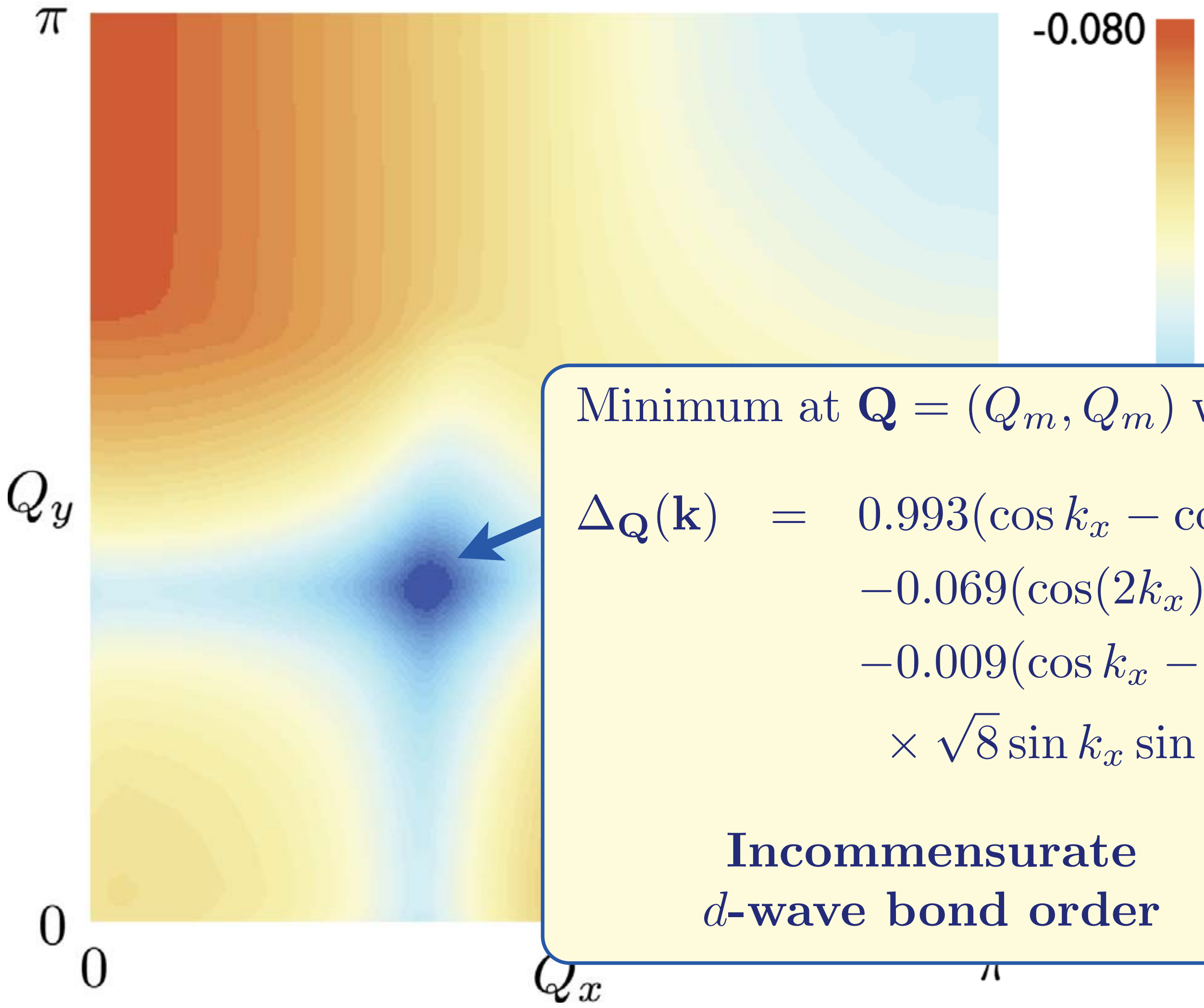
$$F = F_0 + \sum_{\mathbf{k},\mathbf{Q}} \Delta^*_{\mathbf{Q}}(\mathbf{k}) \mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') \Delta_{\mathbf{Q}}(\mathbf{k}')$$

We compute the eigenvalues, $1+\lambda_{\mathbf{Q}}$, and eigenfunctions, $\Delta_{\mathbf{Q}}(\mathbf{k})$ of the kernel $\mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}')$



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114



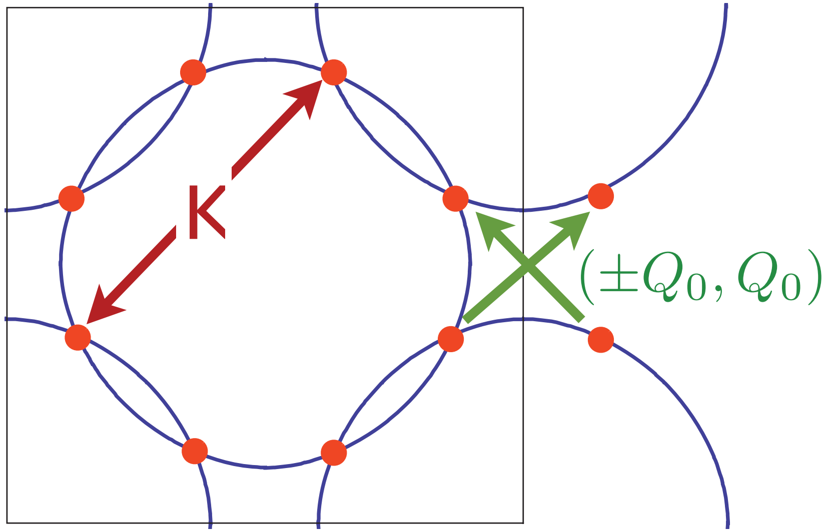
Minimum at $\mathbf{Q} = (Q_m, Q_m)$ with

$$\begin{aligned}\Delta_{\mathbf{Q}}(\mathbf{k}) \quad = \quad & 0.993(\cos k_x - \cos k_y) \\ & -0.069(\cos(2k_x) - \cos(2k_y)) \\ & -0.009(\cos k_x - \cos k_y) \\ & \times \sqrt{8} \sin k_x \sin k_y\end{aligned}$$

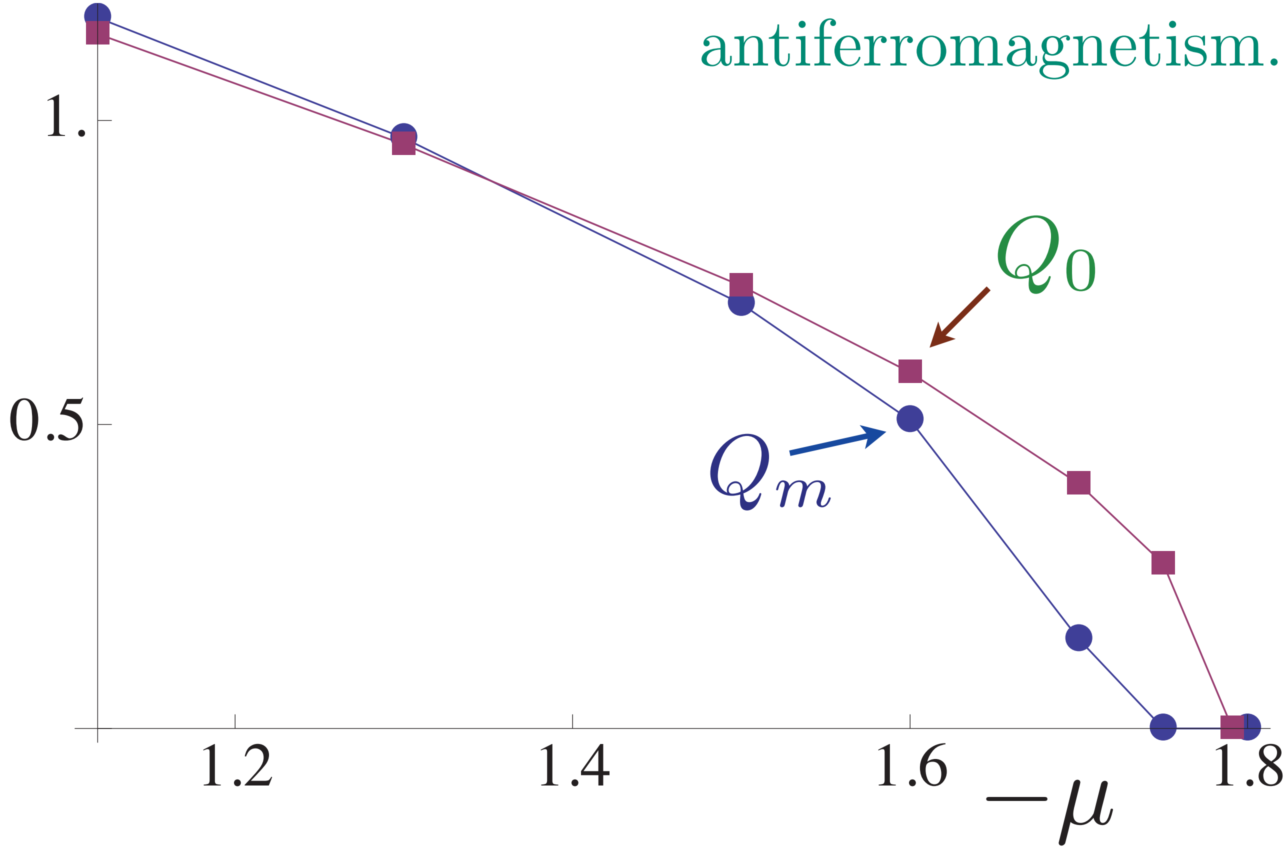
Incommensurate
***d*-wave bond order**

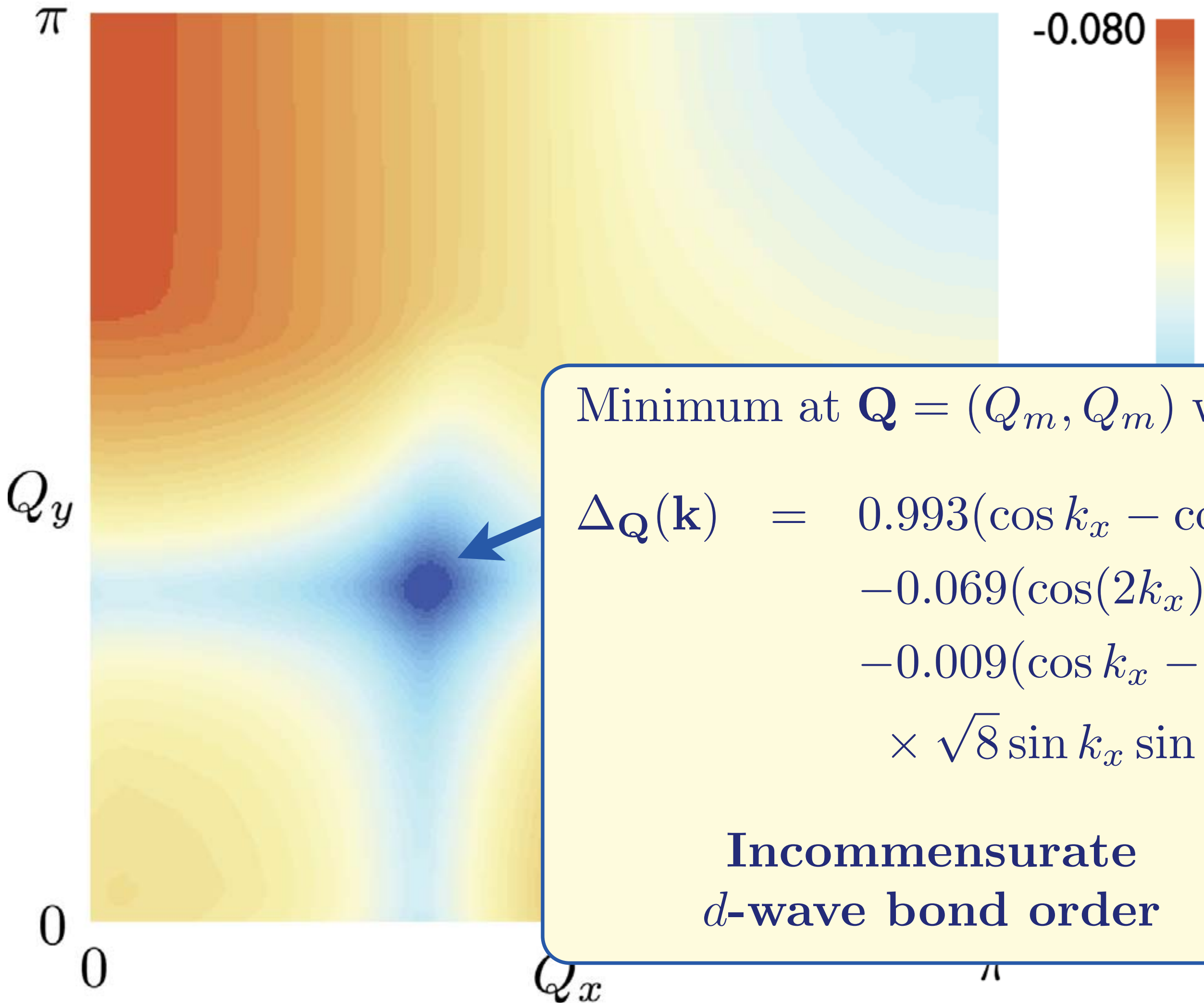
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114



Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.





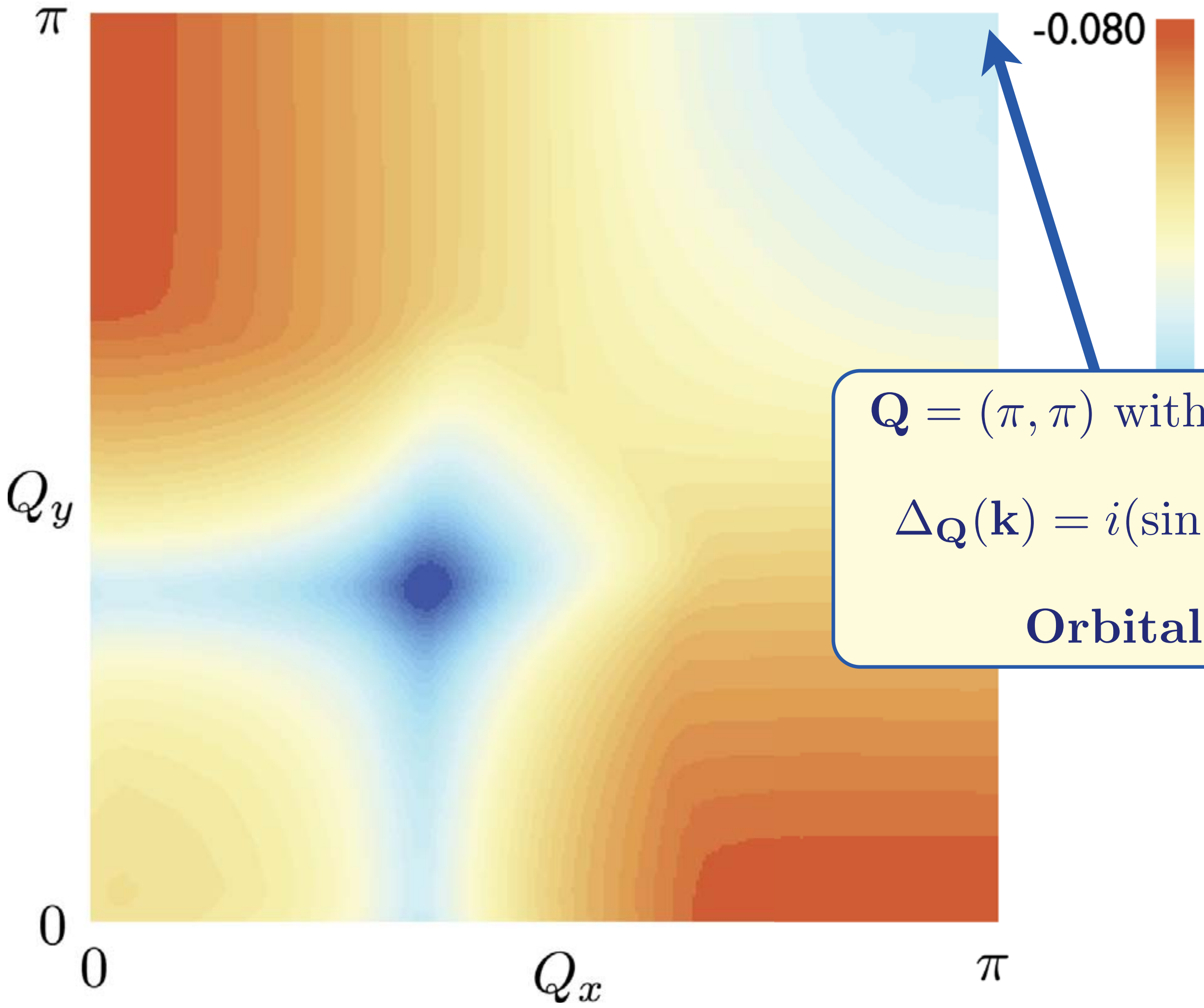
Minimum at $\mathbf{Q} = (Q_m, Q_m)$ with

$$\begin{aligned}\Delta_{\mathbf{Q}}(\mathbf{k}) \;=\; & 0.993(\cos k_x - \cos k_y) \\ & -0.069(\cos(2k_x) - \cos(2k_y)) \\ & -0.009(\cos k_x - \cos k_y) \\ & \;\times \sqrt{8}\sin k_x \sin k_y\end{aligned}$$

Incommensurate
***d*-wave bond order**

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114



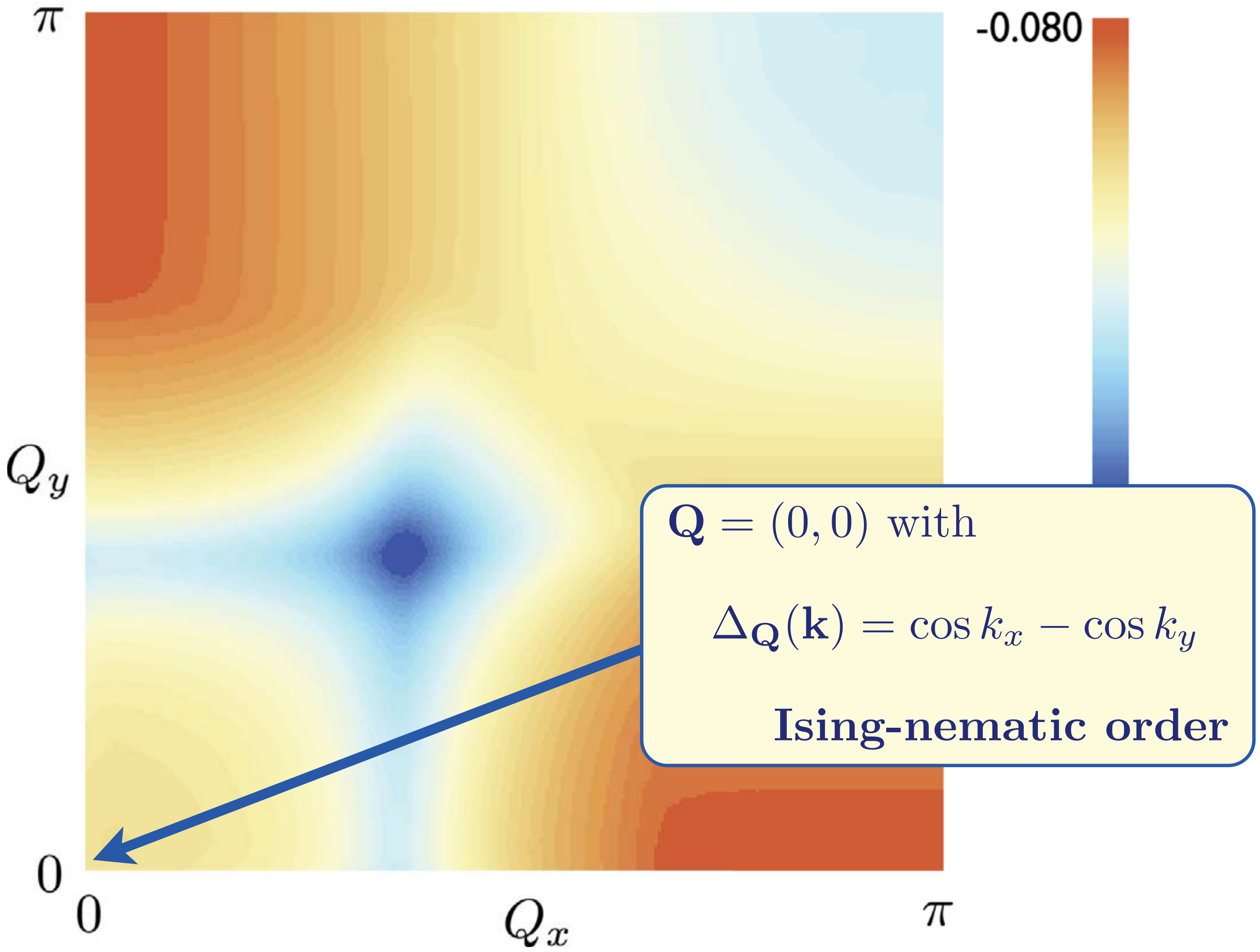
$\mathbf{Q} = (\pi, \pi)$ with

$\Delta_{\mathbf{Q}}(\mathbf{k}) = i(\sin k_x - \sin k_y)$

Orbital currents

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114



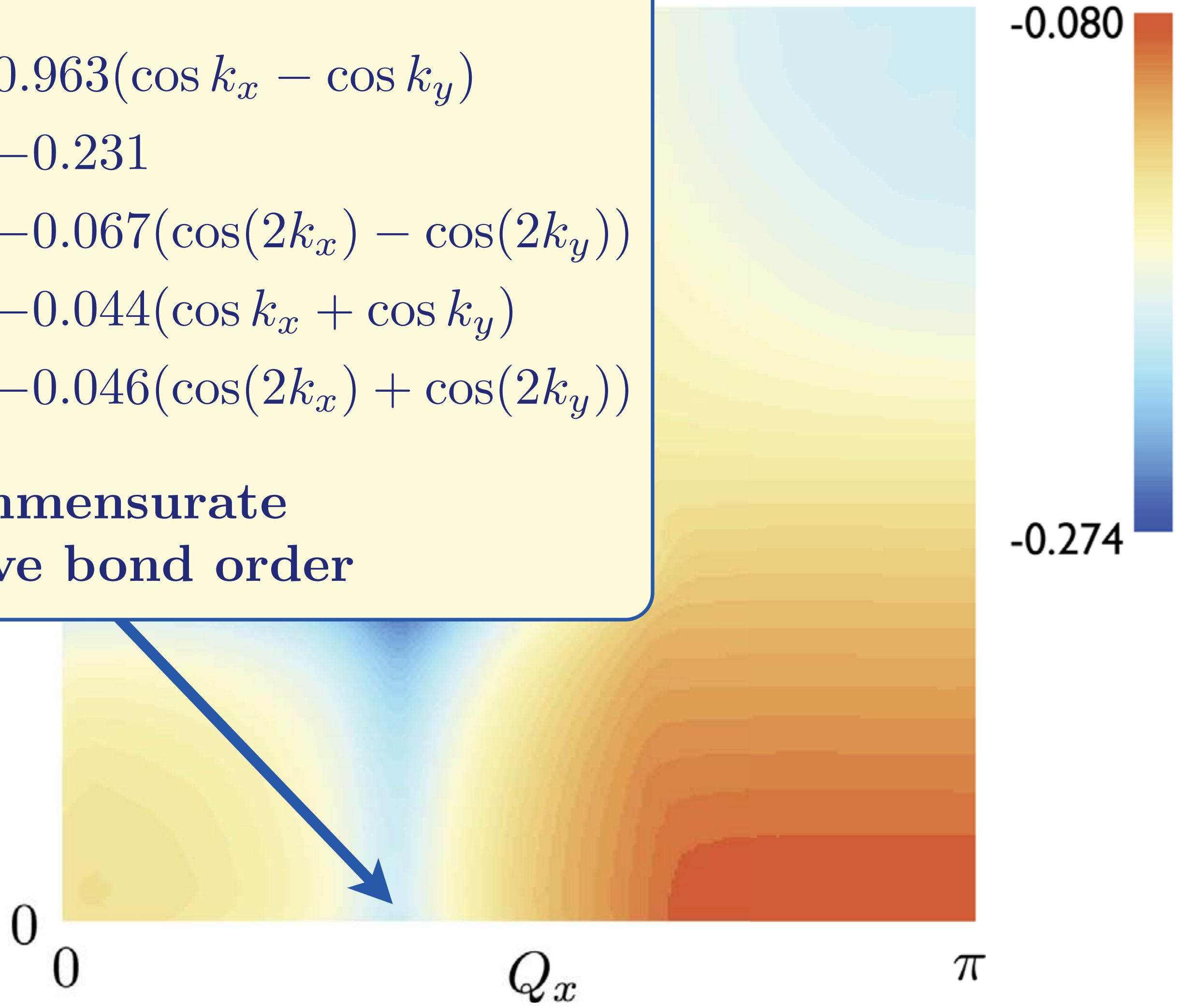
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114

$\mathbf{Q} = (Q_m, 0)$ with

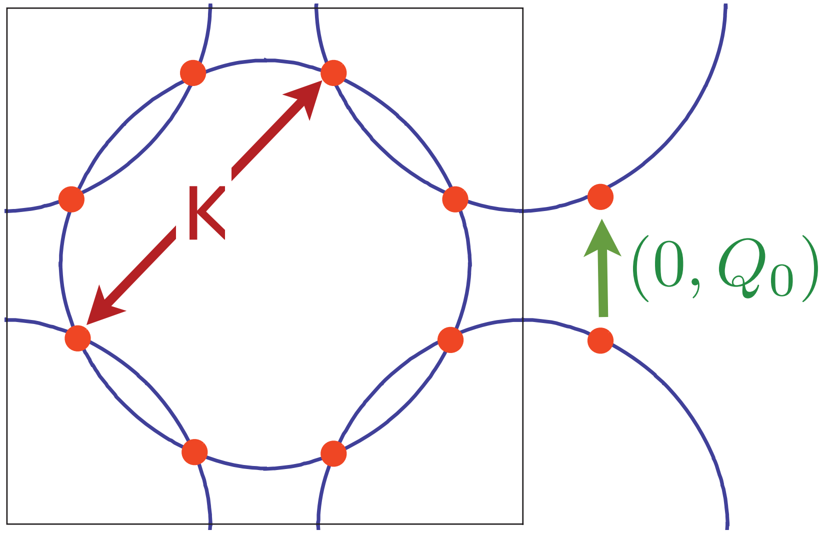
$$
\begin{aligned}
\Delta_{\mathbf{Q}}(\mathbf{k}) \quad = \quad & 0.963(\cos k_x - \cos k_y) \\
& -0.231 \\
& -0.067(\cos(2k_x) - \cos(2k_y)) \\
& -0.044(\cos k_x + \cos k_y) \\
& -0.046(\cos(2k_x) + \cos(2k_y))
\end{aligned}
$$

Incommensurate
d_{+s} **-wave bond order**

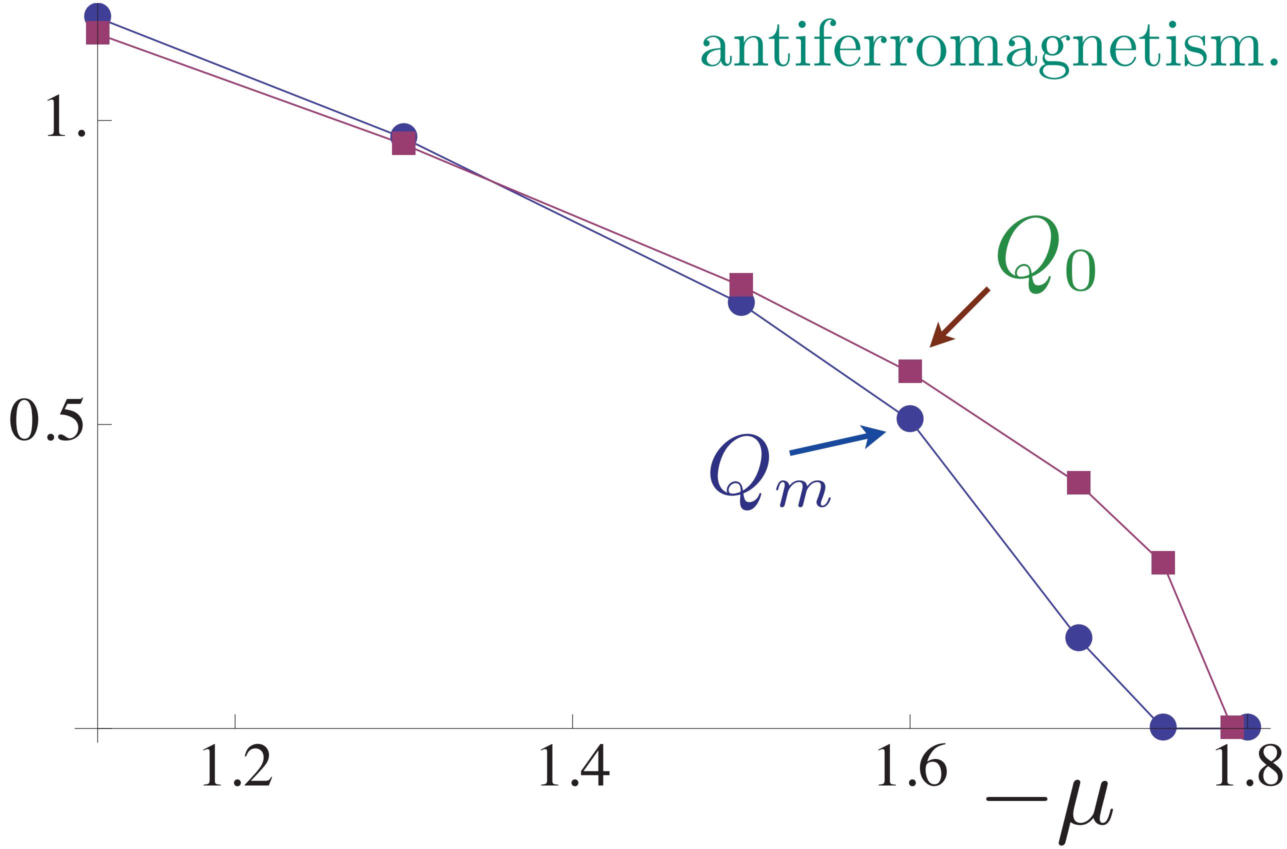


Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

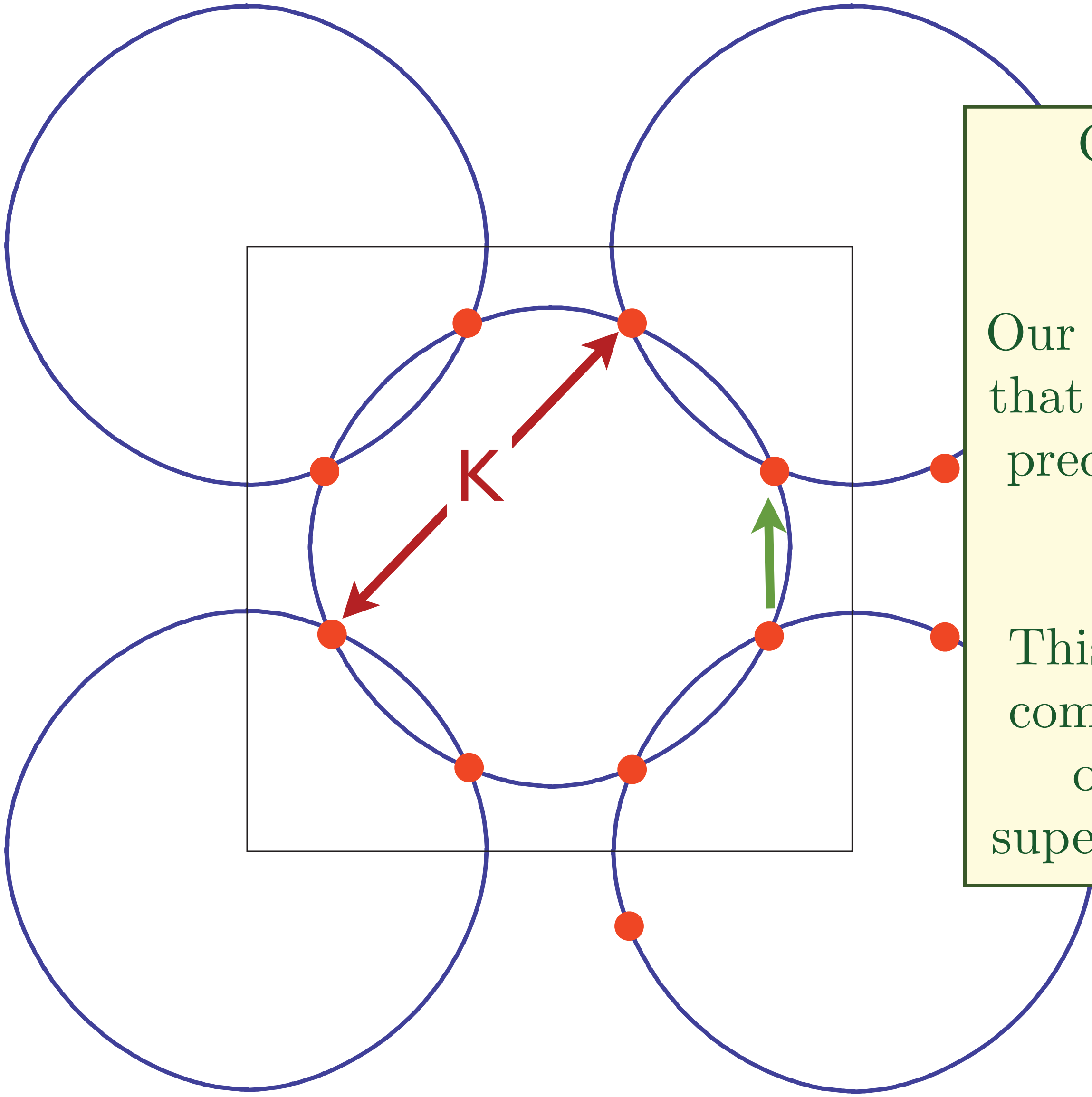
S. Sachdev and R. La Placa, Phys. Rev. Lett. in press, arXiv:1303.2114



Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.



Incommensurate d-wave bond order



Observed low T ordering.

Our computations show that the charge order is predominantly d-wave also at this $\mathbf{Q}$.

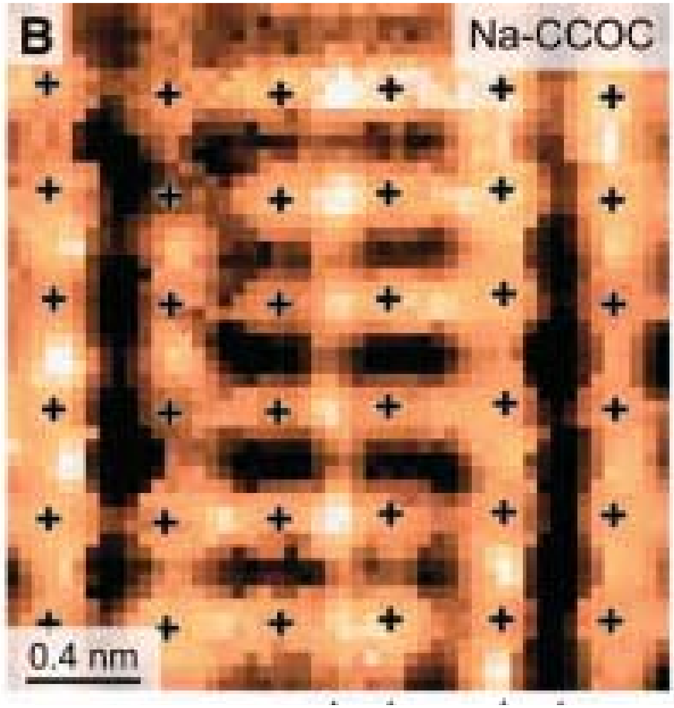
This $\mathbf{Q}$ is preferred in computations of bond order within the superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters in press; arXiv:1303.2114
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)

An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8]*

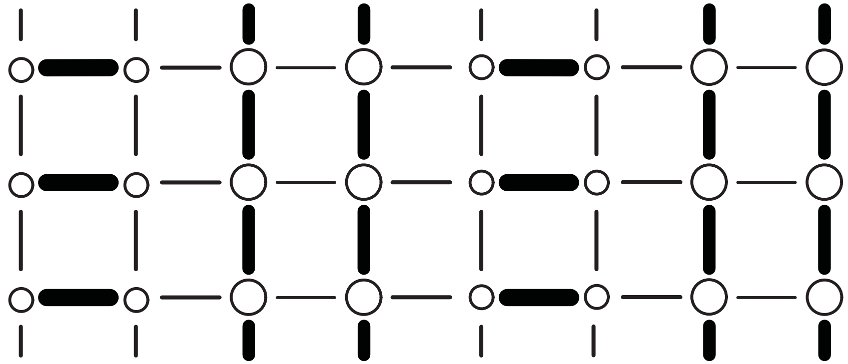
9 MARCH 2007 VOL 315 SCIENCE



PHYSICAL REVIEW B **77**, 094504 (2008)

Superconducting *d*-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch

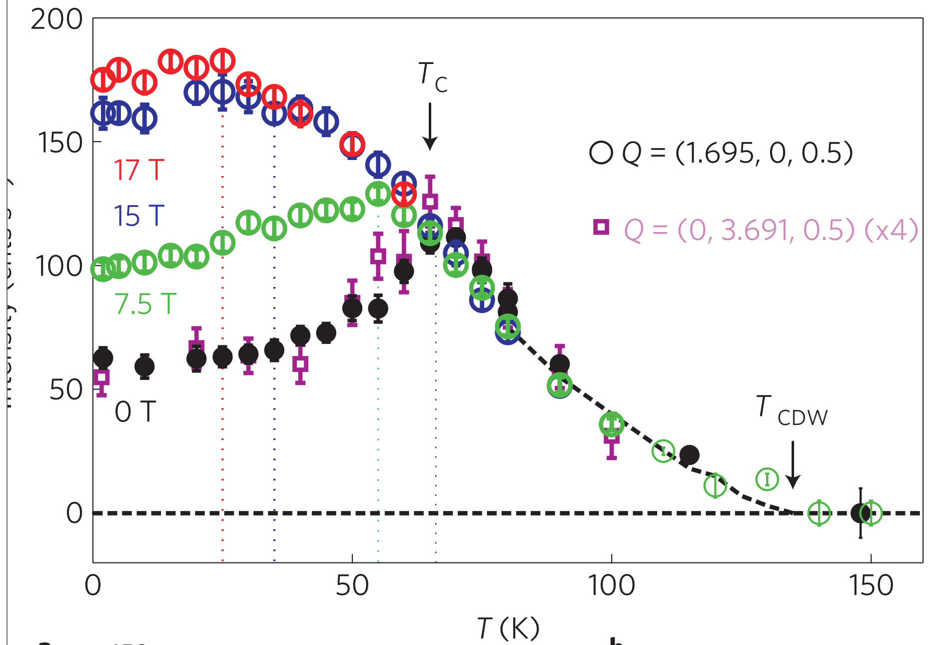


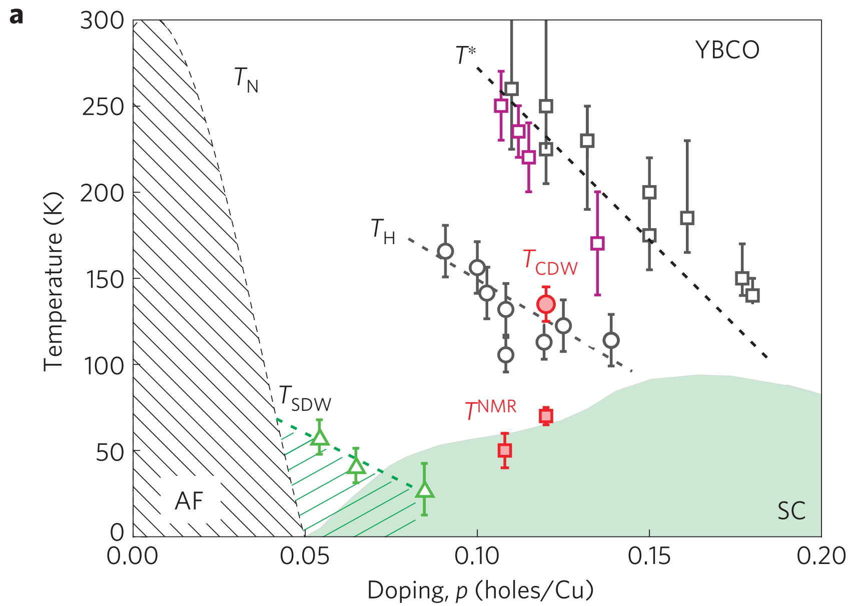
We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both *s*- and *d*-wave channels, with nonlocal Coulomb repulsion suppressing the *s*-wave component.

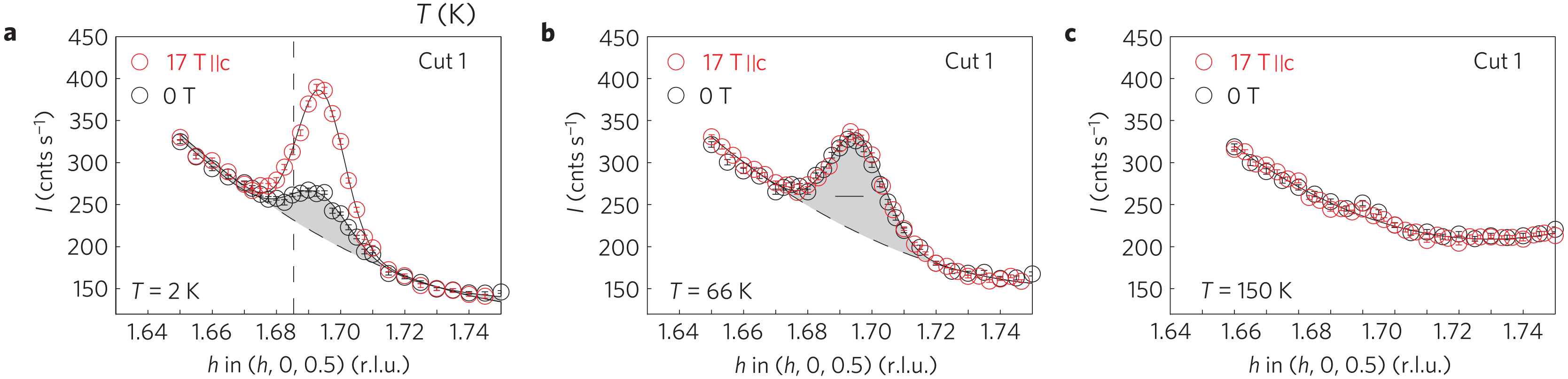
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]



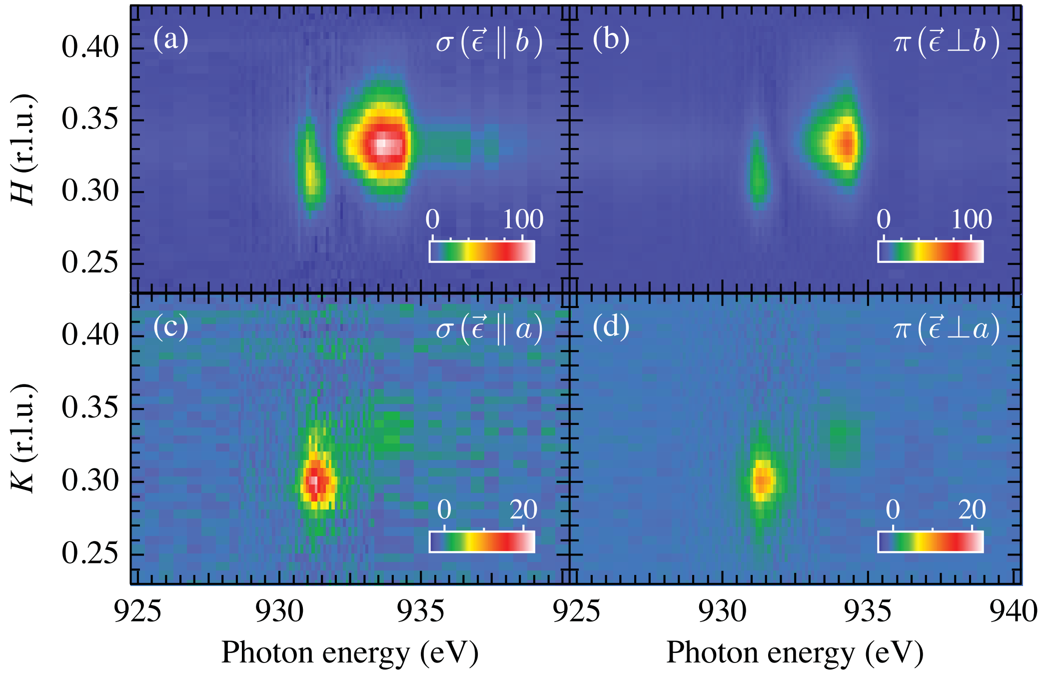




Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

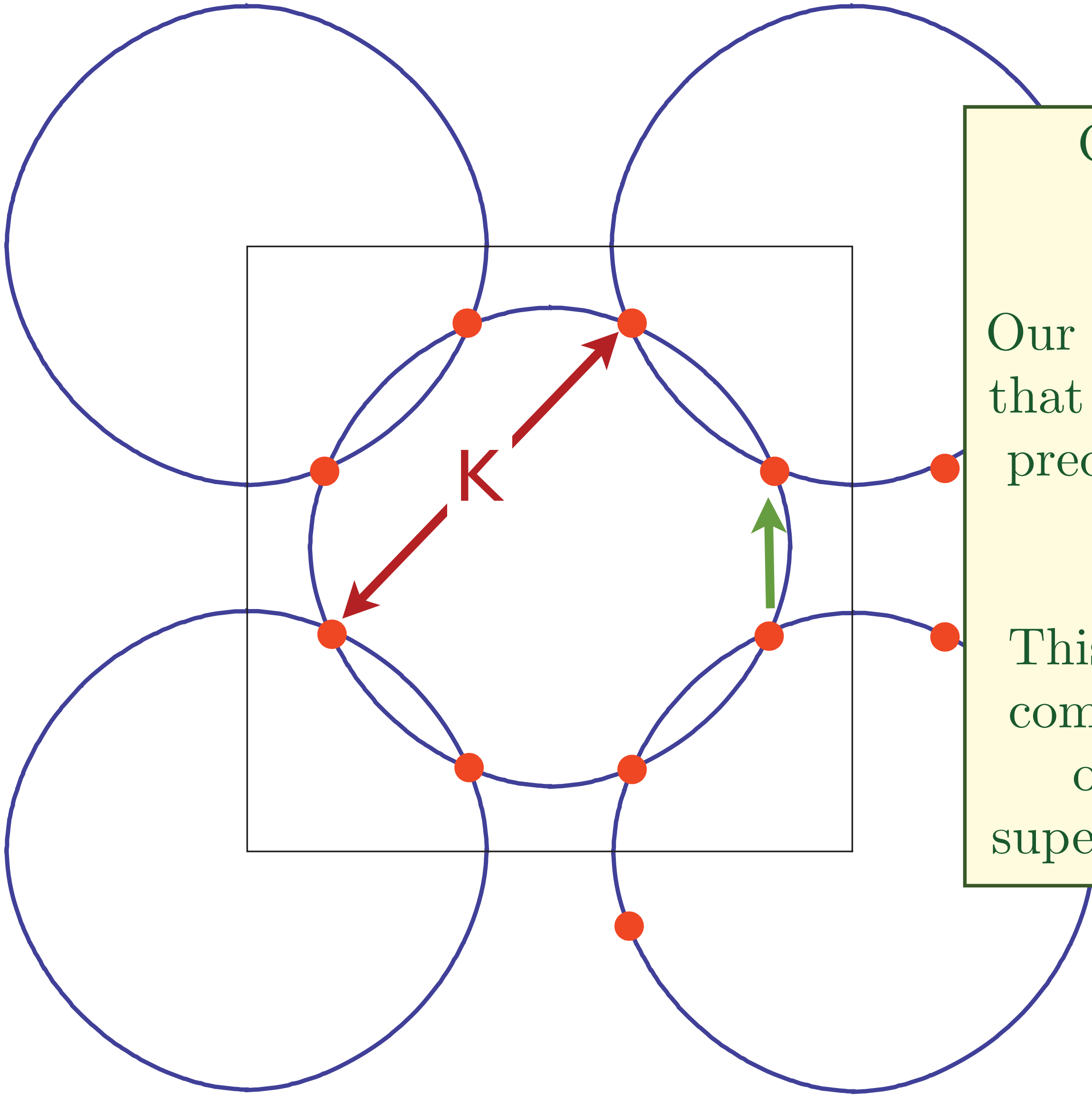
PRL **109,** 167001 (2012)



Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu $2p$ to $3d_{x^2-y^2}$ transition, similar to stripe-ordered 214 cuprates.

These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.

Incommensurate *d*-wave bond order



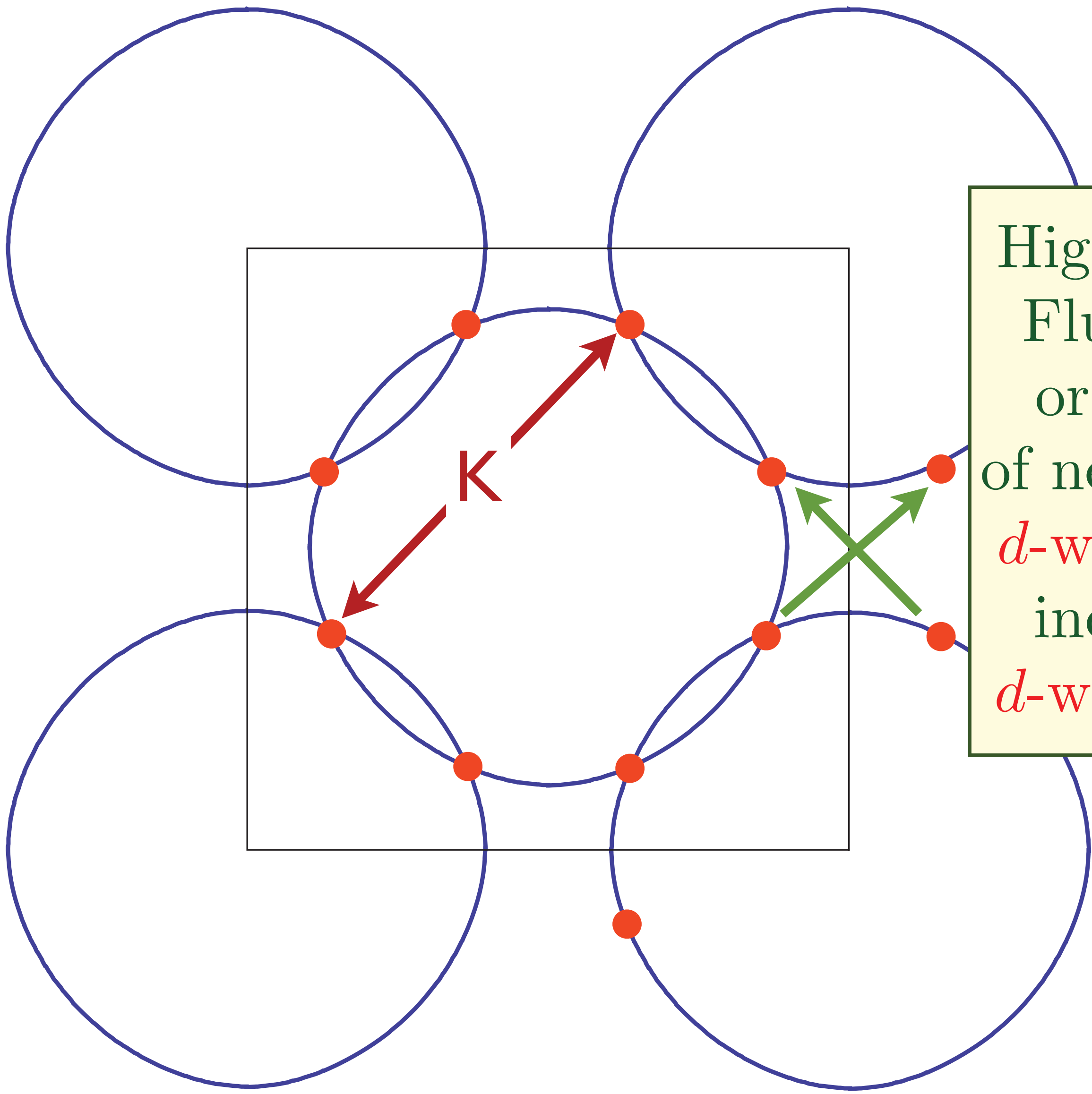
Observed low T ordering.

Our computations show that the charge order is predominantly d-wave also at this $\mathbf{Q}$.

This $\mathbf{Q}$ is preferred in computations of bond order within the superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters in press; arXiv:1303.2114
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)

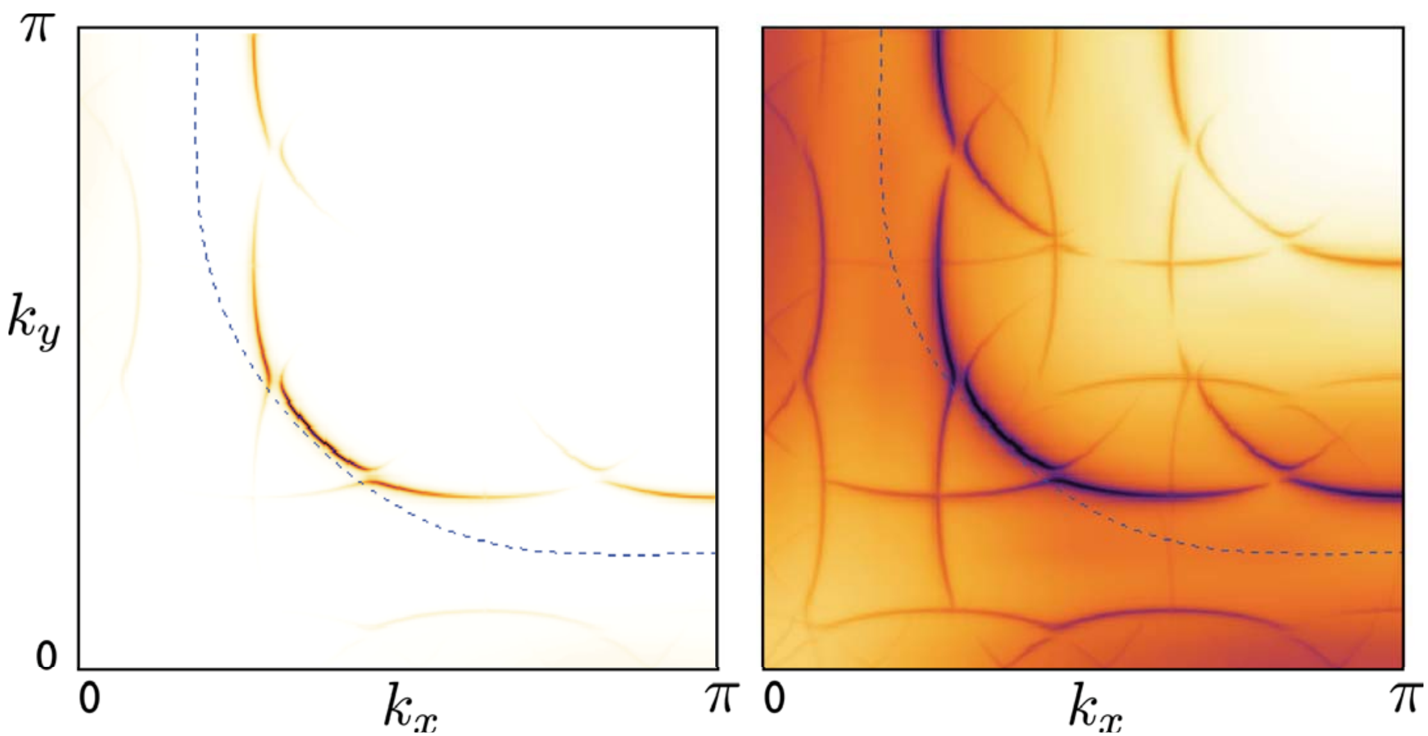
Incommensurate *d*-wave bond order



High T pseudogap: Fluctuating O(4) order parameter of nearly degenerate *d*-wave pairing and incommensurate *d*-wave bond order.

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Electron spectral function



$\mathrm{Im}G(\mathbf{k}, \omega + i\eta)$ $\quad\quad$ $\log \mathrm{Im}G(\mathbf{k}, \omega + i\eta)$

$$\left\langle c^\dagger_{\boldsymbol{k}-\boldsymbol{Q}/2,\alpha} c_{\boldsymbol{k}+\boldsymbol{Q}/2,\alpha} \right\rangle \propto \Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \begin{cases} \Delta_s + \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (\pm Q_0, 0) \\ \Delta_s - \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (0, \pm Q_0) \end{cases}$$

$$\text{with } \Delta_s/\Delta_d = -0.234.$$

S. Sachdev and R. La Placa, arXiv:1303.2114

Summary

Antiferromagnetism in metals and the high temperature superconductors

- Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)

- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to *d*-wave superconductivity, and to a charge density wave with a *d*-wave form factor. This is a promising explanation of the pseudogap regime.