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Towards controlled non-Fermi liquid fixed points

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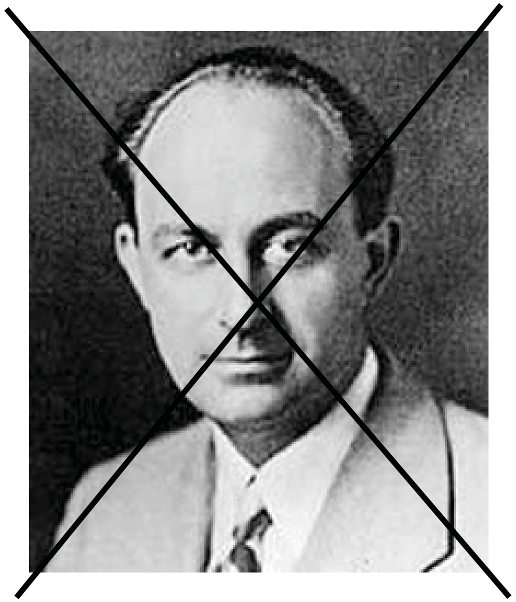
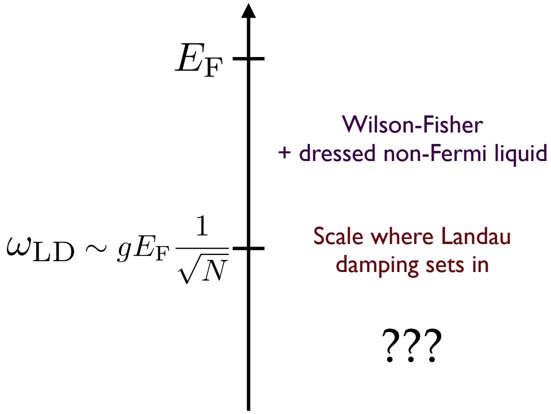




Shamit Kachru (Stanford & SLAC)

This talk is based entirely on work/discussions with:

A.L. Fitzpatrick

J. Kaplan

S. Raghu

Any major conceptual errors or calculational mistakes are attributable entirely to them

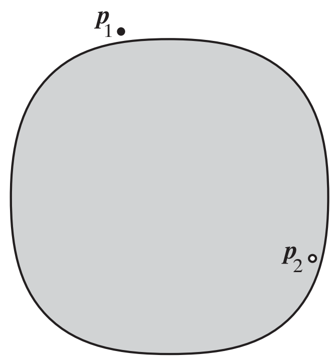
1. Introduction

The Fermi liquid fixed point is a stable IR fixed point (modulo the BCS instability).

Shankar, Polchinski

Its starting point is the free fermion action:

$$\int dt\, d^3\mathbf{p} \left\{ i\psi^\dagger_\sigma(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - (\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}})\psi^\dagger_\sigma(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}.$$



The scaling which governs the Fermi liquid fixed point, determined by the free action, is:

$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

$$\varepsilon(\mathbf{p}) - \varepsilon_{\mathrm{F}} = l v_{\mathrm{F}}(\mathbf{k}) + O(l^2),$$

$$dt \to s^{-1} dt, \quad d\mathbf{k} \to d\mathbf{k}, \quad d\mathbf{l} \to s\, d\mathbf{l}, \quad \partial_t \to s\partial_t, \quad l \to sl,$$

so examining the action

$$\int dt\, d^2\mathbf{k}\, d\mathbf{l} \left\{ i\psi_\sigma^\dagger(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - l v_{\mathrm{F}}(\mathbf{k})\psi_\sigma^\dagger(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}$$

we see that the Fermi field should scale as:

$$\psi \to s^{-1/2}\psi \ .$$

The first interaction, which determines the (in)stability of this fixed point, is:

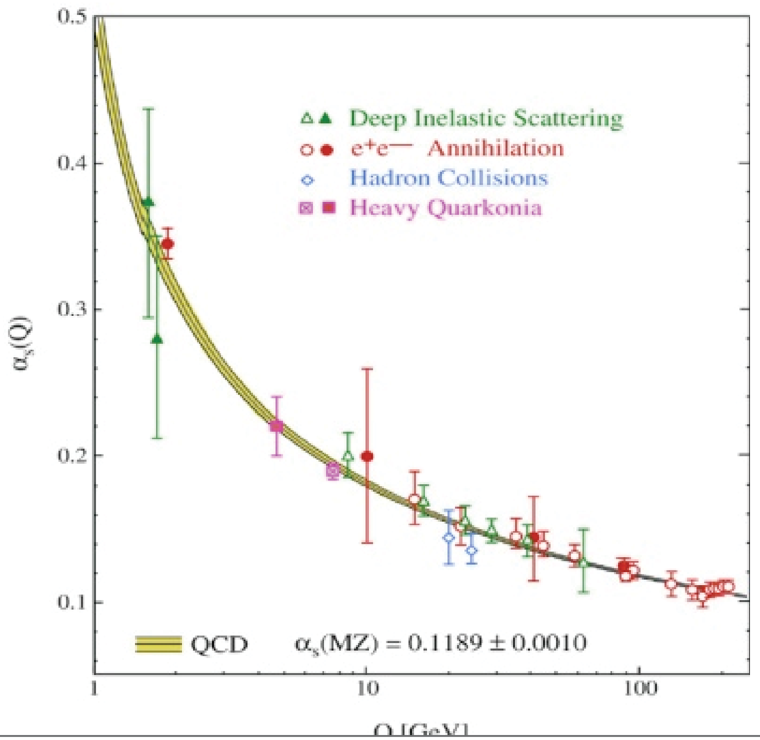
$$\int dt\, d^2\mathbf{k}_1\, d\mathbf{l}_1\, d^2\mathbf{k}_2\, d\mathbf{l}_2\, d^2\mathbf{k}_3\, d\mathbf{l}_3\, d^2\mathbf{k}_4\, d\mathbf{l}_4\, V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$
$$\psi^\dagger_\sigma(\mathbf{p}_1)\psi_\sigma(\mathbf{p}_3)\psi^\dagger_{\sigma'}(\mathbf{p}_2)\psi_{\sigma'}(\mathbf{p}_4)\delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

It naively scales like s and is irrelevant, but for special kinematic configurations the delta function scales and we get marginal forward scattering and BCS couplings.

As it has no other obvious interesting perturbations, it is seemingly not a good starting point to describe the non-Fermi liquid physics which is apparently seen in many real systems.

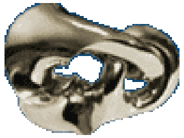
In fact, the theoretical description of non-Fermi liquid fixed points remains a subject of active interest.

As a high energy theorist, I am going to borrow a lesson from our study of QCD.



The effective coupling runs strong in the IR. The emergent physics includes chiral symmetry breaking and confinement of quarks.

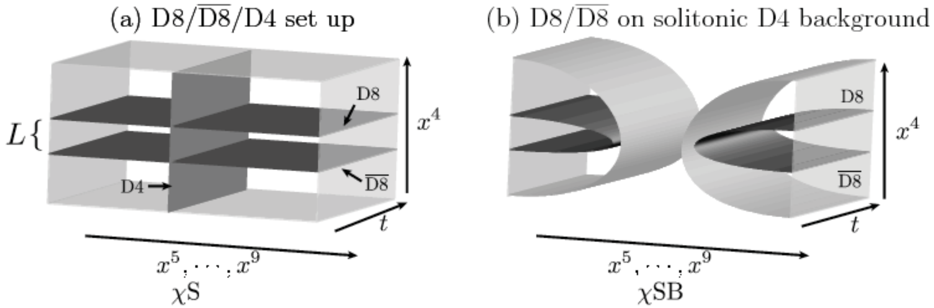
The brute force understanding of this behavior remains an open problem:



Clay Millenium Problem: prove mass gap in Yang-Mills theory.

However, through use of:

* Large N methods parametrized by (N_c, N_f)



* Study of supersymmetric toy models, similarly parametrized by (N_c, N_f)

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we have come to understand a rich set of possible behaviors of non-Abelian gauge theories, including:

* strongly coupled theories with free magnetic duals

* confinement without chiral symmetry breaking

* Banks/Zaks and other conformal fixed points

...and of course, some phases with qualitatively similar behavior to real-world QCD.

The lesson I will take from this is the following: it will be useful to find controlled approaches to non-Fermi liquid fixed points using toy models, even unrealistic toy models.

II. Our toy model & RG philosophy

We will study the theory with UV action:

$$\mathcal{S} = \int d\tau \int d^d x \; \mathcal{L} = S_\psi + S_\phi + S_{\psi-\phi}$$

$$\mathcal{L}_\psi = \bar{\psi}_\sigma \left[\partial_\tau + \mu - \epsilon(i\nabla)\right] \psi_\sigma + \lambda_\psi \bar{\psi}_\sigma \bar{\psi}_{\sigma'} \psi_{\sigma'} \psi_\sigma$$

$$\mathcal{L}_\phi = m_\phi^2 \phi^2 + (\partial_\tau \phi)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 + \frac{\lambda_\phi}{4!}\phi^4$$

$$S_{\psi,\phi} = \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2(d+1)}} g(k,q) \bar{\psi}(k) \psi(k+q) \phi(q),$$

i.e. a bosonic "order parameter field" coupled to a conventional Fermi liquid, in the tuned limit:

$$m_\phi \to 0 \ .$$

* We will do perturbative RG in the Yukawa and quartic couplings.

* We will introduce two additional control parameters:

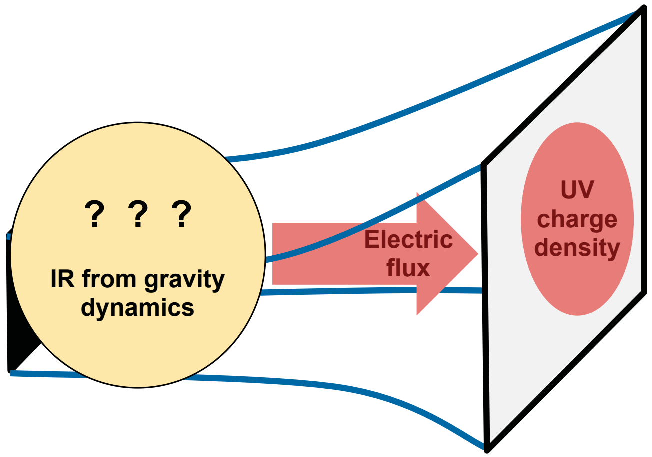
$$\epsilon = 3 - d$$

$N \longleftarrow$ matrix boson, 1 "flavor" of fermions

The unconventional large N we use here is in part inspired by AdS/CFT examples.

In general, in that setting, the question of finite charge density dynamics is mapped to a geometry question:

$$S = \int \, d^4x \, \sqrt{-g} \left(R - 2\Lambda + F^2 + \cdots \right)$$



Considering "probe fermions" in the geometries:

$$ds^2 = -r^{2z}dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2}$$

which are supported by the doped large N CFT, has given rise to some interesting non-Fermi liquids.

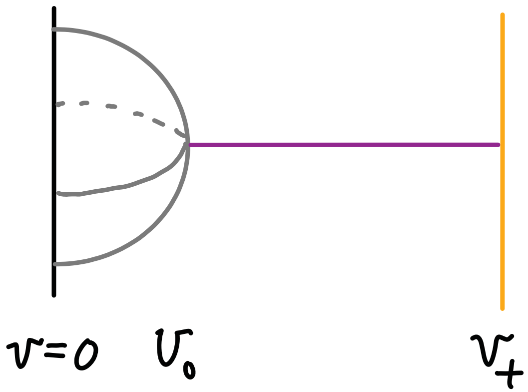
1. $$z = \infty \rightarrow AdS2 \times R^2$$

S.S. Lee;
Liu, McGreevy,...;
Cubrovic, Schalm, Zaanen

1/N non-Fermi liquids which can exhibit local quantum criticality and other interesting behaviors.

2. $z = 2 \rightarrow$ brane probe fermions

Hartnoll, Polchinski,
Silverstein, Tong



$$\rho \propto \frac{T^{2/z}}{J^t}\,.$$

In both cases, the large N bath dresses the fermions very effectively.

The theories I discuss today are simple field theories that should serve as good starting points for finding similar phenomena in more conventional models.

With our detour into geometry at an end, lets return to the field theory.

What scaling do we use?

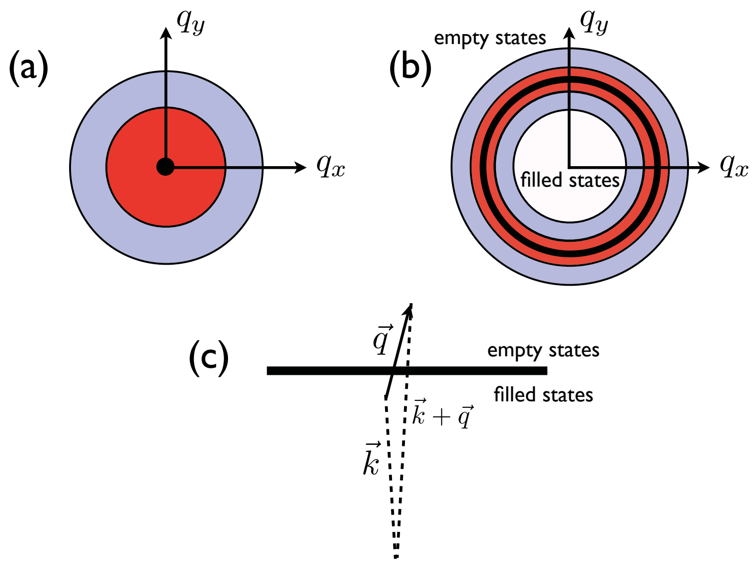


FIG. 1. Summary of tree-level scaling. High energy modes (blue) are integrated out at tree level and remaining low energy modes (red) are rescaled so as to preserve the boson and fermion kinetic terms. The boson modes (a) have the low energy locus at a point whereas the fermion modes (b) have their low energy locus on the Fermi surface. The most relevant Yukawa coupling (c) connects particle-hole states nearly perpendicular to the Fermi surface; all other couplings are irrelevant under the scaling.

We expand fermion momenta about the closest point on the Fermi surface

$$\boldsymbol{k} = \hat{\Omega}(k_F + \ell).$$

and do Polchinski scaling:

$$k_0' = e^t k_0, \ \boldsymbol{k}_F' = \boldsymbol{k}_F, \ \ell' = e^t \ell$$

In contrast, we scale boson momenta isotropically:

$$k_0' = e^t k_0, \ \boldsymbol{k}' = e^t \boldsymbol{k}$$

This implies the following scaling of fields:

$$\psi' = e^{-3t/2}\psi,\ \phi' = e^{-\frac{(d+3)}{2}t}\phi$$

With the Polchinski scaling, we know the BCS four-Fermi interaction will remain marginal at tree level in all d. The bosonic quartic coupling scales as:

$$\lambda'_\phi = e^{(3-d)t}\lambda_\phi$$

So d=3 is the upper critical dimension.

Finally, expanding the momentum-dependent Yukawa coupling around the Fermi surface:

$$g(k,q) = g(\boldsymbol{k}_F, 0) + a_1\ell + a_2 q + \cdots$$

we see that:

$$\lim_{\boldsymbol{k}\to\boldsymbol{k}_F} \lim_{q\to 0} g'(\boldsymbol{k}', \boldsymbol{q}') = e^{\frac{3-d}{2}t} g(\boldsymbol{k}, \boldsymbol{q})$$

This single term in the Yukawa coupling remains marginal in d=3; the further terms in the Taylor expansion are irrelevant.

We note that there are different (reasonable) possible choices for Fermi surface "patch" scaling. This scaling has been chosen for the following merits:

* It reverts to the obvious scalings as one decouples the boson and fermion, as perturbation theory should.

* BCS and forward scattering interactions of fermions remain obviously marginal at tree level.

While all of this is the obvious procedure starting from the decoupled UV fixed point, it is distinct from the analysis done in some of the recent literature.

Instead, starting with Hertz, the notion that "Landau damping" of the boson by the Fermi liquid is going to be the most important effect is often fed in as the crucial determinant of low-energy physics.

$$S_\phi^{\mathrm{Hertz}} = S_\phi + g^2 m_\psi^2 \int \frac{d^{d+1}q}{(2\pi)^d} \frac{|q_0|}{|q|} \theta(|q| - |q_0|) \phi_{\boldsymbol{q}} \phi_{-\boldsymbol{q}}$$

The non-analyticity arises from integrating out gapless fermions.

The fermion correction to the boson propagator:

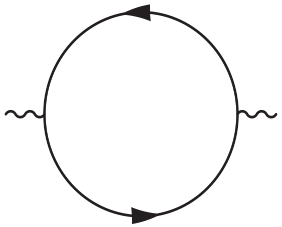


FIG. 2: The one-loop boson self energy.

$$D(q) = \frac{1}{N}\left(c_b \frac{|q_\tau|}{|q_y|} + \frac{q_y^2}{e^2} + r\right)^{-1}.$$

yields a theory with z=3 dynamical scaling.

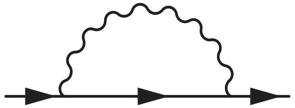


FIG. 3: The one-loop fermion self energy. Here the boson propagator is a dressed propagator which include the one-loop self energy correction in Fig. 2.

Feeding this back in to the fermionic sector yields a non-Fermi liquid.

S.S. Lee;
Metlitski, Sachdev;
....

While this may be true at N=1, I wish to instead develop a systematic RG starting from a local high energy theory. What emerges in the IR is determined by decimation. If there is an intermediate fixed point, the IR behavior may be different than has been expected.

So, I follow a "radically conservative" approach and avoid integrating out gapless modes. The effects of Landau damping will be visible in correlators, and are part of our theory, but only arise in determining the RG fixed point as decimation of high energy modes would have them.

III. RG in the epsilon expansion

Because we have couplings that are marginal in d=3, we can hope to find controlled fixed points in the epsilon expansion.

So, we do the RG. Our procedure will be to decimate in energy intervals, while integrating over all spatial momenta.

$$\Lambda e^{-t} < E < \Lambda$$

$$\text{external legs} : E < \Lambda e^{-t}$$

The diagrams that arise at one-loop order are:

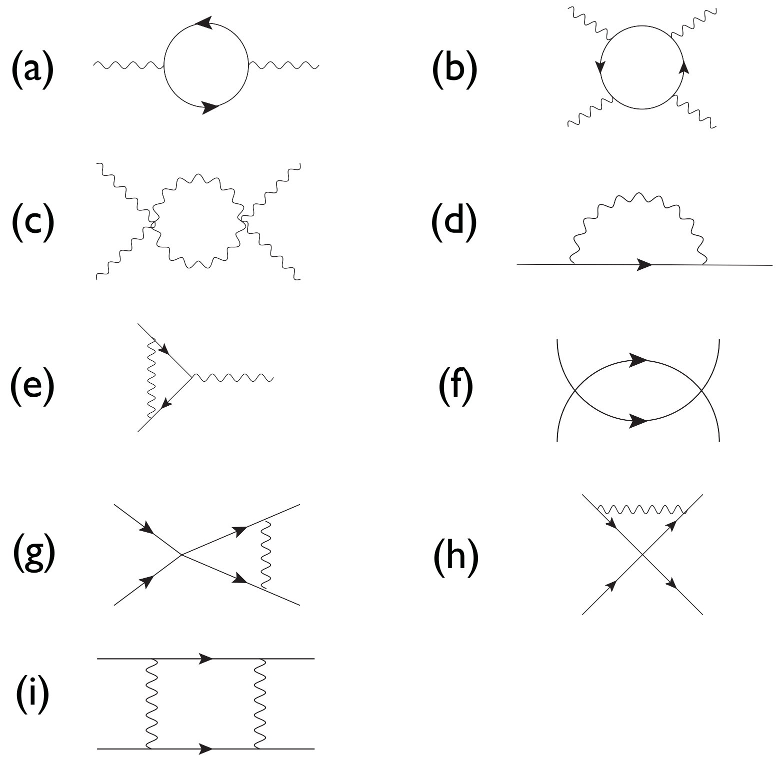
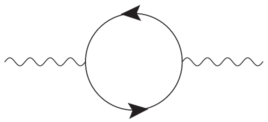
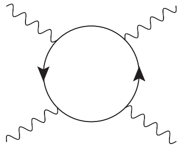


FIG. 2. One-loop diagrams. The boson self-energy (a), boson self-interactions (b,c), fermion self-energy (d), vertex correction (e) and particle-hole scattering (f). Diagrams (a) and (b) do not contribute to the renormalization group flow while (c) produces the ordinary Wilson-Fisher fixed point for bosons. Diagram (d) gives rise to fermion wave-function renormalization and (e) yields logarithmic Yukawa coupling constant renormalization. The usual marginal BCS interaction Fermi liquid theory (f) is altered by fermion wave-function renormalization as well as by diagram (g), both of which make the BCS interaction irrelevant.

An immediate and interesting result is that the diagrams (a) and (b) vanish:

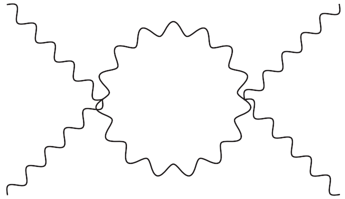
(a)  (b) 

$$\Pi_{d\Lambda}(p) \propto \int_{d\Lambda} dq_0 \left[\mathrm{sgn}(q_0) - \mathrm{sgn}(q_0 + p_0)\right] = 0,$$

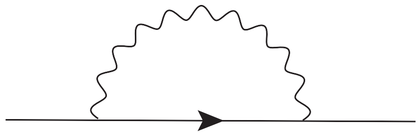
So the diagram which normally yields the dramatic effect of Hertz-Millis theory, does not contribute in the RG.

I emphasize that Landau-damping does appear in correlation functions at the resulting fixed point.

The diagram (c) yields the standard contribution arising in study of the Wilson-Fisher fixed point:

(c) 

$$\frac{d\lambda_\phi}{dt} = \epsilon\lambda_\phi - \underset{\text{positive constant}}{a_{\lambda_\phi}}\lambda_\phi^2.$$

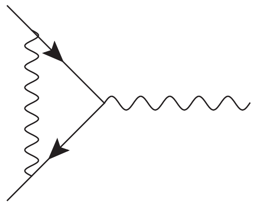
(d) 

(d) gives a nontrivial fermion wavefunction renormalization:

$$\Sigma_{d\Lambda}(k) = \left(ik_0 g^2 \underset{\text{another positive constant}}{a_g}\right) d\log\Lambda,$$

The beta function for the Yukawa coupling arises from both the fermion wave-function renormalization, and the vertex correction. (The boson wave-function renormalization fortuitously vanishes at 1-loop).

The vertex correction is:

(e) 

$$\frac{\delta g_{d\Lambda}(k,0)}{g} = -C_3 \frac{\partial \Sigma_{d\Lambda}(k)}{\partial\,(ik_0)} = -g^2 C_3 a_g d \log \Lambda$$

The resulting flow equation is:

$$\frac{dg}{dt} = \frac{\epsilon}{2} g - (1 + C_3) g^3 a_g + \mathcal{O}(g^3 \epsilon)$$

with a fixed point: $g^2 = \frac{\epsilon}{2(1+C_3)a_g}$.

1 in basic theory
1/N at large N

Naive fixed point structure:

$$\lambda_\phi^* \sim \epsilon$$

$$g^* \sim \sqrt{\epsilon}$$

The Wilson-Fisher boson is unaffected at this order, but dresses the Fermi liquid into a non-Fermi liquid

The anomalous dimension of the fermion at this order is:

$$\gamma_\psi = \frac{\epsilon}{8} \ .$$

The fermion Green's function takes the form:

$$G(\omega, \ell) \sim \frac{1}{(i\omega - v_F \ell)^{1-2\gamma_\psi}}.$$

In particular, the lifetime is governed by:

$$\mathrm{Im}(\Sigma) \sim (g^*)^2 \omega^{1-\frac{\epsilon}{4}} \ .$$

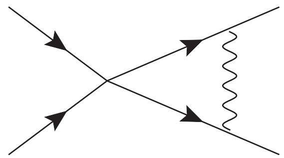
Contrast with the standard approach, which gives:

$$\mathrm{Im}(\Sigma) \sim \omega^{\frac{d}{3}} = \omega^{1-\frac{\epsilon}{3}} \ .$$

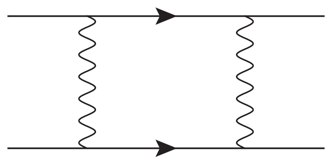
IV. Subtleties with fermion self-coupling, and large N

There are subtleties with the RG for the four-fermion term. The diagrams:

(g)



(i)



naively have log squared divergences, which would put explicit dependence on the t parameter on the RHS of the RG equations.

For instance, one gets integrals of the form:

$$g^2\lambda \int \frac{d\omega d\ell d^{d-1}k_{\|}}{(2\pi)^{d+1}} \left(\frac{1}{\omega^2 + c_s^2(k_{\|}^2 + \ell^2)}\right)\left(\frac{1}{i\omega - v_F\ell}\right)\left(\frac{1}{-i\omega - v_F\ell}\right)$$

Performing two of the integrals yields:

$$g^2\lambda \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{\log\left(\frac{c_s^2 v_F^2 k^2}{(c_s+v_F)^2}\right)}{4\pi k^2 c_s^2 v_F}$$

and expanding around d=3, the final integral gives a UV divergence of the form:

$$\frac{g^2\lambda}{4\pi^2 c_s^2 v_F} \log(\Lambda_{UV}) \log\left(\frac{c_s v_F \Lambda_{UV}}{c_s + v_F}\right)$$

I don't have anything useful to say about these yet. C.f. discussion in the finite density QCD literature by Son and by Schafer and Wilczek. We plan to return to a systematic treatment of this issue in the future.

Instead, we now resort to large N. Consider the theory:

$$\mathcal{L}_\psi = \bar{\psi}^i \left[\partial_\tau + \mu - \epsilon(i\nabla)\right]\psi_i + \frac{\lambda_\psi}{N}\bar{\psi}^i\psi_i\bar{\psi}^j\psi_j$$

$$\mathcal{L}_\phi = \mathrm{tr}\left(m_\phi^2\phi^2 + (\partial_\tau\phi)^2 + c^2\left(\vec{\nabla}\phi\right)^2\right)$$

$$+\frac{\lambda_\phi^{(1)}}{8N}\mathrm{tr}(\phi^4) + \frac{\lambda_\phi^{(2)}}{8N^2}(\mathrm{tr}(\phi^2))^2$$

$$\mathcal{L}_{\psi,\phi} = \frac{g}{\sqrt{N}}\bar{\psi}^i\psi_j\phi_i^j$$

One can set:

$$\lambda_\phi^{(1)} = 0$$

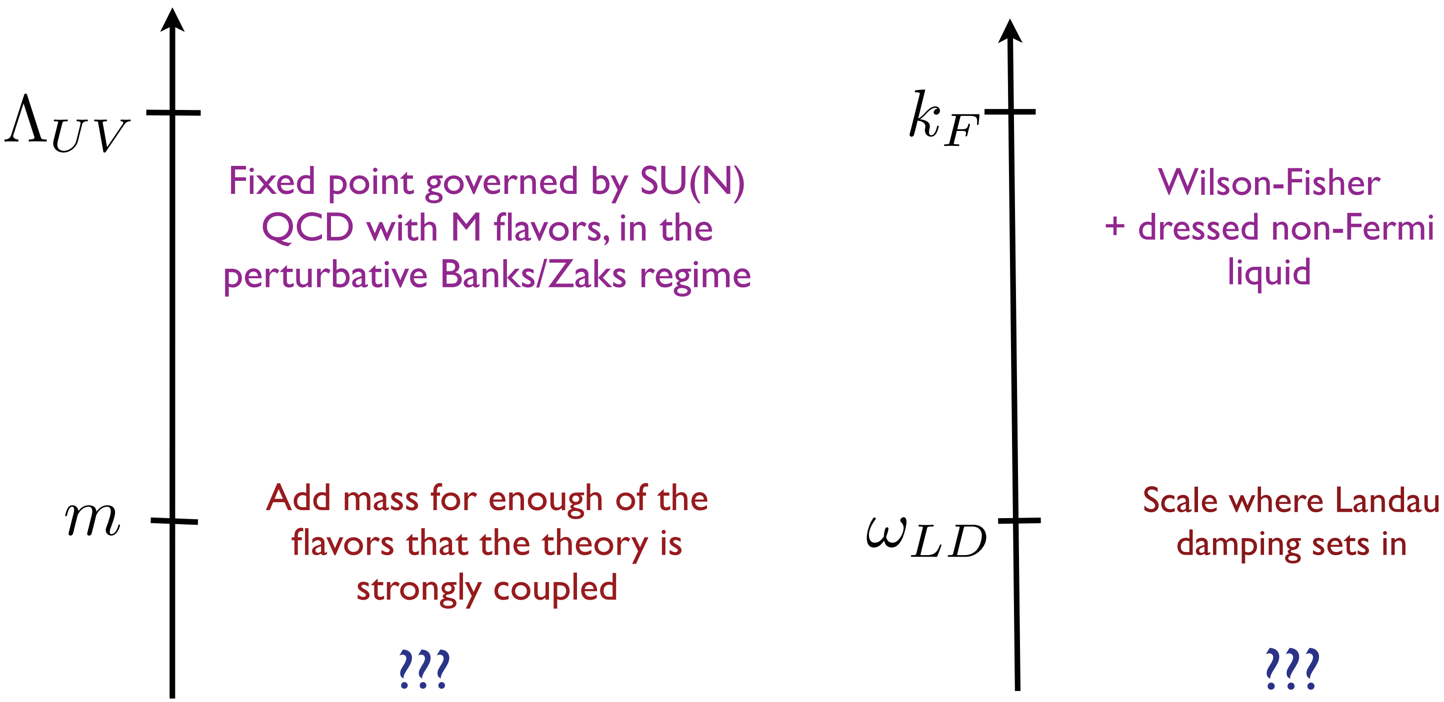
in a natural way (in the sense of `t Hooft). We do so.

In this theory, the only contribution to the four-Fermi beta function comes from the wavefunction renormalization of the fermion. One finds:

$$\frac{d}{dt}\lambda_\psi = -2a_g g^2 \lambda_\psi .$$

The result is a Wilson-Fisher boson which dresses a fermion into a non-Fermi liquid, stable against superconductivity.

Because the theory still "sees" Landau damping, one might worry about what happens in the deep IR. We believe an analogy to a better-understood situation is helpful in understanding the possible uses of the fixed point:



Work in progress:

* We would like to find a systematic treatment of the log squared divergences in the RG. In previous work these have led to enhanced pairing, but a systematic framework is absent.

* We had two control parameters: ϵ, N . Hopefully, one is sufficient. We are currently in the process of solving the theory at $\epsilon = 1, N \to \infty$. We find integral equations for the self-energies which we can solve self-consistently in the large N limit.