The Abdus Salam
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for Theoretical Physics

2469-13

Workshop and Conference on Geometrical Aspects of Quantum States in Condensed Matter

1 - 5 July 2013

Novel Topological Phases and Surface States in Interacting Systems

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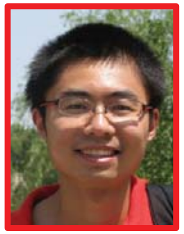
Acknowledgements



YuanMing Lu
(Berkeley)

arXiv:1205.3156 (PRB 2012)
2D Bosonic Topological Phases
Yuan-Ming Lu, AV

T. Senthil
(MIT)

arXiv:1209.3058 (PRX 2013)
3D bosonic topological phases:
AV, T. Senthil

Xie Chen
(Berkeley)

Lukasz Fidkowski
(SUNY)

Fiona Burnell
(Oxford)

Surface Topological Order

arXiv:1302.7072 Bosonic Topological Sc.

arXiv:1305.5851 Electronic Topological Sc.

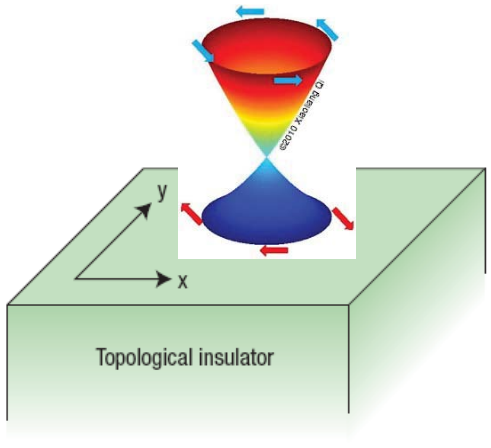
arXiv:1306.3250 Electronic Topological Ins.

Introduction - Free Fermion Topological Insulators

3D Topological Insulators

(Fu-Kane-Mele; Moore-Balents; Fu-Kane; Roy; Hsieh et al.; Xia et al.; Chen et al.; Qi-Hughes-Zhang)

- Bulk gapped.
- Gapless `Dirac' surface state, not realizable in a 2D system
- Protected by Time reversal and charge conservation symmetry.



Interaction Effects:

1. NEW phases analogous to topological insulators that only appear in interacting systems. (Boson/Spin)

2. NEW surface termination of fermion T.Ins./T.Sc. with interactions.

Similarly 3D Topological Sc. (DIII)

- Gapless Majorana surface state Protected by Time reversal.
- Realized in He^3 B-phase

Free Fermion Topological Phases

– Complete classification & several experimental examples.

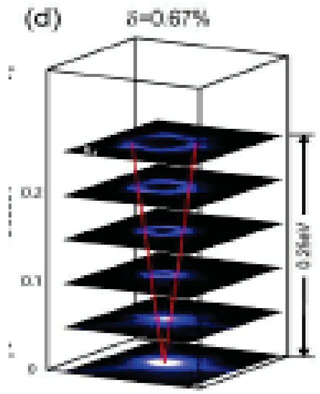
– Only `anti-unitary symmetries’ need to be considered (T and C). Stable to disorder.

Cartan	d: 1	2	3
Complex case:			
A	0	$\mathbb{Z}$	0
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$
Real case:			
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$2\mathbb{Z}$	0
CI	0	0	$2\mathbb{Z}$

(Ryu, Schnyder, Furusaki, Ludwig; Kitaev))

3D Topological superconductor. B-phase of superfluid He_3 (Volovik, Roy, Schneider et al.)

Z_2 Topological Insulators eg Bi_2Se_3



New Phenomena WITH Interactions?

- Free fermion classification is reduced – phases that were distinct now can be connected without a phase transition. $Z \rightarrow Z_8$
- *New Phases with interactions?*

Cartan	d = 1	2	3
Complex case:			
A	0	$\mathbb{Z}$	0
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$
Real case:			
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$2\mathbb{Z}$	0
CI	0	0	$2\mathbb{Z}$

(Ryu, Schnyder, Furusaki, Ludwig; Kitaev))

Bosonic Analogs of Topological Insulators

- Bulk is Short range entangled
  - gapped
  - **no** topological order
    - Unique ground state with periodic boundary conditions
    - No topological entanglement entropy
    - No anyon excitations in the bulk (2D)

 Laughlin Fractional Quantum Hall

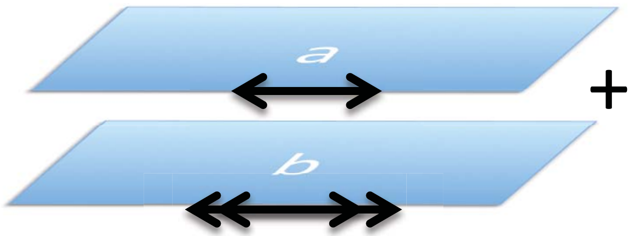
- BUT
  - Exotic surface states (edge degeneracy) protected by internal symmetry (eg. T, U(1) . Not translation/point group).

- <u>Many body ground state </u>(NOT 1 Photon/Phonon/Magnon dispersion)

General Considerations

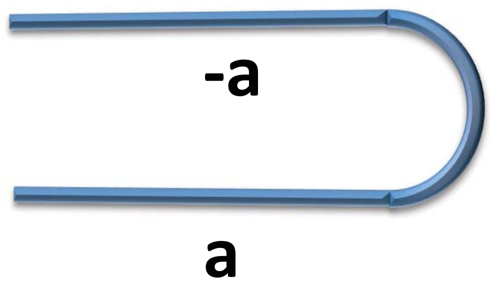
- Set of (SRE) topological phases in `d' dimensions protected by symmetry G must be an Abelian group.

Can add phases:



Can subtract phases:

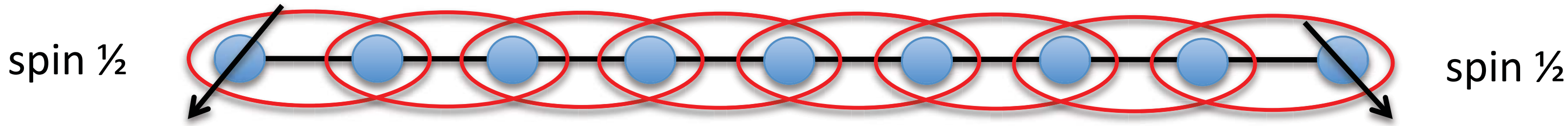
(x, y,..) -> (-x, y,..)



True for both free and interacting phases with short ranged entanglement.

Bosonic Analogs of Topological Insulators

- Topological Insulators of bosons/spins – exotic surfaces
  - 1D Example – AKLT/Haldane spin-1 chain. Eg. Y_2BaNiO_5



Any Examples in 2D/3D? YES! Chen&Gu&Lu&Wen; Kitaev; Levin&Gu; Y. M. Lu&AV. AV& T. Senthil.

Realization in cold atoms or quantum magnets?

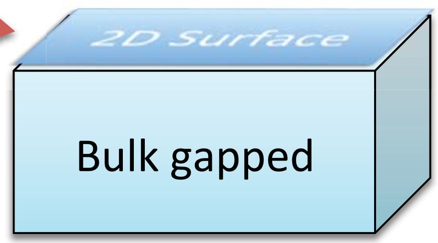
Higher Dimensional Realizations?

- *Proposed Classification* d=1, 2,3. Chen, Gu, Liu, Wen (2011)

- Bosons with Symmetry G.
  - Set of topological phases in d dim: group cohomology.
    - Abelian group $H^{d+1}(G, U(1))$

Symmetry Dimension	Time Rev. T	Charge Consv. U(1)	Charge Consv. +Time Rev. U(1) |x T	NO Symmetry
d=1	Z_2		Z_2	
d=2		**Z**	**Z_2**	**Z**
d=3	**Z_2 x Z_2**		**Z_2 x Z_2**	

Physical properties? Nature of surface states?



1D: Projective representations of G –edge states.
2D: Lu and AV
3D: AV and Senthil –
Bonus: *New states beyond cohomology*

Field Theory Approach to 2D Symmetry Protected Topological Phases

2D Topological Phases: Chern-Simons Approach

- K-matrix description of topological phases.

  (eg. Hierarchy quantum Hall states, bilayer states etc.).

$$(\rho, j_x, j_y) = \frac{\nabla \times a}{2\pi} \quad \text{where} \quad (\partial_t, \partial_x, \partial_y) = \nabla$$

- Multiple components I,J=1,2.
- Mutual statistics

Theoretically `cheaper' method than cohomology.

Physical properties are transparent.

2D Topological Phases: Chern-Simons Approach

- K-matrix directly related to edge states:

$$\mathcal{L} = \sum_{I,J} \frac{K_{IJ}}{4\pi} a^I \cdot \nabla \times a^J \quad \Rightarrow \quad \mathcal{L}_{\text{edge}} = \frac{1}{4\pi} K_{IJ} \dot{\phi}_I \partial_x \phi_J$$

- K: symmetric integer matrix.
- |Det K| = torus degeneracy. Demand:
  1. |Det K|=1 (no topological order)
  2. Diagonal entries are even. (only bosonic excitations)

K=1 not allowed.

‘Bosonic’ Topological Superconductor in d=2.

1. Bosonic ‘topological Sc.’
(like p+ip)

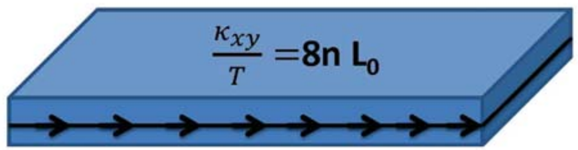
- chiral state, no symmetry.

MIN Dimension: 8x8

- 8 chiral edge modes.

- K matrix (E_8 Cartan matrix)
- Kitaev E_8 state.

$$K^{E_8} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$



1. No Symmetry.
   Z classes. Chiral Edge States.
   Quantized Thermal Hall Conductivity

Bosonic Integer Quantum Hall Phase

Simplest non-chiral state: $K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad q = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\mathcal{L} = \frac{1}{2\pi} a_1 \cdot \nabla \times a_2 + \frac{\nabla \times A}{2\pi}(a_1 + a_2)$$

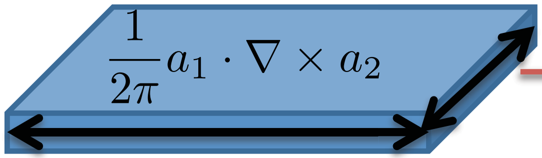
2. Bosonic 'Integer Hall' state: U(1) symmetry. Charge vector: q= (1 n)

- Quantized σ_{xy}=2n (q^2/h).
- EVEN Integers only! (Lu-AV 2012, Senthil-Levin)

Edge States and Symmetry

- Edge - 1D Luttinger Liquid

  But Symmetries act in a way that is *impossible* in 1D. (Lu-AV, Chen-Lu-Wen.)



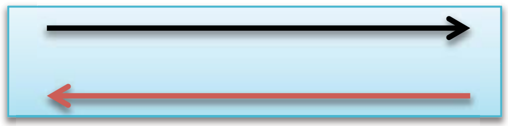
$$\mathcal{L}_{\text{edge}} = \dot{\phi}_1 \frac{\partial_x \phi_2}{2\pi}$$

ϕ_1 Boson phase,

$\rho = \frac{\partial_x \phi_2}{2\pi}$ Boson density.

Usual Tomonaga-Luttinger Liquid	Edge Tomonaga-Luttinger Liquid
(φ) $\phi_1 \to \phi_1 + \epsilon$	$\phi_1 \to \phi_1 + \epsilon$
(2θ) $\phi_2 \to \phi_2$	$\phi_2 \to \phi_2 + \epsilon$
Δ L=Cos ϕ_2 (can be gapped)	*No gapping term*

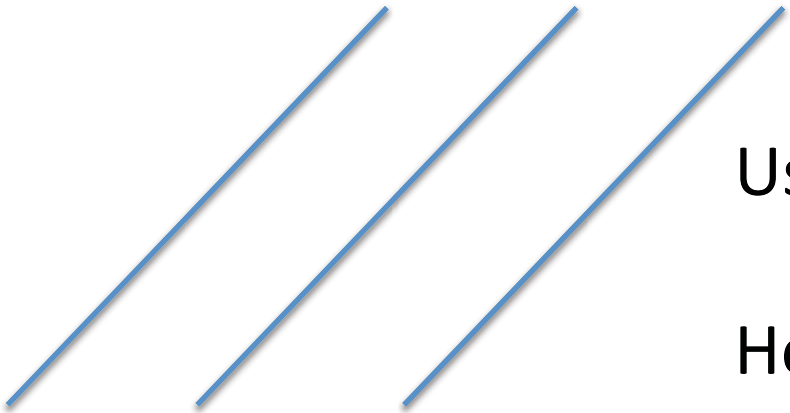
(1+2) CHARGED



(1-2) NEUTRAL

Microscopic Model

- Coupled Wire Construction – Luttinger Liquids (ϕ,θ).
  (Yakovenko et al, Yang-Sondhi, Mukhopadhya-Lubensky-Kane, Lu-AV)



Usual Insulator - $-\cos(2\theta_i)$

Here condense Charged vortex :
$-\cos(\phi_{i-1} - \phi_{i+1} + 2\theta_i)$

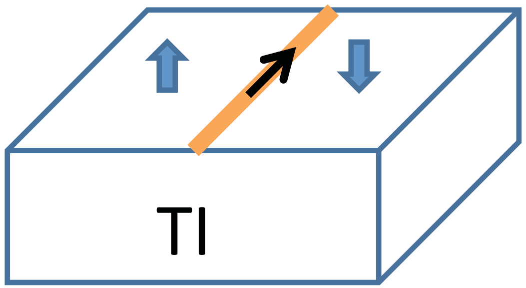
-

edge states:
$\phi_0, \theta_0 - \phi_1/2$

Field Theory of 3D Symmetry Protected Topological Phases

Physical Description of 3D Bosonic Topological Phases

Free fermion Topological Ins.
Break T symmetry – insulating surface
Domain wall - Edge of ν=1 Integer Quantum Hall
Each Domain – σ_{xy} =1/2; $\Theta=\pi$.

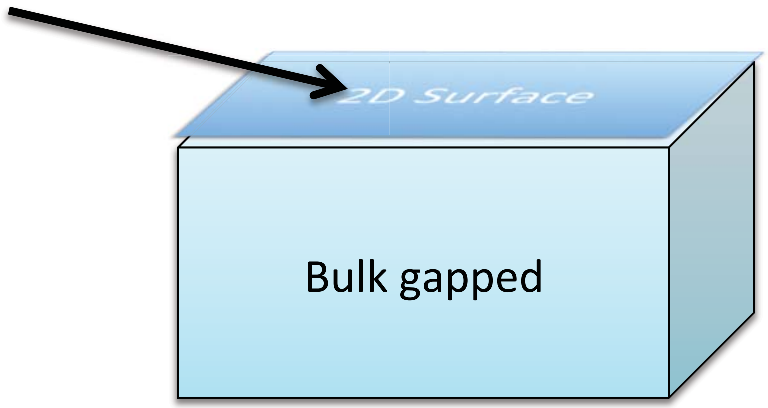


Fu, Kane, 2007; Qi, Hughes, Zhang 2008

Bosonic Topological Ins.
Domain wall - Edge of σ_{xy}=2 Integer Quantum Hall.
Each Domain – σ_{xy} =1; $\Theta=2\pi$.

3D BOSONIC `TOPOLOGICAL INSULATORS'

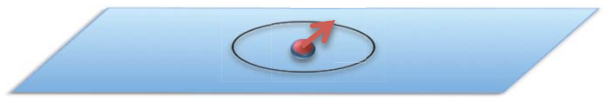
- Surface: 2D Bosons but symmetries act in a special way.



- Example: Bulk paramagnet.
  - Surface: 2D XY model BUT unusual symmetry action.

- Describe an SPT phase with:
  - break T on surface.

    Insulator with σ_{xy}=1
  - Symmetry: U(1) (S_z) + *T*

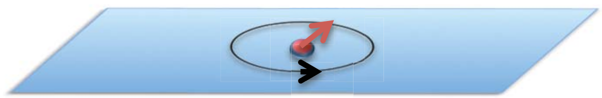


$$\text{vortex} = (\psi_\uparrow, \psi_\downarrow)$$

3D BOSONIC `TOPOLOGICAL INSULATORS' & Magneto-electric Effect

- Consider [U(1) x U(1)] x *T*
  - *Break* U(1). XY order.
  - Vortices transform projectively under: U(1) x *T*
  - Boson-Vortex duality:
    - *Vortices coupled to gauge field `a'.*
    - *`External' Fields: A, A*

$$\text{vortex} = (\psi_\uparrow, \psi_\downarrow)\; S_z = \pm\frac{1}{2}$$



$$\frac{\nabla \times a}{2\pi} = n$$

$$\mathcal{L}_{\text{edge}} = |(\partial - ia + i\frac{\sigma}{2}A)\psi_\sigma|^2 + \frac{\nabla \times a}{2\pi} \cdot A + \mathcal{L}_a$$

Condense: $\psi_\uparrow$ *then* $a = \frac{1}{2}A$ *and* $\mathcal{L}_{em} = \frac{1}{4\pi} A \cdot \nabla \times A$

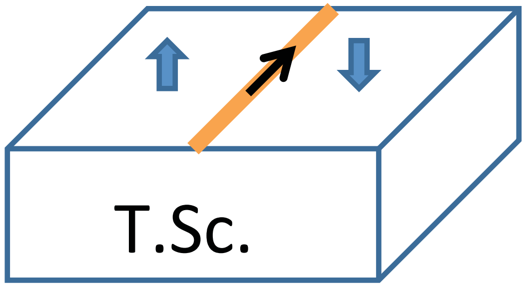
Break T

$$\sigma_{xy} = 1 \;\; (\Theta = 2\pi)$$

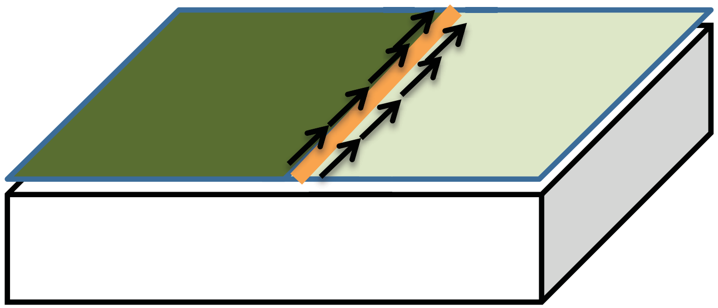
Two U(1)s needed to obtain fermionic vortices – `framing'.

Xu and Senthil

Physical Description of 3D Bosonic Topological Superconductor



Free fermion Topological Superconductor.
Break T symmetry –surface gapped
Domain wall – Chiral majorana mode
Each Domain – κ_{xy} /T=1/4;

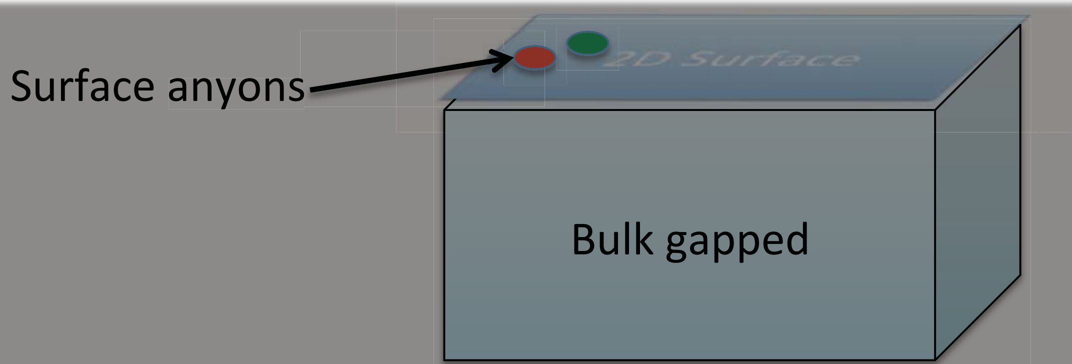


Bosonic* Topological Superconductor? *T symm.
T breaking Domain wall - Edge of Kitaev E8 State.
Each Domain – κ_{xy} /T=4;

Beyond Group cohomology (AV and Senthil)

Topological Order on the Surface of Bosonic SPT phases

- 2D Surface of a 3D Symmetry Protected topological phase must be either:
  1. Gapless (like Dirac cone metallic state) OR
  2. Breaks symmetry and is gapped OR



  3. Is Symmetric and Gapped – but develops *surface* Topological Order.
     - This Topological Order cannot be realized in 2D *with* this symmetry
     - Vortices transform projectively – but can condense pairs of vortices => Z_2 gauge theory

Example: Topological Order on the `Beyond Group Cohomology' 3D State

- Physical Requirements:
  - Not realizable in 2D with time reversal symmetry
  - On breaking T and confining, should give E8 domain wall state.

Candidate State: 3-fermion Z2 topological order

Z2 topological order (Toric code): { 1, e, m, f} [Z2 gauge theory: e, m boson.
f is fermion mutual π statisitics]

3-fermion Z2 topological order: {1, f1, f2, f3} [all fermions with mutual π statistics.
#8 in Kitaev 16 fold way]

3- Fermion Model as a Surface State

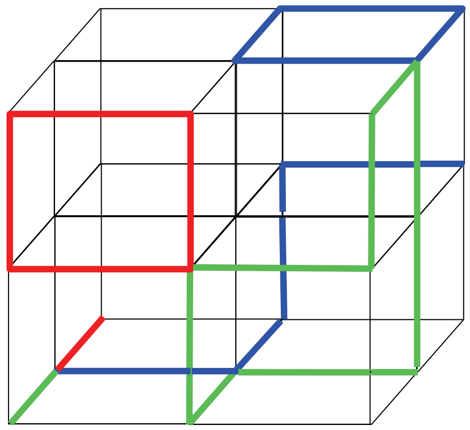
- 3-fermion model `appears' T symmetric (statistics) {1, f1, f2, f3}
- When realized in 2D *always* chiral edge states (c_-=4). Must break T in 2D.

$$\frac{1}{D}\sum_a d_a^2\theta_a = e^{i2\pi c_-/8}$$

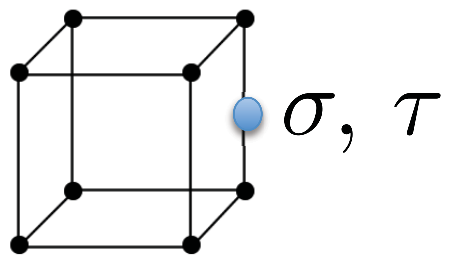
- However, on the boundary of 3D – no edge. Can be T symmetric.

3D Bosonic Topological Superconductor

- A 3D exactly soluble model with confined bulk excitations *but* deconfined excitations at the edge:
  (Walker-Wang; Keyserlingk-Burnell-Simon)
  - Edge topological order: 3-fermion Z2 gauge theory
  - Time reversal symmetric
  - Confined bulk

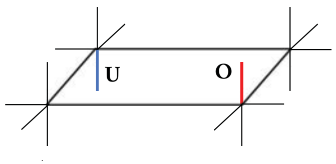


3D Bosonic Topological Superconductor: Exactly Soluble Model

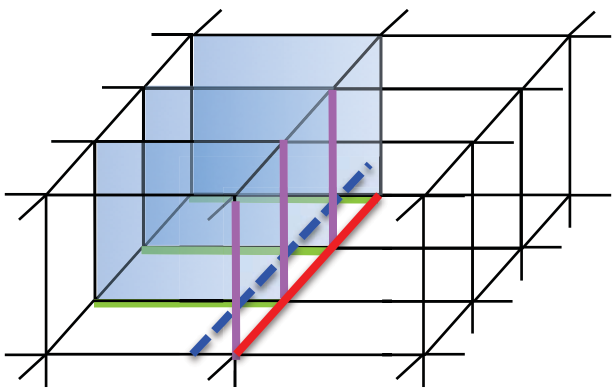


$$H = -\sum_V A_V - \sum_P B_P$$

$$A_V = \left(\prod_i \sigma_i^x + \prod_i \tau_i^x \right)$$

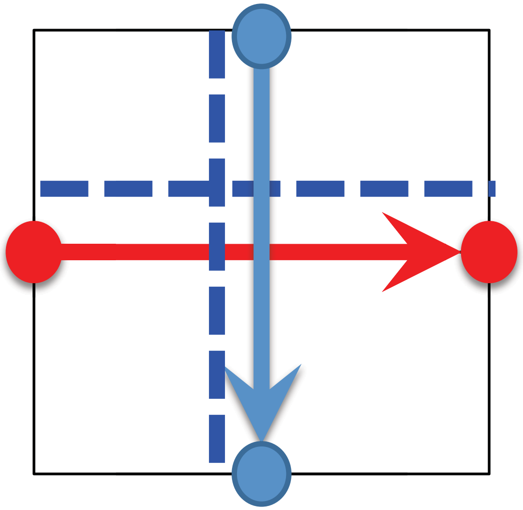


$$B_P = [\sigma^x]_{\mathrm{O}} \prod_{\Box} \sigma^z [\sigma^x \tau^x]_{\mathrm{U}} + [\sigma^x \tau^x]_{\mathrm{O}} \prod_{\Box} \tau^z [\tau^x]_{\mathrm{U}}$$



Point defects *confined* in the bulk but *deconfined* on surface.

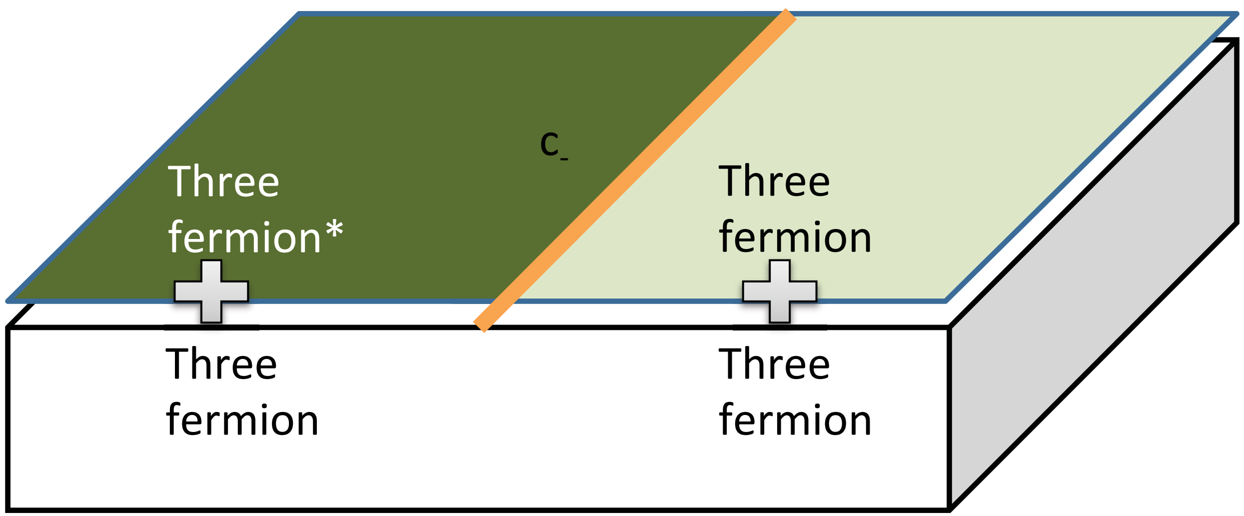
3D Bosonic Topological Superconductor: Exactly Soluble Model



Verify Semionic Mutual statisitics and fermionic self statistics

3-fermion Z_2 surface state is realized.

BUT H is $\mathcal{T}$ symmetric.



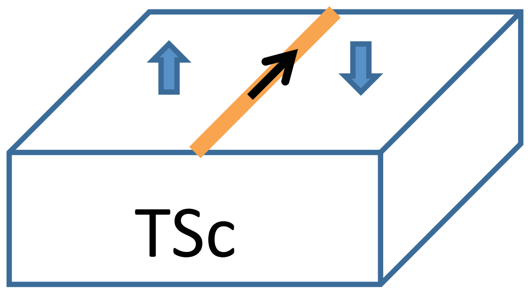
Confine surface by breaking T – (add a 2D 3-fermion state)

Surface domain wall hosts 4+4=8 chiral edge states.

Topologically Ordered Surface States of Electronic Topological Phases

Surface Topological Order of Fermionic Topological Phases?

- Eg. 3D Topological Superconductors (superfluid He_3 B phase)
  - Single Majorana-cone on surface.
  - T breaking domain wall – chiral Majorana: c_-=1/2
    - Each domain `c=1/4 + Integer'.



Surface Topological Order MUST

1. be non-Abelian (!)
2. T symmetric in 3D surface but not in 2D.
3. contain fundamental electron

Topological Superconductor Surface Topological Order

Particle	0	1	2	3	
θ	1	i	-i	-1	Electron

Fusion rules :

$$s \times s = \tilde{s} \times \tilde{s} = 1 + s + \tilde{s}$$
$$s \times \tilde{s} = e + s + \tilde{s}$$

Quantum dimensions and topological spins :

$$d_1 = d_e = 1, \;\; d_s = d_{\tilde{s}} = 1 + \sqrt{2}$$
$$\theta_1 = 1, \;\; \theta_e = -1, \;\; \theta_s = i, \;\; \theta_{\tilde{s}} = -i$$

- Simplest non-Abelian topological order with elementary fermion (eg. Read-Moore 12 ptcls)
- c_-=9/4.
- Breaks T in 2D, but may be T symmetric as surface state?
- Yes – but ONLY if electron transforms as Kramers pair (T^2=-1)

Surface Topological Order of Topological Insulator

"T - Pfaffian": a variation of Read-Moore: Ising * X $U(1)_8$

Also: Bondersan, Qi, Nayak

	0	e/4	e/2	3e/4	e	5e/4	3e/2	7e/4
I	1		i		1		i	
σ		1		-1		-1		1
ψ	-1		-i		-1		-i	

- Has $\sigma_{xy}=1/2$; $\kappa_{xy}/T=1/2$ as required by TI surface state ($\theta=\pi$).
- Has Z2 `ness' : 2 copies are trivial.
- Electron must transform as $T^2=-1$

BUT

- Cannot be obviously connected to TI Superconductor surface.
- Connection to Metlitskii-Kane-Fisher-Wang_Potter-Senthil state?

Abelian Surface Topological Order for *Even* `*N*' T.Sc.

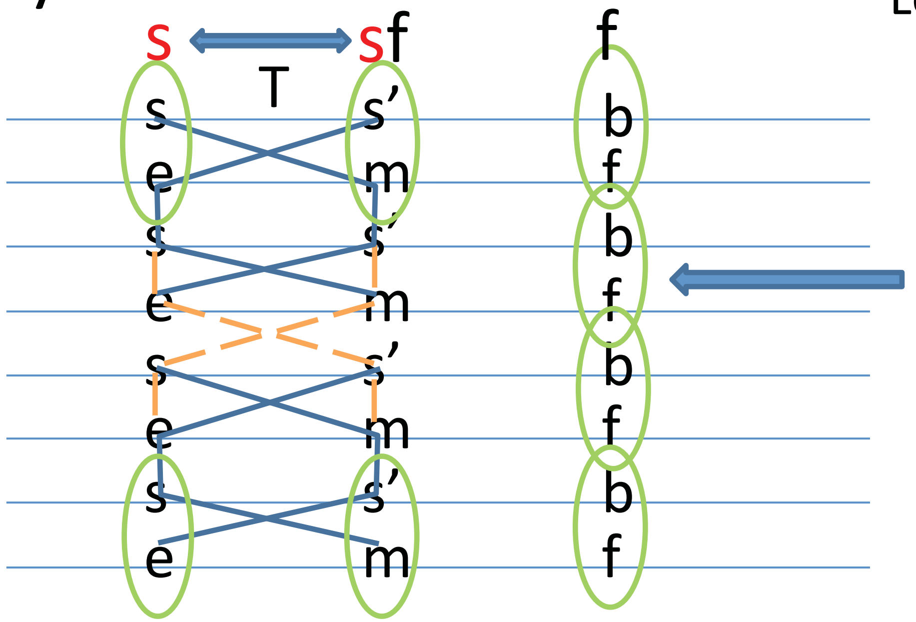
fermion X semion state:

	I	s	sf	f
θ	1	i	-i	-1

T

Local fermion

Coupled layer construction



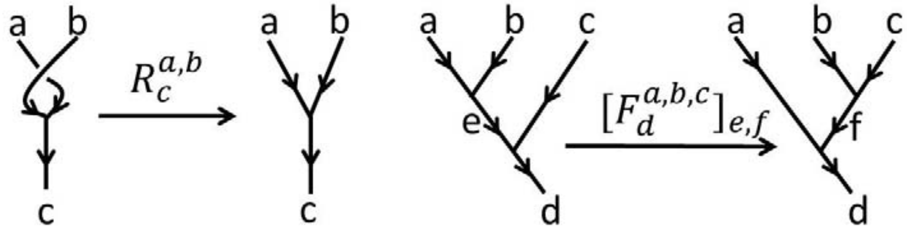
Bulk fermion

Double semion theory

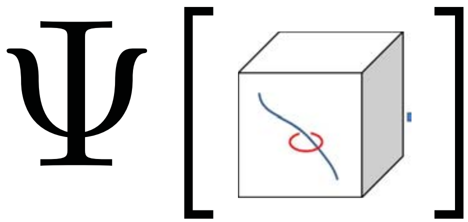
Z2 gauge theory with 2D TSc.

General Picture – Walker Wang Model

Use Algebraic Description of surface topological order: Anyons a,b,c..

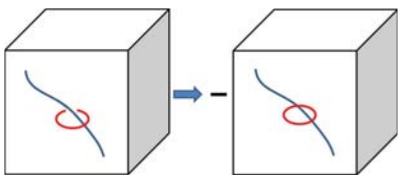


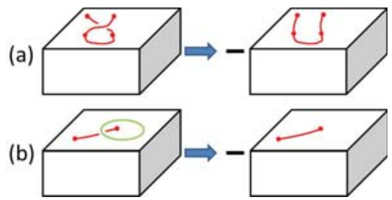
Walker Wang Ground State Wavefunction:



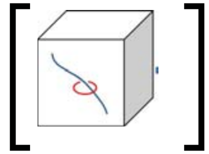
=> 2D projection of 3D lattice.
Use these to obtain amplitude

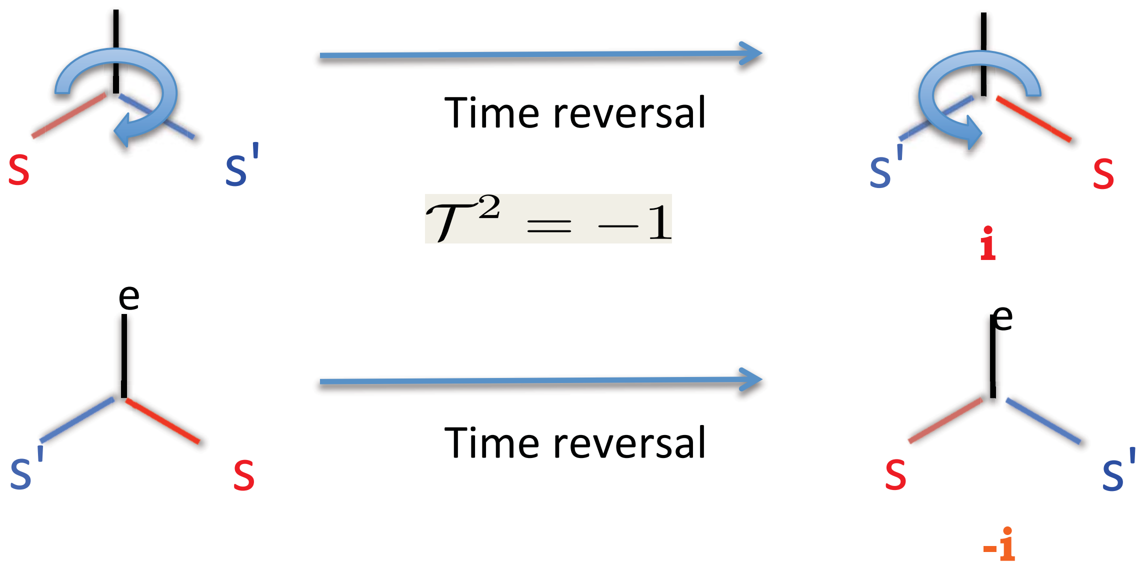
Ensures Bulk Confined and Surface Topological Order:





Time Reversal Symmetry and Surface Topological Order of TSc

- Walker Wang Wavefunction complex – Ψ  invariant under complex conjugation and exchanging particle types?
- No – requires additional phase factors on basis states.



Conclusions and Future Directions

- Several Interacting Bosonic Topological phases even with SRE.
  - Experimental realization? (lowest Landau level – Senthil-Levin)

- New surface termination of topological phases– surface topological order.
  - For some fermionic phases *must be* non-Abelian. Gapped, disordered phase=> non-Abelions.

- Classifying SRE phases of interacting fermions?
  - Not free fermion/not bosonic SPT?