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Topological Order at the Surface of a Topological Insulator

Charles Kane

University of Pennsylvania
USA

Strada Costiera, 11 - 34151 - Trieste - Italy • Tel. +39 0402240111 • Fax. +39 040224163 • sci_info@ictp.it • www.ictp.it
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C.L. Kane
University of Pennsylvania

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Acknowledgments

Max Metlitski, KITP

Matthew Fisher, UCSB

Metlitski, Kane, Fisher arXiv: 1302:6535
arXiv: 1306:3286

Closely Related Recent Work :

Bonderson, Nayak, Qi arXiv: 1306: 3230
Chen, Fidkowski, Vishwanath arXiv: 1306: 3250

Wang, Potter, Senthil arXiv: 1306: 3223

Topological Order at the Surface of a Topological Insulator

I. Introduction

- Topological Insulators
- Broken symmetry Surface States

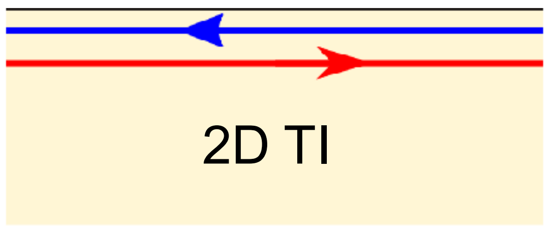
II. A few comments about interacting symmetry protected topological states

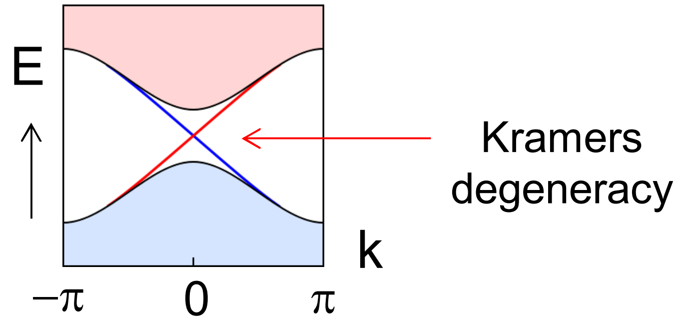
III. A symmetry respecting gapped state of a TI surface with intrinsic topological order

- Construction via quantum disordered superconductor
- Properties of the surface state
- Open questions

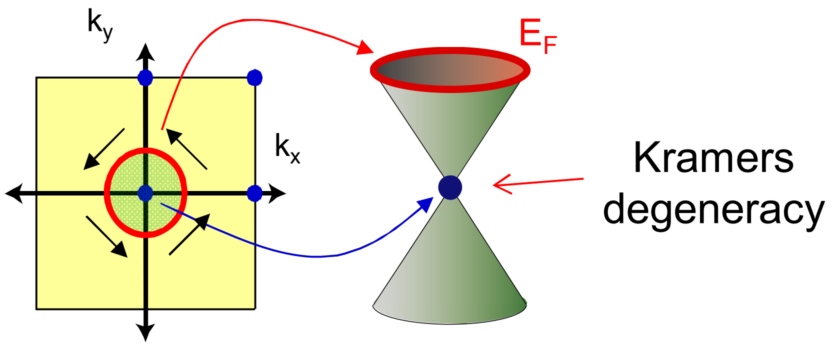
Topological Insulator

d=2 : Edge states protected by time reversal symmetry



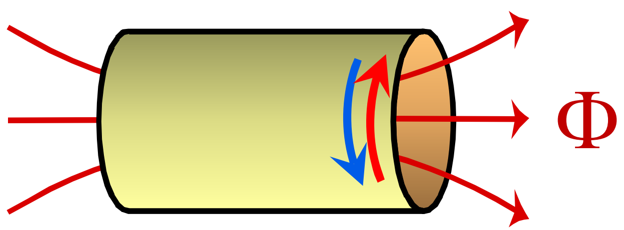


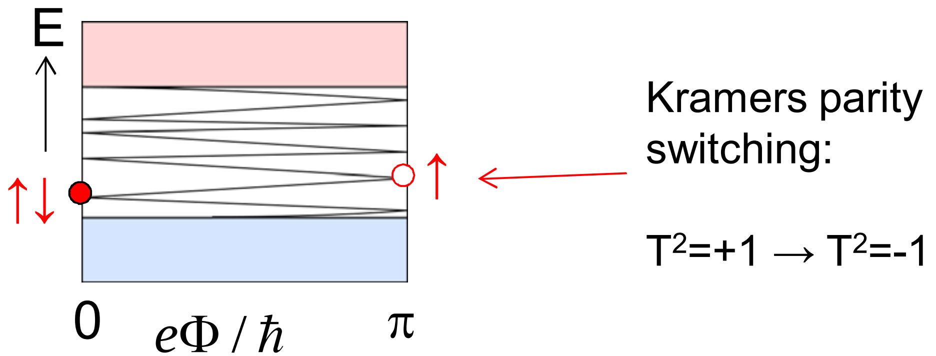
d=3 : Single Surface Dirac cone protected by Time reversal



Stability to Interactions and Disorder:

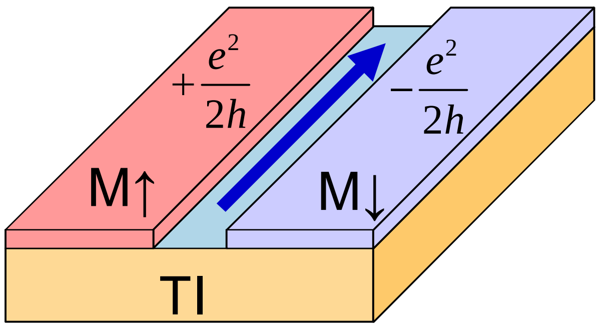
symmetry forbids trivially gapped surface termination

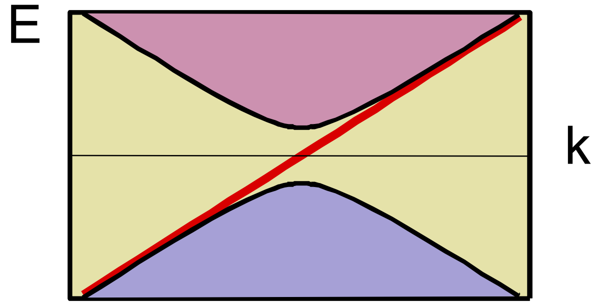




Break Symmetry: Open Surface Energy Gap

Break Time Reversal : Quantum Hall State $\sigma_{xy} = \frac{e^2}{h}\left(n + \frac{1}{2}\right)$

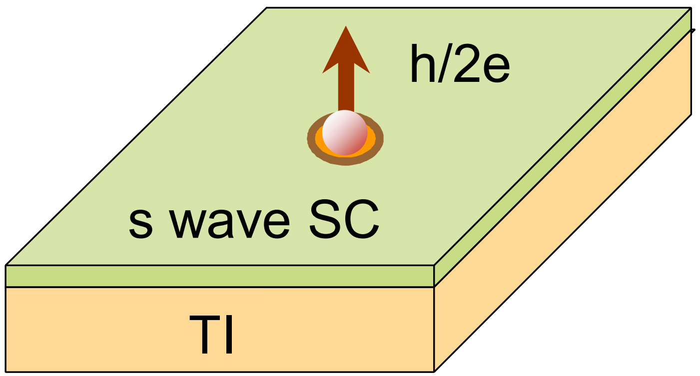


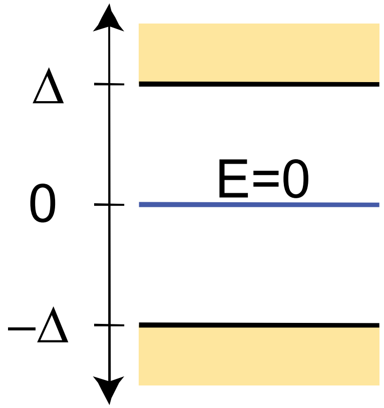


Chiral Dirac fermion mode

$\nu = 1$, $c = 1$

Break U(1) gauge symmetry: Surface Superconductor





Majorana zero mode

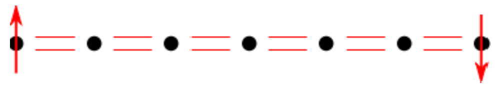
Symmetry Protected Topological States

In the absence of U(1) gauge symmetry or T symmetry, topological insulators are trivial insulators with no intrinsic topological order.

SPT's beyond the free fermion paradigm :

Cohomology classification Chen, Gu, Wen '11

Haldane (AKLT) spin 1 chain



Bosonic Topological Insulators Vishwanath, Senthil, '13
Lu, Vishwanath '13

A powerful tool for diagnosing symmetry protected states :

Gauge the symmetry, characterize elementary quasiparticles Levin, Gu ' PRB 12

Topological Insulator coupled to compact U(1) gauge field, A

Low energy theory for A : θ term

Qi, Hughes and Zhang '08

$$S = i\theta N \qquad\qquad N = \frac{1}{32\pi^2}\int d^3x d\tau \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \in \mathbb{Z}$$

θ is defined mod 2π. Time reversal symmetry requires $\theta = 0$ or π.

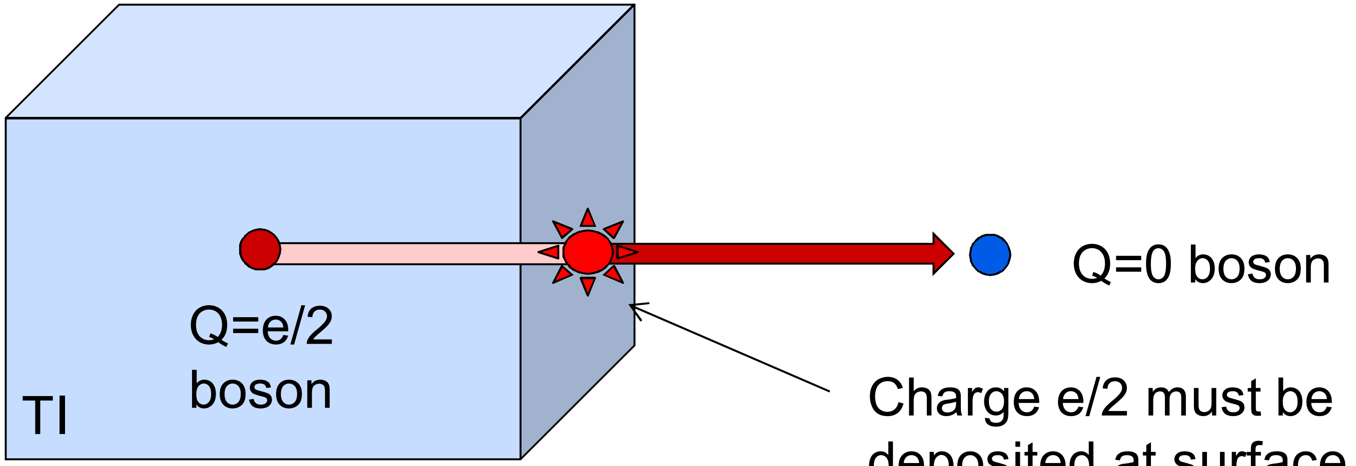
Witten Effect: Magnetic Monopoles acquire a polarization charge $Q = \frac{\theta}{2\pi} e$

In a gauged topological insulator $\theta=\pi$:

Monopoles (or dyons) are bosons with charge $Q = e\left(n + \frac{1}{2}\right)$

Can the surface of a TI be gapped without breaking symmetry ?

Pass monopole from inside to outside of TI :



Break T at surface:

$\sigma_{xy} = e^2/2h$: charge e/2 flows away on surface

Superconductor at surface:

Charge conservation is violated at surface

Keep U(1) and T at surface :

Charge e/2 stays at surface : Requires a topologically ordered surface state with e/2 quasiparticle

Digression: 3D Bosonic Topological Insulator

θ is defined mod 4π for bosons :

Vishwanath, Senthil, '13

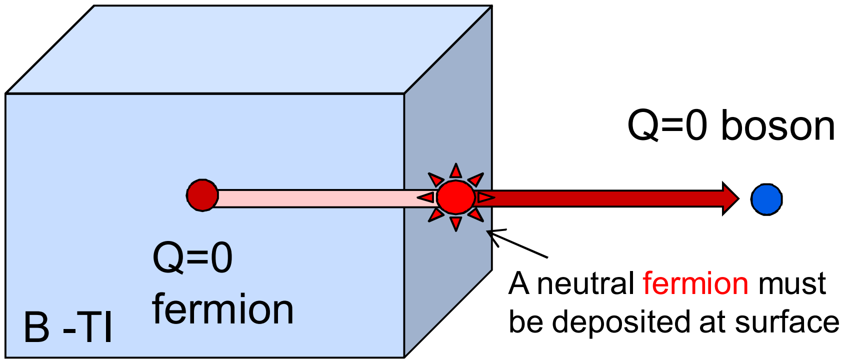
$$S = i\theta N \qquad N = \frac{1}{32\pi^2}\int d^3x d\tau \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \in \mathbb{Z} \quad \text{OR} \quad \mathbb{Z}+1/2$$

N can be half integer on a manifold without a spin structure (eg CP^2)

Monopole (or dyon): Charge Q = $(\theta/2\pi + n)e$ $\begin{cases} \text{n even : boson} \\ \text{n odd : fermion} \end{cases}$

Bosonic TI has $\theta = 2\pi$. Monopole is neutral fermion, or charge e boson.

Trivial gapped surface is impossible:



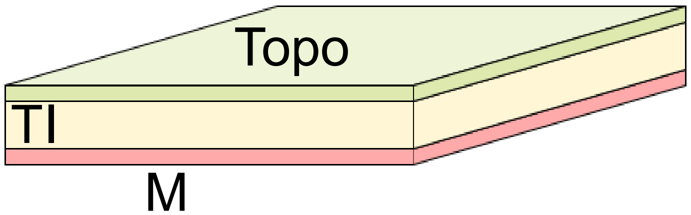
Possible Surface Terminations:

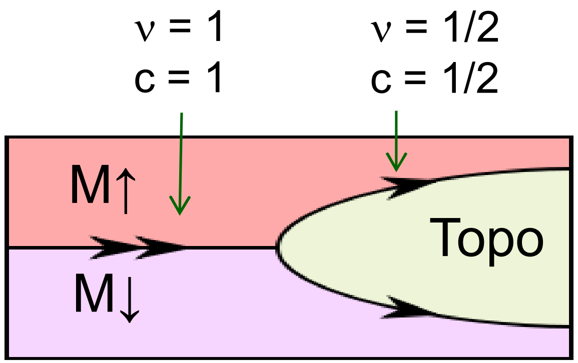
- Magnetic Gap
- Superconducting Gap
- Gapped state with toric code topological order

Requirements for a Topological Surface Phase on fermion TI

It should be impossible in 2D if symmetry is preserved, but if symmetry is broken there should be a 2D state with the same topological order

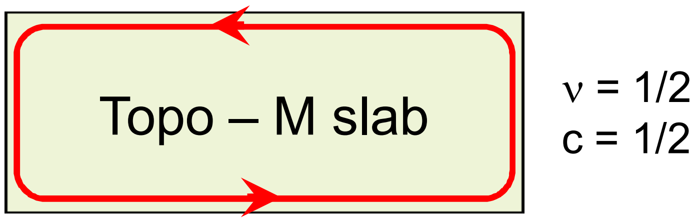
Broken T : Topo – M slab





Non-Abelian quantum Hall state with

- Hall conductance ν e2/h; $\nu = 1/2$
- Thermal Hall cond. $c\, \pi^2 k_B^2/6h$; $c = 1/2$



Broken U(1) : Topo – SC slab

c = 0 : edge states not necessary

Theory of topological surface state

Levels of theory :

- Hamiltonian : (eg IQHE, TI)
- Wavefunction : (eg Laughlin, Moore-Read)
- Characterize elementary quasiparticles: (TQFT)

  Braiding, Fusion, Action of symmetry

Strategy :

Begin with T invariant surface superconductor. Restore U(1) symmetry by condensing vortices.

Resulting Theory : $[\text{ Ising} \times U(1)_8\ ]_R \times U(1)_{-2}$

Moore Read State
$\nu = 1/2$, $c = 3/2$

Neutral anti-semion
$\nu = 0$, $c = -1$

Ingredients in the theory :

1. Abelian Chern Simons Model : $U(1)_k$

Describes Laughlin state of bosons (k even) at filling $\nu = 1/k$

k quasiparticle types : $I_m = e^{im\phi}, \quad m = 0, \ldots, k-1$

Fusion: $I_m \times I_{m'} = I_{m+m'}$

Topological Spin: $\theta_m = e^{i\pi m^2/k}$

Chiral central charge: $c = \mathrm{sgn}(k) = \pm 1$

2. Ising Anyon Model

Describes

Non-Abelian phase of Kitaev honeycomb model

Gauged p+ip superconductor

3 quasiparticle types : $I \qquad \sigma \qquad \psi$

Fusion: $I \times a = a; \quad \psi \times \psi = I; \quad \psi \times \sigma = \sigma; \quad \sigma \times \sigma = I + \psi$

Topological spin: $\theta_I = 1 \quad ; \quad \theta_\psi = -1 \quad ; \quad \theta_\sigma = e^{i\pi/8}$

Central Charge: $c = 1/2$

Moore Read State

[Ising x $U(1)_8$ $]_R$: A subset of Ising x $U(1)_8$

$$a_m = a e^{im\phi}$$

m even : $a = 1, \psi$

m odd : $a = \sigma$

Charge Q_m = me/4

Central charge : $c = 1/2 + 1 = 3/2$

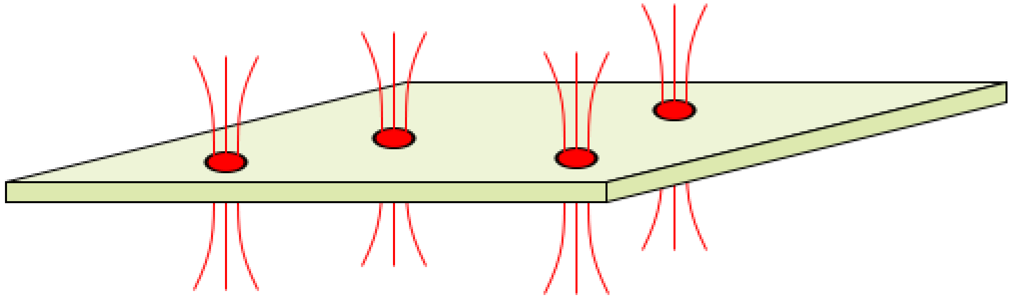
m	0	1	2	3	4	5	6	7
I_m	1		i		1		i	
σ_m		$e^{i\pi/4}$		$-e^{i\pi/4}$	e	$-e^{i\pi/4}$		$e^{i\pi/4}$
ψ_m	-1		-i		-1		-i	

Moore Read – Anti Semion Theory [Ising x $U(1)_8$ $]_R$ x $U(1)_{-2}$

$U(1)_{-2}$: $\{I, \overline{s}\}$ neutral anti semion

Central charge : $c = 3/2 - 1 = 1/2$

Superconductor – Insulator Transition via vortex condensation



Condense single (h/2e) vortices in conventional 2D superconductor:

- Leads to conventional insulating phase

Condense double (h/e) vortices:

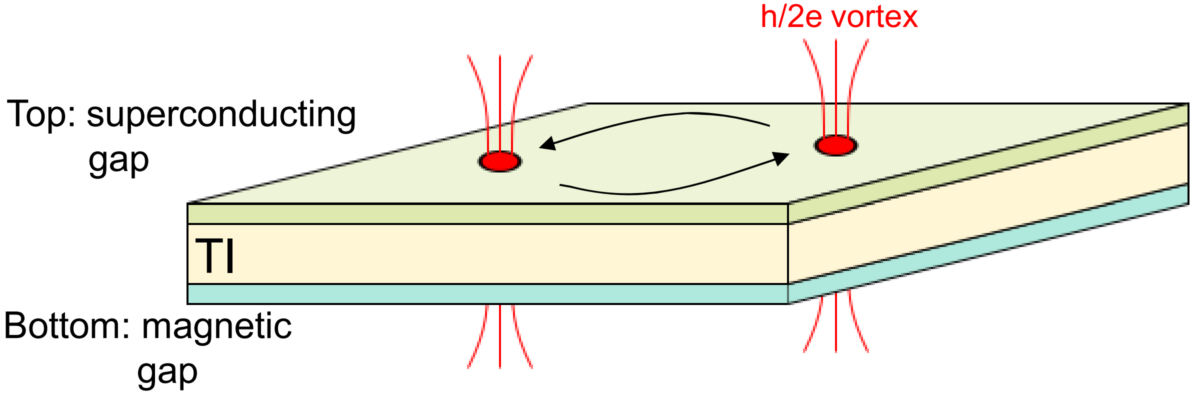
- Single vortex remains as a deconfined anyon.
- Leads to insulator with “toric code” topological order :

p+ip Topological Superconductor

- Single vortices bind a Majorana fermion mode, can’t condense.
- Double vortex condensation leads to insulator with Ising topological order

Vortices in the Surface Superconductor

Gauge the U(1) symmetry, so that vortices have well defined statistics



SC-M slab = p+ip superconductor : $S_{\text{slab}} = S_{\text{Ising}}[j_v, j_\psi]$

$$S_{\text{slab}} = S_{\text{top}}[j_v, j_\psi] + S_{\text{bottom}}[A]$$

$$S_{\text{bottom}}[A] = S_{\nu=1/2}[A]$$

$$S_{\text{top}}[j_v, j_\psi] = S_{\text{Ising}}[j_v, j_\psi] - S_{\text{bottom}}[A]$$

$$= S_{\text{Ising}}[j_v, j_\psi] + S_{U(1)_{-8}}[j_v]$$

$$\nabla \times \mathbf{A} = \frac{h}{2e} \mathbf{j}_v$$

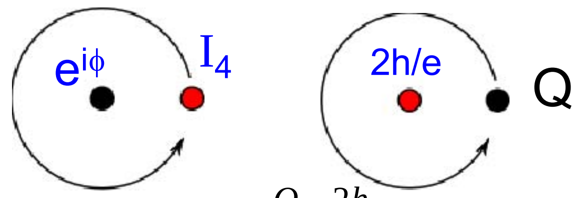
Gauged surface superconductor : $[\text{ Ising x U}(1)_{-8}\ ]_R$

[Ising x U(1)$_{-8}$]$_R$ $\quad a_l = a e^{il\theta}$

l	0	1	2	3	4	5	6	7
I_l	1		-i		1		-i	
σ_l		1		-1		-1		1
ψ_l	-1		i		-1		i	

Vortex Condensation : Restore U(1) symmetry

- π (h/2e) vortex σ_1 : Non Abelian X
- 2π (h/e) vortex I_2 : Anti Semion X
- 4π (2h/e) vortex I_4 : Boson Condense



phase $2\pi = \frac{Q}{\hbar} \times \frac{2h}{e}$

New particle: “vortex in vortex” $e^{i\phi}$: charge Q = e/2 boson

I_2, ψ_0, ψ_2 remains as a deconfined anyons : I_2 = neutral anti semion $\bar{s}$

σ_l binds “half” vortex (due to -1 mutual statistics with I_4): charge e/4

Moore Read – Anti Semion Theory $[\,\text{Ising} \times U(1)_8\,]_R \times U(1)_{-2}$

Moore-Read: $[\,\text{Ising} \times U(1)_8\,]_R$ = subset of $\text{Ising} \times U(1)_8$

$$a_m = a e^{im\phi}$$

m even : $a = 1, \psi$

m odd : $a = \sigma$

charge $Q_m = me/4$

$U(1)_{-2}$: $\{I, \bar{s}\}$ neutral anti semion

Central charge : $c = (1/2 + 1) - 1 = 1/2$

12 distinct quasiparticles (not counting electron)

m	0	1	2	3	4	5	6	7
I_m	1		i		1		i	
σ_m		$e^{i\pi/4}$		$-e^{i\pi/4}$	*e*	$-e^{i\pi/4}$		$e^{i\pi/4}$
ψ_m	-1		-i		-1		-i	
$\bar{s}\, I_m$	-i		1		-i		1	
$\bar{s}\, \sigma_m$		$e^{-i\pi/4}$		$-e^{-i\pi/4}$		$-e^{-i\pi/4}$		$e^{-i\pi/4}$
$\bar{s}\, \psi_m$	i		-1		i		-1	

Transition back to superconductor

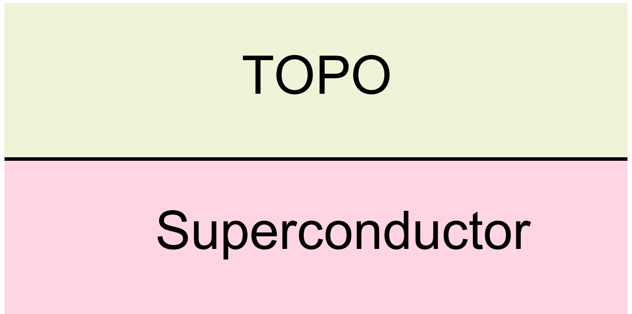
m	0	1	2	3	4	5	6	7
I_m	1		i		1		i	
σ_m		$e^{i\pi/4}$		$-e^{i\pi/4}$	*e*	$-e^{i\pi/4}$		$e^{i\pi/4}$
ψ_m	-1		-i		-1		-i	
			b					
$\bar{s}\, I_m$	-i		1		-i		1	
$\bar{s}\, \sigma_m$		$e^{-i\pi/4}$		$-e^{-i\pi/4}$		$-e^{-i\pi/4}$		$e^{-i\pi/4}$
$\bar{s}\, \psi_m$	i		-1		i		-1	

Condense charge e/2 boson $\bar{s}I_2$

- $\bar{s}I_2$ has non trivial mutual statistics with all other particles (except e)
- Confines all anyons.
- Reproduces surface superconductor (without topological order)
- Transition in XY universality class

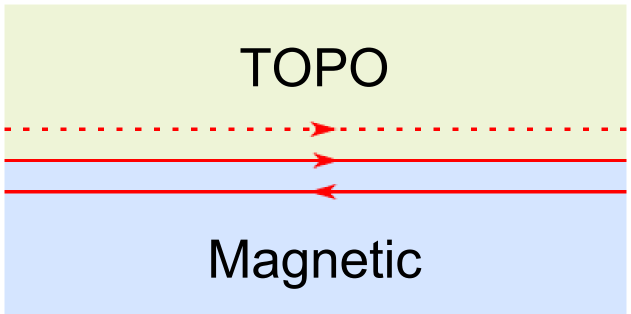
Edge States

Topo. – SC :



No gapless edge states.
Similar to edge of ordinary SC

Topo. – M :



Majorana mode f
Charge mode ϕ_ρ
upstream neutral mode θ
c=1/2

electron: $\psi_e^\dagger \sim f e^{4i\phi_\rho} e^{4i\theta}$

e/4 qp: $\psi_{e/4}^\dagger \sim \sigma e^{i\phi_\rho}$

Edge state tunneling, noise, interferometry expts can in principle probe the non-Abelian topological order.

Is the Moore-Read – Anti Semion the unique (or simplest) Topo state?

Alternative (possible) topological surface state:

T – Pfaffian state : $[\text{ Ising}^* \times U(1)_8]_R$

Bonderson, Nayak, Qi '13
Chen, Fidkowski, Vishwanath '13

m	0	1	2	3	4	5	6	7
I_m	1		i		1		i	
σ_m		1	⇕	-1		-1	⇕	1
ψ_m	-1		-i		-1		-i	

Similar to description of gauged surface superconductor, except T and U(1) symmetries act differently.

Condensation of I_4 boson does not confine all anyons. Gives superconductor with topological order and broken T (since $T^2=-1$).

Different edge state structure from MR-AS state.

This state does not appear to break any rules, but it is not clear how it is related to the free electron topological insulator.

Conclusion

- A symmetry preserving gapped state of a 3D topological insulator is possible provided the state has intrinsic topological order

- Candidate state: Moore-Read + anti-semion state.

  - 12 distinct quasiparticles, respects T

  - Explicit contact with electron TI : direct transition to and from surface superconducting state

- Many open questions :

  - What is the relationship to the T – Pfaffian state?

  - Can a topological state of a TI be engineered ?